# *Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1 Problem 1*

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If $f(x) = x^2 - 4x$, find

a) $f(-5)$

b) $f\left(y^2 + 1\right)$

c) $f(x + \Delta x)$

d) $f(x + 1) - f(x - 1)$

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## *SOLUTION:*

**Part a**

$$f(-5) = (-5)^2 - 4(-5)$$

$$= 25 + 20$$

$$= 45$$

**Part b**

$$f\left(y^2 + 1\right) = \left(y^2 + 1\right)^2 - 4\left(y^2 + 1\right)$$

$$= y^4 + 2y^2 + 1 - 4y^2 - 4$$

$$= y^4 - 2y^2 - 3$$

**Part c**

$$f(x + \Delta x) = (x + \Delta x)^2 - 4(x + \Delta x)$$

$$= (x + \Delta x)[(x + \Delta x) - 4]$$

$$= (x + \Delta x)(x + \Delta x - 4)$$

**Part d**

$$f(x+1) - f(x-1) = \left[(x+1)^2 - 4(x+1)\right] - \left[(x-1)^2 - 4(x-1)\right]$$

$$= \left[x^2 + 2x + 1 - 4x - 4\right] - \left[x^2 - 2x + 1 - 4x + 4\right]$$

$$= x^2 - x^2 + 2x - 4x + 2x + 4x + 1 - 4 - 1 - 4$$

$$= 4x - 8$$

$$= 4(x - 2)$$

# Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1, Problem 2

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If $y = \dfrac{x^2 + 3}{x}$, find x as a function of y.

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## Solution:

$$y = \frac{x^2 + 3}{x}$$

$$xy = x^2 + 3$$

$$x^2 - xy + 3 = 0$$

Solve for x using quadratic formula. We have

# Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1 Problem 3

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If $y = tan\,(x + \pi)$, find x as a function of y.

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## Solution:

$$y = tan\,(x + \pi)$$

$$x + \pi = tan^{-1} y$$

$$x = tan^{-1} y - \pi$$

# Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1 Problem 4

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Express the distance D traveled in t hr by a car whose speed is 60 km/hr.

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## Solution:

$$Distance = Rate \times Time$$

$$D = (60\ km/hr) \cdot t$$

$$D = 60t$$

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# Exercise 1.1
# Problem 5

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*Express the area A of an equilateral triangle as a function of its side x.*

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# Solution:

From the formula of the area of a triangle, $A = \frac{1}{2} \cdot a \cdot b \cdot sin\,(\theta)$. Also, we know that an interior angle of an equilateral triangle is 60 degrees, and $sin\,60° = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2} \cdot x \cdot x \cdot sin\,60°$$

$$A = \frac{1}{2} \cdot x^2 \cdot \frac{\sqrt{3}}{2}$$

$$A = \frac{\sqrt{3}}{4} x^2$$

# *Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1 Problem 6*

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*The stiffness of a beam of rectangular cross section is proportional to the breadth and the cube of the depth. If the breadth is 20 cm, express the stiffness as a function of the depth.*

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## *Solution:*

Let S=stiffness, b=breadth, and d=depth

$$S = bd^3$$

$$S = 20d^3$$

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# Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1, Problem 7

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A right circular cylinder, radius of base x, height y, is inscribed in a right circular cone, radius of base r and a height h. Express y as a function of x (r and h are constants).

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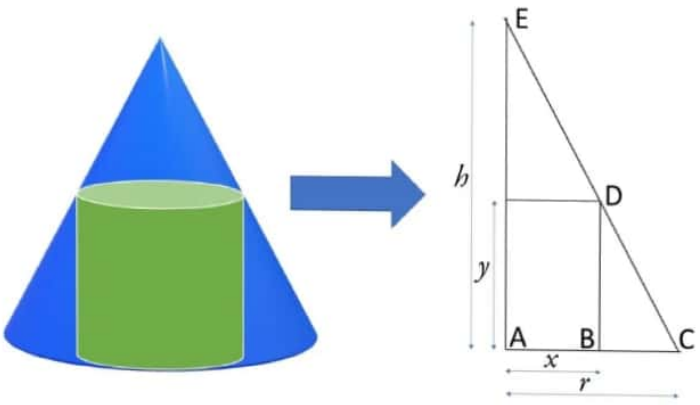
## Solution:

Refer to the figure below.




By ratio and proportion of two similar triangles ΔBCD and ΔACE, we have

$$\frac{y}{r-x} = \frac{h}{r}$$

$$y = \frac{h\,(r-x)}{r}$$

# *Differential and Integral Calculus by Feliciano and Uy, Exercise 1.1, Problem 8*

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If $f\left(x\right)=x^2+1$, find $\dfrac{f\left(x+h\right)-f\left(x\right)}{h},\ h\neq 0$.

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## Solution:

$$\frac{f\left(x+h\right)-f\left(x\right)}{h}=\frac{\left[\left(x+h\right)^2+1\right]-\left(x^2+1\right)}{h}$$

$$=\frac{x^2+2xh+h^2+1-x^2-1}{h}$$

$$=\frac{2xh+h^2}{h}$$

$$=\frac{h\left(2x+h\right)}{h}$$

$$=2x+h$$

# Integral Calculus by Feliciano and Uy, Exercise 1.1, Problem 9

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If $f(x) = 3x^2 - 4x + 1$, find $\dfrac{f(h+3) - f(3)}{h}$, $h \neq 0$.

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## Solution:

$$\frac{f(h+3) - f(3)}{h} = \frac{\left[3(h+3)^2 - 4(h+3) + 1\right] - \left[3(3)^2 - 4(3) + 1\right]}{h}$$

$$= \frac{3\left(h^2 + 6h + 9\right) - 4h - 12 + 1 - 16}{h}$$

$$= \frac{3h^2 + 18h + 27 - 4h - 12 + 1 - 16}{h}$$

$$= \frac{3h^2 + 14h}{h}$$

$$= \frac{h(3h + 14)}{h}$$

$$= 3h + 14$$

# Differential and Integral Calculus| Feliciano and Uy| Exercise 1.1| Problem 10

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If $f(x) = \dfrac{4}{x+3}$ and $g(x) = x^2 - 3$, find $f[g(x)]$ and $g[f(x)]$

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## Solution:

$$f[g(x)] = \frac{4}{(x^2+3)+3}$$

$$= \frac{4}{x^2}$$

$$g[f(x)] = \left(\frac{4}{x+3}\right)^2 - 3$$

$$= \frac{16}{(x+3)^2} - 3$$

$$= \frac{16 - 3(x+3)^2}{(x+3)^2}$$

$$= \frac{16 - 3\left(x^2+6x+9\right)}{(x+3)^2}$$

$$= \frac{16 - 3x^2 - 18x - 27}{(x+3)^2}$$

$$= \frac{-3x^2 - 18x - 11}{(x+3)^2}$$

# Differential and Integral Calculus| Feliciano and Uy| Limit of a Function| Exercise 1.2| Problem 1

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Evaluate $\lim_{x \to 2} \left(x^2 - 4x + 3\right)$.

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## Solution:

$$\lim_{x \to 2} \left(x^2 - 4x + 3\right) = \lim_{x \to 2} \left(x^2\right) - \lim_{x \to 2} \left(4x\right) + \lim_{x \to 2} \left(3\right)$$

$$= \left[\lim_{x \to 2} \left(x\right)\right]^2 - 4 \lim_{x \to 2} \left(x\right) + 3$$

$$= \left(2\right)^2 - 4\left(2\right) + 3$$

$$= -1$$

# Exercise 1.2| Problem 2

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Evaluate $\lim_{x\to 3}\left(\frac{4x+2}{x+4}\right)$.

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## Solution:

$$\lim_{x\to 3}\left(\frac{4x+2}{x+4}\right) = \frac{\lim_{x\to 3}(4x+2)}{\lim_{x\to 3}(x+4)}$$

$$= \frac{\lim_{x\to 3}(4x) + \lim_{x\to 3}(2)}{\lim_{x\to 3}(x) + \lim_{x\to 3}(4)}$$

$$= \frac{4\cdot\lim_{x\to 3}(x) + 2}{3+4}$$

$$= \frac{4\cdot 3+2}{3+4}$$

$$= \frac{12+2}{7}$$

$$= \frac{14}{7}$$

$$= 2$$

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# Feliciano and Uy|Limit of a Function| Exercise 1.2| Problem 3

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Evaluate $\lim_{x \to \frac{\pi}{4}} (tan\ x + sin\ x)$

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## Solution:

$$\lim_{x \to \frac{\pi}{4}} (tan\ x + sin\ x) = \lim_{x \to \frac{\pi}{4}} (tan\ x) + \lim_{x \to \frac{\pi}{4}} (sin\ x)$$

$$= tan\ \frac{\pi}{4} + sin\ \frac{\pi}{4}$$

$$= 1 + \frac{\sqrt{2}}{2}$$

$$= \frac{2 + \sqrt{2}}{2}$$

# Feliciano and Uy| Limit of a Function| Exercise 1.2| Problem 4

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Evaluate $\lim_{x \to \frac{\pi}{3}} \left( \frac{sin\, 2x}{sin\, x} \right)$.

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## Solution:

Plug in the value $x = \frac{\pi}{3}$

$$\lim_{x \to \frac{\pi}{3}} \left( \frac{sin\, 2x}{sin\, x} \right) = \frac{sin\left(2 \cdot \frac{\pi}{3}\right)}{sin\, \left(\frac{\pi}{3}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}$$

$$= 1$$

# Limit of a Function| Exercise 1.2| Problem 5

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Evaluate $\lim_{x \to 8} \left(2x + \sqrt[3]{x} - 4\right)$.

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# Solution:

Plug in the value x=8.

$$= \left[2\,(8) + \sqrt[3]{8} - 4\right] = [16 + 2 - 4] = 14$$

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# Exercise 1.2| Problem 6

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Evaluate $\lim_{x \to 2} (4x - 3)(x^2 + 5)$

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# Solution:

Plug in the value x=2.

$$\lim_{x \to 2} (4x - 3)(x^2 + 5) = [(4 \cdot 2) - 3]\left[(2)^2 + 5\right]$$

$$= [8 - 3][4 + 5]$$

$$= (5)(9)$$

$$= 45$$

# Limit of a Function| Exercise 1.2| Problem 7

Evaluate $\lim_{x\to 3}\left(\frac{\sqrt{3x}}{x\sqrt{x+1}}\right)$.

**SOLUTION:**

Plug in the value x=3.

$$\lim_{x\to 3}\left(\frac{\sqrt{3x}}{x\sqrt{x+1}}\right)=\frac{\sqrt{3(3)}}{3\sqrt{3+1}}$$

$$=\frac{\sqrt{9}}{3\sqrt{4}}$$

$$=\frac{3}{3\cdot 2}$$

$$=\frac{3}{6}$$

$$=\frac{1}{2}$$

# Integral Calculus| Feliciano and Uy| Limit of a Function| Exercise 1.2| Problem 8

Evaluate $\lim_{x \to 0} \left( \frac{3x+2}{x^2-2x+4} \right)$.

**SOLUTION:**

Plug in the value x=0.

$$\lim_{x \to 0} \left( \frac{3x+2}{x^2-2x+4} \right) = \frac{3(0)+2}{(0)^2-2(0)+4}$$

$$= \frac{0+2}{0-0+4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

# Differential and Integral Calculus| Feliciano and Uy| Limit of a Function in Indeterminate Form| Exercise 1.3| Problem 1

**Evaluate**

$$\lim_{x\to 4}\left(\frac{x^3-64}{x^2-16}\right)$$

**SOLUTION:**

A straight substitution of $x = 4$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 4}\left(\frac{x^3-64}{x^2-16}\right) = \lim_{x\to 4}\left(\frac{(x-4)\left(x^2+4x+16\right)}{(x+4)(x-4)}\right)$$

$$= \lim_{x\to 4}\left(\frac{x^2+4x+16}{x+4}\right)$$

$$= \frac{(4)^2+4(4)+16}{4+4}$$

$$= \frac{48}{8}$$

$$= 6$$

# Form| Exercise 1.3| Problem 2|

## Evaluate

$$\lim_{x\to 2}\left(\frac{x^2+2x-8}{3x-6}\right)$$

**SOLUTION:**

A straight substitution of $x = 2$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 2}\left(\frac{x^2+2x-8}{3x-6}\right) = \lim_{x\to 2}\left(\frac{(x+4)(x-2)}{3(x-2)}\right)$$

$$= \lim_{x\to 2}\left(\frac{x+4}{3}\right)$$

$$= \frac{2+4}{3}$$

$$= \frac{6}{3}$$

$$= 2$$

# Form|Exercise 1.3| Problem 3|

## Evaluate

$$\lim_{x\to 3}\left(\frac{x^3-13x+12}{x^3-14x+15}\right)$$

**SOLUTION:**

A straight substitution of $x=3$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 3}\left(\frac{x^3-13x+12}{x^3-14x+15}\right)=\lim_{x\to 3}\left(\frac{(x-3)\left(x^2+3x-4\right)}{(x-3)\left(x^2+3x-5\right)}\right)$$

$$=\lim_{x\to 3}\left(\frac{x^2+3x-4}{x^2+3x-5}\right)$$

$$=\frac{(3)^2+3(3)-4}{(3)^2+3(3)-5}$$

$$=\frac{9+9-4}{9+9-5}$$

$$=\frac{14}{13}$$

# Cy|Limit of a Function in Indeterminate Form| Exercise 1.3| Problem 4|

## Evaluate

$$\lim_{x\to 2}\left(\frac{x^3-x^2-x-2}{2x^3-5x^2+5x-6}\right)$$

**SOLUTION:**

A straight substitution of $x = 2$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 2}\left(\frac{x^3-x^2-x-2}{2x^3-5x^2+5x-6}\right) = \lim_{x\to 2}\left(\frac{(x-2)\left(x^2+x+1\right)}{(x-2)\left(2x^2-x+3\right)}\right)$$

$$= \lim_{x\to 2}\left(\frac{x^2+x+1}{2x^2-x+3}\right)$$

$$= \frac{2^2+2+1}{2(2)^2-2+3}$$

$$= \frac{4+2+1}{8-2+3}$$

$$= \frac{7}{9}$$

# Problem 5|

## Evaluate

$$\lim_{x\to 0} \frac{(x+3)^2-9}{2x}$$

**SOLUTION:**

A straight substitution of $x = 0$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 0} \frac{(x+3)^2-9}{2x} = \lim_{x\to 0} \frac{(x+3)^2-(3)^2}{2x}$$

$$= \lim_{x\to 0} \frac{(x+3-3)(x+3+3)}{2x}$$

$$= \lim_{x\to 0} \frac{x(x+6)}{2x}$$

$$= \lim_{x\to 0} \frac{x+6}{2}$$

$$= \frac{0+6}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

# Problem 6|

## Evaluate

$$\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x}$$

**SOLUTION:**

A straight substitution of $x = 0$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x} = \lim_{x\to 0} \frac{\sqrt{x+16}-4}{x} \cdot \frac{\sqrt{x+16}+4}{\sqrt{x+16}+4}$$

$$= \lim_{x\to 0} \frac{(x+16)-4^2}{x\left(\sqrt{x+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{x+16-16}{x\left(\sqrt{x+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+16}+4}$$

$$= \frac{1}{\sqrt{0+16}+4}$$

$$= \frac{1}{4+4}$$

$$= \frac{1}{8}$$

# Problem 7|

## Evaluate

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$$

**SOLUTION:**

A straight substitution of $x = 1$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} = \lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$= \lim_{x\to 1} \frac{(x-1)\left(\sqrt{x+3}+2\right)}{(x+3)-2^2}$$

$$= \lim_{x\to 1} \frac{(x-1)\left(\sqrt{x+3}+2\right)}{x-1}$$

$$= \lim_{x\to 1} \sqrt{x+3}+2$$

$$= \sqrt{1+3}+2$$

$$= \sqrt{4}+2$$

$$= 2+2$$

$$= 4$$

## Evaluate

$$\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8}$$

▼ **SOLUTION**

▼ Details

A straight substitution of $x = 8$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8} = \lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8} \cdot \frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}$$

$$= \lim_{x\to 8} \frac{x-8}{(x-8)\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}$$

$$= \lim_{x\to 8} \frac{1}{\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}$$

$$= \frac{1}{\left(\sqrt[3]{8^2}+2\sqrt[3]{8}+4\right)}$$

$$= \frac{1}{4+4+4}$$

$$= \frac{1}{12}$$

# Indeterminate Form|Exercise 1.3| Problem 9|

## Evaluate

$$\lim_{x\to 4}\left(\frac{\frac{1}{x}-\frac{1}{4}}{x-4}\right)$$

▼ **SOLUTION**

A straight substitution of $x = 4$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 4}\left(\frac{\frac{1}{x}-\frac{1}{4}}{x-4}\right) = \lim_{x\to 4}\left(\frac{\frac{4-x}{4x}}{x-4}\right)$$

$$= \lim_{x\to 4}\frac{4-x}{4x(x-4)}$$

$$= \lim_{x\to 4}\left(\frac{4-x}{-4x(4-x)}\right)$$

$$= \lim_{x\to 4} -\frac{1}{4x}$$

$$= -\frac{1}{4\cdot 4}$$

$$= -\frac{1}{16}$$

# Calculus| Feliciano and Uy| Exercise 1.3, Problem 10|

## Evaluate

$$\lim_{x\to 2}\left(\frac{x^3-8}{x^2-4}\right)$$

▼ **SOLUTION**

A straight substitution of $x = 2$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 2}\left(\frac{x^3-8}{x^2-4}\right) = \lim_{x\to 2}\left(\frac{(x-2)\left(x^2+2x+4\right)}{(x+2)(x-2)}\right)$$

$$= \lim_{x\to 2}\left(\frac{\left(x^2+2x+4\right)}{(x+2)}\right)$$

$$= \frac{2^2+2\cdot 2+4}{2+2}$$

$$= 3$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 11|*

## *Evaluate*

$$\lim_{x\to 3}\left(\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right)$$

▼ **SOLUTION**

▼ Details

A straight substitution of $x=3$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

▼ Details

Therefore, to evaluate the limit of the given function, we proceed as follows

▼ Details

$$\lim_{x\to 3}\left(\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right)=\lim_{x\to 3}\left(\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right)\cdot\frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}}$$

$$=\lim_{x\to 3}\left(\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{\left(\sqrt{x-2}-\sqrt{4-x}\right)\left(\sqrt{x-2}+\sqrt{4-x}\right)}\right)$$

$$=\lim_{x\to 3}\left(\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2x-6}\right)$$

$$=\lim_{x\to 3}\left(\frac{(x-3)\left(\sqrt{x-2}+\sqrt{-x+4}\right)}{2(x-3)}\right)$$

$$=\lim_{x\to 3}\left(\frac{\sqrt{x-2}+\sqrt{4-x}}{2}\right)$$

$$=\frac{\sqrt{3-2}+\sqrt{4-3}}{2}$$

$$=1$$

# Calculus| Feliciano and Uy| Exercise 1.3, Problem 12|

## Evaluate

$$\lim_{x\to 0}\left(\frac{1}{x}\left(\frac{1}{3}-\frac{1}{\sqrt{x+9}}\right)\right)$$

**SOLUTION:**

A straight substitution of $x=0$ leads to the indeterminate form $0\cdot 0$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 0}\left(\frac{1}{x}\left(\frac{1}{3}-\frac{1}{\sqrt{x+9}}\right)\right)=\lim_{x\to 0}\left(\frac{1\cdot\left(\frac{1}{3}-\frac{1}{\sqrt{x+9}}\right)}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\frac{\sqrt{x+9}-3}{3\sqrt{x+9}}}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sqrt{x+9}-3}{3x\sqrt{x+9}}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sqrt{x+9}-3}{3x\sqrt{x+9}}\right)\cdot\frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}$$

$$=\lim_{x\to 0}\left(\frac{x}{3x\left(\sqrt{x+9}+3\right)\sqrt{x+9}}\right)$$

$$=\lim_{x\to 0}\left(\frac{1}{3\left(\sqrt{x+9}+3\right)\sqrt{x+9}}\right)$$

$$=\frac{1}{3\left(\sqrt{0+9}+3\right)\sqrt{0+9}}$$

$$=\frac{1}{54}$$

# Calculus| Feliciano and Uy| Exercise 1.3, Problem 13|

## Evaluate

$$\lim_{x\to 3}\left(\frac{\sqrt{x^2-9}}{x-3}\right)$$

**SOLUTION:**

A straight substitution of $x = 3$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 3}\left(\frac{\sqrt{x^2-9}}{x-3}\right) = \lim_{x\to 3}\left(\frac{\sqrt{x^2-9}}{x-3}\cdot\frac{\sqrt{x^2-9}}{\sqrt{x^2-9}}\right)$$

$$= \lim_{x\to 3}\left(\frac{x^2-9}{(x-3)\sqrt{x^2-9}}\right)$$

$$= \lim_{x\to 3}\left(\frac{(x+3)(x-3)}{(x-3)\sqrt{x^2-9}}\right)$$

$$= \lim_{x\to 3}\left(\frac{x+3}{\sqrt{x^2-9}}\right)$$

$$= \frac{3+3}{\sqrt{3^2-9}}$$

$$= \frac{6}{0}$$

$$= \infty$$

# Calculus| Feliciano and Uy| Exercise 1.3, Problem 14|

## Evaluate

$$\lim_{x \to \frac{\pi}{4}} \left( \frac{tan\, 2x}{sec\, 2x} \right)$$

**SOLUTION:**

A straight substitution of $x = \frac{\pi}{4}$ leads to the indeterminate form $\frac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x \to \frac{\pi}{4}} \left( \frac{tan\, 2x}{sec\, 2x} \right) = \lim_{x \to \frac{\pi}{4}} \left( \frac{\frac{sin\, 2x}{cos\, 2x}}{\frac{1}{cos\, 2x}} \right)$$

$$= \lim_{x \to \frac{\pi}{4}} (sin\, 2x)$$

$$= sin\, \left( 2 \cdot \frac{\pi}{4} \right)$$

$$= sin\, \frac{\pi}{2}$$

$$= 1$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 15|*

## *Evaluate*

$$\lim_{x\to 0}\left(\frac{sin^3x}{sin\,x-tan\,x}\right)$$

**SOLUTION:**

A straight substitution of $x=\dfrac{\pi}{4}$ leads to the indeterminate form $\dfrac{0}{0}$ which is meaningless.

Therefore, to evaluate the limit of the given function, we proceed as follows

$$\lim_{x\to 0}\left(\frac{sin^3x}{sin\,x-tan\,x}\right)=\lim_{x\to 0}\left(\frac{sin^3x}{sin\,x-\frac{sin\,x}{cos\,x}}\right)$$

$$=\lim_{x\to 0}\left(\frac{sin^3\left(x\right)}{\frac{sin(x)cos(x)-sin(x)}{cos(x)}}\right)$$

$$=\lim_{x\to 0}\left(\frac{sin^3\left(x\right)cos\left(x\right)}{sin\left(x\right)cos\left(x\right)-sin\left(x\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{sin^3\left(x\right)cos\left(x\right)}{sin\left(x\right)\left(cos\left(x\right)-1\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{sin^2\left(x\right)cos\left(x\right)}{\left(cos\left(x\right)-1\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{\left(1-cos^2\left(x\right)\right)cos\left(x\right)}{-\left(1-cos\left(x\right)\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{\left(1+cos\left(x\right)\left(1-cos\left(x\right)\right)\right)cos\left(x\right)}{-\left(1-cos\left(x\right)\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{\left(1+cos\left(x\right)\right)cos\left(x\right)}{-1}\right)$$

$$=-1\cdot\lim_{x\to 0}\left(\left(1+cos\left(x\right)\right)cos\left(x\right)\right)$$

$$=-1\cdot\left(1+cos\left(0\right)\right)cos\left(0\right)$$

$$=-1\cdot\left(1+1\right)\cdot 1$$

$$=-2$$

# and Uy| Exercise 1.3, Problem 16|

Evaluate

$$\lim_{x \to 0} \left( \frac{1 - cos^2(x)}{1 + cos(x)} \right)$$

**SOLUTION:**

This problem can be solved using a direct substitution of $x = 0$. That is

$$\lim_{x \to 0} \left( \frac{1 - cos^2(x)}{1 + cos(x)} \right) = \frac{1 - cos^2(0)}{1 + cos(0)}$$

$$= \frac{1 - 1}{1 + 1}$$

$$= 0$$

# Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 17|

## Evaluate

$$\lim_{x\to 0}\left(\frac{sin(x)sin(2x)}{1-cos(x)}\right)$$

**SOLUTION:**

Direct substitution of $x = 0$ gives the indeterminate form $\frac{0}{0}$. Therefore, we should apply trigonometric identities.

We know that $sin\,(2x) = 2sin\,(x)\,cos\,(x)$, so we can rewrite the original function as

$$\lim_{x\to 0}\left(\frac{sin(x)sin(2x)}{1-cos(x)}\right) = \lim_{x\to 0}\left(\frac{sin(x)2(sin(x)cos(x))}{1-cos(x)}\right)$$

$$= 2\cdot\lim_{x\to 0}\left(\frac{sin^2(x)cos(x)}{1-cos(x)}\right)$$

We also know the pythagorean identity, $sin^2\,(x) = 1 - cos^2\,(x)$. So,

$$= 2\cdot\lim_{x\to 0}\left(\frac{\left(1-cos^2(x)\right)cos(x)}{1-cos(x)}\right)$$

$$= 2\cdot\lim_{x\to 0}\left(\frac{(1+cos(x))(1-cos(x))cos(x)}{1-cos(x)}\right)$$

$$= 2\cdot\lim_{x\to 0}\left((1+cos\,(x))\,cos\,(x)\right)$$

$$= 2\cdot\left((1+cos\,(0))\,cos\,(0)\right)$$

$$= 2\cdot((1+1)\cdot 1)$$

$$= 4$$

# Calculus| Feliciano and Uy| Exercise 1.3, Problem 18|

## Evaluate

$$\lim_{x \to \pi} \left( \frac{sin^2(x)}{1+cos(x)} \right)$$

**SOLUTION:**

Direct substitution of $x = 0$ gives the indeterminate form $\frac{0}{0}$. Therefore, we should apply trigonometric identities.

We know the pythagorean identity, $sin^2\left(x\right) = 1 - cos^2\left(x\right)$. So,

$$\lim_{x \to \pi} \left( \frac{sin^2(x)}{1+cos(x)} \right) = \lim_{x \to \pi} \left( \frac{1-cos^2(x)}{1+cos(x)} \right)$$

$$= \lim_{x \to \pi} \left( \frac{(1+cos(x))(1-cos(x))}{1+cos(x)} \right)$$

$$= \lim_{x \to \pi} \left(1 - cos\left(x\right)\right)$$

$$= \left(1 - cos\left(\pi\right)\right)$$

$$= \left(1 - (-1)\right)$$

$$= 2$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 19|*

*If* $f\,(x) = \sqrt{x}$, *find*

$$\lim_{x \to 4} \left( \frac{f(x) - f\,(4)}{x - 4} \right)$$

**SOLUTION:**

$$\lim_{x \to 4} \left( \frac{f(x) - f(4)}{x - 4} \right) = \lim_{x \to 4} \left( \frac{\sqrt{x} - \sqrt{4}}{x - 4} \right)$$

Direct substitution of $x = 4$ gives the indeterminate form $\frac{0}{0}$. Therefore, we proceed by rationalizing the numerator

$$= \lim_{x \to 4} \left( \frac{\sqrt{x} - 2}{x - 4} \right) \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$

$$= \lim_{x \to 4} \left( \frac{x - 4}{(x - 4)(\sqrt{x} + 2)} \right)$$

$$= \lim_{x \to 4} \left( \frac{1}{\sqrt{x} + 2} \right)$$

$$= \left( \frac{1}{\sqrt{4} + 2} \right)$$

$$= \frac{1}{4}$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 20|*

*If* $f\,(x) = \sqrt{x}$, *find*

$$\lim_{x\to 0}\left(\frac{f(9+x)-f(9)}{x}\right)$$

**SOLUTION:**

$$\lim_{x\to 0}\left(\frac{f(9+x)-f(9)}{x}\right) = \lim_{x\to 0}\left(\frac{\sqrt{9+x}-\sqrt{9}}{x}\right)$$

Direct substitution of $x = 0$ gives the indeterminate form $\frac{0}{0}$. Therefore, we proceed by rationalizing the numerator

$$= \lim_{x\to 0}\left(\frac{\sqrt{9+x}-3}{x}\cdot\frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}\right)$$

$$= \lim_{x\to 0}\left(\frac{9+x-9}{x\left(\sqrt{9+x}+3\right)}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{x\left(\sqrt{9+x}+3\right)}\right)$$

$$= \lim_{x\to 0}\left(\frac{1}{\left(\sqrt{9+x}+3\right)}\right)$$

$$= \left(\frac{1}{\left(\sqrt{9+0}+3\right)}\right)$$

$$= \frac{1}{6}$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 21|*

If $f\left(x\right)=x^2-2x+3$, find

$$\lim_{x\to 2}\left(\frac{f(x)-f\left(2\right)}{x-2}\right)$$

**SOLUTION:**

$$\lim_{x\to 2}\left(\frac{f(x)-f(2)}{x-2}\right)=\lim_{x\to 2}\left(\frac{\left(x^2-2x+3\right)-\left(2^2-2\cdot 2+3\right)}{x-2}\right)$$

$$=\lim_{x\to 2}\left(\frac{\left(x^2-2x+3\right)-3}{x-2}\right)$$

$$=\lim_{x\to 2}\left(\frac{x^2-2x}{x-2}\right)$$

Direct substitution of $x=2$ gives the indeterminate form $\frac{0}{0}$. Therefore, we proceed by factoring the numerator

$$=\lim_{x\to 2}\left(\frac{x(x-2)}{x-2}\right)$$

$$=\lim_{x\to 2}\left(x\right)$$

$$=2$$

# *Limit of a Function in Indeterminate Form| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.3, Problem 22|*

If $f\left(x\right)=x^2-2x+3$, find

$$\lim_{x\to 0}\left(\frac{f(x+2)-f(2)}{x}\right)$$

**SOLUTION:**

$$\lim_{x\to 0}\left(\frac{f(x+2)-f(2)}{x}\right)=\lim_{x\to 0}\left(\frac{\left((x+2)^2-2(x+2)+3\right)-\left(2^2-2\cdot 2+3\right)}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\left((x+2)^2-2(x+2)+3\right)-(3)}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\left((x+2)^2-2(x+2)\right)}{x}\right)$$

Direct substitution of $x=0$ gives the indeterminate form $\frac{0}{0}$. Therefore, we proceed by factoring the numerator

$$=\lim_{x\to 0}\frac{(x+2)(x+2-2)}{x}$$

$$=\lim_{x\to 0}\frac{(x+2)(x)}{x}$$

$$=\lim_{x\to 0}\left(x+2\right)$$

$$=0+2$$

$$=2$$

# Feliciano and Uy| Exercise 1.4, Problem 1|

## Evaluate

$$\lim_{x\to\infty}\left(\frac{6x^3+4x^2+5}{8x^3+7x-3}\right)$$

**SOLUTION:**

Divide by the highest denominator power

$$\lim_{x\to\infty}\left(\frac{6x^3+4x^2+5}{8x^3+7x-3}\right)=\lim_{x\to\infty}\left(\frac{6x^3+4x^2+5}{8x^3+7x-3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{6+\frac{4}{x}+\frac{5}{x^3}}{8+\frac{7}{x^2}-\frac{3}{x^3}}\right)$$

$$=\frac{\lim\limits_{x\to\infty}\left(6+\frac{4}{x}+\frac{5}{x^3}\right)}{\lim\limits_{x\to\infty}\left(8+\frac{7}{x^2}-\frac{3}{x^3}\right)}$$

$$=\frac{6+0+0}{8+0-0}$$

$$=\frac{6}{8}$$

$$=\frac{3}{4}$$

# Limit at Infinity| Differential and Integral Calculus| Feliciano and Uy| Exercise 1.4, Problem 2|

Evaluate

$$\lim_{x\to\infty}\left(\frac{3x^2+x+2}{x^3+8x+1}\right)$$

**SOLUTION:**

Divide by the highest denominator power

$$\lim_{x\to\infty}\left(\frac{3x^2+x+2}{x^3+8x+1}\right)=\lim_{x\to\infty}\left(\frac{3x^2+x+2}{x^3+8x+1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{3x^2}{x^3}+\frac{x}{x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{8x}{x^3}+\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{3}{x}+\frac{1}{x^2}+\frac{2}{x^3}}{1+\frac{8}{x^2}+\frac{1}{x^3}}\right)$$

$$=\frac{0+0+0}{1+0+0}$$

$$=0$$

# Integral Calculus| Feliciano and Uy| Exercise 1.4, Problem 3|

## Evaluate

$$\lim_{x\to\infty}\left(\frac{4x+5}{x^2+1}\right)$$

**SOLUTION:**

Divide by the highest denominator power

$$\lim_{x\to\infty}\left(\frac{4x+5}{x^2+1}\right)=\lim_{x\to\infty}\left(\frac{4x+5}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{4x}{x^2}+\frac{5}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{4}{x}+\frac{5}{x^2}}{1+\frac{1}{x^2}}\right)$$

$$=\frac{0+0}{1+0}$$

$$=0$$

# Differential and Integral Calculus| Feliciano and Uy| Exercise 1.4, Problem 4|

## Evaluate

$$\lim_{x\to\infty}\left(\frac{x^3+x+2}{x^2-1}\right)$$

**SOLUTION:**

Divide by the highest denominator power

$$\lim_{x\to\infty}\left(\frac{x^3+x+2}{x^2-1}\right)=\lim_{x\to\infty}\left(\frac{x^3+x+2}{x^2-1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{x^3}{x^3}+\frac{x}{x^3}+\frac{2}{x^3}}{\frac{x^2}{x^3}-\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{1+\frac{1}{x^2}+\frac{2}{x^3}}{\frac{1}{x}-\frac{1}{x^3}}\right)$$

$$=\frac{1+0+0}{0-0}$$

$$=\infty$$

# Feliciano and Uy| Exercise 1.4, Problem 5|

## Evaluate

$$\lim_{x\to\infty}\left(\frac{8x-5}{\sqrt{4x^2+3}}\right)$$

**SOLUTION:**

Divide by the highest denominator power

$$\lim_{x\to\infty}\left(\frac{8x-5}{\sqrt{4x^2+3}}\right)=\lim_{x\to\infty}\left(\frac{8x-5}{\sqrt{4x^2+3}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{8x}{x}-\frac{5}{x}}{\sqrt{\frac{4x^2}{x^2}+\frac{3}{x^2}}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{8-\frac{5}{x}}{\sqrt{4+\frac{3}{x^2}}}\right)$$

$$=\frac{8-0}{\sqrt{4+0}}$$

$$=\frac{8}{2}$$

$$=4$$

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