Civil Service Exam Reviewer 2021

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➢VERBAL

Simple Tenses: Summary and Quiz

To recapitulate, simple tenses consist of **present tense**, **past tense**and **future tense**. The simple present tense is formed using the **base form of the verb** such as **talk, walk, etc**. (for plural noun or pronoun) or **the s-form of the verb (talks, walks, etc**) for singular noun or pronoun. Other forms of the verbs like the **"be" verb such as is/ are**;**auxiliary verbs like has / have, do / does** can be used. These forms of the verb are used to express actions that are habitually done, to state a fact or general truth.

The past tense is formed with the following verbs like **was (singular) / were (plural), had and did** (both for singular and plural). For **regular verbs** you just add **–ed (talked, walked, etc**) or **a change in spelling is needed for irregular verbs (eat = ate, write =wrote, etc**). This tense of the verb is used for actions that happened or completed in a definite past time. **Time expressions like yesterday, last month, few days ago, etc. are used.**

For actions that are intended to be completed or done in a particular time in the future, Future tense of the verb is used. Here, e**xpressions like tomorrow, next year, next summer, etc. will signify futurity.** Though, **will /shall + base form of the verb** can be used most of the time will + base form of the verb is preferred by most people both in oral / written communication.

To evaluate mastery of these simple tenses, exercises below are designed for this purpose.

Choose the appropriate verb in the following sentences. Check your answers below.

1. Citizens in a democratic country (has, have) to select a leader through election process.
2. Pump prices (go, goes) up rapidly depending on the scarcity of supply.
3. Terrorists groups (behead, beheaded) foreign journalists 7 years ago.
4. Transport group (express, expresses) fare hike if oil prices (will continue, continues) to rise next week.
5. Overseas workers (is expected, are expected) to go back home in the coming years.
6. Today’s youth (know, knows) the difference between real friendships from mere acquaintances.
7. Typhoon Hagupit (was, were) the most destructive typhoon last month.
8. Merchandises from China (will continue, continue) to flock the global market.
9. Persona non-grata (is, are) declared whenever a person / an individual utters or behaves inappropriately in a particular place or country.
10. Global warming (become, becomes) an alarming problem worldwide.

Answer Key

1. Have
2. Go
3. Beheaded
4. Expresses
5. Are expected
6. Knows
7. Was
8. Will continue
9. Is
10. Becomes

Grammar Tutorial: Simple Future Tense

The simple future tense indicates that the action is in the future relative to the speaker. Verbs in the future tense are not changed (or inflected), instead, helping verbs such as *will* and *shall* are added before the base form of the verb.

Examples

I *will* buy a computer tomorrow.

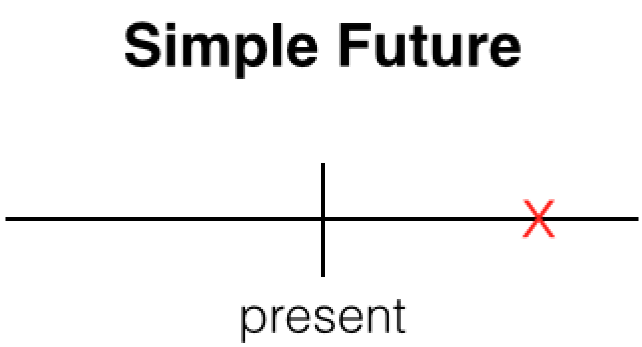
I *shall* return.

Shall we dance?

Will you help me?

In the first example, the helping verb *will* is added before buy which is a verb in base form. In the second sentence, the helping verb *shall* is added before the verb return. The future tenses in question are also shown above.

If we use a diagram, the simple future tense can be represented in the timeline as the red x below.



Aside from will and shall, the form *will be*, *shall be*, *be + going to* are also several ways of expressing things in simple future.

I *will* make a sandwiches for breakfast.

I *will be* in Tokyo next week.
I am *going to* buy a dictionary tomorrow.
It is *going to be* sunny tomorrow.
I *shall be* back in a month's time.
As shown above, some of the common usage of future sentences include: voluntary action (sentence 1), expressing a plan (sentence 2) or prediction (sentence 3), and making a promise (sentence 4).

Take note however that the form of the verb is changed to past tense in passive simple future sentences.

Active: Annie *will* finish the job at 6pm.

Passive: The work *will be* finished by Annie at 6pm.

Grammar Tutorial: Simple Past Tense

Simple past tense is used when the action referred to happened in the past.

Example: They walked to the police station yesterday.
In this example, the verb walk is added with "ed" since the situation happened the day before. This is indicated by "yesterday."
Rules in Forming the Verbs
a.) Verbs ending in *e* are usually just appended by -d.
Examples

- dive – dived
- tie – tied
- carve – carved

b.) Verbs ending in a consonant preceded by a single stressed vowel double the consonant in the past tense.

Examples

- hop – hopped
- stop – stopped
- fit – fitted

b.) Most verbs that end in *y* change the *y* into -ied

Examples

- Hurry – hurried
- Bury – buried
- Reply – replied
- Worry – worried

c.) In regular verbs, the past simple ends in -ed

Examples

walk – walked
talk – talked
worship – worshiped
wash – washed

d.) Some verbs are irregular

Examples

- write – wrote
- see – saw
- go – went
- throw – threw
- cost – cost

- hit – hit

Quiz for simple past tense

1. Linda ______ in my house last week.

a. stay
b. stayd
c. stayed

Answer

c. stayed

2. I ______ the bunny at Nordstrom's.

a. buyed
b. bawt
c. bought

Answer

c. bought

3. He just ______ "Hi."

a. said
b. sayed
c. sought

Answer

a. said

4. I ______ the article in the newspaper.

a. read
b. red
c. readed

Answer

a. read

5. She _______ 10 gifts.

a. choosed

b. chosed

c. chose

Answer

c. chose

6. The water rise during the 1962 floods _______ about 10 feet.

a. rise

b. rised

c. rose

Answer

c. rose

7. Andy _______four chicken legs and one chicken breast.

a. fried

b. freid

c. freed

Answer

a. fried

8. He _______ the glass by accidentally knocking it over with his elbow.

a. broke

b. brake

c. broked

Answer

a. broke

9. How did they send the letter?

They ______ the letter by airmail.

a. saint

b. sent

c. sunt

Answer

b. sent

10. I ______ that Japanese would be hard for me to learn.

a. knew

b. know

c. knowed

Answer

a. knew

Grammar Tutorial: Simple Present Tense

We use the simple present tense when expressing action in the present taking place once, never or several times, facts, actions taking place one after another, and action set by a timetable or schedule

The simple present tense obeys the subject verb agreement and, of course, the verb is in present tense.

Simple present tense are used in the following situations.

(a) Facts and generalizations

1.) The sun *rises* from East.

2.) The dog *barks*.

(b) Repeated actions, customs, and habits

1.) People *celebrate* Christmas on 25th December.

2.) Kenyans *go* for elections every five years.

(c) Things happening now.

(1) He *balances* on a rope.

(2) He *begins* his speech now.

(d) Things happening in the near future.

(1) The train *leaves* in 2 minutes.

(2) The program *starts* in 1 hour.

When asking questions in this tense, the auxiliary must reflect the number.

e.g. Does he walk to school ?(singular subject)

Do they walk to school? (plural subject)

If you are not sure whether it is in present tense or not, there words we would consider markers of present tense, for example: always, every …, never, normally, often, seldom, sometimes, usually… e.t.c

QUIZZ FOR PRESENT SIMPLE TENSE

1.) The food in Japan is expensive. It ___ a lot to live there.

a. cost

b. costs

c. costing

Answer

b. costs

2.) His job is great because he ___ a lot of people.

a. meet

b. meets

c. is meeting

Answer

b. meets

3.) He always ___ his car on Sundays.

a. wash

b. washes

c. is washing

Answer

4. My watch is broken and it ___ to be fixed again.

a. need

b. needs

c. needed

Answer

b. needs

5. I ___ to watch movies.

a. love

b. loves

c. loving

Answer

love

6. I ___ to the cinema at least once a week.

a. go

b. goes

c. went

Answer

a. go

7. They never ___ tea in the morning.

a. drink

b. drinks

c. drunk

Answer

a. drink

8. We both ___ to the radio in the morning.

a. listen

b. listens

c. listened

Answer

a. listen

9. He ___ a big wedding.

a. want

b. wants

c. wanted

Answer

b. wants

10. George ___ too much so he’s getting fat.

a. eat

b. eats

c. eaten

Answer

b. eats

Perfect Tenses: Summary and Quiz

In the previous posts, I have summarized the**simple tenses**. In this post, we summarize the perfect tenses. Review exercises are provided below to assess your mastery of the lesson.

Perfect tenses have three types: **Present perfect**, **Past perfect** and **future perfect**.

The **present perfect tense** denotes actions that began in the past and continues up to the present time. It is also used to suggest events that happened at unspecific time before now. Has (singular) and have (plural) + past participle of the given verb are used to form the present perfect tense.

The **past perfect tense** of the verb is formed with Had (for singular and plural noun)+ past participle of the verb. This tense of the verb is used to express an action that happened before another past action occurred. Always remember that the second past action must use the simple past tense of the verb.

Similarly, for the **future perfect tense** two actions/events are required here. But, these actions are intended to be completed in the future. Expressions such as by tomorrow, by next year, ten years from now, etc. are commonly used plus the future perfect tense (will have + past participle). This is to suggest that the action is completed before a certain time.

Practice Quiz

Choose the correct form of the perfect tense for each of the following sentences.

1.) Corona virus (has, have) spread in countries like Philippines.

2.) The reinforcement team (arrived, had arrived) after the forty-four Special Action force members (has died, had died) in the encounter.

3.) The country (will have experienced, will experience) drought before the summer comes next year.

5.) The government of China (expressed, has expressed) its desire to end the territorial row with the Philippines.

6) Melinda (will have become, had become) a lawyer before her mother retires.
World Health Organization (WHO) (warned, had warned) the public about the Covid-19 before it became widespread.
7.) The government (ordered, has ordered) recall of a certain brand of apples in the market because of its toxic contamination.

8.) Food and Drug Administration (has advised, have advised) the public against the proliferation of untested diet pills in the market.

9.) The US government (has tested, had tested) all its local produce before it reached the market.

10.) Two years from now, Reymart (will have been, will become) a licensed physical therapist.

Answer key

1. Has
2. Arrived, had died

3. Will have experienced
4. Has experienced
5. Will have become
6. Had warned
7. Has ordered
8. Has advised
9. Had tested
10. Will have been

Grammar Rules: Future Perfect Tense

In the previous posts, we have discussed **Present Perfect Tense** and **Past Perfect Tense**. In this post, we are going to discuss Future Perfect Tense. **Future perfect** expresses the idea that something will happen before another action in the future. It also shows that something will happen before a specific time in the future. Just like the past perfect tense there must be two actions. In this case, these actions should be completed or done in the future. The action that will happen first, though in the future, will follow the **will + have + past participle of the verb**. Then, the other future action will use the simple present tense.

Form: will + have +past participle of the verb

The common time expressions are: before, by tomorrow/ 7 o'clock / next month, etc., until or till.

Examples:

1. By next year, I ***will have moved*** to my new house.
2. By the time the husband gets home, the wife ***will have cooked*** dinner.
3. He ***will have perfected*** his English by the time he moves to the US.

Exercise: Use the future perfect form of the verb in the following sentences.

1. They (graduate) from college by April 2025.
2. Melissa (turn) 40 by the end of the year.
3. The students (submit) the report until the teacher (arrive).
4. I (complete) the needed reports by the time the president (need) them.
5. A better model of cellular phones (release) in the market by next month.

Answers:

1. They will have graduated from college by April 2025. This means that come 2025 the students are definitely done with their studies.

2. Melissa will have turned 40 by the end of the year. This statement tells us that before the end of December Melissa will have celebrated her 40th birthday. Therefore, the birthday comes first before the year ends.

3. The students will have submitted the report until the teacher arrives. The idea here is that when the teacher comes to school the students have already submitted the report. Therefore, she doesn't have to wait for them because she has it probably on her table.

4. I will have completed the reports by the time the president needs it. The two future actions here are the completion of the reports and the time when the report is needed. In this sentence the completion happened first even before the president asks the secretary probably, to hand him the copy of the reports.

5. A better model of cellular phones will have released in the market by next month. In this number the two future events are the release of the new cellular phones and next month. Therefore the market is expected to have the new unit of cell phones before we turn to another month.

Grammar Rules: Past Perfect Tense

In the previous post, we have discussed present perfect tense. Now, we continue with the rules on past perfect tense.

Past perfect tense is used to express an action that took place before another past action. The auxiliary verb **had** + **past participle**of the verb is used to form the past perfect tense.   This tense is also called the **had tense.**

Examples

1. I **had prepared** the salad when my nephews arrived home.
2. Frieda **had entered** the grades before Melissa started to compute her grades.
3. The student **had received** the report card before her mother asked for it.
4. Kristine **had never** complained before yesterday.
5. Julius **had studied** Thai before he moved to Thailand.

Exercise: Underline the correct form of the verb for each sentence.

1. We (waited, had waited) at the mall for half an hour when my friend (came, had came) into view.

2. The family of the fallen policemen (receive, had received) financial help from NGOs before the government (announced, had announced) its monetary assistance.

3. Josefa (had worked, worked) as a secretary before she (became, had become) the company president.

4. Rumors about the wedding (had come, came) out when people (saw, had seen) them together from the church.

5. There (was, is) heavy traffic because it (had rained, rained) so much the whole day.

Answer:

1. The correct answer here is **had waited** and **came**. This means that the waiting part took place before the arrival of her friend.
2. **Had received, announced**
3. **Had worked, became**
4. **Came, had seen**
5. **Was, had rained**

Grammar Rules: Present Perfect Tense

Like **simple tenses,** perfect or sometimes called compound tenses have three categories namely: Present Perfect, Past Perfect, and Future Perfect. Each of these has a corresponding usage depending on the time of action is completed or intended to be done.

PRESENT PERFECT TENSE

Present Perfect Tense is used to express an action happened at an unspecific time before now. The exact time is not important. Unlike the simple past tense, the action is done at a particular time. Hence, time expression such as yesterday, last month, etc. must be stated. The only time expressions accepted in this tense are: ever, never, once, many times, several times, before, so far, already, yet, etc.

FORM: for singular subject =Has + past participle of the given verb

For plural subject = Have + past participle of the given verb

***

Example 1: I **have seen** the movie Serendipity more than ten times.
Present perfect tense is also used to talk about change that has happened over a period of time.
Example 2: My English **has improved** since I migrated to America.
We also use the present perfect tense of the verb to tell an action that began in the past but continues up to the present.
Example 3: I **have been** in Japan since October.
Exercise: Choose the correct form of the present perfect tense in the following sentences.

1. My friend Claire (has been, have been, was) in England for six months.
2. Many policemen (have died, has died, died) in the Mindanao siege.
3. The army (has attacked, have attacked, attacked) that city five times.
4. The principal (has been, have been, was) in the meeting since this morning.
5. The baby (has grown, have grown, grew) so fast!

Answer:

1.) The correct answer to this number is ***Has been*** for the following reasons: first, the subject, **Claire** is singular that is why we use HAS not HAVE. Second, the exact time she moved to England was not stated. But, the action began six months ago and until now she is still in England.
2.) The answer here is ***Have died***. The subject is plural (policemen) therefore, **have** must be used together with the past participle of the verb die. We cannot use the simple past tense here (died) because the specific time is not mentioned.
3.) The answer is ***has attacked*. **Again, there is no specific time when the attacked happened. But the idea here is that from the first time the city was attacked until now it happened only six times.

4.) The correct answer is ***Has been***. The subject (principal) is singular so, has been is used. This sentence means that the meeting started in the morning until the time of speaking the meeting is still on-going.
5.) The sentence tells us that the change happened for a period of time (but unspecified).

Grammar Tutorial: Future Perfect Progressive Tense

Future perfect progressive is used to state an action that is perceived to be continuously happening when a certain time comes.

Form: **will have been + present participle**

Examples:

1. By this afternoon, the teachers will have been arriving to attend the meeting.
2. Few months from now, politicians will have been preparing for the forthcoming national election.

Exercises:

Write the correct future perfect progressive form of the given verb inside the parentheses.

1. By tomorrow, the patient (**recover**) from the trauma.
2. Four years from now, my friend (**complete**) her papers to Australia.
3. By the next decades, scientists (**explore**) different planets.
4. A few weeks from now, teachers (**train**) for the Online Class for this coming school year.
5. By tomorrow, Myrna (**visit**) her sick mother for a long time.

Answers

1. Will have been recovering

This means that before tomorrow ends the patient already recovered from trauma.

2. will have been completing
The idea here is that before the four Years of preparation ends my friend already completed all papers document for Australia.

3. Will have been exploring
This sentence tells us that in the coming decades scientists will try exploring different planets.

4. will have been training

In this particular sentence, it gives us a picture of the teachers in the Online Class which will continue until Covid-19 pandemic ends..

5. Will have been visiting
This means that before the day tomorrow ends Myrna already came to visit her ailing mother for a long time.

Understanding Conditionals I: First Conditional

We use First Conditionals to talk about events which are possible.

The Conditionalclause can refer to the present or the future.

Conditional clause main clause

If + Present Simple – will + bare infinitive

1. If we hurry, we'll catch the bus.
2. If we miss it, there'll be another one.

The Conditional clause can come before or after the main clause. We use a comma at the end of the Conditional clause when it comes first.

1. If I hear any news, I'll phone you.
2. I will phone you if I hear any news.

Other structures are possible, depending on what you want to say.

Conditional clause main clause

If + Present Simple – modal verb

If + Present Simple – be going to (future)

If + Present Simple – Imperative

If + Present Continuous – will + bare infinitive

If + Present Perfect – will + bare infinitive

If + Present Perfect – modal verb

Imperative – and/or + will

Exercises: Put the verbs in brackets into the correct form.

1. If you (see) Tom (tell) him I have message for him.
2. If you'd like some ice I (get) some from the fridge.
3. That book is overdue. If you (not take) it back to the library tomorrow you (have) to pay a fine.
4. If you (want) to see some of his drawings I (send) them round to your office.
5. (take) more exercise and you'll soon feel better.

Answers:

1. If you see Tom tell him I have message for him.
2. If you'd like some ice I will get some from the fridge.
3. That book is overdue. If you don't take it back to the library tomorrow you will have to pay a fine.
4. If you want to see some of his drawings I will send them round to your office.
5. Take more exercise and you'll soon feel better.

Note: Conditional clauses are often used in imperative structures. Present Simple inConditional clause and imperative in the main clause.

When you are talking about a possible situation in the present, or a possible future occurrence, you usually use the simple present tense in the conditional clause and the simple future tense in the main clause.
If the sentence starts with the Imperative verb, you use simple future tense in the main clause.

Understanding Conditionals IV: Mixed Conditionals

There are two mixed types of sentences of unreal condition:

1.) If – clause refers to the present and the main clause refers to the past.
e.g. If he were a fast runner, he would have won the race.

If – clause refers to the past and the main clause refers to the present.
e.g. If he had found a job, he wouldn't be searching for one now.

Sometimes we make sentences which mix Second and Third Conditionals, especially when a past event has an effect in the present.

Example:
a.) If you hadn't invited me, I wouldn't have gone to the party. (=I did go to the party – Third Conditional).
If you hadn't invited me, I wouldn't be here now. (=I'm at the party now. – Third + Second Conditionals)
b.) If you had planned things properly, you wouldn't have got into a mess. (=You didn't plan – Third Conditional)

If you had planned things at the start, we wouldn't be in this mess now (=We are in trouble now – Third + Second Conditionals)
All types of conditionals can be mixed. Any tense combination is possible if the context permits it.

Conditional clause main clause
If nobody phoned him he won't come to the meeting.
If he knew her, he would have spoken to her.
If he had found a job, he wouldn't be searching for one now.

Exercises: Put the verbs in brackets into the correct form.

1. If you (not spend) so much money, I (not be) angry now.
2. If they (post) the parcel yesterday, it (get) here before Friday.
3. If you (not wake) me up in the middle of the night, I (not feel) so tired now.
4. If Tom (be) a bit more ambitious, he (find) himself a better job years ago.
5. If you (know) me better, you (say) that.

1. If you hadn't spent so much money, I wouldn't be angry now.
2. If they posted the parcel yesterday, it won't get here before Friday.
3. If you hadn't woken me up in the middle of the night, I wouldn't feel so tired now.
4. If Tom was a bit more ambitious, he would have found himself a better job years ago.
5. If you knew me better, you wouldn't have said that.

Undrestanding Conditionals II: Second Conditional

In the previous post, we have written about**First Conditional**. In this post, we continue this series by talking about Second Conditional.
We use Second Conditional for unlikely situations in the present or future:
Conditional clause – Main clause

If + Past Simple – would
If I had a million pounds, I would probably buy a yacht.

The if – clause is usually past simple. However, we can also use the past continuous, could, or were/was to:

If you were coming with me, I'd give you a lift.
If I could have the day off, I'd come with you.
If you were to ask John, I'm sure he would do it.

In the conditional clause, `were' is sometimes used instead of `was', especially after `I'.

If I were as big as you, I would kill you.
If I were asked to define my condition, I'd say `bored'.

The main clause often has 'would'. We can also use 'could' or 'might'.

If we had a calculator, we could work this out a lot quicker.
If she worked harder, she might do even better at her studies.
Exercises: Put the verbs in brackets into the correct form.

1. If we (work) all night we (finish) in time, but we have no intention of working all night.

2. If someone (ring) my doorbell at 3 a.m. I (be) very unwilling to open the door.
3. If I (have) heaps of money I (drink) champagne with every meal.
4. If the earth suddenly (stop) spinning we all (fly) off it.
5. Of course I'm not going to give her a diamond ring. If I (give) her a diamond ring she (sell) it.

Answers:

1. If we worked all night we would finish in time, but we have no intention of working all night.
2. If someone rang my doorbell at 3 a.m. I would be very unwilling to open the door.
3. If I had heaps of money I would drink champagne with every meal.
4. If the earth suddenly stopped spinning, we would all fly off it.
5. Of course I'm not going to give her a diamond ring. If I gave her a diamond ring she would sell it.

Note: When you are talking about an unlikely situation, you use the simple past tense in the conditional clause, and `would' in the main clause.

Understanding Conditionals III: Third Conditional

We use the Third Conditional to talk about an event or situation that did not happen in the past:

Conditional clause main clause
If + Past Perfect – would + Perfect

If David had been more careful, he wouldn't have fallen.
If you hadn't made the mistake, you'd have passed your test.

We can use could + perfect in the if-clause.
If I could have warned you in time, I would have done.

We can use other modal verbs such as could or might+ perfect in the main clause.
If I'd written the address down, I could have saved myself some trouble.

The plan might not have worked if we hadn't had one great piece of luck.
We can also use continuous forms.

If he hadn't been evicted by his landlord, he wouldn't have been sleeping in the streets.
If he had been traveling in that car, he would have been killed too.

Exercises: Put the verbs in brackets into the correct tenses.

1. If he (not take) his gloves off he (not get) frost bitten.
2. He didn't tell me that he was a vegetarian till halfway through the meal. If he (tell) me earlier I (cook) him something more suitable.
3. I had no map; that's why I got lost. If I (had) a map I (be) all right.
4. It's a pity he never patented his invention. If he (patent) it he (make) a lot of money.
5. The club secretary is useless. He never tells anybody anything. We (not know) about this meeting if the chairman (not tell) us.

1. If he had not taken his gloves off, he wouldn't have got frost bitten.
2. He didn't tell me that he was a vegetarian till halfway through the meal. If he had told me earlier I would have cooked him something more suitable.
3. I had no map; that's why I got lost. If I had had a map I would have been all right.
4. It's a pity he never patented his invention. If he had patented it he would have made a lot of money.

5. The club secretary is useless. He never tells anybody anything. We wouldn't have known about this meeting if the chairman hadn't told us.

Note: When you are talking about something that might have happened in the past but did not happen, you use the past perfect tense in the conditional clause. In the main clause, you use `would have'.

5 Effective Ways to Improve Your Vocabulary

English Proficiency makes up about 50% of the Philippine Civil Service Examinations both Professional and Subprofessional. Vocabulary test items such as *word meaning* and *analogy*are included in these exams. For a higher probability of passing, it is important that you have a wide vocabulary when taking these examinations. In this post, I am going to give you some tips on how to improve yourvocabulary in English based on personal experience. I think these tips will not only improve your vocabulary, but also will improve your command of the English language in general. One thing to remember though: a wide vocabulary does not just develop overnight. It takes time.

1. Read, Read, and Read

Reading is one of the best ways to improve your vocabulary and you should do this everyday. In reading, you do not only meet new words, you see how they are used in sentences. Try reading an article or two a day and you will see your progress in a few months.

Reading novels will introduce you to conversational English and reading news will introduce you to technical English. If you are observant with sentence structure, reading will also introduce you to correct grammar and usage. In effect, if you have not learned grammar formally, you'll have an idea if a sentence is constructed correctly or not.

2. Play Word Games

Playing word games especially Scrabble against a computer will surely introduce you to new words. If you own a gadget, download a word game app and play against a computer.Download also a dictionary app so that you can look up the new words that you have learned.

In playing word games, you will not just enjoy, you will also learn.

3. Watch English Movies

Watching English movies can also help improve vocabulary. If you are not native English speaker (I assume that this is also read in other countries), I recommend that you watch movies with subtitles. In so doing, you will most likely meet new words that are used in daily conversations especially in movies with modern settings. Be warned though that in order to make the movies more realistic, some dialogues may be grammatically incorrect. This is because it happens in reality.

4. Take Notes

Whether reading, playing word games, or watching movies, you might also want to take note of the new words that you encounter. Look up their meaning in a dictionary and use them in sentences. After writing a sentence using the new words, you might also want to look at online dictionaries. Most online dictionaries nowadays (such asTheFreeDictionary.com) have sample sentences on how to use words.

5. Write, Write, and Write

Writing a blog or a journal can also help improve your vocabulary. In writing your thoughts, you search for appropriate words to use. This will also help you better remember those words.

After all I said, you are probably asking the following questions in your mind.

How about memorizing?

What if I randomly look up a word at a dictionary and memorize them?

Well, picking some words and memorizing their meaning will help you a little. This strategy based on personal experience is not effective. Most word meanings that are memorized without context are easily forgotten.

How to Use Contractions

We use **contractions** to make our conversations more fluid. It can also be engaging and user-friendly. I am sure there are more reasons but these are just mainly what contractions are for.
Although grammatically correct, these are only use informally.

Example instead of saying :

1. “We have arrived.” Use instead “We’ve arrived.”
2. “I am not eating the leftovers!” You can use “I ain’t eating the leftovers.”

Contractions may appear in the Grammar and Correct Usage part of the Civil Service Examination. Be sure that you are familiar with them.
Exercises:

1. I am –

Answer

I’m

2. Does not –

Answer

Doesn’t

3. Shall not –

Answer

Shan’t

4. There is –

Answer

There’s

5. You are –

Answer

You’re

6. I do –

Answer

I’d

7. She will –

Answer

She’ll

8. We have –

Answer

We’ve

9. What shall –

Answer

What’ll

10. Where has –

Answer

Where's

How to Use the Articles A, An, and The

A, *an*, and *the* are called **articles**. The definite article t*he* is used to refer to a specific noun, while the indefinite articles *a* and *an* are used to refer to non-specific nouns. Consider the following examples.

1.) Please give me the notebook.

2.) Please give me a notebook.

In the first sentence, a specific notebook is referred to. It is assumed that both the speaker and the listener know which notebook is referred to. On the other hand, in the second sentence, the speaker asks for a notebook, any notebook will do.

Rules in Using A and An

A and *an* are both used to refer to non-specific nouns but there are rules that you must remember to use them.

Rule 1a: *a* is used before a SINGULAR noun beginning with a CONSONANT letter (with exception of Rule 1b).

Examples: *a* laptop, *a* car, *a* mirror, *a* pen

Rule 1b: *a* is used before a SINGULAR noun beginning with a VOWEL letter that sounds like aconsonant

Examples: *a* user (sounds like yooser), *a* university (yooniversity), *a* unique (yoonik) trait

In the previous examples, *u* sounds like *yu* which starts with *y*.

Rule 2: *an* is used before a SINGULAR non beginning with a VOWEL

Examples: *an* English teacher, *an* umbrella, *an* amicable settlement
Rule 3: *A* and *an* are both used before words starting with *h*. If *h* sounds like a CONSONANT, *a* is used. If it sounds like a VOWEL, *an* is used.
Examples: *a* hammer, *a* human being, *an* hour (sound like "our"), *an* honorable man (sounds like "onorable")

In Grammar Quiz 4, number 3, you were asked the following question. If you got this item wrong, now, you should be able to answer it correctly. 🙂
3. __ hour ago, I met ___ beautiful woman.
a.) A, a
b.) An, a
c.) A, an
d.) An, an
Your answer?

Identifying Nouns and Pronouns

In this post, we are going to learn how to identify nouns and pronouns. You might say that this is very basic, but it is important since nouns and pronouns are usually the subject of the sentence. Identifying the subject can be useful. For example, once you identified the subject, you will now which correct verb to use in the subject-verb agreement.

Nouns are proper names that usually start with a capital letter. **Pronouns** are substitute for proper names.
He, she, it, they, us, we – are examples of pronouns.
Example: **George** and **Martha** are **Dennis** the Menace's neighbors.
(In bold letters are nouns)

They are Dennis the Menace's neighbors. (They is a pronoun which is a substitute for George and Martha collectively.)

1. I would love to be with him.

Answer

I , him – pronouns

2. Mr. de Veyra will be right over.

Answer

Mr. De Veyra – noun

3. Luis Suarez made FIFA 2014 popular because of the biting incident.

Luis Suarez – noun

4. She will have none of that nonsense.

Answer

She – pronoun

5. We will follow soon.

Answer

We – pronoun

6. There they are with bags full of groceries.

Answer

They – pronoun

7. Tsuey Lin does not like it.

Answer

Tsuey Lin – noun

8. Ben will be there in half an hour.

Answer

Ben – noun

9. Be careful of the cutter.

Answer

Cutter – noun

10. His printer is next to the door.

Answer

His – pronoun

When to Use the Personal Pronouns I and Me

When do we use the personal pronouns I or me?
We use the word “I” when the pronoun is the subject of the verb. Meanwhile, we use the “me” if we are referring to the object of the verb.

Quick and dirty tip!

If you are confused, try to temporarily take out the other subject and read out loud. If it sounds nice, then it should be the correct one.

Example: (I, me) will go to Dumaguete with Jhody.

1. I will go to will go to Dumaguete with Jhody.
2. Me will go to Dumaguete with Jhody.

Exercise:

1. Joy and (I, me) went out for lunch.
2. (I, me) will go with Richard.
3. That is for (I, me), thank you very much!
4. Stephen and (I, me) are going to see the latest installment of the Transformers later.
5. Do you have anything for (I, me)?
6. Raymond and (I, me) love strawberries.
7. It’s you and (I, me) against the world, baby!
8. Sometimes my husband drives (I, me) crazy.
9. Those apples over there at the counter are for you and (I, me).
10. (I, me) beg to differ!

Answers:

1. I
2. I
3. Me
4. I
5. Me
6. I
7. Me
8. Me
9. Me
10. I

Grammar Tutorial: Subject-Verb Agreement

Subject-verb agreement:

If the subject is singular the verb should have “s” at the end. If the subject is plural there is no “s” at the end of the verb, except for the word “I” which always take the plural form of the verb.

Example: Jenna writes a poem.

In this sentence, the subject *Jenna* is singular, so the verb *writes* has “s” at the end.
Example: We visit our grandparents every Sunday.

In this sentence, the subject *we* is plural, so the verb *visit* has no “s.”
Now, it’s your turn! Choose the correct verb.

1.)Yen (call, calls) Socks the cat to come inside the house.

Answer

calls

2.) Gaute (cook, cooks) his own dinner.

Answer

cooks

3.) They (come, comes) in time for the movie.

Answer

come

4.) She (eat, eats) apple for breakfast.

Answer

eats

5.) Mickey and Minnie (is, are) on the cover of the Kleenex tissue.

Answer

are

6.) I (love, loves) cheesecake.

Answer

love

7.) Ryan (run, runs) to the nearest exit.

Answer

runs

8.) Justin (go, goes) for work again.

Answer

9.) Vivian (offer, offers) me a drink.

Answer

offers

10.) We (celebrate, celebrates) the fourth on July with a barbeque on the front porch.

Answer

celebrate

The Subject-Verb Agreement Rules Part 1

Subject-verb agreement means that the subject and verb endings agree in number. Determining singular or plural endings can be confusing because an -s ending on a noun indicates plural, whereas an -s ending on a verb indicates singular form. The subject of every sentence is either singular or plural, and that determines the ending of the verb.

In the examples below, the subjects in the sentences are <u>underlined</u> the verbs are *italicized*.

Rule 1: Singular nouns (usually without s) take singular verbs (usually with s).
Plural nouns (usually with s) take plural verbs (usually without s).
Examples:

The <u>bee</u> *buzzes* every night. (One bee = singular verb)
The <u>bees</u> *buzz* every night. (More than one bee = plural verb)
The <u>stamps</u> *stick*.
<u>Time</u> *flies* so fast.
Note: The nouns “I” and “you” always take a plural verb.

<u>I</u> *eat* a lot.
<u>You</u> *are* so beautiful.
Rule 2: Compound subjects or subjects joined by **and** take a plural verb.

Example:

My father and my brother *visit* me every year.

Rule 3: The conjunction 'or' does not conjoin like 'and.' When you use or, the verb takes the the number of the closest subject.

Your father or his sisters *are* going to take care of Anna.

Your sisters or your father *is* going to take care of Anna.

Rule 4: Just like in Rule 3, when the subject words are joined by *either ... or, neither . . . nor, or not only ... but* , the verb agrees with the subject closest to it.

Examples:

Either her friend or her mother *has* the money.

Neither her uncle nor her aunts *have* the money.

In sentence 1, the verb "has" which is singular agrees with the subject "mother" (singular). In the second sentence, the verb "have" (plural) agrees with the subject "aunts" (plural).

Rule 5: The indefinite pronouns *no one*, *anyone*, *everyone*, *someone*, *anybody*, *everybody*,*somebody*, and *nobody* are always singular. They take singular verbs.

Examples:

No one *is* above the law.

Everyone *was* happy.

Rule 6: When word groups or modifiers separate the subject and the verb, locate the subject word to determine whether to use a singular or plural verb.

Examples:

The flowers in the pot on the balcony *need* watering.

An apple a day *keeps* the doctor away.

Rule 7: Phrases starting with the following words are normally not part of the subject: *along with, together with, accompanied by, in addition to, as well as*, except, with, no less than.

Risa, together with her friends, *goes* to a party every weekend.

Dana and Gemma, together with their father, *go* to church every weekend.
That's all for now, we will discuss more rules in the next post.

The Subject-Verb Agreement Rules Part 2

In the previous post, we have learned seven**rules of the subject-verb agreement**. We now continue with the 8th rule.

Rule 8: Modifiers between the subject and the verb does not affect the number of the subject.

Jason, who is a father of four, *is* currently suffering liver cancer.

In this sentence, the phrase "who is a father of four" is a modifier of Jason. It does not affect Jason as a subject and therefore takes a singular verb 'is.'

Rule 9: Some nouns (collective nouns) can be used as singular or plural depending on the context and usage.

Rica's family *plans* to go on a vacation this summer.

The staff *have* gone their separate ways after the meeting.

In the first sentence, the family is a collective noun and functions as one group. In the second sentence, the staff refers to the persons individually.

Rule 10: Uncountable nouns or nouns that can't be counted takes singular verbs.

Too much sugar *was* put in this coffee.

Money *is* the root of all evils."

In this sentence, sugar is an uncountable noun, so we used 'was' instead of 'were.' It is the same with the second sentence.

Rule 11: There are words that end in s that are always considered as singular.

The news about her death *is* spreading very fast.

Mathematics *is* a very difficult subject.

Diabetes *is* not a curable disease.

Rule 12: Fractional expressions 'half of,' 'part of,' 'portion of' may take singular or plural verbs depending on the context.

Half of the audience *are* asleep because of his boring speech.

A portion of his wealth *was* donated to cancer patients.

Rule 13: In inverted sentences especially those that use here and there, the subject follows the verb.

Here *are* the towels.

A good snack *is* a salad.

Now, that you know the basics, we are no ready to discuss the tenses. More to come

Verbs and Tenses: An Introduction

In the next few weeks, we will be looking at*tenses*. When we talk about the tenses, we cannot look at them in isolation. We are actually looking at *verbs*.

In this post we will review the basics of verbs. It is through them that we will tell the tense in which a sentence is.

What are verbs?

Verbs are **actions** in sentences. A verb denotes the action or state of being of the subject in a sentence. The actions can be physical or mental.

Examples

She *went* to the party.

Anna *considers* the job easy.

In the sentences above, *went* and *considers* are the verbs of the subjects *she* and *Anna*respectively. The verb *went* refers to physical action, while the verb *considers* refers to mental action.

There is also a group of verbs that you often use but *probably* don't know that they are verbs. They denote **state of being**. The base of these verbs is *to be*. They are the following:*am*, *is*, *are*, *was*, *were*, and *will be*.

When we talk about verbs, we also want to indicate the time they happened. Is it the past, present, or future? This is called **tense** of the verbs. Even if you are not a native English speaker, you would now that you should use *was* when the event has already happened, use *is* if it is happening, and use *will be* if the event is going to happen in the future.

Examples

Carrie *was* with Emmy yesterday.

I *am* tired.

My son *will be* here tomorrow.

As you already know, verbs are also affected by the subject. Singular subjects take singular verbs, while plural subject take plural verbs. For example, in the first sentence above, if Carrie has another companion, say, Abby,

then *was* will become *were*. That is,

Carrie and Abby *were* with Emmy yesterday.

This is called the subject-verb agreement. This is also an important concept which we will discuss in the next post because we start with simple tenses.

Answers to Grammar Practice Test 1

Below are the answers and the explanations to the **Grammar and Correct Usage Practice Test 1**. The incorrect word or phrase in the sentence is highlighted red, while the correct word or phrase is highlighted green.

1. The inventor stood to ***except*** the award.

Correct Sentence: The inventor stood to***accept*** the award.
Explanation: Accept means “to receive”, except means “to exclude.”

2. ***Between*** the three of us, I think I am the slowest runner.
Correct Sentence: ***Among*** the three of us, I think I am the slowest runner.
Explanation: Among is used to refer to 3 or more members of the group, while between is used to refer to two member of the group.

3. There are ***scarcely no more*** birds in this city.
Correct sentence: There are ***scarcely any*** birds in this city.
Explanation: Double negative. Scarcely and no more are both negative word/phrase.

4. This fruit contains ***fewer*** sugar.
Correct sentence: This fruit contains ***less*** sugar.
Explanation: Fewer is used to describe plural nouns (how many) while less is used to describe singular nouns (how much). Sugar is singular.

5. I have ***never seen nothing*** as beautiful as this city.
Correct sentence: I have ***never seen anything*** as beautiful as this city.
Explanation: Double negative. Never seen and nothing are both negative word/phrase.

6. Place the mirror on the wall to give you an ***allusion*** of bigger room.
Correct sentence: Place the mirror on the wall to give you an ***illusion*** of bigger room.
Explanation: Allusion means “a passing or casual reference,” while illusion means “something that deceives by producing a false or misleading impression of reality.”

7. We were lucky we didn’t have typhoon this month. (No Error)

8. My favorite vegetable ***are*** peas.
Correct sentence: May favorite vegetable ***is*** peas.
Explanation: The subject is *favorite* is singular, therefore the verb are should be *is*.

9. Either James or John ***are*** going to lead the choir in the recital ***tommorow***.

Correct sentence: Either James or John ***is*** going to lead the choir in their recital ***tomorrow***.
Explanation: In *either or* statements, the verb considers the subject after the or statement. The subject *John* is singular, so the verb should be *is*. The correct spelling is *tomorrow*.
10. The additional supplies that we need to bring are**:** band aids, cottons, alcohol, and gauze.
Correct sentences:

The additional supplies that we need to bring are band aids, cottons, alcohol, and gauze. (The colon was deleted).
The additional supplies that we need to bring ***are the following***: band aids, cottons, alcohol, and gauze.
Explanation: The colon is used before a list of items especially after expressions like *the following* and *as follows*. Do not use a colon before a verb or a preposition.
11. All the students ***has*** finished their report.
Correct sentence: All the students ***have*** finished their report.
Explanation: In this sentence, “all” refers to many students, therefore “have” is the correct verb.
12. He was ***a*** honorable man.
Correct sentence: He was ***an*** honorable man.
Explanation: *A* is used before words beginning with a consonant sound (a car, a pencil), while*an* is used to before words beginning with vowel sounds (an ant, an egg). *An* is also used to before words beginning with the consonant *h* when *h* is not pronounced (such as*honorable*).
13. The recently heavy flooding ***effected*** the crops of farmers.

Correct sentence: The recently heavy flooding ***affected*** the crops of farmers.

Explanation: Affect means "to influence" while effect means "the result of some actions."

14. Emmanuel ***could of*** passed the examinations if he had studied hard enough.

Correct sentence: Emmanuel ***could have*** passed the examinations if he studied hard enough.

Explanation: The word "of" is not used after the verb could.

15. I believe that ***were*** going to have a prosperous new year.

Correct sentence: I believe that ***we're*** going to have a prosperous new year.
Explanation: The word *were* is the past tense of *was* while *we're* is the contraction of *we are*.

Subject-Verb Agreement Practice Test 1 Answers

Below are the answers to the **Subject-Verb Agreement Practice Test 1**. The incorrectverb in each sentence is highlighted ***red*** and the correct verb in the corrected sentence is highlighted ***green***. An explanation follows every correction.

Subject-Verb Agreement Practice Test 1 Answers

1. My brother or my sister ***are*** arriving tomorrow.

Correct sentence: My brother or my sister ***is*** arriving tomorrow

Explanation: Two singular subjects connected by *or* require a singular verb. In this sentence, there *brother* and *sister* are both singular, so the sentence should use the singular verb ***is***.

2. Neither Ella nor her friends ***is*** available to assist you.

Correct sentence: Neither Ella nor her friends ***are*** available to assist you.

When a singular subject and a plural subject are connected by *neither/nor* or *either/or*, the verb agrees with the subject nearest to it. In the sentence above, the plural verb ***are*** is used because it is nearer to the plural subject *friends*.

3. Armand, together with his friends, ***are*** going on a camping trip tomorrow.

Correct sentence: Armand, together with his friends, ***is*** going on a camping trip tomorrow.

Explanation: In general, phrases such as *together with*, *along with*, *as well as*, should be ignored when considering the verb of the sentence. The verb should always agree with the subject. In this sentence, Armand is the singular subject, so the singular verb ***is*** is used.

4. Jim and Mike ***are*** going to take the Subprofessional Civil Service Examination next month.

Explanation: No error. Jim and Mike is a plural subject, so the plural verb ***are*** is used.

5. Each of the boys ***play*** piano well.

Correct sentence: Each of the boys ***plays*** piano well.

Explanation: The pronouns *each*, *anyone*, *anybody*, *everyone*, *every one*, *everybody* are singular and require singular verbs. In the sentence, *each* is singular so, the singular verb***plays*** must be used.

6. Neither of them ***are*** available to meet you at the airport.

Correct sentence: Neither of them ***is*** available to meet you at the airport.

Explanation: Sentences that use e*ither* or *neither (*without or) always require singular verb.

7. Ten years ***are*** such a long time to wait my love.

Correct sentence: Ten years ***is*** such a long time to wait my love.

Explanation: Periods of time take singular verb.

8. Five thousand pesos ***are*** a high price to pay for a single T-shirt.

Correct sentence: Five thousand pesos ***is*** a high price to pay for a single T-shirt.

Explanation: Amount of money takes singular verb.

9. Fifty percent of the cake ***have*** disappeared after just two minutes.

Correct sentence: Fifty percent of the cake ***has*** disappeared after just two minutes.

Explanation: The verb used in words that indicate portions such as *percent*, *fraction*, *part*, *all*,*some*, etc. depends on the noun in the "of phrase." In the sentence above, the cake in "of the cake" phrase is singular, therefore, the singular verb ***has*** should be used.

10. Everybody ***is*** happy about the result of the examination

Explanation: No Error. See no. 5 for explanation.

11. One-tenth of the people in this city ***are*** unemployed.

Explanation: No error. See no. 9 for explanation.

12. He is one of the men who ***does*** the work without complaining.

Correct sentence: He is one of the men who ***do*** the work without complaining.

Explanation: The pronouns *who*, *which*, and *that* which is the subject of the verb in the middle of the sentence become singular or plural according to the noun directly in front of them. In the sentence above, the noun in front of *who* is *men* which is plural, so the plural verb ***do*** must be used.

13. The team ***are*** going to practice tomorrow for the final competition

Correct sentence: The team ***is*** going to practice tomorrow for the final competition

Explanation: Collective nouns such as *team* and *staff* may either be singular or plural depending on their use in the sentence. In the sentence above, the team acts as a unit and therefore requires a singular verb.

14. The committee ***are*** in disagreement whether to use the fund in a feeding program or donate it to a hospital.

Explanation: No error. Unlike in number 13, the *committee* in this sentence act as individuals and not as a unit.

15. Nina, as well as his brother and sister, ***is*** attending a birthday party.

Explanation: No error. See no. 3 for explanation.

Subject-Verb Agreement Practice Test 1

The subject-verb agreement is one of the basic rules in grammar and correct usage. It is important that you master this rule if you want to pass the Civil Service Examination.

The basic rule in the subject-verb agreementis that a singular subject requires a singular verb and a plural subject requires a plural verb. Of course, to be able to answer correctly, you must be able to identify the subject of the sentence and the verb. As a review, a subject is the noun or pronoun that performs the verb. A verb on the other hand, is a word that shows action. We will have a separate discussion on these topics. For now, just answer the practice test below and see how much do you remember of the subject-verb agreement.

Subject-Verb Agreement Practice Test 1

1. My brother or my sister are arriving tomorrow.

2. Neither Ella nor her friends is available to assist you.
3. Armand, together with his friends, are going on a camping trip tomorrow.
4. Jim and Mike are going to take the Subprofessional Civil Service Examination next month.
5. Each of the boys play piano well.
6. Neither of them are available to meet you at the airport.

7. Ten years are such a long time to wait my love.
8. Five thousand pesos are a high price to pay for a single T-shirt.
9. Fifty percent of the cake have disappeared after just two minutes.
10. Everybody is happy about the result of the examination
11. One-tenth of the people in this city are unemployed.
12. He is one of the men who does the work without complaining.
13. The team are going to practice tomorrow for the final competition
14. The committee are in disagreement whether to use the fund in a feeding program or donate it to a hospital.
15. Nina, as well as his brother and sister, is attending a birthday party.

How to Answer Paragraph Organization Tests Part 1

Paragraph Organization or arranging separate sentences into a coherent paragraph composition is probably one of the most difficult types of test in the Civil Service Examination. In this series, I am going to show that it is actually not that hard.

In this post, I am going to show you how to analyze in details a Paragraph Organizationsample question. The task is to arrange the five sentences below in correct order. Please read the sentences thoroughly before you continue.

A. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes.
B. In fact, even my father who is already 40 years old has a big Transformer robot.
C. Collecting action figures has become a popular hobby for many Filipinos nowadays
D. Indeed, it's true what they say, a hobby is a hobby and has nothing to do with age.
E. Not only that, our rich neighbor who is already in his late 50's just came back from Japan, bought a life-size statue of Vegeta.

Locating the First Sentence

Knowing the first sentence is very crucial in Paragraph Organizaiton questions. Alwaysremember that the first sentence is a **topic introduction**. From the sentences above, B, C, and E cannot be topic introductions because they begin with "in fact," "indeed" and "not only that." These are clauses that refer to 'something' that has already been introduced. Therefore, we are only left with A and C as possible topic introduction.

It looks like A and C are both good candidates for introduction, but notice that all of the sentences talk about action figures which is obviously the topic. In A, the subject of the sentence is "small children, adult, and adults" which is not the topic. In C on the other hand, the sentence talks about collecting action figures. Therefore, the first sentence is C.

Locating the Second Sentence

The second sentence usually elaborates the first sentence. Looking from the paragraph, A and B are the only candidates (Why?). However, notice that in the first sentence, we are talking about many Filipinos. Now, who is 'closer' to "many Filipinos"? The author's father, or the *children, teenagers, and adults*? In addition, observe below that it is a bit "strange" if we place B as the second sentence. Further, if we place B as the second sentence, we would not find any place for A later. Therefore, the correct answer is A.

Correct

Collecting action figures has become a popular hobby for many Filipinos nowadays. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes.

Incorrect

Collecting action figures has become a popular hobby for many Filipinos nowadays. In fact, even my father who is already 40 years old has a big Transformer robot.

Third Sentence

In the third sentence, the author may talk about his father (B) or his neighbor (E). This is logical because we talked about many Filipinos, then children, teenager adults. Notice that we are talking about a topic from general to specific.

Now, B and E may be interchangeable, but looking at the beginning of the sentences, the word “in fact” is more appropriate as the third sentence. The phrase “not only that” if put on the third sentence would reinforce the idea of “small children, teenagers, and adults” which is not connected to “neighbor.” Read the paragraphs below and you will see that putting E on the third sentence makes the flow of the paragraph disconnected. Therefore, the correct answer is B.

Correct

Collecting action figures has become a popular hobby for many Filipinos nowadays. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes. In fact, even my father who is already 40 years old has a big Transformer robot.

Incorrect

Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes. Not only that, our rich neighbor who is already in his late 50's just came back from Japan, bought a life-size statue of Vegeta.

Fourth and Fifth Sentences

It is clear after we have chosen the third sentence that D is the conclusion of the paragraph. You will also see that the phrase "not only that" in E reinforces the idea of old people (ages 40 and 50's) collecting action figures.

Collecting action figures has become a popular hobby for many Filipinos nowadays. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes. In fact, even my father who is already 40 years old has a big Transformer robot. Not only that, our rich neighbor who is already in his late 50's just came back from Japan, bought a life-size statue of Vegeta.

Therefore, the correct order of the above question is C-A-B-E-D.

This is probably what you are thinking now:

What the heck, you said it is not that hard. This is so freaking hard!

In the next part of this series, I will tell you why Paragraph Organization questions are easier to answer than other question types.

How to Answer Paragraph Organization Tests Part 2

In the previous post, we have discussed in detail some strategies on how to arrange shuffled sentences into a coherent paragraph. Although I have mentioned in that post that it was not that hard, it appeared to be the opposite. This time, we discuss why is it actually not as hard as you think.

In actual examinations, what makes a Paragraph Organization test a bit easy is the availability of choices. For instance, let us answer the question in the previous post — this time with choices. Use Sample Choices 1 in the table below.

A. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes.
B. In fact, even my father who is already 40 years old has a big Transformer robot.
C. Collecting action figures has become a popular hobby for many Filipinos nowadays
D. Indeed, it's true what they say, a hobby is a hobby and has nothing to do with age.
E. Not only that, our rich neighbor who is already in his late 50's just came back from Japan, bought a life-size statue of Vegeta.

If the person who constructed the test used Sample Choices 1, then it is very easy to answer because once you know that C is the first sentence, then you don't have to read the whole paragraph. There is only one choice which has C as the first sentence; that is, *b*.

Sample Choices 1	Sample Choices 2	Sample Choices 3
a. E-D-C-A-B	a. A-C-B-E-D	a. A-C-B-E-D
b. C-A-B-E-D	b. C-A-B-E-D	b. A-B-C-E-D
c. A-C-B-D-E	c. D-B-A-E-B	c. C-A-B-E-D
d. D-C-A-B-E	d. C-E-B-A-D	d. C-B-A-E-D
e. B-A-C-E-D	e. E-B-A-C-D	e. C-E-B-A-D

Now, suppose the person who constructed the example used Sample Choices 2. In thechoices, there are only two sample choices which begins with C (b and d), so still, you eliminate the three wrong answers.

Sample Choices 3 is well-thought because each choice might be equally likely to be chosen. A and C are good candidates as the first sentene and D is a very good candidate for a conclusion. Still, even though Sample Choices 3 is better made (on the perspective of the examinee), knowing the first sentence will still eliminate three choices.

That is the reason why I was saying that Paragraph Organization is not that hard. This is because in Paragraph Organization questions, once you know the first sentence (or sometimes the last), it is possible for you to eliminate the other wrong answers. The more wrong answers you eliminate, the higher is the chance of getting the correct answer.

Let’s Do the Math!

If you know the first sentence in a Paragraph Organization question, then

- you have a 100% chance of getting the correct answer in Sample Choices 1
- you have a 50% chance of getting the correct answer in Sample Choices 2
- you have a 33.33% chance of getting the correct answer in Sample Choices 3.

In other types of exams with 5 choices where you don’t know the answer and you just guess, you only get 1/5 chance of getting the correct answer or 20%. Since it is likely that you will know the introduction sentence in a Paragraph Organization test based on the tips that I have given you, I think it is quite reasonable to use the calculation above.

That is why Paragraph Organization is easier than many multiple-choice type exams.

How to Answer Paragraph Organization Tests Part 3

This is the third part and the conclusion of *How to Answer Paragraph Organization Tests* Series. In the first part, we have learned how to strategically order random sentences into a coherent paragraph, and, in the second part, we have learned how to make use of the choices in Paragraph Organization questions to increase the chance of getting the correct answer.

In this post, I will share with you a personal strategy, the things I usually do when I answer Paragraph Organization questions. Note, however, that different strategies work for different persons, so be careful. You should try out the strategy first before using it in actual exams.

Let us use the question below for discussion.

A. Small children, teenagers, and even adults would spend money just to get ahold of their favorite action heroes.
B. In fact, even my father who is already 40 years old has a big Transformer robot.
C. Collecting action figures has become a popular hobby for many Filipinos nowadays
D. Indeed, it's true what they say, a hobby is a hobby and has nothing to do with age.
E. Not only that, our rich neighbor who is already in his late 50's just came back from Japan, bought a life-size statue of Vegeta.

Choices a. A-C-B-E-D b. C-A-B-E-D c. C-E-B-A-D d. D-B-A-E-C e. E-D-B-A-C

This is what I do when I answer Paragraph Organization questions:

1. I read the question thoroughly by reading all the sentences.
2. I look for the sentence that might be a candidate as a first sentence in the paragraph. This is usually easy to see since it introduces the topic.

3. I look at the choices and see which of them has my choice as first sentence. For example, in the paragraph above, if I know that the first sentence is C, then I only have to consider b and c. This narrows the choices to 2 instead of 5.
4. Once I already narrowed the choices, I look for the sentence that might be a conclusion. In the case above it's D. Note that both b and c has D as the last sentence, so I have now to choose between b and c.
5. I read the paragraph in the order of b and c and see check which is more coherent.

Note that the strategy above happens very fast because of time constraint. You have to read as fast as you can.

Now, all you have to do is to practice it and see if it works for you. Remember, just knowing the first sentence already increases your chance of getting the correct answer.

Strategies in Answering Paragraph Organization Tests

This is the summary of the last three posts about answering paragraph organization tests.

The **first part** discusses strategies on how to order random sentences into a coherent paragraph. The sentences that can easily be seen in terms of order is the first sentence which usually introduces the topic and the last sentences which is the conclusion. As for the second sentence, it usually explains, supports, or elucidates the first sentence.

The **second part** discusses what makes paragraph organization easy. Although in the first part, ordering the sentence are somewhat difficult, this is complemented by the available choices. Reading all the sentences and looking at the options will likely narrow the choices for the correct answer. This gives you a higher probability of getting the correct answer.

The **third part** concludes the advantages of paragraph organization over other multiple choice questions. Knowing the beginning of the paragraph, which is likely to happen, reduces the chance of getting the wrong answer. In this part, I have also given some personal tips on how I answer questions. Note that these tips are based on my own experience and may or may NOT work with others. There is no harm in trying though, but you have to practice it first before using in actual examinations.

➢ANALYTICAL

How to Answer Word Analogy Questions Part 2

This is the second post in the **Word Analogy Tutorial Series**. In the **previous post**, we have discussed a double word analogy question. In this post, we are going to look at a single word analogy question and discuss how to answer it. In single word analogy, we are just looking for one word, not a pair of words. Consider the example below.

Question: **[____ : launch] [breakfast:lunch]**
Choices:

a. sandwich

b. dinner

c. eggs

d. countdown

Analysis

One of the best ways to answer analogy questions is to know the meaning of the words. If you do not know the meaning of the words, then it is likely that you will just have to rely on your best guess. That is why it is important to always **improve your vocabulary** by reading, writing, etc.

In the example above, I think we all understand the meaning of lunch and breakfast, so we only need to know about "launch." Well, launch is not really a difficult word. Companies launch products, scientist launch rockets. So, to launch means to start something (start a product, start a rocket so it can take off). Now, that we know the meaning of all the words, we analyze what goes into the blank.

First, it is quite tempting to choose *sandwich* and *eggs* because they are related to breakfast and lunch. However, choosing sandwich or eggs would mean that "launch" should be strongly related to them as breakfast relates to lunch. Looking closely, it is easy to see there is none. So, it seems that (a) and (c) are not the correct answer. Therefore, we are left with *dinner* (b) and *countdown* (d).

Let's try putting *dinner* in the blank. Well, it has some connection. You probably have dinner if you launch something. But then, breakfast and lunch are both 'eating events' and so there is no connection between the launch and dinner. Launch is not necessarily an eating event!

Remember that we are looking for the relationship between [breakfast:lunch] that is the same as the relationship of [dinner:launch] and looks like we can't find any. Can you think of one?

Now let's put *countdown*. Can you see the relationship between countdown and launch?

[*countdown*: launch] [breakfast:lunch]

When there is a launch, there is usually a countdown. And when does the countdown happens? A *countdown* **happens before** a *launch*. Is this relationship connected to [breakfast:lunch]. Oh yes, *breakfast* **happens before** *lunch*. So, we can see that the relationship between the pair words is the order in which they happen.

Hmmm, you can probably argue that [dinner:launch] is also a correct answer since it is possible to have dinner before a launch. Yes, it's possible, but it's not absolute. Meaning, we can have dinner after a launch. But breakfast ALWAYS **happens before** dinner. And nobody does a countdown after a launch — it's ALWAYS before. So, the correct answer is (d) for countdown.

There are many relationships that can exist between pair of words, so it is impossible to discuss them all. However, in the **next post**, we are going to discuss the common relationships that are easy to spot. So share this post and keep posted.

How to Answer Word Analogy Questions Part 3

This is the third and the last part of the **Word Analogy Tutorial Series**. In the **first part** and **second part** of this word analogy series, I have given you examples on how to analyze word analogy (or verbal analogy) questions. Those examples are, of course, not enough as there are many relationships that can exist between and among words. In this post, we are going to look at some of the most common word analogy relationships.

1. Definition

brother: sibling:: mother: ______

a. father
b. sister
c. parent
d. daughter

Correct Answer: c. parent

Explanation: A brother is a sibling by definition. A mother is a parent by definition.

2. Synonyms or Sameness

pity: compassion:: grief:_____

a. sorrow
b. comfort
c. blunt
d. regret

Correct Answer: a. sorrow

Explanation: Pity and compassion are synonyms. Grief and sorrow are synonyms.

3. Antonyms or Oppositeness

attentive:careless::firm: _____

a. stiff
b. flexible
c. substantial
d. alert

Correct Answer: flexible

Explanation: Attentive and careless are antonyms. Firm and flexible are also antonyms.

4. Cause and Effect

rain: flood::smoke: _____

a. pollution
b. fire
c. inhale
d. heat

Correct Answer: a. pollution
Explanation: Rain is the cause of flood. Smoke is the cause of pollution.

5. Whole and Part

flower: petal: computer: _____

a. printer
b. keyboard
c. scanner
d. spreadsheet

Answer: b. keyboard
Explanation: A petal is an essential part of a flower. A keyboard is an essential part of a computer. Other parts such as pinter and scanner are just accessories. A spreadsheet is a software and is not included as a hardware.

6. Objects and Their Functions

This is the example in **first part** of this series.

7. Objects and Their Characteristics

ball:round::paper: ____

a. light
b. white

c. rectangular

d. smooth

Answer: c. rectangular

Explanation: Although all the choices are characteristics of a paper, round describes shape.

8. Objects and Their Groups

fish:school::cow: _____

a. flock

b. herd

c. pack

d. pride

Answer: herd

Explanation: School is the term used to call a group of fish and herd is used for cows, antelopes, etc. The term pack is used for wolves, coyote, etc. while pride is used for lions.

Aside from the examples above, you should also watch out for things that go together (spoon:fork::bread:butter), problem and solution (thirsty:drink::tired:rest), verb tenses (walk:walked::seek:sought), and performer and their actions (singer:sings::actor:acts).

There are so many relationships that could exist between words, so it is important that you analyze the words carefully before answering word analogy questions. That's it. In the next post, we are going to summarize what we have learned in this series.

The Word Analogy Tutorial Series

In the previous three posts, we have discussed methods and strategies on answering word analogy questions. In this post, we are going to summarize what we have learned.

This is the **Word Analogy Tutorial Series**.

How to Answer Word Analogy Questions Part 1 uses double word analogy question as an example to answer basic questions on verbal analogy. It uses the strategy of putting the words in sentences in order to see the relationship easily. It also teaches a strategy like looking at the words if they are noun, verb, etc. to identify the answer.

How to Answer Word Analogy Questions Part 2 uses single word analogy as example and discusses in-depth analysis in finding the relationships between words. This includes step-by-step analogy and elimination.

How to Answer Word Analogy Questions Part 3 lists the most common relationships that usually appear in single word analogy and double word analogy exams. This includes synonyms, antonyms, cause and effect, whole and part, and many others.

That's it! With all the knowledge that you have gained from this series, I hope that you would be able to use them in answering questions in Civil Service Examinations and other examinations as well. Good luck.

Civil Service Exam Vocabulary Review 1

As in any examination, of course, the sets of questions change, so it is impossible to know what are the vocabulary words that will appear in the next Civil Service Exam. However, in vocabulary exams, there are also some word favorites. In this series ofvocabulary review, I am going to discuss some of the words that usually appear inexaminations. I suggest that you do not just memorize them. To remember them better, write your own sentence using these words and speak them whenever you have a chance.

Civil Service Exam Vocabulary Review

1. alleviate – to make easier to endure, to lessen

Sample Sentence: After this operation, take three tablets everyday to *alleviate* the pain.

2. amicable – friendly, showing good will

Sample Sentence: After so many years of battle in courts, the two families finally agreed on an *amicable* settlement.

3. benevolent – showing good will, desiring to help others

Sample Sentence: The first time I saw him, I knew that he was a *benevolent* person.

4. inevitable – sure to happen, unavoidable, certain

Sample Sentence: Death is *inevitable*. Taxes too.

5. scrutinize – to examine in detail

Sample Sentence: The guy who entered the building was suspicious, so the police *scrutinized*belongings.

6. tenacious – persistent, stubborn, obstinate, retentive

Sample Sentences

- That guy has been courting me for 5 years. He was so *tenacious* that I finaly fell in love with him.
- I have quite a *tenacious* memory. I can't forget a face.

7. disdain – to scorn, to treat with contempt, despise

After the death of his wife, he was offered a large amount of money by his boss, but he refused it with *disdain*.

8. evident – plain or clear to the sight or understanding

Sample Sentence: Even when he was young, it was *evident* that he will do great things.

9. frugal – not wasteful, thrifty

Sample Sentence: All his life, he had been *frugal* in his expenses. No wonder he has a lot of money.

10. superficial – lacking in content, shallow

Sample Sentence: Sometimes, his writings seems *superficial*, but when you look at them deeply, you will see the real meaning.

I hope you have learned something in this post. We will have some more words in the next posts.

Civil Service Exam Vocabulary Review 2

Vocabulary is one of the main parts of theCivil Service Examination. In this Civil Service Exam Vocabulary Review series of posts, I am going to list the most popular vocabulary words used in examinations and their meaning. I have also written at least one sample sentence for each word.

Commonly Used Words in Examinations

1. **abdicate –** to give up a throne or right, power, or claim in a formal manner.
Sample Sentence: Edward VIII *abdicated* his throne to be with the woman he loved.
2. **candor** – the quality of being frank, sincere, and honest
Sample sentence: Everyone was surprised by the *candor* of his speech because he usually evades questions.
3. **connive –** to conspire; to secretly help someone do something dishonest
Sample Sentence: Two thieves *connived* with a bank employee and robbed the bank before it closed yesterday.
4. **eloquent –** showing the ability to use the language clearly and effectively
Sample Sentence: Ninoy Aquino's eloquent and lively speeches is one of the reasons why he always spoke at the end of every event.
5. **forsake** – to quit or abandon entirely
Sample Sentence: Disability should not be the reason why you should *forsake* your dreams.
6. **inhibit –** to restrain or prohibit; to keep someone from doing something
Sample: The presence of CCTV did not *inhibit* the driver from beating the red light.

7. **modicum** – small amount or portion

Sample Sentence: He did not even show a *modicum* of guilt after saying lots of bad things to everyone.

8. **nuance** – small difference

Sample Sentence: The *nuances* in our beliefs shouldn't be a hindrance to keep us united.

9. **penchant –** a strong liking or inclination

Sample Sentence: I have a *penchant* for classical music; Pavarotti was one of my favorite classical singers.

10. **zenith –** the highest point or state

Sample Sentence: The *zenith* of the human mind is around at age 30. After that, it begins to decline.

***

Note that I have simplified the meaning of some words above from the references listed below. I have personally created the sample sentences though.

Civil Service Exam Vocabulary Review 3

This is the third of the Civil Service Exam Vocabulary review series. In this series, I am going to discuss the most popular words used in vocabulary examinations. Hopefully, some of these words will come up the next time you take the Civil Service Examination.

Aside from memorizing these words, I suggest that you use them in sentences. Write sentences using these words. If you will memorize these words without using them, it is likely that you will forget them sooner or later.

1. **boisterous –** noisy, rough, and energetic
Sample Sentence: The sound of her *boisterous* laughter was disturbing everyone.
2. **camaraderie** – a feeling of friendship to a group; good-fellowship
Sample Sentence: One of the most valuable things you learn in school aside from academic lessons is *camaraderie*.
3. **conundrum** – a difficult problem having only a guess as an answer; a riddle or puzzle
Sample Sentence: The origin of the universe has always been a *conundrum* to humankind.
4. **divergent –** to follow to different directions; deviates from the standard
Sample Sentence: My business partners and I have *divergent* ideas on how to run our company.
5. **foster –** to help grow or develop
Sample Sentence: The UN has helped *foster* peace and equality among all the nations in the world.

6. **intuitive** – having to know or understand by means of feelings (and not by facts)
Sample Sentence: Most people have the *intuitive* knowledge of right and wrong.
7. **mundane** – worldly, common, ordinary
Sample Sentence: His *mundane* desires make people think that he is a shallow person.
8. **opulent –** expensive and luxurious
Sample Sentence: Before he went bankrupt, he had an *opulent* lifestyle.
9. **procrastinate –** to delay to do something because it is boring; to delay doing something out of laziness

Sample Sentence: Even if you *procrastinate*, you will still make a decision, so I suggest that you make up your mind once and for all.

10. **spontaneous** – done or said without lots of thoughts and planning

Sample Sentence: My friend has a habit of *spontaneous* visiting even in late hours of the evening.

Please stay tuned for more vocabulary review.

Civil Service Exam Vocabulary Review 4

This is the fourth Civil Service ExamVocabulary Review. This review series consists of the most commonly used words in vocabulary exams both local an international. I have simplified their meanings and use them in a sentence. Although you can memorize their meanings, it is more effective if you use them in a sentence whenever possible and appropriate. Get a notebook and write your sentence or use the comment box below.

1. **arid** – with very little rain or moisture, extremely dry

*In the recent climate change, in some countries, the farmers could not grown anything on the lands that had become **arid**. In some countries, though, it has been raining and flooding a lot.*

2. **conformist** – a person who complies or follows tradition and usual practices (manners, religion, dress, etc).

*Being a **conformist** is good most of the time. But sometimes you must learn how to take risks.*

3. **deleterious –** harmful, can cause injury

The rapid increase of factories and cards made *the* air more ***deleterious** than ever.*

4. **elucidate –** clarify, explain, make clear

*Maybe he will **elucidate** his actions later. Don't judge him just yet.*

5. **ephemeral** – short lived, lasting for a short time

*Sometimes, the **ephemeral** fame and the lack of investment of movie stars make them poor later in their life.*

6. **intrepid** – brave, courageous, fearless

*Christopher Columbus' **intrepid** explorations made him discover America.*

7. **jeopardy** – exposure to harm; danger of injury, loss, death

*Nobody ever wins in wars. They just put everyone in **jeopardy**: even children.*

8. **prudent –** careful, exercising good judgement, cautious

*Even though his boss yelled at him, he gave a **prudent** reply.*

9. **subtle –** not obvious, mysterious, something that requires discernment

*My absence during the meeting was a **subtle** protest to the management's new implemented rules.*

10. **tactful** – kind, considerate to others, someone who thinks of other people's feelings

*He is a very **tactful** young man. It's hard to find such nowadays.*

Stay tuned. I am listing more words as soon as possible.

Civil Service Exam Vocabulary Review 5

This is the fifth of the Civil Service ExamVocabulary Series. This series provides words that are usually used in examinations. Included in this reviewer are the meaning of the words as well as a sample sentence using these words. The complete list of this series can be found in the **English** page.

Civil Service Exam Vocabulary Review 5

1. affable – friendly, easy to talk to

Sample Sentence: He is a very good person, always welcoming, always *affable*.

2. docile – easily taught, obedient, ready to learn

Sample Sentences

- I am glad I have found a *docile* dog.
- His students are *docile* and hardworking.

3. enthrall – to captivate or charm, to hold the attention of someone by being exciting or interesting

Sample Sentence: I was entirely *enthralled* by listening to her song that I lost track of time.

4. fraud – a deception or trickery

Sample Sentence: A lot of people fall into different online *fraud* and lose a lot of money.

5. lucrative – money making, profitable

Sample Sentence: Online selling has become a *lucrative* business since the invention of social media websites such as Facebook.

6. reclusive – a person who lives alone and withdrawn from society

After reading these examples, use them in your own sentences to remember them better

Civil Service Exam Vocabulary Review 6

This is the sixth part of the Civil Service Exam Vocabulary Series. This series provides words that are usually used in examinations both local and abroad. Included in this reviewer are the meaning of the words as well as a sample sentence using these words. The complete list of this series can be found in the **English** page.

Civil Service Exam Vocabulary Review 6

1. **brusque –** unfriendly, rude, rough
Sample Sentence: Even some educated people cannot hide their *brusque* manner sometimes.

2. **deprive –** to take something away, to withhold something from the enjoyment or possession
Sample Sentence: Many parents deprive their kids of junk foods until the age of 10.

3. **exploit –** a notable deed or heroic act (noun), to utilize especially for profit (verb), to use selfishly for one's own gain (verb).
Sample Sentences

- The *exploits* of Achilles is known to many people, not just the Greeks.
- Many companies *exploit* their employees because they know that it is hard to find a job nowadays.

4. **endeavor –** to attempt with effort, to strive to achieve or reach
Sample Sentence: He *endeavored* to learn Kanji characters for 3 years in order to read Japanese manga.

5. **grumble –** to complain about something, to utter complain in a low voice
Sample Sentence: She always *grumbles* when you tell her to do something.

6. **insatiable** – incapable of being satisfied
Sample Sentence: His *insatiable* appetite for power and money made him sacrifice all of his loved ones.
7. **meddle –** to involve in a matter without right or invitation, to change or handle something in a way that is unwanted or harmful
Sample Sentence: If people just don't *meddle* with other people's business, this world will be a better place to live in.
8. **obese** – very fat, overweight
Sample Sentence: Most of the things that can be bought in fast food chains today can make you *obese*.
9. **presumptuous –** to be confident in a way that is rude
Sample Sentence: He does not care who is he talking to; he is always *presumptuous*.
10. **tantalize –** to tempt, to cause someone to feel interest or excitement
Sample Sentence: The President *tantalized* him to a position in the board and now he has abandoned his principles.

That's all for now. Practice by using the words in sentences. Use the comment box below and I'll see if the sentence is correct.

Civil Service Exam Vocabulary Review 7

This is the seventh part of the Civil Service Vocabulary Review series of **Ph Civil Service Reviewer**. In this series, we discuss the words that commonly appear in English vocabulary examinations. The other parts of this series can be found in this blog's English page.

Civil Service Exam Vocabulary Review 7

1. **diligent –** hardworking; constant in effort to accomplish something

Sample Sentence: That student is very *diligent*. He always submits his homework on time and does extra work.

2. **emulate** – to try to equal or excel; to surpass

Sample Sentence: Many tenor singers nowadays are trying to *emulate* Pavarotti, but I think we only have one Pavarotti in this generation.

3. **haughty –** proud, snobbish, arrogant

Sample Sentence: His *haughty* attitude makes everyone hate him.

4. **incompatible** – not to be able to exist together without conflict; not able to be used together

Sample Sentences

- Many stories tells how *incompatible* a prince and a pauper, but who cares about status anyway.
- This plug is *incompatible* with the socket. Kindly buy another one.

5. **novice** – a person who has just started doing something

Sample Sentence: For *novice* boxers, head gears are needed.

6. **orator** – an eloquent public speaker; a person who can make speeches and is very good in making them

Sample Sentence: Martin Luther King Jr. was one of the greatest orators of his time.

7. **prosperity –** the condition of being successful especially financially

Sample Sentence: "I wish you good luck, happiness, and *prosperity* in your newfound life."

8. **resilient –** recovering readily from illness, depression, etc.

Sample Sentence: He was *resilient* enough to cope up with the tragedy that happened to his family.

9. **submissive –** obedient

Sample Sentence: During the feudal times in Japan, the women were expected to be totally submissive to their husband.

10. **substantiate** – to establish proof or complete evidence

Sample Sentence: There were many allegations of corruption against him, but no one was able to *substantiate* their claims.

That's all for now, see you in the next vocabulary review.

Civil Service Exam Vocabulary Review 8

This is the eighth part of the Civil Service Vocabulary Review series of **Ph Civil Service Reviewer**. In this series, we discuss the words that commonly appear inEnglish vocabulary examinations. The other parts of this series can be found in this blog's English page.

Civil Service Exam Vocabulary Review 8

1. **bias –** the action of supporting/opposing a particular person or thing in an unfair way.

Sample Sentence: Good judges must be objective about their decisions and must uphold the law without *bias*.

2. **contradiction –** a disagreement between two things which means that both cannot be true; inconsistency

Sample Sentence: How can anybody believe him? His statements are full of *contradictions*.

3. **debris –** the remains of something broken or destroyed; rubble, ruins

Sample Sentence: After the earthquake, two cars were hit by falling *debris* from a 21-story building.

4. **elicit –** to draw or bring out, to get or produce something (especially information)
Sample Sentence: If you ask questions, you will surely *elicit* answers.
5. **mediocre –** ordinary, of moderate quality
Sample Sentence: In our country, there are many blockbuster movies whose quality is less than *mediocre*.
6. **mendicant –** someone who asks people whom they do not know for money
Sample Sentence: I am a spiritual person, but I do not really favor *mendicants* on the streets preaching with their donation pouches displayed beside them.

7. **nuisance** – someone or something that is annoying
Sample Sentence: In my country, when you do not have money and you run for elections, they call you a *nuisance* candidate.
8. **provocative –** serving to provoke, incite, or excite
Sample Sentences

- In hostage situations, the hostages are advised to refrain from making *provocative*actions that will anger the hostage takers.
- Some believe that one of the causes of rape are the women's *provocative* outfits.

9. **redundant –** unnecessary repetition
Sample Sentence: "Kindly edit this article. Many of the sentences are *redundant*."
10. **sporadic –** happening sometimes, not regularly
Sample Sentence: Ten kids died this month because of the *sporadic* cases of a new strain of influenza.

Civil Service Exam Vocabulary Review 9

This is the ninth part of the Civil Service Vocabulary Review series of **Ph Civil Service Reviewer**. In this series, we discuss the words that commonly appear inEnglish vocabulary examinations. The other parts of this series can be found in this blog's English page.

Civil Service Exam Vocabulary Review 9

1. **affliction –** state of pain, distress, or grief

Sample Sentence: We must not abandon our brothers and sisters in the time of their *affliction*.

2. **cajole –** to persuade someone to do something or to give you something by making promises or saying nice things

Sample Sentence: Anna, the favorite daughter in the family, *cajoled* her father to buy her a new computer.

3. **drought –** a long period of dry weather

Sample Sentence: Many crops were damaged because of the 5-month *drought*.

4. **dumbfounded –** so shocked that you cannot speech

Sample Sentence: Maria was *dumbfounded* when her boyfriend proposed to her in front of so many people.

5. **extol –** to praise highly

Sample Sentence: His first movie was *extolled* by critics for its profound originality and musical scoring.

6. **illicit –** not legally authorized; not allowed

Sample: A teacher and student relationship in the same school is an *illicit* affair. -_-

7. **harangue –** a scolding or a long verbal attack; a long and passionate speech

Sample Sentence: He delivered his speech in a *harangue*: full of passion, vehemence, and discourse.

8. **reverberate –** to continue in a series of repeated sounds

Sample Sentence: The sound of his voice *reverberates* across the room.

9. **succumb –** to give away to a superior force; to yield to disease, or wounds (die)

Sample Sentence: After 3 years of suffering, she finally *succumbed* to cancer.

10. **vehement –** strongly emotional, passionate, zealous

Sample Sentence: Despite the vehement protest of the employees, the company did not give in to increasing their salary or other benefits.

That's all for now, I hope you have learned something. If you have any questions, just use the comment box below.

Civil Service Exam Vocabulary Review 10

This is the tenth part of the Civil Service Vocabulary Review series of **Ph Civil Service Reviewer**. In this series, we discuss the words that commonly appear in English vocabulary examinations. The other parts of this series can be found in this blog's English page.

The Civil Service Exam Vocabulary Review 10

1. **belligerent –** angry and aggressive; feeling or showing readiness to fight

Sample Sentence: I don't like the *belligerent* tone of his speech.

2. **convergent –** tending to come together; tending to move toward one point

Sample Sentence: Most of the speakers have *convergent* views toward the use of contraceptives.

3. **delusion** – belief in something that is not true

Sample Sentence: I think you are living in a *delusion* that you are better than everyone else.

4. **gullible** – too willing to believe everything that other people say; easy to deceive or trick

Sample Sentence: I think the voters now are wiser. Unlike before, many people are so *gullible*that they believe almost every promise of any politician.

5. **juxtapose** – to put things that are not similar next to each other

Sample Sentence: The exhibit *juxtaposed* M.C. Escher's earlier artworks and his later masterpieces.

6. **maxim** – a short statement about a general truth; a saying

Sample Sentence: It is a common *maxim* that "a book should not be judged by its cover."

7. **nullify** – to cause something to have no value or effect; to declare to be legally void

Sample Sentence: The court did *nullify* their marriage because it was discovered that his husband was married to another woman.

8. **odyssey** – a long journey which is usually marked by notable experiences, adventures, and hardships

Sample Sentence: Billy Beane has indeed an exciting *odyssey* before he became a well-known baseball manager.

9. **pacify** – to bring or restore to a state of peace; to cause someone who is angry to be calm or quite

Sample Sentence: A charming airline attendant came out to *pacify* the crowd of people who were complaining about the delayed flight.

10. **recant –** to announce in public that the past beliefs or statements were wrong and that you do not agree with them anymore
Sample Sentence: After too much pressure from his family, he recanted his former statements favoring the law on abortion.

That's it for this post, I hope that you have learned something.

Civil Service Exam Vocabulary Review 11

This is the eleventh part of the Civil Service Vocabulary Review series of **Ph Civil Service Reviewer**. In this series, we discuss the words that commonly appear in English vocabulary examinations. The other parts of this series can be found in this blog's English page.
Most of the meanings of the words below were taken from Dictionary.com and Merriam Webster. I have simplified some of them for ease of reading.
Vocabulary Review for the Civil Service Exam
1. appease – to ease, to calm, to satisfy

To *appease* the angry passengers due to a 5-hour flight delay, the airline gave them free meals.
2. coerce – to force someone to do something by threat, intimidation, or authority
He was just *coerced* to sign the document, so the court did not honor the document as evidence.
3. confidante – somebody entrusted with secrets

Jean is my only *confidante*. She is the only one who understands my problems.
4. demure – shy, modest, reserved

She was not chosen by the panel because she was so *demure* during the interview. She didn't look confident.

5. erudite – characterized by great knowledge; learned or scholarly

After one audience asked an interesting question, the speaker gave a 10-minute *erudite*lecture about it.

6. fabricate – to make or build; to fake or forge

The parts of many products are *fabricated* in first world countries but they are usually assembled in developing countries.

They try to *fabricate* a story in order to sell their magazine.

7. jubilant – showing great happiness, joy, or triumph

The *jubilant* coach ran around and shout loudly after his team won the championship game.

8. nadir – lowest point, point of greatest adversity or despair

The *nadir* of his career was the time when he was caught having an affair with his cousin.

9. parody – a piece of writing, music, etc. that imitates someone else in a funny or an amusing way

The students made a *parody* of how their teacher teach in a play.

10. vex – to annoy or irritate

My boyfriend keep on *vexing* me about getting married. I'm too young for that!

➢NUMERICAL

How to Get the Greatest Common Factor of Numbers

The numbers that can divide an integer is called its factor or divisor. For example, the factors of 4 are 1, 2, and 4 because these are the numbers that divide 4 without having a remainder. Another example is 6 which has factors 1, 2, 3, and 6. It is clear that each number has always 1 and itself as factors. Note that in this discussion, when I say number, I mean positive integer.

If we select more than one number, we can observe that they have common factors (just like having common multiples). Let's have the following examples.

How to Get the Greatest Common Factor of Numbers

Example 1**:** What are the common factors of 12 and 18?

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 18: 1, 2, 3, 6, 9, 18

If we examine the factors of 12 and 18, we see that there are 4 common factors: 1, 2, 3 and 6. Among the factors, 6 is the largest. Therefore, we say that 6 is the **greatest common factor** (GCF) or **greatest common divisor** (GCD) of 12 and 18. **Example 2 :** Find the GCF of 20, 32, 28.

Factors of 20: 1, 2, 3, 4, 5, 10, 20

Factors of 32: 1, 2, 4, 8, 16, 32

Factors of 28: 1, 2, 4, 7, 14, 28

As we can see, the common factors of 20, 32, and 28 are 1, 2, and 4. The GCD or GCF of the three numbers is 4.

Another way to get the greatest common factor of numbers is to write their prime factorization. Prime factorization is the process of expressing a number as product of prime numbers. A prime number is a number which is only divisible by 1 and itself (read**Introduction to Prime Numbers** if you don't know what is a prime number). The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

We will use the examples above and use prime factorization in order to get their greatest common factor.

Example 3: Find the GCF of 12 and 18 using prime factorization.

Prime factorization of 12: $2 \times 2 \times 3$

Prime Factorization of 18: $2 \times 3 \times 3$

Now to get the greatest common factor, we multiply the common factors to both numbers. The common factors to both are 2 and 3, therefore, the greatest common factor of 12 and 18 is $2 \times 3 = 6$.

Example 4: Find the GCF of 12 and 18 using prime factorization.

Prime factorization of 20: $2 \times 2 \times 5$

Prime factorization of 32: $2 \times 2 \times 2 \times 2$

Prime factorization of 28: $2 \times 2 \times 2$

In this example, 2 and 2 are common to all the three numbers, so the GCD or GCD of these three numbers is 2×2 which is equal to 4.

The difference between the two methods is that in the first method, you list all the factors and find the largest number. In the second method, you list the prime factorization and the multiply the factors that are common to all numbers.

What's the use of greatest common factor?

Well, GCF are used a lot in mathematics, but in the Civil Service Exam, you will use it when you reduce fractions to lowest terms. For example, your final answer is

$$\frac{12}{18}$$

and $\frac{12}{18}$ is not on the choices. Then, you know that you have to get the greatest common factor of 12 and 18 and divide both the numerator and denominator by it. So, the answer is

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

How to Get the Least Common Multiple of Numbers

In mathematics, a multiple is a product of any number and an integer. The numbers 16, -48 and 72 are multiples of 8 because 8 x 2 = 16, 8 x -3 = -48 and 8 x 9 = 72. Similarly, the first five positive multiples of 7 are the following:

7, 14, 21, 28, 35.

In this post, we will particularly talk about positive integers and positive multiples. This is in preparation for the discussions on addition and subtraction of fractions.

We can always find a common multiple given two or more numbers. For example, if we list all the positive multiples of 2 and 3, we have

2, 4, **6**, 8, 10, **12**, 14, 16, **18**, 20

and

3, **6**, 9, **12**, 15, **18**, 21, 24, 27, 30.

As we can see, in the list, 6, 12 and 18 are common multiples of 2 and 3. If we continue further, there are still other multiples, and in fact, we will never run out of multiples.

Can you predict the next five multiples of 2 and 3 without listing?

The most important among the multiples is the **least common multiple**. The least common multiple is the smallest among all the multiples. Clearly, the least common multiple of 2 and 3 is 6. Here are some examples.

*Example 1***:** Find the least common multiple of 3 and 5

Multiples of 3: 3, 6, 9. 12, **15,** 18

Multiples of 5: 5, 10, **15**, 20, 25,30

As we can see, **15** appeared as the first common multiple, so 15 is the least common multiple of 3 and 5.

Example 2: Find the least common multiple of 3, 4, and 6.

In this example, we find the least multiple that are common to the three numbers.

Multiples of 3: 3, 6, 9, **12**, 15

Multiples of 4: 4, 8, **12**, 16, 20

Multiples of 6: 6, **12**, 18, 24, 30

So, the least common multiple of 3, 4, and 6 is **12**.

Example 3: Find the least common multiple of 3, 8 and 12.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, **24**

Multiples of 4: 4, 8, 12, 16, 20, **24**,

Mulitples of 12: 12, **24**, 36, 48, 60

So, the least common multiple of 3, 4 and 6 is **24**.

In the next part of this **series**, we will discuss about How to Add Fractions.

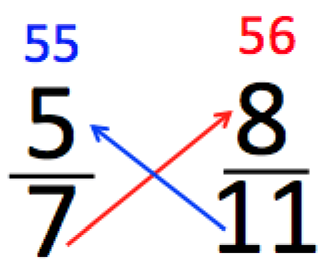
3 Helpful Strategies in Comparing Fractions

There are questions in Civil Service Examinations that may require you to compare fractions or even arrange them in order. In this post, I am going to teach you three strategies in comparing fractions.

Strategy 1: Cross Multiplication
Which is greater, 5/7 or 8/11?

If only two fractions are compared, the easiest way is to cross multiply. However, take note of the following:

1.) You multiply the denominator of the fraction to the numerator of the other fraction.
2.) Place the product above the numerator.



The larger product is the larger fraction. As shown in the example above, 56 is larger than 55, therefore, 8/11 is larger than 5/7.

Strategy 2: Converting to Similar Fractions
Sample Question: Which is the largest: 13/16, 5/8, 3/4?

We can get the least common denominators of these fractions. Now, the LCM of these denominators is 16. So, we convert everything to fractions whose denominators is 16.

To convert 5/8 to something over 16, we divide 16 by 8 then multiply by 5 which gives us 10. So, 5/8 is equal to 10/16.

To convert ¾ to 16, we divide 16 by 4, then multiply by 3. This gives us 12.

So, we have converted all fractions to fractions whose denominator is 16.

We have 13/16, 10/16, and 12/16. Obviously, the largest is 13/16. Note that using thisstrategy does not only tell us which is the largest. In fact, we can order the fractions from smallest to largest or vice versa.

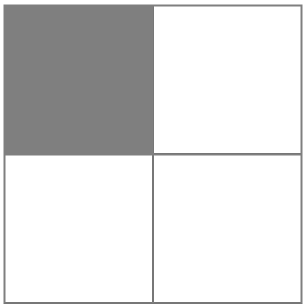
Strategy 3: Converting to Decimals
Which is larger: 2/5, 3/4, or 7/10.

We can convert them to decimal by manually dividing the numerator by the denominator (watch video above). The equivalent of 2/5 = 0.4, 3/4 is 0.75 and 7/10 = 70.
The strategies above can be used effectively by looking at the fractions. If two fractions are compared, use Strategy 1. If the numerators are not very large, you can use strategy 2 or 3.

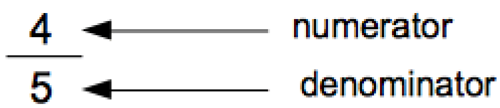
A Gentle Introduction to Fractions

Fractions is one of the mathematics topics that many people have difficulty with. However, unfortunately, it is also one of the most important topics that must be mastered. This is because examination questions in mathematics always include fractions. For example, in the Civil Service Review Numerical Reasoning tests, fractions appear in almost every test: basic arithmetic, number sequences, equations and problem solving.
In this post, we are going to discuss the basics about fractions particularly about the terminologies used. Of course, you don't really have to memorize them now, but you can refer to this post in the following discussions. In the future discussions, you will use the vocabulary that you have learned here.

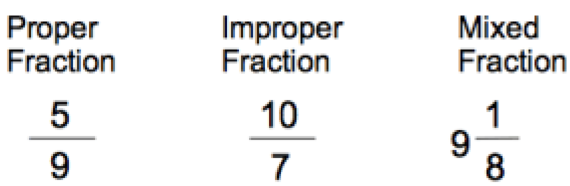
Introduction to Fractions
In layman's language, a fraction is really a **part of a whole**. In the figure below, the part which is shaded is one out of four, so we say that ¼ of the square is shaded. We can also say that three out of four or ¾ of the square is not shaded. We can also say that adding ¼ and ¾ equals one whole.



Fractions can also be a **subset of a set.** If 3 out of 10 students are girls, then we say that 3/10 of the students are girls. A fraction could also mean division. For example, wen we say 7/10, we can also mean, 7 divided by 10.



A fraction is composed of a **numerator**, the number above the bar, and a **denominator**, the number below the bar. . Fractions whose numerator are less than the denominator are called **proper fractions**. Fractions whose numerator are greater than the numerator are called **improper fractions**. Improper fractions can be converted to **mixed fractions** or fractions that contain whole numbers.



Just like other numbers, we can perform operations on fractions. In the next four posts, we will be discussing the different operations on fractions. We will learn how to add, subtract, multiply, and divide fractions.

Exercises on Converting Fractions to Lowest Terms

In the previous post, we learned how toconvert fractions to lowest terms. In this post, I have created 15 exercises for you to practice.

Convert the following fractions to lowest terms. In case the fraction is improper,convert it to mixed form. Be sure that thefraction part is in lowest terms.

1. $\frac{12}{15}$

2. $\frac{18}{24}$

3. $\frac{21}{49}$

4. $\frac{56}{72}$

5. $\frac{26}{65}$

6. $\frac{18}{32}$

7. $\frac{38}{95}$

8. $\frac{32}{12}$

9. $\frac{16}{84}$

10. $\frac{39}{24}$

11. $\frac{15}{45}$

12. $\frac{51}{85}$

13. $\frac{18}{54}$

14. $\frac{35}{49}$

15. $\frac{74}{24}$

I will be posting the solutions and answers to these problems in the next article.

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11. $\frac{15}{45}$

12. $\frac{51}{85}$

13. $\frac{18}{54}$

14. $\frac{35}{49}$

15. $\frac{74}{24}$

I will be posting the solutions and answers to these problems in the next article.

Solution to the Exercises on Reducing Fractions to Lowest Terms

Below are the complete solutions and answers to the exercises on **reducing fractions to lowest terms**. I will not give any tips or methods of shortcuts on doing this because teaching you shortcuts will give you problems in case you forget them. The best thing that you can do is to solve as many related problems as you can and develop shortcuts that work for you. Each person has his own preference in solving procedural problems such as these, so it is important that you discover what's best for you.

For converting improper fractions to mixed form, I will discuss it in a separate post. Try to see the solutions below and see if you can use these solutions to develop your own method. Honestly, the three examples below on converting improper fractions to mixed form should be enough to teach you how to do it yourself. 🙂

Reducing Fractions to Lowest Terms

1. $\frac{12}{15}$

Solution

$$\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

2. $\frac{18}{24}$

Solution

$$\frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

3. $\frac{21}{49}$

Solution

$$\frac{21 \div 7}{49 \div 7} = \frac{3}{7}$$

4. $\frac{56}{72}$

Solution

$$\frac{56 \div 8}{72 \div 8} = \frac{7}{9}$$

5. $\frac{26}{65}$

Solution

$$\frac{26 \div 13}{65 \div 13} = \frac{2}{5}$$

6. $\frac{18}{32}$

Solution

$$\frac{18 \div 2}{32 \div 2} = \frac{9}{16}$$

7. $\frac{38}{95}$

Solution

$$\frac{38 \div 19}{95 \div 19} = \frac{2}{5}$$

8. $\frac{32}{12}$

Solution

First, convert to lowest terms:

$$\frac{32 \div 4}{12 \div 4} = \frac{8}{3}$$

Second, convert to mixed form. Eight divided by 3 is 2 remainder 3. So 2 becomes the whole number, 2 (the remainder) becomes the numerator and 8 becomes the denominator. Therefore, the answer is $2\frac{2}{3}$.

9. $\frac{16}{84}$

Solution

$$\frac{16 \div 4}{84 \div 4} = \frac{4}{21}$$

10. $\frac{39}{24}$

Solution

First, reduce to lowest terms.

$$\frac{39 \div 3}{24 \div 3} = \frac{13}{8}$$

Second, convert the answer to mixed form. Thirteen divided by 8 is 1 remainder 5. So 1 becomes the whole number, 5 (the remainder) becomes the numerator of the fraction and 8 becomes the denominator. So the correct answer is $1\frac{5}{8}$.

11. $\frac{15}{45}$

Solution

$$\frac{15 \div 15}{45 \div 15} = \frac{1}{3}.$$

12. $\frac{51}{85}$

Solution

$$\frac{51 \div 17}{85 \div 17} = \frac{3}{5}$$

13. $\frac{18}{54}$

Solution

$$\frac{18 \div 18}{54 \div 18} = \frac{1}{3}$$

14. $\frac{35}{49}$

Solution

$$\frac{35 \div 7}{49 \div 7} = \frac{5}{7}$$

15. $\frac{74}{24}$

Solution

First, reduce to lowest terms.

$$\frac{74 \div 2}{24 \div 2} = \frac{37}{12}$$

Second, divide 37 by 12. The answer is 3 remainder 1. Now, 3 becomes the whole number, 1 becomes the numerator of the fraction, and 12 becomes the denominator. So, the correct answer is $3\frac{1}{12}$.

In the next post, we will be talking about multiplying and dividing fractions.

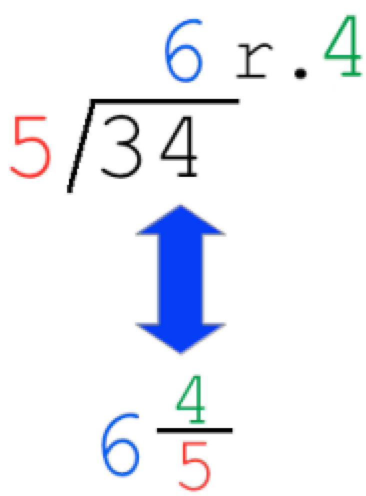
How to Convert Improper Fractions to Mixed Forms

In **Introduction to Functions**, we have learned about proper and improper fractions. A fraction whose numerator (the number above the fraction bar) is less than its denominator (the number below the fractionbar) is called a **proper fraction**. Therefore, $\frac{1}{3}$, $\frac{2}{5}$and $\frac{11}{20}$ are proper fractions.

On the other hand, a fraction whose numerator is greater than its denominator is called an **improper fraction**. Therefore thefractions $\frac{21}{7}$, $\frac{8}{3}$ and $\frac{67}{5}$ are improper fractions.

In the Civil Service Examinations, some fractions need to be converted from one form to another. For example, in answering a number series test, you might need to convert an improper fraction to mixed form in order to compare it to other fractions in mixed form. In this post, we learn this method: how to convert an improper fraction to mixed form.

In converting improper fractions to mixed form you will just have to divide the fraction, find its quotient and its remainder. Remember that the fraction $\frac{34}{5}$ also means 34 divided by 5.



When we divide 34 by 5, we call 5 the divisor. The quotient to this division is 6 with a remainder of 4. From the method, we can observe the following:

- The quotient 6 is the whole number on the mixed fraction.

- The divisor 5 is the denominator of the mixed fraction.
- The remainder 4 goes to the numerator of the mixed fraction.

Now, for the second example, let us convert $\frac{28}{3}$ into mixed fraction. If we divide 28 by 3, the divisor is 3, the quotient is 9 and the remainder is 1. Therefore, the equivalent of the improper fraction $\frac{28}{3}$ is

$$9\frac{1}{3}.$$

Practice Test on Converting Improper Fractions to Mixed Number

In the previous post, we have learned how to**convert improper fractions to mixed number** . Now, try the following exercises. Allthe answers must also be reduced to lowest terms. Good luck.

1.) 22/7

2.) 81/6

3.) 55/10

4.) 76/32

5.) 34/16

6.) 89/35

7.) 114/6

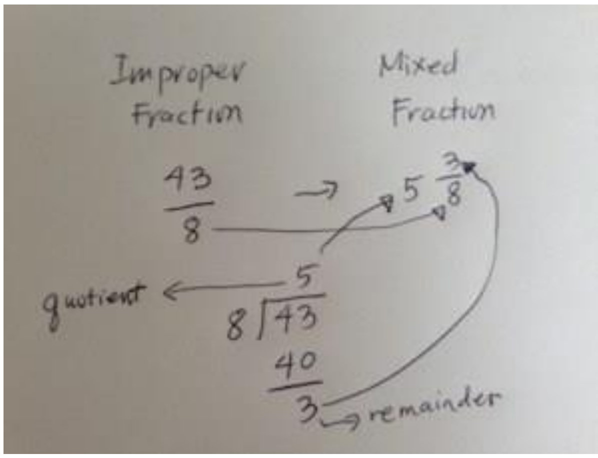
8.) 81/33

9).) 76/15

10.) 19/3

Answers to Practice Test on Converting Improper Fraction to Mixed Number

This is the complete solutions and answers to the **Practice Test on Converting Improper Fraction to Mixed Number**. As illustrated in the image below, the quotient in the division becomes the whole number in the mixed fraction, the remainder in the division becomes the numerator of the fraction part of mixed fraction, and the denominator from the improper fraction becomes the denominator of the fractional part of the mixed fraction.



In the solutions below, all answers were also **reduced to lowest terms**.

Solutions and Answers

1.) 22/7

Solution:

Quotient: 3

Remainder: 1

Denominator: 7

Answer: $3\frac{1}{7}$

2.) 81/6

Solution:

Quotient: 13
Remainder: 3
Denominator: 6

Answer: $13\frac{3}{6}$
Answer in lowest terms: $13\frac{1}{2}$

3.) 55/10

Solution:
Quotient: 5
Remainder: 5
Denominator: 10

Answer: $5\frac{5}{10}$
Answer in lowest terms: $5\frac{1}{2}$

4.) 76/32

Solution:
Quotient: 2
Remainder: 12
Denominator: 32

Answer: $2\frac{12}{32}$
Answer in lowest terms: $2\frac{3}{8}$

5.) 34/16

Solution:
Quotient: 2
Remainder: 2
Denominator: 16

Answer: $2\frac{2}{16}$
Answer in lowest terms: $2\frac{1}{8}$

6.) 89/35

Solution:
Quotient: 2
Remainder: 19
Denominator: 35

Answer: $2\frac{19}{35}$

7.) 114/6

Solution:
Quotient: 19
Remainder: 0
Denominator: 6

Answer: $19\frac{0}{6}$ or simply 19.

8.) 81/33

Solution:
Quotient: 2
Remainder: 15
Denominator: 33

Answer: $2\frac{15}{33}$ or $2\frac{5}{11}$.

9.) 76/15

Solution:
Quotient: 5
Remainder: 1
Denominator: 15

Answer: $5\frac{1}{15}$

10.) 19/3

Solution:
Quotient: 6
Remainder: 1
Denominator: 3

Answer: $6\frac{1}{3}$

How to Convert Mixed Fractions to Improper Fractions

We have already learned how to convert**improper fractions to mixed fractions**. In this post, we are going to learn how to convert mixed fractions to improper fractions. In converting mixed fractions to improper fractions, the denominator stays as it is. You only have to calculate for the numerator. To get the numerator of the improper fraction, multiply the denominator to the whole number and then add the numerator of the mixed fraction.
Let’s have three examples.
Example 1
Convert $6\dfrac{2}{5}$ to improper fraction.
Solution
Denominator: 5

Numerator: $5 \times 6 + 2$

Final Answer: $\dfrac{32}{5}$

Example 2
Convert $4\dfrac{2}{3}$ to improper fraction.
Solution
Denominator: 3

Numerator: $3 \times 4 + 2 = 14$

Final Answer: $\dfrac{14}{3}$

Example 3
Convert $8\frac{21}{28}$ to improper fraction.

Solution
We can reduce $\frac{21}{28}$ to $\frac{3}{4}$, so given fraction can be converted to $8\frac{3}{4}$. Now, we can now convert the mixed fraction to improper fraction.
Denominator: 4

Numerator: $4 \times 8 + 3 = 35$

Final Answer: $\frac{35}{4}$

From the pattern above, the fraction $c\dfrac{a}{b}$, where c is the whole number, a is the numerator and b is the denominator can be converted to the improper fraction

$$\frac{b \times c + a}{b}.$$

How to Divide Fractions

We have already discussed **addition** and **multiplication of fractions** and what we have left are subtraction and division. In this post, we learn how to divide fractions.
To divide fractions, we must get the reciprocal of the divisor. This is just the same as swapping the numerator and the denominator. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. After getting the reciprocal, just multiply the fractions.

Example 1
$\dfrac{3}{5} \div \dfrac{2}{3}$

Solution

First, we get the reciprocal of $\frac{2}{3}$, the divisor. This is $\frac{3}{2}$. Then, we multiply the fractions.

$$\frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

Answer: $\frac{9}{10}$

Example 2

$$\frac{5}{6} \div \frac{10}{7}$$

Solution

First, we get the reciprocal of $\frac{10}{7}$ which is $\frac{7}{10}$. Multiplying the fractions, we have

$$\frac{5}{6} \times \frac{7}{10} = \frac{35}{60}$$

We **reduce the answer to lowest terms** by dividing both the numerator and denominator by 5 resulting to $\frac{7}{12}$.

Answer: $\frac{7}{12}$

Example 3

$$5\frac{3}{4} \div \frac{4}{5}$$

Solution

In dividing fractions, the dividend and the divisor must not be mixed fractions. Therefore, we need to convert the **mixed fraction to improper fraction**. To do this, we multiply 4 by 5 and then add 3. The result becomes the numerator of the mixed fraction. So, the the equivalent of $5\frac{3}{4}$ is $\frac{23}{4}$.

Multiplying the fractions, we have

$$\frac{23}{4} \times \frac{5}{4} = \frac{115}{16}$$

We can convert the **improper fraction to mixed f**orm which is equal to

$$7\frac{3}{16}$$

Answer: $7\frac{3}{16}$

Example 4
$\frac{7}{8} \div 4$.

Solution
If the divisor is a whole number, the reciprocal will be 1 "over" that number. In the given, the reciprocal of 4 is $\frac{1}{4}$. After getting the reciprocal of the divisor, we multiply the two fractions:

$$\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}.$$

Answer: $\frac{7}{32}$

To test your understanding of division of fractions, you can answer this Practice Test and read the Answer Key with solutions after.

Practice Test on Dividing Fractions

Divide the following fractions and reduce your answers to lowest terms. Convert all answers that are improper fractions to mixedfractions.
1.) $\frac{4}{5} \div \frac{2}{3}$.

2.) $\frac{2}{7} \div \frac{5}{21}$

3.) $8 \div \frac{4}{5}$

4.) $\frac{3}{5} \div 12$

5.) $15\frac{2}{3}$

6.) $3\frac{2}{5} \div \frac{3}{4}$
7.) $\frac{3}{4} \div 2\frac{1}{9}$.

8.) $7\frac{2}{3} \div 7\frac{1}{2}$

9.) $\frac{2\frac{3}{5}}{4}$

10.) $\frac{2\frac{1}{2}}{\frac{8}{3}}$

How to Get the Least Common Multiple of Numbers

In mathematics, a multiple is a product of any number and an integer. The numbers 16, -48 and 72 are multiples of 8 because 8 x 2 = 16, 8 x -3 = -48 and 8 x 9 = 72. Similarly, the first five positive multiples of 7 are the following:

7, 14, 21, 28, 35.

In this post, we will particularly talk about positive integers and positive multiples. This is in preparation for the discussions on addition and subtraction of fractions.

We can always find a common multiple given two or more numbers. For example, if we list all the positive multiples of 2 and 3, we have

2, 4, **6**, 8, 10, **12**, 14, 16, **18**, 20

and

3, **6**, 9, **12**, 15, **18**, 21, 24, 27, 30.

As we can see, in the list, 6, 12 and 18 are common multiples of 2 and 3. If we continue further, there are still other multiples, and in fact, we will never run out of multiples.

Can you predict the next five multiples of 2 and 3 without listing?

The most important among the multiples is the **least common multiple**. The least common multiple is the smallest among all the multiples. Clearly, the least common multiple of 2 and 3 is 6. Here are some examples.

Example 1: Find the least common multiple of 3 and 5

Multiples of 3: 3, 6, 9. 12, **15,** 18

Multiples of 5: 5, 10, **15**, 20, 25,30

As we can see, **15** appeared as the first common multiple, so 15 is the least common multiple of 3 and 5.

Example 2: Find the least common multiple of 3, 4, and 6.

In this example, we find the least multiple that are common to the three numbers.

Multiples of 3: 3, 6, 9, **12**, 15

Multiples of 4: 4, 8, **12**, 16, 20

Multiples of 6: 6, **12**, 18, 24, 30

So, the least common multiple of 3, 4, and 6 is **12**.

Example 3: Find the least common multiple of 3, 8 and 12.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, **24**

Multiples of 4: 4, 8, 12, 16, 20, **24**,

Mulitples of 12: 12, **24**, 36, 48, 60

So, the least common multiple of 3, 4 and 6 is **24**.

In the next part of this **series**, we will discuss about How to Add Fractions.

How to Multiply Fractions

Among the four fundamental operations on fractions, multiplication is the easiest. It is just simple. Multiply the numerator and then the denominator. Of course, if the given fractions can be **converted to lowest terms**, the easier the multiplication will be.

In this post, we are going to learn how to multiply fractions. You must master this operation, as well as other fundamental operations on fractions because you will use them in higher mathematics and solving word problems. Below are some examples.

Example 1

$$\frac{4}{5} \times \frac{1}{3}$$

Solution

$$\frac{4}{5} \times \frac{1}{3} = \frac{4 \times 1}{5 \times 3} = \frac{4}{15}.$$

Answer: $\frac{4}{15}$.

Example 2

$\frac{2}{3} \times \frac{5}{6}$

Solution

$\frac{2}{3} \times \frac{5}{6} = \frac{10}{18}$

We reduce the fraction to lowest term by dividing both the numerator and the denominator by 2. This results to $latex $\frac{5}{9}$ which is the final answer.

Answer: $\frac{5}{9}$

Example 3

$\frac{12}{15} \times \frac{2}{3}$

Solution

First, we reduce $\frac{12}{15}$ by dividing both the numerator and the denominator by 3. This results to $\frac{4}{5}$. We now multiply:

$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15.}$

Answer: $\frac{8}{45.}$

Example 4

$3\frac{1}{2} \times \frac{1}{4}$

Solution

In this example, we need to convert the mixed fraction into improper fraction. To do this, we multiply the denominator of the mixed fraction to the whole number and the product to the denominator. That is

$$\frac{2 \times 3 + 1}{2} = \frac{7}{2.}$$

Now, let us multiply the two fractions.

$\frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$

Answer: $\frac{7}{8}$

Now, test your knowledge on multiplication of fractions by answering the Practice Exercises and then read the Answer Key with solution after.

Practice Test on Multiplying Fractions

In the previous post, we have learned **how to multiply fractions**. We have learned that it is the easiest operation on fractions. To multiply fraction, we just have to multiply the numerators and then the denominators. That is a fraction $\frac{a}{b}$ multiplied by $\frac{c}{d}$ is equal to $\frac{a \times c}{b \times d}$.

Practice Test on Multiplying Fractions

Below are the exercises on multiplying fractions. Multiply the fractions and reduce your answers to the lowest terms. If the answer is an improper fraction, convert the improper fraction to mixed fraction.

1. $\frac{2}{3} \times \frac{4}{5}$
2. $\frac{3}{4} \times \frac{5}{6}$
3. $\frac{3}{5} \times \frac{5}{7}$
4. $\frac{8}{16} \times \frac{2}{3}$
5. $\frac{6}{15} \times \frac{1}{4}$
6. $\frac{11}{12} \times \frac{5}{22}$
7. $\frac{8}{15} \times 9$

8. $\frac{2}{3} \times \frac{6}{5}$

9. $\frac{15}{4} \times \frac{3}{18}$

10. $\frac{1}{8} \times 1\frac{2}{9}$

11. $1\frac{5}{9} \times 3\frac{2}{7}$

In the next post, we will discuss the solutions of the multiplication problems above. Goodluck!

Answers to the Multiplying Fractions Practice Test

In the previous post, we have learned **how to multiply fractions**. We have learned that it is Below are the solutions and answers to thePractice Test on Multiplying Fractions. If you have forgotten the methods of calculation, you can read How to Multiply Fractions.

The methods shown in some of the solutions below is only one among the many. I have mentioned some tips, but I don't want to fill the solution with short cuts because there are times that when you forget the shortcut, you are not able to solve the problem. My advice if you want to pass the Civil Service Examination for Numerical Literacy is to master the basics, practice a lot, and develop your own shortcuts.

1. $\frac{2}{3} \times \frac{4}{5}$

Solution

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

2. $\frac{3}{4} \times \frac{5}{6}$

Solution

$$\frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24}$$

Now, reducing to lowest term we have

$$\frac{15 \div 3}{24 \div 3} = \frac{5}{8}.$$

3. $\frac{3}{5} \times \frac{5}{7}$

Solution

$$\frac{3}{5} \times \frac{5}{7} = \frac{3 \times 5}{5 \times 7} = \frac{15}{35}$$

Reducing to lowest terms, we have

$$\frac{15 \div 5}{35 \div 5} = \frac{3}{7}.$$

Note: Notice that the numerator and the denominator of both fractions have 5's. Since we are multiplying them, we can actually *cancel* 5 from the start of the calculation making the answer $\frac{3}{7}$.

4. $\frac{8}{16} \times \frac{2}{3}$

Solution

In computations, if some fractions can be reduced to the lowest term before starting the calculation, the better. In this case, $\frac{8}{16}$ can be reduced to $\frac{1}{2}$, so we just multiply $\frac{1}{2}$ and $\frac{2}{3}$.

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6}.$$

Dividing both the numerator and the denominator by two reduces $\frac{2}{6}$ to $\frac{1}{3}$ which is the final answer.

5. $\frac{6}{15} \times \frac{1}{4}$

Solution

First, we reduce first $\frac{6}{15}$ to $\frac{2}{5}$ by dividing both the numerator and denominator by 3. We then multiply the two fractions.

$$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20}.$$

We reduce to lowest terms by dividing both the numerator and the denominator by 2 which results to $\frac{1}{10}$.

6. $\frac{11}{12} \times \frac{5}{22}$

Solution

In this example, 11 and 22 are both multiples of 11. Eleven is a numerator and 22 is a other one is in the denominator. This way, you can cancel them by dividing both sides by 11. This makes the first fraction $\frac{1}{12}$ and the second fraction $\frac{5}{2}$. That is,

.

This gives the final product $\frac{5}{24}$.

7. $\frac{8}{15} \times 9$

Solution

When multiplying whole numbers with fractions, just put 1 as the denominator of the whole numbers.

$$\frac{8}{15} \times \frac{9}{1} = \frac{72}{15}$$

Dividing both the numerator and denominator by 3, we have $\frac{24}{5}$ or $4\frac{4}{5}$ in mixed fraction form.

8. $\frac{2}{3} \times \frac{6}{5}$

Solution

$$\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$$

Dividing the numerator and the denominator by 3, we have $\frac{4}{5}$.

9. $\frac{15}{4} \times \frac{3}{18}$

Solution

$$\frac{15}{4} \times \frac{3}{18} = \frac{45}{72}$$

Dividing both the numerator and the denominator by 9 gives $\frac{5}{8}$ as the final answer.

10. $\frac{1}{8} \times 1\frac{2}{9}$

Solution

First we convert $1\frac{2}{9}$ to improper fraction. That is,

$$1\frac{2}{9} = \frac{(9 \times 1) + 2}{9} = \frac{11}{9}.$$

Then we multiply:

$$\frac{1 \times 11}{8 \times 9} = \frac{11}{72}.$$

The correct answer is $\frac{11}{72}$.

11. $1\dfrac{5}{9} \times 3\dfrac{2}{7}$

Solution

In this example, we convert $3\frac{2}{7}$ first to improper fraction. To convert, multiply the denominator by the whole number and then add the numerator to the product. This will be the numerator of the mixed fraction as shown in the following computation.

$$3\frac{2}{7} = \frac{7 \times 3 + 2}{7} = \frac{23}{7}.$$

Now, converting $1\frac{5}{9}$ to mixed fraction gives us \frac{14}{9}$.

Multiplying the fractions, we have

$$\frac{14}{9} \times \frac{23}{7}.$$

We can reduce the fraction to $\frac{2}{9} \times 23$ by dividing 14 and 7 by 7. Therefore, the final answer is $\frac{46}{9}$ or

$$5\frac{1}{9}$$

In the next post, we are going to learn how to convert mixed form to improper fraction.

How to Subtract Fractions

We have already learned the three operations on fractions namely **addition**, **multiplication**, and **division**. In this post, we are going to learn the last elementary operation: subtraction. If you have mastered **addition of fractions**, this will not be a problem for you because the process is just the same. Let's subtract fractions!

Example 1: $\dfrac{8}{15} - \dfrac{3}{15}$.

Solution

The given is a similar fraction (fraction whose denominators are the same), so just like in addition, we just perform the operation on the numerators. Therefore, we just have to subtract the numerator and copy the denominator. That is,

$$\frac{8}{15} - \frac{3}{15} = \frac{5}{15}.$$

We **reduce to lowest term** by dividing both the numerator and denominator of $\frac{5}{15}$ by 5. This results to $\frac{1}{3}$ which is the final answer.

Example 2: $\frac{3}{5} - \frac{1}{2}.$

Solution

The two fractions are dissimilar, so we must find their least common denominator. To do this, we **find the least common multiple** of 2 and 5. The common multiples of 2 are

$2, 4, 6, 8, 10, 12$ and so on

and the common multiples of 5 are

$5, 10, 15, 20, 25$ and so on.

As we can see from the lists above, 10 is the least common multiple of 2 and 5.

We now change the denominator of both fractions to 10.

First, we find the equivalent fraction of $\frac{3}{5}$. That is,

$$\frac{3}{5} = \frac{x}{10}.$$

To find the value of x, divide 10 by 5 and then multiply to 3. The result is 6 which becomes the numerator of the equivalent fraction. So, the equivalent fraction of $\frac{3}{5}$ is $\frac{6}{10}$. If you are confused with this process, please read **How to Add Fractions**.

Now, we get the equivalent fraction of $\frac{1}{2}$ or we find the value of y in $\frac{1}{2} = \frac{y}{10}$. We divide 10 by $2$and then multiply it by 1, which gives us 5. So, the equivalent fraction of $\frac{1}{2}$ is $\frac{5}{10}$.

We now subtract the fractions.

$$\frac{6}{10} - \frac{5}{10} = \frac{1}{10}.$$

The final answer is $\frac{1}{10}$.

Example 3: $6\frac{3}{4} - \frac{1}{5}$
Solution
First, we convert $6\frac{3}{4}$ to improper fraction. That is,

$$\frac{4 \times 6 + 3}{4} = \frac{27}{4}.$$

to get

$$\frac{27}{4} - \frac{1}{5}.$$

The least common multiple of 5 and 4 is 20 (try listing as in example 2).

Now, to get the equivalent fraction, we have $\frac{27}{4} = \frac{a}{20}$. Now, $(20 \div 4) \times 27 = 135$. This means, the equivalent fraction

$$\frac{27}{4} = \frac{135}{20}.$$

We also convert $\frac{1}{5}$ to $\frac{b}{20}$ which is equal to $\frac{4}{20}$.

Now, we subtract the fractions.

$$\frac{135}{20} - \frac{4}{20} = \frac{131}{20}.$$

Converting the answer which is an **improper fraction to mixed number**, we have $\frac{131}{20} = 6\frac{11}{20}$.

There is another way to make the solution of the third examples shorter. We will discuss this in the next post which is subtraction of fraction involving mixed fractions.

Practice Test on Subtraction of Fractions

A month ago, I have discussed the first part of the **subtraction of fraction** tutorials and I apologize for the delay of the exercises. Sharpen what you have learned from thepractice test below.

Practice Test on Subtraction of Fractions

1. $\frac{13}{17} - \frac{2}{17}$.

2. $\frac{8}{15} - \frac{4}{15}$.

3. $\frac{5}{8} - \frac{1}{2}$

4. $\frac{4}{5} - \frac{1}{3}$

5. $2\frac{3}{5} - \frac{1}{4}$

6. $4\frac{5}{6} - 2\frac{1}{6}$
7. $4\frac{3}{4} - 2\frac{1}{3}$

8. $\frac{6}{13} - \frac{3}{10}$

9. $4\frac{8}{15} - 2\frac{3}{5}$

10. $8\frac{1}{3} - 4$

Solutions and Answers to Subtraction of Fractions Practice Test

Below are the complete solutions and answers to the **Practice Test on Subtraction of Fractions**. If you do not know how to do it or you have forgotten the methods, please read **How to Subtract Fractions.**

Practice Test on Subtraction of Fractions

1. $\frac{13}{17} - \frac{2}{17}$.

Solution
The given fractions are similar, so we just subtract the numerators and copy the denominator.

$$\frac{13}{17} - \frac{2}{17} = \frac{11}{17}.$$

Answer: $\frac{11}{17}$.
***

2. $\frac{8}{15} - \frac{4}{15}$.

This is similar to number 1. They are similar fractions.

$$\frac{8}{15} - \frac{4}{15} = \frac{4}{15}.$$

Answer: $\frac{4}{15}$

***

3. $\frac{5}{8} - \frac{1}{2}$

As we have mentioned in **How to Add Fractions** and **How to Subtract Fractions**, we need to make the dissimilar fractions similar in order to perform addition and subtraction. In this case, we need to make their denominators the same. We need to make $\frac{1}{2}$ as $\frac{n}{8}$, and clearly $n = 4$. So,

$$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$$

Answer: $\frac{1}{8}$

***

4. $\frac{4}{5} - \frac{1}{3}$

First, we find the Least Common Multiple (LCM) of 3 and 5 by listing:

Common Multiples of 3: 3, 6, 9, 12, **15**, 18, …
Common Multiples of 5: 5, 10, **15**, 20, 25

So, the LCM of 3 and 5 is 15.

Next, we convert the given to fractions whose denominator is 15.

$\frac{4}{5} = \frac{x}{15}$ which means that $x = (15 \div 5) \times 4 = 12$.

$\frac{1}{3} = \frac{y}{15}$ which means that $y = (15 \div 3) \times 1 = 5$.

So, $\frac{4}{5} = \frac{12}{15}$ and $\frac{1}{3} = \frac{5}{15}$.

Performing the subtraction, we have

$$\frac{12}{15} - \frac{5}{15} = \frac{7}{15}.$$

Answer: $\frac{7}{15}$

***

5. $2\frac{3}{5} - \frac{1}{4}$

First, we get the LCM of 4 and 5 which is 20 (try listing as shown in the previous question.

Second, we we convert the mixed fraction to improper fraction. That is,

$2\frac{3}{5} = \frac{5 \times 2 + 3}{5} = \frac{13}{5}$.

Third, we get the equivalent fractions of $\frac{13}{5}$ and $\frac{1}{4}$ whose denominator is 20. Clearly, $\frac{1}{4} = \frac{5}{20}$, so we are only left with $\frac{13}{5}$.

$\frac{13}{5} = \frac{n}{20}$

We solve for n: $(20 \div 5) \times 13 = 52$. This means that

$$\frac{13}{5} = \frac{52}{20}.$$

Performing the subtraction, we have

$$\frac{52}{20} - \frac{5}{20} = \frac{47}{20}.$$

Converting this **improper fraction to mixed form**, we have
$2\frac{7}{20}$ as the answer.

Answer: $2\frac{7}{20}$.

***

6. $4\frac{5}{6} - 2\frac{1}{6}$

This is one of the cases where you can separate the whole number and the fraction in subtraction. I will discuss about this later. Here, we just subtract the whole numbers $4 - 2 = 2$ and then subtract the fraction $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$. So, the answer is $2\frac{4}{6}$ or $2\frac{2}{3}$ when reduced to lowest terms.
Answer: $2\frac{2}{3}$.

***

7. $4\frac{3}{4} - 2\frac{1}{3}$

First, we get the LCM of 4 and 3, which is clearly 12 (try listing as shown in No. 4).

Second, we convert the mixed fractions into improper fractions.

$4\frac{3}{4} = \frac{4 \times 4 + 3}{4} = \frac{19}{4}$

$2\frac{1}{3} = \frac{3 \times 2 + 1}{3} = \frac{7}{3}$

Now, we convert the given fractions to fractions whose denominator is 12.

$\frac{19}{4} = \frac{m}{12}$, $m = (12 \div 4) \times 19 = 57$.

$\frac{7}{3} = \frac{n}{12}$, $n = (12 \div 3) \times 7 = 28$.

This means that $\frac{19}{4} = \frac{57}{12}$ and $\frac{7}{3} = \frac{28}{12}$.

Now, subtracting the two fractions, we have

$$\frac{57}{12} - \frac{28}{12} = \frac{29}{12}.$$

Converting the improper fraction to mixed form, we have $2\frac{5}{12}$.

Answer: $2\frac{5}{12}$.

***

8. $\frac{6}{13} - \frac{3}{10}$

The Least Common Multiple of 10 and 13 is 130. Converting both fractions and and subtracting, we have

$$\frac{60}{130} - \frac{39}{130} = \frac{21}{130}.$$

Answer: $\frac{21}{130}$.

***

9. $4\frac{8}{15} - 2\frac{3}{5}$

The Least Common Multiple of 15 and 5 is 15.

Now, converting the mixed fractions to improper fractions gives us $\frac{68}{15}$ and $\frac{13}{5}$.

We only need to convert $\frac{13}{5}$ to $\frac{n}{15}$ since the denominator of the other fraction is already 15. Now,

$\frac{13}{5} = \frac{39}{15}$. Subtracting the two fractions, we have

$$\frac{68}{15} - \frac{39}{15} = \frac{29}{15}.$$

Converting the improper fraction to mixed number, we have

$$\frac{29}{15} = 1\frac{14}{15}.$$

Answer: $1\frac{14}{15}$.

***

10. $8\frac{1}{3} - 4$

In this case, we just subtract the whole numbers which leaves an answer of $4\frac{1}{3}$.

Answer: $4\frac{1}{3}$

So far, we have finished all the operations on fractions. In the next posts, we will be discussing operations on decimals and number series (or number patterns).

How to Subtract Positive and Negative Integers

This is the continuation of the series of Civil Service review in mathematics particularly on operations of integers. In this post, we are going to discuss the most complicated operation on integers. I have taught people ofall ages about this topic and it seems that for many, this is the most difficult among the four operations. In this post, we are going to learn how to subtract positive and negative integers or signed numbers. Note that in subtracting integers, there are only four forms. If *a* and *b* are positive, the subtraction are of the following forms.

Case 1: positive minus positive ($a - b$)
Case 2: negative minus positive ($^{-}a - b$)
Case 3: positive minus negative ($a - {}^{-}b$)
Case 4: negative minus negative ($^{-}a - {}^{-}b$)

How to Subtract Positive and Negative Integers

What most people don't know that $a - b$ is the same as $a + {}^{-}b$, or subtracting a number is the same as adding its negative. That means that you only have to memorize the steps inaddition of integers. Given a subtraction sentence, you then transform it into addition. Here are a few examples.

Case 1 Exampe 1: 5 – 8

Subtracting is the same as adding its negative, so $5 - 8 = 5 + {}^{-}8$. Note that $5 + {}^{-}8$ is already addition and $5 + {}^{-}8 = {}^{-}3$.

Case 2 Example: $^{-}10 - 4$

The expression $^{-}10 - 4$ is the same as $^{-}10 + {}^{-}4 = {}^{-}14$.

Remember also that if you see two consecutive – signs or a minus and a negative sign, you can transform it to +. That is, $-(^{-}a) = + a$ and $-(^{-}a) + a$. In most exam, the negative signs are not usually superscript, so you will likely $-(-a)$.

Case 3 Example: $5 - {}^{-}6$

The above expression might be written in 5- -6 or 5-(-6). In any case, two negative signs, a minus and a negative sign can be transformed into a plus sign so, 5 – (–6) = 5 + 6 = 11. Notice that the last equation is also an addition sentence.

Case 4 Example: $^{-}8 - {}^{-}6$

The expression $^{-}8 - {}^{-}6 = {}^{-}8 + 6 = {}^{-}2$.

Observe that the four forms are already completed in the examles. From the strategy above, we only remember two strategies: (1) transform any subtraction sentence to addition sentence and (2) replace two consecutive negatives or a minus and a negative with + sign.

Civil Service Practice Test on Subtracting Integers

Three days ago, we have learned how to subtract positive and negative integers orsigned numbers. In this post, I am going to give you a practice test on subtracting integers. For convenience, I have colored the negative signs blue instead of raising them to an exponent.

1. 12 – 23

2. –21 -(–4)
3. 17 – (–36)
4. –18 – 13
5. 22 – 35

6. –34 -(– 21)
7. 29 -(– 6)
8. 98 – 14

9. – 87 – 53
10. 63 – 92

Solutions to Subtracting Integers Practice Test

Blow are the solutions to the Civil Service Exam **subtracting integers practice test**. As I have mentioned before, you do not have to memorize the rules of subtraction. All you have to do is convert it to addition. We can do this by converting a subtraction of integers problem to addition of integers by changing the signs: minus "-: to plus negative "+ (-" and minus negative "– –" or "– (-" to plus "+". Observe how the subtraction problems are converted into addition problems in the solutions below. If you have any questions, please use the comment box below.

1. 12 – 23 = 12 + (-23) = -11
2. -21 – (-4) = -21 + 4 = -17
3. 17 – (-36) = 17 + 36
4. -18 – 13 = -18 + (-13) =-31
5. 22 – 35 = 22 + (-35) = -13
6. -34 – (-21) = -34 + 21 = – 13
7. 29 – (-6) = 29 + 6 = 35
8. 98 – 14 = 84 (no need to change signs, this is ordinary subtraction since the minuend is larger than the subtrahend).
9. -87 – 53 = -87 + (-53) = -140
10. 63 – 92 = 63 + (-92) = -29

In the next post, we are going to discuss about multiplication and division of integers.

Subtraction of Integers Quiz 1

Welcome to the first quiz on Subtraction of Integers. If you have forgotten how to subtract integers, please read *How to Subtract Integers*. You can also answer the Practice Exercise first (with full solutions) before answering the quiz below. Note that in some mobile devices, the answers maybe shown, so it is better to answer this quiz using a PC.

Instruction: Perform the subtraction. To check if your answer is correct, click the + button. You may want check your answers and share with us your score. 🙂

Subtraction of Integers Quiz 1

1. What is $13 - 8$?

Answer

5

2. What is $7 - 20$?

Answer

-13

3. What is $-1 - 11$?

Answer

-12

4. What is $21 - (-4)$?

Answer

25

5. What is $-6 - 5$?

Answer

-11

6. What is $0-(-8)$?

Answer

8

7. What is $15-12$?

Answer

3

8. What is $9-18$?

Answer

-9

9. What is $-6-(-4)$?

Answer

-2

10. What is $-10-12$?

Answer

-22

11. What is $-7-(-8)$?

Answer

1

12. What is $3-6$?

Answer

-3

13. What is $14-(-8)$?

Answer

22

14. What is $-16-6$?

Answer

-22

15. What is $8-(-8)$?

Answer

16

How to Add Numbers with Decimals

Now that we have finished learning fractions, we proceed to the learning about decimals. Adecimal is another representation of numbers. For instance, the fraction $\frac{1}{2}$ can be represented with 0.5 and the decimal $3\frac{1}{5}$ can be represented with 3.2. In this post, we are going to learn about addition of decimals. In learning decimals, there is just one simple rules: write the numbers, one under the other such that thedecimal points are aligned.

Example 1: 3.02 + 80.401

In adding 3.02 and 80.401, the decimal part of the 80.401 has more digits that that of 3.02. But there is no number to the right of 2, so we can just put 0.

$$\begin{array}{r} 3.020 \\ 80.401 \\ \hline 83.421 \end{array}$$

Remember that adding 0 to the right hand size of the last number in the decimal does not change its value. That is, 0.02, 0.020, 0.0200, 0.02000 are just the same, so we did not change its value.

Example 2: 1.95 + 180.2 + 20.003

$$\begin{array}{r} 1.950 \\ 180.200 \\ 20.003 \\ \hline 202.153 \end{array}$$

Example 3: 5.5 + 2.21 + 3.891

Well, I think you can do this on your own. The answer is 11.601.

Of course, in the actual Civil Service Exam where time is essential, you do not really to put all the 0's as I have done above. You can just align the decimal points and add the corresponding columns. In case there is just one number in one column, just copy the number on the sum.

How to Divide Numbers with Decimals

This is the fourth part and the conclusion of the Operations on Decimals series. In this post, we are going to discuss how to divide numbers with decimals.

In the examples below, it is assumed that you already know how to divide decimal numbers by whole numbers. Therefore, the basic idea is to eliminate the decimal point of the divisor. It can done by multiplying both the divisor and the dividend by powers of 10.

Example 1: What is 18.5 divided by 0.2?

To get rid of the decimal point in 0.2, we multiply it by 0. If we do this, we also multiply 8.5 by 10. This gives us 185 divided by 2 which 92.5.

Example 2: What is 4.26 divided by 0.3?

To get rid of the decimal point in 0.3, we multiply it with 10. We also multiply 4.26 by 10. This gives us 42.6 divided by 3. Well, we can actually do this mentally: 42 divided by 3 is 14 and 0.6 divided by 3 is 0.2. So, the correct answer is 14.2

Example 3: What is 32.85 divided by 0.203?

Well, just multiply 0.203 by 1000; this results to 203. Now, multiply 32.85 by 1000, this gives us 32850. So, the new given now is, 32850 divided by 203. Well, I'm sure you can do that.

Why does multiplying by powers of 10 works?

If you divide a by b, then you have the fraction $\frac{a}{b}$. Now, when we multiply the dividend and divisor with the same number, we are actually multiplying the numerator and denominator with that number. For instance, if we multiply a and b by 10, we have

$$\frac{a \times 10}{b \times 10} = \frac{10a}{10b} = \frac{a}{b}$$

we are not actually changing its value of the fraction. Therefore, we are still dividing the same numbers.

Now that concludes our series. In the next post, we will be discussing about percent.

How to Multiply Numbers with Decimals

This is the third part of the Operations on Decimals Series and in this post, we discuss about Multiplication of Decimals. In multiplying decimals, the decimal point in the product has something to do with the number of decimals of the factors.

How to Multiply Numbers with DecimalsExamples

Example 1: What is 3.6×4?

In this example, only one factor has a decimal number, the other is a whole number. To multiply, first, ignore the decimal point and then just multiply the numbers:

$$36 \times 4 = 144.$$

After multiplying, count the number of decimal numbers (numbers after on the right hand side of the decimal point) of the factors. There is only **one** decimal number which is 6. So, in the product, starting from the right, count **one** number and then place the decimal point before that number making it 14.4.

So, the final answer is 14.4.

Example 2: What is 8.3×4.2?

Again, ignore the decimal points and multiply the numbers:

$$83 \times 42 = 3486.$$

There is one decimal number in the first factor and one in the second factor. Therefore, there are **two** decimal numbers. Now, count **two** numbers from the right, and place the decimal point before the last number on your count.

Therefore, the correct answer is 34.86.

Example 3: What is 3.28×0.01?

Now, $328 \times 1 = 328$. Notice that there are only three numbers in the product, but there are**four** decimal numbers in the two factors. So, in the product, we count **three** numbers from the right hand side and then add **one** 0 before 3 to make the number of decimals **four**. So, the correct answer is $.0328$ or 0.0328.

Multiplying Decimal Numbers by 10

In multiplying decimal numbers by 10 or its powers, just count the number of zeroes and move the decimal point to the right hand side the number of zeroes appear.

Example 1: What is 3.45×10?

Ten has one zero, so, we move the decimal point one place to the right hand side. Therefore, the correct answer is 34.5.

Example 2: What is 76.98301×100?

There are two zeros, so we move the decimal point two digits to the right hand side. Moving the decimal points gives us 7698.301

Example 3: What is 34.7×1000?

There are three zeros, however, only one decimal point. So, we move the decimal point one time to the right of seven, and add two zeros. Therefore, the final answer is 34700.

In the next post, we will be discussing about division of decimal number

How to Subtract Numbers with Decimals

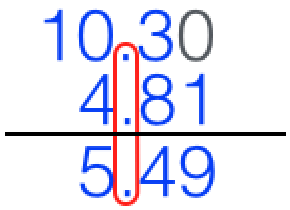
This is the second part of the DecimalOperations series and in this post, we are going to discuss subtraction of numbers withdecimals. This operation is very much the same with addition of decimals.

How to Subtract Numbers with Decimals

The rule in subtraction of decimals is the same with addition of decimals. First, position the numbers such that the decimal points are aligned. Then, add zeros to make the number of decimal places the same. Lastly, perform subtraction.

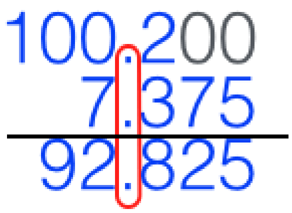
Example 1: 10.3 – 4.81

In the first example, we add 0 to the minuend 10.3 to make it 10.30. This way, we can subtract 1 from 0.



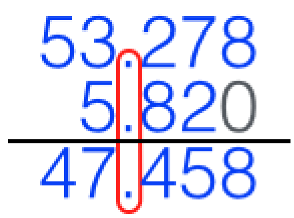
Example 2: 100.2 – 7.375

In example 2, we add two zeros to 100.2 so that we can subtract the
three decimalnumbers.



Example 3: 53.278 – 5.82

In the third example, we add 0 to the decimal part of the subtrahend to make the number ofdecimal places equal.



Now, try practicing subtraction and use a calculator to see if your answer is correct. If you have questions, please use the comment box below.

The Operations on Decimals Series

After the **Operations on Fractions**, we have finally finished **Operations on Decimals**. This series discusses the methods and procedures on how to calculate numbers with decimals using the four fundamental operations.

Operations on Decimals

Dividing Positive and Negative Integers

In the previous post on integers, we have learned the **rules in multiplying positive integers and negative integers**. In this post, we are going to learn how to divide positive and negative integers.

If you have observed, in the post on**subtracting integers**, we have converted the "minus sign" to a "plus negative sign." I think it is safe for us to say that subtraction is some sort of "disguised addition." Similarly, we can also convert a division expression to multiplication. For example, we can turn

$$\frac{5}{3} \text{ to } \left(5 \times \frac{1}{3}\right).$$

In general, the division

$$\frac{a}{b} \text{ to } \left(a \times \frac{1}{b}\right).$$

From the discussion above, we can ask the following question:

Can we use the rules in multiplying integers when dividing integers?

The answer is a big YES. The rules are very much related.

positive integer ÷ positive integer = positive integer
positive integer ÷ negative integer = negative integer
negative integer ÷ positive integer = negative integer
negative integer ÷ negative integer = positive integer

Notice that they are very similar to the rules in multiplying integers.

positive integer x positive integer = positive integer
positive integer x negative integer = negative integer
negative integer x positive integer = negative integer
negative integer x negative integer = positive integer

Here are some examples worked examples.

1. 18 ÷ 3 = 6

2.36 ÷ -12 = – 3

3. -15 ÷ 2 = – 7.5

4.- 8 ÷ -4 = 2

From the discussion and the worked examples above, we can therefore conclude that in dividing positive and negative integers, we only need to memorize the rules in multiplying integers and apply them in dividing integers.

Practice Test on Dividing Integers

In the previous post we have discussed how to divide integers. Operations on real numbers, particularly integers, is one of the scopes of the Civil Service Examinations both Professional and Subprofessional. You must master these operations because you will use them in solving equations and word problemsin Algebra.

Test your skill by answering the exercises below. Recall that *a* divided by *b* is the same as *a* times reciprocal of *b*.

Practice Test **on Dividing Integers**

1.) -35 ÷ 7

2.) 38 ÷ -19

3.) 56 ÷ 8

4.) -84 ÷ -12

5.) 5. -28 ÷ -1

6.) 0 ÷ -5

7.) 47 ÷ -47

8.) -156 ÷ 12

9.) -34 ÷ -17

10.) -180 ÷ 9

To view the answers and explanations of the practice test above, read the Answers to Practice Test on Dividing Integers.

Answers to Practice Test on Dividing Integers

Below are the answers and the explanationsof the **Practice Test on Dividing Integers**. Note that as mentioned in the post **Dividing Positive and Negative Integers**, the rules in dividing integers as well as real numbers are the following:

(1) positive number ÷ positive number = positive number

(2) positive number ÷ negative number = negative number

(3) negative number ÷ positive number = negative number

(4) negative number ÷ negative number = positive number.

Answers and Explanation

1.) -35 ÷ 7 = -5 (by rule 3)

2.) 38 ÷ -19 = -2 (by rule 2)

3.) 56 ÷ 8 = 7 (by rule 1)

4.) -84 ÷ -12 = 7 (by rule 4)

5.) -28 ÷ -1 = 28 (by rule 4)

6.) 0 ÷ -5 = 0 (0 divided by any number is 0)

7.) 47 ÷ -47 = -1 (by rule 2)

8.) -156 ÷ 12 = -13 (by rule 3)

9.) -34 ÷ -17 = 2 (by rule 4)

10.) -180 ÷ 9 = -20 (by rule 3)

I hope you got all the answers correct. In the next post, we will be discussing about the order of operations. This is simplifying numbers with more than one operations.

Division of Integers Quiz 1

This is the first of a series of quiz on division of integers. If you have forgotten how to divide integers, please read *How to Divide Integers*. You may also want to answer thepractice exercise with full solutions. Note that in some mobile devices, the answers maybe shown, so it is better to answer this quiz using a PC or a tablet.
Instruction: Perform the multiplication. To check if your answer is correct, click the +button. You may want check your answersand share with us your score. 🙂

Division of Integers Quiz 1

1. What is $15 \div (-5)$?

Answer

-3

2. What is $(-45) \div 9$?

Answer

-5

3. What is $18 \div (-6)$?

Answer

-3

4. What is $16 \div (-4)$?

Answer

-4

5. What is $(-25) \div (-5)$

Answer

5

6. What is $18 \div (-3)$?

Answer

-6

7. What is $-56 \div (-8)$?

Answer

7

8. What is $36 \div (-6)$?

Answer

-6

9. What is $(-120) \div (-20)$?

Answer

6

10. What is $0 \div (-5)$?

Answer

0

11. What is $(-72) \div 9$?

Answer

-8

12. What is $64 \div 16$?

Answer

4

13. What is $49 \div (-7)$?

Answer

-7

14. What is $(-63) \div 9$?

Answer

-7

15. What is $(-47) \div 47$?

Answer

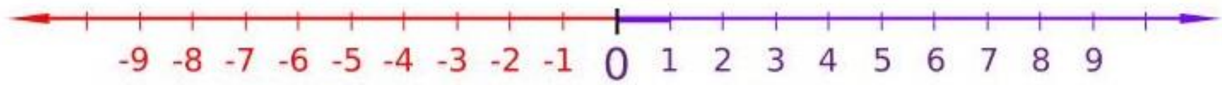
-1

Did you get a perfect score? For more quizzes and practice test, visit the Practice Test page.

How to Add Positive and Negative Integers

One of the topics in basic mathematics that will likely be included in the the Philippine Civil Service Exam both professional and subprofessional are operations on integers. Although a few Civil Service test items may be given from this topic, it is important that you master it because a lot of calculation in other topics will need knowledge of integers and its operations (addition, subtraction, multiplication, division). For example, solving some word problems in mathematics and solving equations will need knowledge on operations of integers.

Integers are whole numbers that are either positive or negative. Examples of integers are -5, 6, 0, and 10. If we place this on the number line, negative integers are the integers that are below 0 (left of 0), while the positive integers are the integers above 0 (right of 0).



Adding Integers that Are Both Positive

When you add integers that are both positive, it is just like adding whole numbers. Below are the examples.

Example 1: $^{+}2 + ^{+}4 = ^{+}6$
Example 2: $^{+}9 + ^{+}41 + ^{+}9 + = ^{+}56$
Example 3: $^{+}120 + ^{+}13 + ^{+}12 + = ^{+}145$
Although we have created a small + before the number to indicate that it is positive, in reality, only negative numbers have signs. This means that $^{+}2 + ^{+}4 = ^{+}6$ is just written as $2 + 4 = 6$.

Adding Integers that Are Both Negative

Adding number that are both negative is just the same as adding numbers that are both positive. The only difference is that if you add two negative numbers, the result is negative.

Example 1: $^{-}5 + ^{-}8 = ^{-}13$
Example 2: $^{-}10 + ^{-}18 + ^{-}32 + = ^{-}60$
Example 3: $^{-}220 + ^{-}11 + ^{-}16 + = ^{-}247$

How to Add Positive and Negative Integers

Before adding, you should always remember that $^{+}1$ and $^{-}1$ cancel out each other, or $^{+}1 + ^{-}1$ is 0. So the strategy is to pair the positive and negative numbers and take out what's left.
Example 1: What is $^{+}13 + ^{-}8$?
Solution:

We pair 8 positives and 8 negatives to cancel out. Then what's left is of $^{+}13$ is $^{+}5$. In equation form, we have

Example 2: What is $^{+}17 + {}^{-}20$?

Solution:

We pair 17 negatives and 17 positives. What's left of $^{-}20$ is $^{-}3$. In equation form, we have

$$^{+}17 + {}^{-}20 = {}^{+}17 + ({}^{-}17 + {}^{-}3) = ({}^{+}17 + {}^{-}17) + {}^{-}3 = 0 + {}^{-}3 = {}^{-}3$$

Example 3: What is $^{+}16 + {}^{+}37 + {}^{-}20 + {}^{-}3 + {}^{-}9$?

In answering questions with multiple addends, combine all the positives and the negatives then add.

That is $^{+}16 + {}^{+}37 = {}^{+}53$ and $^{-}20 + {}^{-}3 + {}^{-}9 = {}^{-}32$.

So, the final equation is $^{+}53 + {}^{-}32$. We pair 32 positives and 32 negatives leaving 21 positives.

In equation form, we have

$$^{+}53 + {}^{-}32 = {}^{+}21 + {}^{+}32 + {}^{-}32 = {}^{+}21 + ({}^{+}32 + {}^{-}32) = {}^{+}21 + 0 = {}^{+}21$$

Practice Exercises on Addition of Integers

In the previous post, we have learned about adding positive and negative integers particularly on how to add integers with different signs. In this post, I am going to give you 10 exercises on adding integers. I will give the answers below for you to be able to check if your answers are correct. Also, I am going to omit the + sign before the positive integers because this is not usually shown in the exam. This means that 3 will automatically mean +3 unless a – sign precedes it.

Exercises on Addition of Integers

1. $28 + 12$
2. ${}^{-}14 + {}^{-}11$
3. $24 + {}^{-}15$
4. ${}^{-}16 + 31$
5. $23 + 46 + {}^{-}15$
6. $45 + {}^{-}12 + {}^{-}16$
7. ${}^{-}12 + {}^{-}15 + {}^{-}62$
8. $22 + {}^{-}36 + 36$
9. $12 + 18 + {}^{-}12 + {}^{-}18$
10. $31 + 55 + {}^{-}41 + {}^{-}32 + {}^{-}10$

Solutions to Practice Exercises on Addition of Integers

We have learned how to add integers and in the previous post, I have given you practice exercises that you can use to evaluate your understanding of the topic. Below is the complete solutions to the practice exercises on adding integers. Share to me how many did you get right.

Solutions to Practice Exercises on Addition of Integers

1. $28 + 12 = 40$.

2. ${}^{-}14 + {}^{-}11 = {}^{-}25$

3. $24 + {}^{-}15 = 9$

Solution: We pair 15 and -15 to get 0. We have 9 left. So, the correct answer is 9.

4. -16 + 31 = 15

We pair -16 and 16 to get 0. We are left with positive 15.

5. 23 + 46 + -15 = 54

We add the two positive numbers first: 23 + 46 = 69. Next, we add 69 and -15. We get 15 from 69 and pair it with -15 resulting to 0, so we have 54 left.

6. 45 + -12 + -16 = 17

We add the negative numbers first: -12 + -16 = -28. We add the result to 45. We get 28 from 45 and pair it with -28 to get 0 leaving 17.

7. -12 + -15 + -62 = -89

Explanation: They are negative, so we just added them. Of course negative added to negative is always negative.

8. 22 + -36 + 36 = 22

Explanation: -36 + 36 = 0, so we are left with 22.

9. 12 + 18 + -12 + -18 = 0

Explanation: 12 and -12 and 18 and -18 = 0.

10. 31 + 55 + -41 + - 32 + -10 = 3

Explanation: Adding the positive integers, we have 31 + 55 = 86. Adding the negative integers, we have -41 + - 32 + -10 = -83. Now, 86 + -83 = 3

Addition of Integers Quiz 1

We have already learned how to operate with integers (addition, subtraction, multiplication, and division) as well as its order of operation. Test your skills in addition of fraction in this quiz below. Fill in the blanks with the correct answer. Good luck!

Addition of Integers Quiz 1

1. Question 1 of 10

   1. Question

   2 points

   23 + 31

   ▪

   Incorrect

   The correct answer is 54. 

How to Multiply Signed Numbers

In the two previous post in Mathematics, we have discussed how to add and subtractsigned numbers. In this post, we are going to learn how to multiply signed numbers particularly integers. Signed means positive and negative.

Positive Integer x Positive Integer

Clearly, the product is positive. We had been multiplying positive integers since Grade school and we all know that the product is positive.

Positive Integer x Negative Integer

When you multiply, notice that you are actually adding repeated. When we say 2 x 3, we are actually saying twice three or 2 groups of 3 or 3 + 3. When we say, thrice 11, we are saying 11 + 11 + 11. With this in mind, 3 x – 5 = -5 +

-5 + -5. Since we are adding integers which are negative, the sum is also negative or -15. This means that 3 x -5 = -15. If we generalize this, we can say that the product of a positive integer and a negative integer is negative.

Negative Integer x Positive Integer

If you can remember, multiplication is commutative. This means that the order of the number you multiply does not matter, their product will always be the same. For example, 4 x 3 x 5 is equal to 5 x 4 x 3 or 3 x 4 x 5 or any other arrangement using the three numbers. This means that 3 x -5 = -5 x 3. So, a negative integer multiplied by a positive integer is also negative.

Negative Integer x Negative Integer

For multiplication of two negative integers, we can use patterns to know their product and generalize.

-3 x 2 = -6

-3 x 1 = -3

– 3 x 0 = 0

Now, what is -3 x -1?

If we look at the pattern in the product, we are actually adding by 3 each step so the next number is 3. All other numbers from -1, -2, -3 and so on will be positive (Why?). Therefore, the product of two negative numbers is positive.

Summary: Rules on how to Multiply Signed Numbers

From the above discussion, we summarize the multiplication of integers.

Positive Integer x Positive Integer = Positive Integer

Positive Integer x Negative Integer = Negative Integer

Negative Integer x Positive Integer = Negative Integer

Negative Integer x Negative Integer = Positive Integer

We can also say that if we multiply two numbers with the same sign, the answer is positive. If we multiply two numbers with different signs, the answer is negative.

Practice Test on Multiplying Integers

In the previous post, we have learned therules in multiplying integers. Below is a 10-item practice test on multiplying integers. Note that in multiplication, we can use the x and () symbols in multiplying two numbers. This means that 3 x 4 is the same as 3 (4) and (3)(4). We will use both symbols in the practice test below to familiarize yourselves with them. Any of the two symbols can be used in the Civil Service exams.

Practice Test on Multiplying Integers

1. -4 x -8

2. 3 x -7

3. -12 x -7

4. -8 x -1 x -4
5. (3 x 4)(-5)

6. 4 x -3 x -2

7. -7 x -2 x 3

8. (-8)(-3)(2)

9. (-6)(-7)(-2)

10. (2)(-3)(5)(-2)

In the next post, I am going to post the solution and the answers to the practice test above.

Solutions to Practice Test on Multiplying Integers

Recall the rules on multiplying integers. Multiplying two with the like signs (negative and negative or positive and positive) gives a positive product. Multiplying two numbers with unlike signs gives a negative product.

1. -4 x -8 = 32

2. 3 x -7 = -21

3. -12 x -7 = 84

4. -8 x -1 x -4
Group any two first: -8 x -1 = 8. Then 8 x -4 = -32.
5. (3 x 4)(-5) = (12)(-5) = -60
6. 4 x -3 x -2
Group any two first: 4 x -3 = –12. Then -12 x -2 = 24
7. -7 x -2 x 3
Group any two first: -2 x 3= -6.. Then, -6 x -7 = 42.
8. (-8)(-3)(2) = (24)(2) = 48
9. (-6)(-7)(-2) = (42)(-2) = -84,
10. (2)(-3)(5)(-2) = (-6)(-10) = 60

By now you will have realized that no matter how many integers you multiply, you just count the number of negative integers. An even number of negative integers (2, 4, 6, 8 and so on) gives a positive product. An odd number (1, 3, 5, 7, and so on) of negative integers gives a negative product.

Multiplications of Integers Quiz 1

This is a quiz on multiplication of integers. As we have learned, the following rules are applied in multiplication of integers.
a. positive x positive = positive

b. positive x negative = negative

c. negative x positive = negative

d. negative x negative = positive.

In short, if the signs are the same, the product is positive. If the signs are different, the product is negative.
Good luck!

Multiplications of Integers Quiz 1

Question 1 of 10

1. Question
2 points

13×3

- ◉ 39
- ○ -39

Question 2 of 10

2. Question
2 points

$12 \times (-3)$

- ○ -15
- ◉ 36
- ○ -36
- ○ 9

3. Question
2 points

-8×5

- -40
- 13
- -3
- 40

4. Question
2 points

-3×-4

- -7
- 12
- -12
- -1

5. Question
2 points

$6 \times (-3)$

- 18
- 9
- -18
- 3

6. Question
2 points

-7×-3

- -10
- 4

- ◉ 21
- ○ -21

7. Question
2 points

$(2)(5)(-3)$

- ○ 30
- ◉ -30

8. Question
3 points

$-(2 \times 3)(-3 \times 2)$

- ◉ 36
- ○ -36

Question 9 of 10

9. Question
3 points

$-3 \times -2 \times -5$

- ◉ -30
- ○ 30

10. Question
3 points

$(2 \times -3) \times (2 \times -1) \times -2$

- ○ 24
- ◉ -24

Multiplication of Integers Quiz 2

Welcome to the first quiz on Multiplication of Integers. If you have forgotten how to subtract integers, please read *How to Multiply Integers*. A Practice Exercise with full solution is also available. Note that in some mobile devices, the answers maybe shown, so it is better to answer this quiz using a PC or a tablet.

Instruction: Perform the multiplication. To check if your answer is correct, click the + button. You may want check your answers and share with us your score. 🙂

Multiplication of Integers Quiz 1

1. What is 3×5?

Answer

15

2. What is 7×-8?

Answer

-56

3. What is $(-3)(-11)$?

Answer

33

4. What is 4×-4?

Answer

-16

5. What is -6×-5?

Answer

30

6. What is 0×-8?

Answer

0

7. What is 6×8?

Answer

48

8. What is $9(-10)$?

Answer

-90

9. What is $(-9)(3)$?

Answer

-27

10. What is $(-10)(-12)$?

Answer

120

11. What is $(-2)(8)$?

Answer

-16

12. What is $(3)(-6)(-2)$?

Answer

36

13. What is $(2)(3)(-8)$?

Answer

-48

14. What is $(-5)(-5)(-1)$?

Answer

-25

15. What is $(-2)(-3)(-4)(-5)$?

Answer

120

How to Subtract Positive and Negative Integers

This is the continuation of the series of Civil Service review in mathematics particularly on operations of integers. In this post, we are going to discuss the most complicated operation on integers. I have taught people of all ages about this topic and it seems that for many, this is the most difficult among the four operations. In this post, we are going to learn how to subtract positive and negative integers or signed numbers. Note that in subtracting integers, there are only four forms. If a and b are positive, the subtraction are of the following forms.

Case 1: positive minus positive ($a - b$)
Case 2: negative minus positive (${}^{-}a - b$)
Case 3: positive minus negative ($a - {}^{-}b$)
Case 4: negative minus negative (${}^{-}a - {}^{-}b$)

How to Subtract Positive and Negative Integers

What most people don't know that $a - b$ is the same as $a + {}^{-}b$, or subtracting a number is the same as adding its negative. That means that you only have to memorize the steps in addition of integers. Given a subtraction sentence, you then transform it into addition. Here are a few examples.

Case 1 Exampe 1: 5 – 8

Subtracting is the same as adding its negative, so 5 – 8 = 5 + $^{-}$8. Note that 5 + $^{-}$8 is already addition and 5 + $^{-}$8 = $^{-}$3.

Case 2 Example: $^{-}$10 – 4

The expression $^{-}$10 – 4 is the same as $^{-}$10 + $^{-}$4 = $^{-}$14.

Remember also that if you see two consecutive – signs or a minus and a negative sign, you can transform it to +. That is, -(^{-}a) = + a and -(^{-}a) + a. In most exam, the negative signs are not usually superscript, so you will likely -(-a).

Case 3 Example: 5 – $^{-}$6

The above expression might be written in 5- -6 or 5-(-6). In any case, two negative signs, a minus and a negative sign can be transformed into a plus sign so, 5 – (–6) = 5 + 6 = 11. Notice that the last equation is also an addition sentence.

Case 4 Example: $^{-}$8 – $^{-}$6

The expression $^{-}$8 – $^{-}$6 = $^{-}$8 + 6 = $^{-}$2.

Observe that the four forms are already completed in the examles. From the strategy above, we only remember two strategies: (1) transform any subtraction sentence to addition sentence and (2) replace two consecutive negatives or a minus and a negative with + sign.

Civil Service Practice Test on Subtracting Integers

Three days ago, we have learned how to subtract positive and negative integers orsigned numbers. In this post, I am going to give you a practice test on subtracting integers. For convenience, I have colored the negative signs blue instead of raising them to an exponent.

1. 12 – 23
2. –21 -(–4)
3. 17 – (–36)
4. –18 – 13
5. 22 – 35
6. –34 -(– 21)
7. 29 -(– 6)
8. 98 – 14
9. – 87 – 53
10. 63 – 92

Solutions to Subtracting Integers Practice Test

Blow are the solutions to the Civil Service Exam **subtracting integers practice test**. As I have mentioned before, you do not have to memorize the rules of subtraction. All you have to do is convert it to addition. We can do this by converting a subtraction of integers problem to addition of integers by changing the signs: minus "-: to plus negative "+ (-" and minus negative "– –" or "– (-" to plus "+". Observe how the subtraction problems are converted into addition problems in the solutions below. If you have any questions, please use the comment box below.

1. 12 – 23 = 12 + (-23) = -11
2. -21 – (-4) = -21 + 4 = -17
3. 17 – (-36) = 17 + 36
4. -18 – 13 = -18 + (-13) =-31
5. 22 – 35 = 22 + (-35) = -13
6. -34 – (-21) = -34 + 21 = – 13
7. 29 – (-6) = 29 + 6 = 35
8. 98 – 14 = 84 (no need to change signs, this is ordinary subtraction since the minuend is larger than the subtrahend).
9. -87 – 53 = -87 + (-53) = -140
10. 63 – 92 = 63 + (-92) = -29

Subtraction of Integers Quiz 1

Welcome to the first quiz on Subtraction of Integers. If you have forgotten how to subtract integers, please read *How to Subtract Integers*. You can also answer the Practice Exercise first (with full solutions) before answering the quiz below. Note that in some mobile devices, the answers maybe shown, so it is better to answer this quiz using a PC.

Instruction: Perform the subtraction. To check if your answer is correct, click the + button. You may want check your answers and share with us your score. 🙂

Subtraction of Integers Quiz 1

1. What is $13 - 8$?

Answer

5

2. What is $7 - 20$?

Answer

-13

3. What is $-1 - 11$?

Answer

-12

4. What is $21 - (-4)$?

Answer

25

5. What is $-6 - 5$?

Answer

-11

6. What is $0 - (-8)$?

Answer

8

7. What is $15 - 12$?

Answer

3

8. What is $9 - 18$?

Answer

-9

9. What is $-6 - (-4)$?

Answer

-2

10. What is $-10 - 12$?

Answer

-22

11. What is $-7 - (-8)$?

Answer

1

12. What is $3 - 6$?

Answer

-3

13. What is $14 - (-8)$?

Answer

22

14. What is $-16 - 6$?

Answer

-22

15. What is $8-(-8)$?

Answer

16

How to Convert Fraction to Percent Part 1

In the previous post, we have learned how to convert percent to fraction. In these series of posts, we learn the opposite: how to convert fraction to percent. I am going to teach you three methods, the last one would be used if you forgot the other two methods, or if the first two methods would not work. Please be reminded though to understand the concept (please do not just memorize).

The first method can be used for fractions whose denominators can be easily related to 100 by **multiplication** or **division**. Recall that from **Converting Percent to Fraction**, I have mentioned that when we say *percent* it means "per hundred." In effect, n% can be represented by n/100. Therefore, if you have a fraction and you can *turn* it into n/100 (by multiplication/division), then you have turned it into percent.

Example 1: What is the equivalent of 1/5 in percent?

How do we relate the denominator 5 to 100? By multiplying it by 20. Therefore, we also multiply its numerator by 20:

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$$

Now, since we have 100 as denominator, the answer in percent is therefore the numerator. Therefore, the equivalent of 1/5 in percent is 20%.

Example 2: What is 3/25 in percent?

Again, how do you related 25 to 100? By multiplying it by 4. Therefore,

$$\frac{3 \times 4}{25 \times 4} = \frac{12}{100}$$

Therefore, the equivalent of 3/25 in percent is 12%.

Example 3: What is 23/200 in percent?
In this example, we can relate 200 to 100 by dividing it by 2. So, we also divide the numerator by 2. That is

$$\frac{23 \div 2}{200 \div 2} = \frac{11.5}{100}$$

Therefore, the answer is 11.5%

There are two important things to remember in using the method above.

(1) in changing the form the fractions to n/100, the only operations that you can use are multiplication and division and
(2) whatever you do to the numerator, you also do to the denominator.

Note that multiplying the denominator (or dividing it) by the same number does not change its value, it only change its representation (fraction, percent or decimal).

Why It Works
When you are relating a fraction a/b to n/100, you are actually using ratio and proprotion. For example, in the first example, you are actually solving the equation

$$\frac{1}{5} = \frac{n}{100}.$$

The equation will result to $n = \frac{100}{5}$ which is equal to 20. Now, this is just the same as multiplying both the numerator and the denominator by 20.

Note that the method of "relating to 100 by multiplication or division" can only work easily for denominators that divides 100 or can be divided by 100. Other fractions (try 1/7), you have to use ratio and proportion and manual division.

How to Convert Fraction to Percent Part 2

In the Part 1, we have learned how to convert fraction to percent by relating the denominator to 100 by multiplication or division. In this post, we do its 'algebraic version.' This method is a generalized method to the previous post especially for numbers that do not divide 100 or cannot be divided by 100 easily. However, to see the relationship between the two methods, let us do the first example in Part 1 of this series.

Example 1: What is the equivalent of 1/5 in percent.

Recall that in Part 1, we multiplied both the numerator and the denominator by 20, to make the denominator 100. That is,

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$$

Now, notice how it is related to the new method. In this method, we related 1/5 to n/100. That is, what is the value of n in

$$\frac{1}{5} = \frac{n}{100}.$$

To simplify the equation, we multiply both sides of the equation by 100, and we get

$$\frac{100}{5} = \frac{100n}{100}$$

Simplifying and switching the position of the expressions, we get the $n = 20$. This means that $\frac{1}{5} = 20$.

Of course, Part 1 seems to be easier, but the good thing about putting it into equation is that it applies to all fractions. For instance, it is quite hard to convert 7/12 using the method in part 1.

Example 2: What is the equivalent of $\frac{7}{12}$ in percent?

We set up the equation with $\frac{n}{100}$ on the left.

$$\frac{n}{100} = \frac{7}{12}$$

To eliminate the fraction, multiply both sides by denominator. This results to

$$n = \frac{7}{12}(100) = \frac{700}{12} \approx 58.33$$

or about 58.33%.

The curly equal sign means approximately equal to since 3 is a non-terminating decimal.

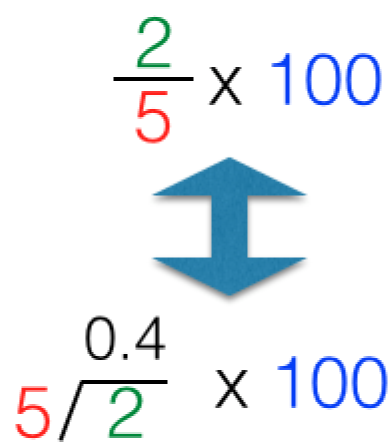
Now, try to examine the expression

$$\frac{7}{12}(100)$$

because this is where they derived the rule. Recall the rule in converting fraction to percent:**Divide the fraction and then multiply the result to 100**. That is exactly it. So, when you have the fraction, $2/5$ just divide it manually, and then multiply the result to 100. That is,

$$\frac{2}{5}(100) = \frac{200}{5} = 40.$$

Do not forget though that the divisor during division is the denominator (5 in 2/5). as shown below.



That's it. I think we don't have to have the third part, since we already derived the rule here.

How to Convert Percent to Fraction

In Civil Service Examinations, as well as other examinations in basic mathematics, knowing how to convert percent, fractions, anddecimals to each other is very advantageous especially if you can do it mentally. Let us try with the following example.

A P640 shirt is marked 25% discount. How much will you have to pay for it?

It seems that you need a pencil for this problem, but you can actually do it in your head. Read it to believe it.

The equivalent of 25% in fraction is 1/4, therefore, you have to take away the fourth of the price. Now, 1/4 of 640 seems difficult but what if we try to split it to 600 + 40? Now, 1/4 of 600 is 150, which means that from the 600, you have 450 left. Now, 1/4 of 40 is 10, which means that you have 30 left. So, 450 + 30 is **480** and that is the discounted price of the t-shirt.

Now, with a little bit of practice, you would be able to do this on your own and you won't have to use a pen to perform calculations for problems such as this.

How to Convert Percent to Fraction

There is one important concept to remember when converting percent to fraction. That is, when you say percent, it means per hundred. The word *cent* comes from the Latin word*centum* which means "hundred". In effect, when you say, 60%, it means 60 per hundred, 0.4% means 0.4 per hundred, 125% means 125 per hundred. When you say *x* per hundred, you can also represent it by the fraction x/100. This means that the percentages above can be represented as

$$\frac{60}{100}, \frac{0.4}{100}, \frac{125}{100}$$

respectively. Now, all we have left to do is to convert these fractions to lowest terms.

Example 1: $\frac{60}{100}$

Recall that to convert a fraction to lowest terms, we find the greatest common factor (GCF) of its numerator and denominator and then divide them both by the GCF. The GCF of 60 and 100 is 20, so

$$\frac{60 \div 20}{100 \div 20} = \frac{3}{5}$$

Therefore, the equivalent of 60% in fraction is $\frac{3}{5}$.

Example 2: $\frac{0.4}{100}$

In this example, we have a decimal point at the numerator and a whole number at the denominator. We have to "get rid" of the decimal point. To do this, we can multiply both the numerator and the denominator by 10 (since 0.4 x 10 = 4). Therefore, we have

$$\frac{0.4 \times 10}{100 \times 10} = \frac{4}{1000}.$$

Now, the greatest common factor of 4 and 1000 is 4, so we divide both the numerator and the denominator by 4. The final result is $\frac{1}{250}$.

Therefore, the equivalent fraction of 0.4% is $\frac{1}{250}$.

Example 3: $\frac{125}{100}$

The greatest common factor of 125 and 100 is 25, so we divide both the numerator and the denominator by 25. In doing this, we get $\frac{5}{4}$.

Therefore, the equivalent fraction of 125% is $\frac{5}{4}$

Summary

There are three steps to remember in converting percent to fractions.

1. Make a fraction from the given percent with the given as numerator and 100 as denominator.
2. Eliminate the decimal points (if there are any) by multiplying the numerator and denominator by the same number which is a power of 10 (10, 100, 1000 and so on).
3. Reduce the resulting fraction to lowest terms.

That's it. You can now convert any given percent to fraction.

Introduction to the Concept of Percentage

Now that we have already studied **fractions**and **decimals**, we discuss percentage. You are likely to be aware that the concept of percentage is very useful in daily life. We always go to stores where there are discounts and we do not want loans with high interest. These calculations involve the concept of percentage.

What is percentage really?

Percentage is a number ratio expressed as a fraction of 100. When we say 10 percent, what we really mean is 10 out of 100, or in fraction notation 10/100. Therefore, when we see that a shirt is sold for a 50 percent discount, we actually say 50 out of 100 or 50/100. Notice that 50/100 when reduced to lowest terms is 1/2 which means that we only have to pay half of the price of the shirt. As we all know, we use the symbol % to denote percent.

Converting Percent to Fractions for Faster Calculations

Numbers in their percent form can be converted to fractions for quicker calculations. For example, when we say that a Php2400.00 wristwatch has a 25% discount, we can easily calculate by converting 25% to fraction. The equivalent of 25% discount is 1/4 in fraction, so, we deduct 1/4 of 2400 (which is equal to Php600) from Php2400. This means that we can buy the watch for only Php 1800.00

Percents, fractions, and decimals can be converted to one another, to whichever representation is more convenient for calculations. In examinations such as the Civil Service Exam, in most cases, fraction is the easiest to use but the problem is conversion also takes time. Therefore, it is also good to familiarize yourself with the conversion of the most commonly used fractions in problems such as shown in the table below. You can memorize them if you want, but the conversion method is fairly easy that you can do them mentally.

Percent	Fraction	Percent	Fraction
10%	1/10	50%	1/2
20%	1/5	60%	3/5
25%	1/4	75%	3/4
40%	2/5	80%	4/5

In the next few post, will discuss how to convert fractions, percents, and decimals to one another. Then, we will also discuss common percentage problems like discounts and interests. These types of problems usually appear in the Math Word Problem Solving section of the Civil Service Examination.

A Review on Operations on Real Numbers

We had just finished discussing the different operations on integers: addition, subtraction,multiplication and division. Since you are already familiar with these operations onintegers, the operations on real numbers (integers, decimals, andfractions) will be very easy for you becausethe process is just the same. For example, positive a real number 0.4 is multiplied by a negative real number 0.1, the result is negative just like multiplying positive and negative integers.

Below are worked examples on the operations on real numbers. I made the examples easy so that you can recognize the pattern even if you forgot the rules on operating with decimals and fractions. Do not worry though if you have forgotten the rules because I will have separate posts about them. For now, try to solve and by compare your answer with the calculated results below.

Addition

Rules:

- Positive Number + Positive Number = Positive Number
- Negative Number + Negative Number = Negative Number
- Positive Number + Negative Number: Subtract, and then take the sign of the larger number when the negative sign is disregarded.

Worked Examples on Addition of Real Numbers
1. 4.3 + 2.5 = 6.8

2. 8.7 + -3.7 = 5 (8.7 is larger whether the negative sign is disregarded or not)

3. -1/5 + -2/5 = -3/5

4. -10.3 + 2.2 = 10.3 – 2.2 = -8.1 (-10.1 is larger when the negative sign is disregarded, so the answer is negative)

Subtraction
Rule: Change the subtraction to addition, then use the addition rules.

Worked Examples on Subtraction of Real Numbers
1. 8.9 – 7.2 = 1.7

2. 4.3 – (-3) = 4.3 + 3 = 7.3

3. -2/5 – 1/5 = -2/5 + -1/5 = -3/5

4. -2/7 – (-1/7) = -2/7 + 1/7 = -1/7

Multiplication
Rules

- Positive Number x Positive Number = Positive Number
- Positive Number x Negative Number = Negative Number
- Negative Number x Positive Number = Negative Number
- Negative Number x Negative Number = Positive Number

Worked Examples on Multiplication of Real Numbers
1. 2.3 x 1.2 = 2.76

2. 1.6 x -3 = -4.8

3. -2.5 x 4 = -10

4. – 0.5 x -1.2 = 0.6

Division

Rules

- Positive Number ÷ Positive Number = Positive Number
- Positive Number ÷ Negative Number = Negative Number
- Negative Number ÷ Positive Number = Negative Number
- Negative Number ÷ Negative Number = Positive Number

Worked Examples on Division of Real Numbers

1. 4/7 ÷ 2/7 = 2

2. 5.2 ÷ -2 = -2.6

3. -3.2 ÷ 1.6 = -2

4. -7.5 ÷ -2.5 = 3

That completes are worked examples on operations on integers. In the next posts, we will be discussing operations on fractions and decimals.

PEMDAS Rules and Operations on Real Numbers

Now that you have already learned the four fundamental operations on real numbers – addition, subtraction, multiplication, division – it is time to combine these operations into a single problem. In the Philippine Civil Service Examination, most of the problems on operations on real numbers have at least two or more operations involved. If you can recall, we call these operations MDAS in theelementary grades and later and PE making it PEMDAS. PEMDAS is the acronym for Parenthesis, Exponent, Multiplication, Division, Addition and Subtraction. This is basically the order of operations when you calculate an arithmetic problem involving two or more operations.

PEMDAS RULES

Calculate in the following order.

1. the expressions inside the **P**arentheses.

2. the expression with **E**xponents.
3. If no operation separates **M**ultiplication and **D**ivision, perform from left hand side to right whichever comes first.
4. If no operation separates **A**ddition and **S**ubtraction, perform from left hand side to right whichever comes first.

EXAMPLES

Example 1: 4 + 3 x 5

Perform multiplication first before addition since M comes before A in PEMDAS.

Multiply: 4 + 3 x 5 = 4 + 15
Add: 4 + 15 = 19.

Example 2: (3 + 3) x 5

Simplify the expression inside the parenthesis first before multiplying since P comes before M in PEMDAS.

Parenthesis: (3 + 3) x 5 = 6 x 5
Multiply: 6 x 5 = 30.

Example 3: 8 + 4^2 x 3

Simplifying the expression with exponent first, the multiply, and then add.

Exponent: 8 + 4^2 x 3 = 8 + 16 x 3
Multiply: 8 + 16 x 3 = 8 + 48
Add: 8 + 48 = 56

Example 4: 3 x 4 + 6 x 2 – 5

Perform multiplication simultaneously, add, and then subtract.

Multiply: 3 x 4 + 6 x 2 – 5 = 12 + 12 – 5
Add: 12 + 12 – 5 = 24 – 5
Subtract: 24 – 5 = 19

Example 5: (4 + 5) x $(8 – 2)^2$ ÷ 2

Perform the operations inside the parentheses simultaneously, simplify the operation with exponent, multiply, and then divide.

Parentheses: $(4 + 5) \times (8 - 2)^2 \div 2 = 9 \times 6^2 \div 2$
Exponent: $9 \times 6^2 \div 2 = 9 \times 36 \div 2$
Multiplication: $9 \times 36 \div 2 = 324 \div 2$
Divide: $324 \div 2 = 162$

Example 6: $(5 + 8)^2 - 18 \div 6 \times 2$
Parenthesis: $(5 + 8)^2 - 18 \div 6 \times 2 = 13^2 - 18 \div 6 \times 2$
Exponent: $13^2 - 18 \div 6 \times 2 = 169 - 18 \div 6 \times 2$
*Divide: $169 - 18 \div 6 \times 2 = 169 - 3 \times 2$
*Multiply: $169 - 3 \times 2 = 169 - 6$
Subtract: $169 - 6 = 163$

COMMON MISTAKES in PEMDAS
I have placed * in the division and multiplication operations in the last example above because this is one of the misconceptions of a lot of people. Many think that since M comes before D, multiplication must always be performed first before division.
That is NOT ALWAYS the case.

If multiplication and division operations are **NOT SEPARATED** by other operations or grouping symbols, you must simplify from the left hand side to the right hand side whichever operation comes first. For example, the expression $6 \div 3 \times 2$ has no +, – or () in between. So what is the answer? If your answer is 4, you are correct. Why? Notice that if multiplication is performed first, then the expression becomes $6 \div 6 = 1$. But the correct answer is 4 because $6 \div 3 = 2$ and $2 \times 2 = 4$. If you don't believe me, try to key in the expression $6 \div 3 \times 2$ in a calculator and then press the equal sign.

PEMDAS Rules Practice 1 Solutions

Below are the solutions and answers to the problems in PEMDAS Rules Practice 1. Notice that I have color coded the solution to guide you which operation results to which answer. I have also varied the notations like / and ÷ to familiarize you with both of them. In addition, I have also included operations on fractionswith expressions in the numerator and the denominator. In a fraction whose numerator and/or denominator contains one or more operations , you have to simplify first both the numerator and the denominator before dividing. The methods in calculating fractionsare shown in numbers 7, 9 and 10.

PEMDAS Rules Practice 1 Solutions

1. $2 \times 3 + 4 \times 6$

Solution:

Multiply: 2 x 3 + 4 x 6 = 6 + 24
Add: 6 + 24 = 30
Answer: 30

2. $(-3)(2) + 18 \div 3$

Solution:

Multiply: $(-3)(2) + 18 \div 3 = -6 + 18 \div 3$

Divide: $-6 + 18 \div 3 = -6 + 6$

Add: $-6 + 6 = 0$

Answer: 0

3. $4 + (6 - 2)^2 + 1$

Solution:

Parenthesis: $4 + (6 - 2)^2 + 1 = 4 + 4^2 + 1$
Exponent: $4 + 4^2 + 1 = 4 + 16 + 1$
Add: 4 + 16 + 1 = 21

Answer: 21

4. $8(6 - 2) \div 2(5 - 3)$

Solution:

Parenthesis: 8(6 – 2) ÷ 2(5 – 3) = 8(4) ÷ 2(2)
Multiply: 8(4) ÷ 2(2) = 32 ÷ 2(2)*
Divide: 32 ÷ 2(2)= 16(2)
Multiply: 16(2) = 32

Answer: 32
*This is the case mentioned in the PEMDAS Rules that when multiplication and division are performed consecutively (without any other operations or grouping symbols in between), the perform the operations from the left hand side to the right hand side.

5. $(-12)(-3) + 8^2$

Solution:

Exponent: (-12)(-3) + 8^2 = 36 + 64
Multiply: (-12)(-3) + 64= 36 + 64
Add: 36 +64 = 100

Answer: 100

6. $4 \div 5 \times 25 + 2$
Solution:

Divide: 4/5 x 25 + 2 = 0.8 x 25 + 2*
Multiply: 0.8 x 25 + 2 = 20 + 2
Add: 20 + 2 = 22

Answer: 22
*This is the case mentioned in the PEMDAS Rules that when multiplication and division are performed consecutively (without any other operations or grouping symbols in between), the perform the operations from the left hand side to the right hand side.

7. $\frac{-9(2+1)}{-2(-2-1)}$

Solution:

Numerator:
Parenthesis: -9(2 + 1) = -9(3)
Multiply: -9(3) = -27

Denominator:
Parenthesis: -2(-2 – 1) = -2(-3)
Multiply: -2(-3) = 6

Divide the numerator by the denominator: -27/6 = -4.5

Answer: -4.5

8. $4(3+1)-2(5-2)$

Solution:

Parenthesis: 4(3 + 1) – 2(5 -2) = 4(4) – 2(3)
Multiply: 4(4) – 2(3) = 16 – 6
Subtract: 16 – 6 = 10

Answer: 10

9. $\frac{14}{-3-4}$

Solution

Denominator: -3 – 4 = -7

Divide the numerator by the denominator: 14 ÷ -7 = -2

Answer: -2

10. $\dfrac{2^2 - 4^2}{-3-1}$

Numerator:
Exponent: $2^2 - 4^2 = 4 - 16$
Subtract: 4 – 16 = -12

Denominator: – 3 – 1 = -4

Divide the numerator by the denominator: -12 ÷ -4 = 3

Answer: 3

11. $-(-3) + 8 \div 4$

Solution:

We know that -(-3) = 3, so we only have two operations to perform.

Divide: -(-3) + 8 ÷ 4 = 3 + 2
Add: 3 + 2 = 5

Answer: 5

12. $9^2 - 8 - 2^3$

Solution:

Exponent: $9^2 - 8 - 2^3 = 81 - 8 - 8$
Subtract: 81 – 8 – 8= 73 – 8
Subtract: 73 – 8 = 65

Answer: 65

13. $(-7-9)(8-4)+4^3 \div 8$

Parenthesis: (-7 – 9) (8 – 4) + 4^3 ÷ 8 = (-16)(4) + 4^3 ÷ 8
Exponent: (-16)(4) + 4^3 ÷ 8 = (-16)(4) + 64 ÷ 8
Multiply: (-16)(4) + 64 ÷ 8 = -64 + 64 ÷ 8
Divide: -64 + 64 ÷ 8 = -64 + 8
Add: -64 + 8 = 56

Answer: -56

14. $6+3\times 2-12\div 4$

Multiply: 6 + 3 x 2 – 12/4 = 6 + 6 – 12/4
Divide: 6 + 6 – 12/4 = 6 + 6 – 3
Perform Addition and Subtraction from left to right: 6 + 6 – 3 = 9

Answer: 9

15. $7\times(3+2)-5$

Parenthesis: 7 x (3 + 2) – 5 = 7 x 5 – 5
Multiply: 7 x 5 – 5 = 35 – 5
Subtract: 35 – 5 = 30

Answer: 30

Real Number Operations and PEMDAS Practice Test 1

In the previous post, you have learned thePEMDAS rules or the rules in performing arithmetic operations namely addition,subtraction, multiplication and division. In this post, you will practice to see if you have mastered these rules. I have mixed the notations so that you will be familiarized with all of them. For example, $4\times 8+3\times -2$ can also be written as $4(8)+3(-2)$ or $(4)(8)+(3)(-2)$.

You should also be familiar with division where in the expression

$$\frac{(3+2)\times 3}{3+5}$$

the operation in both the numerator and numerator are simplified first before dividing the expressions. This is equivalent to $(3+2)(3) \div (3+5)$. Do not worry though regarding the use of parentheses, we will discuss them in the next topic. For now, answer the questions to the best of your abilities.

PEMDAS Practice Test 1

1. $2 \times 3 + 4 \times 6$

2. $(-3)(2) + 18 \div 3$

3. $4 + (6-2)^2 + 1$

4. $8(6-2) \div 2(5-3)$

5. $(-12)(-3) + 8^2$

6. $4 \div 5 \times 25 + 2$

7. $\frac{-9(2+1)}{-2(-2-1)}$

8. $4(3+1) - 2(5-2)$

9. $\frac{14}{-3-4}$

10. $\frac{2^2-4^2}{-3-1}$

11. $-(-3) + 8 \div 4$

12. $9^2 - 8 - 2^3$

13. $(-7-9)(8-4) + 4^3 \div 8$

14. $6 + 3 \times 2 - 12 \div 4$

15. $7 \times (3 + 2) - 5$

In the next post, I will give the solutions to the practice test above. After that, I will also discuss on how to calculate expressions with nested parentheses.

How to Convert Decimal Numbers to Percent

Conversions of decimals, fractions, and percent is a very important basic skill inmathematics and many problems in the Civil Exams require this skill. Being able to convert from one form to another will help you speed up in calculations.   For example, instead of multiplying a number by 25%, you just have to get its 1/4 or simply divide it by 4.

Percent usually appears in **discount** and interest problems while fractions and decimals appear in various types of problems.

How to Convert Decimals to Percent

To convert decimal percent, you just have to multiply the decimal by 100.

Example 1

What is 0.25 in percent?

Solution

0.25 × 100 = 25

So, the answer is 25%.

Example 2

What is 0.08 in percent?

0.08 × 100 = 8

Therefore, the answer is 8%.

Of course, there are cases that the given is more than one such as the next example

Example 3

What is 1.8 in percent?

Solution
1.8 × 100 = 180

Therefore, the answer is 180%.

Example 4
What is 0.009 in percent?

Solution
0.009× 100 = 0.9%

Notice that some percent can also have decimal point such as shown in Example 4. In dealing with many decimals, if we multiply them with 100, we just move two decimal places to the right.

In the next post, we are going to discuss the other way around. That is, how to convert, percent to decimals.

How to Convert Decimals to Fractions Part 1

We have learned how to **convert fractions to decimals** and in this post, we are going to learn how to convert decimals to fractions. Before doing this, we need to review the meaning of **place value**. In the decimal number 0.532, 5 is the *tenths place*, 3 is the*hundredths place*, 2 is the *thousandths place.*
The number 5 tenths is the same as $5 \times \frac{1}{10}$, 3 hundredths is the same as $3 \times \frac{1}{100}$ and 2 thousandths is the same as $2 \times \frac{1}{1000}$. In converting decimals to fractions, we have to see the place value of the last digit of the decimal place.

Example 1
Convert 0.7 to fraction.

Solution

0.7 is 7 tenths or $7 \times \frac{1}{10} = \frac{7}{10}$.

Therefore, the equivalent of 0.7 in fraction is the same as $\frac{7}{10}$

Example 2

Convert 0.6 to fraction.

0.6 is $6 \times \frac{1}{10} = \frac{6}{10}$

We **reduce the fraction to lowest terms** by dividing both the numerator and the denominator by the **greatest common factor** of 6 and 10 which is 2.

$$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Therefore, the equivalent fraction of 0.6 is $\frac{3}{5}$.

Example 3

Convert 0.12 to fraction

The last digit of the decimal is in the hundredths place, so we can read this as 12 hundredths.

Twelve hundredths is $12 \times \frac{1}{100} = \frac{12}{100}$.

We convert this fraction to lowest terms by dividing both the numerator and denominator by the greatest common factor of 12 and 100 which is equal to 4. So,

$$\frac{12 \div 4}{100 \div 4} = \frac{3}{25}.$$

Therefore, the equivalent of 0.12 in fraction is $\frac{3}{25}$

Example 4

Convert 0.375 to fraction.

Solution

The last digit of the decimal number above is in the thousandths place. So, we can read it as 375 thousandths.

Now, 375 thousandths is the same as $375 \times \frac{1}{1000} = \frac{375}{1000}$.

We convert 375 thousandths to lowest terms by dividing both its numerator and denominator by the greatest common factor of 375 and 1000 which is equal to 125. That is,

$$\frac{375 \div 125}{1000 \div 125} = \frac{3}{8}.$$

Therefore, the equivalent fraction of 0.375 is $\frac{3}{8}$

How to Convert Fraction to Percent Part 1

In the previous post, we have learned how to convert percent to fraction. In these series of posts, we learn the opposite: how to convert fraction to percent. I am going to teach you three methods, the last one would be used if you forgot the other two methods, or if the first two methods would not work. Please be reminded though to understand the concept (please do not just memorize).

The first method can be used for fractions whose denominators can be easily related to 100 by **multiplication** or **division**. Recall that from **Converting Percent to Fraction**, I have mentioned that when we say *percent* it means "per hundred." In effect, n% can be represented by n/100. Therefore, if you have a fraction and you can *turn* it into n/100 (by multiplication/division), then you have turned it into percent.

Example 1: What is the equivalent of 1/5 in percent?

How do we relate the denominator 5 to 100? By multiplying it by 20. Therefore, we also multiply its numerator by 20:

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$$

Now, since we have 100 as denominator, the answer in percent is therefore the numerator. Therefore, the equivalent of 1/5 in percent is 20%.

Example 2: What is 3/25 in percent?
Again, how do you related 25 to 100? By multiplying it by 4. Therefore,

$$\frac{3 \times 4}{25 \times 4} = \frac{12}{100}$$

Therefore, the equivalent of 3/25 in percent is 12%.

Example 3: What is 23/200 in percent?
In this example, we can relate 200 to 100 by dividing it by 2. So, we also divide the numerator by 2. That is

$$\frac{23 \div 2}{200 \div 2} = \frac{11.5}{100}$$

Therefore, the answer is 11.5%

There are two important things to remember in using the method above.

(1) in changing the form the fractions to n/100, the only operations that you can use are multiplication and division and

(2) whatever you do to the numerator, you also do to the denominator.

Note that multiplying the denominator (or dividing it) by the same number does not change its value, it only change its representation (fraction, percent or decimal).

Why It Works
When you are relating a fraction a/b to n/100, you are actually using ratio and proprotion. For example, in the first example, you are actually solving the equation

$$\frac{1}{5} = \frac{n}{100}.$$

The equation will result to $n = \frac{100}{5}$ which is equal to 20. Now, this is just the same as multiplying both the numerator and the denominator by 20.

Note that the method of "relating to 100 by multiplication or division" can only work easily for denominators that divides 100 or can be divided by 100. Other fractions (try 1/7), you have to use ratio and proportion and manual division.

How to Convert Fraction to Percent Part 2

In the Part 1, we have learned how to convert fraction to percent by relating the denominator to 100 by multiplication or division. In this post, we do its 'algebraic version.' This method is a generalized methodto the previous post especially for numbers that do not divide 100 or cannot be divided by 100 easily. However, to see the relationship between the two methods, let us do the first example in Part 1 of this series.

Example 1: What is the equivalent of 1/5 in percent.

Recall that in Part 1, we multiplied both the numerator and the denominator by 20, to make the denominator 100. That is,

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$$

Now, notice how it is related to the new method. In this method, we related 1/5 to n/100. That is, what is the value of n in

$$\frac{1}{5} = \frac{n}{100}.$$

To simplify the equation, we multiply both sides of the equation by 100, and we get

$$\frac{100}{5} = \frac{100n}{100}$$

Simplifying and switching the position of the expressions, we get the $n = 20$. This means that $\frac{1}{5} = 20$.

Of course, Part 1 seems to be easier, but the good thing about putting it into equation is that it applies to all fractions. For instance, it is quite hard to convert 7/12 using the methodin part 1.

Example 2: What is the equivalent of $\frac{7}{12}$ in percent?

We set up the equation with $\frac{n}{100}$ on the left.

$$\frac{n}{100} = \frac{7}{12}$$

To eliminate the fraction, multiply both sides by denominator. This results to

$$n = \frac{7}{12}(100) = \frac{700}{12} \approx 58.33$$

or about 58.33%.

The curly equal sign means approximately equal to since 3 is a non-terminating decimal.

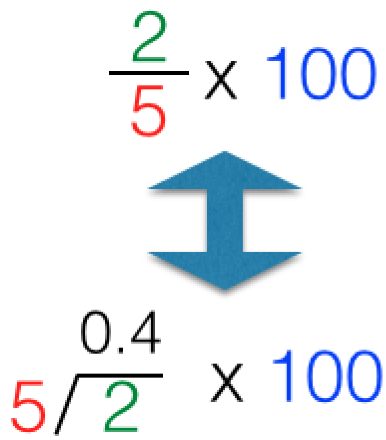
Now, try to examine the expression

$$\frac{7}{12}(100)$$

because this is where they derived the rule. Recall the rule in converting fraction to percent:**Divide the fraction and then multiply the result to 100**. That is exactly it. So, when you have the fraction, $2/5$ just divide it manually, and then multiply the result to 100. That is,

$$\frac{2}{5}(100) = \frac{200}{5} = 40.$$

Do not forget though that the divisor during division is the denominator (5 in 2/5). as shown below.

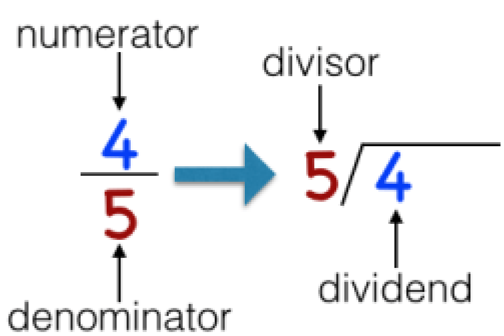


That’s it. I think we don’t have to have the third part, since we already derived the rule here.

How to Convert Fractions to Decimals

Converting fractions to decimals is one of the basic skills in mathematics that you should learn in order to pass the Civil Service Examination. Being able to convert numbers to fractions, decimals, and percents, will give you an advantage to solve problems better and faster. In this post, we are going to discuss how to convert fractions to decimals.

Recall that in fractions, the number at the top of the fraction bar is called the **numerator**and the number at the bottom of the fraction bar is called the **denominator**. In converting fractions to decimals we divide: the numerator becomes the dividend and the denominator becomes the divisor (don’t switch!).



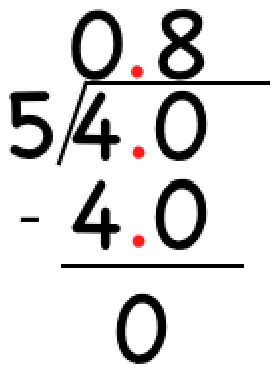
In converting fractions to decimals, you should divide the numerator by the denominator manually. Take note of this step because most solvers switch their places.

Example 1: Convert $\frac{4}{5}$ to decimals.
First, 4 divided by 5 cannot be done, so we place 0 in the quotient.

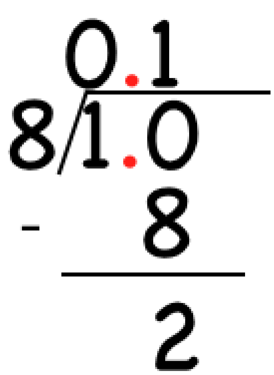
Second, we add the decimal point and place 0 after the decimal point in the dividend. We also add the decimal point to the quotient aligned with the first decimal point.

Third, ignoring the decimal point, we divide 40 by 5, which gives us 8. We write 8 at the right of the decimal point and continue our calculation.

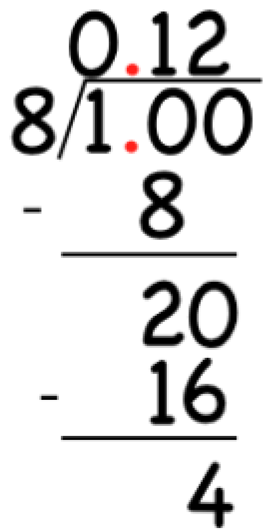
So, $\frac{4}{5}$ in decimals is 0.8.



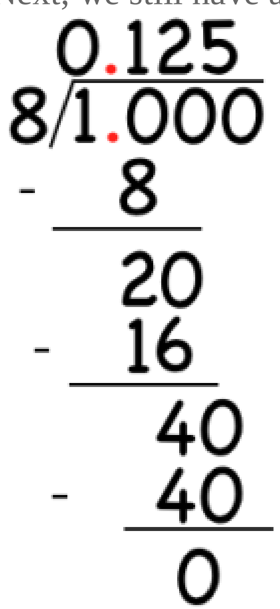
Example 2: Convert $\frac{1}{8}$ to decimals.
Again, we align the decimals and divide 1 with 8 which cannot be, so we place 0 in the quotient. Next, we add the decimal point and 0 to the dividend. Now dividing 10 by 8, we get 1 a quotient as shown below.



After subtraction, we still have a remainder. So, we add another 0 in the dividend as shown. Performing division, we have the following calculation.



Next, we still have a remainder. Adding 0, we have the following calculation.



Therefore $\frac{1}{8} = 0.125$.

Example 3: There are cases that the decimal in non-terminating such as $\frac{1}{3}$. If you calculate this fraction, it will give you 0.333333 with never ending 3's. So, you can just round to 0.33 or depending on the number of decimal places required.
Example 4: There are cases that the decimals are repeating. For example, if we convert $\frac{1}{7}$ to fractions, we get 0.142857142857 with 142857 repeating. Again, in examinations, they usually tell you to round your answers to the nearest place values.

Example 5: For mixed fractions, you can just ignore the whole number, and then convert the fraction to decimals. After you have calculated the decimal, add the whole number.
For example, how do we convert $9\frac{4}{5}$ to decimals.

First, we ignore the whole number. Then, we convert $\frac{4}{5}$ to decimals which is 0.8 in Example 1. Lastly, we add 9 and 0.8 which is equal to 9.8.
$9\frac{4}{5} = 9.8$.

Practice Quiz on Converting Fractions to Decimals

In the previous post, we have learned **how to convert fractions to decimals**. Now, let's see what you have learned in converting fractions to decimals by answering the following quiz.
Convert the following fractions to decimals.

1.) 1/5

2.) 3/4

3.) 2/3

4.) 3/8

5.) 5/6

6.) 7/10

7.) 7/15

8.) 5/8

9.) 5/7

10.) 9/20

11.) $8\frac{3}{4}$

12.) $12\frac{7}{9}$

Answer key

1.) 0.2

2.) 0.75

3.) 0.666… (repeating never ending 6’s) or 0.67 if rounded to nearest hundredths

4.) 0.375

5.) 0.8333… (repeating never ending 3’s)

6.) 0.7

7.) 0.4666 (repeating never ending 6’s)

8.) 0.625

9.) 0.714285714285 (repeating never ending 714285’s)

10.) 0.45

11.) 8.75

12) 12.777… (repeating never ending 7’s)

How to Convert Percent to Decimals

In the previous post, we have learned **how to convert decimals to percent**. In this post, we learn the opposite of this procedure. We learn how to convert percent to decimals.

If you can remember from the previous post, we convert decimals to percent by multiplying the decimal by 100. So, in this case, we divide percent by 100 in order to get the decimal value. Remember: division is the inverse operation of multiplication.

Example 1
Convert 85% to decimals.

Solution
We divide 85% by 100 which means that we will move two decimal places to the let. Note that the decimal point is on the immediate right of the ones place (in this case 5). So, if we move the decimal point two places to the left, we have .85 or 0.85. Note that we usually add one 0 to the left of the decimal point if there is no whole number.

Answer: 0.85

Example 2
What is 40% in decimal?

Solution
Again, it is a whole number, so the decimal point is at the right of 0. Moving the decimal point two places to the left, we have .40 or 0.40

Answer: 0.40 or 0.4 (0 at the right of the decimal numbers may be omitted)

Example 3
Convert 65.2% to decimal.

Solution
This is not a whole number. We can see the decimal point between 5 and 2. Moving the decimal two places to the left, we end up with .652 or 0.652.

Answer: 0.652

Example 4
Convert 2.5% to decimal.

Solution
There is only one number to the left of the decimal place. But we need to move two places, so, we add 0. That becomes .025 or 0.025

Answer: 0.025

Example 5
What is 124% to decimal.

Solution

This is a whole number, so the decimal point is at the right of 4. Moving the decimal point to the left we have 1.24.

Answer: 1.24

Example 6
What is 0.8% in decimal?

Solution
Moving 2 decimal places to the left, we have .008. So the answer is 0.008.

Answer: 0.008
Example 7
A t-shirt worth P600 has a 15% discount. How much is the discount?

Solution
The equivalent of 15% to decimal is 0.15

Now, 600 × 0.15 = 90

Therefore, the discount is Php90.

How to Convert Fractions to Decimals

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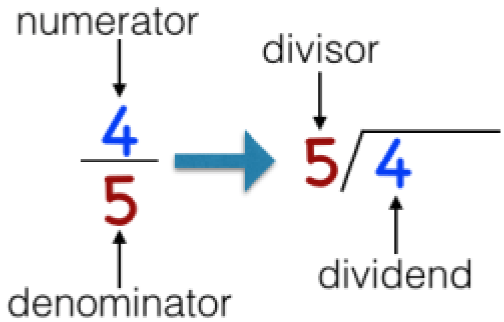

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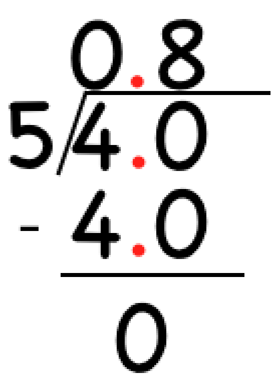

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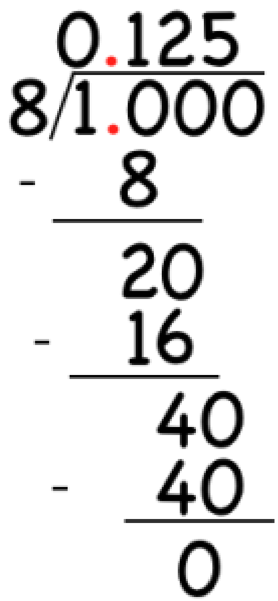
Again, we align the decimals and divide 1 with 8 which cannot be, so we place 0 in the quotient. Next, we add the decimal point and 0 to the dividend. Now dividing 10 by 8, we get 1 a quotient as shown below.

$$\begin{array}{r} 0.1 \\ 8\overline{)1.0} \\ -8 \\ \hline 2 \end{array}$$

After subtraction, we still have a remainder. So, we add another 0 in the dividend as shown. Performing division, we have the following calculation.

$$\begin{array}{r} 0.12 \\ 8\overline{)1.00} \\ -8 \\ \hline 20 \\ -16 \\ \hline 4 \end{array}$$

Next, we still have a remainder. Adding 0, we have the following calculation.



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9.) 0.714285714285 (repeating never ending 714285's)

10.) 0.45

11.) 8.75

12) 12.777… (repeating never ending 7's)

A Teaser on Answering Number Series Questions

First of all, I would like to point out the term series in the "Number Series" questions in theCivil Service Examinations is a bit incorrect. Technically, the list of numbers in the examinations is actually called a **sequence.** A**series** is a sequence of sums — well, I will not go into details since it is not included in the examinations. You can click the link though if you want to know about it.

Second, this is quite a premature discussion since I have only written a few posts about integers. I planned to write about this later, but I thought that a teaser would be nice. In this post, I will show you that it is a must to master all the topics in mathematics because they are all connected. We will not discuss the strategies on how to answer the sequence problems here; I will have a separate post about them later. Don't stop reading though because you are going to miss half of your life if you do (kidding).

A **sequence** or a **progression** is an ordered list of objects which can be numbers, letters, or symbols. The list 3, 7, 11, 15, 19 is a sequence where 3 is the first term and 19 is the fifth term. Of course, it is easy to see the sixth term is 23 since each term is the sum of 4 and term before it.

There are also sequences that are in decreasing order such as 12, 5, -2, -9, -16 and so on. As you can observe, to get the next term, 7 is subtracted from the term before it. Notice also that this sequence needs knowledge on subtraction of negative integers.

The list

$$\frac{3}{5}, \frac{11}{10}, \frac{8}{5}, \frac{21}{10}, \frac{13}{5}$$

is also an example of a sequence. This sequence involves addition of fractions. The next term can be easily solved by converting the given into similar fractions which when done will result to

$$\frac{6}{10}, \frac{11}{10}, \frac{16}{10}, \frac{21}{10}, \frac{26}{10}.$$

Clearly, we only need to add $\frac{5}{10}$ to the last term to get the next term which equals $\frac{31}{10}$.

In the sequences above, we have only used two number representations (integers and fractions) and two operations (addition and subtraction). In the actual Civil Service Examinations, the sequences can also include one or a mixture of other number representations such as percent, decimal, mixed numbers, and a combination of these representations. They can also include the four fundamental operations (addition, subtraction multiplication and division). When I took the Civil Service Examination in 2002 and 2003, there are fractions, whole numbers, and decimals in a single given number sequence. I know that 2002 was a long time ago, but the format of the examination had not changed since.

For now, we will abandon sequences and return to basic Mathematics and English in the next few posts. When all the pre-requisite knowledge are discussed, we will learn the strategies on answering number sequence questions.

How to Solve Civil Service Exam Number Series Problems 1

First of all let me clarify that what you are solving in the Civil Service Examination are number sequences (or letter sequences) and not a number series. A series has a different meaning in mathematics.

Before proceeding with the discussion below, first, try to find the next term in the following sequences.

1. 4, 7, 10, 13, ___

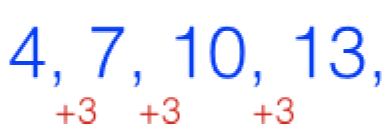
2. 17, 11, 5, -1, ___

3. C, F, I, L, ___

4. $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{11}{6}.$

Solution and Explanation

Numbers 1 and 2 are the easiest type of sequence to solve. This is because they are integers and you just add (or subtract) a constant number to each term to get the next term. In solving this type of sequence, you can see this pattern by subtracting adjacent terms (13 – 10 = 3, 10 – 7 = 3, 7- 4 = 3) to see if the difference is constant. If it is, then you will know that you will just have to add the same number to get the next term. Therefore, the next term to the first sequence is 13 + 3 = 16.



Of course, sequences can also be decreasing. In the second example, the difference is 6 or it means that 6 is subtracted from a number to get the next term (see Subtraction of Integers). Therefore, the next term is -1 – 6 = -7.

17, 11, 5, -1
-6 -6 -6

The third example is composed of letters but the principle is the same: constant difference or constant skips. C and F, for instance has two letters in between. This is also true between F and I and I and L. Therefore, the next letter in the sequence is O (L, M, N, O).

C, D, E, F, G, H, I, J, K, L

Fractions and decimals are also included in the sequence problems, so it is important that you master them. In the following example, one half is added each time. As you can see, it is not easy to find the next term of this sequence without manually solving it. The firststrategy in solving fraction sequences problems is to subtract the adjacent terms such as
$\frac{11}{6} - \frac{4}{3} = \frac{3}{6}$ which is equal to $\frac{1}{2}$ (since $\frac{4}{3} = \frac{8}{6}$I

$\frac{4}{3} - \frac{5}{6} = \frac{3}{6}$ which is equal to $\frac{1}{2}$

$\frac{5}{6} - \frac{1}{3} = \frac{3}{6}$ which is equal to $\frac{1}{2}$ since $\frac{1}{3} = \frac{2}{6}$.

There is however a better strategy than subtracting the adjacent terms when it comes to sequences on fractions. Sometimes, it is easier to see the pattern if you convert them to similar fractions (fractions with the same denominator). Converting the sequence above to similar fractions gives us

$$\frac{2}{6}, \frac{5}{6}, \frac{8}{6}, \frac{11}{6}.$$

From here, it is clear that the next term in the sequence is $\frac{14}{6}$. Note however that thisstrategy is best only for sequences with constant difference and may be difficult to use in other types of sequences.

In the next post, we are going to discuss about another type of sequence.

How to Solve Civil Service Exam Number Series Problems 2

In the previous post, we have learned how to solve number sequence (for the Civil Service Exam Number Series test) and letter sequence problems that involves constant difference or constant skips. In this post, we are going to discuss another type of sequence. Before we discuss, see if you can find the next term of the following sequences.

1. 3, 6, 12, 24, 48, ___

2. 18, 6, 2, 0.66…, ___

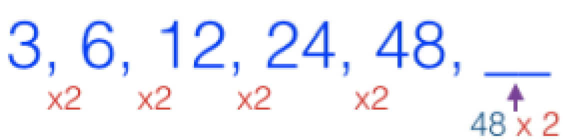
3. 1, 4, 9, 16, 25, ___

4. 3, 12, 27, 48, 75, ___

Solution and Explanation

First Sequence: 3, 6, 12, 24, 48, ___

In the first sequence, the first that you will notice is that the second term is twice the first term. So, the next thing that you should ask is, "Is the third term twice the second term?" Yes, 12 is twice 6. What about the next term? Yes. So, each term in the sequence is multiplied to 2 to get the next term. Therefore, the missing term is 96 which is 48 multiplied by 2.



If we look at the difference of the numbers in the sequence above, we can see that the number we add is also increasing twice. To get 6, we added 3. To get 12, we added 6. To get24, we add 12 and so on. As we can see, the sequence of the numbers we add (the numbers in red color) is the same as the original sequence (numbers in blue color).

3, 6, 12, 24, 48, ___

+3 +6 +12 +24 +48

Second Sequence: 18, 6, 2, 0.66…, ___

In the second sequence, the number is reduced each time. Since they are integers, it can either be subtraction or division. As we can see, 6 is a third of 18. This means that to get 6, 18 is divided by 3. Now, look at the next term. It's 2. So it is also a third of 6. Can you see the pattern now?

Each term is divided by 3 to get the next term. So, we must divide 0.66… by 3. therefore, the next term is 0.22… The three dots means that the 2's are infinitely many.

Third Sequence: 1, 4, 9, 16, 25, ___

What is familiar with this sequence? They are all square numbers! That is,

$$1^2, 2^2, 3^2, 4^2 \text{ and } 5^2.$$

So the next term is 6^2 which is 36.

Fourth Sequence: 3, 12, 27, 48, 75, ___

The fourth sequence seems difficult, but I have just multiplied each number in the third sequence by 3. So, if the sequence is not familiar, try to see if you can divide it by any number. As you can see,

$$3, 12, 27, 48, 75 = 3\,(1, 4, 9, 16, 25)$$

or the product of 3 and the square numbers.

In the next post, we are going to discuss "alternating sequences."

How to Solve Civil Service Exam Number Series Problems 3

This is the third part of the solving number series problems. The first part includes dealing with patterns that contains addition and subtraction and the second partdiscusses patterns that contains multiplication or division.

In this post, we are going to learn some "alternating sequences." I put a quote inalternating sequence because in mathematics, it has a slightly different meaning. Note that it is likely that these type of sequence will appear in examinations such as the Civil Service Exam.

Before we continue with the discussion, try to see if you can answer the following questions.

1. 2, -5, 4, -8, 6, -11, 8, -14, ___, ___

2. 4, 7, 12, 15, 20, 23, 28, ____

3. A, 3, D, 8, G, 13, ___, ___

4. $\frac{1}{2}$, 5, 1, 9, $\frac{3}{2}$, 13, ____, _____

Solutions and Explanations

First Sequence: 2, -5, 4, -8, 6, -11, 8, -14, ___, ___

The first sequence seems hard, but it is actually easy. If you perform addition and subtraction among consecutive terms, you will surely see a pattern (left as an exercise). However, before doing it, notice that the sign of the numbers are alternating: that is, positive, then negative, then positive, and so on.

Now, what if, we separate the two sequences? What if we treat the positive numbers as a sequence, and the negative numbers as another sequence. Well, we just put different colors on them, so it is easy to see the pattern.

2, -5, 4, -8, 6, -11, 8, -14, ___, ___

Do you see now? Can you answer the problem?

As you can see, the red numbers are just increasing by 2 and the blue numbers are decreasing by 3. Therefore, the next numbers are 10 and -17.

Second Sequence:. 4, 7, 12, 15, 20, 23, 28, _____

In the second sequence, 4 is increased by 3 to become 7. Then, 7 is increased by 5. The increase in the numbers are also in alternating pattern. So the correct answer is 31 which is equal to 28 + 3.

Further, what is interesting is that the "coloring strategy" that we used in the first sequence can be also used in this sequence. As you can see in the colored numbers below, it becomes two sequences as well. The sequence composed of blue numbers and the other red. In both sequences, the numbers is increased by 8. Since the next number is blue, then it is equal to23 + 8 = 31.

4, 7, 12, 15, 20, 23, 28, _____

Third Sequence: A, 3, D, 8, G, 13, ____, ____

In the third sequence, the answers are already obvious after learning the strategy above. There are two letters in between the letter terms in the sequence (*A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*). Further, each number term is 5 greater greater than the previous number term. So, the correct answer answers are ***J***, *18*.

A, 3, D, 8, G, 13, ____, ____

Fourth Sequence: $\frac{1}{2}$, 5, 1, 9, $\frac{3}{2}$, 13, _____, ______

Sequence 4 is alternating addition. The red numbers as shown in the next figure are added by 1/2 to get the next term while the blue numbers are added by 4.

We have done several examples and it is impossible for us to exhaust all patterns, so it is up to you to be able to spot them. The patterns could be different, but the principle of solving them is the same.

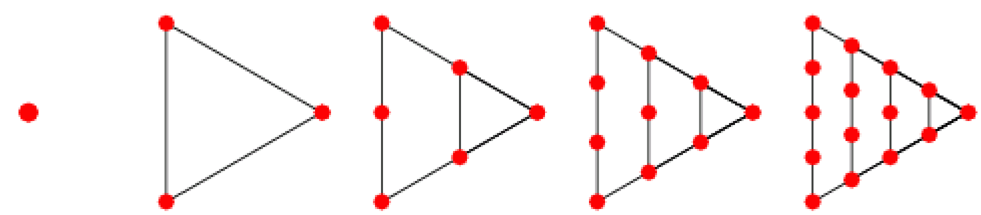
In the next post, we are going to look at some of the special sequences that may also appear in the Civil Service Exam.

How to Solve Civil Service Exam Number Series Problems 4

This is the fourth part of the solving number series problems. The first part discussed patterns that contains addition and subtraction and the second part discusses patterns that contains multiplication or division. The third part was about alternating patterns.
In this post, we are going to discuss some special number patterns. Although there is a small probability that these types of patterns will appear in the Civil Service Examination (I didn't see any when I took the exams, both professional and subprofessional), it is better that you know that such patterns exist.

Triangular Numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, …

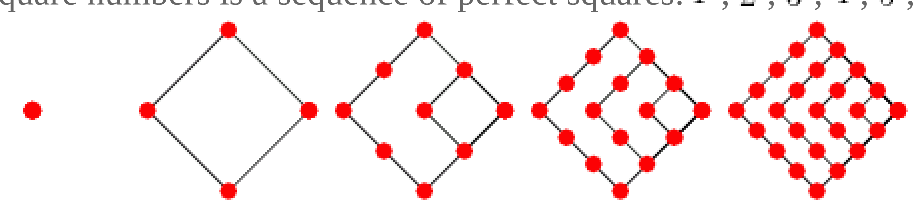


Triangular numbers are numbers that are formed by arranging dots in triangular patterns. Therefore, the first term is 1, the second term is 1 + 2, the third term is 1 + 2 + 3 and so on.

Square Numbers

1, 4, 9, 16, 25, 36, …

The square numbers is a sequence of perfect squares: 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, and so on.



Cube Numbers
1, 8, 27, 64, 125, …

Well, from square numbers, you surely have guessed what are cube numbers. They are asequence of cube of integers.

$1^3, 2^3, 3^3, 4^3, \ldots$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, …

Technically, a Fibonacci sequence is a sequence that starts with (0, 1), or (1, 1), and each term is the sum of the previous two. For example, in the sequence above, 5 is the sum of 2 and 3, while 21 is the sum of 8 and 13. In the actual examination, they may give Fibonacci-like sequences (technically called Lucas Sequence) where they start with two different numbers. For example, a Lucas sequence that starts with 1 and 3 will generate

1, 3, 4, 7, 11, 18, …

Of course, they can also combine positive and negative numbers to create such sequences. For example, a Lucas sequence that starts with -8 and 3 will generate the sequence

-8, 3, -5, -2, -7, -9, …

Well, this looks like a difficult sequence, but remember that if you can see the pattern, it is easy to look for the next terms.

Images Credit: Math World

How to Calculate the Area of a Circle

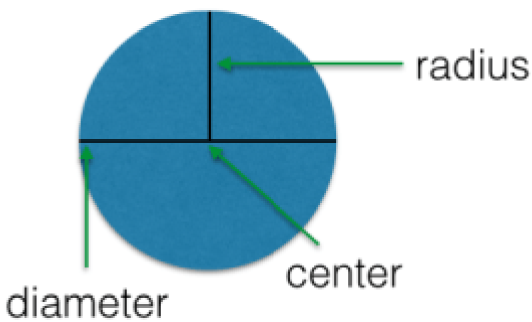
Last week, we have discussed **how to calculate the circumference of a circle**. In this post, we learn how to calculate the area of a circle. The area of a circle which we will denote by A is equal to the product of π and the square of its radius r. Putting it in equation, we have

$$A = \pi r^2.$$

In the examinations, the value of π is specified. They usually use 3.14, 3.1416 or $\frac{22}{7}$.

If you can recall, the radius is the segment from the center to the point on the circle as shown below. The radius is half the diameter. The diameter is the longest segment that you can draw from one point on the circle to another. It always passes through the center.

Note: We also use the term **radius** to refer to the *length of the radius* and **diameter** as the*length of the diameter*.



Now that we have reviewed the **basic terminologies**, let us have
some examples on how to calculate the area of a circle.

Example 1

What is the area of a circle with radius 8 centimeters.Use $\pi = 3.14$.

Solution

$A = \pi r^2$

$A = (3.14)(8^2)$

$A = (3.14)(64)$

$A = 200.96$

So, the area of the circle is 200.96 **square centimeters** (sometimes abbreviated as sq. cm.)

Be Careful! Length is measured in **units** and area is measured in **square units**. For example, the radius given is in inches (length), the answer for area is in square inches. So, since theCivil Service Exam is multiple choice, the examiner could place units and square units in the choices.

Example 2

Find the area of a circle with diameter 14 centimeters. Use $\pi = \frac{22}{7}$.

Solution

Notice that the given is the diameter, so we find the radius. Since the diameter is twice its radius, we divide 14 centimeters by 2 giving us 7 centimeters as the radius. Now, let's calculate the area.

$A = \pi r^2$

$A = (\frac{22}{7})(7^2)$

$A = 22(7)$

$A = 154$ square centimeters.

Example 3

Find the radius of a circle with area 6.28 square meters. Use $\pi = 3.14$.

Solution

In this problem, area is given. We are looking for the radius. We still use the original formula and make algebraic manipulations later, so we don't have to memorize a lot of formulas.

$A = \pi r^2$

We substitute the value of area and π.

$6.28 = 3.14r^2$

We are looking for r, so we isolate r to the right side (recall how to solve equations).

$$\frac{6.28}{3.14} = \frac{3.14r^2}{3.14}$$

$$2 = r^2$$

Since, we have a square, we get the square root of both sides. That is

$$\sqrt{2} = \sqrt{r^2}$$

$$\sqrt{2} = r$$

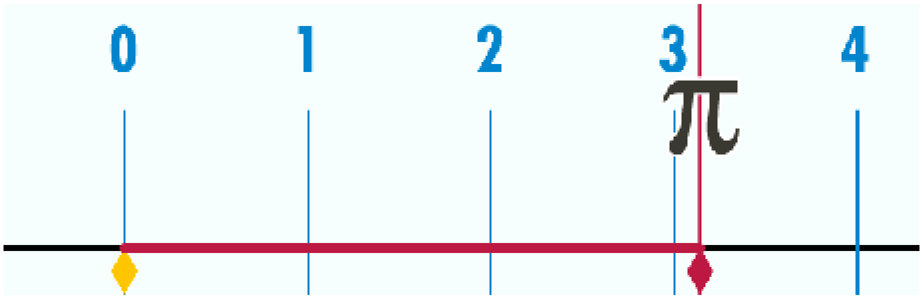
So, radius is square root of 2 meters or about 1.41 meters.

In this calculation, 2 is not a perfect square. Since **you are not allowed to use calculator**, they probably won't let you calculate for the square root of number. So, in this case, the final answer is that the radius of the circle is square root of 2 meters (meters, not square meters).
That's all for now. In the next post, we will be working on problems involving area of a circle.

How to Calculate the Circumference of a Circle

In the previous post, we have learned about the **basic terminologies about circles**. We continue this series by understanding the meaning of circumference of a circle. The circumference of a circle is basically the distance around the circle itself. If you want to find the circumference of a can, for example, you can get a measuring tape and wrap around it.

The animation below shows, the meaning of circumference. As we can see, the circle with diameter 1 has circumference π or approximately 3.14.



Note: If you want to know where π came from, read Calculating the Value of Pi.

Example 1

What is the perimeter of a circle with diameter 1 unit?

Solution

The formula of finding the circumference of a circle is with circumference C and diameter d is $C = \pi d$. So,

$$C = \pi d = \pi(1) = \pi.$$

Example 2

Find the circumference of a circle with radius 2.5 cm.

Solution

The circumference C of a circle with radius r is

$C = 2\pi r$

So, $C = 2(3.14)(2.5) = 15.7$

Therefore, the circumference of a circle with radius 2.5 cm is 15.7 cm.

Example 3

Find the radius of a circle with a circumference 18.84 cm. Use $\pi = 3.14$.

Solution

$C = 2\pi r$

$18.84 = 2(3.14)r$

$18.84 = 6.28r$

Dividing both sides by 6.28, we have

$3 = r.$

Therefore, the radius of a circle with circumference 18.84 cm is 3 cm.

Example 4

Mike was jogging in circular park. Halfway completing the circle, he went back to where he started through a straight path. If he traveled a total distance of 514 meters, what is the total distance if he jogged around the park once? (Use $\pi = 3.14$).

Solution

The distance traveled by Mike is equal to half the circumference of the circular park and its diameter. Since the circumference of a circle is $2\pi r$ and the diameter is equal to $2r$, the distance *D* traveled by Mike is
So, $D = \frac{1}{2}(2\pi r) + 2r$.

Substituting, we have $514 = \pi r + 2r$.

Factoring out r, we have $514 = r(\pi + 2)$

$514 = r(3.14 + 2)$

$514 = r(5.14)$.

Dividing both sides by 5.14, we get

$r = 100$.

Now, we are looking for the distance around the park (cirumfrence of the circle). That is,

$C = 2\pi r = 2(3.14)(100)$

$C = 628$ meters.

In the next post, we will discuss how to calculate the area of a circle.

Introduction to the Basic Concepts of Circles

The Civil Service Exams also contain geometry problems, and so far, our discussions are mostly algebra problems. In this new series of posts, we will discuss how to solve geometry and measurement problems particularly about circles. However, before we start solving problems, let us firstdiscuss the basic terminologies about circles.

A **circle** is a set of points equidistant to a point called the **center** of the circle. As I go around to give trainings and lectures , I usually hear the wrong definition below. I am not sure where this definition originated, but this is wrong.

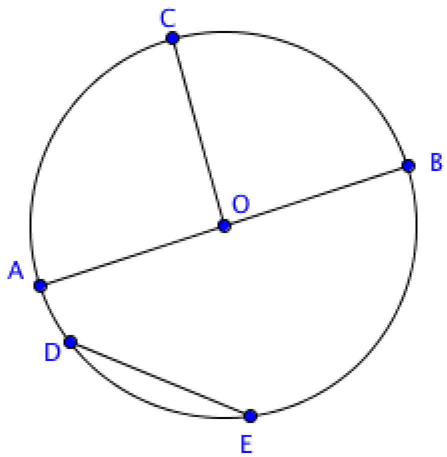
Wrong Definition: A circle is a polygon with **infinite** number of sides.

If you look at the definition of a polygon (see Wikipedia), it is a region bounded with a **finite**number of straight lines (sides). So, if you say that a circle is a polygon with infinite number of sides, it is already a contradiction. Therefore, remember from now on that **a circle is NOT a polygon and it has no side**.

Parts of a Circle

Below is a circle with center *O*. A circle is usually named using its center, so we can call it *circle O*.

A segment from the center of a circle to a point on the circle such as $\overline{OC}$ is called **radius** (plural is radii, pronounced as raid-yay). A segment joining two points on the circle such as $\overline{DE}$ is called **chord**.



The longest chord that can be made in a circle passes through the center. This chord is called **diameter**. In the figure above, $\overline{AB}$ is a diameter of circle *O*.

Some Basic Facts About Circles

Notice that diameter $\overline{AB}$ is composed of two radii, $\overline{OA}$ and $\overline{OB}$. Therefore, the diameter of a circle is twice its radius. So, if we let the diameter *D*, and radius *r*, we can say that

$$D = 2r.$$

If we measure the length of the circle, that is if we start from *B*, *go around along the circle*until we reach *B* again, the distance we would have traveled is called its *circumference*. The formula circumference *C* is

$$C = 2\pi r$$

where π is approximately 3.1416.

Since $2\pi r = \pi(2r)$ and $D = 2r$, we can also say that $C = \pi D$. Note that the circle itself (the path itself from *B* going around back to *B*) is also called circumference.

In this series, we will also learn how to calculate the area *A* of a circle in this series which has formula

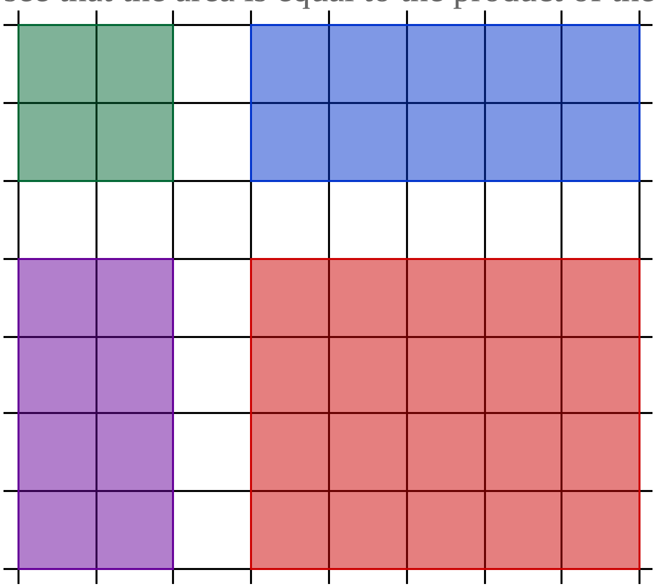
$$A = \pi r^2.$$

In the next post, we are going to discuss how to calculate the **circumference of a circle**.

Calculating Areas of Geometric Figures

Area of geometric figures are very common in Civil Service Exams and also other typesofexaminations. Area is basically the number of square units that can fit inside a closed region. In a closed region, if all the unit squares fit exactly, you can just count them and the number of squares is the area. For example, the areas of the figures below are 4, 10, 8 and 20 square units.

The figures blow are rectangles (yes, **a square is a rectangle**!). Counting the figures and observing the relationship between their side lengths and their areas, it is easy to see that the area is equal to the product of the length and the width (Why?).



The blue rectangle has length 5 and width 2, and counting the number of squares, we have 10. Of course, it is easy to see that we can group the squares into two groups of 5, or five groups of 2. From this grouping, we can justify why the formula for the area of a rectangle is described by the formula

$$A_R = l \times w$$

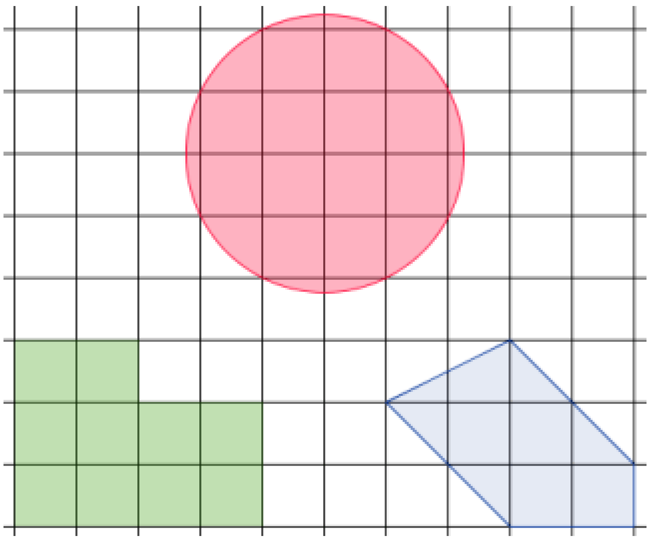
where A_R is the area of the rectangle, l is the length and w is the width. Since the square has the same side length, we can say that

$$A_S = s \times s = s^2$$

where A_S is its area and s is its side length.

There are also certain figures whose areas are difficult to calculate intuitively such as the area of a circle, but mathematicians have already found ways to calculate the areas for these figures.

Challenge: Find the area of the green and blue figure below and estimate the area of the circle.



Below are some formulas for the most common shapes used in examinations. Don't worry because we will discuss them one by one.

Triangle: $A = \frac{1}{2}bh$, b is base, h is height.

Parallelogram: $A = bh$, b is base, h is height

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$, b_1 and b_2 are the base, h is the height

Circle: $A = \pi r^2$ r is radius

In this series, we are going to discuss the areas of the most commonly used figure inexaminations and we will discuss various problems in calculating areas of geometric figures. We are also going to discuss word problems about them. Questions like the number of tiles that can be used to tile a room is actually an area problem.

How to Solve Rectangle Area Problems Part 1

The area of a rectangle including square are the simplest to calculate. As we have discussed in the **previous post**, they can be calculated by multiplying their length and the width. That is if a rectangle has area A, length l and width w, then,

$A = l \times w$ or simply $A = lw$.

In this post, we are going to solve various problem involving area of rectangles.

Problem 1
The length of a rectangle is 12 centimeters and its width is 5 centimeters. What is its area?
Solution
Using the representation above, $l = 12$ and $w = 5$. Calculating the area, we have

$$A = 12(5) = 60.$$

The area is 60 square centimeters.
Problem 2
The area of a rectangular garden is 20 square meters. Its width is 2.5 meters. What is its length?

Solution
In this problem, the missing is the length and the given are the area and the width. So, $A = 20$ and $w = 2.5$. Using the formula, we have

$$A = lw.$$

Substituting the values of A and w, we have

$$20 = l(2.5).$$

Since we are looking for l, we divide both sides of equation by 2.5. That is

$$\frac{20}{2.5} = \frac{l(2.5)}{2.5}.$$

Simplifying, we have $8 = l$.

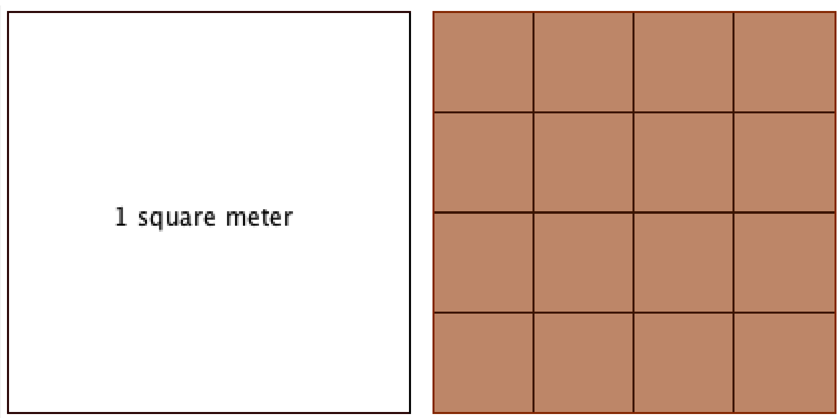
Therefore, the length of the rectangular garden is equal to 8 meters.

Problem 3
The floor of a room 8 meters by 6 meters is to be covered with square tiles. The tiles dimensions is 25 centimeters by 25 centimeters. How many tiles are needed to covered the room? Note: 1 meter = 100 centimeters

Solution
This problem has at least two solutions. I will show one solution and leave you to look for another solution. Using the area formula, we can calculate the area of the room in square meters. That is,

$$A = lw = 6(8) = 48.$$

So, the area of the room is 48 square meters. However, we are looking for the number of tiles that can cover the room and not the area in square meters. Now, the easiest solution is to find the number of tiles that can fit inside 1 square meter. Since the side of a square is 1 meter which is equal to 100 centimeters, it can fit 4 tiles as shown below.



1 square meter contains 16 tiles

Now, four tiles at the side means 1 square meter contains $4(4) = 16$ square tiles. Since there are 48 square meters, the number of tiles needed is

$$16 \times 48 = 768.$$

Therefore, we need at least 768 square tiles to cover the entire floor.

How to Solve Rectangle Area Problems Part 2

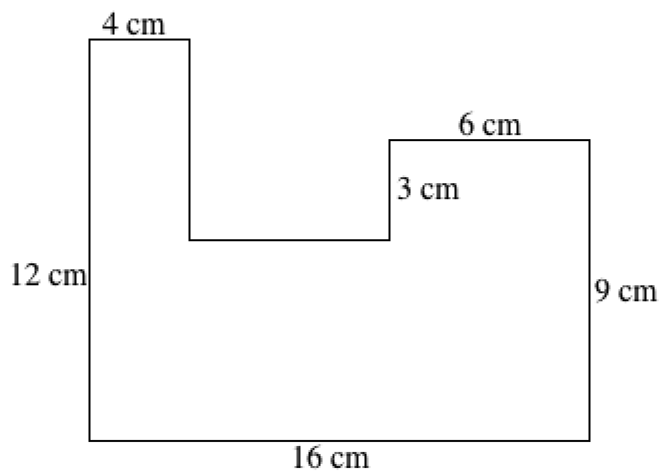
We have already learned the concept of area of a rectangle and solved sample problemsabout it. In this post, we continue the rectangle area problems series. We discuss three more problems about rectangle area.

The fourth problem below involves area preservation, the fifth is calculating the area given its perimeter, and the sixth requiring the use of quadratic equations.

Let’s begin.

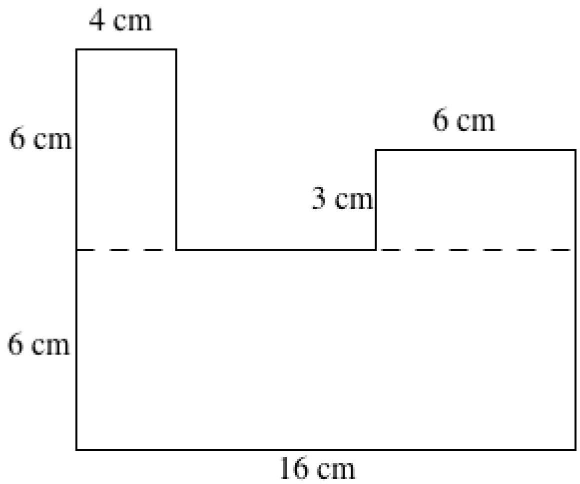
Problem 4

What is the area of the figure below?



Solution

The figure above can be divided into 3 rectangles. One way to do this is to draw the dashed line below (can you find other ways?). Notice that drawing the lines give us rectangles with dimensions 6c m by 4 cm, 6 cm by 3 cm, and 6cm by 16 cm.



Now, the area of the figure is the sum of the areas of the three rectangles.

Area of a 6 cm by 4 cm is 6 cm × 4 cm = 24 square cm.
Area of a 6 cm by 4 cm is 6 cm × 3 cm = 18 square cm.
Area of a 6 cm by 4 cm is 6 cm × 16 cm = 96 square cm
So, the area of the figure is 24 + 18 + 96 = **138 sq. cm.**

Problem 5

The perimeter of a rectangle 54 cm. Its length is twice than its width. What is its area?

Solution

We have already discussed how to calculate the perimeter of a rectangle and we have learned its formula. A rectangle with perimeter P, length l and width w has perimeter

$$P = 2l + 2w.$$

Now, we let the width be equal to x. Since the length is twice, it is $2x$. Substituting them to the formula above, we have

$$54 = 2(2x) + 2x.$$

Simplifying, we have

$$54 = 6x$$

resulting to $x = 9$. Therefore, the width is 9 and the length which is twice the width is 18. So, the area is 9(18) = **162 sq. cm.**

Problem 6

The length of a rectangle is 5 more than its width. Its area is 84 square centimeters. What are its dimensions?

Solution

Guess and Check

This problem can be solved using guess and check but I wouldn't recommend it. For example, you can choose two numbers where one is 5 greater than the other and find their product. Choosing 4 and 9 results to the product 36. It is quite small, so you might want to try 10 and 15 but the product is 150, quite large, so, you can go down, and you will eventually find 7 and 12 which is the correct answer. Another guess and check strategy in this problem is to find the factors of 84 (left as an exercise).

Now, remember that guess and check does not always work and it takes time, so you better learn the solution below.

Algebra (Quadratic Equation)

If we let x be the width of the triangle, then it's length is 5 greater than the width, so it is therefore, $x+5$. Since the area of a rectangle is the product of its length and width, so,

$$A = x(x+5) = 84.$$

This results to the quadratic equation $x(x+5) = 84$ which is equivalent to

$$x^2 + 5x - 84 = 0.$$

If you still remember factoring, then this is an easy problem to factor. This gives us

$$(x+12)(x-7) = 0$$

which gives us $x = 7$ which is its width. This also gives us the length x + 5 = 12.

This solution which uses quadratic equation is a bit advanced, but there is no way that you can solve problems like the one above if you don't know it. I am afraid that you have to learn it again if you have forgotten it. You must practice factoring and memorize the quadratic formula (I will discuss this after this series). Then and only then, that you would be able to solve such problems with better speed and accuracy.

Exam Tip

If you encounter problems such as this and you don't know what to do, it is important that you do not spend too much time on them. Just guess the answer first, mark them, and come back to them when you still have time at the end of the exam. However, be sure not to skip too many items.

In the **next post**, we will have a quiz on solving rectangle area problems.

Rectangle Area Quiz

This is the conclusion of the S**olving Problems on Rectangle Area Series**. In thefirst part, we have discussed the intuition basics of rectangle area formula and solved basic problems about it. In the second part, we have solved more complicated rectanglearea problems. In this post, you are allowed to test what you have learned in the previous parts of the series.
Ideal Time Limit: 15 minutes

Rectangle Area Quiz

1. The length of a rectangle is 8 cm and its width is 7 cm. What is its area?

Answer

56 sq cm

2. Fill in the blank: A rectangular pool has area 180 square meters. Its dimensions are 12m by ___ m.

Answer

15m

3. The bottom of a rectangular pool is 18m by 25m is to be covered with 50 cm by 50 cm tiles. How many tiles are needed to combined the bottom of the pool?

Answer

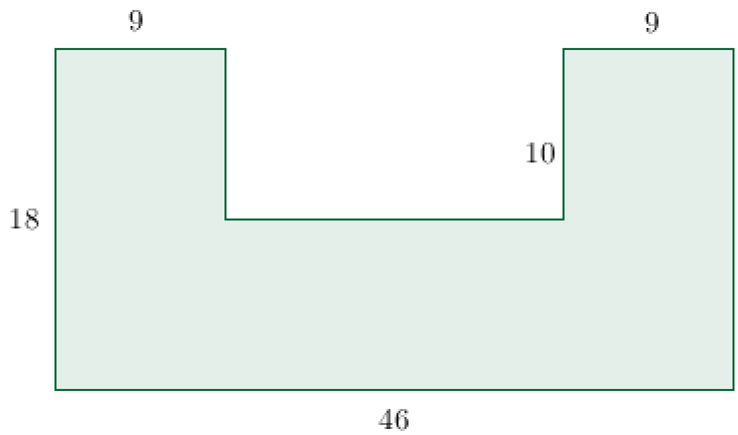
1800 tiles

4. The length of a rectangular rose garden is 40 meters. Its area is 1020 square metes. What is its width?

Answer

25.5 meters

5. What is the area of the figure below?



Answer

548 square units

6. The length of a rectangle is 5 cm more than its width. Its perimeter is 26 cm. What is its area?

Answer

40 sq cm

7. Anna wants to frame her picture with a 1 inch margin on its side. If her picture is 12 inches by 15 inches, what is the area of the frame?

Answer

238 sq in

8. Fill in the blanks: A theater stage is covered with with 187 tiles with no tiles cut. The dimensions of the stage in terms of tiles are _____ by _____ tiles.

Answer

11 by 17

9. What is the area of a rectangle with length 8.5 cm and width 4.5 cm?

Answer

38.25

10. What is the least dimensions of a gift wrapper that can cover a box measuring 6 cm by 8 cm by 10cm?

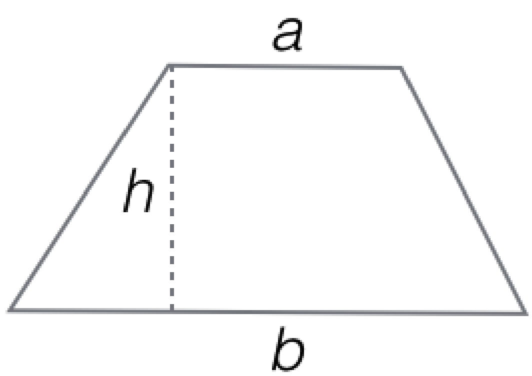
Answer

376 sq cm

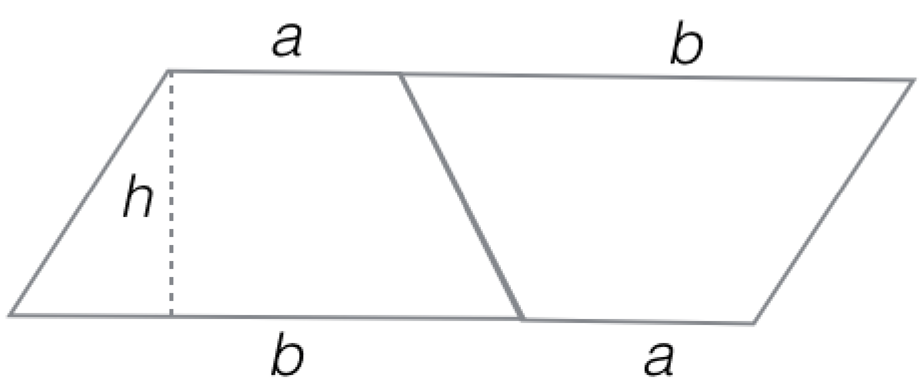
How to Find the Area of a Trapezoid

We have learned how to calculate the areas of a **square, rectangle**, **parallelogram**, and**circle**. In this post, we are going to learn how to find the area of a trapezoid. This is the first post of **Finding the Area of a Trapezoid Series**.

A trapezoid is a polygon whose exactly one pair of sides are parallel*. The figure below is a trapezoid where sides *a* and *b* are parallel.



Notice that if we make another trapezoid which has the same size and shape as above, flip one trapezoid, and make one pair of the non-parallel sides meet, we can form the figure below. That figure is a parallelogram. Can you see why?



Now, observe that the *base* of the parallelogram from the figure is *a* + *b*. Its height is *h*.

We have learned that the **area of a parallelogram** is the product of its base and height. So, the expression that describes its area is

$$h(a+b).$$

Now, when we calculated for the area of the parallelogram above, we actually calculated the area of two trapezoids. Therefore, to get the area of a trapezoid, the have divide the formula above by 2 or multiply it by $\frac{1}{2}$. That is, if we let A be the area of a trapezoid is

$$A = \frac{1}{2}h(a+b)$$

where *a* and *b* are the base (parallel sides) and *h* is the height.

*Please take note that there are other definitions of this polygon. In some books, it is defined as polygons whose at least one pair of sides are parallel.

Example 1

What is the area of a trapezoid whose base are 12 cm and 18 cm and whose height is 15 cm.

Solution

Using the notation above, in this problem we have $a = 12$, $b = 18$ and $h = 15$?

The formula for area is

$$A = \frac{1}{2}h(a+b)$$

So, substituting we have

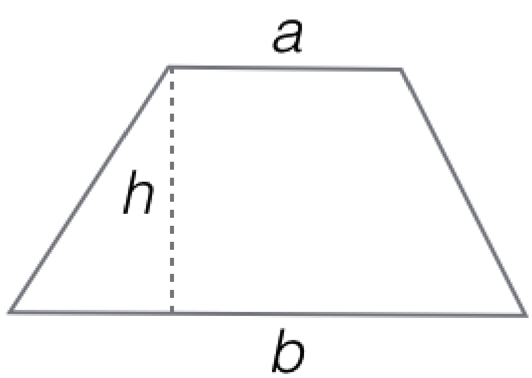
$$A = \frac{1}{2}(15)(18+12) = 225$$

So, the area of the trapezoid is 225 square units.

How to Find the Area of a Trapezoid Part 2

In the previous post, we have learned the formula for **finding the area of a trapezoid**. We derived that the formula for the area A of a trapezoid with base a and b (the base are the parallel sides), and height h is $A = \frac{1}{2}h(a+b)$

In this post, which is the second part of**Finding the Area of a Trapezoid Series**, we are going to continue with some examples. We will not only find the area of a trapezoid, but other missing dimensions such as base and height. Now, get your paper and pencils and try to solve the problems on your own before reading the solution.



We have already discussed one example in the previous post, so we start with the second example.

Example 2

What is the area of a trapezoid whose parallel sides measure 6 cm and 8 cm and whosealtitude is 2.5 cm?

Solution

In this example, the parallel sides are the base, so we can substitute them to a and b. Since we are looking for the sum of a and b, we can substitute them interchangeably. The term*altitude* is also another term for height. So, $a = 6$, $b = 8$ and $h = 2.5$.

We now substitute.

$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2}(2.5)(6+8)$$

$A = \frac{1}{2}(2.5)(14)$

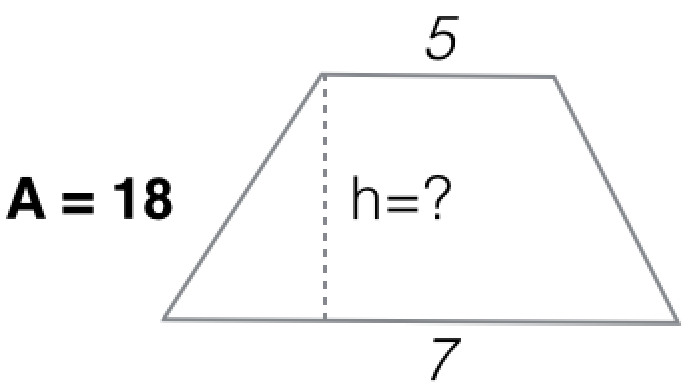
$A = 17.5$

So the area is 17.5 square centimeters.

Be Careful!: Again, remember that if we talk about area, we are talking about square units, and in this case square centimeters. If you choose an option which is **17.5 centimeters**, then it is WRONG. It should be **7.5 square centimeters!**

Example 3

Find the height of a trapezoid whose base lengths are 5 and 8 units and whose area is 18 square units.



Solution

In this problem, we look for the height. But don't worry, we will still use the same formula, and manipulate the equation later to find h. So here, we have $A = 18$, $a = 5$, and $b = 7$.

$A = \frac{1}{2}h(a + b)$

$18 = \frac{1}{2}h(5 + 7)$

$18 = \frac{1}{2}h(12)$

Multiplying 1/2 and 12, we have

$18 = 6h.$

We are looking for h, so to eliminate 6, we divide both equations by 6. That is,

$\frac{18}{6} = \frac{6h}{6}$

$3 = h.$

So, the height of the trapezoid is 3 units (not square units!).

In the **next post**, we will have two more examples.
Note: If you have a hard time understanding the solution, or particularly solving equations, you should read the series on **solving equations**.

How to Find the Area of a Trapezoid Part 3

This the third part of **a series** on finding thearea of a trapezoid here in **PH Civil Service Review**. In the first post, we discussed thederivation of the area of a trapezoid and give a worked example. In the second post, we discussed how to find the area given the base and the height as well as to find the height given the area and the base.
In this post, we are going to find the base, given the height and the area. We continue with the fourth example.

Example 4
A trapezoid has area 65 square centimeters, height 13 cm, and base of 4 cm. Find the other base.

Solution
In this example, we have $A = 65$, $h = 13$ and $a = 4$. We are looking for b.
$A = \frac{1}{2}h(a + b)$

$65 = \frac{1}{2}(13)(4 + b)$

In equations with fractions, we always want to eliminate the fractions. In the equation above, we can do this by multiplying both sides of the equation by 2. That is,

$2(65) = 2(\frac{1}{2})(13)(4+b)$.

The product of 2 and 1/2 is 1, so,

$130 = 13(4+b)$.

Next, we use distributive property on the right hand side. Recall: $a(b+c) = ab + ac$.

$130 = 13(4) + 13(b)$

$130 = 52 + 13b$.

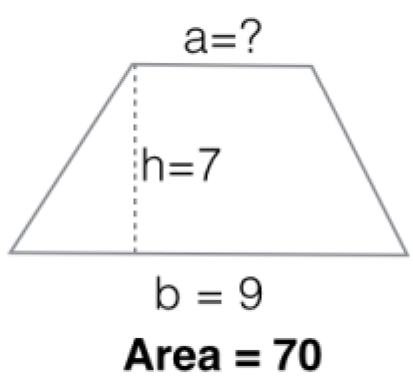
We want to find *b*, so we subtract 52 from both sides giving us
$78 = 13b$.

Next, we divide both sides by 13

$6 = b$.

So, the other base is 6 centimeters which is our answer to the problem.

Example 5
The figure below is a trapezoid. Find the value of a.



Solution
$A = \frac{1}{2}h(a+b)$

$70 = \frac{1}{2}(7)(a+9)$

We eliminate the fraction by multiplying both sides by 2 to get

$140 = 7(a+9)$.

Note: It will be shorter if we divide both sides of equation by 7. You might want to try it.

Using the distributive property, we have

$140 = 7(a) + 7(9)$

$140 = 7a + 63$.

Subtracting 63 from both sides, we have

$77 = 7a$.

Dividing both sides by 7, we have

$11 = a$,

So, the other base of the trapezoid is 11 units.

In the next post, we are going to summarize what we have learned from the this series.

How to Solve Age Problems Part 1

After a series of tutorials on word problemsinvolving numbers, we now move to learning on how to solve word problems involving age. Age problems are very similar to number problems, so if you have finished reading The Number Word Problem Series, then it will be easier for you to solve the following age problems.

Example 1

Benjie is thrice as old as his son Cedric. The sum of their ages is 64. How old are both of them?

Scratch Work

This is one of those age problems that are very similar to number problems. Let's take a specific case. If Cedric is say 8 years old, then Benji is $3(8)$ years old. This means that if Cedric is x years old, then Benjieis $3x$. If we add their ages, the result is 64.

Solution

Let x be the age of Cedric and $3x$ be the age of Benjie.

Cedric's Age + Benjie's Age = 64

$x + 3x = 64$.

$4x = 64$

Dividing both sides of the equation by 4 gives us $x = 16$.
Therefore, Cedric is 16 and Benjie is $3(16) = 48$ year sold.

Check

48 is thrice 16 and $48 + 16 = 64$. So, we are correct.

Example 2

Karen is 6 years older than Nina. Five years from now, the sum of their ages will be 52. How old are both of them?

Scratch Work

If Nina is x years old, then Karen is 6 years older, so her age will be $x + 6$. Five years from now, both of their ages will increase by 5 as shown on the table below.

	now	5 years from now
Nina	x	x+5
Karen	x+6	x + 6 + 5

Therefore, 5 years from now, the sum of their ages will be equal to

$(x + 5) + x + 6 + 5 = 2x + 16$. Now this sum is equal to 52.

Solution

Let x be Nina's age and $x+5$ be Karen's age. In 5 years, Nina will be $x+5$ years old and Karen will be $x+6+5$ years old.

Now, $(x+5)+(x+6+5)=52$

$2x+16=52$

Subtracting 16 from both sides of the equation, we have

$2x=36$

Dividing both sides by 2 we have

$x=18$.

This means that Nina is 18 and Karen is 24.

Check

24 is 6 more than 18 and five years from now, $(18+5)+(24+5)=52$. Therefore, we are correct.

Example 3

Sarah is twice as old as Jimmy. Three years ago, the sum of their ages is 39. How old are both of them now?

Scratch Work

If Jimmy is x years old, then Sarah's age is twice his age, so Sarah is $2x$ years old. Three years ago, both are younger by 3 years, so both their ages must be subtracted by 3.

	now	**3 years ago**
Jimmy	x	x - 3
Sarah	2x	2x - 3

Three years ago, the sum of their ages is 39. So, we add $x-3$ and $2x-3$ and equate it to 39

Solution
Let x be Jimmy's age and $2x$ be Sarah's age.

Three years ago, Jimmy was $x-3$ years old and Sarah was $2x-3$ years old.

Three years ago, the sum of Jimmy's and Sarah's age is

$(x-3)+(2x-3)=39$.

$3x-6=39$

Adding 6 to both sides of the equation results to

$$3x=45$$

Dividing both sides by 3, we have

$$x=15.$$

So, Jimmy is 15 and Sarah is 30.

Check
Three years ago, Jimmy was $15-3=12$ years old and Sarah was $30-3=27$ years old. The sum of their ages was $12+27=39$.

How to Solve Age Problems Part 2

This is the second part of the Solving Age Problem Series. We will continue solving age problems that are slightly more complicatedthat the first part. We have already discussed 3 problems in the first part of this series, so we continue with the fourth problem.

Problem 4

Simon is four years older than Jim. The sum of their ages is 52. How old is Simon?

Scratch Work
This problem is a sort of review of first part of this series. Simon is older than Jim by 4 years. So, if Jim is x years old, then Simon is $x+4$ years old. The sum of their ages is 52. This means that if add x and $x+4$, then the sum is 52. That is the equation.
Solution
Let x be Jim's age and $x+4$ be Simon's age.

Now,

Jim's age + Simon's age = 52 which means that

$x+(x+4)=52$.

Simplifying we have

$$2x+4=52.$$

Subtracting 4 from both sides, we have

$$2x=48$$

$$x=24$$

So, Jim is 24 years old. Now, the question asks for the age of Simon. Simon is $24+4=28$years old.

Check
Simon is 28 and Jim is 24, so he is indeed four years older. The sum of their ages is $28+24=52$ which agrees with the given in the problem. Therefore, we are correct.

Problem 5
Allan is 5 times as old as Leah. Five years from now, he will be 3 times as old. How old isAllan?
Scratch Work

Now, if Leah is, for example, 7 years old, then Allan is $5(7)$ years old. This means that if Leah is x years old, then Allan is $5x$ years old. Five years from now, Leah will be $x + 5$ years old and Allan will be $5x + 5$ years old as shown on the table below.

	now	5 years from now
Leah	x	x + 5
Allan	5x	5x + 5

Note that $latex $5 years from now, Allan will be three times as old as Leah. This means that if we multiply Leah's age by 3, then, their ages will be equal. That is, if we multiply $x + 5$by 3, it will be equal to $5x + 5$. In equation form,

$$3(x + 5) = 5x + 5$$

which is the final equation.

Solution
Let x be Leah's age and $5x$ be Allan's age.

Five years from now, Leah will be $x + 5$ years old and Allan will be $5x + 5$ years old.

Now, we multiply Leah's age and equate it to that of Allan's

$$3(x + 5) = 5x + 5.$$

By Distributive Property, we have

$$3x + 15 = 5x + 5.$$

Putting all x's to the right and all numbers to the left, we have

$$15 - 5 = 5x - 3x$$

$$10 = 2x.$$

Dividing both sides by 2, we have

$$5 = x.$$

So, Leah is 5 years old and Allan is $5(5) = 25$

Check

Allan is 25 and Leah is 5 so he is indeed 5 times as old. In 5 years, Allan will be 30 and Leah will be 10. Thirty is indeed three times 10, so we are correct.

Problem 6

Philip is twice as old as Ben. If 5 is subtracted from Philip's age and 10 is added to Ben's age, then their ages will be equal. How old are both of them?

Scratch Work

Ben is x years old and Philip is $2x$. If we subtract 5 from Philip's age, it will be come $2x - 5$. If we add 10 to Ben's age, it will be $x + 10$. Now, after the results to these operations, their ages will be equal or

$$2x - 5 = x + 10$$

Solution

Let x be Ben's age and $2x$ be Philip's age.

$2x - 5 = x + 10$

$2x - x = 10 + 5$

$x = 15.$

So, Ben is 15 and Philip is 30.

Check

Philip is 30 and Ben is 15 so, he is twice as old. Subtracting 10 from Philip's age results to 20. Adding 5 to Ben's age is 20. Well, 20 equals 20, so we are correct.

In the next post, we will be discussing more age problems. Please keep posted.

How to Solve Age Problems Part 3

This is the third part of the Solving Age Problems Series. In this part, we will solve age problems with a variety of formats and difficulty that are not discussed in the first two parts. We have already solved six problems in the first and second part, so we start with the seventh problem.

Example 7

Bill is four times as old as Carol. One fifth of Bill's age added to one half Carol's age is equal to 13 years. How old are both of them?

Scratch work

Bill is older than Carol and he is four times older. This means that if Carol is x years old, then Bill is $4x$ years old. Now, one fifth of Bill's age is $\frac{1}{5}(4x)$ and one half of Carol's age is $\frac{1}{2}x$. Add these together and you get 13. Now, we have an equation.

Solution

Let x be Carol's age and $4x$ be Bill's age.

$$\frac{1}{5}(4x) + \frac{1}{2}x = 13.$$

Simplifying, we have

$$\frac{4}{5}x + \frac{1}{2}x = 13.$$

Since we have a fraction, we can eliminate the denominator by multiplying everything with the least common multiple of 5 and 2 which is 10. Multiplying both sides of the equation by 10, we have

$$\frac{40}{5}x + \frac{10}{2}x = 130.$$

$$8x + 5x = 30$$

$$13x = 30$$

$$x = 10.$$

This means that Carol is 10 and Bill is 40.

Check

Bill is 40 and Carol is 10. Yes, Bill is four times as old as Carol. One fifth of 40 is 8. One half of 10 is 5 and $8+5=13$. So, we are correct.

Example 8

When a really smart math kid was asked about his age, he said:

"I am one fifth as old as my mother. In six years, I will be one-third as old."

How old is the kid and his mother?

Scratch Work

The kids is one fifth as old as his mother. So, if the mother is x years old, then the kid is $\frac{1}{5}x$Six years from now, the ages of the mother and the kid respectively are $x+6$ and $\frac{1}{5}x+6$ as shown in the table below.

	now	6 years from now
Mother	x	x + 6
Kid	(1/5)x	(1/5)x + 6

As the kid said, in 6 years, his age will be a third of his mother. This means that if we multiply his age by $latex3$, then it will equal the age of his mother. In equation form, we have

$$3(\frac{1}{5}x+6)=x+6.$$

Now, we write the solution.

Solution

Let x be the mother's age and $\frac{1}{5}x$ be the kid's age.

$$x+6=3(\frac{1}{5}x+6)$$

We simplify the right hand side by Distributive Property. This gives us

$$x+6=\frac{3}{5}x+18$$

Now, to eliminate the fraction, we multiply both sides of the equation by 5.

$$5(x+6) = 3x + 90$$

Again, by distributive property, we have

$$5x + 30 = 3x + 90$$

Putting all the x's on the left hand side and all the numbers on the right hand side, we have

$$5x - 3x = 90 - 30$$

$$2x = 60$$

$$x = 30.$$

So, the mother and 30 and the kid is $\frac{1}{5}(30) = 6$. A smart kid indeed, giving problems such as this at age 6.

Check
Left as an exercise.

Example 9
Donna is 6 years older than Demi. One fifth of Donna's age a year ago added to three fourth of Demi's age is equal to Demi's age. How old is Donna?

Scratch Work

Demi is x years old and Donna is $x+6$. Now, Donna's age a year a go is $x+6-1$ which is equal to $x+5$. How, one fifth of Donna's age a year ago is $\frac{1}{5}(x+5)$ and one fourth of Demi's age is $\frac{1}{4}x$.

Now, these ages if added equal's Donna's age which is x. Therefore, the equation is

$$\frac{1}{5}(x+5) + \frac{3}{4}x = x$$

Solution

Let x be Demi's age and $x+6$ be Donna's age

$\frac{1}{5}(x+5)+\frac{3}{4}x=x$

Simplifying, the left hand side by distributive property, we have

$\frac{1}{5}x+1+\frac{3}{4}x=x$.

Now, to eliminate the fractions, we multiply both sides of the equation by the least common multiple of 5 and 4 which is 20. This will result to

$\frac{20}{5}x+20+\frac{60}{4}x=20x$

$4x+20+15x=20x$

$19x+20=20x$

$20=x$

Therefore, Demi is 20 and Donna is 26.

The Age Problem Solving Series

One of the common types of word problemsin mathematics and in many examinations is about Age Problems. This series discusses various problem styles involving age problems and explains in details how they are solved.

The Age Problem Solving Series

How to Solve Age Problems Part 1 discusses simple 2-person problems particularly present-past and present-future age relationships.

How to Solve Age Problems Part 2 discusses a slightly more difficult 2-person problems particularly present-past and present-future age relationships.

How to Solve Age Problems Part 3 discusses age problems that involves fractions.

I am planning to write a fourth part for this series in the near future, but for now, I will focus more on yet uncovered topics.

Introduction to Coin Problems

Coin problems is one of the word problem types that may also appear in the next Civil Service Examination. Coin problems may refer problems regarding actual coins or even problems involving bills. Although the Civil Service Examination is solely for Filipinos, nobody will prevent the creators of the exams using American terms such as pennies, nickels, and dimes. In case you do not know, or you have forgotten, a *penny* is equivalent to 1 cent, a *nickel* is *equivalent* to 5 cents, and a *dime* is equivalent to 10 cents.

Let us try to solve two problems as a teaser to this series.

Problem 1

Bingbong has 18 coins in his pocket. Three of them are nickels and five of them are pennies. If the remaining coins are dimes, how much money does Bingbong in his pocket?

Solution

There are 3 nickels and a nickel is 5 cents, so the three nickels are worth 15 cents.

There are 5 pennies and each penny is 1 cent, so 5 pennies are worth 5 cents.

There are 10 coins left, each of which is a dime or 10 cents. Therefore, there is 1.00 peso.

So, Bingbong has 0.15 + 0.05 + 1.00 = P1.20.

Therefore, Bingbong has one peso and 20 cents.

Problem 2

Jamie has 18 bills in her wallet worth 20 pesos and 50 pesos. If the bills totaled to 660 pesos, how many 20-peso and 50-peso are there?

Solution

There are 18 bills and for example, there are five 20-peso bills, then we will be left with $18 - 5 = 13$ fifty-peso bills. This means that if there are x 20-peso bills, then there are $(18 - x)$ 50-peso bills.

bills	amount	number	total
20 pesos	20	x	20x
50 pesos	50	18 - x	50(18-x)

Now, if we multiply the amount and the number of coins, we have $20x$ for the 20-peso coin and $50(18-x)$ for the 50-peso coin. If we add these total amounts, we have 660 pesos. Therefore, we can form the equation

$$20x + 50(18-x) = 660$$

By distributive property, we have

$$20x + 900 - 50x = 660$$

Simplifying, we have

$$-30x + 550 = 660$$

Subtracting 550 from both sides, we have

$$-30x = -240$$

Dividing both sides by -30, we have $x = 8$.

Therefore there are 8 twenty-peso coins and 10 fifty-peso bills.

Check
The total amount for the 20-peso bill is 8(20) = 160.

The total amount for the 50-peso bill is 10(50) = 500.

In the next post in this series, we will be discussing more coin problems.

How to Solve Coin Problems Part 2

I have already introduced how to solve coin problems, so in this post, we solve more problems about it. Coin problems involve problems consisting coins, bills, and of course, any object of value. We have already solved 2 problems in the previous post, so we continue with the third problem.

Problem 3

A box contains 32 bills consisting of Php20 and Php50. The total amount of money in the box is 1000 pesos. How many bills of each kind are there?

Note: For my readers from other countries, Php means Philippine pesos.

Solution

This problem is similar to problem 2, so we will not be solving it in details.

Let x be the number of 50-peso bills. Since there are 32 bills, then the number of 20-peso bills is $32 - x$ Now, if we multiply the number of 50-peso bills by 50 pesos and multiply the number of 20-peso bills by 20 pesos, we have

$$50x \text{ and } 20(32 - x)$$

respectively.

If we add them, the total is 1000 pesos. That is,

$$50x + 20(32 - x) = 1000.$$

Simplifying and solving for x, we have

$$50x + 640 - 20x = 1000$$

$$30x + 640 = 1000$$

Subtracting both sides by 640, we have

$$30x = 360.$$

Dividing both sides by 30, we have

$$x = 12.$$

Therefore, the number of 50-peso bills is 12. Now, since there are 32 bills, the number of 20-peso bills is 32-12 = 20.

Check
12(Php50) + 20(Php20) = Php600 + Php400 = Php1000

Problem 4
In a charity musical show, there are the same number of tickets sold worth $20, $50, and $100. The total cost of the tickets is $5100.

Solution
Let x be the number of tickets sold. Since there are the same number of tickets, if we multiply the number of ticket to each of the price, we have

$$20x, \ 50x \text{ and } 100x$$

which are the total cost of each kind. If we add them all together, then, it is the total cost of all the tickets which is $5100. That is,

$$20x + 50x + 100x = 5100.$$

Simplifying, we have

$$170x = 5100$$

Dividing both sides by 170, we have

$$x = 30.$$

This means that there are 30 tickets of each price that were sold.

Check
$20(30) + $50(30) + $100(30) = $600 + $1500 + $3000 = $5100.

In the next part of this post, we will solve two more problems to end this series.

The Solving Consecutive Number Problems Series

The Solving Consecutive Number Problems Series is a series of post discussing how to solve consecutive number word problems in Algebra. Consecutive number problems are very common in many exams including the Subprofessional and Professional Civil Service Exams. Below is the list of posts including their descriptions.

How to Solve Consecutive Number Problems Part 1 is an introduction to the concept and algebraic representation of numbers. This post discusses the difference between consecutive integers, consecutive odd integers, and consecutive even integers. Two sample problems with complete and detailed solutions were discussed in this post.

How to Solve Consecutive Number Problems Part 2 discusses more examples about consecutive numbers and consecutive odd numbers.

How to Solve Consecutive Number Problems Part 3 discusses examples about consecutive odd numbers and consecutive even numbers.

Each of this posts has a video from Youtube that you can watch if you are not fond of reading.

I hope you enjoy these posts.

If you want me to discuss a particular topic, please comment them below.

How to Solve Consecutive Number Problems Part 1

This is the first of the **Solving Consecutive Number Series**, a series of post discussingword problems about consecutive numbers.

Consecutive numbers are numbers that follow each other in order. In number problems in Algebra, consecutive numbers usually have difference 1 or 2. Below are the types of consecutive numbers, consecutive numbers – 5, 6, 7, 8, …

consecutive even numbers – 16, 18, 20, 22…

consecutive odd numbers – 3, 5, 7, 8, …

The symbol … means that the list may be continued.
Notice that consecutive numbers always increase by 1 in each term. If we make 5 as point of reference, then, we can write the numbers above as

$$5, 5 + 1, 5 + 2, 5 + 3.$$

That means that if our first number is x, then the list above can be written as

$$x, (x + 1), (x + 2), (x + 3)$$

and so on.
As for the consecutive even and consecutive odd numbers above, with the smallest numbers as point of reference, they can be written as

$$16, 16 + 2, 16 + 4, 16 + 6.$$

and

$$3, 3 + 2, 3 + 4, 3 + 6.$$

Notice that both consecutive odd and consecutive integers increase by 2 in each time. So, if we let x be the first number, the terms can be written as

$$x, (x + 2), (x + 4), (x + 6)$$

and so on.

Now, that we know how to represent consecutive numbers, let us solve our first problem.

Example 1
The sum of two consecutive numbers is 81. What are the numbers?

Solution
Since there is no mention of odd or even, the terms only increase by 1. So, let

x = the first number

$x + 1$ = the second number.
The word sum means we have to add and the phrase "is 81" means that we have to equate the sum to 81. That is

first number + second number = 81.

Since the first number is x and the second number is $(x + 1)$,

$$x + x + 1 = 81.$$

Solving the equation, we have

$$2x + 1 = 81.$$

Subtracting 1 from both sides, we have

$$2x = 80.$$

Dividing both sides by 2 results to

$$x = 40.$$

So, the smaller number is 40, and the larger number is 40 + 1 = 41. The consecutive numbers are 40, 41 and their sum is 81.

Example 2
The sum of three consecutive numbers is 42. What are the numbers?

Solution
In this example, we have 3 consecutive even numbers. Recall that form above, consecutive even numbers increase by 2 each time. So, let

x = first number
$x + 2$ = second number
$x + 4$ = second number.
Again, the problem mentioned the word sum, so we have to add. That is,

first number + second number + third number = 42.

Substituting the algebraic representation above, we have

$$x + (x + 2) + (x + 4) = 42$$

Solving the equation,

$$3x + 6 = 42.$$

Subtracting 6 from both sides, we have

$$3x = 36.$$

Dividing both sides of the equation by 3 results to

$$x = 12.$$

So, 12 is the smallest number, 12 + 2 = 14 is the second number and 12 + 4 = 16 is the largest number. The consecutive numbers are 12, 14, and 16 and their sum is 42.

How to Solve Consecutive Number Problems Part 2

In the **previous** post, we have discussed the basics of consecutive number problems. We have learned that in word problems in Algebra, consecutive numbers usually mean numbers increasing by 1. Consecutive even numbers and consecutive odd numbers increase by 2. So, consecutive numbers whose smallest is x are x, x + 1 and x + 2 and so on, while consecutive odd/even numbers whose smallest number is y are y, y + 2, y + 4 and so on.

In this post, we begin with the third example in the series since we already had 2 examples in the previous post.

Example 3

The sum of four consecutive numbers is 70. What are the numbers?

Solution

Let

x = first number

$x + 1$ = second number

$x + 2$ = third number

$x + 3$ = fourth number

Since we are talking about the sum of the four numbers, we add them. That is,

sum of four numbers = 70

$x + (x + 1) + (x + 2) + (x + 3) = 70$

Simplifying, we have

$4x + 6 = 70$

$4x = 70 - 6$

$4x = 64$

$4x/4 = 64/4$

$x = 16$.

So, the smallest number is 16. Therefore, the four consecutive numbers are 16, 17, 18, and 19.

Check: $16 + 17 + 18 + 19 = 70$

Example 4
The sum of 3 consecutive odd numbers is equal to 51. What are the numbers?

Solution

As we have discussed above, odd numbers increase by 2 each time (like 5, 7, 9, 11), so we let

x = first number

$x + 2$ = second number

$x + 4$ = third number

Now, we add the numbers and equate to 51.

$x + (x + 2) + (x + 4) = 51$

$3x + 6 = 51$

$3x = 51 - 6$

$3x = 45$

$3x/3 = 45/3$

$x = 15$

So the smallest odd number is 15. Therefore, the three consecutive odd numbers are 15, 17, and 19.

Check: $15 + 17 + 19 = 51$

How to Solve Consecutive Number Problems Part 3

This is the third part of the Solving Consecutive Number Problems Series. In this post, we solve more problems about consecutive numbers. We have already discussed four problems in the **first part** and**second part** of this series, so we start with the fifth example.

Example 5

There are 3 consecutive odd numbers. Twice the smallest number is one more than the largest. What are the numbers?

Solution

In the first post in this series, we have learned that odd numbers increase by 3 (e.g. 7, 9, and 11). So, let

x = the smallest odd number

$x+2$ = the second odd number

$x+4$ = the largest odd number.

Now that we have represented the numbers, we now go to the second sentence. The second sentence says that twice the smallest number is one more than the largest. The smallest number is x, so twice the smallest number is $2x$. Now, $2x$ is one more than the largest number $x+4$. This means that if we add 1 to the largest number, then they will be equal. That is,

$$2x = (x+4)+1.$$

Solving, we have

$2x = x+5$

$x = 5.$

So the consecutive numbers are 5, 7, and 9.

Check: Twice the smallest is 2(5) = 10 is one more than the largest which is 9. Therefore, ouranswers are correct.

Example 6

There are three consecutive even integers. The sum of the first two integers is 16 more than the largest. What are the numbers?

Solution

As we have discussed in the previous posts, the representations of consecutive odd numbers and consecutive even numbers are the same. Consecutive even numbers such as 18, 20, 22 increase by 2 inch time.

x = the smallest even number

$x+2$ = the second even number

$x+4$ = the largest even number

The second sentence states that the sum of the first two integers is 16 more than the largest. The sum of the first two integers is $x+(x+2) = 2x+2$ and the largest integer is $x + 4$. Since the sum of the first two integers is 16 more than the largest integer, if we add 16 to the largest integer, then they will be equal. That is,

$$x + (x + 2) = (x + 4) + 16$$

Simplifying,

$2x + 2 = x + 20$

$2x = x + 20 - 2$

$2x = x + 18$

$2x - x = 18$

$x = 18$.

Therefore, the consecutive integers are 18, 20, 22$.

Check: The sum of the first two integers 18 + 20 = 38 is 16 more than 22. Therefore, we are correct.

How to Solve Digit Problems Part I

Digit Problems is one of the **word problems**in Algebra. To be able to solve this problem, you must understand how our number system works. Our number system is called the decimal number system because the numbers in each place value is multiplied by powers of 10 (deci means 10). For instance, the number 284 has digits 2, 8, and 4 but has a value of 200 + 80 + 4. That is,

$$(100 times 2) + (10 times 8) + (4 times 1) = 284.$$

As you can observe, when our number system is expanded, the hundreds digit is multiplied by 100, the tens digit is multiplied by 10, and the units digit (or the ones digit) is multiplied by 1. Then, all those numbers are added. The numbers 100, 10, and 1 are powers of 10: $10^2 = 100$, $10^1 = 10$, and $10^0 = 1$. So, numbers with h, t, and u as hundreds, tens, units digitsrespectively has value

$$100h + 10t + u.$$

It is clear that this is also true for higher number of digits such as thousands, ten thousands, hundred thousands, and so on.

Many of the given numbers in this type of problem have their digits reversed. As we can see, if 10t + u is reversed, then it becomes $10u + t$. For instance, $32 = 10(3) + 1(2)$ when reversed is $23 = 10(2) + 1(3)$. Now, that we have already learned the basics, we proceed to our sample problem.

Worked Example

The tens digit of a number is twice the units digit. If the digits are reversed, the new number is 18 less than the original. What are the numbers?

Solution and Explanation

The tens digit of a number is twice the unit digit. This means that if we let the units digit be x, then the tens digit is $2x$. As we have mentioned above, we multiply the tens digit with 10 and the units digit with 1. So, the number is

$$(10)(2x) + x.$$

Now, when the digits are reversed, then x becomes the tens digit and $2x$ becomes the ones digit. So, the value of the number is

$$(10)(x) + 2x.$$

From the problem above, the number with reversed digit is 18 less than the original number. That means, that if we subtract 18 from original number, it will equal the new number. That is,

$(10)(2x) + x - 18 = 10(x) + 2x$
$20x + x - 18 = 12x$
$21x - 18 = 12x$
$9x = 18$
$x = 2$
$2x = 4$

So, the number is 42 and the reversed number is 24.

Check: 42 – 24 = 18.

How to Solve Digit Problems Part II

In the previous post, we have discussed the**basics of digit problems**. We have learned the decimal number system or the number system that we use everyday. In this system, each digit is multiplied by powers of 10. For instance, 871 means

$$(8 \times 10^2) + (7 \times 10^1) + (1 \times 10^0).$$

Recall that $10^0 = 1$.

In this post, we continue this series by providing another detailed example.

Problem
The sum of the digits of a 2-digit number is 9. If the digits are reversed, the new number is 45 more than the original number. What are the numbers?
Solution and Discussion
If the tens digit of the number is x, then the ones digit is $9 - x$ (can you see why?).

Since the tens digit is multiplied by 10, the original number can be represented as

$$10x + (9 - x).$$

Simplifying the previous expression, we have **10x – x + 9 = 9x + 9.**
Now, if we reverse the number, then $9 - x$ becomes the tens digit and the ones digit becomes x. So, multiplying the tens digit by 10, we have

$$10(9 - x) + x.$$

Simplifying the expression we have **10 – 10x + x = 90 – 9x**.
As shown in the problem, the new number (the reversed number) is 45 more than the original number. Therefore,

reversed number – **original number** = 45.

Substituting the expressions above, we have

$$90 - 9x - (9x + 9) = 45.$$

Simplifying, we have

$90 - 9x - 9x - 9 = 45$
$81 - 18x = 45$
$18x = 81 - 45$
$18x = 36$
$x = 2.$

Therefore, the tens digit of the original number is 2 and the ones digit is $9 - 2 = 7$.

So, the original number is 27 and the reversed number is 72.

Now, the problem says that the new number is 45 more than the original number. And this is correct since $72 - 27 = 45$.

Discount Problem Quiz

After learning the how to solve discount problems and the strategies how to solve them, it is now your turn to solve discount problems.

Discount Problem Quiz

Solve each problem as fast as you can. You can check your answer by clicking the red + button after each question.

1. You were walking and saw the following sign in a shoe shop. How much is the discount and sale price of a shoes worth Php1600.00?

Answer

discount is 640 pesos, sale price is 960 pesos

2. A coffee shop label its 1 cup of frappuccino 10% discount. If 1 cup of frappucino costs Php150, how much will the customer pay upon ordering?

Answer

Php135.00

3. After a 10% discount has been made, a necklace costs Php5400.00. What is the original price of the necklace before the discount has been deducted?

Answer

Php6000.00

4. A popular mall placed a big “up to 40% off” streamer in front of its entrance. You are planning to buy a television worth Php21,550. What is the lowest possible amount that you will pay if you buy the television?

Answer

Php12930.00

5. You want to buy a mountain in a bicycle shop where you saw a “15% off flyer.” The price of the bike was Php26350.00. You asked the owner what will be price after the discount and he replied that the tag price was the discounted price. What was the original price of the bike?

Answer

Php31000.00

6. What is the sale price of a laptop worth Php42000.00 which has a 5% discount?

Answer

Php39900.00

7. A plate is worth Php120 each. The shop owner told you that he will give you a 10% discount of the total price if you buy 1 dozen. How much will you have to pay if you buy one dozen of the plates?

Answer

Php1296.00

8. After getting an 8% discount, a jacket costs 690. What is the original price?

Answer

Php750.00

9. You saw a cellular phone company placed a "5% off on all items"commercial on television. How much will you have to pay for a cellular phone whose price is Php5250.00?

Answer

Php4987.50

10. You saved 200 pesos for buying an external hard drive which is marked "5% discount." How much was the price of the hard without the discount?

Answer

Php4000.00

Did you enjoy the quiz? For more quizzes and practice test, visit the Practice Test page.

How to Solve Discount Problems

The Civil Service Examinations offer various types of math problems which may change from one examination to another. Two types of math problems that will likely appear are discount and interest problems. In this post, we will tackle discount problems.

The tag price that you see on items are their*marked price*. The *sale price* is the price that you pay after the discount has been made. If an item costs Php100 (marked price) and has a 10% discount, then you have to subtract the 10% of 100 from 100. Therefore, that item will cost Php100 – Php10 = Php90. Php90 is the sale price.

Note that in solving discount problems, you must know how to convert percent to decimals. You need to convert percent to decimals (just divide by 100) in order to perform the calculation.

Sample Problem 1

A movie DVD which costs 600 is marked “25% off.” What is the discount? What is its sale price?

Solution

Discount = 25% of Php600 or (Php600 × 0.25) = Php150.00
Sale price = Php600 – Php150 = Php450.00

So, the discount is Php150 and the sale price is Php450.

Note that 0.25 is the decimal equivalent of 25%.

Sample Problem 2

Anna shops in an international store. A t-shirt with a tag price $42 is marked “save 20%.” How much will Anna have to pay for the t-shirt if she were to buy it?

Solution

Discount = 20% of $42 or ($42 × 0.20) = $8.40
Sale Price = $42.00 – $8.40 = $33.60

Therefore, Anna will have to pay $33.60 if she wants to buy the t-shirt.

Although it seldom happens in the real world, a discounted price might also be given in aCivil Service Exam problem. In this type of problem, the task is to find the original price (marked price) like the problem below.

Sample Problem 3

After getting a 10% discount, Nina bought a sofa for only 7200. What was the original price of the sofa?

Solution

We can use this equation to solve the problem above.

Marked Price – Discount Price = Sale Price

Now, if we let x be the marked price of the sofa, then the discount price is 10% multiplied by x or $0.1x$.

So, substituting to the equation above, we have

$x - 0.1x = 7200$

$0.9x = 7200$

To eliminate the decimal point, multiply both sides by 10.

$9x = 72000$

Dividing both sides by 9, we have 8000.

Therefore, the marked price of the sofa is Php8000. In the next post, we will discuss about strategies and short-cuts in solving discount problems.

Now that you have understood that concept of discount, you may want to read about thestrategies and shortcuts on how to solve discount problems or take a quiz on discount problems.

Strategies in Solving Discount Problems

In the previous post, we have learned how to solve discount problems. In this post, I am going to teach you some strategies that will make solving faster. We know the Civil Service Examinations, as well as other examinations, are always under time pressure. Being able to solve problems fast will be a great advantage.

We are going to solve the same problems, only this time, we are going to use strategies that would be able to make solving discount problems faster. You are probably wondering why I didn't teach this strategy the first time. The answer is, you have to know the basics first, so if you forgot your strategy or shortcut, you can always go back to the long method.

Sample Problem 1

A movie DVD which costs 600 is marked “25% off.” What is the discount? What is its sale price?

Discussion

In the previous post, we used decimals to solve this problem. However, percentages like 25%, 50%, 75%, 10%, 75% and the like are easy to convert to fractions. If you are familiar with their equivalent fractions, it is easier to solve the problem above.

Solution

The equivalent of 25% is 1/4 and 1/4 is half of half. So, half of 600 is 300 and half of 300 is 150. Therefore, the discount is P150 and the sale price is P600-150 = Php450.

You see, you can solve the problem above mentally.

Sample Problem 2

Anna shops in an international store. A t-shirt with a tag price $42 is marked “save 20%.” How much will Anna have to pay for the t-shirt if she were to buy it?

Discussion

In the previous post, we multiplied $42 by 0.2 (or 20%), then subtract the result from 42. Note that if you subtract the percentage first, the calculation will be easier. That is, if the discount is 20%, then, we only have 20 pay 80%. Therefore, we just have to multiply 0.8 by 42.

Solution

The discount is 20% so we only need to pay 80% of the $42. So, (42)(0.8) = 33.60. This means that the sale price is $33.60.

Sample Problem 3

After getting a 10% discount, Nina bought a sofa for only 7200. What was the original price of the sofa?

Discussion

Again, like in Problem 1, it is faster to convert 10% to fraction. The discount is 1/10, so this means that Nina only paid 9/10 of the original price. So, we can set up the equation $\frac{9}{10}x = 7200$.

Solution

The discount of the sofa was 1/10, therefore, Nica paid 9/10 of the price which is 7200. Setting up the equation, we have

$\frac{9}{10}x = 7200$

$9x = 72000$

$x = 8000$

So, the sofa cost Php8000.00

That's it. In the next post, I will post some sample problems discount problems for your practice.

The Solving Discount Problems Series

This is the summary of the series on **Solving Discount Problems Series**. This series is is divided into three parts: a note on solving discount problems, the strategies to use, and a quiz to assess your knowledge about the topic.

1. **How to Solve Discount Problems**discusses the basic concepts of discount problems, the common terminologies used, and some worked examples on how to solve such problems.
2. **Strategies in Solving Discount Problems** discusses some of the strategies and shortcuts to solve discount problems. Knowing the basic and these strategies will make you solve discount problems much faster.
3**. Discount Problem Quiz** serves as a practice test to those who want to test their understanding about the previous two articles. The questions in this quiz all have answers.

In future posts, we will also be discussing problems about interest, as well as problems on rate, base, and percentage.

How to Solve Simple Interest Problems Part 1

This is the first part of the **Solving Simple Interest Problem Series** for the Civil Service Examination.
Simple interest problems are usually included in many examinations such as the Civil Service Exams. It is important that you practice solving these types of problems in order to increase your chance of passing the exams.

Before solving simple interest problems, let us familiarize ourselves with the terms used in simple interest problems. These are the money invested which is called the principal, the rate of interest which is the percent and the interest which is the income or return of investment, and time. Time may vary depending on the investment. It can range from months to years.
Example 1
Mr. Reyes invested Php50,000 at an interest rate of 3% per year.

a.) Identify the principal and rate of interest.
b.) Calculate the interest earned after 1 year.

Solution
For (a)
The money invested or principal is Php50,000, the interest rate is 3%, and the time is 1 year.

For (b)
We want to calculate 3% of Php50,000. To multiply, we must convert 3 percent to decimal which is equal to 0.03.

interest = Php50,000 × 0.03
interest = Php1500

This means that for a year, the money earned Php1500.

Example 2
Ms. Gutierrez invested Php60,000 at a simple interest of 4% per year for 4 years.

a.) Identify the principal, rate of interest, and time.
b.) How much money will Ms. Gutierrez have after four years?

Solution
For (a),
The principal or money invested is Php60,000.
The rate of interest is 4%.
The time is 4 years.

For (b),
We need to calculate 4% of Php60,000. Just like above, we must first convert 4 percent to decimal which is equal to 0.04.

Now,
interest (1 year) = P60,000 × 0.04 = 2,400
That is the interest for 1 year. To be able to calculate the interest for four years, we have

interest (4 years) = 2,400 × 4 = 9600.

So, the money Ms. Gutierrez will have by the end of four years is the Principal which is 60,000 and the interest for 4 years which is 9600. So, in total, her money will be 69,600.

***

From the two problems above, we can see the interest (I) is the product of the principal (P), the rate (R), and the time(T). Therefore, we can have the formula.

$I = P \times R \times T$

or simply

I = PRT.

How to Solve Simple Interest Problems Part 2

This is the second part of the **Solving Simple Interest Problems Series**. In the **previous pos**t, we have discussed the basics of simple interest problems. We have learned that the simple interest (I) is equal to the product of the amount of money invested or the principal (P), the percentage of interest or the rate (R) and the time (T). Therefore, we can use the following formula:
I = PRT.

In this post, we are going to discuss more problems particularly interests that are not yearly and finding unknowns other than interest.

Example 3
Dr. Lopez invested his Php120,000 in a bank that gives 2% interest every quarter. What is the interest of his money if he is to invest it for 1 years?

Solution and Explanation
Notice that the interest is applied quarterly and not every year. Quarterly means every three months and therefore it will be applied four times a year since there are four quarters every year. So in this case, the time is 4. So,

P = Php120,000
R = 2%
T = 4
I = ?

Note that the rate **percent must be converted to decimal** by dividing it by 100. So 2% equals 0.02. Now, using the formula, we have
I = PRT
I = (Php120,000)(.02)(4)
I = Php9600.00

So, the interest of the money for 1 year is Php9600.

Example 4

Danica invested here money amounting to Php150,000 in a bank that offers a 5% simple interest every year. She went abroad and never made any deposit or withdrawal in her account. After coming back, she immediately checked her account and found out that her money got an interest of Php37,500. How many years was the money invested?

Solution and Explanation

In this problem, interest is given and time is unknown. Assigning the values we have

I = Php37,500
P = Php150,000
R = 5%
T = ?.

Using the formula, we have

I = PRT

Converting 5 percent to decimal and substituting, we have

37,500 = (150,000)(.05)(T)
37,500 = 7500(T).

Dividing both sides by 7500, we have

5 = T.

That means that the money was invested for 5 years.

In the **next post**, we will be solving simple interest problems whose uknowns are rate and principal.

How to Solve Simple Interest Problems Part 3

This is the third and last part of the **Solving Simple Interest Problems Series**. In the **first**and **second** part of this series, we have learned how to solve simple interest problems. In this post, we continue with two more problems, one with the rate missing and the other one with the principal missing.

Example 5

Mr. Wong deposited \$15,000 in a bank for a certain rate of interest per year. After two years, the interest was \$900. What was the rate of interest in percent?

Solution and Explanation

We have the following given:

Principal (P) = \$15,000
Interest (I) = 900
Time = 2 (years)
Rate (R) = ?

As we have learned in the previous posts, simple interest is the product of the principal, the rate, and time or

I = PRT.

Substituting the given we have

900 = (15000)(R)(2)
900 = 30000R

Dividing both sides by 30000, we have

R = 900/30000
R = 0.03

The rate is 0.03 which is we need to convert to percent by multiplying it by 100. Therefore, the rate is 3%.

Example 6

Mrs. Lansangan invested a certain amount of money in a bank that gives 4% interest per year. She got an interest of Php2400 after 3 years.

Solution and Explanation

Given
I = 2400
R = 4%
T = 3 years

Using the simple interest formula mentioned above, we have

I = PRT
2400 = (P)(4%)(3)

Converting 4 percent to decimal, we have

2400 = (P)(0.04)(3)
2400 = 0.12P

Dividing both sides by 0.12, we have

20000 = P

Therefore, Mrs. Lansangan invested Php20000.

That’s it for our series on how to solve simple interest problems. I hope you have learned well. Good luck for the exams.

The Solving Simple Interest Problems Series

The Solving Simple Interest Problems is a series of tutorials on how to solve simple interest problems. In solving interest problems you need to know the following terms:

the **principal (P)** is the money invested, the**rate (R)** of interest is the percentage of interest (that is the number with percent sign), the **time (T)** and the **interest (I)** is the earnings or return of investment. The interest is the product of the principal, the rate, and the time, or I = PRT is explained in the first part of the series.

How to Solve Simple Interest Problems Part 1 discusses the basics of simple interest problems and the terms used in such problems. Two examples worked examples are solvedin which interests are both unknowns.

How to Solve Simple Interest Problems Part 2 is a continuation of the simple interest discussion. One problem is an example of a rate of interest which is not given yearly and the other one is an investment made for more than a year.

How to Solve Simple Interest Problems Part 3 is a discussion of simple interest problems where the unknowns are the principal and the rate.

I will be posting exercises and problems with solutions about this type of problems soon, so keep posted.

How to Solve Investment Word Problems in Algebra

Investment word problems in Algebra is one of the types of problems that usually come out in the Civil Service Exam. In solving investment word problems, you should know the basic terms used. Some of these terms are principal (P) or the money invested, the rate (R) or the percent of interest, the interest (I) or the return of investment (profit), and the time or how long the money is invested. Investment is the product of the principal, the rate, and the time, and therefore, we have the formula

I = PRT.

This tutorial series discusses the different types of problems in investment and discussed the method and strategies used in solving them.

How to Solve Investment Problems Part 1 discusses the common terminology used in investment problems. It also discusses an investment problem where the principal is invested at two different interest rates.
How to Solve Investment Problems Part 2 is a discussion of another investment problem just like in part 1. In the problem, the principal is invested at two different interest rates and the interest in one investment is larger than the other.
How to Solve Investment Problems Part 3 is very similar to part 2, only that the smaller interest amount is described.
How to Solve Investment Problems Part 4 discusses an investment problem with a given interest in one investment and an unknown amount of investment at another rate to satisfy a percentage of interest for the entire investment.

How to Solve Investment Problems Part 1

In the **Simple Interest Problems Series**, we have learned how to calculate the interest, rate, or time of the principal invested. In that series, the principal is invested to a single bank or company. In this series, we will learnhow to calculate the interest of money invested at different companies (different rates). Before starting with our first example, familiarize yourself with the following terms:
principal – the amount of money invested
rate of interest – the percent of interest yearly or any period of time (e.g. monthly, quarterly)
interest – income or return of investment

Hence, if Php10000 is invested at a bank with 3% interest is 300, Php10000 is the principal, 3% is the rate of interest, and 300 is the interest.

<u>Example 1</u>
Mr. Molina invested Php100,000.00. A part of it was invested in a bank at 4% yearly interest and another part of it at a credit cooperative at 7% yearly interest. How much

investment he made in each if his yearly income from the two investments is Php5950.00?

Scratch Work

If we let x be the money invested at a bank, then 100000 – x is the amount invested at the credit cooperative. To calculate for the interest, we must apply the percent of each interest at each amount. That is

(4%)(x) = yearly interest from at the bank
(7%)(100000 – x) = yearly interest from the credit cooperative

If we add the interest, it will amount to Php5950. So, here's our solution.

Solution

Let x = amount of money invested at the bank
100000 – x = amount of money invested at the credit cooperative

Total Interest = Interest From the Bank + Interest from the Credit Cooperative
5950 = (4%)(x) + (7%)(100000 – x)

We need to convert percent to decimals in order to multiply. We do this by dividing the percentage by 100. So, 4% = 0.04 and 7% = 0.07. Substituting to the previous equation, we have
5950 = (0.04)(x) + (0.07)(100000 – x)
5950 = 0.04x + 7000 – 0.07x
5950 – 7000 = -0.03x
-1050 = -0.03x.

We can eliminate the decimals by multiplying by 100.

-105000 = -3x
Dividing both sides by -3, we have

35000 = x.

That means that Mr. Molina invested Php35000 in the bank and Php100000 – Php35000 = Php65000 in the credit cooperative.

Check
(4%)(35000) + (7%)(65000) = (.04)(35000) + (.07)(65000)
= 1400 + 4550 = 5950

As we can see, our interest from the two investments is Php5950.00

How to Solve Investment Problems Part 2

This is the second part of the Solving Investment Problems Series. In the **first part**, we discussed in detail the solution of a problem at two different rates of interest. In this post, we discuss another problem.

Problem
Mr. Reyes invested a part of Php70000 at 3% yearly interest and the remaining part at a 5% yearly interest. The annual interest on the 3% investment is Php100 more than the annual interest on the 5% investment. How much was invested at each rate?

Solution
If we let x be the amount invested at 3%, then, 70000 – x is the amount invested at 5%. The yearly interest is the product of the rates and the amount invested so,
(3%)(x) = yearly interest of the amount invested at 3%
(5%)(70000 – x) = yearly interest of the amount invested at 5%

Now, the annual interest at 3% is 100 more than the annual interest at 5%. This means that if we add 100 to the yearly interest at 5%, the interests will be equal. That is,

(3%)(x) = (5%)(70000 – x) + 100.

Next, we convert percent to decimal by dividing the percentage by 100. So,
(0.03)(x) = (0.05)(70000 – x) + 100.

Simplifying, we have

0.03x = 3500 – 0.05x + 100
0.03x = 3600 – 0.05x
0.03x + 0.05x = 3600
0.08x = 3600.

Tip: You can calculate better by eliminating the decimal. You can do this by multiplying both sides by 100.

Dividing both sides by 0.08, we have

x = 45000.

This means that 45000 is invested at 3% yearly interest.

Now, the remaining amount is 70000 – 45000 = 25000.

This means that 25000 is invested at 5%.

Check:
Yearly interest of 45000 at 3% interest = (45000 x 0.03) = 1350

Yearly interest of 25000 at 5% interest = (25000 x 0.05) = 1250

As we can see, the interest at 45000 at 3% interest is 100 more than the interest of 25000 at 5% interest.

Therefore, we are correct.

How to Solve Investment Problems Part 3

This is the third part of the Solving Investment Problems Series. In this part, we discuss solve invest problem which is very similar to the second part. We discuss an investment at two different interests.

Problem

A government employee invested a part of Php60000 in bonds at 6% yearly interest and the remaining part in stocks at a 5% yearly interest. The annual interest in stocks is Php850 less than the annual interest in bonds. How much was invested at each rate?

Solution and Explanation

Let x = amount invested on bonds (6% yearly interest)
60,000 – x = amount invested on stocks (5% yearly interest).

The interest in bonds is 6% per year times the amount invested or

(0.06)(x)

when the percentage is converted to decimals.

In addition, the interest in stocks is 5% per year times the amount invested or

(0.05)(60000 – x)

when the percentage is converted to decimals.

Now, the interest in stocks is 850 less than the interest in bonds which means that if we subtract 850 from the interest in bonds they will be equal. That is

interest in bonds – 850 = interest in stocks.

Substituting the expressions of each, we have

(0.06)(x) – 850 = (0.05)(60000 – x).

Simplifying, we have

0.06x – 850 = 3000 – 0.05x.

To eliminate the decimal numbers, we multiply everything by 100. The equation becomes

6x – 85000 = 300000 – 5x

We simplify by adding 85000 and 5x to both sides. The equation becomes.

11x = 385000.

Dividing both sides by 11, we have
x = 35000.

So, 35000 was invested in bonds and 60000-35000 = 25000 was invested in stocks.

Check:

35000 × 0.06 = 2100
25000 × 0.05 = 1250

Indeed, the amount invested in bonds is 2100 – 1200 = 850 less than the interest in stocks.

How to Solve Investment Problems Part 4

This is the fourth part of the Solving Investment Problems Series. In this part, we discuss a problem which is very similar to the third part. We discuss an investment at two different interest rates.

Problem
Mr. Garett invested a part of $20 000 at a bank at 4% yearly interest. How much does he have to invest at another bank at a 8% yearly interest so that the total interest of the money is 7%.

Solution and Explanation
Let x be the money invested at 8%
(1) We know that the interest of 20,000 invested at 4% yearly interest is

20,000(0.04)

(2) We also know that the interest of the money invested at 8% is

(0.08)(x)

(3) The interest of total amount of money invested is 7%. So,

(20,000 + x)(0.07)

Now, the interest in (1) added to the interest in (2) is equal to the interest in (3). Therefore,

20,000(0.04) + (0.08)(x) = (20,000 + x)(0.07)

Simplifying, we have

800 + 0.08x = 1400 + 0.07x

To eliminate the decimal point, we multiply both sides by 100. That is

80000 + 8x = 140000 + 7x
8x – 7x = 140000 – 80000
x = 60000

This means that he has to invest $60,000 at 8% interest in order for the total to be 7% of the entire investment.

Check:
$20,000 x 0.04 = $800
$60,000 x 0.08 = 4800

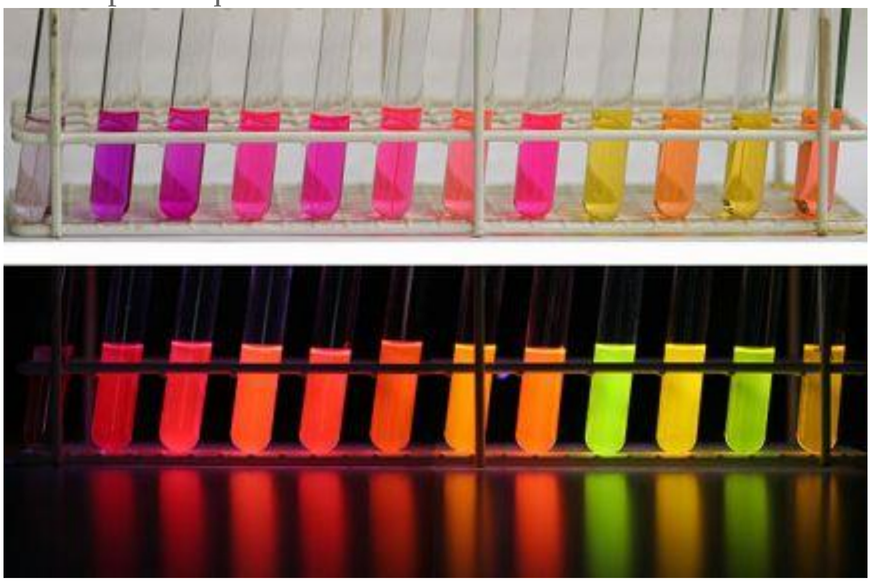
Adding the two interest, we have $5600. We check if this is really 7% of the total investment.
Our total investment is $80,000.

Now, $80,000 x 0.07 = $5600.

You might also like:

How to Solve Mixture Problems Part 1

If you have followed this blog, then you would know that we have been tackling a lot of**math word problems**. In this series, we will learn to solve another type of math word problem called mixture problems. Mixture problems are easy if you know how to set up the equation.



Mixture problems can be classified into two, those which deals with percent and the other which deals with price. In any case, the method of solving is almost the same. Of course, in working with percent, you must be able to know the basics of percentage problems. Let's have our first example.

Example 1
How many ml of alcohol does a 80 ml mixture if it contains 12% alcohol?

Solution and Explanation
In this example,the word mixture means that alcohol is mixed with another liquid (we don't know what it is and we don't need to know). The total mixture contains 80 ml and we are looking for the pure alcohol content which is 12% of the entire mixture. Therefore,

$$\text{pure alcohol content} = 12\% \times 80 \text{ ml} = 0.12 \times 80 \text{ ml} = 9.6 \text{ ml}$$

In the calculation above, we converted percent to decimal (12% to 0.12) and then multiply it with 80. This means that in the 80 ml alcohol, 9.6 ml is pure alcohol.

Example 2
What is the total alcohol content of an 80 ml mixture containing 12% alcohol and a 110 ml mixture containing 8% alcohol?

Solution and Explanation
In this problem, we need to "extract" the pure alcohol content of both mixtures and add them in order to find the volume of the pure alcohol content of both mixtures.

In Example 1, we already know that it contains 9.6 ml of alcohol, therefore, we only need to solve for the second mixture.

$$\text{Pure alcohol content of Example 2} = 8\% \times 110 \text{ ml} = 0.08 \text{ x } 110 \text{ ml} = 8.8 \text{ ml}$$

In the calculation above, we converted 8% to decimal so it became 0.08. Multiplying 0.08 by 110 gives us 8.8 ml. Therefore,

$$\text{total alcohol content} = 9.6 \text{ ml} + 8.8 \text{ ml} = 18.4 \text{ ml.}$$

That means that the two mixtures contain 18.4 ml of pure alcohol in total.

The second problem shows that if we have more than one mixture, and we want to find the total amount of pure content, then we need to add the pure contents in the mixtures. Now, does the percentages add up? Will the total mixture contain 20% alcohol? Let's see.

The total amount of liquid = 110 ml + 80 ml = 190 ml
Total amount of alcohol = 18.4 ml

$$18.4 \text{ ml} / 190 \text{ ml} = 0.0968$$

As we can see, if we convert 0.0968 to percent, it becomes 9.68% and not 20%. In what case will they add up?
We will use the concepts we have learned in these two problems to solve more complicated problems in the next post. Stay tuned by subscribing in the email subscription box on the right part of the page.

How to Solve Mixture Problems Part 2

In the **previous post**, we have learned the basics of mixture problems. We have learned that if solutions are added, then the pure content of the combined solution is equal to the sum of the all the amount of pure content in the added solutions.
In this post, we are going to discuss two more mixture problems. We have already finished two examples in the previous part, so we start with Example 3.

Example 3

How many liters of 80% alcohol solution must be added to 60 liters of 40% alcohol solution to produce a 50% alcohol solution.

Solution and Explanation

The first thing that you will notice is that we don't know the amount of liquid with 80% alcohol solution. So, if we let x = volume of the solution of liquid with 80% alcohol content. So, the amount of pure alcohol content is 80% times x.
We also know that the amount of alcohol in the second solution is 60% times 40.

Now, if we let solution 1 be equal to the solution with 80% alcohol, solution 2 with 40% alcohol, and solution 3 be the combined solutions, we have

amt of alchohol in solution 1 = 0.8x
amt of alcohol in solution 2 = (0.4)(60)
amt of alchohol in solution 3 = 0.50(0.8x + 60)

Note that we have already **converted the percentages to decimals** in the calculation above: 80% = 0.8, 40% = 0.4, and 50% = 0.5.
amt. of alcohol in solution 1 + amt. of alcohol in solution 2 = amt. of alcohol in solution 3

Solving, we have

$0.8x + (0.4)(60) = 0.50(x + 60)$

$0.8x + 24 = 0.5x + 30$

$0.8x – 0.5x = 30 – 24$

$0.3x = 6$

To get rid of the decimal, we multiply both sides by 10.

$3x = 60$

$x = 60/3$

$x = 20.$

This means that we need 20 liters of solution 1, the solution containing 80% alcohol. We need to combine this to solution 2, to get solution 3 which has a 50% alcohol content.

In problems like this, you can check your answers by substituting the value of x to the original equation.

$0.8x + (0.4)(60) = 0.50(x + 60)$

$0.8(20)+ (0.4)(60) = 0.50(20 + 60)$

$16 + 24 = 0.50(80)$
$40 = 40$

Indeed, the amount of alcohol in the left hand side is the same as the amount of alcohol in the right hand side.

How to Solve Mixture Problems Part 3

In the previous post, we have discussed threeexamples on how to solve mixture problems. In this post, we are going to learn how to set up the givens in a mixture problem in a table, so it is easier to solve. Let's have the fourth example.
Example 4

How many liters of pure water must be added to 15 liters of a 20% salt solution to make a 5% salt solution?

Solution and Explanation

In the first column, we placed the liquid or solution. We have three kinds: water, the liquid with 20% salt, and liquid with 5% salt. In the second column, we place the volumes of ths liquid. We do not know the volume of water, so we represent it with x. Since we have to combine water with 20% salt solution, we have to add the volumes of the two liquid. This makes sense since if we add more water, the amount of salt is getting lesser in relation to the total volume of the liquid.

Liquid/Solution	volume of liquid (L)	% of salt	total amount of salt
water	x	0%	0
20% salt solution	15	20%	(15 L)(0.20) = 3
5 % salt solution	x+ 15	5%	0.05 (x + 15)

Again, the total amount of salt when water is combined with 20% salt solution should also be equal to the total amount of salt in the 5% salt solution. That means that in the last column, we have to add the first and the second row and then equated to the third row. That is,

$3 = 0.05(x + 15)$

$3 = 0.05x + 0.75$

$2.25 = 0.05x$

$45 = x$

That means that we need 45L of water to turn a 20% salt solution to a 5% solution.

Check:

$3 = 0.05(x + 15)$

$3 = 0.05(45 + 15)$

$3 = 0.05(60)$

$3 = 3$

How to Solve Mixture Problems Part 4

This is the fifth post of the **Solving Mixture Problems Series** on PH Civil Service Review. In this post, we are going to solve a problem in which only the total amount of mixture is given.

Problem

A chemist creates a mixture with 5% boric acid and combined it with another mixture containing 40% boric acid to obtain a 800 ml of mixture with 12% boric acid. How much of each mixture did he use?

Solution and Explanation

Let

x = mixture with 5% boric acid

800 – x = mixture with 40% boric acid.

Note that if these two mixtures are combined, we will produce a mixture with 800 ml of solution with 12% boric acid. As we have learned in the previous tutorials, the amount of boric acid in the first mixture added to the amount of boric acid in the second mixture is equal to the amount of boric acid in the combined mixtures. That is,

(5%)(x) + (40%)(800-x) = (12%)(800).

Take note that x, 800 – x, and 800 are amount of the mixture and if these are multiplied by the percentage of boric acid, then we will get the exact amount of pure boric acid.

Converting percent to decimals, we have

(0.05)(x) + (0.4)(800-x) = (0.12)(800)

0.05x + 320 – 0.4x = 96.

Simplifying, we have

-0.35x = -224

x= 640.

That means that we need 640 ml of mixture with 5% boric acid and 800 – 640 = 160 ml of mixture with 40% boric acid.

Check:
Amount of boric acid in mixture with 5% boric acid: (640 ml)(0.05) = 32ml
Amount of boric acid in mixture with 40% boric acid: (160 ml)(0.4) = 64ml
Amount of boric acid in mixture with 12% boric acid: (800 ml)(0.12) = 96ml

As we can see, 32 ml + 64 ml = 96 ml which means that we are correct.

How to Solve Mixture Problems Part 5

In the previous posts, we have learned how to solve mixture problems involving percentages and liquid mixture problems. In this post, we are going to solve mixture problems involving prices. Although these two types of problems are different, they are very similar when you set up the equation. Below is our first problem.

Problem
A seller mixes 20 kilograms of candy worth 80 pesos per kilogram to candies worth 50 pesos per kilogram. He sold at 60 pesos per kilogram.

After selling all the candies he discovered that he had no gain or loss. How much of the 50-pesos per kilogram candies did he use?

Solution and Explanation
Let x = number of kilograms of candy worth 50-pesos per kilogram.

The total price of 20 kilograms of candy at 80 pesos per kilogram is (20 kg)(80 pesos/kg) = 1600 pesos.

The total price of x kilograms of candy at 50 pesos per kilogram is (x kg)(50 pesos/kg) pesos.

When we add these 2, the number of kilograms of candy is x + 20 (can you see why?) and it is sold at 60 pesos. So, its total price is (x + 20)(60 pesos/kg).

Using these facts, we have the following equation:

total price of 80pesos/kg candy + total price of 50pesos/kg candies = total price of 60pesos/kg candies.

Substituting the expressions above, we have

(20 kg)(80 pesos/kg) + (x kg)(50 pesos/kg) = (x + 20 kg)(60 pesos/kg)

$1600 + 50x = 60(x + 20)$
$1600 + 50x = 60x + 1200$
$1600 – 1200 = 60x – 50x$
$400 = 10x$
$40 = x.$

Therefore, he used 40 kilograms of candy worth 50 pesos per kilogram.

Check:

$1600 + 50(40) = 60(40 + 20)$
$1600 + 2000 = 60(60)$
$3600 = 3600.$

This means that we are correct.

How to Solve Mixture Problems Part 6

This is the 6th part and last part of the**Solving Mixture Problems Series**. In the previous 4 parts, we have learned how to solve mixture problems involving percent and in part 5, we have learned how to solve problems involving percents. In this post, we solve another problem involving percent.
A bakery owner wants to box assorted chocolates. Each box is to be made up of packs of black chocolates worth \$10 per pack and packs of ordinary chocolates worth \$7 each pack. How many packs of each kind should he use to make 15 packs which he can sell for \$8 per pack?

Solution
Let x = number of \$10 packs
15 – x = number of \$7 packs

Multiplying the cost per pack and the number of packs we have

(\$10)(x) = total cost of \$10 packs
(\$7)(15 – x) = total cost of the \$7 packs
(\$8)(15) = total cost of all the chocolates

Now, we know that

total cost of \$10 packs + total cost of the \$7 packs = total cost of all the chocolates.

Substituting the values, we have

(\$10)(x) + (\$7)(15 – x) = (\$8)(15).

Eliminating the dollar sign and solving for x, we have

(10)(x) + (7)(15 – x) = (8)(15)
10x + 105 – 7x = 120
3x + 105 = 120
3x = 120 – 105
3x = 15

x = 5.
This means that we need 5 packs of $10 and 10 packs of $7 chocolates.

Check:
($10)(5) + ($7)(10) = ($8)(15)
$50 + $70 = $120
$120 = $120
Therefore, we are correct.

Introduction to Motion Problems

We have already finished learning how to solve number problems and age problems, so we continue with learning motion problems. Motion problems deal with moving objects such as cars, planes, boats, etc and their speed, distance traveled and time spent traveling. Walking and running are also usually asked in motion problems. Motion problems also include headwind, tailwind, for planes and other flying objects and upstream and downstream problems on boats traveling in rivers.

Average Speed and Actual Speed

In motion problems, you can usually read phrases like the "speed of a car is 60 kilometers per hour." In many of such problems, it appears that the cars are traveling at a constant speed. In reality, of course, speed is not usually constant. In the course of the travel, a car will accelerate, decelerate, or even stop at an intersection if the light is red. This means that when we talk about "60 kilometers per hour" in moition problems, in reality, they mean average speed. That is, in one hour, a car usually travels 60 kilometers given normal traffic conditions. Usually, in math problems, rate and speed means the same thing.

Distance = Rate Times Time

A car that travels 70 kilometers per hour can travel 140 kilometers in two hours. We can write this as an equation shown below

140km = (70 km/hr)(2hrs).

Since 140 kilometers is the *distance (d)*, 70 km/hr is the *rate (r)*, and 2 hours is the *time (t)*, it follows that

$$\text{distance} = \text{rate} \times \text{time}$$

$$\text{or } d = rt.$$

This is a basic formula and you don't really need to memorize it if you understand the concept. Now, for the examples, let's solve two basic motion problems.

Problem 1

Aron goes to a convenient store every week using a bicycle. If his average speed using a bicycle is 30 km/hr, how far is the convenient store if it takes him 15 minutes to go there?

Solution 1

This problem can be solved by inspection, but we will setup the equation in solution 2. For inspection, 15 minutes is 1/4 of an hour, so if it takes Aron to travel 30 km per hour, then he will travel 1/4 of it given 1/4 of an hour. Therefore, 1/4 of 30 kilometers is 7.5 kilometers.

Solution 2

The second solution is needed so you would learn how to set up equations. Later problems will be a lot harder than this, so you will need to set up the equations correctly.

distance = ?

rate = 30 km/h

time 1/4 hr

$d = rt$

$d = 30(\frac{1}{4}) = 7.5$ km.

Problem 2

It took 4.5 hours from Ninoy Aquino International Airport (Philippines) to Narita Airport (Japan). If the route of the plane is around 2250 kilometers, what is its speed?

Solution

$t = 4.5$ hours

$r = ?$

$d = 2250$ kilometers

From the original equation, we have

$d = rt$

$2250 = 4.5(r)$

To get r, divide both sides of the equation by 4.5 giving us

$r = 500.$

So, the airplane is traveling 500 kilometers per hour.

That's it! In the next post, we discuss how to solve more motion problems.

You might also like:

How to Solve Motion Problems Part 1

Now that I have introduced to you motion problems, let us solve the type of problems that usually appear in textbooks as well asexaminations. In this part of the series, we will learn how to analyze and solve a problem involving objects moving in the same direction. This is our third problem in the **How to Solve Motion Problems Series**.

Problem 3

Car *A* left Math City going to English City at an average speed of 40 kilometers per hour. Two hours later, Car B traveling 60 kilometers per hour leaves the same place for English City. In how many hours, will the car B overtake Car A?

Solution 1

This problem can be solved manually by creating the table as shown in the next figure. In the first column, we have the cars, and in the first row the number of hours.

Car A traveled 40 kilometers and Car B travels 60 kilometers per hour. Since Car B only started traveling after two hours, so, after three hours, Car A has traveled 120 kilometers and Car B just 60 kilometers. Now, since Car B is faster, it will eventually overtake Car A which is after 6 hours shown below. Having the same distance traveled in this problem means that Car B overtakes Car A since they are traveling the same route.

Car/Hr	1	2	3	4	5	6
Car A	40	80	120	160	200	240
Car B	0	0	60	120	180	240

Now notice that the question in "how many hours" means that the counting starts when Car B left. Therefore, the answer is 4 hours because in 4 hours, car B traveled 240 kilometers, the same distance traveled by Car A.

Solution 2

In solving Motion problems, it is always helpful to create a table of the given distance, rate, and time. The rate r of Car A is 40km/hr and Car B is 60 km/hr. Now, for the time, Car A started two hours earlier, so if, for example, it has traveled 5 hours, Car B has only traveled 3 hours. It means that the time Car A traveled is 2 hours more than that of Car B. This means that if Car B traveled x hours, then car A traveled $x+2$ hours. Now, as we have discussed in the previous post, $d=rt$, so for column d in the table below, we just multiply the rate (r) and the time (t). Now what to do next?

	r (km/hr)	t (hrs)	d (km)
Car A	40	x + 2	40(x+2)
Car B	60	x	60x

At the exact time Car B overtakes Car A, which is what is asked above, their distance traveled will be equal. This means that

distance traveled by Car A = distance traveled by Car B

or using the expressions above

$$40(x+2) = 60x$$

Note that the x in this equation is the time from the table above, so the answer will be in hours. We solve the equation by simplifying the left hand side first by distributive property

$$40x + 80 = 60x.$$

Subtracting $40x$ from both sides results to

$80 = 20x$ which simplifies to

$$4 = x.$$

This means that Car A overtakes car B in 4 hours after Car B left. This confirms the answer in Solution 1.

This post is a bit detailed in order to let you understand how to solve the equations. The next two problems will be shorter.

How to Solve Motion Problems Part 2

This is the second part of the How to Solve Motion Word Problem series. I suggest that you read first the introduction to this series as well as the first part in order to understand better.

In the first part of this series, we discusses about a faster object overtaking a slower one who left first. In this post, we continue to solve motion problems involving two objects traveling in the same direction and one object faster than the other; this time, they left at the same time. We want to know that given a particular time when they are a number of kilometers apart.

Problem 4

A freight train and a passenger train left the same station and traveled to the same direction. The passenger train travels at an average speed of 80 kilometers per hour while the freight train travels at an average speed of 65 kilometers per hour. In how many hours will they be 75 kilometers apart?

Solution 1

Like in the first part of this series, the problem can be solved using a table. The table below shows the distance traveled by the train after each hour. In the 5th hour, the passenger train traveled 400 km while the freight train traveled 325 kilometers. This means that after 5 hours, the distance between them is 75 kilometers.

	Distance Traveled AFTER				
	1h	2h	3h	4h	5h
Passenger Train	80	160	240	320	400
Freight Train	65	130	195	260	325
Distance Between	15	30	45	60	75

Solution 2

An algebraic solution can also be done to solve the problem above. Let us create a table like what we have done in the first part of this series. As shown in the table below, the rate of the passenger train is 80 kilometers per hour and the rate of the freight train is 65 kilometers per hour. The time traveled is the same since they left the same location at the same time. The distance is the product of the rate and time so we multiply them as shown below.

Trains	r (km/hr)	t (hrs)	d (km)
Passenger	80	x	80x
Freight	65	x	65x

Now, that we have all the given in place, let us analyze the problem. We are asked for the number of hours when the trains are 75 kilometers part. The phrase "75 kilometers apart" means the

d traveled by the passenger train – *d* traveled by the freight train = 75

or

$$80x - 65x = 75$$

$$15x = 75$$

Dividing both sides by 15, we have

$$x = 5.$$

Note that x is the number of hours in the table, therefore, in 5 hours, the trains will be 75 kilometers apart.

How to Solve Motion Problems Part 3

This is the third part of the How to Solve Motion Problems Series, a part of the Word Problem Solving Series of Ph Civil Service Reviewer. In Part 1 and Part 2 of this series, we discussed objects moving in the same direction. In this part, we are going to discuss objects moving toward each other. We have already discussed four problems in the previous parts of this series, so, we solve the fifth problem.

Problem 5

A car leaves City A and travels towards City B at an average speed of 60 kilometers per hour. At the same time, another car leaves City B and travels towards city A at an average speed of 70 kilometers per hour. If the two cars use the same route, and if the distance between two cities is 520 kilometers, how many hours before they meet?

Solution 1

Again, this problem can be solved using a table. By now, you would have realized that most motion problems can be solved by creating a table. You can use the table solution in case you cannot solve the motion problem algebraically.

	1h	2h	3h	4h
Car from City A	60	120	180	240
Car from City B	70	140	210	280
Total Distance	130	260	390	520

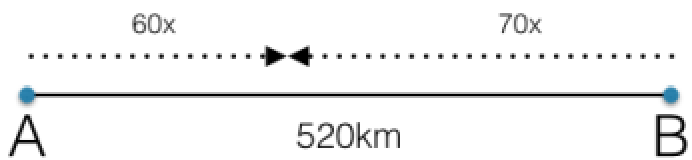
As we can see in the table above, for one hour, the two cars have traveled 130 kilometers toward each other. Since the distance between the two cities is 520 kilometers, it will take them four hours.

Solution 2

In the algebraic solution, it is also important to create a table as shown below. Shown on the columns are the rate, time, and distance. The car from City A travels at 60 kilometers per hour and the car from City B 70 kilometers per hour. The time they spend on the road is the same since they left the cities at the same time. Then, the distance they traveled is the product of the rate and the time.

Car	r (km/hr)	t (hrs)	d (km)
From City A	60	x	60x
From City B	70	x	70x

Note that at the exact time they meet, the would have traveled the total distance from A to B which is 520 kilometers. This means that if we add the distance they traveled, then the sum is 520 kilometers.



From the discussion above, the equation is

d traveled by car from City A + d traveled by car from City B = 520 km.

Substituting the expressions on the table, we have

$$60x + 70x = 520$$

$$130x = 520$$

Dividing both sides by 4, we have

$$x = 4$$

Therefore, the two cars will meet in **<u>4 hours</u>** after they have left the two cities.

Check

By the time they meet, the car from City A will have traveled 60(4) = 240 kilometers and car from City B will have traveled 70(4) = 280 kilometers. If you add the distance traveled by both cards, the answer is 520 kilometers. Therefore, we are correct.

How to Solve Motion Problems Part 4

In Part 1, Part 2, and Part 3 of the How to Solve Motion Problems Series, we have learned how to solve problems involving objects moving in the same direction as well as those which move toward each other. In this post, we are going to learn about objects which move on opposite directions. Themethod in solving this problem is very similar to the method used in Part 3 of this series.

We now solve the sixth problem in this series.

Problem 6

Two jet planes left Naria Airport at 9:00 am and travels in opposite directions. One jet travels at an average speed of 450 kilometers per hour and the other jet travels at an average speed of 550 kilometers per hour. By what time will the two jets be 2500 kilometers apart?

Solution 1

Just like in the previous parts, we can solve this problem using a table. As we can see, in the first hour, the jets will be 1000 kilometers apart. After three hours they will be 3000 kilometers apart.

	Distance Traveled After		
	1h	2h	3h
Plane A	450	900	1350
Plane B	550	1100	1650
Total	1000	2000	3000

The question, however, is the time when they are 2500 kilometers. From the table above, since after each hour, the distance traveled is 1000 kilometers, then 2500 kilometers will require 2 and a half hours. Now, 2 and a half hours after 9 am is 11:30 am.

Solution 2

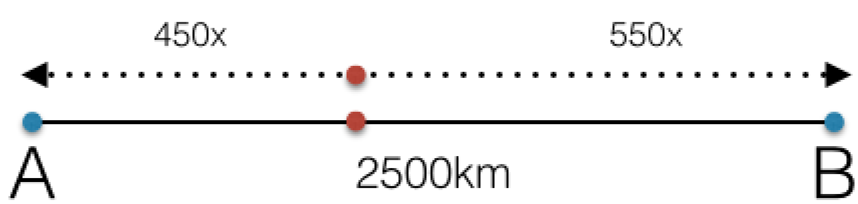
We name the jet planes A and B as shown below. Plane A travels at 450 kilometers per hour and plane B at 550 kilometers per hour. The time traveled by both planes, we let x and since they both start and the same time, they will have the same time (duration) traveled. The distance *d* is the product of the rate and the time.

Note: The time of departure so (9:00 am) is irrelevant at first in solving the problem. It can only be used after the answer is obtained.

Plane	r (km/hr)	t (hrs)	d (km)
A	450	x	450x
B	550	x	550x

Now that we have the table, let us examine the figure below. In the question, how many hours will the two planes be 2500 kilometers apart. Since the planes are traveling in opposite directions, the word "apart" in the problem means the distance traveled by the first plane (450x) and the distance traveled by the second plane (550x). Therefore, we can form the equation

d traveled by Plane A + d traveled by plane B = 2500



If we substitute the values in the table in the preceding equation, then we have

$450x + 550x = 2500$

$1000x = 2500$

$x = 2.5$ hours

Meaning, if the planes are 2500 kilometers apart, two and a half hours would have past. Therefore, 2 and a half hours from the time of the departure which is 9:00 am is 11:30 am.

Therefore the correct answer is 11:30 am. This confirms the answer in Solution 1.

How to Solve Word Problems Involving Ratio Part 1

In a dance school, 18 girls and 8 boys are enrolled. We can say that the ratio of girls to boys is 18:8 (read as 18 is to 8). Ratio can also be expressed as fraction so we can say that the ratio is 18/8. Since we can reduce fractions to lowest terms, we can also say that the ratio is 9/4 or 9:4. So, ratio can be a relationship between two quantities. It can also be ratio between two numbers like 4:3 which is the ratio of the width and height of a television screen.

Problem 1

The ratio of boys and girls in a dance club is 4:5. The total number of students is 63. How many girls and boys are there in the club?

Solution and Explanation

The ratio of boys is 4:5 means that for every 4 boys, there are 5 girls. That means that if there are 2 groups of 4 boys, there are also 2 groups of 5 girls. So by calculating them and adding, we have

4 + 5 = 9
4(2) +5(2) =18
4(3) +5(3) =27
4(4) +5(4) = 36
4(5) +5(5) = 45
4(6) +5(6) =54
4(7) +5(7) =63

As we can see, we are looking for the number of groups of 4 and, and the answer is 7 groups of each. So there are 4(7) = 28 boys and 5(7) = 35 girls.

As you can observe, the number of groups of 4 is the same as the number of groups of 5. Therefore, the question above is equivalent to finding the number of groups (of 4 and 5), whose total number of persons add up to 63.

Algebraically, if we let x be the number of groups of 4, then it is also the number of groups of 5. So, we can make the following equation.
4 x number of groups + 5 x number of groups of 5 = 63

Or

$4x + 5x = 63$.

Simplifying, we have

$9x = 63$
$x = 7$.

So there are 4(7) = 28 boys and 5(7) = 35 girls. As we can see, we confirmed the answer above using algebraic methods.

How to Solve Word Problems Involving Ratio Part 2

This is the second part of a series of post on Solving Ratio Problems. In the **first part**, we have learned how to solve intuitively and algebraically problems involving ratio of two quantities. In this post, we are going to learn how to solve a ratio problem involving 3 quantities.
Problem 2
The ratio of the red, green, and blue balls in a box is 2:3:1. If there are 36 balls in the box, how many green balls are there?
Solution and Explanation
From the previous, post we have already learned the algebraic solutions of problems like the one shown above. So, we can have the following:

Let x be the number of grous of balls per color.

$2x + 3x + x = 36$

$6x = 36$

$x = 6$

So, there are 6 groups. Now, since we are looking for the number of green balls, we multiply x by 3.

So, there are 6 groups (3 green balls per group) = 18 green balls.

Check:
From above, $x = 6(1)$ is the number of blue balls. The expression 2x represent the number of red balls, so we have 2x = 2(6) = 12 balls. Therefore, we have 12 red balls, 18 green balls, and 6 blue balls.

We can check by adding them: 12 + 18 + 6 = 36.

This satisfies the condition above that there are 36 balls in all. Therefore, we are correct.

How to Solve Word Problems Involving Ratio Part 3

In the previous two posts, we have learned how to solve word problems involving ratio with **two** and **three** quantities. In posts, we are going to learn how to solve a slightly different problem where both numbers are increased.
Problem

The ratio of two numbers is 3:5 and their sum is 48. What must be added to both numbers so that the ratio becomes 3:4?

Solution and Explanation

First, let us solve the first sentence. We need to find the two numbers whose ratio is 3:5 and whose sum is 48.

Now, let x be the number of sets of 3 and 5.

$3x + 5x = 48$
$8x = 48$
$x = 6$

Now, this means that the numbers are 3(6) = 18 and 5(6) = 30.

Now if the same number is added to both numbers, then the ratio becomes 3:4.

Recall that in the previous posts, we have discussed that ratio can also be represented by fraction. So, we can represent 18:30 as $\frac{18}{30}$. Now, if we add the same number to both numbers (the numerator and the denominator), we get $\frac{3}{4}$. If we let that number y, then

$\frac{18+y}{30+y} = \frac{3}{4}$.

Cross multiplying, we have

$4(18+y) = 3(30+y)$.

By the distributive property,

$72 + 4y = 90 + 3y$

$4y - 3y = 90 - 72$

$y = 18$.

So, we add 18 to both the numerator and denominator of $\frac{18}{30}$. That is,

$$\frac{18+18}{30+18} = \frac{36}{48}.$$

Now, to check, is $\frac{36}{48} = \frac{3}{4}$? Yes, it is. Divide both the numerator and the denominator by 12 to reduce the fraction to lowest terms.

How to Solve Word Problems Involving Ratio Part 4

This is the fourth and the last part of thesolving problems involving ratio series. In this post, we are going to solve another ratio word problem.
Problem

The ratio of two numbers 1:3. Their difference is 36. What is the larger number?

Solution and Explanation

Let x be the smaller number and 3x be the larger number.
3x – x = 36
2x = 36
x = 18

So, the smaller number is 18 and the larger number is 3(18) = 54.

Check:

The ratio of 18:54 is 1:3? Yes, 3 times 18 equals 54.
Is their difference 36? Yes, 54 – 18 = 36.

Therefore, we are correct.

How to Solve Venn Diagram Problems Part 1

According to examinees, some of the items that were included in the recently concluded Civil Service Exams are problems involving Venn Diagrams. So, in this series, we will discuss in detail how to solve these types of problems. If you are not familiar with Venn diagrams, please read the **Introduction to Venn Diagrams**.
Problems involving Venn Diagrams usually discusses choices of groups of people.
Let’s have the first example.

Problem 1

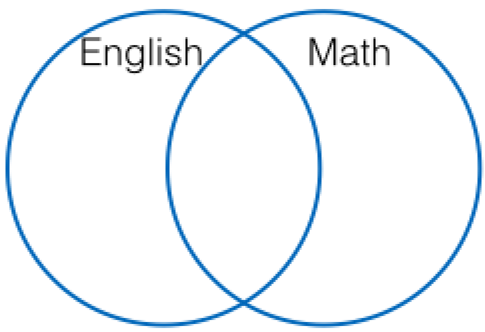
In a class of 40 students, 25 are taking English and 17 are taking Mathematics. If 7 are taking both subjects,

(a) How many students are not taking English, and at the same time not taking Math?

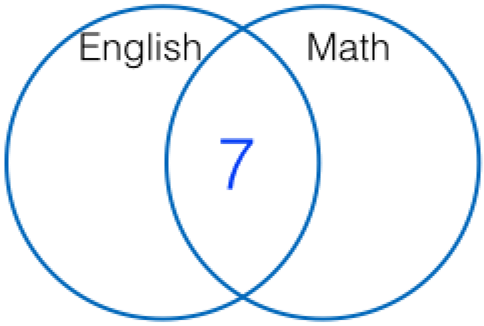
(b) How many students are in English class only? in Math class only?

Solution

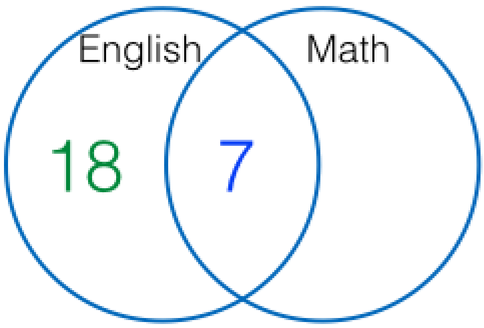
The most effective strategy in solving problems such as this is to create overlapping circles which is usually known as a Venn Diagram. Venn diagrams are used to represent sets. In this problem, we are actually talking about sets, their union, and their intersection. Below is the Venn diagram of the problem above.



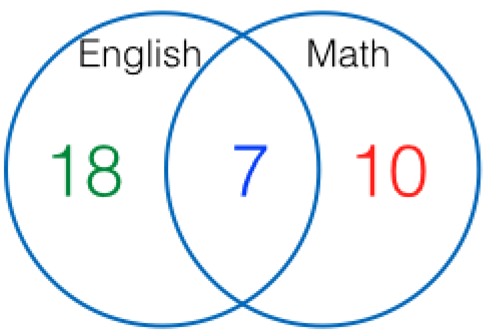
First, notice that the overlapped part belongs to both Math and English, so we can place the 7 students who are taking both subjects in that overlap.



Second, the problem says, there are 25 students taking English. But from the diagram, **7**are taking both subjects. So, the only number of students who are taking English and not taking Math is 25 – 7 = **18**.

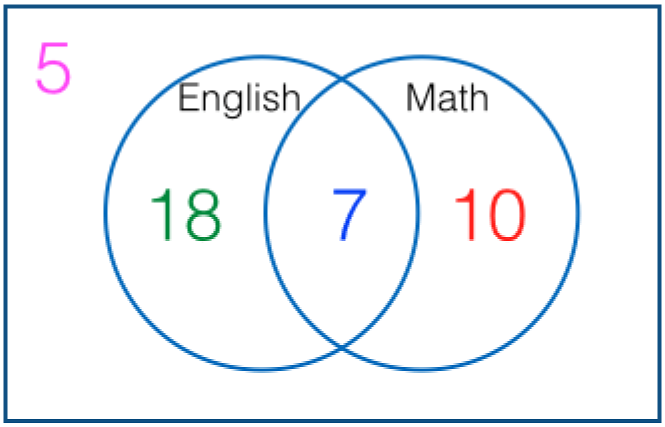


Third, there are 17 students who take Math, but 7 of those students are also taking English. Therefore, there are only **10** students who take Math and not English.



So, all in all, there are 18 + 7 + 10 = 35 students who are either in English, Math or both. Since there are 40 students in all, 40 – 35 = **5** of them are not taking English and at the same time not taking Math. This answers the first question.

If we want to represent the universal set which is all the 40 students, we can place the overlapping circles inside a rectangle (or any other shape) and place 5 inside that rectangle but outside the circles as shown below.



From above, it is also clear that 18 are only taking English and not Math, and 10 are taking Math and not English. This answers the second question.

How to Solve Venn Diagram Problems Part 2

In the previous posts, the **introduction** and the **second part** of this series, we have learned the basics of Venn Diagrams as well as solving the 2-circle Venn Diagram problem. In this post, we are going to solve a more complicated problem which is composed of 3-circle Venn diagram problem.

Venn Diagram Problem

There are 100 students surveyed and asked which of the following subjects they take thissemester: Mathematics, English, or Biology. Below is the result of the survey.

- 35 responded English
- 50 responded Mathematics
- 29 responded Biology
- 12 responded Mathematics and English
- 8 responded English and Biology
- 11 responded Biology and Math

- 5 responded all

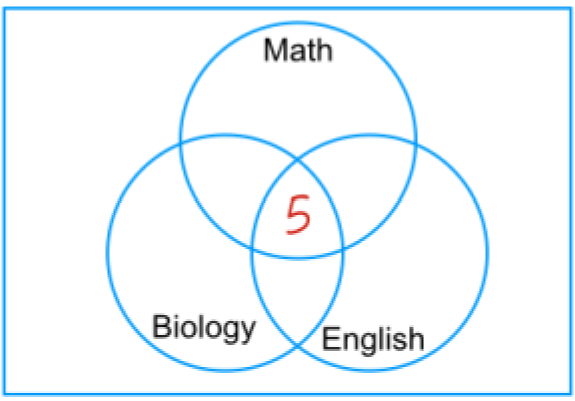
Questions

1. How many students are not taking any of the three subjects?
2. How many students take Math, but not Biology or English?
3. How many students take Math and English, but not Biology?

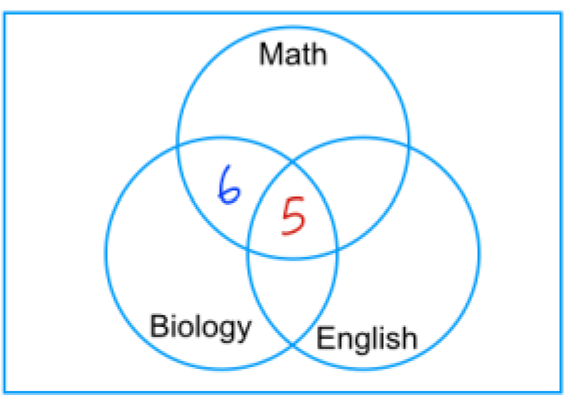
Solution

As we have done in **Solving Venn Diagram Problems Part 1**, we started with the overlapping circles. In this problem, we have four regions which overlap. The the easiest strategy is to start at the center, the part where the three circles overlap. In short, we start from bottom to the top in the result above.

(1). Five students responded that they took all the subjects, so we put 5 at the center.

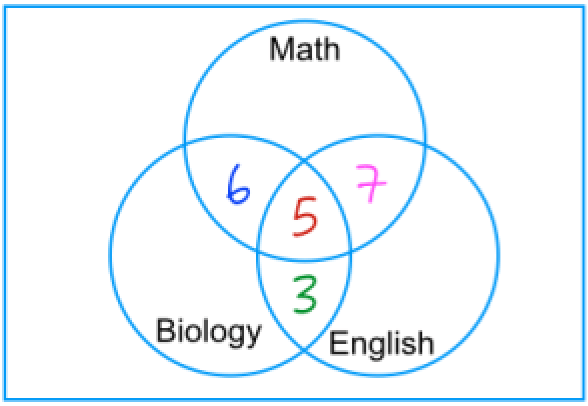


(2). Eleven responded Biology and Math. So, we should put it in the Biology-Math overlap. However, of the 11 who takes Biology and Math, 5 were also taking English as shown in (1). So, there are 11-5 = 6 students in the Math-Biology overlap.

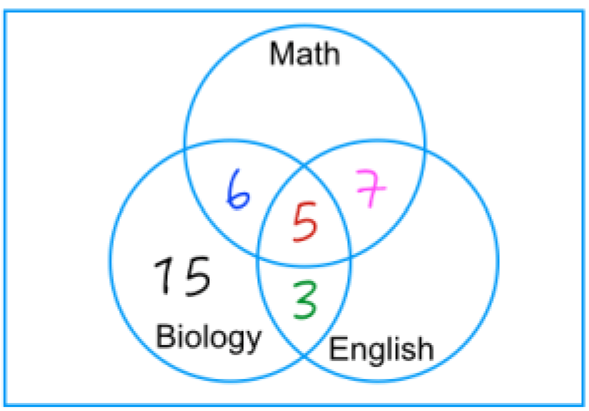


(3). Eight responded Biology and English. But of those 8 taking Biology and English, 5 are also taking Math. So, there are 8 – 5 = 3 students who are taking Biology and English.

Also, there are 12 students who are taking Math and English, and of those 12, 5 are also taking Biology, so there are 12 – 5 = 7 who are taking Math and English. Now see the next figure to see how the Venn diagram should look like after this step.



(4). Next, 29 students responded Biology. But notice that 6, 5, and 3 are already in the Biology circle. So, we subtract those students from 29. That is, 29 – (6 + 5 + 3) = 29 – 14 = 15. So, there are 15 students who take only Biology.

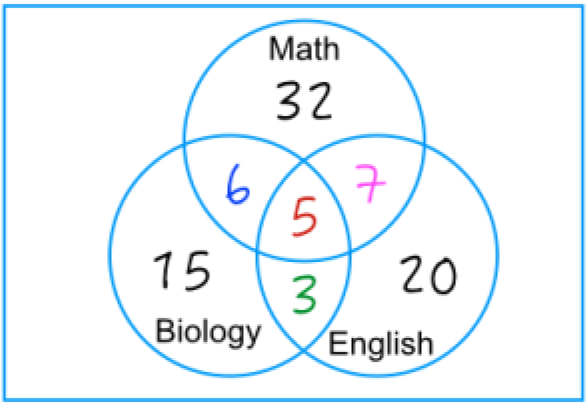


(5). Lastly, there are 50 students who are taking Math and 35 who are taking English. But in the Math circle there are 6 + 5 + 7 students who are also taking the other subjects and in the English circle, there are 3 + 5 + 7 students who are also taking the other subjects. Therefore, we can have the following calculations

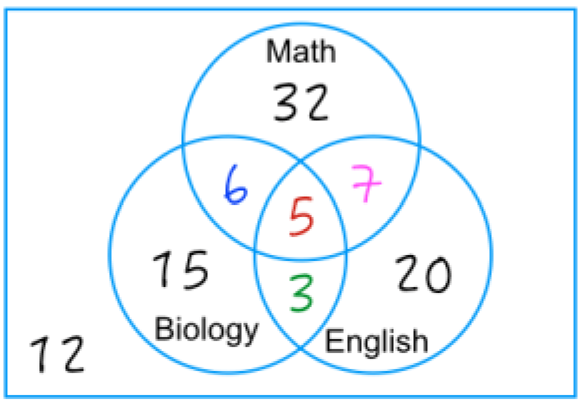
Number of Students Who Take Only Math = 50 – (6 + 5 + 7) = 32

Number of Students Who Take Only English = 35 – (3 + 5 + 7) = 20

After answering the problems above, the Venn diagram should look like below.



Now, to answer the first question, how many students did not take any of the three subjects, recall that there are 100 students who were surveyed. If we add all the numbers in the diagram, 15 + 32 + 20 + 6 + 7 + 3 + 5, the sum is only 88. Therefore, 100 – 88 = 12 students did not take any of the three subjects.



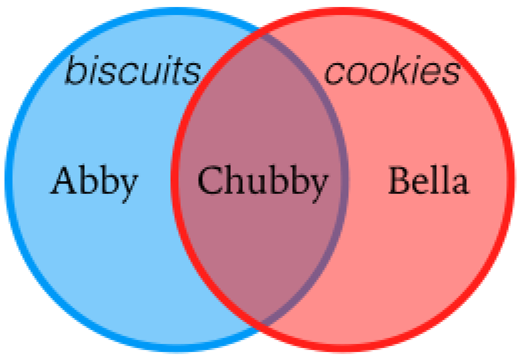
For question 2, another way to rephrase it is how many students take only Math, so the answer is 32.

For question 3, the answer is 7 (can you see why?)
Now, that's it for the Venn Diagram series. We will have practice problems in the next posts

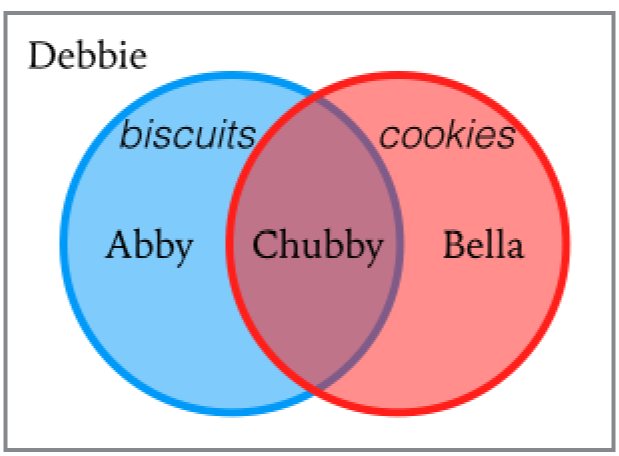
Introduction to Venn Diagrams

According to some examinees, there are several items in the October 2014 Civil Service Exam that requires the use of Venn diagrams, so I will discuss in details how to use Venn diagrams in a series of posts. In this post, I am going to introduce its concept and its use as *layman* as possible. I will limit the discussion to the preparation of solvingword problems involving Venn diagrams.

Venn diagrams are used to represent logical relations between sets. In word problemsinvolving diagrams, elements of sets are usually people who choose or prefer a particular thing (e.g. color, food, hobbies). For instance, we have two available desserts, biscuits and cookies, and then Abby ate a biscuit and Bella ate a cake. Chubby, being extremely hungry, ate a biscuit and a cake. So, the situation can be represented as follows using a Venndiagram.

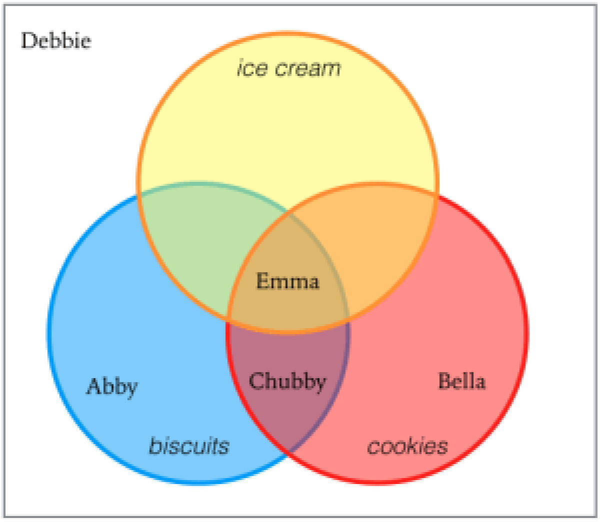


Note that Abby is in the blue area which means that Abby chose biscuits only. Bella is in the red area which means that she chose cookies only. Chubby is in the overlap of the two circles, which means that she chose both. Now, if Debbie is on diet and she didn't eat any dessert, then the Venn diagram would look like below.

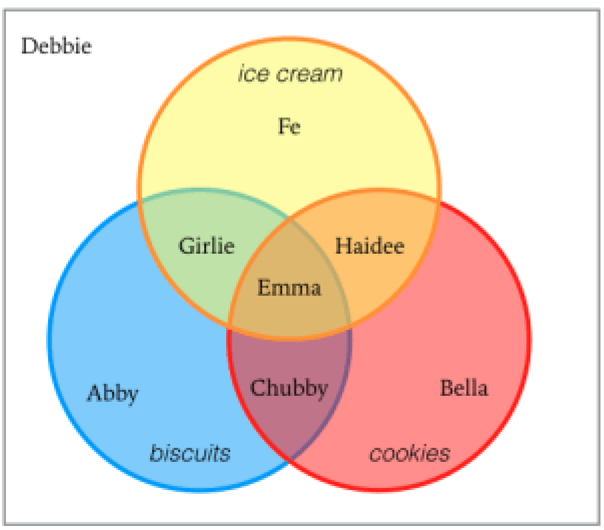


The rectangle which includes all four friends, those who ate dessert and those who did not, is called the *universal set*. Debbie's name is outside the circles because she did not eat cookies and biscuits.

Now suppose there is a third dessert which is ice cream, and Emma ate all three, then thediagram would look like the one in the next figure.



Suppose Fe, Girlie, and Haidee arrived. Fe ate ice cream, Girlie ate biscuits and ice cream, and Haidee ate ice cream and cookies, the diagram would look like the one shown in the next figure.



Once you have learned to use Venn diagrams correctly, it would be easier for you to solve problems involving it. The next step is to read **How to Solve Venn Diagram Problems Part 1**.

The Venn Diagram Problem Solving Tutorial Series

According to some readers who took the Civil Service Examination last April and October, some of the questions include Venn diagram problems, so I wrote a series on how to solve problems about it. The Venn Diagram Problem Solving Tutorial Series is a series of detailed tutorials on how to solve Venn Diagram Word problems. It is recommended that you read the tutorial in the following order.

1. Introduction to Venn Diagrams defines and introduces the use of Venn Diagrams. This includes how to set up problems involving 2-circle Venn Diagrams and 3-circle Venn Diagrams.
2. How to Solve Venn Diagram Problems Part 1 is a detailed tutorial on how to solve 2-circle diagram problems.
3. How to Solve Venn Diagram Problems Part 2 is a detailed tutorial on how to solve 3-circle diagram problems.

If you have questions, please use the comment box below.

How to Solve Work Problems Part 1

This is the first part of the **Solving Working Problems Series**. In this post, we are going to discuss in details the basics of work problems.

Work problems usually involve the time for two or more persons or machines to complete the same job given the *rate* that they can work. For discussion purposes, let us have the following example.

Work problem:

Ariel can paint a house in 5 days and Ben can do the same job in 6 days. In how many days can they complete the job if they work together?

Discussion and Scratch Work

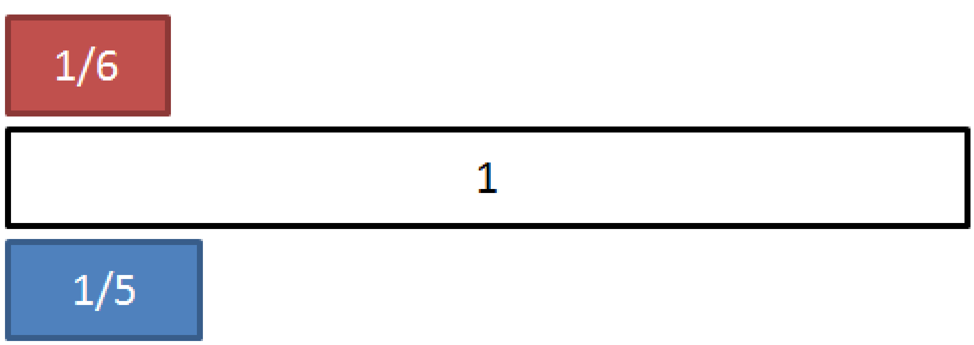
If Ariel can finish the job in 5 days, then if he were to work one day, he would have completed 1/5 of the job. If he works for two days, then he would have completed 2/5 of the job. Similarly, if Ben can finish the same job in 6 days, if he were to work for one day, then he would have completed 1/6 of the job. If he works for 2 days, he would have completed 2/6 of the job (or 1/3 of the job if **reduced to lowest terms**).
Suppose Ariel and Ben work together starting on a Monday. Then their progress can be described as shown in the table below.

Now, since Ariel can finish the job in 5 days, with Ben working with him even at a slower rate, the job can be finished less than 5 days (working together makes the completion shorter!). Looking at their progress in the table below, it is quite clear that the job can be finished in less than 3 days (can you see why?). Examine the table and see why this is so.

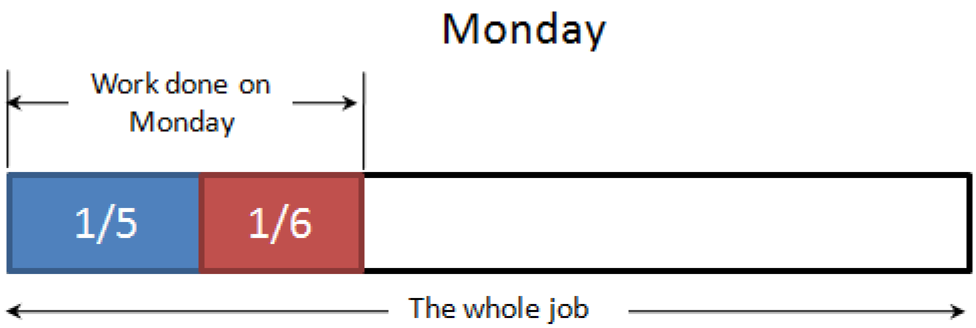
Name/Day	Mon (Day 1)	Tues (Day 2)	Wed (Day 3)
Ariel	1/5	2/5	3/5
Ben	1/6	2/6	3/6

The Geometric Representation of Working Together

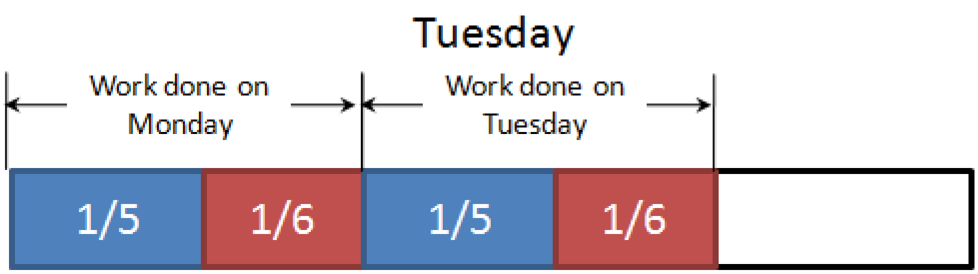
The rates of work of Ariel and Ben can be represented geometrically by comparing them to a length of a rectangle. The length of the red rectangle (Ariel's rate per day) is 1/6 the length the white rectangle, while the length of the blue rectangle (Ben's rate per day) is 1/5 the length of the white rectangle. Since we used the white rectangle as our unit of measure, its length is equal to 1. In this problem, 1 represents the completed job.



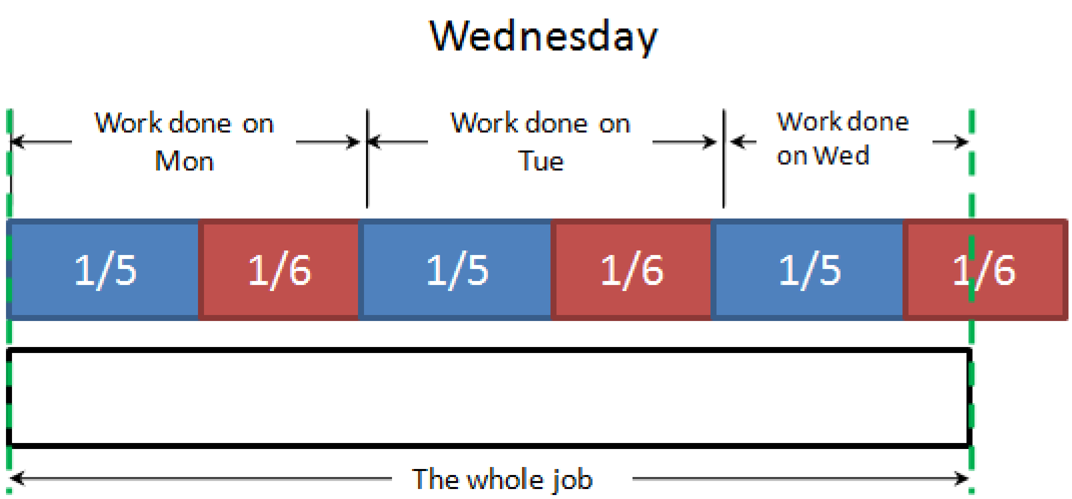
By the end of Monday (see next figure), the combined work done is shown as the length of the two colored rectangles. So, we can say that the number these **pairs** that can fit horizontally into the white rectangle is the number of days that the job will be completed.



By the end Tuesday, both of them have worked 1/5(2) + 1/6(2) as shown in the figure below, more than half of the job.



In the next figure, notice that the job can be completed even if Ben work less than his usual rate. Of course it is also possible to lessen Ariel's work or lessen both of their work.



Since the pair of blue and red rectangle represents one day, this means that the job can be completed in less than 3 days if they both work together.

The Tabular Representation of Working Together

In the first part of this post, I have mentioned that from the table, it can be seen that the job can be completed in less than 3 days. Why? Because if you can see, 3/6 is already half of the job (50%), while 3/5 is more than half (60%). This means that on the third day, they would have completed 110% of the job. Therefore, the job will take less than 3 days.

The 110% is confirmed by adding the fractions. By the end of the Wednesday, they would have completed 33/30 which is equal to 110%.

Name/Day	Mon	Tues	Wed
Ariel	1/5	2/5	3/5
Ben	1/6	2/6	3/6
Together	1/5 + 1/6 = 11/30	2/5 + 2/6 = 22/30	3/5 + 3/6 = 33/30

Notice that both the tabular and geometric interpretation only gave us an approximation. This is why we need Algebra to solve this type of problem. So, how many days will it take to finish the job if they work together?

The Algebraic Representation of Working Together

We can form the equation using the table above. If they both work for one day, then they have worked $\frac{1}{5}(1) + \frac{1}{6}(1)$. If they both worked for two days, they have finished $\frac{1}{5}(2) + \frac{1}{6}(2)$ of the job. Since we are looking for the number of days that they have worked together, which we represent with x days, we can form the expression $\frac{1}{5}(x) + \frac{1}{6}(x)$. And since we are looking for the complete job, we will equate the expression with 1. That is,

$$\frac{1}{5}(x) + \frac{1}{6}(x) = 1.$$

Finalizing the Solution

Now, we have an equation with fractions. As we have learned in **solving equations** with fractions, we need to eliminate the denominators. To eliminate the denominators of the fractions, we multiply everything with the **least common denominator** of 1/5 and 1/6 which is 30. That is,

$30(\frac{1}{5}x) + 30(\frac{1}{6}x) = 30(1)$

$\frac{30}{5}x + \frac{30}{6}x = 30$

$6x + 5x = 30$

$11x = 30$

$x = 2\frac{30}{11}$

$x = 2\frac{8}{11}$.

Again this confirms the tabular and geometric representations that the job can be completed in less than 3 days.

How to Solve Work Problem Part 2

This is the second part of the **Solving Work Problems Series**. In the **previous post**, we have discussed in detail the concept behind how to solve work problems. In this post, we are going to learn more examples and solve more complicated problems.

Problem 2

A hose can fill a pool in 3 hours, while a smaller hose can fill it in 5 hours. If the hoses are opened together the same time, how many hours will they be able to fill the pool?

Solution

House A can fill the pool in 3 hours, so it can fill 1/3 of the pool in 1 hour.

House A can fill the pool in 5 hours, so it can fill 1/5 of the pool in 1 hour.

Together, they can fill 1/3 + 1/5 of the pool in 1 hour.

Let x be the number of hours to fill the pool. As we have done in the previous post, we set up the following equation (read the previous post for details). That is,

$$\frac{1}{3}x + \frac{1}{5}x = 1$$

Multiplying both sides by 15, the least common denominator of 1/5 and 1/3, we have

$15(\frac{1}{3}x) + 15(\frac{1}{5}x) = 15(1)$

$\frac{15}{3}x + \frac{15}{5}x = 15$

$5x + 3x = 15$

$8x = 15$

$x = \frac{15}{8}$

This means that the two hoses will fill the pool in 15/8 or 1 and 7/8 hours.

Problem 3

Chloe and Diane are gown designers in a prestigious company. Chloe and Diane can embellish a gown in 4 hours. Chloe alone can do the same task in 6 hours. How long will Diane be able to do the same task if she were to work alone?

Solution

Chloe and Diane can finish the task in 4 hours, so they can finish 1/4 of the task in 1 hour.

Chloe alone can finish the task in 6 hours, so she can finish 1/6 of the task in 1 hour.

Diane can finish the task in *x* hours, so she can finish 1/*x* of the task in 1 hour.
Note that if we combine the work of Chloe (1/6) and Diane (1/*x*), their rate is 1/4 of the task. That is

$\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$

We multiply the equation by 24*x*, the **least common denominator** of 6, 4, and *x*, we have

$24x(\frac{1}{6}) + 24x(\frac{1}{6}) = 24x(\frac{1}{4})$

$\frac{24x}{6} + \frac{24x}{x} = \frac{24x}{4}$

$4x + 24 = 6x$

$2x = 24$

$x = 12$

This means that Diane can finish the task alone in 12 hours.

How to Solve Work Problems Part 3

This is the third part of the **Solving Work Problems Series**. The first part of this series discussed in detail the **concept behind work problems** and the second part discussed the**basic work problems** and their solutions.
In this post, we discuss two more work problems. The first problem is about two persons who started to work together and after a while, the other person stopped. The second problem is about filling a pool whose outlet pipe is left open.

Sample **Problem 4**
Jack can dig a ditch alone in 5 days, while John alone can do it in 8 days. The two of them started working together, but after two days, Jack left the job. How many more days do John need to work to finish the job alone?
Solution
Jack can finish the job in 5 days, so he can finish 1/5 of the job in 1 day.

John can finish the job in 8 days, so he can finish 1/8 of the job in 1 day.

In 1 day, both of them can finish 1/5 + 1/8 of the job.

Since both of them worked for two days, they have worked 2(1/5) + 2(1/8) before Jack left.
Now, let *x* be the number of days John need to finish the job, so he needs to work 1/8(x) more days. The whole job is equal to 1, so we form the following equation.
$2(\frac{1}{5}) + 2(\frac{1}{8}) + \frac{1}{8}(x) = 1$

$\frac{2}{5} + \frac{2}{8} + \frac{1}{8}x = 1$

Multiplying both sides by 40, the **least common denominator** of the three fractions, we have

$40(\frac{2}{5}) + 40(\frac{2}{8}) + 40(\frac{1}{8}x) = 40(1)$

$\frac{80}{5} + \frac{80}{8} + \frac{40}{8}x = 40$

$16 + 10 + 5x = 40$

$26 + 5x = 40$

$5x = 40 - 26$

$5x = 14$

$x = \frac{14}{5}$.

So, John still needs $\frac{14}{5}$ days or about $2\frac{4}{5}$.

Sample Problem 5

A swimming pool can be filled with water using an inlet pipe in 6 hours. It can be emptied using an outlet pipe in 8 hours. One day, after emptying the pool, the owner opened the inlet pipe but forgot to close the outlet pipe. How many hours will it take to fill the pool with both pipes open?

Solution

The inlet pipe can fill the pool in 6 hours, so it can fill 1/6 of the pool in 1 hour.

The outlet pipe can empty the pool in 8 hours, so it can empty 1/8 of the pool in 1 hour.

If both the inlet and outlet pipes are opened, then in 1 hour the pool is 1/6 filled but emptied 1/8 of water. So, the remaining water is 1/6 – 1/8.

Therefore, if we let *x* be the number of hours, then in *x* hours, the pool is filled with $\frac{1}{6}x - \frac{1}{8}x$.

So, to fill the entire pool, we equate the preceding equation with 1 (can you see why?).

$\frac{1}{6}x - \frac{1}{8}x = 1$.

Multiplying both sides of the equation by 24 which is the **least common denominator** of 1/6 and 1/8, we have

$\frac{24}{6}x - \frac{24}{8}x = 24(1)$

$4x - 3x = 24$

$x = 24$.

Therefore, with both pipes open, the pool can be filled in 24 hours.