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MEGA REVIEW

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PROBLEM SET 1

(Binomial and Multimonomial Expansion)

Note: For 991 ES USERS:
FOR FACTORIAL NUMBERS (!) ENTER: SHIFT x^{-1}
FOR COMBINATION: SHIFT ÷
FOR REGRESSION: USE SHIFT 1 5 FOR REGRESSION

1. Find the 6th term of the expansion of $\left(\frac{1}{2a} - 3\right)^{16}$.

   a. $-\frac{66339}{124a^{11}}$ b. $-\frac{66339}{125a^{11}}$

   c. $-\frac{66339}{123a^{11}}$ d. $-\frac{66339}{128a^{11}}$

2. Find the 5th term of the expansion of $(a - 2b)^{20}$.

   a. $77520a^{16}y^4$ b. $77542\ a^{16}y^4$

   c. $87520a^{16}y^4$ d. $97520a^{16}y^4$

3. Find the coefficient of the term involving x^4 in the expansion of $(3x - \frac{1}{x})^{10}$.

   a. -262444 c. -252440

   b. -262440 d. -362440

4. Find the term involving x^{13} in the expansion of $(4x^2 + \frac{1}{x})^{14}$

   a. 524812288 b. 524812286

   c. 544812288 d. 624812288

5. Find the 6th term of the expansion of $(3x - \frac{1}{x^2})^{10}$

   a. $-\frac{51236}{x^5}$ b. $-\frac{31236}{x^4}$

   c. $-\frac{61236}{x^4}$ d. $-\frac{61236}{x^5}$

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6. Find the constant term in the expansion of $(x^2 - \frac{1}{x})^9$

a. 83 b. 82
c. 84 d. 85

7. Find the term involving x^6y^{12} in the expansion of $(3x^2 - 4y^3)^7$.

a. 241920 b. 231920
c. 231224 d. 251234

8. Find the term involving x^3y^2 in the expansion of $(4x + 2y - 3z)^5$.

a. 2568 b. 2560
c. 2342 d. 3214

9. Find the term involving x^4y^2 in the expansion of $(4x - y + 2z)^6$.

a. 3840 b. 3920
c. 4120 d. 4312

10. Find the term involving x^2yz in the expansion of $(2x + y + 5z)^4$
.

a. 320 b. 240
c. 260 d. 280

11. Find the sum of the coefficients of $(a + 2b)^6$.

a. 612 b. 711
c. 729 d. 741

12. Find the sum of the coefficients $(3x + 1)^4$.

a. 242 b. 243
c. 244 d. 255

13. Find the sum of the coefficients of $(2x - 3)^8$.

a. -5660 b. -6560
c. -6780 d. -5120

14. Find the sum of the coefficients of $(3x + 2y + 1)^5$

a. 7776 b. 7774
c. 7773 d. 7775

15. Find the sum of the exponents of $(x + y)^6$.

a. 40 b. 41
c. 42 d. 43

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16. Find the sum of the exponents $(3x^3 + 2y^4)^{10}$
    a. 384 b. 385
    c. 389 d. 390
17. Find the sum of the coefficients of $(3x + 2y -2)^8$.
    a. 6302 b. 6305
    c. 6304 d. 6306
18. Find the sum of the exponents of $(4x^2 + 5y^7)^3$?
    a. 54 b. 56
    c. 58 d. 58
19. The term involving x^9 in the expansion of $(x^2 + 2/x)^{12}$
    a. $24534x^9$ b. $25344x^9$
    c. $23455x^9$ d. $24544x^9$
20. What is the sum of the coefficients of the expansion of $(2x -1)^{20}$?
    a. 1 b. 0
    c. 2 d. 125

Solution:

Note: nCr = n!/(n-r)! r!

1. 6^{th} term = $16C(5)(1/2a)^{16-5}(-3)^5$
   $= 4368(1/2)^{11}(-3)^5 /a^{11}$
   $= - 66339/128/ a^{11}$ (d)
2. 5^{th} term = $20C4 (a)^{20-4}(-2b)^4$
   $= 20 C4 (-2)^{16} a^{16} b^4$
   $= 77520 a^{16} b^4$ (a)
3. r th term =
   $10 C (r – 1) (3x)^{10-(r-1)} (-x^{-1})^{r-1}$ = Constant (x^4)
   Then: $10-(r-1) -1(r-1) = 4$ then $r = 4$
   4^{th} term = $10 C3 (3x)^{10-3}(-1/x)^3$
   $= (10C3)(3^7) (-1)^3 (x^4)$
   $= -262440x^4$ (b)
4. r th term = $14 C (r -1) (4x^2)^{14-(r-1)} (x^{-1})^{r-1}$ = Constant (x^{13})
   Equate the two powers. Disregard Constat:
   $x^{2(14-(r-1)} x^{-1(r-1)} = x^{13}$
   Then: $2 [14 – (r -1)] + (-1)(r – 1) = 13; r = 6$

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6^{th} term = $14\,C\,5\,(4x^2)^{14-5}\,(x^{-1})^5 = (14\,C_5)(4^9)\,x^{13}$

$= 524812288x^{13}$ (a)

5. 6^{th} term = $10C5\,(3x)^{10-5}(-x^{-2})^5$

$= 10\,C5\,(3)^5\,(-1)\,x^5\,x^{-10} = 61236/\,x^5$ (d)

6. r th term = $9C\,(r-1)\,(x^2)^{9-(r-1)}(-x^{-1})^{r-1}$ = Constant (x^0)

Then: $2[\,9 - (r-1)\,] - (r-1) = 0;\ r = 7$

7^{th} term = $9C6 = 84$

7. Divide power of x^6 by power of $3x^2$ $\quad 6/2 = 3$

Divide power of y^{12} by power of $4y^3$ $\quad 12/3 = 4$

coefficient of $3x^2 = 3$

coefficient of $-4y^3 = -4$

Term containing $x^6\,y^{12} = \dfrac{7!}{(3!)(4!)}(3^3)(-4)^4 = 241920$

8. Term involving $x^3y^2 = \dfrac{5!}{3!2!}(4)^3(2)^2 = 2560$

Note: 5 is the power of the given.

3! comes from x^3

2! comes from y^2

4 is the coefficient of 4x

2 is the coefficient of 2y

9. Term involving $x^4y^2 = \dfrac{6!}{4!2!}(4)^4(-1)^2 = 3840$

Note: 6 is the power of the given.

4! comes from x^4

2! comes from y^2

4 and -1 are coefficients of 4x and –y resp.

10. Term involving $x^2yz = \dfrac{4!}{2!1!1!}(2)^2(1)^1(5)^1 = 240$

Note: 2 , 1 and 5 are coefficients in 2x, 1y, 5z respectively.

The denominator and powers are powers in $x^2y^1z^1$ respectively.

11. Let $f(a,b) = (a + 2b)^6$

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Then the sum of coefficients = $f(1,1) = (1 + 2)^6 = 729$

12. Let $f(x) = (3x + 1)^4$
    Then the sum of coefficients = $f(1) - f(0) = (3(1) + 1)^4 - 1^4 = 255$
13. Let $f(x) = (2x-3)^8$
    Then the sum of coefficients = $f(1) - f(0) = -6560$
    Note: $f(1) = (2(1) - 3)^8 = 1$
    $f(0) = (-3)^8 = 6561$
14. Let $f(x,y) = (3x + 2y + 1)^5$
    Then the sum of coefficients = $f(1,1) - f(0,0)$
    $= (3 + 2 + 1)^5 - 1^5 = 7775$
15. Sum of exponents = $(1 + 1)\ 7\ C2 = 42$
    Note: sum of exponents of $(x^a + y^b)^n = (a + b)\ (n+1)C2$
16. Sum of exponents = $(3 + 4)\ (11C2) = 385$
17. Let $f(x,y) = (3x + 2y - 2)^8$
    Then the sum of coefficients = $f(1,1) - f(0,0)$
    $= (3 + 2 - 2)^8 - (-2)^8$
    $= 6305$
18. Sum of exponents = $(2 + 7)\ (4\ C2) = 54$
19. r th term = $12\ C(r-1)\ (x^2)^{12-(r-1)}\ (2x^{-1})^{r-1} = \text{Constant}(x^9)$
    Then: $2[12 - (r-1)] - (r-1) = 9;\ r = 6$
    6^{th} term = $12\ C5\ (x^2)^{12-5}(2/x)^5 = 12\ C5(2)^5\ x^9$
    $= 25344$ (b)
20. Let $f(x) = (2x - 1)^{20}$
    Then sum of coefficients = $f(1) - f(0)$
    $= (2(1) - 1)^{20} - (-1)^{20}$
    $= 0$

Problem Set 2 (Digit Problems, Mixture Problems, Age Problems, Work Problems, Clock Problems,

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Remainder Theorem)

1. The sum of the digits of a 3 digit number is 17, the hundreds digit is twice the unit digit. If 396 is subtracted from the number, the order of the digits will be reversed. Find the units digit.

a. 2 b. 3
c. 4 d. 5

2. 2000 kg of steel containing 8% nickel is to be made by mixing a steel containing 14% nickel with another containing 6% nickel. How much of the 14% nickel is needed?

a. 1000 b. 700
c. 500 d. 400

3. A chemist of a distillery experimented on two alcohol solutions of different strength , 35% alchol and 50% alcohol respectively. How many cubic meters of 35% strength must he use in order to produce a mixture of 60 cubic meters that contain 40% alcohol.

a. 40 b. 30
c. 35 d. 20

4. How many liters of water must be added to 80 liters of a 40% salt solution to produce a solution that is 25% salt?

a. 48 b. 45
c. 44 d. 40

5. One number is 5 less than the other. If their sum is 135, what is the product of the numbers?

a. 4550 b.4135
c. 2140 d. 7150

6. The sum of two numbers is 21 and one number is twice the other. Determine product of the numbers.

a. 100 b. 112
c. 98 d. 94

7 The denominator of a certain fraction is 3 more than twice the numerator. If 7 is added to both terms of the fraction, the resulting fraction is 3/5. Find the original fraction

a. 4/13 b. 5/13
c. 6/13 d. 7/13

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8. The sum of the digits of a 3 digit number is 14. The hundreds digit being 4 times the unit digit. If 594 is subtracted from the number, the order of the digits will be reversed. Find the ten's digit of the number.

a. 4 b. 6
c. 8 d. 2

9. Six years ago, Jun was 4 times as old as John. In 4 years, he would be twice as old as John. How is Jun now?

a. 12 b. 13
c. 21 d. 26

10. In 5 years, Jose would be twice the age of Ana. Five years ago, Jose was 4 times as old as Ana. Find the sum of their present ages.

a. 35 b. 45
c. 40 d. 50

11. Jun can finish an accounting work in 8 hrs. Leo can finish the same work in 6 hrs. After 2 hrs of working together, Jun left Leo for lunch and Leo finished the job. How long does it take Leo to finish the job?

a. 3 hrs b. 2.5 hrs
c. 3.5 hrs d. 4 hrs

12. A can do a piece of work in 10 days. After he has worked 2 days, B came to help him and together, they finish the job in 3 days. In how many days could B alone do the work?

a. 5 days b. 6 days
c. 7 days d. 8 days

13. A and B can do a piece of work in 42 days, B and C in 31 days and A and C in 20 days. Working together, how many days can all of them finish the work?

a. 19.23 days b. 18.87 days
c. 15.34 days d. 13.56 days

14. A, B and C can finish the same job is 7, 9 and 12 days respectively. All of them worked for 2 days after which C was left to finish the job? How long did C work?

a. 5.9 days b. 6.2 days
c. 6.3 days d. 5.5 days

15. In how many minutes after 2 PM will the hands of the clock be at extend in opposite direction for the first time?

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a. 45.6 min b. 43.6 min
c. 47.8 min d. 44.3 min

16. At what time a between 3 PM to 4 PM will the hands of the clock be at right angle ?

a. 3 32 8/11 PM b. 3: 33 7/11
c. 3: 32 5/11 d. 3:33 5/11

17. In how many minutes after 3 PM will the hands of the clock be together for the first time.

a. 16.45 min b. 16.36 min
c. 16.21 min d. 16.66 min

18. In how many minutes after 7 PM will the hands of the clock be together for the first time?

a. 38.22 min b. 38.18
c. 39.22 min d. 39.44 min

19. At what time after 12 noon will the hour hand and minute hand of the clock form an angle of 120^0?

a. 21.818 min b. 22.34 min
c. 32.342 min d. 41.23 min

20. Find the quotient and remainder when $4y^3 + 18y^2 + 8y - 4$ is divided by $2y + 3$?.

a. $2y^2 + 6y - 5$ rem 11 b. $2y^2 - 6y - 5$ rem 12
c. $2y^2 + 6y + 5$ rem 11 d. $2y^2 + 3y - 5$ rem 12

21. What is the remainder when $3x^4 + 2x^3 - 5x^2 + x + 7$ is divided by $x - 3$?

a. 234 b. 262
c. 311 d. 312

22. What is the remainder when $3x^4 + 3x^3 - 5x^2 - 5x + 7$ is divided by $x + 2i$?

a. 12+3i b. 34 + 12i
c. 8-25i d. 75 + 22i

23. Let f(x) = $x^5 + ax^4 - 3x^3 + bx - 4$. If f(x) is divided by x + 7 , the remainder is -3805 , when divided by x + 1 the remainder is -1. What is the value of a?

a. 2 b. 3
c. 4 d. 5

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24. A polynomial has an equation $x^5 - 5x^4 + 5x^3 + 15x^2 + 36x + 20$. How many rational roots does it have?

a. 0 b. 1
c. 2 d. 3

25. How many real roots are there for a polynomial $7x^7 + 5x^5 + 3x^3 + x$?

a. 0 b. 1
c. 2 d. 3

26. Find the least possible number of positive real zeros of the polynomial $P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$.

a. 1 b. 2
c. 3 d. 4

27. Given $P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$, what is the maximum possible number of real zeros in P(x)?

a. 3 b. 4
c. 5 d. 6

28. Find the value of k so that $k - 3$ is a factor of $x^4 - k^2x^2 - kx - 39 = 0$

a. -7/3 b. 3
c. 5/3 d. -3

29. Find the remainder when $x^3 + 3x^2 - 5x + 7$ is divided by $x + 1 - 3i$?

a. 14-31i b. 14+ 51i
c. 18-51i d. 14-51i

30. Find the quotient when $x^3 - x^2 + 3x - 6$ is divided by $x + 2i$.

a. $x^2 + (-1 -2i)x -1 + 2i$ b. $x^2 + (-1 +2i)x -1 + 2i$
c. $x^2 + (-1 -3i)x -1 + 2i$ d. $x^2 + (1 -2i)x -1 + 2i$

31. Find the upper bound of the roots of $x^4 - x^3 - 2x^2 - 4x - 24$.

a. 1 b. 2
c. 3 d. 4

32. Find the lower bound of the roots of $x^4 - x^3 - 2x^2 - 4x - 24 = 0$.

a. -1 b. -2
c. -3 d. -4

33. In an organization, there are CE, ME, and EE. The sum of all their ages is 2160. Their average is 36. The average of CE and ME

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IS 39. The ME and the EE is 32 8/11 and the CE and EE is 36 2/3. If each CE had been one year older, each ME 6 year older and each ME 6 years older, their average age will increase by 5. Find the number of CE in the organization. PAST CE BOARD

a. 16 b. 18
c. 23 d. 30

34. Find the product of 2 numbers such that their sum multiplied by the sum of their squares is 5500 and their difference multiplied by the difference of their squares is 352. (PAST CE BOARD)

a. 121 b. 132
c. 117 d. 361

35. An alloy of silver and gold weigh 15 oz in air and 14 oz in water. Assume that silver loses 1/10 of its weight in water and gold loses 1/18 of its weight, how many gold is in the alloy? PAST ECE BOARD

a. 11.25 oz b. 10.35 oz
c. 12.50 oz d. 10.25 oz

36. A certain loan association has invested P 500,000 in 3 different transactions. First investment is in real earning 9% interest annually; second investment is in loans earning 6% annually, and the 3rd investment in bonds earning 4% annually. The total annual income in interest is P 34,000, but the annual interest in loans is 3 times that in bonds. How much in the investment in real estate. PAST ECE BOARD

a. 150000 b. 200000
c. 180000 d. 125000

37. Find the smallest positive number among the 4 numbers such that the sum of the 1st , 3rd and 4th exceeds the second by 8; the sum of the squares of the 1st and the 2nd exceeds the sum of the squares of the 3rd and the 4th by 36; the sum of the products of the 1st and the 2nd and 3rd and 4th is 42; the cube of the 1st equals the sum of the cubes of the 2nd , 3rd and 4th . PAST CE BOARD

a. 1 b. 2
c. 3 d. 4

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38. It is now between 9 and 10 o'clock. In 4 minutes, the hour hand will be directly opposite the position occupied by the minute hand 3 minutes ago. What time is it? PAST ECE BOARD

a. 9:19 b. 9:20
c. 9:21 d. 9:22

39. At what time after 2 o'clock will the angle between the hands of the clock be bisected by the line connecting the center of the clock and the 3 o'clock mark? PAST ECE BOARD

a. 2: 18.33 b. 2:18.56
c. 2:18.46 d. 2:18.23

40. The time is past 2 PM. In 10 minutes, the minute hand will be as much as the hr hand as it is now behind it. What time is it?

a. 2:05.909 b. 2:05.809
c. 2:05.709 d. 2:05.701

Solution:

1. Let x = hundred digit y = ten's digit z = unit digit
   No = 100 x + 10y + z Reversed Number = 100z + 10y + x
   x + y + z = 17 (1)
   x = 2z (2)
   100z + 10y + x = 100 x + 10y + z - 396 (3)
   Arrange:
   x + y + z = 15
   x - 2z = 0
   -99x +99z = -396 (MODE 5 2)
   So, x = 8 y = 3 and z = 4
   Ans. z = 4
2. Let x = amount of 14% nickel
   y = amount of 6% nickel
   Then: x + y = 2000
   0.14 x + 0.06 y = 2000(0.08) = 16 (MODE 5 1)
   x = 500, y = 1500
   Ans. x = 500
3. Let x = amount of 35% alcohol

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y = amount of 50% alcohol

Then: x + y = 60

0.35x + 0.50 y = 0.4(60) = 24 (MODE 5 1)

Ans.: x = 40, y = 20

4. Let X = liters of water to be added.

Then: (X + 80)(0.25) = 80(0.4)

Ans.: X = 48

5. Let X = one number

Y = the other number

Then: X + Y = 135

X = Y – 5

Arrange:

X + Y = 135

X – Y = - 5 (MODE 5 1)

X = 65, Y = 70

Ans,: Product = 65 x 70 = 4550

6. Let X = one number

Y = the other number

Then: X = 2 Y → X – 2Y = 0

X + Y = 2 (MODE 5 1)

X = 14, Y = 7

Ans.: XY = 98

7. Let X = numerator

Y = denominator

Y = 3 + 2X ► -2X + Y = 3 (1)

$\frac{x+7}{y+7}=\frac{3}{5}$ ► 5(X +7) = 3(Y + 7) 5X – 3Y = -14 (2)

Solve (1) and (2)

X = 5, Y = 13

Ans. 5/13

8. Let X = hundred's digit

Y = ten's digit

Z = unit digit

No = 100 x + 10y + z

Reversed Number = 100z + 10y + x

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X + Y + Z = 14
X = 4Z ► X – 4Z = 0
100z + 10y + x = 100x + 10y + z – 594
► -99x + 99 z = - 594
The equations:
X + Y + Z = 14
X - 4Z = 0
-99X +99Z = -594 (MODE 5 2)
X = 8 Y = 4 Z = 2
Ans. 4

9.

6 years ago	present age	4 years from now
x -6	x(Jun's age)	x + 4
y -6	y(John's age)	y + 4

So, x – 6 = 4(y – 6) ► x – 4y = -18
x + 4 = 2(y + 4) ► x – 2y = 4 (MODE 5 1)
Ans. X = 26, Y = 11

10.

5 years ago	present age	5 years from now
X -5	X (Jose)	X + 5
Y -5	Y (Ana)	Y + 5

So, X + 5 = 2(Y + 5) ► X – 2Y = 5
X – 5 = 4(Y – 5) ► X – 4Y = -15 (MODE 5 1)
X = 25, Y = 10
Ans.: X + Y = 35

11. Let x = time Leo will finish the job after working for 2 hrs.
Then: 2/8 + 2/6 + x/ 6 = 1
Ans.: x = 2.5

12. Let X = time B can do the work alone
Then: 2/10 + 3/10 + 3/x = 1
Ans,: x = 6

13. Let X , Y and Z = A, B and C can finish the job alone resp.
Then: 42/x + 42/y= 1
31/y + 31/z =1
20/x + 20/y = 1 (MODE 5 2)

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So, 1/x = 541/26040; 1/y = 79/26040; 1/z = 761/26040
Then: 1/z = 1/x + 1/y + 1/z = 1381/26040
z = 18.855 days

14. 2/7 + 2/9 + 2/12 + x/12 = 1
x = 82/11
x + 2 = 5.90476 days

15. ENTER: MODE 3 2
Input the given coordinates MODE 3 2
X Y
0 -60 (2 x -30)
60 270 (-60 + 330)
ENTER: AC
compute $180\hat{x}$ (180 SHIFT 1 5 4)
Ans. 43.6363

16. Input the given coordinates MODE 3 2
X Y
0 -90 (3 x -30)
60 240 (-90 + 330)
ENTER: AC
Compute: 90 $\hat{x}$ (90 SHIFT 1 5 4) SHIFT S↔D
Ans. 32 8/11

17. SAME COORDINATES AS 16
COMPUTE: 0 $\hat{x}$ (0 SHIFT 1 5 4)
Ans. 16.363636

18. ENTER: MODE 3 2
INPUT THE GIVEN COORDINATES
X Y
0 -210 (7 x -30)
60 120 (-210 + 330)
ENTER: AC
ENTER: 0 $\hat{x}$ (0 SHIFT 1 5 4)
Ans. 38.18 min

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19. ENTER: MODE 3 2
INPUT THE GIVEN COORDINATES

x	y
0	0
60	330

ENTER: AC
Ans. 21.818 min

20. Using Synthetic Division

$$\begin{array}{r|rrrr} & 4 & 18 & 8 & -4 \\ -3/2 & & -6 & -18 & 15 \\ \hline & 4 & 12 & -10 & 11 \end{array}$$

$(4y^2 + 12y – 10)/2$ rem 11
or $2y^2 + 6y – 5$ rem 11

21. Let $f(x) = 3x^4 + 2x^3 – 5x^2 + x + 7$
$f(3) = 262$
Note: Enter $3x^4 + 2x^3 – 5x^2 + x + 7$ CALC 3 =

22. Let $f(x) = 3x^4 + 3x^3 – 5x^2 – 5x + 7$
$f(-2i) = 3(-2i)^3(-2i) + 3(2i)^3 – 5(2i)^2 -5(2i) + 7$
ENTER: MODE 2
ENTER: $3x^3$ x + $3x^3 – 5x^2 – 5x + 7$ CALC 2i =
Ans. 75 + 22i

23. $f(x) = x^5 + ax^4 – 3x^3 + bx – 4$
$f(-7) = -3805$ $-3805 = (-7)^5 + a(-7)^4 – 3(-7)^3 + b(-7) – 4$
$7^4a – 7b = 11977$ (1)
$f(-1) = -1$ $-1 = (-1)^5 + a(-1)^4 -3(-1)^3 + b(-1) – 4$
$a – b = 1$ (2)
ENTER: MODE 5 1
Ans. a = 5, b = 4

24. The possible rational roots using the rational root theorem are factors of 20 which are 1, -1, 2, -2, 4, -4, 5, -5, 10, -10, 20, -20:
ENTER: MODE 7 $f(x) = x^5 – 5x^4 + 5x^3 + 15x^2 + 36x + 20$
ENTER: START = -20 END = 0 STEP = 1

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Note: That the column for f(x) has no roots from -20 to 0.
ENTER: AC Edit START = 0 END = 20 STEP = 1
The column for f(x) has no zero.
Therefore: No rational roots. (a)

25. $7x^7 + 5x^5 + 3x^3 + x = x(7x^6 + 5x^4 + 3x^2 + 1)$
So one real root = 0.
For $7x^6 + 5x^4 + 3x^2 + 1 = 0$
Using Descartes Rule of Signs
7 5 3 1 (There is no shift in sign)
+ + + + (No positive real root)
$f(-x) = 7(-x)^7 + 5(-x)^5 + 3(-x)^3 + -x$
$= -7x^7 – 5x^5 – 3x^3 - x$
-7 -5 -3 -1 (No shift in sign)
(No negative real roots)
The only real root is x = 0 Ans (b)

26. Using Descartes Rule of Signs:
3 4 3 -1 - 3
+ + + - - (There is only 1 change of sign
From 3 to -1 . Answer (1)

27. $F(-x) = 3x^6 – 4x^5 – 3x^3 + x - 3$
3 - 4 - 3 1 - 3
+ - - + - (3 changes in signs)
Possible number of negative real roots are 3 o r 1.
From problem 26, there is only 1 possible real root.
Therefore, the maximum possible number of real roots
1 + 3 = 4 (b)

28. Let $f(x) = x^4 – k^2x^2 – kx – 39$ $f(3) = 0$
$0 = 3^4 – k^2(3)^2 – k(3) – 39$ ► $9k^2 + 3k - 42 = 0$
ENTER: MODE 5 3
Ans. X = 2 and X = -7/3

29. Let $f(x) = x^3 + 3x^2 - 5x + 7$
$f(-1 + 3i) = 14-51i$

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Note: ENTER: MODE 2:
ENTER: $x^3 + 3x^2 - 5x + 7$
ENTER: CALC -1 + 3i =

30. Find the quotient when $x^3 - x^2 + 3x - 6$ is divided by x + 2i.
Use synthetic division.

1	- 1	3	- 6	| -2i
	-2i	-4 +2i	4+ 2i	
1	-2i-1	-1 + 2i	-2 + 2i	

Ans. $x^2 + (-1 -2i)x -1 + 2i$

31. Use synthetic division

1	-1	- 2	-4	- 24	| 1
	1	0	- 2	- 6	
1	0	-2	- 6	-30	

1	-1	-2	-4	-24	|3
	3	6	12	24	
1	2	4	8	0	

Upper bound (Third line are all positive or zero)
Ans. 3

32. To get the lower bound
Use Synthetic Division:

1	- 1	-2	-4	- 24	| -2
	-2	6	-8	24	
1	-3	4	-12	0	

The signs alternate, therefore -2 is a lower bound.

33. Let x = number of CE; y = number of ME; z = number of EE
A = ave. age of CE B = ave. age of ME and C = ave. age of EE

Then :x A + y B + z C = 2160 (1)
(x A + y B + z C)/(x + y + z) = 36 (2)
(x A + y B)(x + y) = 39 (3)
(y B + z C)/(y + z) = 32 8/11 (4)
(x A + z C)/(x + z) = 36 2/3 (5)
(1x + 6y + 7z)/(x + y + z) = 5 (6)

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From 2 , $x A + y B + z C = 36(x + y + z)$
Substitute this to eq. 1
$36(x + y + z) = 2160$
$x + y + z = 60$ (7)
From 6:
$x + 6y + 7z = 5x + 5y + 5z$
$-4x + y + 2z = 0$ (8)
Add eq. 3, 4 and 5.
$x A + y B = 39x + 39y$
$y B + z C = 360/11 y + 360/11 z$
$xA + z C = 110/3x + 110/3z$
$2x A + 2y B + 2z C = 227/3x + 789/11y + 2290/33 z$
$2(x A + y B + z C) = 2(2160)$
Then: $227/3x + 789/11y + 2290/33 z = 4320$ (9)
Solve 7 , 8 and 9 (MODE 5 2)
$x = 16$ $y = 24$ $z = 20$

34. Let x and y be the numbers
Then : $(x + y)(x^2 + y^2) = 5500$ (1)
$(x – y)(x^2 - y^2) = 352$ (2)
From: Eq. (2) $(x – y)(x – y)(x + y) = 352$
$(x – y)^2(x + y) = 352$ (3)
Divide Eq. 3 by Eq. 1.
$(x – y)^2 / (x^2 + y^2) = 352/5500 = 8/125$
$(x^2 – 2xy + y^2)/(x^2 + y^2) = 8/125$
$125 (x^2 – 2xy + y^2) = 8(x^2 + y^2)$
$117x^2 – 250xy + 117y^2 = 0$
Factor: Use MODE 5 3
Input 117 - 250 117
Answers 13/9 , 9/13
$(13x – 9y)(9x – 13y) = 0$
$x = 9y/13$ or $x = 13y/9$
Use Eq. 1
$(9y/13 + y)([9y\13]^2 + y^2) = 5500$
Use Shift CALC:
$y = 13$ and $x = 9(y)/13 = 9$

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Ans. Product = 13 x 9 = 117

35. Let x = amount of silver in the alloy
y = amount of gold in the alloy
Then: x + y = 15
x/10 + y/18 = 15 -14 = 1
USE MODE 5 1
Ans. x = 3.75; y = 11.25

36. Let x = investment in real eanings.
y = investment in loans earnings
z = investment in bonds
Then: x + y + z = 500,000
0.09x + 0.06y + 0.04z = 34,000
0.06x = 3(0.04z) → 0.06x + 0y - 0.12 z = 0
USE MODE 5 2
Ans.: x = 200,000 y = 200,000 z = 100000

37. Let x , y , z and w be the numbers.
Then: $x + w + z - y = 8$ (1)
$x^2 + y^2 - z^2 - w^2 = 36$ (2)
$xy + zw = 42$ (3)
$y^3 + w^3 + z^3 = x^3$ (4)

From (3) : -2xy - 2zw = -84 Add this to (2)
$x^2 - 2xy + y^2 - z^2 - 2zw - w^2 = 36 - 84 = -48$
$(x - y)^2 - (z + w)^2 = - 48$
$[(x - y) - (z + w)] [(x - y) + (z + w)] = - 48$
Divide this to eq 1.
$x - y - z - w = - 6$ (5)
Add eq (1) and eq (5)
$2x - 2y = 2$ or $x - y = 1$ (6)
Subtract eq 1 and eq 5
$2w + 2z = 14$ or $w + z = 7$ (7)

Note: $(x - y)^3 = x^3 - 3x^2 y + 3xy^2 - y^3$
$x^3 - y^3 = (x - y)^3 + 3x^2y - 3xy^2$
$= (x - y)^3 + 3xy (x - y)$

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Thus: $x^3 - y^3 = 1^3 + 3xy$

$x^3 - y^3 = 1 + 3xy$ (8)

Also $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$w^3 + z^3 = (w + z)^3 - 3wz(w + z)$

$w^3 + z^3 = 7^3 - (3wz)(7)$

$w^3 + z^3 = 7^3 - 21wz$ (9)

But from (4) $x^3 - y^3 = w^3 + z^3$

Then $1 + 3xy = 7^3 - 21wz$

$3xy + 21wz = 342$ (10)

$xy + wz = 42$ (3)

USE MODE 5 1

$xy = 30$	$wz = 12$
$x - y = 1$	$w + z = 7$
$x = y + 1$	$w = 7 - z$
$(y)(y + 1) = 30$	$(7 - z)(z) = 12$
$y^2 + y - 30 = 0$	$-z^2 + 7z - 12 = 0$
USE MODE 5 3	USE MODE 5 3
$y = 5, -6$	$z = 4, 3$
$x = 6, -5$	$w = 3, 4$

38. Let X = position of minute hand after 9 PM (in minutes)
Then 45 + X/12 = position of the hr hand.
Position occupied by the minute hand 3 min ago = X - 3
Position occupied by the hr hand in 4 min = 45 + (X + 4)/12 + 30
So, X – 3 + 30 = 45 + (X + 4)/12
Ans. 20

39. Let X = position of the minute hand after 2 o'clock (in minutes)
Then: 10 + X/12 = position of the hr hand.
Angle between them after X min = X – (10 + X/12)
Half of this angle = (X - 10 - X/12)/2 = X - 15
X = 18.46

40. Let X = position of the minute hand (in min) after 2 PM.
Then: 10 + X/12 = position of the hr hand.
In 10 min, angle between the min hand and the hr hand

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= X + 10 - (10 + (X + 10)/ 12) = X – (X + 10)/ 12
Angle now = 10 + X/12 - X
Thus: 10 + X/12 – X = X - (X + 10)/ 12
Ans.: X = 5.909

PROBLEM SET 3

Arithmetic and Geometric Progression, Diophantine Equation, Number Sequence)

1. Find the 30th term of an A.P. 4, 7, 10 …. PAST ECE BOARD
   a. 88 b. 75
   c. 91 d. 95
2. Find the 100th term of the sequence 1.01, 1.00, 0.99
   a. 0.05 b. 0.03
   c. 0.04 d. 0.02
3. Find the 4th term of the progression 1/2, 0.2, 0.125 (ECE Nov)
   a. 0.102 b. 1/11
   c. 1/10 d. 0.099
4. The 5th term of an A.P is 123 and the 30th term is 245. What is the 12th term? PAST CE BOARD
   a. 3929/25 b. 3412/25
   c. 3372/25 d. 3312/25
5. The 3rd term of a harmonic progression is 15 and the 9th term is 6. Find the 11th term
   a. 4 b. 5
   c. 6 d. 7
6. How many terms of the sequence -9, -6, -3 … must be taken so that the sum is 66? ECE Nov 1996
   a. 13 b. 11
   c. 12 d. 16
7 A besiege fortress is held by 5700 men who have provisions for 66 days. If the garrison loses 20 men each day, how many days can the provision hold out? PAST CE BOARD EXAM
   a. 60 b. 76

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c. 82 d. 72

8. In the Bosnia conflict, the NATO forces captured 6400 soldiers. The provisions on hand will last for 216 meals while feeding 3 meals a day. The provisions lasted 9 more days because of daily deaths. At an average, how many died per day?(PAST CE BOARD)

a. 15.2 b. 18.3
c. 17.8 d. 19.4

9. What is the 11^{th} term of the harmonic progression if the 1^{st} and the 3^{rd} term are 1/2 and 1/6 respectively? ECE BOARD Nov 2003.

a. 1/20 b. 1/4
c. 1/12 d. 1/22

10. A company sells 80 units and makes P80 profit. It sells 110 units and makes P 140 profit. If the profit is a linear function of the number of units sold, what is the average profit per unit if the company sells 250 units? ECE APRIL 2003

a. P 1.76 b. P 1.68
c. P 1.66 d. P 1.86

11. What are the 1^{st} four terms of the sequence whose general term is $n^2 + 1$? ECE April 2004

a. 1 4 9 16 b. 2 5 10 17
c. 5 10 17 26 d. 2 4 6 8

12. What are the 1^{st} 4 terms of the sequence whose general term is $n^3 – 2n^2 + 1$?

a. 0 1 10 37 b. 1 2 11 39
c. 0 1 10 33 d. 1 3 11 39

13 To build a dam, 60 men must work 72 days. If all 60 men are employed at the start but the number is decreased by 5 men at the end of each 12 day period. How long will it take to complete the dam?

a. 108 days b. 94 days
c. 100 days d. 122 days

14. Solve for x in the following equation. ECE April 2004

$x + 4x + 7x + \ldots. 64x = 1430$

a. 1 b. 2
c. 3 d. 4

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15. The sum of 3 numbers in AP is 45. If 2 is added to the 1st number, 3 to the second and 7 to the 3rd, the new numbers will be in a geometric progression. Find the common difference in AP. PAST CE BOARD

a. -5 b. 10
c. 6 d. 5

16. A merchant has 3 items on sale, namely item A for P 50, a citem B for P 30 and item C for P 1.00. At the end of the day, he has sold a total of 100 items and has taken exactly P 1000 on the total sales. How many item A did he sell?

a. 16 b. 18
c. 20 d. 22

17. A stack of bricks has 61 bricks in the bottom layer, 58 bricks in the second layer, 55 bricks in the 3rd layer and so on until there are 10 bricks in the last layer. How many bricks are there together? PAST ME BOARD

a. 638 b. 640
c. 637 d. 639

18. Gravity causes a body to fall 16.1 ft in the 1st sec, 48.3 in the 2nd, 80.5 in the 3rd sec and so on. How far did the body fall during the 10th sec? PASt ME BOARD

a. 243.1 ft b. 305.9 ft
c. 133.4 d. 412.1 ft

19. Find the sum of the sequence 25, 30, 35 ..

a. $5/2n^2 + 45n/3$ b. $5/2n^2 + 45/2n$
c. $7/2n^2 + 45/4n$ d. $7/2n^2 + 45n/2$

20. The 5th term of a geometric progression is 162 and the 10th term is 39366. What is the 3rd term? PAST CE BOARD

a. 21 b. 18
c. 36 d. 24

21. The 4th term of a geometric progression is 189 and the 6th term is 1701. What is the 12th term?

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a. 1240029 b. 1310049
c. 1521011 d. 1812221

SITUATION 16- 19

If the 3rd term of a GP is 28 and the 5th term is 112, find

22. 9th term of the sequence.
a. 1433 b. 1812
c.1792 d. 1344

23. Find the 1st term in the sequence.
a. 4 b. 5
c. 6 d. 7

24. the nth term of the sequence is 3584. n is
a. 9 b. 10
c. 11 d. 12

25. The sum of the first 10 terms.
a. 7161 b. 7321
c. 7781 d. 7323

26. The sequence x, 2x + 7, 10x – 7 forms a geometric progression. The sum of the first 10 terms of the geometric progression.
a. 206,668 b. 210,228
c. 234,122 d. 321,134

27. Four positive integers form an arithmetic progression. If the product of the 1st and the last term is 70 and the 2nd and the 3rd term is 88, find the 1st term.
a. 6 b. 7
c. 5 d. 8

28. What is the sum of all even integers from 10 to 500?
ECE Nov
a. 87,950 b. 124,950
c. 62,370 d. 65,955

29. What is the sum of all odd integers between 10 and 500?
ECE BOARD
a. 87,950 b. 124,950
c. 62,475 d. 65,955

30. How many terms of the progression 3, 5, 7 should there be so that their sum will be 2600? ECE April 2005
a. 45 b. 50

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c. 60 d. 48

31. The total amount collected from the contribuitions of 50 people was P 1,630. If each man contributed P 50.00 , each woman P 25.00 and each child P 3.00, how many women are there?

a. 20 b. 23
c. 16 d. 18

32. If you divide 136 into two parts, one of which when divided by 5 leaves a remainder 2 and the other divided by 8 leaves a remainder 3, then the product of the two parts is

a. 2223 b. 3412
c. 4561 d. 9822

Solution:

1. $a_{30} = 4 + (30-1)3 = 91$
   CALCULATOR SOLN.
   ENTER: MODE 3 2
   Input:

X	Y
1	4
2	7

   ENTER: AC
   $30\ \hat{y} = 91$

2. $a_{100} = 1.01 + (100-1)(-0.01) = 0.02$
   CALCULATOR SOLN.
   ENTER: MODE 3 2
   Input:

X	Y
1	1.01
2	1.00

   ENTER: AC
   $100\ \hat{y} = 0.02$

3. The series is a harmonic:
   1/2 1/5 1/8
   4^{th} term is $1/(8 + 3) = 1/11$

4. $a_5 = 123$ $a_{30} = 245$

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123 = a_1 + 4d
245 = a_1 + 29 d
USE MODE 5 1
a_1 = 2587/25 d = 122/25
a_{12} = 2587/25 + 11(122/25) = 3929/25
CAL TECHNIQUE (MODE 3 2)
X Y
5 123
30 245
12 $\hat{y}$ = 3929/25

5. 3rd term AP = 1/15
9th term AP = 1/6
ENTER: MODE 3 2
x y
3 1/15
9 1/6
Compute: 11 $\hat{y}$ = 1/5 Ans. 5

6. S_n = 0.5n(2a_1+ (n-1)d)
66 = 0.5n(-18 + (n-1)(3))
n = 11
CAL TECHNIQUE
ENTER: MODE 3 3
Input:
X Y
0 0
1 -9
2 -9-6 = -15
$66\hat{X}1$ Ans. 11

7. Day 1 5700
Day 2 5680
Day 3 5660
….. …..
5700 x 66 = 0.5n(2(5700) + (n-1)(-20))
n = 76
CAL TECHNIQUE:

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ENTER MODE 3 3

X	Y
0	0
1	5700
2	5700 + 5680

ENTER: (57000 x 66) $66\hat{X}1$; Ans. 76

8. Let x = number who died per day

216/3 = 72 days (number of days the food will last if there are no deaths)

72 + 9 = 81 days (actual number of days the food lasted)

	Amount Consumed
Day 1	6400 x 3 = 19200 meal days
Day 2	3 (6400 - x)
. …	
Day 81	3 (6400 – 80x)

$S_n = n/2 (a_1 + a_n)$

6400(216) = [19200 + 3(6400-80x)] (81/2)

x = 17.78

9. ENTER: MODE 3 2

Input:

X	Y
1	2
3	6

Then: 11 $\hat{y}$ = 22 and reciprocal = 1/22

10. Y = A + BX

when X = 110 Y = 140

X = 80 Y = 80

Then: A + 110B = 140

A + 80B = 80

ENTER: MODE 5 1

A = -80 B = 2

Then Y = -80 + 2x

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when X = 250 Y = 420
420/250 = 1.68
CAL TECHNIQUE: MODE 3 2

X	Y
110	140
80	80

$\frac{\widehat{250}y}{250} = 1.68$

11. ENTER: MODE 7
Input f(x) = X^2+ 1 =
START = 1 END = 4 STEP = 1
Answers: 2 5 10 17
12. ENTER: MODE 7
Input: f(x) = $x^3 - 2x^2 + 1$
START = 1 END = 4 STEP = 1
Answers: 0 1 10 33

13.

	Man Days Consumed
1st 12 days	60 x 12 = 720
2nd 12 days	55 x 12 = 648
xth 12 days	(60 – 6(x-1)) x 12

……

Let x = number of 12 days to finish the job.
60 x 72 = [720 + [60 – 5(x -1)] x 12] (x/2)
x = 9 Thus no of days = 9 x 12 = 108

CAL TECHNIQUE: MODE 3 3
Input:

X	Y
0	0
12	720
24	720 + 660

(60 X 72) $\hat{X}1$ = 108

14. x (1 + 4 + 7 + …. 64) = 1430

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$a_n = a_1 + (n-1)d$

$64 = 1 + (n-1)(3)$ $\quad n = 22$

$1 + 4 + 7 + .. \ 64 = (1 + 64)(22/2) = 715$

$x = 1430/715 = 2$

15. Let $a - d = 1^{st}$ number

$a = 2^{nd}$ number

$a + d = 3^{rd}$ number

Then: $a - d + a + a + d = 45$

$3a = 45; \ a = 15$

$15 - d + 2 = 17 - d$ (1^{st} number in GP)

$15 + 3 = 18$ (2^{nd} number in GP)

$15 + d + 7 = 22 + d$ (3^{rd} number in GP)

Then : $\dfrac{22+d}{18} = \dfrac{18}{17-d}$

So, $(22 + d)(17-d) = 18^2$; $d = 5$

16. X = number of item A

Y = number of item B

Z = number of item C

$X + Y + Z = 100$ or $Z = 100 - X - Y$

$50X + 30Y + Z = 1000$

Substitute Z: $50X + 30Y + 100 - X - Y = 1000$

$49X + 29Y = 900$

$Y = \dfrac{900 - 49X}{29}$

ENTER: MODE 7

Input $f(x) = \dfrac{900 - 49X}{29}$ START = 0 END = 20 STEP = 1

The only exact value: when x = 16 F(x) = 4 Ans. 16 (a)

17. $61 + 58 + 55 + ... \ 10 = ?$

$10 = 61 + (n-1)(-3)$; $n = 18$

Sum = $(61 + 10)/2 \ (18) = 639$

18. 16.1 48.3 80.5 (AP)

ENTER: MODE 3 2

Input

X Y

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1 16.1
2 48.3
ENTER: 10 $\hat{y}$ Ans. 305.9

19. $a_1 = 25$ $d = 5$ $S_n = (n/2)(2a_1 + (n -1) d)$
$= (n/2 (2(25) + (n -1) 5)$
$= n/2 (45 + 5n^2)$
$= 5/2n^2 + 45n/2$

CAL TECHNIQUE
ENTER: MODE 3 3
Input:

X	Y
0	0
1	25
2	25 + 30 = 55

Get A: SHIFT 1 5 1 = (A = 0) (or 1 5 1 ES +)
B: SHIFT 1 5 2 = (B = 22.5)
C: SHIFT 1 5 3 = (C = 2.5)

$Y = A + Bn + Cn^2 = 0 + 22.5n + 2.5n^2$

20. $a_n = a_1 r^{n-1} \rightarrow 162 = a_1r^4$
$39366 = a_1r^9$; $r = (39366/162)^{1/5} = 3$
$a_1 = 162/3^4 = 2$
$a_3 = 2(3)^2 = 18$

CALCULATOR SOLUTION:
ENTER: MODE 3 6
Input

X	Y
5	162
10	39366

COMPUTE 3 $\hat{y}$ Ans. 18

21. CALCULATOR SOLUTION.
ENTER: MODE 3 6
Input

X	Y
4	189
6	1701

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Compute $12\,\hat{y}$ Ans. 1240029

22. CALCULATOR SOLUTION.

ENTER: MODE 3 6

x	y
3	28
5	112

Then: $9\,\hat{y}$ = 1792

23. $1\,\hat{y}$ = 7

24. $3584\,\hat{x}$ = 10

25. A = 3.5 (Shift 1 5 1)
B = 2 (Shift 1 5 2)
Compute $\sum(AB^x,1,10) = 7161$

26. Solve for x first.

$$\frac{2x+7}{x} = \frac{10x-7}{2x+7}$$ OR (2x +7)(2x + 7) = x(10x – 7) X = 7

ENTER: MODE 3 6
Input:

X	Y
1	7
2	21

A = (Shift 1 71) = 7/3
B = (SHIFT 1 5 2) = 3
COMPUTE: Σ (7/3 $(3)^X$, 1, 10)
Ans. 206,668

27. Let: a = first term
a+ d = 2nd term
a + 2d = 3rd term
a + 3d = 4th term.
Then: (a)(a + 3d) = 70 (1)
(a + d)(a + 2d) = 88 (2)
From 1: a + 3d = 70/a 3d = 70/a – a
d = (70/a – a)/3 Substitute to eq. 2
[a + (70/a – a)/3] [a + 2(70/a – a)/3] = 88

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USE SHIFT CALC INPUT the given choices for initial value.
a = 5

28. 500 = 10 + (n-1)(2) n = 246
sum = (10 + 500)/2 (246) = 62,370
CAL TECHNIQUE:
Compute how many even integers are between 10 to 500

MODE 3 2
X Y
1 10
2 12
500$\hat{X}$ = 246 (Number of even terms)
ENTER: MODE 3 3
X Y
0 0
1 10
2 10 + 12
246$\hat{y}$ = 62,370

29. First odd integer = 11
Last odd integer = 499
499 = 11 + (n-1)(2) n = 245
Sum = (11 + 499)/2 (245) = 62,475
CAL TECHNIQUE:
ENTER: MODE 3 2
X Y
1 11
2 13
499$\hat{x}$ = 245
ENTER: MODE 3 3
X Y
0 0
1 11
2 11+13
245$\hat{y}$= 62,475

30. $S_n = 0.5n (2a_1 + (n-1)d)$
2600 = 0.5n(6 + (n-1)2); n = 50
CAL TECHNIQUE:

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MODE 3 3

X	Y
0	0
1	3
2	3+5

$2600\hat{X}1 = 50$

31. Let x = amount contributed by each man
y = amount contributed by each woman
z = amount contributed by each child

Then $x + y + z = 50$

$50x + 25y + 3z = 1630$

$z = 50 - x - y$

$50x + 25y + 3(50 - x - y) = 1630$

$47x + 22y = 1480$

$y = (1480 - 47x)/22$

USE MODE 7

Enter f(X) = (1480 – 47x)/22

START = 1 END = 30 STEP = 1

Exact Values for x

X	F(x)	
2	63	Impossible
24	16	Possible

Thus x = 24 and y = 16 (Ans) c

32. Let x be the first part and y be the second part.

(1) $x/5 = a + 2/5$ and (2) $y/8 = b + 3/8$

Then: $x = 5a + 2$ and $y = 8b + 3$

$x + y = 136$

$5a + 2 + 8b + 3 = 136$

$5a + 8b = 131$

$b = (131 - 5a)/8$

USE MODE 7: Input f(x) = (131 – 5x)/8

START? 1 END? 30 STEP? 1

Exact Values

a	b	
7	12	x = 5a + 2 = 37

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		y = 8(b) + 3 = 99
15	7	x = 5a + 2 = 77
		y = 8b + 3 = 59
23	2	x = 5a + 2 = 117
		y = 8b + 3 = 19

Ans. 117 x 19 = 2223

Problem Set 3 Part 2

Arithmetic and Geometric Progression, Rate Problems, Quadratic Equation, Theory of Equations

31. The 1st term of the GP is 27 and the 4th term is -1. Find the 3rd term.

a. 3 b. 2
c. -3 d. -2

32. The sum of the 1st 10 terms of a GP 2, 4, 8, 16….(ECE April 1998)

a. 1024 b. 2046
c. 3024 d. 4024

33. Find the sum 1 , -1/5, 1/25 …..

a. 2/3 b. 5/6
c. 1 d. 7/6

34. The numbers 28, x + 2, 112 form a geometric progression. What is the 10th term? CE May 1995

a. 14536 b. 13463
c. 14336 d. 16344

35. Determine the sum of the infinite series 1/3 + 1/9 + 1/27 + ……..

a. 4/5 b. 3/4
c. 2/3 d. 1/2

36. Find the sum of the infinite series 1 - 1/4 + 1/6 +… (CE May 1998)

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a. 4/5 b. 5/6
c. 6/7 d. 7/8

37. If 1/3 of the air in the tank is removed by each stroke of an air pump, what fractional part of the total air is removed in 6 strokes?

a. 0.7122 b. 0.9122
c. 0.6122 d. 0.8122

SITUATION (Problems 38,39,40) PAST CEBOARD

The following data of road accident vs drivers age form a quadratic function.

Age	Accident per year
20	250
40	150
60	200

38. Find the coefficient of x^2

a. 0.1875 b. 0.2125
c. 0.1575 d. 0.1765

39. Find the coefficient of x.

a. -14.75 b. -17.75
c. -15.25 d. -16.25

40. Find the number of accidents per year for an age of 30.

a. 154.75 b. 189.65
c. 254.25 d. 181.25

41. In a benefit show, a number of wealthy men agreed that the 1st one to arrive would pay 10 centavos to enter and each succeeding arrival will pay twice as much as the preceding man. The total amount collected is P 104, 857.50. How many wealthy men paid?

a. 18 b. 19
c. 20 d. 21

42. What is the equation whose roots are the reciprocal of $2x^2 - 3x - 5 = 0$ PAST ECE BOARD

a. $5x^2 + 3x - 2 = 0$ b. $5x^2 - 3x - 2 = 0$
c. $5x^2 + 3x - 5 = 0$ d. $5x^2 - 3x - 2 = 0$

43. Two engineering students are solving a problem leading to a quadratic equation. One student made a mistake in the coefficient of

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the first degree term and got roots of 2 and - 3. The other student made a mistake in the coefficient of the constant term and got roots of – 1 and 4. What is the correct equation?

a. $x^2 + 3x - 6 = 0$ b. $x^2 – 3x - 9 = 0$

c. $x^2 – 3x - 7 = 0$ d. $x^2 – 3x - 6 = 0$

44. Determine k so that the equation $4x^2 + kx + 1 = 0$ will have just one real solution. PAST ME BOARD

a. 2 b. 3

c. 4 d. 5

45. With a wind velocity of 40 kph, it takes an airplane as long to travel 1,200 km with the wind as 900 km against it. How fast can the airplane travel in still air? PAST ME BOARD

a. 240 kph b. 290 kph

c. 280 kph d. 300 kph

46. A boat travels downstream in 2/3 of the time as it goes going upstream. If the velocity of the rivers current is 8 kph, determine the velocity of the boat in still water.

a. 50 kph b. 40 kph

c. 45 kph d. 50 kph

47. A man leaves his house at 8:00 AM and traveling at an average speed of 2 kph, arrives at his office 3 min ahead of the expected time. Had he left his house at 8:30 am and traveled at an average speed of 3 kph, he will arrive 6 min late of the expected time. Find the distance that he had traveled.

CE BOARD NOV 2005

a. 2.4 km b. 1.8 km

c. 2.1 km d. 2.4 km

48. A ball is dropped from a height of 120 ft and continuously rebounds to 2/3 of the distance it falls. What is the total distance traversed by the ball when it comes to rest?

a. 1000 ft b. 600 ft

c. 800 ft d. 750 ft

49. If one of the roots of $ax^2 + bx + c$ is $3 + \sqrt{2}$. Find the value of b.

a. -6 b. 5

c. 10 d. -8

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50. If one of the roots of $ax^2 + bx + c$ is $2/3 + 5/9i$ where a, b and c are integers, what is the value of c?

a. 62 b. -61
c. 59 d. 61

51. Factor $2x^2 - 5x - 63$

a. $(2x + 7)(x - 9)$ b. $(2x + 9)(x - 7)$
c. $(x + 63)(2x - 1)$ c. $(x + 1)(2x - 63)$

52. If the two roots of the cubic equation $2x^3 + ax^2 + bx + 15$ are 1 and 3, what is the value of a?

a. 2 b. 4
c. -3 d. 5

53. If two of the roots of the cubic equation $ax^3 + bx^2 + cx + d$ are $3/2 + 4/5i$ and 7, and a , b, c and d are integral values, what is the value of c?

a.2145 b. 2211
c. 1239 d. 2389

54. What is the sum of the geometric progression $2x, 4x + 14, 20x - 14$ up to the 10^{th} term? CE Nov 1998

a. 423,114 b. 431,222
c. 413,336 d. 341,116

55. Find the value of x in the equation

$(x + yi)(1 - 2i) = 7 - 4i$ CE NOV 2004

a. 1 b. 3
c.5 d. 2

56. Find the summation of $5k - 3$ from $k = 1$ to $k = 16$.

CE Nov 2005

a. 612 b. 678
c. 632 d. 712

57. The geometric mean of 2 numbers is 8 while the arithmetic mean is 4. The cube of the harmonic mean is CE Nov 2005

a. 4123 b. 4096
c. 5122 d. 5132

(Stuation Problems 58, 59, 60)

You are taking a test in which items of type A are worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer

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each item of type A and 6 minutes for each item of type B. The total time allowed is 60 min, and you may not answer more than 16 questions. Assuming all your answers are correct and you may not answer more than 16 questions. Assume that all you answers are correct.

PAST CE BOARD

58. Determine the number of type A question solved.

a. 13 b. 14
c. 12 d. 15

59. Determine the number of type B question solved.

a. 5 b. 6
c. 4 d. 8

60. The maximum score is

a. 180 b. 200
c. 220 d. 230

60-b. In a pile of log, each layer contains one more log than the layer above and the top contains just one log. If there are 105 logs in the pile, how many layers are there.

a. 12 b. 13
c. 14 d. 15

60-c In a racing contest , there are 240 cars which will have provisions that will last for 15 hrs. Assuming constant hourly consumption for each car, how long will the provisions last if 8 cars withdraw from the race every hr after the 1st?

a. 24 b. 25
c. 26 d. 27

60-d To conserve energy due to present energy crisis, Meralco tried to readjust their charges to electrical energy users who consume more than 2000 kw-hrs. For the 1st 100 kw hr, they charge P 0.40 and increasing at a constant rate more than the preceding one until the 5th 100 kw hr is charge P 0.76. How much is the average charge for the electrical energy per 100 kw hr? PAST CE BOARD

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a. 0.78 b. 0.58
c. 0.32 d. 0.61

60e. A man left his home at past 3:00 PM as indicated in his wall clock. Between 2 to 3 hrs after, he returned home and noticed the hands of the clock interchanged. At what time did he leave his home? PAST ECE BOARD

a. 3:32.17 b. 3:31.47
c. 3:21.33 d. 3:31.88

60f. In a mixed company of Poles, Italians, Greeks, Turks and GermansThe poles are one less than 1/3 of the Germans; and 3 less than half the Italians; the Germans and the Turks outnumber the Greek and the Italians by 3; the Greeks and the Germans form one less than half the company and the Greeks and the Italians form 7/16 of the company. How many Germans are there? PAST CE BOARD EXAM

a. 40 b. 32
c. 24 d. 18

SOLUTION:

31. $a_n = a_1 r^{n-1}$
$-1 = 27r^{4-1}$; $r = (-1/27)^{1/3} = -1/3$
$a_3 = 27(-1/3)^2 = 3$

32'. $S_n = \frac{a_1(1-r^n)}{1-r} = \frac{2(1-2^{10})}{1-2} = 2046$

33. $S_n = \frac{a_1}{1-r} = \frac{a_1}{1-r} = \frac{1}{1-(-1/5)} = 5/6$

34. ENTER: MODE 3 6
Input
X Y
1 28
3 112
$10\,\hat{y} = 14336$

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35. $S_n = \dfrac{a_1}{1-r} = \dfrac{1/3}{1-1/3} = 1/2$

36. $S_n = \dfrac{1}{1-(-1/4)} = 4/5$

37.

	Amount Removed	Amount Remaining
First time	1/3	2/3
2nd time	1/3(2/3)= 2/9	2/3 – 2/9 = 4/9
3rd time	1/3(4/9) = 4/27	4/9 – 4/27 = 8/27

ENTER: MODE 3 6

Input

X	Y
1	2/3
2	4/9

6 $\hat{y}$ = 0.0877914952

38. ENTER: MODE 3 3 ($Y = A + BX + CX^2$)

Input the given data

X	Y
20	250
40	150
60	200

$Y = A + BX + CX^2$

C = SHIFT 1 7 3 = 0.1875 (or SHIFT 1 5 3)

39. B = SHIFT 1 7 2 =-16.25 (or SHIFT 1 5 2)

40. 30 $\hat{y}$ = 181.25

41. 0.10 + 0.20 + 0.40 + … = 104,857.50

$\dfrac{0.1(1-2^n)}{1-2} = 104{,}857.50$ n = 20

42. Solve for x in $2x^2 – 3x – 5 = 0$ (MODE 5 3)

x = 5/2 and - 1

Reciprocal 2/5 and - 1

b = - ($x_1 + x_2$) = 3/5

c = x_1x_2 = 2/5(-1) = -2/5

Equation: $x^2 + 3/5x – 2/5 = 0$ or $5x^2 + 3x – 2 = 0$

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43. For 2 and - 3 $(x – 2)(x + 3)= x^2 + x – 6$ 1x is wrong.
For -1 and 4 $(x +1)(x -4) = x^2 -3x – 4$ -4 is wrong.
Correct equation. $x^2 -3x – 6$

44. Discriminant $b^2 – 4ac = 0$
$k^2 – 4(4)(1) = 0$ k = 4, -4

45. Let v = velocity in still air.
Distance = rate x time
1200 = (v + 40)(t)
900 = (v - 40)(t)
1200/900 = (v + 40)/(v - 40) or: 4/3(v - 40) = v + 40
USE SHIFT CALC. Ans. v = 280

46. Distance = rate x time
Let X = distance t = time V = VELOCITY in still water
X = (V + 8)(2/3t)
X = (V -8) t
(V + 8)(2/3t) = (V -8)t
(V + 8)(2/3) = V – 8 V = 40

47. Let T = expected time.
Distance = rate x time
D = distance from house to office

8	rate = 2 kph	T- 3/60
8 .5	rate = 3 kph	T + 6/60

D = 2(T- 3/60 - 8) (1)
D = 3(T + 6/60 – 8.5) (2)
Arrange:
D – 2T = 2(- 3/60 – 8)
D – 3T = 3(6/60 – 8.5)
USE MODE 5 1
D = 2.1 Y = 9.1 D = 2.1 km Expected time = 9:06

48. Total distance =
120 + 2 [2/3(120) + 2/3(2/3(120) + 2/3(2/3)(2/3) 120 + ….]

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$120 + 2\left(\frac{2/3(120)}{1-2/3}\right) = 600$ ft

49. $[x - (3 + \sqrt{2})][x - (3 - \sqrt{2})] = (x - 3)^2 - (\sqrt{3})^2 = x^2 - 6x + 9 - 3$
$= x^2 - 6x + 6;\quad b = -6$

50. $x_1 + x_2 = -b/a = -b/1$
$-b = -2/3 + 5/9i + 2/3 - 5/9i = 4/3$
$b = -4/3$
$x_1 x_2 = c/a = c \quad c = (2/3 + 5/9i)(2/3 - 5/9i) = 61/81$
The equation is $x^2 - 4/3x + 61/81 = 0$
$81x^2 - 108x + 61 = 0$

51. Solve $2x^2 - 5x - 63 = 0$ using MODE 5 3
Ans. $X_1 = 7$, $X_2 = -9/2$
$(x - 7)(x + 9/2) = 0$ or $(x - 7)(2x + 9)$ are the factors.

52. $f(x) = 2x^3 + ax^2 + bx + 15$
$f(1) = 0 \quad 0 = 2 + a + b + 15 \blacktriangleright a + b = -17$
$f(3) = 0 \quad 0 = 2(3^3) + a(3^2) + 3b + 15$
$9a + 3b = -69$
$a = -3 \quad b = -14$

53. Let $f(x) = ax^3 + bx^2 + cx + d$
If $3/2 + 4/5i$ is a root, the other root is $3/2 - 4/5i$
Form the quadratic equation $a'x^2 + b'x + c' = 0$
$b' = -(x_1 + x_2) = -(3/2 + 4/5i + 3/2 - 4/5i) = -3$
$c' = x_1 x_2 = (3/2 + 4/5i)(3/2 - 4/5i) = 289/100$
So $a'x^2 + b'x + c' = x^2 - 3x + 289/100 = 0$
$100x^2 - 300x + 289 = 0$
Then: $(x - 7)(100x^2 - 300x + 289) = 100x^3 - 1000x^2 + 2389x - 2023$
$c = 2389$

54. Solve for x:
$(4x + 14)/2x = (20x - 14)/(4x + 14)$
$(4x + 14)^2 = (2x)(20x - 14)$
$16x^2 + 112x + 196 = 40x^2 - 28x$
$-24x^2 + 140x + 196 = 0$ (MODE 5 3)
$x = 7$ and $-7/6$
Use $x = 7$

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1st term = 14; 2nd term = 4(7) + 14 = 42

So, r = 42/14 = 3

Then, $S_n = a_1 \frac{1-r^n}{1-r} = \frac{14(1-3^{10})}{1-3}$ = 413336

55. x + yi = (7 – 4i)/(1 – 2i) = 3 + 2i

(Note: ENTER: MODE 5 2 and ENTER:

(7 – 4i)/(1 – 2i) = 3 + 2i

x + yi = 3 + 2i x = 3, y = 2

56. ENTER: $\sum_{1}^{16} 5x-3$ = 632

57. Let X , Y be the numbers.

$\sqrt{xy} = 8$ ► xy = 64

$\frac{x+y}{2} = 4$ ► x + y = 8 ► x + y = 8

Harmonic mean for x and y = $\frac{2}{1/x+1/y} = \frac{2xy}{x+y}$

Cube of harmonic mean = $\frac{8(xy)^3}{(x+y)^3} = \frac{8(64)^3}{8^3} = 4096$

58. 3A + 6B = 60

A + B = 16

A =12; B = 4

59. B = 4

60. Max score = 10A + 15B = 10(12) + 15(4) = 180

60-b . 1 + 2 + 3 + …. n = 105

S_n = n/2 (2 (1) + (n -1)(1) = 105

n = 14

CALCULATOR SOLUTION:

MODE 3 3

x	y
0	0
1	1
2	1+ 2 = 3

$105\widehat{x1}$ = 14

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60c. 240(15) = 240 + 232 + 224 + …

ENTER: MODE 3 3

x	y
0	0
1	240
2	240 + 232

(240 x 15) $\widehat{x1}$ = 25

60d. To get the constant increase:

MODE 3 2

1	0.4
5	0.76

B (SHIFT 1 5 2) = 0.09

Sum = n/2 (a_1 + a_n) = 5/2(0.4 + 0.76) =P 2.9

Average Charge = P 2.9/5 =P 0.58 / 100 kw hr

60e. Let X = position of the minute hand after 3 PM (minutes)

15 + X/12 = position of the hr hand.

Y = Position of the minute hand after 6 PM (observed position)

30 + Y/12 = postion of the hr hand.

But the hands interchanged

X = 30+ Y/12

Y = 15 + X/12

Then, X – Y/12 = 30

-X/12 + Y = 15

USE MODE 5 1 : X = 31.47

60f. Let A = number of Poles

B = numberof Italians

C = number of Greeks

D = number of Turks

E = number of Germans

A = 1/3 E – 1

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A = ½ B - 3
E + D - (C + B) = 3
C + E = ½ (A + B + C + D + E) - 1
C + B = 7/16 (A + B + C + D + E)
The system is

3A = E – 3 E = 3A + 3 (1)
2A = B – 6 B = 6 + 2A (2)
E + D - C – B = 3 (3)
2C + 2E = A + B + C + D + E - 2 (4)
16C + 16B = 7A + 7B + 7C + 7D + 7E (5)

Substitute E = 3A + 3 AND B = 6 + 2A
3A + 3 + D – C - (6 + 2A) = 3
A - C + D = 6 (6)

2C + 2(3A + 3) = A + 6 + 2A + C + D + 3A + 3 - 2
A + C - D = 1 (7)

16C + 16 (6 + 2A) = 7A + 7(6 + 2A) + 7C + 7D + 7(3A + 3)
-10A + 9C – 7D = -33 (8)

MODE 5 2 Solve (6), (7) , (8).
A = 7; D = 15; C = 14; E = 3A+3= 24; B = 6 + 2A = 20

PROBLEM SET 3 (PART 3 CONTINUATION OF PART 2 AND WORK and GEOMETRY PROBLEMS)

61. Two runners A and B complete for a race of 1000 m long. It took 130 sec for A to reach the finish line and for B 138 sec. How far was B behind A when A reaches the finish line? PAST CE BOARD

a. 58.78 m b. 58.02 m
c. 59.21 m d. 53.43 m

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62. A boatman rows to a place 48 km distant and back in 14 hrs. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. Find the rate of the stream.

a. 0.8 km /hr b. 0.9 km/hr
c. 1 km/hr d. 1.1 km/hr

63. The sum of 4 integers in an arithmetic progression is 24 and their product is 945. Find the smallest possible integer.

a. 4 b. 3
c. 5 d. 10

64. If 4, 2, 5 and 18 are added respectively to an arithmetic progression, the resulting series is a geometric progression. What is the highest number in the AP?

a. 23 b. 22
c. 21 d. 20

65. A man set out from a certain point and traveled at the rate of 6 km/hr. After A had gone 2 hrs, another man B set out to overtake him and went 4 km the 1st hr, 5 km the 2nd hr, 6 km the 3rd hr and so on gaining 1 km every hr. After how many hrs will they be together? PAST CE BOARD

a. 7 b. 8
c. 6 d. 9

66. A and B start at the same time from 2 places 91 km apart and they travel toward each other. A travels at a constant rate of 5 1/2 km/hr , while B travels 3 km for the 1st hr, 4.5 km for the 2nd hr, 6 km for the 3rd hr and so on. Where will they meet? PAST CE BOARD

a. 38 km from A b. 38.5 km from A
c. 39 km from A d. 40 km from A

67. A cask containing 20 liters of wine was emptied 1/5 of its contents and then filled with water. If this is done 6 times, how many liters of wine will remain in the cask?

a. 5.242 liters b. 5.811 liters
c. 6.242 liters d. 6.134 liters

68. There are 4 geometric means between 3 and 729. Find the 4th term of the geometric progression. CE Nov 2000

a. 27 b. 81
c. 143 d. 229

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69. Find the 5^{th} term of the series whose sum of n terms is given by 3^{n+2}-6. CE May 2001

a. 1238 b. 1358
c. 1345 d. 1458

70. A messenger travels from point A to point B. If he will leave at 8:00 AM and travel at 2.00 kph, he will arrive 3 minutes earlier than his expected time of arrival. However, if he will leave at 8:30 AM, and travel at 3 kph, he will arrive 6 min later than the expected time. What is the expected time of arrival? PAST CE BOARD

a. 9:03 AM b. 9:04 AM
c. 9:06 AM d. 9:07 AM

SITUATION (PROBLEMS 71, 72, 73)
The 10^{th} term of a GP is 39366 and the 4^{th} term is 54.

71. Find the common ratio.

a.2 b. 3
c.4 d. 5

72. Find the 1^{st} term.

a. 2 b. 3
c. 4 d. 5

73. Find the 7^{th} term

a. 1249 b. 1431
c. 1458 d. 1259

74. The 1^{st} term of an AP is 2 and the 6^{th} term is 12. Find the sum of the 1^{st} 12 terms.

a. 156 b. 158
c. 152 d. 134

75. An audience of 540 person is seated in rows having the same number of persons in each row. If 3 more persons sit in each row, it would require 2 rows less to seat the audience. How many persons were in each row originally?

a. 28 b. 29
c. 27 d. 30

76. Factor $6x^3 + 35x^2 + 21x - 20$

a. $(3x + 4)(2x -5)(x + 1)$ b. $(2x + 1)(3x-5)(x + 4)$
c. $(3x +4)(2x-1)(x +5)$ d. $(3x +1)(2x + 4)(x -5)$

77. How many irrational roots are there in $x^6 - 3x^4 -18x^2 + 40$?

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a. 0 b. 2
c. 3 d. 4

78. From the equation $7x^2 + (2k-1)x - 3k + 2 = 0$, determine the value of k so that the sum and product of the roots are equal. PAST ECE BOARD

a. 2 b. 4
c. 1 d. 2

79. Solve for x if $8^x = 2^{y+2}$ $16^{3x-y} = 4^y$ PAST ECE BOARD

a. 2 b. 4
c. 1 d. 2

80. Determine the sum of the 1^{st} 10 terms if the general term of the sequence is $3^n - 2$. PAST CE BOARD

a. 98221 b. 88552
c. 67112 d. 82133

81. The arithmetic mean and geometric mean of 2 numbers are 10 and 8 respectively. Find the harmonic mean.

a. 6.4 b. 5.7
c. 7.4 d. 3.9

82. An earthquake emits a primary and secondary wave. Near the surface of the earth, the primary wave travels at about 5 miles per second, and the secondary wave travels at about 3 miles per second. Suppose a station measures a time difference of 12 seconds between the arrival of the two waves. How far is the earthquake from the station?

a. 80 miles b. 90 miles
c. 70 miles d. 60 miles

83. A boat takes 1.5 times as long to go 360 miles up a river than to return. If the boat cruises at 15 miles per hr in still water, what is the rate of the current?

a. 2 mph b. 3 mph
c. 4 mph d. 5 mph

84. A plane travels from 2 airports with a distance of 1500 km with a wind along its flight line. It takes the airplane 2 hrs with the tailwind

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and 2.5 hrs with the headwind. Determine the velocity of the airplane in still air.

a. 650 kph b. 660 kph
c. 675 kph d. 600 kph

85. A piece of paper is 0.05 in thick. Each time the paper is folded into half, the thickness is doubled. If the paper was folded 12 times, how thick in feet the folded paper be? PAST ECE BOARD

a. 10.24 b. 12.34
c. 17.10 d. 11.25

86. It takes an airplane 1 hr and 45 min to travel 500 miles against the wind and covers the same distance in one hr and 15 min with the wind. What is the speed of the airplane?

a. 341.45 mph b. 342. 85 mph
c. 371.23 mph d. 355.56 mph

87. A cat is now 50 of her own leaps ahead of a dog which is pursuing her. How many more leaps will the cat take before it is overtaken if she takes 5 leaps while the dog takes 4, but 2 of the dogs leap are equivalent to 3 of the cats leap.

a. 200 b. 220
c. 230 d. 250

88. A policemen is pursuing a thief who is ahead by 72 of his own leaps. The thief takes 6 leaps while the policemen is taking 5 leaps, but 4 leaps of the thief are as long as 3 leaps of the policemen. How many leaps will the policeman make before the thief is caught?

a. 520 b. 530
c. 540 d. 550

89. An oil drilling rig in the gulf of Mexico stands so that 1/5 of it is in sand, 20ft in water and 2/3 of it in still air. What is the total height of the rig?

a. 140 ft b. 150 ft
c. 160 ft d. 170 ft

90. A minor chord is composed of notes whose frequencies are in the ratio 10:12:15. If the 1st note of a minor chord is A, with a frequency of 220 Hz, what is the frequency of the 3rd note?

a. 340 b. 330

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c. 350 d. 380

91. The seating section in a coliseum has 30 seats in the 1st row, 32 seats in the 2nd row, 34 seats in the 3rd row and so on until the 10th row is reached, after which there are 10 rows, each containing 50 seats. Find the total number of seats in the section. ECE April 2001

a. 900 b. 910
c. 910 d. 890

Situation (Problems 92, 93, 94)

A construction job could be finished in 150 days if 50 men are working full time. 60 men started working on the job and after 20 days, 20 more men were added. After 80 days 60 men quit the job.

92. How long could it take for them to finish the job?

a. 145 b. 155
c. 120 d. 115

93. How much penalty would the contractor pay if the contract specifies a penalty of P 10,000 per day being delayed.

a. 40,000 b. 45,000
c. 55,000 d. 50,000

94. What is the labor cost if each worker is paid P 350 a day?

a. 3,234, 000 b. 2,625,000
c. 4,125,000 d. 3,125,000

95. The time required for 2 examinees to solve the same problem differs by 2 minutes. Together, they can solve 32 problems in 1 hr. How long will it take for the slower problem solver to solve the problem?

ECE BOARD Nov 1999

a. 4 min b. 5 min
c. 6 min d. 7 min

96. One pipe can fill the tank in 6 hrs and another can fill the same tank in 3 hrs. Another pipe can drain the tank in 7 hrs. With all the 3 pipes open, how long will it take to fill the pipe? ECE Board April 2001

a. 2.7 hrs b. 2.8 hrs
c. 2.9 hrs d. 3.1 hrs

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97. Twenty eight (28 men) can finish the job in 60 days. At the start of the 16th day 5 men were laid off and after the 45th day 10 more men were hired. How many days were they delayed in finishing the job? CE Nov 2000

a. 2.27 b. 2.45
c. 3.67 d. 1.25

Situation Problems 99, 100

A contractor hired 28 workers with a daily wage of P150 for a job that could be finished in 60 days. He wanted to finish the job earlier so he hired additional 6 workers at the start of the 16th day and 14 more at the start of the 46th day.

98. How many days dit it complete the job?

a. 45 b. 50
c. 55 d. 60

99. If the contractor were given a bonus of P 5000 per day for the number of days he would complete the job earlier, how much bonus did he get?

a, 45,000 b, 50,000
c. 55,000 d. 60,000

100. If the daily wage of the additional workers were P 175, how much did the contractor pay for the salary of all workers for the completion of the project?

a. 256,000 b. 294,000
c. 245,000 d. 259,000

101. A pump can pump out water from a tank in 11 hrs. Another pump can pump out water from the same tank in 20 hrs. How long will it take both pumps to pump out water in the tank? PAST ME BOARD

a. 7.1 hrs b. 6. 1 hrs
c 7.5 hrs d. 6.5 hrs

102. A 400 mm diameter pipe can fill the tank alone in 5 hrs and another 600 mm diameter pipe can fill the tank alone in 4 hrs. A drain

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pipe can empty the tank in 20 hrs. With all the 3 pipes open, how long will it take to fill the tank? PAST CE Nov 1993

a. 2 hrs b. 2.5 hrs
c. 2.25 hrs d. 2.75 hrs

103. Pedro can paint the fence 50% faster that Juan and 20% faster than Pilar and together they can paint a given fence in 4 hrs. How long will it take to paint the same fence if he had to work alone? PAST ECE BOARD

a. 6 b. 8
c. 10 d. 12

104. It takes Butch twice as long to do a certain piece of work compared to Peter. Working together they can do the work in 6 days. How long will it take Peter to do the work alone? PAST CE BOARD

a. 9 days b. 10 days
c. 11 days d. 12 days

105. Crew no 1 can finish installation of an antenna tower in 200 man hrs while Crew no 2 can finish the same job in 300 man hrs. How long will it take both crews to finish the same job working together? PAST ECE BOARD

a. 100 man hr b. 120 man hr
c. 140 man hr d. 160 man hr

106. A man can do job three times as fast as a boy. Working together it would take them 6 hrs to do the same job. How long will it take the man to do the job alone? ECE April 2003

a. 9 hrs b. 8 hrs
c. 7 hrs d. 10 hrs

107. X can do the job 50% faster than y and 20% faster than Z. If they work together, they can finish the job in 4 days. How many days will it take X to finish the job if he is to work alone? ECE Board April 2004

a. 18 b. 10
c. 12 d. 6

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108. The sum of the 1st 5 numbers in AP is 15 and the sum of the 1st 20 terms is -90. What is the 2nd term?

a. 4 b. 10
c. 7 d. 11

109. A and B compete in a race. A runs at a constant rate of 126 m/min while B runs 145 m, the 1st min, 143 m the 2nd min, 141 m the 3rd min and so on. When will A and B be together again. PAST CE BOARD

a. 19 hrs b. 22 hrs
c. 20 hrs d. 21 hrs

110. The side of the square is 6 cm long. A second square is inscribed by joining the midpoints of the sides of the second square and so on. Find the sum of the areas of the infinite number of inscribed squares thus formed. PAST CE BOARD

a. 48 cm^2 b. 72 cm^2
c. 38 cm^2 d. 78 cm^2

111- 112 An equilateral triangle is incscribed within a circle whose diameter is 12 cm. In this triangle, a circle is inscribed and in this circle, another equilateral triangle is inscribed and so on indefinitely.

111. Find the sum of all the perimeters of the triangles

a. $24\sqrt{3}$ b. $36\sqrt{3}$
c. $48\sqrt{3}$ d. $20\sqrt{3}$

112. Find the sum of the areas.

a. $24\sqrt{3}$ b. $36\sqrt{3}$
c. $48\sqrt{3}$ d. $20\sqrt{3}$

113. Two boys A and B runs at a constant rates and in the same direction around a circular track whose circumeference is 40 m. A makes one circuit in 2 sec less time than B , and they are together once every minute. Find the rate of A.

a. 3 m/s b. 5 m/s
c. 4 m/s d. 7 m/s

114. A train , one hour after starting, meets with an accident which detains it an hour , after which it proceeds at 3/5 of its former rate and arrived 3 hrs after the time. Had the accident happened 50 km farther

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on the line, it would have arrived 1.5 hrs sooner. Find the length of the journey. PAST CE BOARD

a. 87.91 km b. 92.23 km
c. 88.89 km d. 83.34 km

115. Two Pals P1 and P2 run at a constant speeds along a circular track 1350 m in circumference. Running in opposite directions, they meet every 3 minutes, while running in the same direction, they are together every 27 minutes. Find the speed of the slower Pal. PAST CE BOARD

a. 100 m/min b. 200 m/min
c. 180 m/min d. 220 m/min

116. Two Cars 2000 m apart are approaching each other. Car A is moving at a constant speed of 10 m/s while Car B is moving at a constant speed of 12 m/s. Super Fly (speed = 15 m/s) started from A and flew back and forth from A to B , B to A and so on until A and B met.

What was the total distance traveled by Superfly? PAST CE BOARD

a. 1234.56 m b. 1457.33 m
c. 1363.63 m d. 1451.23 m

Solution:

61. Time for A to reach the finish line = 130 sec.
Velocity of B = 1000/138 = 500/69 m/s
Distance of B when A reaches the finish line =
500/69 x 130
= 942. 029 m

Distance between A and B = 1000 - 942.029 = 57.97 m = 58 m

62. Distance = rate x time
Let x = rate of stream y = rate of boat in still water
t = time
Upstream: $3 = (y – x) t$
Downstream: $4 = (y + x) t$

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$$\frac{3}{y-x} = \frac{4}{y+x} \qquad 3(y + x) = 4(y - x) \quad (1)$$

Arrange: $y = 7x$

$48 = (y - x)\, t_{up}$ $\quad 48 = (y + x)\, t_{down}$

$t_{up} + t_{down} = 14$

$48/(y - x) + 48/(y + x) = 14 \quad (2)$

Substitute $y = 7x$ to eq. 2.

► $48/(7x - x) + 48/(7x + x) = 14$

$48/(6x) + 48/(8x) = 14$

$8x + 6x = 14 \qquad x = 1$

63. Let a = first integer $\qquad a + d = 2^{nd}$ integer

$a+2d = 3^{rd}$ integer $\qquad a+3d = 4^{th}$ integer

$a + a + d + a + 2d + a + 3d = 24$

$4a + 6d = 24$ ► $2a + 3d = 12$ ► $d = (12 - 2a)/3$

$(a)(a + d)(a + 2d)(a + 3d) = 945$

Use trial and error:

ENTER: $D = 6 - A$: $A(A + D)(A + 2D)(A + 3D)$

CALC 4 = = (1^{ST} choice)

Display: 4/3 10340/9 Wrong!

CALC 3 = = (2^{nd} choice)

DISPLAY: 2 945 Ans. $a = 3$

64. Let $A - 3D$, $A - 2D$, $A - D$, A be the numbers.

$A - 3D + 4$, $A - 2D + 2$, $A - D + 5$, $A + 18$ be the GP

$$\frac{A+18}{A-D+5} = \frac{A-D+5}{A-2D+2} \quad (1) \text{ and } \frac{A-D+5}{A-2D+2} = \frac{A-2D+2}{A-3D+4} \quad (2)$$

From 1 $(A - D + 5)^2 = (A + 18)(A - 2D + 2)$

From 2 $(A - 2D + 2)^2 = (A - D + 5)(A - 3D + 4)$

Using Trial and Error:

Substitute $A = 23$ from 1, then $D = 7.248$

From 2: If $A = 23$ and $D = 7.248$

$(A - 2D + 2)^2 = 110.31 \quad (A - D + 5)(A - 3D + 4) = 109.04$

From 1: If $A = 22$, $D = 7$ or -18

$(A - 2D + 2)^2 = 100 \quad (A - D + 5)(A - 3D + 4) = 100$

$A = 22$ Ans. (c)

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65. Distance = rate x time

A. distance = (t + 2)(6)

B t = 1 D = 4, t = 2, D = 5, t = 3 D = 6

(Arithmetic Progression)

distance = t/2(2(4) + (t -1)1)

= 0.5t(8 + t – 1) = 0.5t(7 + t)

distance A = distance B

(t +2)(6) = 0.5t(7 + t) ► t = 8

CAL TECHNIQUE: ENTER: MODE 3 3

X = TIME of after 2 hrs

Y = DISTANCE BETWEEN THEM AT TIME

X	Y
0	6x2 = 12
1	12 + 6 – 4 = 14
2	14 + 6 - 5 = 15

The distance will be zero at $0\hat{x}_2 = 8$ hrs

66. A -------- 91 km apart ---------- B

$S_A = 5.5t$

For S_B t = 1 S = 3

t = 2 S = 4.5

t = 3 S = 6

$S_B = 0.5t(2(3) + (t-1)(1.5))$

$= 0.5t(6 + 1.5t – 1.5) = 0.5t(4.5 + 1.5t)$

$S_A + S_B = 91$

$5.5t + 0.5t(4.5 + 1.5t) = 91$

t = 7 $S_A = 38.5$

CAL TECHNIQUE:

Let X = time

Y = distance between A and B at time X

MODE 3 3

X	Y
0	91
1	91 – 5.5 – 3 = 82.5
2	82.5 – 5.5 – 4.5 = 72.5

They will meet when the distance is zero.

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$0\hat{X}_2 = 7$

Thus they will meet at 7 x 5.5 = 38.5 Km from A.

67. t = 0 wine = 20 l water = 0
t = 1 wine = 4/5(20) = 16 l water = 4 l
t = 2 wine = 4/5(16) = 12.8 water = 7.2 l
ENTER: MODE 3 6
Input:

X	Y
1	16
2	12.8

Compute: $6\hat{y}$ = 5.24288 liters

68. ENTER: MODE 3 6
Input:

X	Y
1	3
6	729

Compute: $4\hat{y}$ = 81

69. $S_n = 3^{n+2} - 6$
ENTER: MODE 7
$f(x) = 3^{x+2} - 6$
ENTER: START = 1 END = 8 STEP = 1
DISPLAY:

1	21
2	75
3	237
4	723
5	2181
6	6555

5^{TH} term = 2181 – 723 = 1458

70. Let T = expected time.
Let D = distance from A to B
X = rate
Distance = rate x time
D = 2(T – 3/60 – 8) 3 min early
D = 3(T + 6/60 – 8.5) 6 min late

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2(T – 3/60 – 8) = 3(T + 6/60 – 8.5)

T = 9.1 or 9 + 0.1 x 60 = 9: 06

71. ENTER: MODE 3 6

Input

X	Y
10	39366
4	54

Common Ratio = B (SHIFT 1 7 2) = 3

72. 1st term = 1 $\hat{y}$ = 2

73. 7th term = 7 $\hat{y}$ = 1458

74. ENTER: MODE 3 2

Input

X	Y
1	2
6	12

A = 0 B = 2 Sequence = { 2x }

$\sum(2x,1,12) = 156$

75. Let X = number of seats in a row Y = number of rows.

Then: XY = 540 Y = 540/X

(X + 3) (Y -2) = 540

(X + 3)(540/X – 2) = 540

X = 27

76. Solve $6x^3 + 35x^2 + 21x - 20 = 0$ using MODE 5 4

Ans. x = -5 , x = 1/2 , x = - 4/3

(x + 5)(x – 1/2)(x + 4/3) = 0

(x + 5)(2x – 1)(3x + 4) =factors

77. Let $z = x^2$

$z^3 - 3z^2 - 18z + 40 = 0$

Solve using MODE 5 4

Z = -4 , Z = 5 and Z = 2

$Z = x^2$

$x = \sqrt{Z}$ then $x = (+, -) \sqrt{5}$ and $x = (+, -) \sqrt{2}$

There are 4 irrational roots and 2 imaginary roots $\sqrt{-2}, -\sqrt{-2}$

78. For: $ax^2 + bx + c = 0$

$r_1 + r_2 = -b/a$ $r_1 r_2 = c/a$

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-b/a = c/a b = - c

(2k-1) = -(-3k +2) (USE SHIFT CALC) k = 1

79. $8^x = 2^{y+2}$ ▶ $2^{3x} = 2^{y+2}$ ▶ 3x = y + 2

$16^{3x-y} = 2^{2y}$ ▶ $2^{4(3x-y)} = 2^{2y}$ ▶ 4(3x –y) = 2y

Arrange the equations.

3x – y = 2

12x -6y = 0 USE MODE 5 1

x = 2 , y = 4

80. Input: $\sum_{1}^{10}(3^X - 2)$ Ans. 88552

81. Let x and y be the numbers.

$\frac{x+y}{2} = 10$ and $\sqrt{xy} = 8$

x + y = 20 (1) xy = 64 (2)

$HM = \frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2xy}{x+y} = \frac{2xy}{x+y} = \frac{2(64)}{20} = 6.4$

82. Let t_1 = time to to travel (primary wave to station)

t_2 = time to travel (secondary wave to station)

Distance = time x rate time = Distance / rate

$t_1 = D/5$ $t_2 = D/3$ $t_2 - t_1 = 12 = D/3 – D/5$

12 = D/3 – D/5 USE SHIFT CALC D = 90

83. Distance = rate x time

Let X = rate of current Y = time to go downstrem

Upstream: 360 = (15 – X) (1.5 Y)

Downstream: 360 = (15 + X) Y

Divide the 2 equations.

$1 = \frac{(15-X)(1.5)}{15+X}$ OR 15 + X = 1.5(15 – X)

USE SHIFT C ALC , X = 3

84. Distance = rate x time

Let X = velocity of the plane in still air

Y = velocity of the wind

1500 = (X + Y) 2

1500 = (X – Y) 2.5

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Arrange:
2X + 2Y = 1500
2.5X – 2.5Y = 1500 USE MODE 5 1
X = 675 Y = 75

85. ENTER: MODE 3 6
INPUT

X	Y	
1	0.1	(First folding)
2	0.2	(2nd folding)

$12\hat{y}$ = 204.8 in = 17.0666 ft

86. Let X = velocity of airplane Y = velocity of the wind
500 = (X + Y)(1 + 15/60)
500 = (X – Y)(1 + 45/60)
OR
500 = 1.25X + 1.25Y
500 = 1.75 X – 1.75Y USE MODE 5 1
X = 342.857 Y = 57.14

87. ENTER: MODE 3 2
Input (X = leap of the cat,
Y = distance between dog and cat in terms of cats leap)

X	Y
0	50
1	50 + 1 -(4/5)(3/2) = 49.8

COMPUTE x when y = 0 ($0\hat{x}$) = 250 cats leap
Note: 1 cats leap = 4/5 dog leap x (3/2)

88. ENTER MODE 3 2
X = THIEF'S LEAP
Y = distance between thief and policeman in terms of thief's leap
Input

X	Y
0	72
1	72 + 1 – 5/6(4/3) = 71.8888889

COMPUTE: $0\hat{x}$ = 648 leaps of the thief

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or 648 x 5/6 = 540 leaps of the policemen

89. Let X = total height of the rig. X = 1/5 X + 20 + 2/3X
USE SHIFT CALC X = 150

90. 10: 12: 15 is the same as 1: 1.2: 1.5
frequency of the 3rd note = 1.5 x 220 = 330

91. ENTER: MODE 3 3
Input:

X	Y
0	0
1	32 (32 seats in the 1st row)
2	32 + 34 = 66 seats in the 2nd row

Compute $10\hat{y}$ Ans. 410

Total number of seats = 410 + 10(50) = 910

92. Let x = additional number of days to finish the job
60(20) + 80(60) + 20x = 150(50)
x= 75
The job was finished in 80 + 75 = 155 days

93. Penalty =(155 – 150) x 10000 = 50,000

94. Labor cost = 150 x 50 x 350 = 2, 625,000

95. Let X = number of hrs A can solve the problem
Y = number of hrs B can solve the same problem
60/x + 60/y = 32
x – y = 2 or y = x - 2
60/x + 60/(x -2) = 32
USE SHIFT CALC 4 (ENTER THE 1st choice)
Ans. X = 5

96. Let x = number of hrs to fill the tank
X/6 + X/3 – X/7 = 1
X = 2.8 hrs

97. Let X = additional days to finish the job
28 x 15 + 23 x 30 + 33X = 28 x 60
X = 17.27
Days delayed = 15 + 30 + 17.27 – 60 = 2.27

98. Let X = number of days the work can be finished at the start of the
46th day

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28 x 15 + 34 x 30 + 48X = 28 x 60 , X = 5
Number of days finished = 15 + 30 + 5 = 50 days

99. 10 x 5 000 = P 50,000

100. 28 x 60 x 175 = 294,000

101. Let X = total number of hrs to pump out the water
X/ 11 + X/20 = 1
X = 7.1

102. Let X = time to fill the tank
X/5 + X/4 – X/20 = 1 X = 2.5

103. Let X = time for Pedro to finish
Y = time for Juan to finish
Z = time for Pilar to finish
Y = 1.5 X (1)
Z = 1.2 X (2)
4/X + 4/Y + 4/Z = 1 (3)
4/X + 4/(1.5X) + 4/(1.2X) = 1
X = 10

104. Let X = time for Peter to do the work
2X = time for Butch to do the work
6/2X + 6/X = 1 X = 9

105. Let X = number of man hrs to finish the job working together.
X/ 200 + X/300 = 1
X = 120

106. Let X = time for Man to finish the job
3X = time for Boy
6/X + 6/(3X) = 1
X = 8

107. Let X = number of hrs for X to finish
Y = number of hrs for Y to finish
Z = number of hrs for Z to finish

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Y = 1.5 X Z = 1.2 X 4/X + 4/Y + 4/Z = 1
4/(X) + 4/(1.5X) + 4/(1.2X) = 1 X = 10

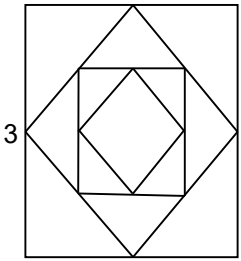
108. ENTER: MODE 3 3

X	Y
0	0
5	15
20	-90

$2\hat{Y} - 1\hat{Y} = 4$ ANS.

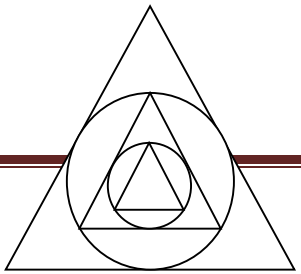
109. ENTER: MODE 3 3
x = time y = distance between them
0 0
1 145 – 126 = 19
2 19 + 143 – 126 = 36
They will be together at 0 $\widehat{X2}$ = 20 hrs

110.



Area of the 1st square = 6 x 6 = 36
The second square has side $\sqrt{3^2 + 3^2} = 3\sqrt{2}$
Area of the second square = ($3\sqrt{2}$)^2 = 18
and so on. ratio = 18/36 = ½
Sum = a_1/(1 – r) = 36/(1 – ½) = 72

111-112



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radius of circumscribed circle = $abc/4\ A_t$

$6 = x^3/4(x^2\sqrt{3}/4)$

$x = 6\sqrt{3}$ (side of the 1st triangle)

Area of the 1st triangle = $x^2\sqrt{3}/4 = 27\sqrt{3}$

Perimeter of the 1st triangle = $3(6\sqrt{3}) = 18\sqrt{3}$

Ratio of Perimeters of 2 successive triangles = ½

Sum of Perimeters = $a/(1 - r) = 18\sqrt{3}/(1 - ½) = 36\sqrt{3}$

Ration of Areas of 2 successive triangles = ¼

Sum of Areas = $27\sqrt{3}/(1 - ¼) = 36\sqrt{3}$

113. Let X = rate of A m/s

Y = rate o B m/s

Then $40/X = 40/Y - 2$ (1)

Distance traveled when they meet again (after 1 min= 60 sec) must differ by 40m.

Then: $60X - 60Y = 40$ (2)

From (2) $3X - 3Y = 2$, $Y = (3X - 2)/3$

Substitute to (1)

$40/X = 40/(3X - 2)/3 - 2$

or $40/X = 120/(3X - 2) - 2$

USE SHIFT CALC with the initial choice of X in the given.

X = 4 m/s

114. Let X = speed of train L = length of the journey.

Distance = rate x time

t = Distance / rate

Condition 1

$1 + 1 + (L - 1x)/(3/5\ x) = L/x + 3$ (1)

Condition 2

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1+ 50/X + 1 + (L – 1x - 50)/(3/5X) = L/X + 1.5 (2)

From 1 : 2x + (L – x)/0.6 = L + 3x
2x – 3x – 1/0.6x + L/0.6 - L = 0
- 8/3x + 2/3L = 0 (3)

From 2
x + 50 + x + (L – 1x - 50)/0.6 = L + 1.5x
2x- 1.5x + L/0.6 – L - 1x/0.6 = - 50 + 50/0.6
-7/6x + 2/3L = 100/3 (4)

MODE 5 1 Solve (3) and (4)
x = 200/9 and L = 800/9

115. Let x = speed of Pal 1 (m/min) y = speed of Pal 2 (m/min)
Then 3x + 3y = 1350 (1)
and 27x – 27y = 1350 (2)
MODE 5 1
x = 250 y = 200

116. Let X = time the two cars met.
Then 10 X + 12 X = 200; X = 90.909 s
Total distance traveled by Superfly = 90.909 (15) = 1363.63 m

PROBLEM SET 4 PARTIAL FRACTIONS

Given (Problems 1 , 2, 3, 4, 5)

$$\frac{2x^4+3x^3+7x^2+10x+10}{(x-1)(x^2+3)^2}=\frac{A}{x-1}+\frac{Bx+C}{x^2+3}+\frac{Dx+E}{(x^2+3)^2}$$

PAST CE Board

1. The value of A is
   a. 1 b. 2
   c. 3 d. 0
2. The value of B is
   a. 0 b. 1
   c. 2 d. 3
3. The value of C is
   a. 1 b. 2
   c. 3 d. 0

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4. The value of D is

a. 1 b.- 2

c. 3 d. 4

5. The value of E is

a. 1 b. -1

c. 2 d. – 2

6. Resolve into Partial Fractions

$$\frac{x+2}{x^2-7x+12}$$ (PAST ME BOARD)

a. $\frac{6}{x-4}-\frac{5}{x-3}$ b. $\frac{5}{x-4}-\frac{3}{x-3}$

c. $\frac{2}{x-4}-\frac{3}{x-3}$ d. $\frac{2}{x-4}-\frac{8}{x-3}$

Given (Problems 7, 8, 9, 10, 11)

$$\frac{2x^4+2x^3-5x^2+10x+9}{(x^2+9)(x^2+2x+5)(x+3)}=\frac{Ax+B}{x^2+9}+\frac{Cx+D}{x^2+2x+5}+\frac{E}{x+3}$$

7. The value of A is

a. 8/31 b. 1/12

c. -8/39 d. 5/48

8. The value of B is

a. -92/13 b. 21/13

c. 27/13 d. -29/4

9. The value of C is

a. 135/104 b. 29/16

c. 199/104 d. 393/16

10. The value of D is

a. 393/104 b. 41/16

c. 291/104 d. 67/16

11. The value of E is

a. 7/24 b. 3/51

c. 2/23 d. 5/48

Problems 12, 13, 14, 15, 16

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Given $\dfrac{x^3+4x^2+5x+3}{(x+1)(x^2+x+1)^2}=\dfrac{A}{x+1}+\dfrac{Bx+C}{(x^2+x+1)}+\dfrac{Dx+E}{(x^2+x+1)^2}$

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12. Find the value of A
    a. 2 b. 1
    c. 3 d. 4
13. Find the value of B
    a. 1 b. -1
    c. 2 d. 3
14. Find the value of C
    a. 2 b. 1
    c. -3 d. 2
15. Find the value of D
    a. 1 b. 2
    c. 3 d. 4
16. Find the value of E
    a. 3 b. 1
    c. 2 d. 3

Given (Problems 17, 18, 19)

$$\frac{x^2+4x+10}{x^3+2x^2+5x}=\frac{A}{x}+\frac{B(2x+2)}{x^2+2x+5}+\frac{C}{x^2+2x+5}$$

17. The value of A is
    a. 1 b. 2
    c. 3 d. 0
18. The value of B is
    a. 1 b. 0
    c. -1 d. 2

19. The value of C is
    a. 1 b.-1
    c.0 d. 2

20. Resolve $\dfrac{3x^2-8x+9}{(x-2)^3}$ into partial fractions.

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a. $3(x+2)-\frac{4}{(x+2)^2}+\frac{5}{(x-3)^3}$

b. $\frac{3}{x-2}+\frac{4}{(x-2)^2}+\frac{5}{(x-2)^3}$

c. $\frac{3}{x-2}-\frac{4}{(x-2)^2}+\frac{5}{(x-2)^3}$

d. $\frac{3}{x-2}-\frac{4}{(x-2)^2}+\frac{6}{(x-2)^3}$

SOLUTION:

1. $2x^4 + 3x^3 + 7x^2 + 10x + 10 =$

$A(x^2 + 3)^2 + (Bx + C) (x -1)(x^2 +3) + (Dx + E)(x -1)$ (1)

$= A(x^4 + 6x^2 + 9) + (Bx + C)(x^3 – x^2 + 3x – 3) + Dx^2 + Ex – E – Dx$

$= A(x^4 + 6x^2 + 9) + Bx^4 – Bx^3 + 3Bx^2 – 3Bx + Cx^3 – Cx^2 + 3Cx – 3C$

$+ Dx^2 + Ex – E – Dx$

$x^4 (A + B) + x^3 (-B + C) + x^2 (6A + 3B – C + D) +$
$x(-3B + 3C + E – D) + 9A – 3C - E$ (2)

Substitute $x = 1$ from `

$32 = A(1 + 3)^2$ $A = 2$

But $A + B = 2$, $B = 0$

$- B + C = 3$, $C = 3$

$6A + 3B – C + D = 7$

$6(2) + 3(0) – 3 + D = 7$ or $D = -2$

$-3B + 3C + E – D = 10$

$-3(0) + 3(3) + E –(-2) = 10$ or $E = -1$

CALCULATOR TECHNIQUE:

Let $f(x) = 2x^4 + 3x^3 + 7x^2 + 10x + 10$

If $x = 1$ $f(1) = A(1^2 + 3)^2 = 32$ then $A = 2$

Let $x = \sqrt{3}i$ (from $x^2 + 3 = 0$)

$f(\sqrt{3}i) = (Dx + E)(x -1) = D(x)(x-1) + E(x-1)$ (2)

GO to complex mode and ENTER:

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$2x^3 x + 3x^3 + 7x^2 + 10x + 10$ CALC $\sqrt{3}i = 7 + \sqrt{3}i$
$x(x-1)$ CALC $\sqrt{3}i = -3 - \sqrt{3}i$
$x - 1$ CALC $\sqrt{3}i = -1 + \sqrt{3}i$

Substitute to eq 2 .

$7 + \sqrt{3}i = D(-3 - \sqrt{3}i) + E(-1 + \sqrt{3}i)$
$7 + \sqrt{3}i = -3D - E + \sqrt{3}(-D + E)$
Then: $-3D - E = 7$ AND $-D + E = 1$
Solve using MODE 5 1 D = -2 E = - 1

To get the values of B and C, (we know already A = 2, D = -2 and E = -1) Substitute any value for x in eq. 1:

Say x = 0
$10 = 9A + [B(0) + C](0-1)(0+3) + (D(0) + E)(0 - 1)$
$10 = 9(2) + C(-3) + -1(-1)$ C = 3
Say x = 2
$2x^4 + 3x^3 + 7x^2 + 10x + 10 =$
$A(x^2 + 3)^2 + (Bx + C)(x - 1)(x^2 + 3) + (Dx + E)(x - 1)$
$114 = A(2^2 + 3)^2 + [B(2) + C](2-1)(2^2 + 3) + (D(2) + E)(2-1)$
Substitute A = 2 , C = 3, D = -2 and E = -1 ► B = 0

2. B = 0
3. C = 3
4. D = - 2
5. E = - 1
6. Factor $x^2 - 7x + 12$ ENTER: MODE 5 3
   roots are 4 and 3
   $x^2 - 7x + 12 = (x - 4)(x - 3)$
   Then: $\frac{x+2}{x^2-7x+12} = \frac{A}{x-4} + \frac{B}{x-3}$
   or $x + 2 = A(x - 3) + B(x - 4)$
   if x = 4 $6 = A(4 - 3)$ A = 6
   If x = 3 $5 = B(3 - 4)$ B = -1
7. A = -8/39
8. B = -92/13
9. C = 199/104
10. D = 393/104

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11. E = 7/24

Solution 7 – 11

$$Ax + B = \frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 2x + 5)(x + 3)} \quad \text{when } x = 3i$$

$$\frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 2x + 5)(x + 3)} \text{ CALC } \quad 3i = \quad -92/13 - 8/13i$$

Then:

A(3i) + B = -92/13 – 8/13i

Thus: B = -92/13 and A = -8/13/ 3 = -8/39

$$Cx + D = \frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 9)(x + 3)} \quad \text{when } x = -1 + 2i$$

(- 1 + 2i is root of $x^2 + 2x + 5 = 0$)

$$\frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 9)(x + 3)} \text{ CALC } -1+ 2i = 97/52 + 199/52i$$

Then: C(-1+2i) + D = 97/52 + 199/52i

-C + D + i(2C) = 97/52 + 199/52i

-C + D = 97/52 and 2C + 0D = 199/52

USE (MODE 5 1)

C = 199/104 and D = 393/104

$$E = \frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 2x + 5)(x^2 + 9)} \quad \text{when } x = -3$$

$$\frac{2x^4 + 2x^3 - 5x^2 + 10x + 9}{(x^2 + 2x + 5)(x^2 + 9)} \text{ CALC } -3 = 7/24$$

E = 7/24

12 A = 1

13. B = -1

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14. C = 1
15. D = 1
16. E = 1

$A = \dfrac{x^3 + 4x^2 + 5x + 3}{(x^2 + x + 1)^2}$ when x = -1 A = 1

(Dx + E = $\dfrac{x^3 + 4x^2 + 5x + 3}{x + 1}$) when x = -1/2 + √3/2 i

D(- 1/2 + √3/2i) + E = 1/2 - √3/2 i

-1/2D + E = 1/2 and -D = - 1 ► D = 1

E = 1/2 + 1/2D = 1/2 + 1/2 = 1

We know already A , D and E

A = 1, D = 1 and E = 1

$$\frac{Bx + C}{x^2 + x + 1} = \frac{x^3 + 4x^2 + 5x + 3}{(x + 1)(x^2 + x + 1)^2} - \frac{A}{x + 1} - \frac{Dx + E}{(x^2 + x + 1)^2}$$

Substitute x = 0. .

$$C = \frac{3}{1} - \frac{1}{1} - \frac{1(0) + 1}{1} \qquad C = 1$$

Substitute x = 2

$$\frac{B(2) + C}{2^2 + 2 + 1} = \left(\frac{x^3 + 4x^2 + 5x + 3}{(x + 1)(x^2 + x + 1)^2} - \frac{A}{x + 1} - \frac{Dx + E}{(x^2 + x + 1)^2}\right) \text{ when } x=2$$

Subst. C = 1, A = 1 , D = 1 and E = 1 and x = 2

(2B + 1)/7 = 37/147 – 1/3 – 3/49

B = - 1

17. A = 2
18. B = -1/2
19. C = 1

A = $\dfrac{x^2 + 4x + 10}{x^2 + 2x + 5}$ when x = 0 ► A = 2

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[B(2x +2) + C = $\frac{x^2+4x+10}{x}$] when x = -1 + 2i

B(4i) + C = 1 – 2i C = 1 4B = -2 B = -1/2

20.

$$\frac{3x^2-8x+9}{(x-2)^3}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$

C = $3x^2 – 8x + 9$ when x = 2 C = 5

B = d/dx($3x^2 – 8x + 9$) when x = 2

= (6x – 8) when x = 2 is 4

A = d/dx(6x – 8) /2! when x = 2 is 3

$$\frac{3x^2-8x+9}{(x-2)^3}=\frac{3}{x-2}+\frac{4}{(x-2)^2}+\frac{5}{(x-2)^3}$$

PROBLEM SET 5
VARIATION PROBLEMS, RECURSION, INVERSE FUNCTION, INEQUALITIES, INVESTMENT

1. Given that w varies directly as the product of x and y and inversely as the square of z and that w = 4, when x = 2, y = 6 and z = 3. Find the value of w when x = 1, y = 4 and z = 2.

PAST CE BOARD

a. 3 b. 4

c. 5 d. 6

2. If x varies directly as y and inversely as z, and when x = 14, y = 7 and z = 2, find the value of x when y = 16 and z = 4 (PAST ECE BOARD)

a. 14 b. 4

c. 16 d. 8

3. The resistance of a wire varies directly with its length and inversely with its area. If a certain piece of wire 10 m long and 0.1 cm in diameter has a resistance of 100 ohms, what will be the resistance

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if it is uniformly stretched so that its length becomes 12 m. PAST ECE BOARD

a. 80 b. 90
c. 144 d. 120

4. The time required for an elevator to lift a weight varies directly with the weight and the distance trough which it is lifted and inversely as the power of the motor. It takes 30 sec for a 10 HP motor to lift 100 lbs trough 50 ft. What size of the motor is required to lift 800 lbs in 40 sec trough 40 ft? PAST ECE BOARD

a. 42 b. 44
c. 46 d. 48

5. If $3x^3 - 4x^2y + 5xy^2 + 6y^3$ is divided by $x^2 - 2xy + 3y^2$
The remainder is

a. 0 b. 1
c. 2 d. 3

6. Solve for x in the system PAST ME BOARD

$y - 3x + 4 = 0$
$y + x^2/y = 24/y$

a. $(-6- 2\sqrt{14})/5$ b. $(6 + 2\sqrt{14})/5$
c. $(2- 6\sqrt{14})/5$ d. $(-2- 6 \sqrt{14})/5$

7. If $-9 < x < -4$ and $-12 < y < -6$ PAST ME BOARD

a. $108 < xy < 24$ b. $108 < xy < 120$
c. $108 > xy > 24$ d. $-21 < x + y < -10$

8. Which of the following is the value of $x^2 + 1/x^2$ if $x + 1/x = 5$? PAST ME BOARD

a. 28 b. 23
c. 24 d. 25

9. If $f(x) = 3x - 1$ then $f^{-1}(x) =$ (PAST ME BOARD)

a. $x - 1/3$ b. $1/3x + 1/3$
c. $3x + 1$ d. $1/x - 1$

10. If $f(x) = x - 2x^2 + 2$ then what is $f(a - 2)$? ME April 2004

a. $9a - 2a^2 - 8$ b. $2a - 9a^2 - 8$
c. $9a - 2a^2 + 8$ d. $2a + 9a^2 - 8$

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11. If $f(x) = x^3 - x - 1$, what is the set of all c if $f(c) = f(-c)$?
ME April 2004

a. all real numbers b. {0}
c. { -1, 0, 1} d. { 0, 1}

12. If $f(x) = 2x^2 + 2$, find the value of $f(x + 4)$ PAST ME BOARD

a. $2x^2 + 16x + 24$ b. $2x^2 + 6x + 4$
c. $2x^2 + 16x$ d. $2x^2 + 16x + 34$

13. If 3, 5, 8.333 and 13.889 are the first 4 terms of the sequence, then which of the following could define the sequence? PAST ME BOARD

a. $A_0 = 3$, $A_n = A_{n-1} + 40/9$
b. $A_0 = 3$, $A_{n+1} = A_{n+2}$
c. $A_0 = 3$, $A_n = 5/3A_{n-1}$
d. $A_0 = 3$, $A_{n+1} = 2\,A_{n+2}$

14. Find the smallest positive integer such that for
$P(x) = x^4 - 2x^3 - 10x^2 + 40x - 90$ this integer is an upper bound.

a. 3 b. 4
c. 5 d. 6

15. Find the largest negative integer such that for
$P(x) = x^4 - 2x^3 - 10x^2 + 40x - 90$ this integer is a lower bound.

a. -2 b. -3
c. – 4 d. - 5

16. Solve $x^2 + x < 12$

a. $-4 < x < 3$ b. $-3 < x < 4$
c. $x < -4$ or $x > 3$ d. $x < -3$ or $x > 4$

17. For what values of x does $f(x) = \sqrt{\frac{x-1}{x+3}}$ represent a real number?

a. $-3 < x <= 1$ b. $x < -3$ or $x >= 1$
c. $x < -1$ or $x >= 3$ d. $-1 < x <= 3$

18. Determine the number of positive roots of the equation. past CE BOARD

$P(x) = x^{2n} - 1$ where $n > 0$

a. 0 b. 1

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c. 2 d. n

19. Solve the inequality $x^3 \leq 4x^2 + 3x$
    a. $0 \leq x \leq \sqrt{7} + 2$ or $x \leq -\sqrt{7} + 2$
    b. $0 \leq x \leq \sqrt{7} - 2$ or $x \leq \sqrt{7} + 2$
    c. $0 \leq -\sqrt{7}-2$ or $x \leq \sqrt{7} - 2$
    d. $-2 - \sqrt{7} \leq x < 2 + \sqrt{7}$

20. Solve the inequality $\frac{3}{x-3} \leq \frac{2}{x+2}$
    a. $-2 < x < 3$ or $x \leq -12$ b. $-2 < x < -12$
    c. $-3 < x < 2$ or $x \leq -12$ d. $3 < x < 12$ or $x \leq -3$

21. Solve the inequality $|3x + 1| \leq 4$
    PAST CE BOARD
    a. $(-\infty, -5/3) \cup (1, \infty)$ b. $[-5/3, 1]$
    c. $(-5/3, 1)$ d. $(-4/3, 1]$

22. Find the solution set of $|2x + 3| > 7$
    PAST CE BOARD
    a. $x < -5$, $x > 2$ b. $x > -5$ $x < 2$
    c. $-5 < x < 2$ d. $-2 < x < 5$

23. Find the domain of $f(x) = \sqrt{9 - x^2}$
    a. $(-3, 3)$ b. $(-\infty, -3) \cup (3, \infty)$
    c. $[-3, 3]$ d. $(-3, 3]$

24. Solve the inequality $|x - 2| < |3x - 5|$
    a. $x < 5/2$ or $x > 7/4$
    b. $x < 3/2$ or $x > 7/4$
    c. $3/2 < x < 7/4$
    d. x is an empty set

25. The graph of $x + y > 1$ is
    a. the area above the line $x + y = 1$
    b. the area below the line $x + y = 1$.
    c. the area above the line $y = x$
    d. the area below the line $y = x$

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26. The vertical asymptotes of the curve $y = \frac{(x+1)}{(x-3)(x+2)}$ are:

a. x = -3 , x = - 2 b. x = 3 , x = - 2
c. x = -1 d. x = -1 , x = 3

27. The hozizontal asymptote of the graph of $y = \frac{(x-1)(x+2)}{(2x+1)(x-3)}$

a. No horizontal asymptote b. y = ½
c. y = 2 d. y = 0

28. The oblique asymptote of the graph $y = \frac{(x-2)(x+3)}{(x+6)}$

a. y = x – 5 b. None
c. y = 5 – x c. y = x + 2

29. If f(3) = 6 and f(8) = 12 and f(x) is a linear function, Find f^{-1} (5).

a. 13/6 b. 12/5
c. 1/3 d. 4/5

30. Find the 1987 th digit in the decimal equivalent of 1785/9999

a. 1 b. 2
c. 8 d. 5

31. The selling price of a TV set is double that of the net cost. If a TV set is sold to a customer at a profit of 25% of the net cost, how much discount was given to the customer? PAST ECE BOARD

a. 33.3% b. 37.5%
c. 22.33 % d. 42.5%

32. A bookstore purchased a book at P 200 per copy. At what price should this book be sold so that giving a 20% discount, the profit is 30%.

a. 357.14 b. 334.56
c. 322.34 d. 356.67

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33. By selling eggs at P 50 per dozen, a vendor gains 20%. The cost price of the egg rises 12.5%. If he sells at the same price as before, find his new gain in percent. PAST ECE BOARD

a. 5.67% b. 6.67%
c. 7.76% d. 8.33%

34. Peter bought a second hand cellphone and then sold it to John at a profit of 40%. John sold the cellphone to Noel at a profit of 20%. If Noel paid P 2,856 more than it cost to Peter, how much did Peter paid for the unit? PAST ECE BOARD

a. 4,200 b. 4,300
c. 4,400 d. 4,500

35. A portion of P 500,000 was invested at 5% in the bank and the remainder at 15% in a survey contract. If the total income from the money is P 55,000, how much was invested at 5% rate. PAST CE BOARD

a. 350,000 b. 400,000
c. 200,000 d. 150,000

36. If f(n+1) = n/f(n) for all positive integers n and f(1) =2 , find f(8).

a. 12/7 b. 13/15
c. 35/32 d. 41/32

37. If today is Monday, what will be the day 2^{234577} days after today.

a. Mon b. Tues
c. Wed d. Thu

Solution:

1. $w = k\frac{xy}{z^2}$ $4 = k\frac{2(6)}{3^2}$ k = 3

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then: $w = 3\frac{1(4)}{2^2}$ w = 3

2. x =k y/z 14 = k(7)/2 k = 4
   x = 4y/z , when y = 16, z = 4 then x = 16

3. R = K L/A = KL/(πD²)
   R = K'L/D²
   100 = K'(10)/0.1² K' = 0.1
   R = 0.1L/D² R = 0.1(12)/0.1² = 120

4. $T = k\frac{WD}{P}$ $30 = k\frac{100(50)}{10}$ k = 3/50 $T = \frac{3}{50}\frac{WD}{P}$

   $40 = \frac{3}{50}\frac{800(40)}{P}$ P = 48

5.
$$\begin{array}{r} 3x + 2y \\ x^2 - 2xy + 3y^2 \;\big|\; 3x^3 - 4x^2y + 5xy^2 + 6y^3 \\ \underline{3x^3 - 6x^2y + 9xy^2} \\ 2x^2y - 4xy^2 + 6y^3 \\ \underline{2x^2y - 4xy^2 + 6y^3} \\ 0 \end{array}$$

   Remainder = 0

6. Y = 3x – 4 Substitute in the next equation.
   (3x -4) + x²/(3x -4) = 24/(3x-4)
   Rewrite as: (3x -4) + x²/(3x -4) - 24/(3x-4)
   Input in the calculator and substitute the given one by one using the CALC Menu.
   Ans. (6 + 2 √14)/ 5

7. Add:
   - 9 < x< - 4
   -12 < y < - 6
   -21 < x + y < -10

8. Input x + 1/x = 5 in the Calculator.
   ENTER SHIFT CALC
   Answer: x = 4.791287847
   ENTER: x² + 1/x²

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Ans. 23

9. $y = 3x - 1$

solve for x: $3x = y + 1 \blacktriangleright x = 1/3y + 1/3$

Replace y by $f^{-1}(x)$ and x by y.

Ans. $f^{-1}(x) = 1/3x + 1/3$

10. $f(x) = x - 2x^2 + 2$ $f(a-2) = (a-2) - 2(a-2)^2 + 2$

$f(a-2) = a - 2 - 2(a^2 - 4a + 4) + 2 = -2a^2 + 9a - 8$

11. $f(c) = c^3 - c - 1$ $f(-c) = (-c)^3 - (-c) - 1$

$= -c^3 + c - 1$

Then: $c^3 - c - 1 = -c^3 + c - 1$

$2c^3 - 2c = 0$ $2c(c^2 - 1) = 0$

$2c(c + 1)(c - 1) = 0$ $c = 0, 1, -1$

12. $f(x) = 2x^2 + 2$

$f(x + 4) = 2(x + 4)^2 + 2 = 2(x^2 + 8x + 16) + 2$

$= 2x^2 + 16x + 34$

13: Note: For c: $A_1 = 5/3A_0$, $n = 1$ $A_1 = 5/3(3) = 5$

$A_2 = 5/3\ A_1$, $n = 2$ $A_2 = 5/3(5) = 25/3$

$A_3 = 5/3A_2$, $n = 3$ $A_3 = 5/3(25/3) = 125/3$

Ans. c

14. For upper bound, when you perform synthetic division , the last row must be all positive or zero.

For 3

1	- 2	-10	40	-90	| 3
	3	3	-21	57	
1	1	-7	19	-33	

(some are negative, not an upper bound)

For 5

1	- 2	-10	40	-90	| 5
	5	15	25	325	
1	3	5	65	235	

(Upper bound = 5)

15. For lower bound, the last row in the synthetic division must alternate in sign.

For – 2:

1	-2	-10	40	- 90	| - 2

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-2 8 4 -88

1 -4 -2 44 -178 (-4 and -2 does not alternate)

For – 5

1 -2 -10 40 - 90 |-5

-5 35 125 -825

1 -7 -25 165 -915 (The signs alternate, -5 is a LOWER bound)

16. Solve $x^2 + x - 12 = 0$ USING MODE 5 3

$x = 3$ and $x = -4$

-4 3

Trial Values -5 0 4

Evaluate $f(x) = x^2 + x - 12$ using trial values.

$f(-5) = 5$ (> 0)

$f(0) = -12$ (< 0)

$f(4) = 8$ (> 0)

Choose the boundary for < 0 .

Ans. $-4 < x < 3$

17. f(x) must be real if $\frac{x-1}{x+3} \geq 0$

The critical points are

$x = 1$ and $x = -3$

- 3 1

Trial Values **-4** **0** **2**

Let $f(x) = \frac{x-1}{x+3}$

f(-4) = 5 (+) f(0) = -1/3 (-) f(2) = 1/5 (+)

Choose on the boundaries for (+)

$x < -3$ or $x \geq 1$ (Note 1 is included but -3 is not)

18. Factor $x^{2n} - 1 = 0$ ► $(x^n - 1)(x^n + 1) = 0$

$x^n = 1$ ► $x = 1$

and $x^n = -1$ ► x cannot be positive.

Ans. $x = 1$, one only.

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19. Solve for $x^3 - 4x^2 - 3x = 0$ using
MODE 5 4
Input [1 - 4 - 3 0] ==
$x = 2 + \sqrt{7}$ $x = 2 - \sqrt{7}$ $x = 0$
Test the boundaries.

$2 - \sqrt{7}$ 0 $2 + \sqrt{7}$

Test values **-1** **-0.1** **2** **5**

Let $f(x) = x^3 - 4x^2 - 3x$
ENTER: $x^3 - 4x^2 - 3x$ in the calculator and use CALC in evaluating
the given functions.
$f(-1) = -2$ $f(-0.1) = 0.259$ $f(2) = -14$ $f(5) = 10$
We need only the negative part since given is $\leq$
Solution: $x \leq 2 - \sqrt{7}$ or $0 \leq x \leq 2 + \sqrt{7}$

20. Rewrite as: $\frac{3(x+2)-2(x-3)}{(x-3)(x+2)} \leq 0$

or $\frac{x+12}{(x-3)(x+2)} \leq 0$

The critical points are: -12, -2, 3
Test the boundaries.

-12 -2 3

Trial Values -14 -5 0 4

$f(x) = \frac{x+12}{(x-3)(x+2)}$

$f(-12) = -1/102$ $f(-5) = -7/24$ $f(0) = -2$ $f(4) = 8/3$
Choose the negative boundaries.
Solution: $x \leq -12$ (f(x) is defined when x = -12)
or $-2 < x < 3$ (f(x) not defined at x = -2 and 3)

21. $|3x + 1| \leq 4$ means
$-4 \leq 3x + 1 \leq 4$ Add -1 to both sides
$-5 \leq 3x \leq 3$ Divide 3 to both sides
$-5/3 \leq x \leq 1$ or [5/3, 1]

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Another Solution:

Find the critical points.

$3x + 1 = 4 \quad x = 1$

$-(3x + 1) = 4 \quad x = -5/3$

Rewrite as $f(x) = |3x + 1| - 4$

Boundary $\quad -5/3 \quad 1$

Test Points $\quad \mathbf{-2} \quad \mathbf{0} \quad \mathbf{2}$

$f(-2) = 1$ (+) $\quad f(0) = -3$ (-) $\quad f(2) = 3$ (+)

We need only the negative part since given is $\leq$

Answer: $-5/3 <= x <= 1$

22.

$|2x + 3| > 7$ means

$2x + 3 < -7$ and $2x + 3 > 7$

$2x < -10 \qquad 2x > 4$

$x < -5 \qquad x > 2$

Solution is $x < -5$ U $x > 2$

23. $9 - x^2 \geq 0$

$(3 - x)(3 + x) \geq 0$

Critical Points are -3 and 3

$-3 \qquad 3$

Test Points $\quad \mathbf{-4} \quad \mathbf{0} \quad \mathbf{4}$

Let $f(x) = 9 - x^2$

$f(-4) = -7$

$f(0) = 9$

$f(4) = -7$

The solution is between -3 and 3

$-3 \leq x \leq 3$

24. $|x - 2| < |3x - 5|$

Find the critical points

Both positive (or both negative)

: $x - 2 = 3x - 5 \quad -2x = -3 \quad x = 3/2$

One is positive and the other negative.

$x - 2 = -(3x - 5) \quad 4x = 7 \quad x = 7/4$

The critical points are 3/2 and 7/4

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3/2 = 1.5 7/4 = 1.75

Test points **1** **1.7** **2**

Let f(x) = | x – 2 | - | 3x – 5 | < 0

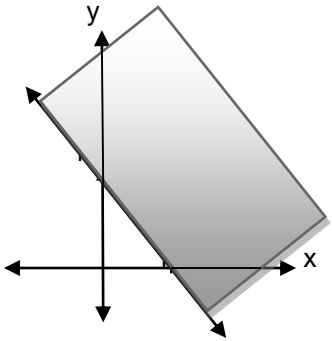
f(1) = -1 f(1.7) = 1/5 f(2) = -1

Since f(x) < 0

Solution is x < 3/2 or x > 7/4

25. Above the line x + y = 1

.



26. Set denominator to zero. x = 3, x = - 2
27. Get the lim of y as x approaches infinity.
    Disregard the constants because x is infinity.
    y = x(x)/(2x·x) , y = ½
28. Divide (x -2)(x+ 3) to x + 6
    (x^2 + x - 6) / (x + 6)
    Result: x – 5 + 24/(x + 6)
    Oblique Asymptote: y = x - 5
29. Let y = f(x) = A + BX
    6 = A + 3B
    12 = A + 8B
    USE MODE 5 1
    A = 12/5 B = 6/5
    Then y = 12/5 + 6/5x
    If y = 5 , then x = 13/6
30. 1785/9999 = 0.178517851785…
    1987/ 4 SHIFT S↔D = 496 ¾
    1 corresponds to 1

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7 corresponds to 2
8 corresponds to 3
5 corresponds to 0 (no remainder)
Ans. 8

31. Let x = net cost
Selling Price = 2x
Profit = .25x
Discount = selling price – actual amount sold
= 2x – 1.25x = 0.75x
Discount Rate = 0.75x/2x = 37.5%

32. Profit = Income – Expenses
Let x = actual selling price
0.8x = discounted selling price
0.3(0.8x) = 0.8x – 200
x = 357.143

33. 50 = x + 1.2x
x = 41.667 Price of eggs/ dozen
Cost of Egg after rise of 12.5% = 41.667 x 1.125 = 46.8753
New gain in % = (50 – 46.8753)/46.8753 = 6.67%

34. Let x = amount Peter Paid for the unit
1.4 x = price paid by John
1.4x (1.2) = price paid by Noel
1.4x(1.2) = 2856 + x
x = 4200

35. Let x = amount invested at 5%
y = amount invested at 15%
x + y = 500000
.05x + .15y = 55,000
MODE 5 1
x = 200,000 y = 300,000

36. n= 1 f(2) = 1/f(1) = ½
n = 2 f(3) = 2/f(2) = 2/(1/2) = 4

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n = 3 f(4) = 3/f(3) = 3/4
n = 4 f(5) = 4/f(4) = 4/(3/4) = 16/3
n = 5 f(6) = 5/f(5) = 5/(16/3) = 15/16
n = 6 f(7) = 6/f(6) = 6/(15/16) = 96/15 = 32/5
n = 7 f(8) = 7/f(7) = 7/(32/5) = 35/32

USING CALCULATOR TECHNIQUE:
ENTER:
C = X/Y: Y = C : D = X + 1: X = D
Explanation:
C is n/f(n) where n = X and f(n) = Y
D = X + 1 , x must be incremented by 1
x = D, the new value of X is D

37. $2^1/7$ = 2/7 remainder = 2
$2^2/7$ = 4/7 remainder = 4
$2^3/7$ = 1 1/7 remainder = 1
$2^4/7$ = 2 2/7 remainder = 2
$2^5/7$ = 4 4/7 remainder = 4
$2^6/7$ = 9 1/7 remainder = 1
Note that the remainder is a sequence of 2 4 1 2 4 1 …
Divide 234577 by 3 = 78912 1/3
Thus 2^{234577} /7 will have a remainder of 2.
2 days after Monday is W

PROBLEM SET 6 PART-1

TRIGONOMETRY

1. Solve for θ in $\cos(6\theta) = \frac{1}{Csc(3\theta + 9)}$, θ in degrees.

PAST CE Board

a. 6 b. 7
c. 8 d. 9

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2. If sin A = 3/5 , θ in the second quadrant and cos B = -1/2, θ is in the 3rd quadrant, what is the value of sin(A + B)?

a. 0.3928 b. 0.4122
c. 0.3411 d. 0.3445

3. If $\cos 65^0 + \cos 55^0 = \cos \theta$, what is θ in radians? PAST CE BOARD

a. 0.0823 b. 0.087
c. 0.922 d. 0.783

4. Find the value of arc tan [2 cos (arcsin (√3/2)]. PAST ECE BOARD

a. π/3 b. π/6
c. π/4 d. π/2

5. Solve for x : $x = (\tan \theta + \cot \theta)^2 \sin^2 \theta - \tan^2 \theta$. PAST CE BOARD

a. sin θ b. 1
c. cos θ d. tan θ

SITUATION (Problems 6, 7, 8)
If A = 3 sin x + 4 cos x and B = 3 cos x – 4 sin x

6. The value of $A^2 + B^2$ is

a. 5 b. 10
c. 0 d. 25

7. Find the value of A if B = 4

a. 3 b. 1
c. -1 d. – 3

8. Find the value of x if B = 4

a. -16.45^0 b. 23.34^0
c. 31.23^0 d. -44.11^0

9. Find the value of θ in coversine θ = 0.134

a. 50^0 b. 60^0
c. 70^0 d. 80^0

10. Versed Sin θ = 0.456 θ = ?

a. 57^0 b. 61^0
c. 63^0 d. 75^0

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11. Solve for θ in $\cosh^2 x - \sin h^2 x = \tan \theta$
    a. π/4 b. π/2
    c. π/3 d. π
12. Find the value of $\sin (90^0 + A)$
    a. cos A b. sin A
    c. – cos A d. – sin A
13. 543 degrees is how many gradians?
    a. 8712/3 b. 1810/3
    c. 7651/3 d. 3312/3
14. PAST CE BOARD : If $\sin 3A = \cos 6B$ then
    a. $A + B = 180^0$ b. $A - 2B = 30^0$
    c. $A + 2B = 30^0$ d. $A + B = 30^0$
15. Find the value of x in
    atan (1 – x) + arctan (1 + x) = arctan (1/8)
    a. 4 b. 6
    c. 6 d. 2
16. If $\sin A = 4/5$, A in quadrant 2 , $\sin B = 7/25$ B in quadrant 1. Find cos (A – B)
    a. 123/25 b. -44/125
    c. 112/25 d. – 23/125

17. sin (arccos 15/17) =
    a. 14/17 b. 18/17
    c. 19/17 d. 20/17
18. What is the first period of $y = 4 \tan 4x$?
    a. π b. π/2
    c. π/4 d. π/8
19. What is the amplitude of $3 \cos x + 5 \sin x$
    a. 5.83 b. 8
    c. -2 d. 15
20. What is the first period of $7 \cos 3x$?
    a. π/3 b. π/6
    c. 2π/3 d. 3π

SOLUTION :

1. $\cos 6\theta = \sin (3\theta + 9) = \cos (90^0 - (3\theta + 9))$

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$6\theta = 90 - 3\theta - 9 \quad \theta = 9$

2. Note Sin A is + in the 1st and sec quadrant
cos B is - in the 2nd and 3rd quadrant.
$A = 180 - \arcsin(3/5) = 143.13^0$
$B = 180 + \arccos(1/2) = 240^0$
$\sin(A + B) = \sin(143.13 + 240) = 0.3928$

3. ENTER: SHIFT MODE 4 (RADIAN MODE)
$\theta = \cos^{-1}(\cos 65^0 + \cos 55^0)$
Note: To input ^{0} ENTER SHIFT ANS 1
Ans. 0.0872664626

4. $\arcsin(\sqrt{3}/2) = \pi/3 \quad \cos \pi/3 = 1/2$
$2 \cos \pi/3 = 2(1/2) = 1 \quad \text{atan } 1 = \pi/4$

5. Let $\theta = 30^0$ (could be any value)
substitute in $(\tan\theta + \cot\theta)^2 \sin^2\theta - \tan^2\theta$
ANS. $x = 1$

6. Assume x be any value say $x = 30^0$
Then $A = 3 \sin 30 + 4 \cos 30 = 4.964$
$B = 3 \cos 30 - 4 \sin 30 = 0.59808$
$A^2 + B^2 = 25$

7. Since $A^2 + B^2 = 25$ and $B = 4$,
then $A = 3$

8. $3 \sin x + 4 \cos x = 3$
$-4 \sin x + 3 \cos x = 4$
USE MODE 5 1
$\sin x = -7/25$ and $\cos x = 24/25$

$x = \text{asin}(-7/25) = -16.26^0$ or 196.26^0
$x = \arccos 24/25 = 16.26^0$ or -16.26^0
Ans. -16.26^0

9. coversed sine $\theta = 1 - \sin\theta = 0.134 \quad \theta = 60^0$

10. Versed Sin $\theta = 1 - \cos\theta = 0.456 \quad \theta = 57^0$

11. Note: $\cosh^2 x - \sinh^2 x = 1$
$\tan\theta = 1 \quad \theta = \pi/4$

12. Assume $A = 10^0 \quad \sin(90 + 10) = 0.9848$
$\cos A = \cos 10 = 0.9848$

13. $400 \text{ grad} = 360^0 \quad 543^0 \times 400 \text{ grad} / 360^0 = 1810/3$

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CAL TECHNIQUE: ENTER: SHIFT MODE 5 543 SHIFT ANS 1 =

14. sin 3A = cos 6B
cos (90 – 3A)= cos 6B ► 90 – 3A = 6B or
3A + 6B = 90 ► A + 2B = 30

15. Rewrite as $\tan^{-1}$ (1 – x) + $\tan^{-1}$ (1 + x) : $\tan^{-1}$(1/8)
and ENTER CALC X? 4
Ans. 4

16. 180 - $\sin^{-1}$ 4/5 store to A. (2nd quadrant)
$\sin^{-1}$ (7/25) store to B (1st quadrant)
ENTER: cos (A –B) = -44/125 Ans.

17. sin ($\cos^{-1}$ (15/17)) = 8/17

18. Period of tan x is π . period of tan 4x = π/4

19. amplitude = $\sqrt{(3^2 + 5^2)}$ = 5.83

20. period = 2π/ 3

PROBLEM SET 6 – PART 2
TRIGONOMETRY

21. Find the value of x in the equation :
(x + yi) (1 – 2i) = 7 – 4i PAST CE BOARD
a. 1 b. 2
c. 3 d. 4

22. $\sqrt[3]{-47-52i}$ is
a. 1+ 2i b. 1 – 3i
c. 2 – 2i d. 1 + 4i

23. The logarithm of the quotient of 2 numbers is 0.255272505 and the logarithm of product of these two numbers is 1.653212514, what is the value of one of the numbers? CE Board Nov 2007
a. 7 b. 8
c. 9 d. 10

24. Find the value of $\log_x 11$ if $\log_x 6 = 1.2925$ PAST CE BOARD
a. 2.13. b. 1.73
c. 3.11 d. 3.22

24-b Find the value of x in terms of a and b if
$2^{2x-y} = 10$

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$5^{x+y} = 100$ where $a = \log 2$, $b = \log 5$

a. $1/2a + 2/3\ b$ b. $1/3a + 2/3b$

c. $2/3\ a + 5/6b$ d. $3/4a + 4/7b$

25. The rectangular coordinates of a point having the polar coordinate
(7, 38^0) is? CE Board Nov 2000

a. (5.52, 4.23) b. (9.8, 4.31)

c. (4.56, 3.45) d. (5.52, 4.31)

26. The polar coordinate of (3, -6) is

a. (6.71, -51.23^0) b. (6.71, -63.43^0)

c. (7.23, 51.23^0) d. (7.23, -51.23^0)

27. Find the value of x if

$x\angle 30^0 + y\angle 45^0 = 9.91788\angle 39.0083^0$

a. 3 b. 4

c. 5 d. 6

28. Mass A moves at 20 kph in the direction N 40^0 E while Mass B moves at 30 kph in the direction of S 30^0 W. What is the relative velocity of B with respect to A ?

a. 49.12 kph b. 48.12 kph

c. 49.82 kph d. 34.23 kph

SITUATION: (Problems 29 – 30)

Car A is moving east at 70 kph while car B is moving in the direction N 45^0 E. For a passenger in Car A, B appears to be moving away in the direction of N 30^0 W. CE Board May 2006

29. What is the velocity that B appears to the passenger in A?

a. 48.23 kph b. 51.24 kph

c. 34.23 kph d. 76.12 kph

30. What is the true velocity of B?

a. 62.76 kph b. 89.12 kph

d. 71.23 kph d. 89.23 kph

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31. CE Board May 2004
The vertices of a triangle have their polar coordinates at $(0,0^0)$, $(6,30^0)$ and $(9, 70^0)$. The perimeter of a triangle is

a. 34.45 b. 20.85
c. 25.67 d. 21.34

32. If a circle of radius 4 cm has a chord of length 3 cm, find the central angle that is opposite the chord.

a. 42^0 b. 43^0
c. 44^0 d. 45^0

33. If sin A = - 4/5 , A is in quadrant 3 , cos B is - 1/5 and B is in quadrant 2 , What is the value of cot (A – B) ?

a. -1.51 b. -1.56
c. -0.991 d. -0.8876

34. $\dfrac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$ is

a. tan 2x b cot 4x
c. tan 4x d. $\cot^2 2x$

35. 2200 mils is how many gradians?

a. 137.5 grad b. 127.5 grad
c. 221.5 grad d. 175.75 grad

36. A swimming pool is shaped from 2 intersecting circles 9 m in radius with their centers 9 m apart. What is the area common to the two circles?

a. 99.1 sq m b. 99.3 sq m
c. 99.5 sq m d. 99.7 sq m

37. A 20 m high mast is placed on the top of the cliff whose height above sea level is unknown. An observer at sea sees the top the mast at an elevation of $46^042'$, the foot at $38^023'$. The height of the cliff is closest to: PAST CE BOARD EXAM

a. 57 m b. 54 m
c. 51 m d. 59 m

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38. A hot air balloon is observed from point A at 22.23^0 angle of elevation and simultaneously from a point B 1500 from A at 48.11^0 elevation. Find the height of the balloon.

a. 968 m b. 976 m
c. 981 m d. 995 m

39. In a right spherical triangle , a = $32^0 16'$ and c = $119^0 23'$ the value of b is:

a. $115^0 15'$ b. $125^0 28'$
c. $131^0 15'$ d. $134^0 12'$

40. In the spherical triangle A = 120^0 B = 135^0 and c = 30^0. What is C ? PAST CE BOARD

a. 79.82^0 b. 67.33^0
b. 46.78^0 d. 98.23^0

SOLUTION: 21- 40

21. x + yi = (7 – 4i)/(1 – 2i) ENTER MODE 2
x + yi = 3 + 2i x = 3 y = 2

22. $(1 + 4i)^3 = - 47 – 52 i$
(Go to MODE 2 and enter $1 + 4i)^2 (1 + 4i)$

23. Let X and Y be the numbers
log X/Y = log X – log Y = -0.255272505
log XY = log X + log Y = 1.653212514
USE MODE 5 1
INPUT 1 - 1 -0.255272505
1 1 1.653212514
log X = 0.700243732 log Y = 0.952968782
$X = 10^{0.700243732} = 5$
$Y = 10^{0.952968782} = 9$

24. Solve for x in $\log_x 6 = 1.2925$ USE SHIFT CALC , x = 4 then enter $\log_4 11$ Ans. 1.73

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24-b

$\log (2^{2x-y}) = \log 10$

$(2x- y)\log 2 = 1$

$2x – y = 1/\log 2 = 1/a$

$\log (5^{x+y}) = \log 100$

$(x + y) \log 5 = 2$

$x + y = 2/\log 5 = 2/b$

$2x – y = 1/a$

$x + y = 2/b$

Add: $3x = 1/a + 2/b$

$x = (1/a + 2/b)/3$

25. ENTER: Rec(7, 38) Ans. X = 5.516 , Y = 4.31

26. ENTER: Pol(3, -6) Ans. r = 6.71 θ = -63.43

27. $x\angle 30^0 + y\angle 45^0 = 9.91788\angle 39.0083^0$

USE MODE 5 1

Input:

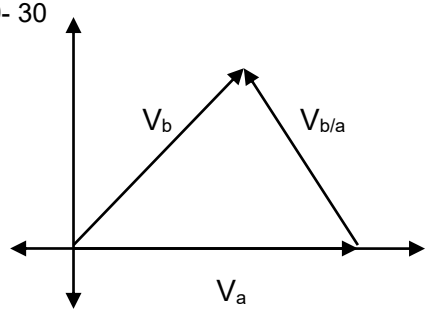
cos 30 cos 45 9.91788cos 39.0083

sin 30 sin 45 9.91788 sin 39.0083

Ans. X = 4 and Y = 6

28. $V_{B/A} = V_B – V_A = 30\angle 240 - 20\angle 50 = 49.82\angle -124^0$

29- 30



Let $V_{B/A}$ = velocity of B as it appears from A

V_B = true velocity of B

Then: $V_{B/A} = V_B - V_A$

$V_{B/A}\angle 120 = V_B\angle 45 - 70\angle 0$

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or $V_B \angle 45 - V_{B/A} \angle 120 = 70 \angle 0$

ENTER: MODE 5 1

Input: cos 45 - cos 120 70cos 0

sin 45 - sin 120 70sin 0

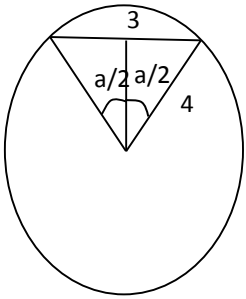
V_B = 62.76 $V_{B/A}$ = 51.24 MPH

31. MODE 2 (NOTE || is shift hyp

ENTER: | 6∟30 | + | 6∟30 - 9∟ 70 | + | 9∟70|

Ans. 20.85

32.



sin a/2 = 1.5/4 a= 44.05^0

33. A = 180 + $\sin^{-1}$ 4/5 = 233.13^0 STORE TO A.

B = 180 – $\cos^{-1}$ 1/5 = 101.53^0 STORE TO B.

cot(A – B) = -0.8876

34. The simplest way is to substitute a value say x = 5^0 in

f(x) = $\frac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$, then f(5) = 0.363937 (Enter f(x) in the calculator and ENTER CALC 5 =) .The choice that will fit in is only tan 4x = tan (4 x 5) = 0.3639 Ans. tan 4x

35. 2200 mils x 360^0/ 6400 mils = 123.75^0

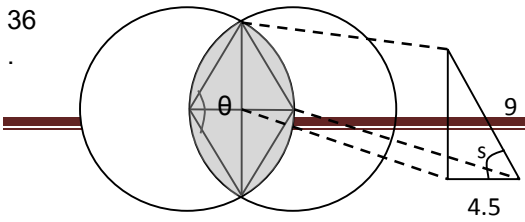
GO TO GRAD MODE.

ENTER: 123.75^0 = (Note: ^{0} is SHIFT ANS 1)

Ans. 137.5 grad

36

.



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cos s = 4.5/9; s = 60; total angle, θ= 120^0

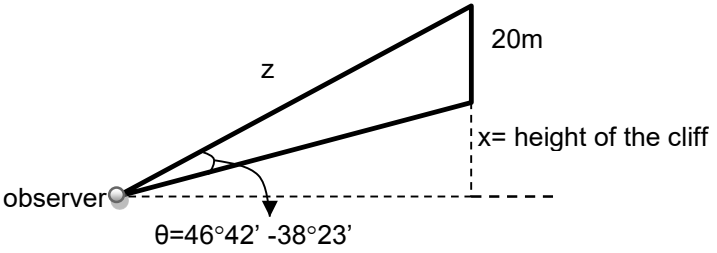
A of one segment = 1/2 R^2 (θ – sin θ)

= 1/2 9^2 (120^0 - sin 120^0) (GO to RAD Mode)

= 49.75

2A = 99.5 sq m.

37.



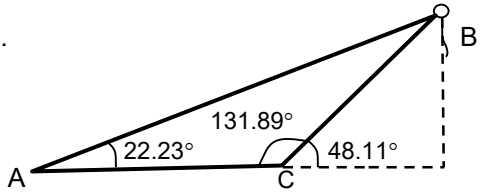
θ = $46^0$42' – $38^0$23' = 8.317^0

Z/ sin (90^0 + $38^0$23') = 20 / sin 8.317^0

Z = 108.38

20 + X = Z sin 46^0 41' X = 58.85 m

38.



USE SIN LAW TECHNIQUE:

ENTER: MODE 5 1

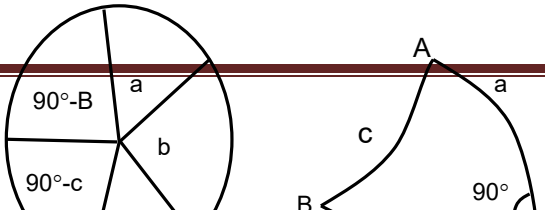
Input:

cos 22.23	cos 131.89	1500
sin 22.23	- sin 131.89	0

AC = 2558.24 and BC = 1300.12 m

height of balloon = 1300.12 sin 48.11 = 967.85 m

39.



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Using Napiers Rule a = $32^0$16' c = $119^0$23' b=?

USE Sin Co Op Rule:

Sin of Any Middle Part = Cos of the Opposite Parts

sin (90 - $119^0$23') = cos a = $32^0$16' cos b

b = 125^0 28'

40. Use Cos Law for Angles

cos C = -cos A cos B + sin A sin B cos c

cos C = -cos 120^0 cos 135^0 + sin 120^0 sin 135^0 cos 30^0

C = 79.82^0

PROBLEM SET 6 PART 3

TRIGONOMETRY

41. A truck travels from point M northward for 30 min then eastward for 1 hr, then shifted N 30^0W . If the constant speed is 40 kph, how far directly from M in km will it be after 2 hrs? PAST CE BOARD

a. 47.88 km b. 34.56 km

c. 41.23 km d. 36.33 km

SITUATION (Problems 42, 43, 44)

If $\sin^{-1}$ (3x – 4y) = π/2 and $\cos^{-1}$ (x – y) = π/3

42. Find the value of x

a. 1 b. 2

c. 3 d. 4

43. Find the value of y

a. 2 b. 1/2

c. 4 d. 1/4

44. Find $\sin^{-1}$ (x – y)

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a. π/6 b. π/8
c. π/3 d. π/96

Situation (Problems 45, 46, 47)
In the triangle ABC, tan A + tan B + tan C = 5.67

45. Find tan A tan B tan C
a. 12.33 b. 6.67
b. 5.67 d. 4.33

46. if tan A = 1.732 , Find tan B tan C = ?
a. 3.274 b. 4.123
c. 6.131 d. 2.451

47. If tan B = 1.915, Find the value of angle C
a. 61.3 b. 69.2
c. 49.1 d. 45.8

48. If the sides of a parallelogram and an included angle are 6, 10 and 100^0 respectively, Find the length of the shorter diagonal. PAST ECE BOARD
a. 10.23 b. 10. 73
c. 11.21 d. 11.34

SITUATION: Problems 49, 50, 51

49. Triangle XYZ has base angles X = 52^0 and Z = 60^0 d, distance XZ = 400 m long. A line AB which is 200 m long is laid out parallel to XZ. What is the sum of the other 2 sides of the triangle?
a. 891.23 b. 713.58
c. 672.34 d. 871.22

50. The area of the triangle XYZ is
a. 58,883 m^2 b. 61,234 m^2
c. 34,123 m^2 d. 91, 234 m^2

51. The area of ABXZ is
a. 42, 134 m^2 b. 44,162 m^2
c. 45,123 m^2 d. 34, 134 m^2

52 What the angle between the hands of the clock at 2:34:12 in radians?
a. 2.236 rad b. 2.122 rad
c. 2.411 rad d. 1.977 rad

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53. Point C is due East of B and 300 m distance apart. A tower not in line in B and C was observed at B and C having vertical angles of 45^0 and 60^0 respectively. The same tower was observed at point D 500 m west of B. The vertical angle of the same tower as observed from D is 30^0.The height of the tower is

a. 567.23 m b. 672.93 m
c. 771.23 m d. 714.51 m

54. The sides of a triangle are 18 cm, 24 cm and 34 cm respectively. Find the length of the median to the 24 cm side in cm. PAST CE BOARD

a. 24.4 cm b. 23.4 cm
c. 21.9 cm d. 20.4 cm

55. For triangle ABC, angle C = 70^0 , angle A = 45^0 and AB = 40 m. Find the length of the median from A to BC.

a. 36.3 b. 35.4
c. 31.2 d. 37.8

56. Three circles are mutually tangent to one another externally. Connecting the 3 centers form a triangle whose sides are 16 cm, 20 cm and 24 cm. What is the area of the smallest circle in cm^2. PAST CE BOARD

a. 100 π b. 144π
c. 36π d. 49π

57. PAST CE BOARD
If tan x = 1/2 and tan y = 1/3 , what is the value of tan (x + y)

a. 1/6 b. 2
c. 1/2 d. 1

58. In triangle DEF, DE = 18 m and EF = 6 m. FD may be PAST CE BOARD)

a. 12 m b. 11m
c. 13 m d. 10 m

59. An equilateral triangular field has sides equal to 2 meter each. If the field is divided into two equal areas by a line parallel to the one side, compute the length of the dividing line.
CE BOARD MAY 2002

a. 1.34 b. 1.41
c. 1.71 d. 1.78

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60. CE BOARD Nov 1995 An earthquake is usually measured by the magnitude M on the Ritcher Scale. The intensity of the earthquake and the magnitude are related by the formula

$M = \log \frac{I}{I_0}$ where I_0 is the intensity of an arbitrary chosen earthquake. The earthquake that hit Kobe Japan measured 5.7 on the Ritcher Scale. The earthquake that hit Baguio Phil measured 7.8. How many times stronger is the earthquake that hit Baguio?

a. 148 times
b. 126 time
c. 137 times
d. 37 times

61. The plane of a small cirle on a sphere of radius 25 cm is 12.5 cm from the center of the sphere. Determine the small circle radius and its polar distance.

a. 24.54, 30^0
b. 21.65, 30^0
c. 31.23, 30^0
d. 21.65, 60^0

62. In a spherical triangle Given b = 60^0 c = 30^0 and A = 45^0 find a.

a. 42.33^0
b. 37.67^0
c. 39.76^0
d. 56.23^0

63. In a spherical triangle Given A = 60^0 B = 60^0 and C = 120^0 , find a

a. 34.33^0
b. 70.53^0
c. 76.44^0
d. 41.45^0

64. Given the right spherical triangle a = 43^0 27' and c = 60^0 24'. Find b.

a. 56.12^0
b. 54.12^0
c. 47.13^0
d. 71.33^0

65. Find the distance in Nautical miles from London (Latitude , $51^0$25'N, Longitude 0^0) to Moscow (Latitude 55^0 45' N , Longitude 37^0 36' E)

a. 1231.44 NM
b. 1331.34 NM
b. 2112.34 NM
c. 1346.78 NM

66. From problem 66, find the course angle from London to Moscow.

a. 69.34^0
b. 62.33^0
c. 64.07^0
d. 63.33^0

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67. A parcel of land in the form of a triangle has sides 312 m long and 485 m long. The included angle is $81^0 30'$. Find the perimeter and area of the triangle.

a. 1, 233.60, 76,334.55 b. 1,333.50, 74, 828.94
c. 1,444.43, 80, 222.33 d. 1,333.67, 79, 447.22

68. The angles of a triangle are $35^0 20'$, $65^0 36'$ and $79^0 04'$. If its area is 1,200 m^2, what is the perimeter?

a. 168.36 b. 133.34 c. 144.55 d. 155.67

69. Given the sides of the quadrantal triangle $a = 75^0$ $b = 85.5^0$ and $c = 90^0$ find angle A

a. 76.71 deg b. 38.78 deg c. 51.21 deg 74.41 deg

70. Given the spherical triangle $A = 112.72^0$ $C = 94.83^0$ $c = 90^0$, find a.

a. 112.23^0 b. 132.11^0 c. 67.22^0 d. 56.23^0

71. Point X is 4 m from A, 3 m from B and 5 m from C. Point X is inside the triangle formed by ABC. If AB = BC and angle B is a right angle, find the length of side AB.

a. 6.211 b. 6.071 c. 7.022 d. 5.922

72. From point O on the ground of area = 160,000 sq ft, the angles of elevations of the 3 flagstaffs of equal heights at 3 consecutive corners of the yard are 45^0, 60^0 and 60^0 respectively. Find the height of each flagstaff.

a. 345.87 m b. 374.95 m c. 411.44 m d. 333.33 m

Two towers A and B are placed at a distance of 100 m apart horizontally. The height of A is 40 m and that of B is 30 m.

73. At what distance vertically above the ground will the intersection of the lines forming the angles of elevation of the two towers are observed from the bases of towers A and B respectively.

a. 12.33 b. 17.14 c. 17.89 d. 13.66

74. At what distance horizontally is this point located from tower A?

a. 57.14 b. 55.78 c. 59.23 d. 61.33

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The angle of elevation of the of the tower from A is 25 deg. From another point B, the angle of elevation of the top of the tower is 56 deg. AB = 300 m and on the same horizontal plane as the foot of the tower. The hroziontal angle subtended by A and B at the foot of the tower is 70^0.

75. What is the height of the tower?
    a. 146.7 m b. 148.8 m c. 134.4 m d. 111.2 m
76. How far is point A from the tower?
    a. 319.1 b. 322.2 c. 341.2 d. 331.2
77. How far is point B from the tower?
    a. 111.1 b. 100.4 c. 122.2 d. 97.6

A plane flies at a speed of 400 Nm from A to B on a direction N 32 deg W with a wind blowing at a speed of 30 Nm on a direction due west.

78. What is the speed of the plane relative to the ground?
    a. 434.22 b. 456.65 c. 416.68 d. 561.22
79. At what angle was the original direction of flight shifted due to the tail wind?
    a. 2.5 deg b. 4.5 deg c. 3.5 deg d. 5.5 deg
80. What is the direction of the plane relative to the ground.
    a. N 35.5 deg W b. N 45.5 deg W
    c. N 32.3 deg W d. N 51.2 deg W

A plane travels N 30 deg W at an air speed of 600 kph. If the wind has a speed of 80 kph on a direction of N 40 deg E.

81. What is the ground speed of the plane?
    a. 456.67 b. 631.85 c. 667.89 d. 811.23
82. What is the angle that the plane shifted from the original course?
    a. 6.83 deg b. 7.81 deg c. 8.32 deg d. 9.11 deg
83. What is the direction of the plane relative to the ground?
    a. N $23^0$10'W b. N 33^0 40' W c. N 51^0 20'W d. N $35^0$12' W

84-85

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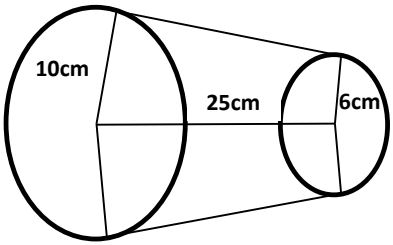
Given a spherical triangle
a = 77^0 36’ b = 63^0 17’ and c = 107^0 23’ with spherical radius of 5m.

84. Find the perimeter?
    a. 21.641 m b. 23.341 m c. 22.333 m d. 28.123 m
85. Find the value of angle A?
    a. 67.89 deg b. 65.83 deg c. 66.78 deg d. 78.23 deg

86- 87
The diagonals of a parallelogram are 87.2 m and 100.4 m respectively and they intersect at an angle of $48^0$18’.

86. Find the perimeter of the parallelogram.
    a. 234.97 b. 248.96 c. 251.23 d. 311.45
87. Find the area of the parallelogram.
    a. 3567 b. 3268 c. 3342 d. 4122

88. Determine the total length of the belt shown which goes around the pulleys A and B of radii 10 cm and 6 cm respectively. The distance between the centers is 25 cm.



    a. 100.91 cm b. 111.94 cm c. 122.23 cm d. 154.34 cm

89. Determine the total length of the crossed belt shown which goes around the pulleys A and B of radii 12 cm and 7 cm respectively. The distance between the pulley centers is 30 cm.
    a. 144.56 cm b. 120.34 cm c. 132.19 cm d. 144.21 cm

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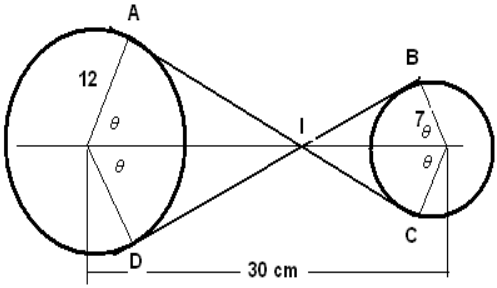


90. A horizontal cylindrical tank, buried underground has a diameter of 1.2 m and a length of 3 m. If it is filled with gasoline to a depth of 0.8 m, find the volume of gasoline in it expressed in liters. 1 liter = 0.001 m^3

a. 2500 liters b. 2400 liters c. 2600 liters d. 2800 liters

91. The bases of a parcel of land in the form of a trapezoid are 92.6 m and 75.8 m respectively. The angles at the extremities of the longer base are 72 deg and 43 deg respectively. Find the perimeter of the parcel of land.

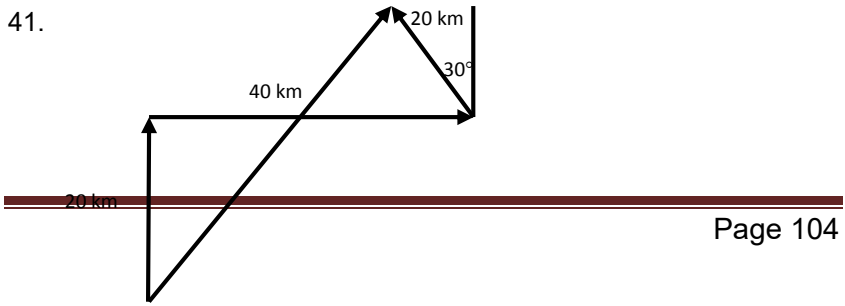
a. 198.67 m b. 201.33 m
c. 123.67 m d. 188.32 m

92. The angles of a triangle are $35^0 20'$ $65^0 36'$ and $79^0 04'$. If its area is 1200 m^2, what is the perimeter of the triangle?

a. 133.45 b. 168.33
c. 155.44 d. 177.56

SOLUTION:

41.



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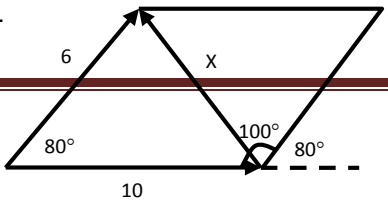
distance = | 20∟90 + 40∟ 0 + 20∟ 120 | = 47.88 km
Note: ENTER MODE 2 and input
SHIFT hyp (20∟90 + 40∟ 0 + 20∟ 120)

42-43-44

3X – 4Y = sin π/2 = 1 X - Y = cos π/3 = 1/2
ENTER: MODE 5 1
Input: 3 - 4 1
1 -1 1/2

42. Ans. X = 1
43. Ans. Y = 1/2
44. $\sin^{-1}(X – Y) = \sin^{-1}(1 – 1/2) = 30^0$ or π/6
45. Note: For a triangle,
If Tan A + tan B + tan C = f , tan A tan B tan C = f
Thus:tan A tan B tan C = 5.67
46. tan A tan B tan C = 5.67
tan B tan C = 5.67/1.732 = 3.274
47. tan A tan B tan C = 5.67
tan A tan C = 5.67/ tan B = 5.67/ 1.915 = 2.961
tan A tan C = 2.961 (1)
tan A + tan B + tan C = 5.67
tan A + tan C = 5.67 – tan B = 5.67 – 1.915
tan A + tan C = 3.754
tan A = 3.754 – tan C (2)
Substitute in 1
(3.754 – tan C)(tan C) = 2.961
$-\tan^2 C + 3.754 \tan C – 2.961 = 0$
tan C = 2.626753 and tan C = 1.127247
C = 69.16^0 or C = 48.42^0

48.



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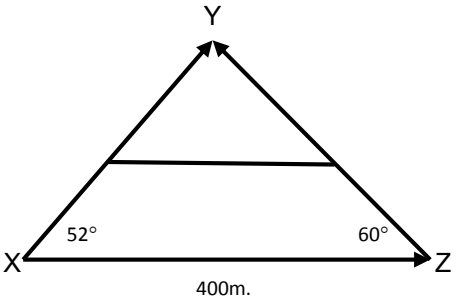
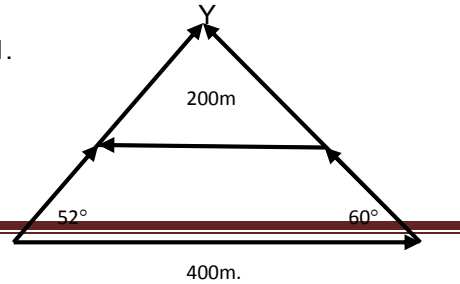
USE COSINE LAW TECHNIQUE: MODE 2
X = | 6∟80 – 10|
= 10.73 1
49.



USE SIN LAW TECHNIQUE:
MODE 5 1
Input
cos 52 cos 60 400
sin 52 - sin 60 0
XY = 373.61 YZ = 373.615
XY = 373.615 ZY = 339.96
XY + ZY = 713.58

50. Area = 1/2 base x height = 1/2 x 400 x 373.615 sin 52^0
= 58,882.53 m^2

51.



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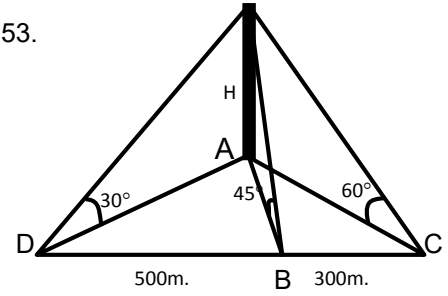
X Z

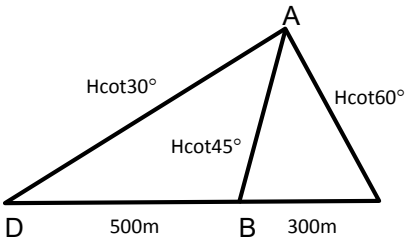
USE SIN LAW TECHIQUE FOR TRAPEZOID ABZX
MODE 5 1
cos 52 cos 60 400 – 200
sin 52 - sin 60 0
AX = 186.808 BZ = 169.98
To get the altitude of the trapezoid:
rec(186.808, 52) Y = 147.208
Area of ABZX = ½ (200 + 400)(147.208) = 44,162.4 m^2

52. ENTER: MODE 3 2
Input:

X Y
2 -60
3 -60+ 330 = 270
ENTER: AC
To get the angle at 2: 34:12
ENTER: 2^0 34^0 12^0 SHIFT 1 7 5 =
Ans. 128.1 degrees.
To convert this to radian : GO to radian mode. ENTER
ENTER:SHIFT MODE 4
ENTER: SHIFT ANS 1 = Ans. 2.235766772 rad

53.





C
Then:
(hcot60)^2 = (hcot 30)^2 + 800^2 – 2(hcot30)(800)cos D (1)

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and $h^2 = (h\cot 30)^2 + 500^2 - 2(h\cot 30)(500)\cos D$ (2)

Using Eq 1. $1/3h^2 = 3h^2 + 800^2 - 2771.28h\cos D$

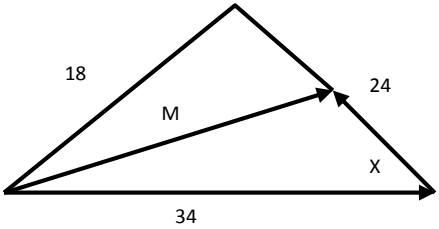
Eq. 2 $h^2 = 3h^2 + 500^2 - 1732.05h\cos D$

Solve for cos D in both equations.

$$\frac{3h^2 + 800^2 - 1/3h^2}{2771.28h} = \frac{3h^2 + 500^2 - h^2}{1732.05h}$$

Use SHIFT CALC h = 672.93

54.



$18^2 = 24^2 + 34^2 - 2(24)(34)\cos X$

Then: $X = 30.374^0$

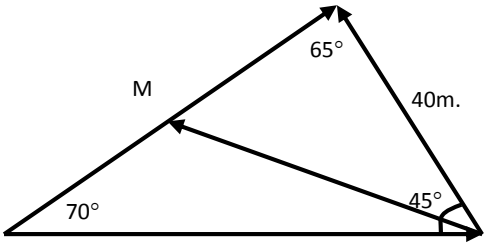
For the median.

USE COSINE LAW TECHNIQUE:

MODE 2:

M = | 34∟30.374 - 12| = 24.4131

55.



USE SINE LAW TECHNIQUE for ABC

MODE 5 1

cos 45 cos 65 40

sin 45 - sin 65 0

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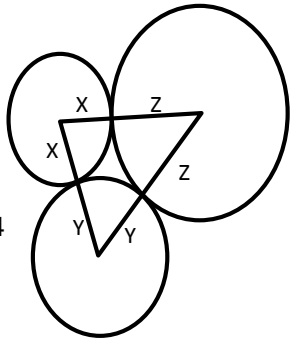
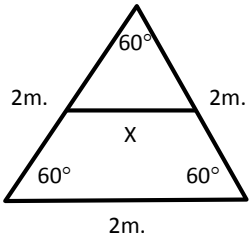
AC = 38.58 BC = 30.1
USE COSINE LAW TECHNIQUE
AM = | 30.1/2 ∟ 70 – 38.58 | = 36.3

56.



X + Y = 16
X + Z = 20
Y + Z = 24
USE MODE 5 2 Ans. X = 6, Y = 10, Z = 14
smallest circle: Area = $\pi(6)^2 = 36\pi$

57. tan (x + y) = tan ($\tan^{-1}$ 1/2 + $\tan^{-1}$ 1/3) = 1
58. In any triangle, the sum of 2 sides must be greater than the third side.
Let X = side FD. Then the possibilities are:
18 + 6 > X or X < 24
18 + X > 6 or X > 6 – 18 or X > -12
6 + X > 18 or X > 18-6 or X > 12
The only value that will fit the 3 conditions is 13.
59.



The area of an equilateral triangle is $A = (x^2\sqrt{3})/4$
Area of the big triangle is $(2^2\sqrt{3})/4 = \sqrt{3}$ m^2
Half of this area is $\sqrt{3}/2$.
The upper triangle is also equilateral of side x whose area is $\sqrt{3}/2$. Then $(x^2\sqrt{3})/4 = \sqrt{3}/2$. or x = 1.414 m
60.

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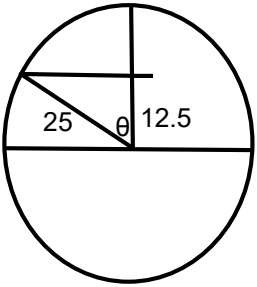
ENTER: MODE 3 6
Input the given data

X	Y
0	1
1	10

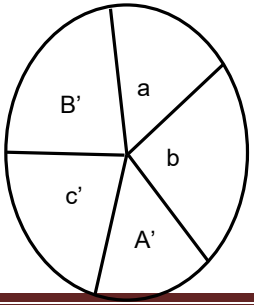
ENTER: AC
The ratio of strength of Baguio Earthquake to Kobe Earthquake is
$7.8\hat{Y} \div 5.7\hat{Y}$
Ans. 125.89 or 126 times.

61.



$r^2 = 25^2 - 12.5^2$ r = 21.65 POL(21.65, 12.5) $\theta = 30^0$
62. cos a = cos b cos c + sin b sin c cos A
b = 60 c = 30 A = 45
$a = 42.33^0$
63. cos A = - cos B cos C + sin B sin C cos a
$A = 60^0$ $B = 60^0$ $C = 120^0$ $a = 70.53^0$

64. Using Napiers Rule



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sin c' = cos a cos b

sin (90 – $60^0$24') = cos 43^0 27' cos b b = 46.13^0

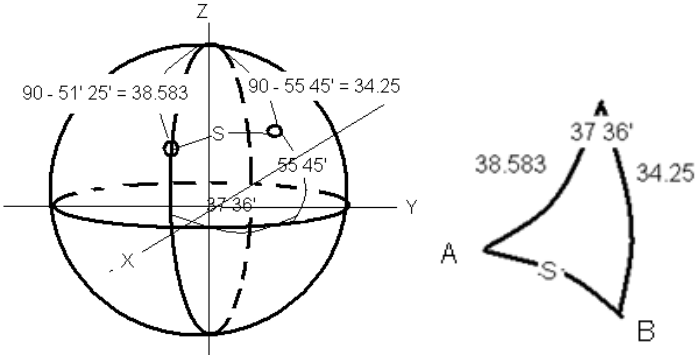
Using COSINE law:

cos c = cos a cos b + sin a sin b cos C cos 90^0 = 0

cos $60^0$24' = cos 43^0 27' cos b

b = 47.13

65.



cos s = cos 38.583 cos 34.25 + sin 38.583 sin 34.25 cos $37^0$36'

s = 22.4464^0 22.4464 x 60 = 1346.78 Nautical Miles

CAL TECHNIQUE. Assume that R = 1

Rec(1, 51^0, 25') X = 0.62365 Y = 0.7817

Coordinate of London (0.62365, 0, 0.7817)

Rec(1, 55^0 45') X = 0.5628 Y = 0.8266

Z coordinate of Moscow = 0.8266

Rec(0.5628, 37^0 36') X = 0.4459 Y = 0.3434

Cordinate of Moscow = (0.4459, 0.3434, 0.8266)

Get the straight line distance of London to Moscow.

Vct A = (0.62365 0 0.7817)

Vct B = (0.4459 0.3434 0.8266)

Angle between Vectors :

cos θ = Vct A dot Vct B/ (Abs(VctA) Abs(Vct B))

θ = 22.447^0 Convert this to Min (multiply by 60)

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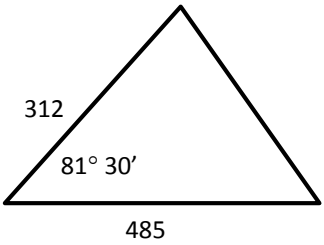
Ans. 1346.78 NM

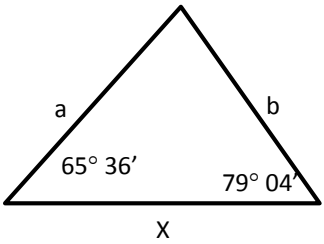
66. To determine the Course Angle (London to Moscow)
Using sin Law
sin A / sin 34.25 = sin 37^0 36' / sin 22.446^0
A = 64.08^0

67.



COMPLEX MODE (MODE 2)
Perimeter = 312 + 485 + | $312\angle 81^0 30'$ – 485 | = 1333.5
Area = 1/2 bh = 1/2 (485) (312 sin 81.5) = 74,828.94

68.



MODE 5 1
cos $65^0 36'$ cos $79^0 04'$ 1
sin $65^0 36'$ -sin $79^0 04'$ 0
a = 1.6977x and b = 1.57467 x
Then Area = 1/2 x (a sin 65^0 36')
1200 = 1/2 (x) (1.6977x) sin 65^0 36'
x = 39.4076
Perimeter = a + b + x =

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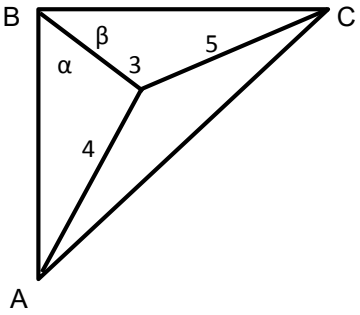
$1.6977x + 1.57467x + x = 168.364$

69.
cos a = cos b cos c + sin a sin b cos A
cos 75^0 = cos 85.5 cos 90 + sin 75 sin 85.5 cos A
A = 74.41^0

70. sin a/ sin A = sin c/ sin C
sin a = sin 112.72^0 (sin 90^0 / sin 94.83^0)
a = 67.77^0 or a = $180 - 67.77^0 = 112.23^0$ Ans.

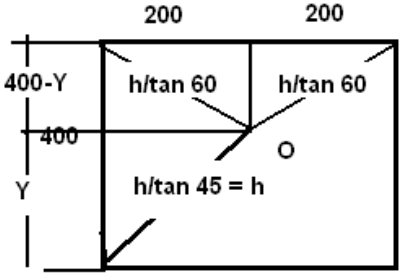
71. We will just use trial and error.
Note that $\alpha + \beta = 90$



Use the Technique: $\cos^{-1}\dfrac{A^2 + B^2 - C^2}{2AB}$

If side AB = BC = 6.071 then $\alpha = 34.949$ and β = 55.069^0
then $\alpha + \beta = 90$ Ans. AB = BC = 6.071

72.



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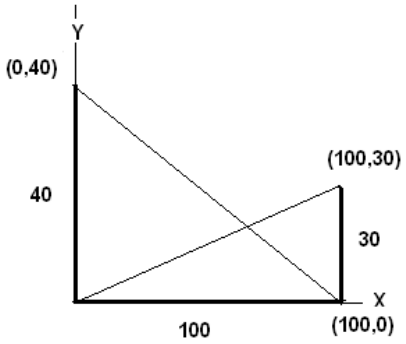
$(400-Y)^2 + (200)^2 = (h/\tan(60))^2$
$(400-Y)^2 + 200^2 = 1/3h^2$
and $y^2 + 200^2 = h^2$
Then: $(400-Y)^2 + 200^2 = 1/3(Y^2 + 200^2)$
USE SHIFT CALC Ans. Y = 317.1573
Then $h = \sqrt{(200^2 + Y^2)} = 374.952$ m
(CAL TECH :USE POL(200,317.1573) h = 374.952

73-74



Get the equation of the line joiing (0, 40) , (100, 0)
MODE 5 1 Input: 0 40 1
100 0 1
Equation: 1/100x + 1/40y = 1
Equation of the line joining (0,0) (100, 30)
y = 30/100x
or 0.30x – y = 0
MODE 5 1
Solve 1/100x + 1/40y = 1
0.3x – y = 0
Ans. x = 57.14 y = 17.1428

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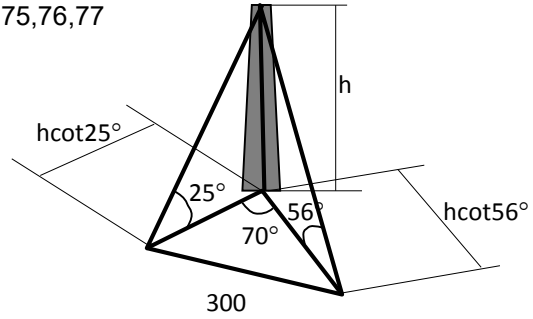
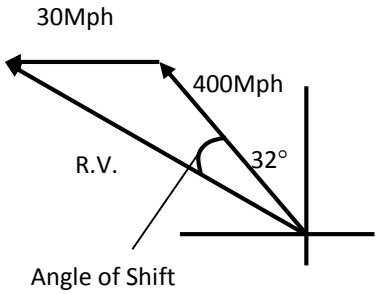
75,76,77



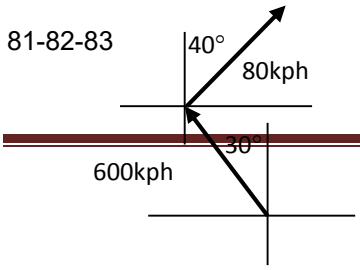
h/tan 25 = 2.1445h h/tan 56^0 = 06745h
USING COSINE LAW TECHNIQE:
|2.1445h∠70 – 0.6745h| = 300
Using trial and error: h = 148.8 (from choices)
2.1445h = 319.1016
0.6745h = 100.37

78-79-80



COMPLEX MODE:
resultant velocity = 400∠(90+ 32) + 30∠(180)
= 416.675∠125.5^0
Angle of shift = 125.5 - (90 + 32) = 3.5 deg
N (125.5 – 90) W or N 35.5 W

81-82-83



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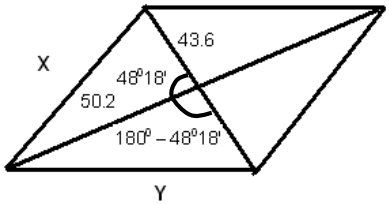
Actual velocity of the plane = $600\angle 120 + 80\angle 50 = 631.85\angle 113.17$
Angle of shift = (90 + 30) – 113.17 = 6.83 deg
True direction = 113.17 deg of N (113.17 – 90) W =
N 23^0 10' W

RADIAN MODE:
84-85
Perimeter =
5 ($77^0 + 36/60^0 + 63^0 + 17/60^0 + 107^0 + 23/60^0$) =
= 21.641 m
cos a = cos b cos c + sin b sin c cos A
cos 77^0 36' = cos $63^0$17' cos $107^0$23' +
sin $63^0$17' sin $107^0$23' cos A
A = 65.83 deg

86-87



MODE 2
X = | $50.2\angle 48^0$18'- 43.6 | = 38.8458 m
Y = | $50.2\angle(180^0 - 48^0$18') – 43.6| = 85.633
P = 2(X + Y) = 248.96
Use Heron's Formula

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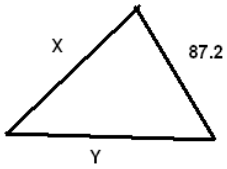
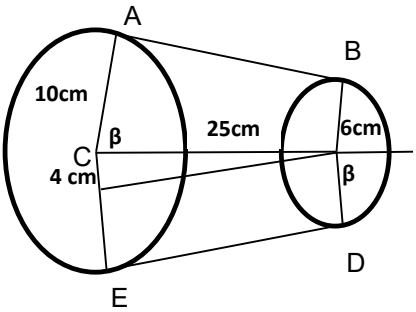


S = (x + y + 87.2)/2 and

1/2 A $= \sqrt{s(s-x)(s-y)(s-87.2)}$

A = 3268

88.



DE = $\sqrt{(\ 25^2 - 4^2\ )}$ = 24.68 = AB = CD

cos β = 4/25 β = 80.793^0

RAD MODE:

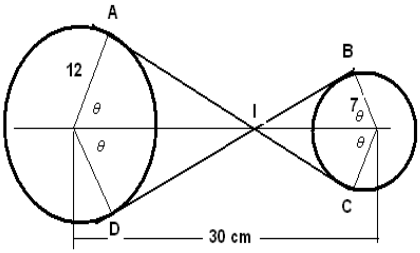
AC = $10(\ 360 - 2(80.793)\ )^0$ = 34.63

BD = $6(\ 2(80.793)\ )^0$ = 16.92

Total length of the belt = 34.63 + 16.92 + 2(24.68)

= 100.91 cm

89.



30 = 12/cosθ + 7/cosθ

30 = 19 sec θ

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$\theta = 50.7035^0$ $2\theta = 101.407^0$

RAD MODE:

Long Arc AD = $12(360 - 101.407)^0 = 54.16$ cm

Long Arc BC = $7(360- 101.407)^0 = 31.593$ cm

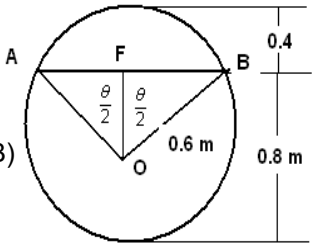
AI = $12 \tan 50.703^0 = 14.663$ cm = DI

BI = $7 \tan 50.703^0 = 8.553$ cm = CI

Length of the Belt = $54.16 + 31.593 + 2(14.663 + 8.553)$

= 132.19 cm

90.



Pol(0.2, $\sqrt{0.6^2 - 0.2^2}$)

θ/2 = 70.5288 , (this is stored to Y)

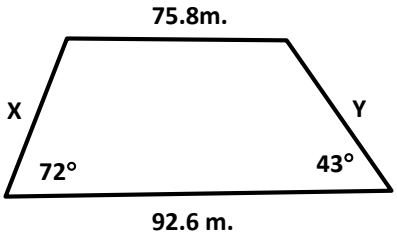
Then α= $360^0 - 2Y = 218.942^0$ (Store it to B)

$A = 1/2r^2 (\theta - \sin \theta)$

$= 0.5(0.6)^2 (B^0 - \sin (B^0)) = 0.801 \ m^2$

Volume = $0.801(3) = 2.4 \ m^3 = 2{,}400$ liters

91.



ENTER: MODE 5 1 and Input the given data:

cos 72 cos 43 92.6 – 75.8

sin 72 - sin 43 0

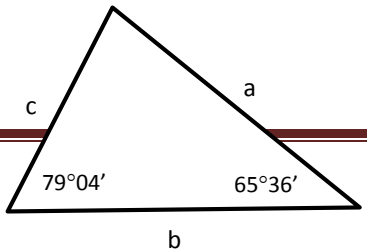
Ouput:

X = 12.642 Y= 17.629

PERIMETER = 75.8 + 92.6 + X + Y

= 198.67 m

92.



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Let b = 1 X :
ENTER: MODE 5 1
Input:
cos $79^0$04’ cos $65^0$36’ 1
sin $79^0$04’ - sin $65^0$36’ 0

X = 1.5746 and Y = 1.6977
Then b = 1 X , c = 1.5746X and a = 1.6977X

Area = ½ b c sin $79^0$04’ =
½ (1X) (1.5746X) sin $79^0$04’ = 1200
X = 39.4
Perimeter = a + b + c = 1.5746X + 1X + 1.6977X
= 168.33

PROBLEM SET 7 – PART 1
(STATISTICS AND PROBABILITY)

Given: 12 34 45 23 87 91 121

1. Find the mean.
   a. 54 **b. 59**
   c. 57 d. 61
2. Find the population standard deviation.
   a. 35.56 b. 33.45
   c. 37.75d d. 38.21
3. Find the sample standard devation.
   a. 40.78 b. 41.22
   c. 43.34 d. 44.34
4. How many 4 digit numbers can be formed by the use of digits 1, 2, 3, 4, 6 and 7 if one of the digits is used only once in one number.
   a. 360 b. 400

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c. 320 d. 420

(Situation - Problems 5, 6)

A Math Professor gives the following scores to his students.

Frequency	1	2	4	6
Numbers	30	42	50	60

5. Find the weighted mean of the scores.

a. 56.41 b. 51.85

c. 56.78 d. 55.34

6. Compute the standard deviation.

a. 8.12 b. 9.13

d. 10.12 d. 8.33

(Problems 7 , 8)

The specification for a job calls for a class “b” mix with a minimum compressive strength of 3,000 psi at 28 days. The result of 125 compressive tests are tabulated in the following table. CE BOARD MAY 1995

28 days compressive strength (psi)	Number of Tests
2800	2
2900	4
3000	6
3100	11
3200	24
3300	37
3400	19
3500	12
3600	6
3700	4

7. The mean strength is

a. 3311.25 b. 3414.34

c. 3289.6 d. 3411.22

8. The standard deviation is

a. 185.33 b. 183.23

c. 195.22 d. 189.45

9. Find the standard deviation of the following sets of numbers.

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Frequency:	1	3	4	12
	21	33	45	14

a. 12.33 b. 12.56
c. 12.67 d. 13.33

10. How many 4 digit numbers can be formed by the use of the digits 1,2,3,4,6 and 7 if one digit is used only once? PAST CE BOARD
a. 360 b. 400
c. 320 d. 420

11. Four different colored flags can be hung in a row to make a coded signal. How many signals can be made if a signal consists of the display of one or more flags?
a. 64 b. 68
c. 62 d. 66

12. In how many ways can a bowling player score in one throw of a bowling ball . (There are 10 pins) CE BOARD Nov 2009
a. 1120 b. 1024
c. 720 d. 30,240

13. What is the number of permutations of the letters in the word BANANA? PAST EE BOARD
a. 36 b. 52
c. 60 d. 42

14. In how many ways can 10 trees be planted in a circular lot?
a. 362,880 b. 3,628,880
c. 432,112 d. 442,122

15. Three boys and three girls sit in a row of 6 chairs. In how many ways can they sit in alternate seats.
a. 36 b. 18
c. 72 d. 144

16. Three boys and 3 girls sit in a row. In how many ways can they sit if the girls are always together?
a. 144 d. 152
c. 100 d. 122

17. A factory building has 8 entrance doors. In how many ways can a person enter and leave by different doors.

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a. 56 b. 70
c. 64 d. 64

18. In how many ways can you invite one or more of your five friends in a party? PAST ME BOARD.

a. 32 b. 30
c. 31 d. 29

19. A semiconductor will hire 7 men and 5 women. In how many ways can the company choose from 9 men and 6 women.

a. 678 b. 216
c. 324 d. 560

20. The lotto uses numbers 1 to 42. A winning number uses 6 different numbers in any order. What is your chance of winning if you bet one ticket? CE BOARD NOV 2009

a. 1/4534568 b. 1/6580668
c. 1/5245786 d. 1/2341668

21. How many triangles are determined by three non collinear points from 10 points? PAST ECE BOARD.

a. 720 b. 120
b. 90 d. 100

22. In a licensure exam, an examinee may select 7 problems from a set of 10 questions. How many choices has he? PAST EE BOARD

a. 720 b. 120
c. 90 d. 100

23. If you roll a pair of dice one time, what is the probability of getting a sum of 9? PAST ECE BOARD

a. 1/9 b. 1/4
c. 1/6 d. 1/3

24. A coin is tossed 5 times. What is the probability of getting 3 heads?

a. 0.3125 b. 0.125
c. 0.5 d. 0.2125

25. A coin is tossed 3 times. What is the probability of getting 3 heads? PAST CE BOARD

a. 1/4 b. 1/8
c. 3/4 d. 1/2

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26. What is the probability of getting at least 3 heads when a fair coin is tossed 6 times.

a. 19/32 b. 17/32
c. 21/32 d. 15/32

27. In a dice game, one fair die is tossed. The player wins P20 if he rolls either 1 or a 6. He loses P10 if he turns up any other face. What is the expected winning for one roll of a die?

a. -2 b. 2
c. 0 d. 1

28. The probability of getting a credit in an examination is 1/3. If three students are selected at random, what is the probability that at least one of them get the credit?

a. 2/3 b. 19/27
c. 20/27 d. 7/9

29. A coin is tossed 10 times. What is the probability of getting 4 tails and 6 heads?

a. 105/512 c. 13/64
d. 51/256 d. 25/128

30. In how many ways can a committee consisting of 3 mean and 2 women be chosen from 7 men and 5 women?

a. 350 b. 400
c. 300 d. 200

31. A die is constructed so that a 1 or a 2 occurs twice as a 3, 4 or a 6 and a 5 occurs thrice as a 1 or a 2. If the die is tossed once. Find the probability that an even number will occur ?

a. 4/9 b. 2/9
c. 1/9 d. 4/13

32. Six married couples are standing in a room. Find the probability that they are married.

a. 1/10 b. 1/9
c. 1/11 d. 1/12

33. A restaurant is giving away 3 different toys with each child's meal. What is the probability of getting all three toys when purchasing 5 meals?

a. 130/243 b. 150/243
c. 130/243 d. 112/243

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34 . What is the probability of a family with 5 children of having 3 boys and 2 girls?

a. 5/16 b. 1/4
c. 3/16 d. 3/8

35 . In a basketball game, the free throw average is 0.65. Find the probability that a player misses one shot of the three free throws?

a. 0.441 b. 0.422
c. 0.444 d. 0.451

36. With 50 questions each of which has 4 given answers, how many possible answer patterns are there? PAST EE BOARD

a. 1.27×10^{30} b. 1.34×10^{30}
c. 1.45×10^{30} d. 1.45×10^{30}

37 From the following tabulation, compute the correlation coefficient between x and y. PAST EE BOARD.

X	80	84	88	92	98	104
Y	4	8	10	8	12	14

a. 0.94 b.0.88
c 0.92 d. 0.88

38. Find the probability that a person flipping a coin gets the 3rd head on the 7th flip.

a. 0.1144 b. 0.1172
c. 0.1221 d. 0.1344

39. Find the probability that a person flipping a coin gets the 1st head on the 4th flip.

a. 0.035 b. 0.0012
c. 0.0625 d. 0.1122

40. The probability that a student pilot passes a written test for his private pilot's licence is 0.7. Find the probability that a person passes the test before the 4th try.

a. 0.973 b. 0.812
c. 0.922 d. 0.954

41. On the average, a certain intersection results in 3 traffic accidents per month. What is the probability that in any given month at this intersection, exactly 5 accidents will occur.

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a. 0.0091 b. 0.2112
c. 0.1221 d. 0.1008

42. Refer to no 41, What is the probability that less than 3 accidents will occur?

a. 0.4232 b. 0.4442
c. 0.3322 d. 0.4276

43. A secretary makes 2 errors per page on the average. What is the probability that the page she makes will have 4 or more errors.

a. 0.1428 b. 0.1234
c. 0.2122 d. 0.1122

44. An urn contains white and black balls. If the probability to pick a white ball is equal to log x and the probability that it will be black is equal to log 2x, what is the value of x? ECE Nov 2002

a. 1.515 b. 2.236
b. 1.732 d. 1.412

45. One bag contains 4 white balls and 3 black balls and a second bag contains 3 white and 5 black balls. One ball is drawn from the second bag and is placed unseen in the first bag. What is the probability that the ball now drawn from the first bag is white?

a. 35/64 b. 33/64
c. 31/64 d. 19/64

46. Box A contains nine cards numbered 1 to 9, and Box B contains 5 cards numbered 1 to 5. A box is chosen at random and a card is drawn. If the number is even, find the probability that it came from box A.

a. 9/19 b.10/19
c. 11/19 d. 21/19

47. During the board exam, there were 350 examinees from Luzon, 250 from Visayas, and 400 from Mindanao. The results of the exams revealed that flunkers from Luzon, Visayas and Mindanao are 3%, 5% and 7%. If a name of a flunker is picked at random, what is the probability that it came from Mindanao? ECE April 2003

a. 0.549 b. 0.553
c. 0.581 d. 0.567

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48. There are 2 copies each of 4 different books. In how many ways can they be arranged in the shelf? ECE April 2003

a. 2340 b. 2520
c. 2321 d. 3410

49. If the probability that a basketball player sinks the basket at a 3 point range is 2/5, determine the probability of shooting 5 out of 8 attempts. ECE April 2003

a. 13.21% b. 11.44%
c. 12.38% d. 11.44%

50. Compute the standard deviation of the following set of numbers. 2, 4, 6, 8, 10, 12
ECE April 2005

a. 3.742 b. 3.451
c. 4.112 d. 4.221

SOLUTION:

Note: For ES plus users SHIFT 1 5 is the same as SHIFT 1 4
For nPr , ENTER n SHIFT x r
For nCr, ENTER n SHIFT ÷ r

1. ENTER: MODE 3 1
Input the given data then ENTER: AC
To get the mean: ENTER: SHIFT 1 5 2
Ans. $\overline{X}$ = 59 (b)
2. ENTER: SHIFT 1 5 3
Ans. Xσn = 37.75 (c)
3. ENTER: SHIFT 1 5 4
Ans. Xσn-1 = 40.78 (a)
4. nPr = 6P4 = 360 (a)
5. ENTER: MODE 3 1
SHOW THE FREQUENCY: ENTER: SHIFT MODE ▼ 4 1
Input the given data.
ENTER: SHIFT 1 5 2

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Ans. $\bar{X}$ = 51. 85

6. ENTER: SHIFT 1 5 3 =
Ans. Xσn = 9.13
7. ENTER: MODE 3 1
SHOW THE FREQUENCY: ENTER: SHIFT MODE ▼ 4 1
Input the given data. ENTER AC
ENTER: SHIFT 1 5 2
Ans. $\bar{X}$ = 3289.6 (c)
8. ENTER: SHIFT 1 5 3
Ans. Xσn = 183.23 (b)
9. ENTER: MODE 3 1
SHOW THE FREQUENCY: ENTER: SHIFT MODE ▼ 4 1
Input the given data. ENTER AC
ENTER: SHIFT 1 5 3
Ans. Ans. Xσn = 12.67 (c)
10. 6P4 = 360
11. 4P1 + 4P2 + 4P3 + 4P4 = 4 + 12 + 24 + 24 = 64 (a)
12. 2^{10} = 1024 (This is the same as 10C0 + 10C1 + 10C2 + ….. 10C10)
13. BANANA 1 B 3 A 2 N
6!/(1!3!2!) = 60 (c)

14. (n-1)! = 9! = 362,880
15. BBBGGG or GGGBBB 2 (3!)(3!) = 72 (c)
16. GGGBBB
BGGGBB
BBGGGB
BBBGGG
4 x 3! x 3! = 144 (a)
17. 8 x 7 = 56
18. 5C1 + 5C2 + 5C3 + 5C4 + 5 C5 = 31 or 2^5 -1 = 31
19. 9C7 x 6C5 = 216 (b)
20. 1/ 42C6 = 1/ 5245786 (c)
21. 10C3 =120 (b)
22. 10C7 = 120 (b)

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23. The possible sums (2 7), (7 2), (4 5), (5 4)
Probability = 4/(6 x 6) = 4/36 = 1/9

24. $(H + T)^5$ You need the coefficient of H^3T^2
Ans. $5C3 (1/2)^3(1/2)^2 = 0.3125$ (a)

25. $(H + T)^3$ coefficient of $H^3 T^3$ is 1.
$1(1/2)^3(1/2)^3 = 1/8$

26. $(H + T)^6$ you need the sum of the coefficients of
H^3T^3, H^4T^2, H^5T , H^6
$(6C3 + 6C4 + 6C5 + 6C6)/ 2^6 = 21/32$ (c)

27. Probability of getting a 1 or a 6 = 2/6
Probability of any other numbers = 4/6
Expectation = 2/6(20) + 4/6(-10) = 0 (c)

28. Let C = with credit N = No credit
$(C + N)^3 = C^3 + 3C^2N + 3CN^2 + N^3$
Probability of getting at least one credit is
$(1/3)^3 + 3(1/3)^2(2/3) + 3(1/3)(2/3)^2 = 19/27$ (b)

29. $(H + T)^{10}$ coefficient of $H^6T^4 = 10C6 = 210$
Ans. $210/2^{10} = 105/512$ (a)

30. $7C3 \times 5C2 = 350$ (a)

31. Let w = probability of getting a 3, 4 or a 6.
then 1 or a 2 has a probability of 2w
Sum of probability = 1
(3, 4, 6) prob = w + w + w = 3w
(1,2) prob = 2w + 2w = 4w
(5) prob = 3(2w) = 6w
Sum = 3w + 4w + 6w = 13w = 1 , w = 1/13

The even numbers are 2, 4 and 6 with probabilities of 2w, w, w resp.
Sum for (2,4,6) = 4w prob = 4(1/13) = 4/13 (d)

32. $6C1 / (12C2) = 1/11$

33. Let X = toy 1, Y = Toy 2 and Z = toy 3.
For $(X + Y + Z)^5$ (5 means five meals), we are looking for the coefficients of
$X^1Y^1Z^3$, $X^1Y^3Z^1$, $X^3Y^1Z^1$, $X^2Y^2Z^1$, $X^2Y^1Z^2$, $X^1Y^1Z^2$

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$$\frac{5!}{1!1!3!}+\frac{5!}{1!3!1!}+\frac{5!}{3!1!1!}+\frac{5!}{2!2!1!}+\frac{5!}{2!1!2!}+\frac{5!}{2!1!2!} = 150$$

Probability = $150/3^5 = 150/243$

34. $(B + G)^5$ Coefficcient of B^3G^2 is $5C3 = 10$

Ans. $10/2^5 = 5/16$ (a)

35. Let X = prob of shooting P(X) = 0.65

Y = prob of missing P(Y) = 0.35

$(X + Y)^3 = X^3 + 3X^2Y + 3XY^2 + Y^3$

Probability of missing one shot

$3X^2Y = 3(0.65)^2(0.35) = 0.443625$ (c)

36. $4^{50} = 1.27 \times 10^{30}$

37. ENTER: MODE 3 2

Input the given data and ENTER: AC

ENTER: SHIFT 1 7 3 =

(FOR ES plus: ENTER: SHIFT 1 5 3)

Ans. r = 0.92166 (c)

38. Possible ways to get 3rd head on the 7th flip = 6C2 = 15

Probability = $15/2^7 = 15/128 = 0.1171875$

39. Possible ways to get 1st head on the 4th flip = 1

Probability = $1/2^4 = 1/16 = 0.0625$ (c)

40. Possible ways. P FP FFP

Probability = 0. 7 + 0.3(0.7) + 0.3(0.3)(0.7)

= 0.973

41. Apply Poisson's Distribution formula.

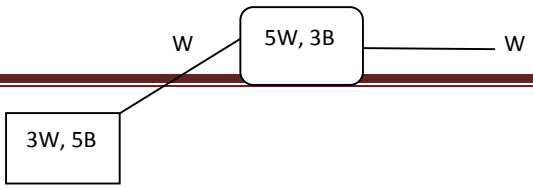
$$P(x) = \frac{e^{-\mu}(\mu)^X}{x!} = \frac{e^{-\mu}(\mu)^X}{x!} = \frac{e^{-3}(3)^5}{5!} = 0.10082 \text{ (d)}$$

42. $P(X < 3) = \sum_{x=0}^{2} \frac{e^{-3}(3)^x}{x!} = 0.4232$

43. $P(X \geq 4) = 1 - F(X < 4) = 1 - \sum_{x=0}^{3} \frac{e^{-2}(2)^x}{x!} = 0.1428$

44. $\log x + \log 2x = 1$ $x = 2.236$ (b) USE SHIFT CALC

45. Bag 1



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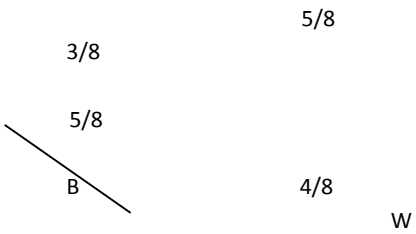
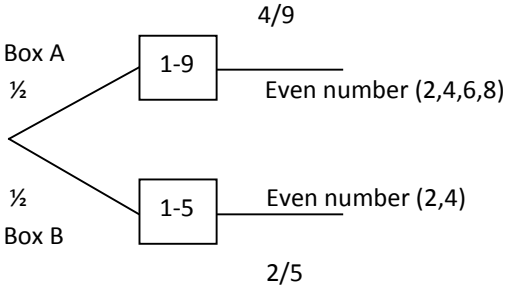


prob of getting white from the first bag =
P(W) = 3/8(5/8) + 5/8(4/8) = 35/64 (a)
46.



Probability of getting even number = 1/2(4/9) + 1/2(2/5) =
19/45
Probability that this number came from Box A =
1/2(4/9) / (19/45) = 10/19 (b)
47. Probability of picking a flunker.
P(F) =
(3% x 350 + 5% x 250 + 7% x 400)/ (350 + 250 + 400) = 51/1000
P (M ∩ F) = (7% x 400)/(350 + 240 + 400) = 7/250
P(M| F) = P (M ∩ F) / P(F) = (7/250)/ (51/1000) = 0.549 (a)
48. 8!/(2!2!2!2!) = 2520
49. Let X = sinking the basket , Y not sinking
P(X) = 2/5 P(Y) = 1 – 2/5 = 3/5
(X + Y) ^{8} The coefficient of $X^5 Y^3$ is 8C5 = 56
Probability of sinking 5 out of 8 is $56(2/5)^5(3/5)^3 = 0.1238$
50. ENTER: MODE 3 2
Input the given data. ENTER AC.
ENTER: SHIFT 1 5 4 =

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Ans, Xσn-1 = 3.74165 (a)

PROBLEM SET 7 PART 2

STATISTICS AND PROBABILITY

51. A bag contains 3 white balls and 5 red balls. If two balls are drawn at random in succession without returning the first ball drawn, what is the probability that the balls drawn are both red? ECE April 2005.

a. 3/14 b. 1/3
c. 5/14 d. 3/7

52. From the digits 0, 1, 2,3, 4, 5, 6, 7, 8, 9, find the number of six digit combinations. PAST ECE BOARD

a. 210 b. 110
c. 220 d. 102

53 Find the number of ways 2 green balls, 4 black balls and 6 yellow balls can be given to 12 children if each child gets one ball.

a. 12,300 b. 13,860
c. 11,200 d. 14,220

54. A class contains 10 men and 20 women at which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.

a. 1/3 b. 1/4
c. 1/6 d. 2/3

55. A point is selected inside a circle. Find the probability that the point is closer to the center of the circle.

a. 1/3 b. 1/2
c. 1/4 d. 1/5

56. The probability that A and B hits a target are 1/4 and 2/5 respectively. If they shoot together, what is the probability that the target will be hit.

a. 11/20 b. 1/2
c. 9/20 d. 7/20

57. A statistics of a machine factory indicates that for every1000 unit it produces , there is one reject unit. If a customer buys 200 units, what is the probability that it will have at least one reject unit. PAST ECE BOARD

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a.0.1233 b. 0.1814
c. 0.2311 d. 0.3122

58. A PSME unit has 10 ME's, 8 PME's, and 6 CPM's. If a committee of 3 members one from each group is to be formed, how many such committees can be formed?

a. 260 b. 480
c. 380 d. 360

59. In how many ways can 4 boys and 4 girls be seated alternately in a row of 8 seats.

a. 1152 b. 576
c. 2304 d. 1120

60. There are 13 teams in a tournament. Each team is to play with each other only once. What is the minimum number of days can they all play without any team playing more than one game in any day. PAST ECE BOARD

a. 11 b. 12
c. 13 d. 14

61. A bag contains 3 white and 5 black balls. If two balls are drawn in succession without replacement, what is the probability that both balls are white?

a. 1/14 b. 3/28
c. 4/7 d. 5/28

62. An urn contains 4 black balls and 6 white balls. What is the probability of getting1 black and 1 white ball in two consecutive draws from the urn? PAST ME BOARD

a. 8/15 b. 7/15
c. 6/15 d. 4/15

63. Find the probability of getting a prime number thrice by tossing a die 5 times.

a. 0.4225 b. 0.3125
c. 0.375 d. 0.1626

64. A janitor with a bunch of 9 keys is to open a door but only one key can open. What is the probability that he will succeed in 3 trials? ECE Nov 2005

a. 1/2 b. 1/4

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c. 2/3 d. 1/3

65. If the odds against event E are 2: 7, find the probability of success.

a. 2/9 b. 7/9

c. 9/14 d. 1/2

66. In a class of 40 students. 27 like calculus and 25 like chemistry. How many like both calculus and chemistry?

a.12 b. 15

c. 18 d. 21

67. A club of 40 executives , 33 like to smoke Marlboro, and 20 like to smoke Philip Morris. How many like both?

a. 12 b. 13

c. 14 d. 15

68. A survey of 500 television viewers produce the following results.

285 watch football games
195 watch hockey games
115 watch basketball games
45 watch football and basketball games
70 watch football and hockey games
50 wacth hockey and basketball games
50 do not watch any of the 3 games.

How many watch Hockey Games only.

a. 75 b. 85

c. 95 d. 105

69. An items cost distribution has a given function of the probability. What is the expected cost?

Cost in Pesos	Probability
1	0.2
2	0.28
3	0.18
4	0.23
5	0.11

a. 2.45 b. 2.77

c. 2.11 d. 2.89

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70. By investing in a particular stock, a person can make in one year P 40,000 with a probability of 0.3 or take a loss of P 10,000 with a probability of 0.7. What is the persons expected gain?

a. 5000 b. 4000
c. 7000 d. 8000

71. A player tosses three fair coins. He wins P 8 if 3 heads occur, P3.00 if 2 heads occur and P 1.00 only if one head occur. If the game is fair, how much should he lose if no heads occur?

a. P 30 b. P 20
c. P 25 d. 11

72. In a survey concerning the smoking habits of consumer, it was found that 55% smoke cigarette A, 50% smoke cigarette B, 40% smoke cigarette C, 30% smoke cigarette A and B, 20% smoke cigarette A and C, 12% smoke cigarette B and C and only 10% smoke all the three cigarettes. What % of the population did not smoke? PAST CE BOARD

a. 7% b. 6%
c. 4% d. 3%

73. Find the number of permutation of letters in the word "TOLENTINO" ?

a. 45,360 b. 42,360
c. 44,160 d. 49,120

74. Three light bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that none is defective.

a. 23/91 b.24/91
c. 20/91 d. 33/91

75. Of 120 students, 60 are studying French, 50 are studying Spanish, and 20 are studying French and Spanish. If the student is chosen at random, find the probability that the student is studying French or Spanish.

a. 1/4 b. 1/2
c. 3/4 d. 4/5

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76. A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

a. 4/9 b.5/9
c. 2/9 d. 3/9

77. Three boys and 3 girls sit in a row. Find the probability that the 3 girls sit together.

a. 1/5 b. 2/5
c. 3/5 d. 2/3

78. A point is selected at random inside an equilateral triangle whose side is 3 units. Find the probability that its distance to any corner is greater than 1.

a. 0.577 b. 0.544
c. 0.597 d. 0.555

79. During a board meeting, each member shakes hands with all other members. If there were a total of 91 handshakes, how many members were in the meeting? ECE NOV 2002

a. 12 b. 13
c. 14 d. 15

80. How many 3 digit area codes are there for a telephone company if the 1st digit may not be 0 or 1 , and the second digit must be 0 or 1?

a. 160 b. 140
c. 210 d. 120

81. In how many ways can 3 marines and 4 armies be seated on a bench if the armies must be seated together? PAST ME BOARD

a. 544 b. 576
c. 466 d. 624

82. There are 5 main roads between cities A and B, and four between B and C. In how many ways can a person drive from A and C and return, going trough B on both trips without driving on the same road twice? PAST ME BOARD

a. 120 b. 240
c. 360 d. 440

83. On a certain examination, the student must answer 8 of the 12 questions, including exactly 5 of the first 6. In how many ways can he write the examination? PAST ME BOARD

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a. 110 b. 120
c. 60 d. 240

84. How many different sums of money can be made from a penny, a nickel, a dime, and a quarter? PAST ME BOARD

a. 12 b. 13
c. 14 d. 15

85. How many line segments can be formed with 6 distinct points , no two of which are collinear?
PAST ME BOARD

a. 15 b. 17
c. 20 d. 22

86. How many triangles are determined by the vertices of a regular hexagon? PAST ME BOARD

a. 40 b. 20
c. 30 d. 10

87. In a toss of a coin, the head is twice as likely to occur as a tail. Find the probability of getting 6 heads if this coin is tossed 15 times.

a. 0.0223 b. 0.0123
c. 0.0723 d. 0.0823

88. If 15 people won prizes in the state lottery, in how many ways can these people win first, second, third, fourth and fifth prizes

a. 360,360 b. 235,360
c. 130,340 d. 245,660

89. A player tosses a fair die. If a prime number occurs, he wins that number of pesos, but if a non prime number occurs, he loses that number of pesos. Find the expected winning.

a. 1/6 b. -1/6
c. 1/3 d. -1/3

90. A sample of 3 items is selected at random from a box containing 12 items of which 3 are defective. Find the expected number of defective items.

a. 1/2 b. 3/4

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c. 4/5 d. 3/8

91. There are 10 points A, B … on a plane. How many triangles are determined by these points.

a. 120 b. 240

c. 720 d. 144

92. From Prob 91, How many of these triangles contains the point A?

a. 44 b. 36

c. 18 d. 44

93. Three machines A , B and C produce respectively 50%, 30% and 20% of the total number of item of a factory. The percentages of defective output of these machines are 3%, 4% and 5%. If an item is selected at random, find the probability that the item is defective.

a.0.037 b. 0.012

c. 0.045 d. 0.047

94. From prob 93: Suppose an item is selected at random and found to be defective, find the probability that the item was produced by machine A.

a. 115/37 b. 12/37

c. 11/37 d. 18/37

95. A player tosses two fair coins. He wins P 1 if 1 head appears, P 2 if 2 heads appear. On the other hand, he loses P 5 if no heads appear. Determine the expected value of the game.

a.-0.50 b. 0.75

c. 0.25 d. -0.25

96. In how many ways can 3 Americans, 4 Frenchmen, 4 Danes and 2 Italians sit in a round table so that the same nationality sit together.

a. 59,232 b. 42,122

c. 41,472 d. 45,122

97. There are 12 students in a class. In how many ways can the 12 sudents take 3 different tests if 4 students are to take the test.

a. 33,450 b. 34,650

c. 31,250 d. 35,450

98. How many 3 digit numbers, each less than 500, can be formed from the digits 1, 3, 4 and 6 and 7.

a. 24 b. 36

c. 18 d. 27

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99. A pack of cards contain 52 cards: 13 spades, 13 clubs, 13 hearts and 13 diamonds. Of the 52 cards, 4 are aces one from each suit. The hearts and the diamonds are colored red , the spades and clubs are black. Four cards are drawn from the pack , each card being returned to
the pack before the next card is drawn. Find the probability that all are clubs.

a. 1/4 b. 1/100
c. 1/256 d. 1/512

100. Refer to problem 99. If 5 cards are drawn simultaneously, find the probability to get all the 4 aces.

a. 1/54145 b. 2/54145
c. 4/54145 d. 9/54145

101. A fair coin is tossed 10 times. Compute the probability of getting at least 7 heads.

a. 9/64 b. 11/64
c. 7/64 d. 13/64

102. A fair die is tossed 8 times. What is the probability of obtaining the faces 5 and 6 twice and each of the other once.

a.0.004 b.0.005
c. 0.006 d. 0.007

103. The painted light bulbs produced by a company are 50% red, 30% blue and 20% green. In a sample of 5 bulbs, find the probability that 2 are red,1 is green and 2 are blue.

a. 0.07 b. 0.08
c. 0.09 d. 0.10

104. Suppose 2% of the people on the average are left handed. Find the probability that exactly 3 are left handed among 100 people.

a. 0.1804 c. 0.1791
c. 0.1922 d. 0.1167

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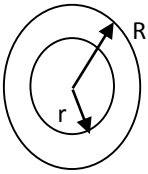
105. Suppose 220 misprints are distributed randomly throughout a book of 200 pages. Find the probability that a given page contains 2 or more misprints.

a. 0.301 b. 0.299
c. 0.276 d. 0.223

SOLUTION

51. Prob = (5C2)/ (8C2) = 5/14
52. 10C6 = 210 (a)

53. 12! /(2!4!6!) = 13,860 (b)
54. P(man) = 10/30 P(brown eyes) = 1/2
P(man $\cap$ brown eyes) = 10/30 x 1/2 = 1/6
P(man $\cup$ brown eyes) =
P(man) + P(brown eyes) - P(man $\cap$ brown eyes)
= 10/30 + 1/2 – 1/6 = 2/3 (d)

55. 

R = 2r

Probability of being closer to the center =
area of the small circle / area of big circle
$\pi r^2 / \pi(2r)^2 = 1/4$ (c)

56. Probability of hitting the target =
1/4(3/5) + 3/4(2/5) + 1/4(2/5) = 11/20
Note: A hit, B miss + A miss, B hit + A hit, B hit = 11/20
57. probability of getting a reject = 1/1000
probability of getting a non reject = 999/1000
probability of getting at least one reject =
$1 – (999/1000)^{200} = 0.18135$ (b)
58. 10 x 8 x 6 = 480 (b)

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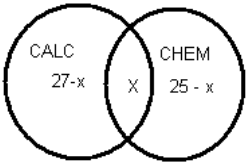
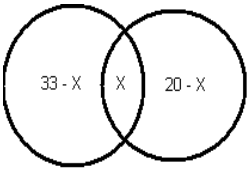
59. 2 x 4! x 4! = 1152 (a)
60. The number of games = 13C2 = 78
61. probability = 3C2 / 8C2 = 3/28 (b)
62. prob = (4C1 x 6C1) / 10C2 = 8/15 (a)
63. Prime = 2, 3 and 5 P(prime) =3/6 = 1/2
    P(not prime) = 1-1/2 = 1/2
    For: (Prime + Not Prime) ^{5} ,
    coefficient of Prime^3 Not Prime^2 = 5C2 = 10
    Probability of getting prime number =
    $10(1/2)^3(/2)^2 = 5/16 = 0.3125$ (b)
64. 1/9 + 1/9 + 1/9 = 1/3 (d)

65. 7/(2 +7) = 7/9 (b)
66.

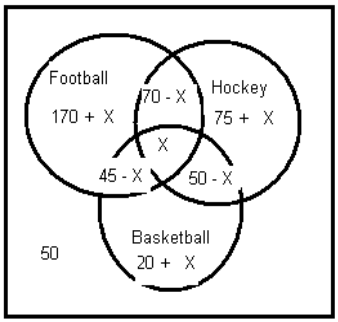


    27 –x + x + 25 – x = 40 ► x = 12 (a)
67.



    33 – x + x + 20 – x = 40 x = 13 (b)

68.



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Note: For football: $285 - (70 - x) - x - (45 - x) = 170 + x$ For Hockey: $195 - (70-x) - x - (50-x) = 75 + x$

$115 - (45-x) - x - (50-x) = 20 + x$

$285 + 75 + x + 50 - x + 20 + x = 450$

$x = 20$

Hockey only: $75 + x = 75 + 20 = 95$ (c)

69. Mean = $1(0.2) + 2(0.28) + 3(0.18) + 4(0.23) + 5(0.11) = 2.77$

70. Mean = $40{,}000(0.3) - 10{,}000(0.7) = 5000$

71. $(T + H)^3 = T^3 + 3T^2H + 3TH^2 + H^3$

for 3 Heads $p = 1/8$

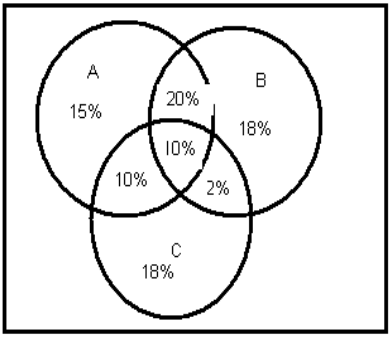
2 Heads $p = 3/8$

1 Head $p = 3/8$

no head $p = 1/8$

Mean = $(1/8)(8) + (3/8)(3) + (1/8)(1) - X(1/8) = 0 \quad X = 11$

72.



The sum of

15% + 20% + 10% + 10% + 18% + 2% + 18% =93%

% that did not smoke = 100 – 93 = 7% (a)

73. TOLENTINO T = 2 N = 2 0 = 2

9!/(2!2!2!) = 45,360 (a)

74. (10C3)/ (15C3) = 24/91 (b) (Note: Choose 3 out of 15 – 5 = 10 non defective)

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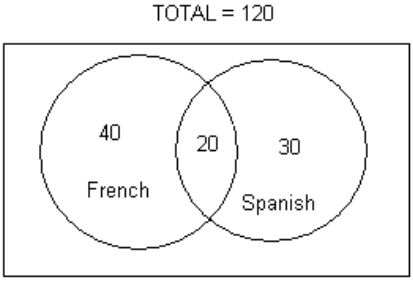
75.



Probability of studying French or Spanish =
(40 + 20 + 30)/ 120 = 3/4 (c)

76. Possible combinations:

The first number should be 5 or 6 and the last number could be any number.

There are 20 possible combinations.

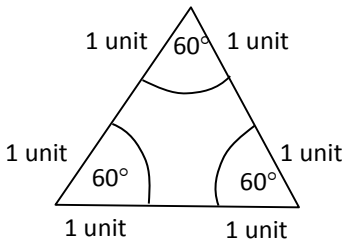
5	1	5	2	5	3	5	4	5	5	5	6
1	5	2	5	3	5	4	5			6	5
6	1	6	2	6	3	6	4	6	6		
1	6	2	6	3	6	4	6				

Ans. 20/36 = 5/9

77. GGGBBB
BGGGBB
BBGGGB
BBBGGG

Possible ways , the girls are together = 4 x 3! x 3! = 144
Prob = 144/6! = 1/5 (a)

78.



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Area of the 3 sectors = $3(\ 1/2(1)^2(\ 60^0)\ ) = \pi/2$
Area of the triangle = $(3)^2\sqrt{3}\ /\ 4 = 9\sqrt{3}/4$
Probability that distance is greater than $\dfrac{9\sqrt{3}\ /\ 4 - \pi\ /\ 2}{9\sqrt{3}\ /\ 4} = 0.597$

79. C(n,2) = 91 Use trial and error for the 4 choices.
n = 14 (c)

80. 8 x 2 x 10 = 160 (a)
81. AAAAMMM
MAAAAMM
MMAAAAM
MMMAAAA
Possible ways the armies are together.
4 x 4! x 3! = 576 (b)
82. For Cities A and B possible ways = 5P2 = 20
For B and C possible ways = 4P2 = 12
Possible ways = 20 x 12 = 240 (b)
83. 6C5 x 4C3 =120 (b) (Note he must answer the 1st 5 , so only 6 questions remain, and he must answer only 3)
84. 4C1 + 4C2 + 4C3 + 4C4 = 2^4 -1 = 15 (d)
85. 6C2 = 15 (a)
86. 6C3 = 20 (b)

87. P(H) = 2/3 P(T) = 1/3
Coefficient of $H^6\ T^9$ in $(H + T)^{15}$ is 15C6 = 5005
Then Required Probability = $5005(2/3)^6(1/3)^9 = 0.0223$
88. 15P5 = 360,360
89.

Number	Prob	Lose
1	1/6	-1 (Not Prime)
2	1/6	2 (Prime)
3	1/6	3 (Prime)
4	1/6	-4 (Not Prime)

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5 1/6 5 (Prime)
6 1/6 6 (Not Prime)

Mean = 1/6(-1) + 1/6(2) + 1/6(3) + 1/6(-4) + 1/6(5) - 1/6(6) = -1/6 (b)

90. Number of possible defectives x = (0, 1, 2, 3)
If x = 0 Prob = (3C0 x 9C3) / 12C3 = 84/220
x = 1 Prob = 3C1 x 9C2 /12C3 = 108/220
x = 2 Prob = (3C2 x 9C1) /12C3 = 27 /220
x = 3 Prob = (3C3 X9C0) /12C3 = 1/220

mean defective = 0 x 84/220 + 1 x 108/220 + 2 x 27/220 + 3 x 1/220 = 3/4 (b)

91. 10C3 = 120 (a)

92. 9C2 = 36 (b)

93. Probability that item is defective =
50% x 3% + 30% x 4% + 20% x 5% = 0.037

94. P(it is produced by machine A $\cap$ defective) =
50% x 3% = 0.015
P(it is produced by machine A given that it is defective) =
0.015/0.037 = 15/37 (a)

95. $(T + H)^2 = T^2 + 2TH + H^2$
1 H prob = 2/4 Prize = P 1
2 H prob = 1/4 Prize = P 2
0H or all T prob= 1/4 Prize = - P 5 (loses)
mean = 1(2/4) + 2(1/4) – 5(1/4) = P -0.25 (d)

96. The four nationalities can be arrange in a circle in 3! ways
The number of ways they can sit together = 3! x (3! x 4! x 4! x 2!) = 41,472 (c)

97. 12! /(4! x 4! x 4!) = 34,650 (b)

98. We can use only 1, 3 , 4 in the 1^{st} 2 digits
4 x 3 x 2 = 24 (a)

99. (13/52)(13/52)(13/52)(13/52) = 1/256 (c)

100. (4C4)(48C1)/ (52C50 = 1/54145

101. (10C7 + 10C8 + 10C9 + 10C10)/2^{10} = 11/64 (b)

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102. prob =

$$\left(\frac{8}{2,2,1,1,1,1}\right)(1/6)^2(1/6)^2(1/6)^1(1/6)^1(1/6)^1(1/6)^1 = \frac{8!}{2!2!}(1/6)^8$$

= 35/5832 = 0.006 (c)

103. prob = $\left(\frac{5}{2,1,2}\right)(0.5)^2(0.3)^1(0.2)^2 = \frac{5!}{2!2!}(0.5)^2(0.3)^1(0.2)^2$ = 0.09(c)

104. mean = 2% x 100 = 2 prob = $\frac{e^{-2}(2)^3}{3!}$ = 0.1804 (a)

105. mean = 220/200 = 1.1

prob that a given page contains 2 or more misprints

= 1- prob that a given page contains 0 or 1 misprint)

= $1 - \left(\frac{e^{-1.1}(1.1)^0}{0!} + \frac{e^{-1.1}(1.1)^1}{1!}\right)$ = 0.301 (a)

PROBLEM SET 8 (COMPLEX NUMBERS)

1. Compute the absolute value of 3 + 4i . PAST CE BOARD
   a. 3 b. 4
   c. 5 d. 6
2. Compute the absolute value of 4 – 5i
   a. 6.4 b. 6.2
   c. 6.3 d. 6.5

3. Find the argument of 4 – 6i
   a. -67.3^0 b. -56.3^0
   c. -67.1^0 d. -77.6^0
4. If A = - 2 – 13i and B = 3 + i4. What is A/B ? PAST EE BOARD
   a. -18/25 + 1/25i b. -58/25 – 31/25i
   c. 58/25 + 1/25i d. 18/25 -13/25i
5. Simplify (3 + 5i)(5 – 3i)(3 + 2i).
   a. 58 – 108i b. 58 + 108i
   c. 45 – 112i d. 56 – 106i
6. Find the value of x that satisfies $x^2 + 36 = 9 – 2x^2$. PAST ME BOARD

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a. $\pm 6i$ b. $\pm 9i$
c. $\pm 3i$ d. $\pm 2i$

7. Convert -5 + 3i to polar form.
   a. $5.83 \angle 139$ b. $5.83 \angle 149$
   c. $5.77 \angle 139$ d. $5.36 \angle 122$
8. Convert $5.67 \angle 23^0$ to rectangular form.
   a. 4.22 + 3.14i b. 4.33 + 2.12i
   c. 5.34 + 3.22i d. 5.22 + 2.22i
9. Reduce $\frac{4(\cos 30 + i \sin 30)}{3(\cos 60 + i \sin 60)}$ to equivalent polar form.
   a. $1.33 \angle 30^0$ b. $1.33 \angle -30^0$
   c. $0.75 \angle 40^0$ d. $0.75 \angle -40^0$
10. $(1 + i)^7$ is
   a. 7+8i b. 8- 8i
   c. 3 + 8i d. 4 – 8i
11. Find the value of y in (3x + 4yi)(6 -7i) = 3 + 5i
   a. 1/20 b. 1/10
   c. 3/20 d. 1/5
12. If $x + yi = 4e^{3i}$, the value of x is
   a. -3.96 b. -4.33
   c. 3.44 d. -4.45
13. ln(3 + 4i) is? PAST EE BOARD
   a. 1.61 – 0.88i b. 1.61 + 0.93i
   c. 1.44 – 0.67i d. 1.45 + 0.93 i
14. The absolute value of 5 + yi is 6.403. What is the value of y ?
   a. 2 b. 3
   c.4 d. 5
15. Simplify $(\sqrt{-9})(\sqrt[3]{-343})$ PAST EE BOARD
   a. 22i b. -23i
   c. -21i d. -22i

16. Simplify $4i^3$ times $2i^2$ PAST EE BOARD
   a. 8i b. 4i

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c. 2i d. 16i

17. Simplify $i^{3219} - i^{427} + i^{18}$ PAST EE BOARD
    a. -1 b. 1
    c. 2 d. - 2
18. Simplify $i^{1997} + i^{1999}$ PAST ECE BOARD
    a. 1 b. 2
    c. -1 d. 0

19. What is the exponential form of 10 + 12i ?
    a. $15.62e^{0.216i}$ b. $15.62e^{0.976i}$.
    c. $15.62e^{0.476i}$ d. $15.62e^{0.876i}$
20. Find the value of x in 3x + 4y + 3yi + 15 – 3i = 0
    a. -6.33 b. 2.33
    c. 1.33 d. -4.33
21. If 3x + 4y – 6 + 7i = 0 , what is the value of (x + yi)(2 + 4i)
    a. 5+11.5i b. -3 + 11.5i
    c. 3 – 11i d. 2 + 11.5i
22. If A = 3 + 2i and $B = 3e^{-3i}$ and x + yi = A/B , what is the value of x?
    a. -1.084 b. 1.112
    c. -1.222 d. 1.322
23. If $A = 3\angle 30^0$ $B = 3e^{-3i}$ and C = 3 – 4i , find the absolute value of ABC.
    a. 45 b. 50
    c. 55 d. 60
24. The third principal of -46 – 9i
    a. 3 – 4i b. 2 – 3i
    c. 3 – 5i d. 4 – 3i
25. The 4^{th} principal of 28-96i
    a. 1 + 2i b. 1 -3i
    c. 1 + 3i d. 2 -3i
26. Let $f(x) = x^5 + 3x^4 + 2x^2$ and f(z) = 1024 + 1232i , the value of z is
    a. 2 + 4i b. 2 – 4i
    c. 3 + 2i d. 1 + 2i

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27. $5 \text{ cis } 40 + 4 \text{ cis } 60 - 7 \text{ cis } 67 = X\angle\theta$, θ is
    a. 4.23 deg b. 3.45 deg
    c. 4.33 deg d. 5.32 deg

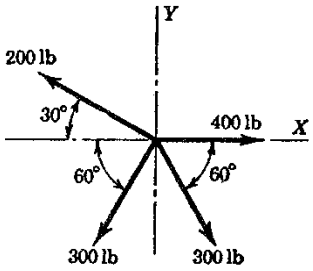
28. The first root of $(1 + i)^{1/5}$ is
    a. 1.03 + 0.133i b. -1.03 + 0.233i
    c. 1.06 + 0.168i d. 1.23 – 1.061i

29. If X is a unit vector at 120 degrees angle, determine the vector sum $1 - X + X^2$ in polar form. PAST EE BOARD
    a. $2\angle -30^0$ b. $2\angle -60^0$
    c. $3\angle -30^0$ d. $3\angle -60^0$

30. If $A = 40e^{j120 \text{ deg}}$ $B = 20\angle -40^0$ and C = 26.46 + j0 , Find the value of A + B + C in polar form.
    a. $30.91\angle 44.82$ b. $33.91\angle 34.82$
    c. $40.91\angle 44.82$ d. $30.91\angle 65.12$

31. Simplify: $\dfrac{(2+3i)(5-i)}{(3-2i)^2}$
    a. $\dfrac{221-9i}{169}$ b. $\dfrac{21+52i}{13}$
    c. $\dfrac{-7+17i}{13}$ d. $\dfrac{-90+220i}{169}$

32. Find the resultant of the system of concurrent forces shown.



    a. 477 N b. 481 N
    c. 511 N d. 233 N

33. From problem 32, find the angle that the resultant makes with the x axis.

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a. 298.4^0 b. 233.1^0
c. 244.3^0 d. 133.4^0

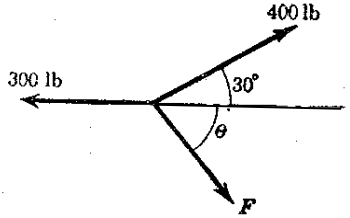
34. EE Board March 1998
Three vectors A, B and C are related as follows:
$\frac{A}{B} = 2$ at 180^0, A + C = - 5 + j 15, C is the conjugate of B. Find A.

a. 5 – j5 b. -10 + 10j
c. 10 – 10j d. 15 + j 15

35. Evaluate the terms of the Fourier Series $2e^{j\pi t} + 2e^{-j\pi t}$ when t =1 PAST EE BOARD

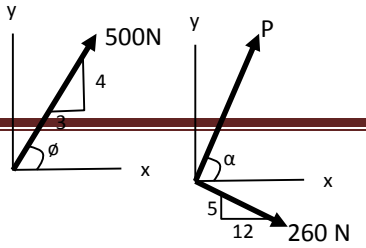
a. 2 + j b. 2
c -4 d. 2 + j2

36. The 300 lb force and the 400 lb force is to be held in equilibrium by a third force acting at an unknown angle with the horizontal. Determine the value of F and θ.



a. $205.31, 103^0$ b. $212.56, 67^0$
c. $222.22, 81^0$ d. $234.12, 111^0$

Problems 37, 38
The force of 500 N is the resultant of the forces P and 260 N acting as shown.
CE BOARD Nov 2006



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37. Find the value of α.

a. 92.12 deg b. 77.23 deg
c. 83.16 deg d. 56.78 deg

38. Find the value of P.

a. 503.59 N b. 511.22 N
c. 489.22 N d. 399.34 N

39. A particle is in equilibrium under the action of 4 N due north, 8 N due west , $5\sqrt{2}$ N South East and P. What is the magnitude of P

a. 2.16 b. 3.16
c. 4.21 d. 3.99

40. From problem 39. What is the direction of P.

a. N 71.6^0 E b. N 34.33^0 W
c. S 33.45^0 W d. N 18.43^0 E

41. Two forces A and B act on the same point. If A = 50 , 30 degrees and B = 40 N, 120 degrees. What is the equilibrant force? ECE APRIL 2004

a. 64 N, 81 deg b. 64 N, 249 deg
c. 106 N, 224 deg d. 106 N, 44 deg

42. Two forces of 30 N at 90 degrees and 40 N at 180 degrees act at the origin. Determine the magnitude and direction of the equlibrant force.

a. 50 N at 143 deg b. 50 N at 53 deg
c. 50 N at 323 deg d. 50 N at 470 deg

43. What is the resultant of a displacement 6 miles North and 9 miles east.

a. 11 miles N 56^0 E b. 11 miles N 54^0E
c. 10 miles N 56^0 E d. 10 miles N 54^0 E

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44. Find the resultant of the following forces : F_1 = 600 N at 40^0 in 1st quadrant, F_2 = 800 N at 20^0 in the second quadrant and F_3 = 200 N at 60^0 in the 4th quadrant. PAST ME BOARD

a. 522.67 at 63.43^0 in the 2nd quadrant
b. 453.45 at 53.34^0 in the 1st quadrant
c. 523.65 N at 56.43^0 in the 3rd quadrant
d. 321.45 N at 34.43^0 in the 4th quadrant

Note: ENTER: MODE 2 (COMPLEX MODE)
For argument: ENTER: SHIFT 2 1
For absolute value: ENTER: SHIFT hyp
To convert from rectangular to polar form: ENTER: SHIFT 2 3
To convert from polar to rectangular:
ENTER: SHIFT 2 4
To input i: ENTER: ENG
To input cis or ∟ ENTER: SHIFT (-)
To input the conjugate. ENTER: SHIFT 2 2

SOLUTION:

1. arg(3 + 4i) Ans. 5 (c)
2. | 4 – 5i| = 6.403 Ans. a
3. arg(4 – 6i) Ans. -56.3 (b)
4. ENTER: $\frac{-2-13i}{3+4i}$ Ans. -58/25 – 31/25i (b)
5. ENTER: (3 + 5i)(5 – 3i)(3 + 2i) Ans. 58 + 108i (b)
6. Rewrite as $3x^2 + 0x + 27 = 0$ USE MODE 5 3
   Ans. 3i, -3i (c)
7. ENTER: -5 + 3i SHIFT 2 3 Ans. 5.83∟149 (b)
8. ENTER: 5.67∟ 23 SHIFT 2 4 Ans. 5.22 + 2.22i (d)

9. 4(cos 30 + i sin 30) = 4∟ 30 and 3 (cos 60 + i sin 60)= 3 ∟ 60
   4∟ 30 ÷ 3 ∟ 60 = 1.33 ∟-30 (b)

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10. ENTER: $(1 + i)^3 (1 + i)^3 (1 + i)$ Ans. $8 – 8i$ (b)

11. $3x + 4yi = \frac{3+5i}{6-7i} = -1/5 + 3/5i \blacktriangleright x = -1/15$
and $y = 3/20$ (c)

12. For $4e^{3i}$: Go to radian mode. ENTER: $4\angle 3$ and convert to rectangular form.
Ans. $x + yi = -3.96 + 0.56448i$ Ans. $x = -3.96$

13. Go to radian mode, Convert $3 + 4i$ to polar form.
polar form = $5\angle 0.9273$
$5\angle 0.9273 = 5e^{0.9273i} \blacktriangleright \ln(5e^{0.9273i}) = \ln 5 + 0.9273i =$
$1.61 + 0.9273i$

14. Use trial and error: $| 5 + 4i| = 6.403 \blacktriangleright y = 4$

15. Just input in the CALCULATOR. Ans. $-21i$

16. Just input in the CALCULATOR. Ans. $8i$

17. Divide 3219 by 4 , remainder = 3 , divide 427 by 4, remainder = 2, divide 18 by 4, remainder = 2 . Then evaluate $i^3 – i^2 + i^2$ Ans. -1

18. Divide 1997 by 4, remainder = 1 , divide 1999 by 4, remainder = 3
$i^1 + i^3 = 0$

19. GO TO radian MODE. Convert $10+ 12i$ to polar form.
Ans. $15.62e^{0.876i}$

20. $3x + 4y + 15 + i(3y – 3) = 0 + 0 i$
$3x + 4y = -15$
$0x + 3y = 3$ Use MODE 5 1
$X = -6.33$ (a) and $y = 1$

21. $3x + 4y – 6 + 7i = 0$
$3x + 4y = 6 – 7i$, then $3x = 6$, $x = 2$ and $4y = -7$,
$y = -7/4$ $x + yi = 2 – 7/4i$
$(x + yi)(2 + 4i) = (2-7/4i)(2+4i) = -3 + 11.5i$

22. $A = 3 + 2i$ $B = 3 e^{-3i}$
(For B go to RADIAN MODE and convert $3\angle -3$ to rectangular form)
$3e^{-3i} = -2.97 -0.4234i$

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Then divide 3 + 2i and - 2.97 – 0.4234i
A/B = -1.084 – 0.519i Thus x = -1.084

23. Go to DEGREE MODE:
INPUT: |(3 ∟30)(3∟ -3^{r})(3 – 4i)|
Ans. 45

24. Use trial and error. (2 -3i)^{3} = -46 – 9i
25. Use trial and error. (1+ 3i)^{3}(1 + 3i) = 28 -96i
26. Use trial and error. ENTER: $x^3x^2 + 3x^2x + 2x^2$
CALC 2 – 4i= 1024 + 1232 i
Ans. (b)

27. INPUT arg(5∟40 + 4∟60 - 7∟67) Ans. 4.333^0
28. Use trial and error.
(1.06 + 0.168i)^{3}(1.06 + 0.168i)^{2} almost 1 + I
29. INPUT 1 – X + X^2 CALC 1∟120 and convert to polar form.
Ans. 2∟-60
30. Input 40∟120 + 20∟-40 + 26.6 + 0i and convert to polar form.
Ans. 30.91∟44.82
31. Just input in the calculator. ANS. -7/13 + 17/13i
32. Input 400∟0 + 200∟ 150 + 300∟240 + 300∟ 300 and convert
to polar form.
Ans. 476.98∟-61.61
33. angle = 360 – 61.61 = 298.39
34. A = (2∟180)B or A = -2B
Let B = x + jy then Conjugate of B = x – jy = C
A = -2(x + jy) = -2x – 2jy
C = x – jy
A + C = - 5 + j 15
A = - C – 5 + J15
A = - (x – jy) – 5 + j15 = - x + jy - 5 + j15
-2x – 2jy = -x + jy - 5 + 15j
-x – 3jy = - 5 + 15j
-x = -5 and -3y = 15
x = 5 y = - 5

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CALCULATOR TECHNIQUE

C = conjugate of B ► A + conjugate B = - 5 + j15

$A = 2B\angle 180$, then $B = \frac{A}{2\angle 180}$

Therefore: $A + \text{conjugate}\left(\frac{A}{2\angle 180}\right) = -5 + j15$

Input : $A + \text{Conjg}\left(\frac{A}{2\angle 180}\right)$ CALC -10 + 10i

(Note: substitute the choices) which is -15 + 15i (b)

35. Input $2\angle \pi^r + 2\angle -\pi^r$ Ans. – 4 (c)

36. For equilibrium, resultant must be zero.

$400\angle 30 + 300\angle 180 + F\angle \alpha = 0$

$F\angle \alpha = -(400\angle 30 + 300\angle 180) = 205.31\angle -103.06^0$

$\theta = 103.06^0$ and R = 205.31 (a)

37. Compute the angle of the 500 N force.

$\theta = \tan^{-1} 4/3 = 53.13^0$

angle of the 260 N is $\tan^{-1} 5/12 = 22.62^0$

Then: $500\angle 53.13^0 = P\angle \alpha + 260\angle -22.62^0$

$P\angle \alpha = 500\angle 53.13^0 - 260\angle -22.62^0$

Input $500\angle 53.13^0 - 260\angle -22.62^0$ and convert to polar form.

Ans. $503.59\angle 83.16$ Ans. 83.16

38. Ans. 503.59 (a)

39. $4\angle 90 + 8\angle 180 + 5\sqrt{2}\angle -45 + P\angle \theta = 0$

$P\angle \theta = -(4\angle 90 + 8\angle 180 + 5\sqrt{2}\angle -45) = 3.16\angle 18.43^0$

Ans. 3.16 (b)

40. Bearing N 71.57° E (a)

41. $50\angle 30 + 40\angle 120 + E\angle \theta = 0$ $E\angle \theta = -(50\angle 30 + 40\angle 120) =$ $64.03\angle -111.34^0$ or $64.03\angle 248.66$ (b)

42. $30\angle 90 + 40\angle 180 + E\angle \theta = 0$

$E\angle \theta = -(30\angle 90 + 40\angle 180) = 50\angle -36.87$ or $50\angle 323.13^0$

43. resultant = $6\angle 90 + 9\angle 0 = 10.8166\angle 33.69$

Ans. 10.82 N 56.31^0 E (a)

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44. Resultant = $600\angle 40 + 800\angle (180-20) + 200\angle (360-60) = 522.67\angle 111.57$ 522.67 at 180 -111.57= 68.43 at 2nd quadrant (a)

PROBLEM SET 9

(VECTORS)

Given Vector A and Vector B A = 6.7i + 8.37j B = -2.53 – 5.55j
PAST CE BOARD

45. The magnitude of the resultant of A and B .
    a. 4.5 b. 3.45
    c. 5.04 d. 7.34
46. The horizontal and vertical components of the resultant vector.
    a.(4.18, 2.82) b. (2.34,2.81)
    c. (3.41,4.21) d. (1.23, -3.4)
47. The angle that the resultant makes with the horizontal.
    a. 37 deg b. 39deg
    c. 34 deg d. 15 deg

Let A = 4i + 2j – 7k and B = 3i – 4j + 5k .
Prob 48 - 58

48. The magnitude of the resultant of A and B is
    a. 7.55 b. 9.21
    c. 8.33 d. 7.67
49. 4A + 5B is
    a. < 31 -12 - 3 > b. < 11 -12 3 >
    c. < 11 -12 - 37> d. < 31 -13 - 3 >
50 . The magnitude of 3A – 2B is
    a. 12.33 b. 34.54
    c. 36.78 d. 44.23
51. The dot product of A and B
    a. 34 b. 45
    c. -31 d. -35
52. The angle between A and B.
    a. 91.2 deg b. 145.4 deg
    c. 121.86 deg d. 81.23 deg
53 The unit vector of A

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a. < 0.68 0.14 -0.84 > b. < 0.37 0.21 0.84 >
c. < 0.48 0.24 -0.84 > d. < 0.41 0.14 -0.84 >

54. The cross product A x B
a. < -18 -41 -22 > b. < -19 21 33 >
c. < -19 21 33 > d. < -19 21 33 >

55. The magnitude of the cross product of A x B
a. 43.67 b. 44.23
c. 49.89 d. 51.23

56. The angle that vector B makes with the y axis.
a. 124.4^0 b. 111.2^0
c. 101.2^0 d. 187.1^0

57. The angle that vector A makes with the z axis.
a. 111.2^0 b. 147.4^0
c. 133.1^0 d. 156.2^0

58 The direction cosine of the resultant of A and B.
a. < 0.9271 -0.265 -0.265 > b. < 0.9571 -0.262 -0.262 >
b. < -0.9971 -0.265 -0.265 > c. < 0.9671 -0.765 -0.265 >

59. Find the magnitude of the resultant F = 3i + 4j + 12k PAST CE BOARD
a. 12 b. 13
c. 14 d. 15

60. Find the length of the vector < 2, 1, 4 > PAST EE BOARD
a. 3.45 b. 3.56
c. 4.58 d. 5.56

61. Add the vectors < -4, 7> + < 5, -9> PAST EE BOARD
a. < -1 2> b. < 1 -2>
c. < 1 -3> d. < 3 1 >

62. What is the cross product A x B of the vectors A = i + 4j + 6k and B = 2i + 3j + 5k . PAST ME BOARD
a. 2i + 7j+ 3k b. 3i – 4j + 4k
c. 4i -4j + 2k d . 2i + 7j – 5k

63. What is the angle between the two vectors A and B if A = 4i + 12j + 6 k and B = 24i -8j + 6k ?
a. 94.56^0 b. 78.33^0
c. 84.32^0 d. 44.56^0

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64. Find the area of the triangle whose vertices are (2, -1,3), Q(1,2,4) and R(3,1,1)?

a. 3.45 b. 7.51
c. 4.74 d. 5.11

65. Find the volume of the parallelepiped whose edges are represented by A = 2i – 3j + 4k, B = i + 2j – k and C = 3i – j + 2k?

a. 4 b. 5
c. 6 d. 7

66. Suppose F has a magnitude of 6 lb and 1/6 π is the radian measure of the angle given its direction. Find the work done by **F** in moving an object along a straight line from the origin to the point (7,1), where distance is measured in feet.

a. 45.2 ft lb b. 34.45 ft lb
c. 39.37 ft lb d. 33.34 ft lb

66. A constant force of 9 N passes trough the point (3, -4) and (10, -2). What is the work done by a force in moving the object from the origin to (-4, -2). Distance is measured in meters.

a. 45.56 Nm b. 71.23 Nm
c. 32.44Nm d. 39.6 Nm

67. A force of F = 3000 lb acts from A(4, -2, 5) to B(3, -5, 7). Find the components of the force.

a. < -801.78 -2405.35 1603.57 >
b. < 56.34 666.12 1012.33>
c. < 891.22 123.23 1111.22 >
d. < 33.67 890.122 – 1012.34>

68. What is the area of the triangle formed by the points A(3, 2, -1) , B(-11, 3, 6) and C(2, -5,9)?

a. 91 b. 90
c. 89 d. 88

69. What is the angle between the vectors A = 3i – 2j and B = 2i + j?

a. 78.12 b. 60.26
c. 71.22 d. 73.22

70. Given the vectors A = -8i +4 j and B = 7i – 6j

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Find the scalar projection of A into B.

a. 6.78 b.- 8.22
c. 9.33 d. -8.68

71. From problem 70, Find the vector projection of A into B

a.<-7.21 4.44> b.< 4.56 7.21>
c.< 2.24 -4.56> d. < -6.59 5.65 >

72. PAST CE BOARD The resultant of the concurrent forces has a magnitude of 1000 kN and acts trough the origin and the points (2,3,4). What is the z component of the resultant force?

a. 742.78 kN b. 778.23 kN
d. 811.22 kN d. 911.22 kN

SITUATION: PROBLEMS 73 – 74

PAST CE BOARD A concurrent force system in space is composed of 3 forces described as follows . A = 100 kN acts through origin and (3,4,2). B = 60 kN and acts through the origin and (4, 1, -2). C = 80 kN and acts trough the origin and
(2, -3, 3).

73. What is the resultant force.

a. 123.4 b. 159.3
c. 122.2 d.111.2

74. What is the y component of the Resultant Force.

a. 36.2 b. 22.2
c. 44.1 d. 55.2

75. What is the perpendicular distance of the line AB that passes from A (3,4,5) to B (4, -4,7) from the point C(6, 1, -7) ?

a. 8.3 b. 11.2
c. 9.8 d. 12.72

76. Find the value of c so that 2i + 4j + 5k and i + cj – 2k are perpendicular.

a. 1 b. 2
c. 3 d. 4

77. Find the volume of the parallelepiped formed by the vectors 1i + 1j + 1k, 2i – j – 3k, 3j – k.

a. 12 b. 14

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c. 16 d. 18

78. The magnitude of a force is 80 kN. The coordinate of the tail is (0,4,3) and that of the tip is (4.5, 0,3). What is the moment of the force about the origin. PAST CE BOARD

a. 341.22 kN m b. 371.23 kN m

c. 338.83 Kn M d. 412.23 kN m

79. A force of 200 N acts from A(3, -2,5) to B(4, 6, -3). What is the moment vector of the 200 N force about (9, 7, -1).

a. 211.3i – 369.8j – 343.38k

b. 111.3i – 369.8j – 343.38k

c. 911.3i – 369.8j + 343.38k

d. 811.3i + 369.8j – 356.38k

80. A constant force of 300 N acts from (3, -4,5) to (6, 7, -1). Find the work done of this force in moving an object from point A(5, 6, -1) to

B(6, 7, -4) in a straight line. The coordinates are in meters.

a. 811.2 Nm b. 745.1 Nm

c. 911.2 Nm d. 911.2 Nm

81. Find the direction cosine of the vector that is perpendicular to the plane defined by

A = 3i + j – k and B = 2i + 2j – k. .

a. 0.31i + 0.32j – 0.5k b. 0.24i + 0.24j + 0.94k

c. 0.71i - 0.24j + 0.94k d. 0.12i + 0.12 j + 0.94k

82 . A force of 300 N passes trough (1,2, -4) to (4, 8, -1). What is the moment of the force about the x axis?

a. 1211.4 Nm b. 3112.22 Nm

c. 411.5 Nm d.1224.7 N m

83. What is the area of the parallelogram formed by the vectors 3i + 2j and 3j – 4k

a. 12 b. 14

c. 16 d. 17

84. Find the distance between (3, 4, -6) and (5, -1, 4)?

a. 13.33 b. 11.36

c. 12.22 d. 14.12

85. Find the distance between (4, 5,6) and (3, 4, 1)? PAST CE BOARD EXAM

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a. 4.5 b. 4.7
c. 8.1 d. 5.2

86. The distance between (4, 6, 7) and (-6, 9, z) is 10.488 Find the value of z. PAST CE BOARD
a. 6 b. 7
c. 8 d. 9

87. A force F represents a force that has a magnitude of 9 lb and 2/3 π is the radian measure of its direction angle. Find the work done by the force in moving the object from the origin to the point (-4, -2). Distance is in ft.
a. 2.41 b. 3.41
c. 6.71 d. 3.56

88. Two particles represented by the vectors $F_1 = 3i – 5j + 7k$ and $F_2 = 3i + 2j – 7k$ act on a particle and causes it to move along the line from the point A(2, 4,1) to the point B(5, 3, -8). Find the work done. F is in Newton and distance is in meters.
a. 34Nm b. 14Nm
c. 21 Nm d. 24 Nm

89. Three concurrent forces P, Q and F have a resultant of 5 N directed forward and up to the right at $\theta_x = 60$ deg , $\theta_y = 60$ deg and $\theta_z = 45^0$. P = 20 lb and passes trough the origin and the point (2, 1,4). Q = 20 lb and passes trough the origin and the point (5,2,3). Determine the magnitude of the 3rd force.
a. 33.7 lb b. 43.3 lb
c. 67.2 lb d. 23.4 lb

90. A force of P is directed from point A (4,1,4) toward a point B(-3, 4, -1). If it causes a moment of Mz = 1900 lbft, determine the moment of P about the X axis.
a. 1400 b. 1500
c. 1700 d. 2100

NOTE: GO TO VECTOR MODE: ENTER MODE 8
1.To store a 2 dim vector to VCT A.
ENTER SHIFT 5 1 1 2 Input the data and enter AC.
2. To store a 3 dim vector to VCT A.

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ENTER: SHIFT 5 1 1 3 Input the data and enter AC.
3. To store a 2 dim vector to VCT B.
ENTER: SHIFT 5 1 2 2 Input the data and enter AC.
4. To store a 3 dim vector to VCT B.
ENTER: SHIFT 5 1 2 1 Input the data and enter AC.
5. To get the dot product, ENTER: SHIFT 5 7
6. To get the cross poduct ENTER x (multiplication sign)

7. To get the absolute value: ENTER: SHIFT hyp
8. To access a VECTOR say VCT A : ENTER: SHIFT 5 3
9. To access VCTans , ENTER: SHIFT 5 6

SOLUTION:

STORE VctA = < 6.71 8.37 > AND VctB = < -2.53 -5.55 >
45. Abs(VctA + VctB) = 5.04 (c)
46. VctA + VctB = < 4.18 2.82 > (a) This is stored in VCTAns
47. VctAns ÷ Abs(VctAns) = < 0.8289 0.5592 >
cos θ_x = 0.8289 θ_x = 34^0 (c)
STORE VctA = < 4 2 -7> VctB = < 3 - 4 5 >
48. abs(VctA + VctB) = 7.55 (a)
49. 4VctA + 5VctB = < 31 -12 - 3 > (a)
50. Abs(3VctA – 2 VctB) = 34.54 (b)
51. VctA●VctB = -31 (c) (Note this is dot product)
52. A ● B = AB cosθ
cos θ = (A ● B) ÷ (Abs(VctA)Abs(VctB))
cos θ = -0.5278
θ = 121.86^0
53. VctA ÷ Abs(VctA) = < 0.4815 0.24077 -0.8427 > (c)
54. VctA x VctB = < -18 -41 -22 > This is stored in VctAns
55. Abs(VctAns) = 49.89 (c)
56. VctB ÷ Abs(VctB) = < 0.4242 -0.5656 0.7071 >
cos θ_y = -0.5656 θ_y = 124.4^0 (a)
57. From problem 53. cos θ_z = -0.8427 θ_z = 147.3^0 (b)
58. (VctA + VctB) ÷ abs(VctA + VctB) = < 0.9271 -0.265 -0.265 > (a)
59. Store VtcA = < 3 4 12 >
Abs(VctA) = 13 (b)

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60. Store VctA = < 2 1 4 >
Abs(VctA) = 4.58 (c)

61. Store VctA = < -4 7> and VctB = < 5 -9>
VctA + VctB = < 1 -2> (b)

62. STORE VctA = < 1 4 6 > and VctB = < 2 3 5 >
VctA x VctB = < 2 7 -5> = 2i + 7j – 5k (d)

63. Store VctA = < 4 12 6 > and VctB = < 24 – 8 6 >
cos θ = (VctA ● VctB) /(abs(VctA)abs(VctB))
θ = 84.32^0

64. Store VctA = < 2 -1 3 > VctB = < 1 2 4 > and VctC = < 3 1 1>
Area = 0.5 abs(VctA xVctB + VctB x VctC + VctC x VctA)
Ans. 4.7434 (c)

65. The volume of the parallepiped is A ● (B x C)
Store VctA = < 2 - 3 4 > VctB = < 1 2 -1> and VctC = < 3 -1 2 >
VctA ● (VctB x VctC) = - 7 Ans. (d)

66. Word by a constant force acting to the line AB is
F●Vector(AB)
Compute 6∟ $(\pi/6)^r$ = 5.196 + 3i
(Go to MODE 2 First)
F = < 5.196 3 > AB = < 7 1 >
Store Vct A = < 5.196 3 > VctB = < 7 1 >
VctA ● VctB = 39.372 ft lb
Store VctA = < 10-3 -2 –(-4) > = < 7 2 >
VctB -= < -4 - 2 >
Work = 9 (VctA ÷ abs(VctA) ● VctB = -39.56 Nm (d)

67. Store Vct A = < 4 -2 5 > VctB = < 3 -5 7 >
F = 3000 (VctB – VctA) ÷ Abs(VctB – VctA)
= < -801.78 -2405.35 1603.57 > (a)

68. STORE VctA = < 3 2 -1 > VctB = < -11 3 6 > VctC = < 2 -5 9 >
Area = 1/2 Abs(VtcA x VctB + VctB x VctC + VctC x Vct A) =88 (d)

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69. STORE VctA = < 3 - 2 > and VctB = < 2 1 >
cos θ = (VctA● VtcB) ÷ (Abs(VctA)Abs(VctB))
θ = 60.255 deg (b)

70. Store VctA = < -8 4 > and VctB = < 7 -6 >
Scalar A●B = ABcosθ scalar projection of A on B is Acosθ
Acosθ = A●B ÷ Abs(B)
(VctA ● Vct B) ÷ Abs(VctB) = -8.67 (d)

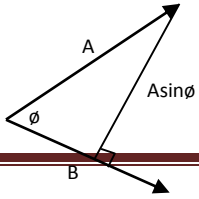
71. Vector projection of A to B is A●B = ABcosθ
Scalar projection of A and B (from prob 70 = -8.67)
Vector projection of A on B is -8.68 (unit vector B)
= 8.68VctB/abs(VctB)
= < -6.59 5.65 > (d)

72. STORE: VctA = < 2 3 4 >
1000VctA/abs(VctA) = < 371.39 557.086 742.781 >
z component = 742.78 kN (a)

73. STORE: VctA = < 3 4 2 > VctB = < 4 1 -2 >
VctC = < 2 -3 3 >
Resultant = 100 VctA /Abs(VctA) + 60 VctB/ Abs(VctB) + 80 VctC/ abs(VctC) = < 142.19 36.203 62.121 >
This is stored in VctAns Abs(VctANs) = 159.33 N (Resultant) (b)

74. From 73 y component = 36.2 (a)

75. Store VctA = < 4 -3 - 4 – 4 7 5 > = < 1 -8 2 >
Vct B = < 3 – 6 4 – 1 5 – (-7) > = < - 3 3 12 >
(Subtract Coordinates of A and C)
A x B = ABsin θ Asin θ is the perpendicular distance of A from
vector B.
Asinθ = (A x B)/abs(B)
perpendicular distance of B on A =
abs (Vct A x Vct B) / abs(Vct A) = 12.723



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76. The dot product of < 2 4 5 > and < 1 c -2 > must be zero.
< 2 4 5 > ● < 1 c - 2 > = 2 x 1 + 4 x c + 5 x -2 = 0 c = 2
(b)

77. Volume of the parallelepiped = A ● (B x C)
Store VctA = < 1 1 1 > VctB = < 2 -1 -3 > VctC = < 0 3 -1 >
VctA ● (VctB x VctC) =18

78. Store VctA = < 4.5 - 0 0 – 4 3 - 3 > = < 4.5 -4 0 >
Store VctB = < 0 4 3 >
Moment of the force about the origin = r x F
= Abs (Vct B x 80 VctA/abs(VctA)) = 338.83 kN m

79. STORE VctA = < 4 – 3 6 –(-2) -3-5 > = <1 8 -8 >
Store VctB = < 3 -9 -2 -7 5 – (-1) > = < -6 -9 6 >
Moment Vector = r x F = VctB x (200VctA/abs(VctA))
= < 211.3 -369.8 -343.38 >
Moment vector = 211.3i – 369.8j – 343.38k (a)

80. Store VctA = < 6 -3 7-(-4) -1 -5 > = < 3 11 -6 >
Store VctB = < 6 -5 7 – 6 - 4-(-1) > = < 1 1 -3 >
Work = Force ● Vct B
= 300(VctA/Abs(VctA)) ● Vct B
= 745.1 Nm (b)

81. Unit vector (Direction Cosine) perpendicular to both A and B is just
the unit cross product
Store VctA = < 3 1 -1> VctB = < 2 2 -1>
Direction Cosine of the vector perpendicular to A and B is
(Vct(A) xVct B)) / abs(Vct A x Vct B) =
< 0.235 0.2357 0.9428 >
Ans. 0.24i + 0.24j + 0.94k (b)

82. STORE VctA = < 4 -1 8 -2 -1 –(-4) > = < 3 6 3 >
STORE Vct B = < 1 2 -4 >
Moment of of the force about the origin =
Vct B x (300 Vct A ÷ abs(VctA)) =
= < 1224.74 -612.3 0 > Mx = 1224.74 N m (d)

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83. Area of the parallelogram with sides of Vectors A and B is Abs(Ax B).
Store VctA = < 3 2 0 > VctB = < 0 3 - 4 >
Area = Abs(VctA x Vct B) = 17 (d)
84. Store VctA = < 3 4 -6> and VctB =< 5 -1 4>
Distance = Abs(VctA - VctB) = 11.36 (b)
85. Store: VctA = < 4 5 6 > VctB = < 3 4 1 >
distance = Abs(VctA - VctB) = 5.196 (d)
86. USE TRIAL AND ERROR:
Store VctA = < 4 6 7> VctB = < - 6 9 z >
Abs(VctA - Vct B) = 10.488
Replace z by the given choices.
The correct choice is z = 8. (c)
87. GO TO Mode 2: $9\angle(2\pi/3)^r$ = -4.5 + 7.794
Then F = < -4.5 7.794 >
Store VctA = < -4.5 7.794 > VctB = < -4 -2 >
Work = VctA ● VctB = 2.412 lb ft (a)
88. Resultant F = < 3 -5 7 > + < 3 2 -7> = < 6 -3 0 >
Vector AB = < 5 – 2 3 – 4 -8 – 1 > = < 3 -1 -9 >
Store VctA = < 6 -3 0 > VctB = < 3 -1 -9 >
Work = VctA ● VctB = 21 Nm (c)
89. Store: VctA = < 5cos 60 5 cos 60 5 cos 45 > =< 2.5 2.5 3.5355 >
Store: VctB = < 2 1 4 > VctC = < 5 2 3 >
20 VctB/Abs(VctB) + 20 VctC/Abs(VctC) + VctD = VctA
VctD = VctA - 20 VctB/Abs(VctB) - 20 VctC/Abs(VctC)
= < -22.45 -8.353 -23.65 > This is stored in VctAns
Abs(VctAns) = 33.665 lb (a)
90. Store: VctB = < -3 – 4 4 -1 -1 – 4 > = < -7 3 -5>
Store: Vct A = < 4 1 4 >
Moment about the origin =
Vct A x P(VctB/Abs(VctB)) = P (VctA x VctB/Abs(VctB))
= P< -1.866 -0.8781 2.0855 >
= < -1.866P -0.8781P 2.0855P >
Thus: Mz = 1900 = 2.0855P P = 911.05
Then: Mx = -1.866P = -1.866(911.05) = -1700 (Ans. (c)

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and My = -0.8781(911.05) = -800

PROBLEM SET 10

MATRICES AND DETERMINANTS

91. CE BOARD MAY 1996

$$B = \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$$

Find the elements of the two matrices BC.

a. $\begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$ b. $\begin{bmatrix} -11 & 8 \\ 19 & -5 \end{bmatrix}$

c. $\begin{bmatrix} -10 & 9 \\ -19 & 6 \end{bmatrix}$ d. $\begin{bmatrix} -11 & 9 \\ -20 & -4 \end{bmatrix}$

92. PAST EE BOARD

$matrix\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} + 2matrix\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$=

a. $\begin{bmatrix} -2 & 4 \\ 2 & 2 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 5 \\ 1 & 5 \end{bmatrix}$

93. Find the element in the third row, third column of the 71 times the inverse matrix of $A = \begin{bmatrix} 1 & 2 & -4 \\ 3 & 1 & 2 \\ -4 & 1 & 5 \end{bmatrix}$

a. 3 b. 4

c. 5 d. 6

94. Evaluate: $\det(\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}^T \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}^2)$

a. 443 b. 546 c. 343 d. 0

95. Find the value of a + 3 b in the matrix equation: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 3 & 3 & 4 \\ 1 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 3 & 4 \\ 1 & 9 & 7 \\ 3 & 1 & 5 \end{bmatrix}$

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a. -0.5 b. 1
c. 1.5 d. 2

96. What is the area of a triangle whose vertices are (3,4) , (-8,21) and (7, 13) ?

a. 78.5 b. 83.5
c. 91.5 d. 94.4

97. PAST CE BOARD Evaluate the determinant:

$$det\begin{bmatrix} 2 & 14 & 3 & 1 \\ 1 & 5 & -1 & 3 \\ 1 & -2 & 2 & -3 \\ 3 & -4 & -3 & -4 \end{bmatrix}$$

a. 489 b. 389
c. 326 d. 452

98. Evaluate the determinant : PAST ECE BOARD

$$\begin{vmatrix} 1 & 6 & 0 \\ 4 & 2 & 7 \\ 0 & 5 & 3 \end{vmatrix}$$

a. 110 b. -101
c. 101 d. -110

99. Given the matrix $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, solve for x and y.

a. -4, 6 b. -4, 2
c. -4, -2 d. -4, -6

100. Evaluate:

$$\det\left[\begin{bmatrix} 3 & 4 & 4 \\ 6 & 6 & 1 \\ 3 & 4 & 8 \end{bmatrix}\begin{bmatrix} 4 & 2 & -1 \\ 1 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix}^T - 3\begin{bmatrix} 4 & 1 & 5 \\ 6 & 1 & 3 \\ -2 & 1 & 4 \end{bmatrix}\right]$$

a. 3412 b. 2212
c. 4312 d. 2172

101. What is the cofactor of element X in the matrix is:

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$$\begin{bmatrix} 2 & 3 & 4 & 8 \\ 5 & -7 & -6 & 3 \\ 7 & 1 & 2 & 3 \\ 5 & 2 & X & -7 \end{bmatrix}$$

a. -401 b. -402
c. 111 d. -111

102 Given: A = $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \\ 4 & 3 & 1 \end{bmatrix}$ and its invers $B = \frac{1}{6}\begin{bmatrix} X & 8 & Y \\ 8 & -10 & 4 \\ 4 & Z & -2 \end{bmatrix}$

Find the value of X , Y and Z

PAST CE Board Nov 1996

a. X = -7, Y = 5, Z = - 2 b. X = 7, Y = 4 , Z = - 2

c. X = 3 , Y = 4, Z = 1 d. X = -9, Y = -5, Z = 0

103. Find the value of x: PAST CE BOARD Exam

$$x = \begin{vmatrix} 4 & -1 & 2 & 3 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$$

a. 27 b. -28
c. 26 d. -29

104. Solve for w in the system of linear equation

x + y + z – 2w = - 4
2y + z + 3w = 4

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$2x + y - z - 2w = 5$
$x - y \quad + w = 4$

a. 1 b. 2
c. 3 d. 4

105. Solve for a in $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 7 & 9 \\ -1 & a & 5 \end{bmatrix} \begin{bmatrix} b \\ c \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$

a. -21 b. 32
c. 45 d. -31

106. Compute the eigenvalues of A = $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

a. 4, -2 b. 5, -1
c. 5, -2 d. 7, 1

107. Compute the eigenvalues of $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

a. 7, -1 b. 4,3
c. -1,7 d. 4,-1

108. Compute the eigenvectors of $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

a. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ b. $\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

c. $\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ d. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

109. Compute the eigenvectors of $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.

a. < 1, 2> and < 4,1 > b. < 2, -1> and < 3, 2>
c. < 4, -1> and < -3,1> d. < 1,-4> and < 4, -2>

110. F = $A^3 + 2A^2 + 3I$ where I is an identity matrix. and

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$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 4 & 2 \\ 5 & 1 & 7 \end{bmatrix},$$

Compute determinant of F.

a. 761211 b. 770612

c. 790122 d.981122

1. To go to MATRIX MODE:

ENTER: MODE 6 AC

2. To store a matrix say 2 x 2 to Matrix A:

ENTER: SHIFT 4 1 1 5 , Input the data and ENTER: AC.

3. To store a matrix say 3 x 3 to Matrix B:

ENTER: SHIFT 4 1 2 1 , Input the data and ENTER: AC.

4. To access a matrix say MATRIX C.

ENTER: SHIFT 4 5

5. To get the determinant of say Matrix A :

ENTER: SHIFT 4 7 SHIFT 4 3

6. To get the Inverse of Matrix B:

ENTER: SHIFT 4 4 x^{-1}

7. To get the transpose of matrix A:

ENTER: SHIFT 4 8 SHIFT 4 3

8. To multiple matrix A and matrix B

ENTER: SHIFT 4 3 SHIFT 4 4

9. To edit the contents of matrix A:

ENTER: SHIFT 4 2 1

10. To access MatAns (this is where your latest matrix calculation result is stored)

ENTER: SHIFT 4 6

SOLUTION:

91. STORE MatB = $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$ and MatC = $\begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$

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ENTER: MatB MatC Ans. $\begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$ (a)

92. Store: MatA = $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ MatB = $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$

ENTER: MatA + 2MatB Ans. $\begin{bmatrix} 0 & 5 \\ 1 & 5 \end{bmatrix}$ (d)

93. Store MatA = $\begin{bmatrix} 1 & 2 & -4 \\ 3 & 1 & 2 \\ -4 & 1 & 5 \end{bmatrix}$ ENTER: 71MatA^{-1}

Ans. $\begin{bmatrix} -3 & 14 & -8 \\ 23 & 11 & 14 \\ -7 & 9 & 5 \end{bmatrix}$ Ans. 5 (c)

94. ENTER:

Store MatA = $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ MatB = $\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$

det(Trn(MatA) MatB^2) Ans. 343 (c)

95. STORE MatA = $\begin{bmatrix} 3 & 3 & 4 \\ 1 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$ MatB = $\begin{bmatrix} 8 & 3 & 4 \\ 1 & 9 & 7 \\ 3 & 1 & 5 \end{bmatrix}$

Then: then $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} MatA = MatB \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = MatB MatA^{-1}$

ENTER: MatB MatA $^{-1}$ Ans. $\begin{bmatrix} 5/2 & -1 & 1/2 \\ 38/5 & -7/5 & -34/5 \\ -21/5 & 9/5 & 23/5 \end{bmatrix}$

Thus: a = 5/2 and b = -1 a + 3b = 5/2 + 3(-1) = -0.5 (a)

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96. The Area of a triangle is $A = 1/2 \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$

STORE: MatA = $\begin{bmatrix} 3 & 4 & 1 \\ -8 & 21 & 1 \\ 7 & 13 & 1 \end{bmatrix}$ ENTER: 0.5 det MatA Ans. -83.5

(disregard sign) (b)

97. CASIO has no direct method to do this. We will apply the pivotal element method where the pivot is the element in the 1st row and 1st column, in this case is 2.

Factor 2 in the determinant. You have to make the pivot (1st row 1st column as 1)

$$\begin{bmatrix} 2 & 14 & 3 & 1 \\ 1 & 5 & -1 & 3 \\ 1 & -2 & 2 & -3 \\ 3 & -4 & -3 & -4 \end{bmatrix} = 2 \begin{bmatrix} 1 & 7 & 3/2 & 1/2 \\ 1 & 5 & -1 & 3 \\ 1 & -2 & 2 & -3 \\ 3 & -4 & -3 & -4 \end{bmatrix}$$

Note: Factor only the 1st row.

Go to MODE 1 :

Compute A – BC base on the table below

A	B	C	Result:
5	1	7	-2
-1	1	3/2	-5/2
3	1	1/2	5/2
-2	1	7	-9
2	1	3/2	1/2
-3	1	1/2	-7/2
-4	3	7	-25
-3	3	3/2	-15/2
-4	3	1/2	-11/2

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STORE MatA = $\begin{bmatrix} -2 & -5/2 & 5/2 \\ -9 & 1/2 & -7/2 \\ -25 & -15/2 & -11/2 \end{bmatrix}$ and ENTER: 2 det (MatA)

Ans. 326

98. Store: MatA = $\begin{bmatrix} 1 & 6 & 0 \\ 4 & 2 & 7 \\ 0 & 5 & 3 \end{bmatrix}$

ENTER: det(MatA) = -101 (b)

99. Multiply the matrix:

$\begin{bmatrix} 1x + 1y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ then x + y = 2 and 3x + 2y = 0

ENTER: MODE 5 1 Ans. X = -4 and Y = 6 (a)

100. Store: MatA = $\begin{bmatrix} 3 & 4 & 4 \\ 6 & 6 & 1 \\ 3 & 4 & 8 \end{bmatrix}$ MatB = $\begin{bmatrix} 4 & 2 & -1 \\ 1 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

MatC = $\begin{bmatrix} 4 & 1 & 5 \\ 6 & 1 & 3 \\ -2 & 1 & 4 \end{bmatrix}$

ENTER: det(MatA Trn(MatB) – 3 MatC) = 2172 (d)

101. The cofactor of a matrix is $(-1)^{i+j}$ Minor of a_{ij} where i is the row and j is the column where the element a_{ij} is found in the matrix.

The cofactor of element X is $(-1)^{4+3}$ $\begin{vmatrix} 2 & 3 & 8 \\ 5 & -7 & 3 \\ 7 & 1 & 3 \end{vmatrix}$

Note: To get the minor of X, delete 4^{th} row and 3^{rd} column.

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Store MatA = $\begin{bmatrix} 2 & 3 & 8 \\ 5 & -7 & 3 \\ 7 & 1 & 3 \end{bmatrix}$

ENTER: $(-1)^{3+4}$ det(MatA) = -402 (c)

102. Since B = A^{-1} then $A^{-1} = \frac{1}{6}\begin{bmatrix} X & 8 & Y \\ 8 & -10 & 4 \\ 4 & Z & -2 \end{bmatrix}$ or

$\begin{bmatrix} X & 8 & Y \\ 8 & -10 & 4 \\ 4 & Z & -2 \end{bmatrix} = 6A^{-1}$ STORE: MatA = $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \\ 4 & 3 & 1 \end{bmatrix}$

ENTER: $6MatA^{-1}$ Ans. $\begin{bmatrix} -7 & 8 & 5 \\ 8 & -10 & -4 \\ 4 & -2 & -2 \end{bmatrix}$

Thus: $\begin{bmatrix} X & 8 & Y \\ 8 & -10 & 4 \\ 4 & Z & -2 \end{bmatrix} = \begin{bmatrix} -7 & 8 & 5 \\ 8 & -10 & -4 \\ 4 & -2 & -2 \end{bmatrix}$

X = -7, Y = 5, Z = - 2 (a)

103. Factor 4: $4\begin{vmatrix} 1 & -1/4 & 2/4 & 3/4 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$

104. Using Cramer's Rule

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$$D = \begin{vmatrix} 1 & 1 & 1 & -2 \\ 0 & 2 & 1 & 3 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & 0 & 1 \end{vmatrix} \quad \text{and} \quad D(w) = \begin{vmatrix} 1 & 1 & 1 & -4 \\ 0 & 2 & 1 & 4 \\ 2 & 1 & -1 & 5 \\ 1 & -1 & 0 & 4 \end{vmatrix}$$

w = D(w)/D

Use Pivotal element method for D

Use A – BC (Use 1st row, 1st column as pivot)

A	B	C	A – BC
2	0	1	2
1	0	1	1
3	0	- 2	3
1	2	1	-1
-1	2	1	-3
-2	2	-2	2
-1	1	1	-2
0	1	1	-1
1	1	-2	3

Store: MatA = $\begin{bmatrix} 2 & 1 & 3 \\ -1 & -3 & 2 \\ -2 & -1 & 3 \end{bmatrix}$ ENTER: det(MatA) Ans. 30.

Use Pivotal element for D(w)

USE A – BC

A	B	C	A-BC
2	0	1	2
1	0	1	1
4	0	-4	4
1	2	1	-1
-1	2	1	-3
5	2	-4	13
-1	1	1	-2
0	1	1	-1

(Use 1st row, 1st column as pivot)

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4 1 -4 8

Store: MatB = $\begin{bmatrix} 2 & 1 & 4 \\ -1 & -3 & 13 \\ -2 & -1 & 8 \end{bmatrix}$ ENTER: det(MatB) = -60

w = D(w)/D = -60/-30 = 2 Ans. 2 (b)

105. The equation can be written as:

1b + 3c + 3(3) = 2 1b + 3c = - 7

3b + 7c + 9(3) = 5 +3b + 7c = -22

-1(b) + ac + 5(3) = 8 -b + ac = - 7

Solve the 1st 2 equations using MODE 5 1

b = -17/2 c = 1/2 then substitute in the 3rd equation: -(-17/2) +

a(1/2) = -7 a = -31 (d)

106. $\begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = 0$

(4 -γ)(-1-γ) -6 = 0 ► γ^2 – 3x -10 = 0 ► γ = 5, -2

Alternative solution to get the Eigenvalues of 2 x 2 matrix A

Let t = eigenvalue.

The required equation for t is t^2 - trace(A)t + det(A=0)

trace of A is the sum of the diagonals = 4 + -1 = 3

det($\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$) = -10

Then: t^2 – 3t – 10 = 0 , (t = 5, -2)

107. Use short cut similar to problem 106.

t^2 – trace(A)t + det(matA) = 0

trace(A) = 1 + 5 = 6

STORE MatA = $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

det(MatA) = -7

then t^2 – 6t – 7 = 0 USE MODE 5 3 t = 7 , -1

108. Use t^2 – trace(A)t + det(A) = 0

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trace(matA) = 4 -1 = 3
det(matA) = -10
Thus: $t^2 - 3t - 10 = 0$ USE MODE 5 3, t = 5, -2
To get the eigenvector corresponding to t = 5.

$$\begin{bmatrix} 4-5 & 2 \\ 3 & -1-5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \blacktriangleright \quad -a + 2b = 0,\ 3a - 6b = 0$$

Assume b = 1 , then a = 2

Eigenvector corresponding to t = 5 is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

To get the eigenvector corresponding to t = -2

$$\begin{bmatrix} 4-(-2) & 2 \\ 3 & -1-(-2) \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \blacktriangleright\ 6a + 2b = 0,\ 3a + b = 0$$

Assume b = 1 , then a = -1/3

Eigenvector corresponding to t = -2 is $\begin{bmatrix} -1/3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ (just multiply by 3 to become integers) Ans. (a)

109. Use $t^2 - \text{trace}(A)t + \det(A) = 0$
trace(A) = 3 – 6 = - 3
det(A) = -10
$t^2 - (-3)t - 10 = 0$, USE MODE 5 3 , t = -5, 2
for t = -5 , subtract the principal diagonal of A by -5.

$$\begin{bmatrix} 3+5 & -4 \\ 2 & -6+5 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 2 & -1 \end{bmatrix}$$ this will correspond to

8a – 4b = 0, 2a – b = 0
Let a = 1, b = 2 Eigenvector = < 1, 2 >
for t = 2 , subtract the principal diagonal of A by 2

$$\begin{bmatrix} 3-2 & -4 \\ 2 & -6-2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 2 & -8 \end{bmatrix}$$ this will correspond

to a – 4b = 0, 2a – 8b = 1
Let a =1 , b = 4 Eigenvector = < 4, 1 >

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110. Store MatA = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 4 & 2 \\ 5 & 1 & 7 \end{bmatrix}$ and MatB = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ENTER: det($MatA^2 + 2MatA^3 + 3\ MatB$)

Ans. 770612 (b)

PROBLEM SET 11
PLANE GEOMETRY

1. The sum of the interior angles of a polygon is 2,520. Find the number of sides.

a. 12 b. 14
c. 16 d. 18

2. How many sides are in an equiangular polygon if each of its interior angle is 165^0?

a. 20 b. 22
c. 24 d. 26

3. The sides of a triangle are 8 cm, 10 cm and 14 cm. Determine the radius of the inscribed circle.

a. 2.45 cm b. 2.56
c. 4.33 d. 1.45

4. The area of a circle circumscribing about an equilateral triangle is 254.47 sq m. What is the area of the triangle?

a. $121.33cm^2$ b. 105.22 cm^2
c. 122.34 cm^2 d. 141.32 cm^2

5. The area of the regular hexagon inscribed in a circle of radius 1 m is?

a. 2.6 m^2 b. 2.5 m^2
c. 2.4 m^2 d. 2.8 m^2

6. The area of the regular octagon inscribed in a circle of radius of radius 10 cm.

a. 433.65 b. 282.85
c. 334.21 d. 431.22

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7. The distance between the centers of the 3 circles which are mutually tangent to each other externally are 10, 12 and 14 units. The area of the largest circle is

a. 100π b. 64π

c. 36π d. 144π

8. The area of the triangle inscribed in a circle is 39.19 cm^2 and the radius of the circumscribed circle is 7.14 cm. If the two sides of the inscribed triangle are 8 cm and 10 cm respectively, find the 3^{rd} side.

a. 14 cm b. 11cm

c. 12 cm d. 15 cm

9 A rhombus has diagonals of 32 and 20 inches. What is the area?

a. 400 b. 240

c. 330 d. 320

10. Find the difference of the area of the square inscribed in a semicircle having a radius of 15 m. The base of the square lies on the diameter of the semicircle.

a. 173.43 m^2 b. 122.34 m^2

c. 154.23 m^2 d. 181.22 m^2

11. The area of the circle is 89.42 sq in. What is the length of the side of a regular hexagon inscribed in a circle?

a. 4.22 b. 6.12

c. 5.89 d. 5.33

12. One side of a regular octagon is 2. Find the area of the octagon.

a. 19.31 sq u b. 13.24 sq u

c. 11.22 sq u d. 14.33 sq u

13. The sum of the interior angles of a regular polygon is 1080^0. How many sides are there?

a. 6 b. 7

c. 8 d 10

14. In a circle of diameter 10 m, a regular 5 pointed star touching its circumference is inscribed. What is the area of that part not covered by the star?

a. 59.2 b. 50.5

c. 49.2 d. 44.5

15. The angle of a sector is 30^0 and the radius is 15 cm. What is the area of the sector in cm^2

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a. 58.9 cm^2 b. 22.34 cm^2
c. 34.56 cm^2 d. 56.21 cm^2

16. The area of a triangle is 65 sq cm and its perimeter is 48 cm. Find the radius of the inscribed circle.

a. 3.41cm b. 3.33 cm
c. 5.55 cm d. 2.71 cm

17.A trapezoid has area of 36 m^2 and an altitude of 2 m. Its two bases have ratio of 4: 5. What are the lengths of the bases. ECE BOARD

a. (16,20) b. (8, 10)
c. (12, 15) d. (14, 10)

18. PAST CE BOARD

Find the area of the quadrilateral having sides AB = 10 cm, BC = 5 cm, CD = 14.14 cm and DA = 15 cm, if the sum of the opposite angles is 225^0.

a. 100 cm^2 b. 120 cm^2
c. 130 cm^2 d. 140 cm^2

19. A circle of radius 8 cm in inscribed in a sector having a central angle of 80^0. What is the area of the sector? PAST CE BOARD.

a. 234.11 cm^2 b. 291.84 cm^2
c. 411.22 cm^2 d. 345.33 cm^2

20. Given a triangle ABC with sides AB = 30 cm, BC = 36 cm and AC = 48 cm. Find the distance of the point of intersection of perpendicular bisectors to side BC. PAST CE BOARDD.

a. 12.34 cm b. 15.92 cm
c. 11.22cm d. 14.33 cm

21. How many diagonals have an undecagon? PAST CE BOARD

a. 22 b. 33
c. 44 d. 55

22. The number of diagonals of a polygon is 275. How many sides are there? CE Board Nov 2009

a. 23 b. 24
c. 25 d. 26

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23. A circle is circumscribed about a hexagon. Determine the area of the hexagon if the area outside the hexagon but inside the circle is 15 sq cm. PAST CE BOARD

a. 71.7 cm^2 b. 89.1 cm^2
c. 91.2 cm^2 d. 81.2 cm^2

24. Determine the area of a regular hexagon inscribed in a circle having an area of 170 sq cm. PAST CE BOARD

a. 122.22 cm^2 b. 147.55 cm^2
c. 134.33 cm^2 d. 171.12 cm^2

25. Find the area of the spherical triangle ABC having the following parts.
Angle A = 140^0 Angle B = 75^0 Angle C = 86^0 Radius of the sphere = 4 m.

a. 33.89 m^2 b. 45.21 m^2
c. 67.12 m^2 d. 87.12 m^2

SITUATION Problems 26, 27
One of the diagonals of the Rhombus is 12 inches. If the area of the Rhombus is 132 sq in, determine the following. (PAST CE BOARD)

26. The length of the other diagonal.

a. 24 cm b. 23 cm
c. 22 cm d. 21 cm

27. The measure of the acute angle between the sides of the Rhombus in degrees.

a. 28.61^0 b. 23.45^0
c. 33.23^0 d. 18.61^0

28. Two sides of parallelogram measure 68 cm and 83 cm and its shorter diagonal is 42 cm. Find the largest interior angle of the parallelogram. PAST CE BOARD

a. 167.2^0 b. 111.2^0
c. 149.7^0 d. 131.1^0

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29. The perimeter of a triangle is 102 cm. If two sides are 24 cm and 32 cm, what is the area of the triangle? PAST CE BOARD

a. 341.22 cm^2 b. 361.68 cm^2

c. 351.21 cm^2 d. 333.12 cm^2

30. The perimeter of a triangle ABC is 400 cm . If angle A is 30^0 and angle B is 58^0, find the measure of side AC. PAST CE BOARD

a. 170.31 cm b. 191.22 cm

c. 122.34 cm d. 150.44 cm

30. A regular hexagon is inscribed in a circle having an area of 158 sq cm. Find the area in the circle not covered by the hexagon. PAST CE BOARD

a. 27.34 cm^2 b. 33.44 cm^2

c. 51.12 cm^2 d. 18.56 cm^2

31. A circle giving an area of 1018 cm^2 is cut into 2 segments by a chord 8 cm from the center. What is the ratio of the area of the smaller segment to the larger segment. ?PAST CE BOARD

a. 0.345 b. 0.293

c. 0.441 d. 0.467

32. The perimeter of a triangle is 271 cm. The interior angles are 50^0, 60^0 and 70^0, respectively. What is the length of the shortest side? PAST CE BOARD

a. 80.71 cm b. 90.22 cm

c. 78.31 cm d. 70.21 cm

33. A polygon has 24 sides. How many are its diagonals? CE Board May 2008

a. 242 b. 252

c. 262 d. 272

34. Two sides of triangle are measuring 20 cm and 28 cm. If the area of the triangle is 227.125 cm^2, how long is the 3^{rd} side? CE BOARD MAY 2008

a. 22 b. 23

c. 24 d. 25

35. How many sides has a polygon is the sum of its exterior angles equals the sum of its interior angles?

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a. 3 b. 4
c. 5 d. 6

36. Find the area of a regular octagon inscribed in a circle of radius 10 cm?

a. 283 b.289
c. 290 d. 291

37. A regular pentagon has sides of 20 cm. An inner pentagon with sides of 10 cm is inside and concentric to the larger pentagon. What is the area inside the larger pentagon and outside the smaller pentagon. PAST EE BOARD

a. 516.14 cm^2 b. 611.22 cm^2
c. 711.33 cm^2 d. 762.23 cm^2

38. The area of a circle circumscribing an equilateral triangle is 254.47 sq m. What is the area of the triangle?

a. 105.22 m^2 b. 111.22 m^2
c. 123.23 m^2 d. 144.34 m^2

39. Find the sum of the interior angles of the vertices of a five pointed star inscribed in a cirlcle.

a. 130^0 b. 180^0
c. 150^0 d. 120^0

40. From a point outside an equilateral triangle, the distances to the vertices are 10 m, 18 m and 10 m, respectively. What is the length of one side of the triangle?

a. 19.95 m b. 21.22 m
c. 18.21 m d. 23.21 m

41. Each angle of a regular dodecagon is

a. 120^0 b. 150^0
c. 130^0 d. 140^0

42. A rectangle ABCD which measures 18 cm by 24 cm is folded once, perpendicular to AC, so that the opposite vertices of A and C coincide. Find the length of the fold.

a. 22.5 cm b. 24 cm
c. 24.5 cm d. 25.5 cm

43. CE BOARD May 2009

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Find the area of the largest circle that can be inscribed in a hexagon of side ‘h’

a. $2.356h^2$ b. $2.441h^2$

c. $3.1416h^2$ d. $1.786h^2$

44. Find the area of the largest equilateral triangle that can be inscribed in a hexagon of side ‘h ‘ .

a. $0.9343\ h^2$ b. $0.9843\ h^2$

c. $0.9243\ h^2$ d. $1.299\ h^2$

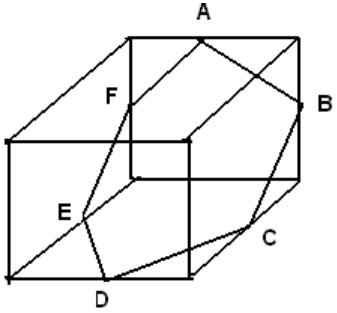
45. Find the area of the largest hexagon that could be inscribed in a hexagon of side ‘h’

a. $1.948h^2$ b. $1.748h^2$

c. $1.148h^2$ d. $1.348h^2$

46. Find the area of the hexagon ABCDEF whose vertices are the midpoints of the respective sides of the cube whose edge is 4 meters.

a. 19.314 b. 21.314 c. 33.423 d. 51.331



The parallel sides of a trapezoidal lot measure 160 m and 240 m are are 40 m apart.

47. Find the distance of the dividing line from the 160 m line parallel to the 2 sides that will divide the trapezoid into 2 equal parts.

a. 23.45 b. 25.33

c. 21.98 d. 22.34

48. Find the length of this dividing line.

a. 203.96 b. 241.33

c. 221.56 d. 222.22

SOLUTION.

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1. (n-2)180 = 2520 n = 16 (c)
2. Let x = number of sides.
   (n-2)180 = 165x n = 24 Ans. c

3. $A = \sqrt{X(X-A)(X-B)(X-C)}$

WHERE x = (A + B + C) / 2 = (8 + 10 + 14)/2 = 16

A = 16√6 and Perimeter = 32

r = 2P/A = 2(16√6)/ 32 = 2.45 (a)

4. $\pi r^2 = 254.47$ r = 9 (radius of the circumscribing triangle)

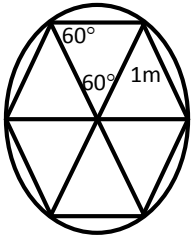
r = abc/(4A) 9 = (s x s x s) /($4s^2\sqrt{3}$ /4) = s/√3

s = 9√3

and Area = $s^2\sqrt{3}$ / 4

Area = (9√3)^2(√3)/ 4 = 105.22 cm^2 Ans. b

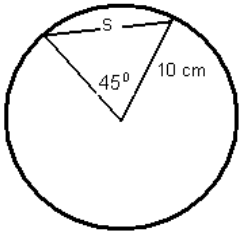
5.



A = 6(s^2 √3 / 4) = 6(1^2) (√3 / 4) = 2.6 m^2

Ans. a

6. Divide 360 by 8 = 45 deg = interior angle.



s/ sin 45 = 10 / sin 67.5 s = 7.654 cm

Area of one triangle = 1/2 x base x height

= 0.5 x 7.654 x 10 cos 22.5 = 35.3569

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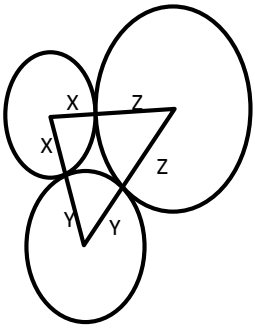
Area of the octagon = 8 x 35.3569 = 282.85 cm^2

7.



x + y = 10 x + z = 12 and y + z = 14

Arrange: x + y + 0z = 10

x + 0y + 1z = 12

0x + 1y + 1z = 14 x = 4, y = 6, z = 8

Largest Circle = $\pi(8)^2 = 64\pi$ (b)

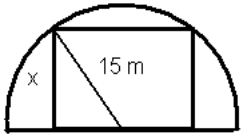
8. r = abc/(4A) 7.14 = 8(10)(c)/ 4(39.19)

c = 14 cm (a)

9. A= $1/2d_1d_2$ = 1/2(32)(20) = 320 in^2 (d)

10.

.

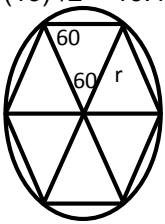


x/2

$x^2 + (x/2)^2 = 15^2$ x = 13.416 m

Difference in Area = $\pi(15)^2/2 - 13.416^2$ = 173.43 m^2 (a)

11.



A=89.42 = πr^2;r = 5.335 in ;since triangle is equilateral, side=5 .335 cm

12. Interior angle = 360/8 = 45^0

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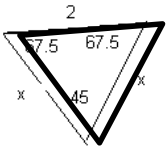


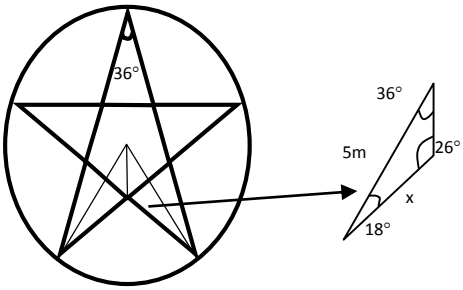
x/sin 67.5 = 2/sin 45 x = 2.613

A = 8 (1/2 base x height) = 8(1/2 x 2 x 2.613 sin 67.5) =19.31 sq u

Ans. 19.313 sq u (a)

13. (n-2)180 = 1080 n = 8 (c)

14.



x/sin 18^0 = 5/sin 126^0 x = 1. 91 m

Altitude of the triangle = x sin 30^0 = 1.91sin 36^0 = 1.122 m

Area of the star = 10 (1/2 (5 x 1.122 m)) = 28.05 m^2

Area not covered by the star

= $\pi(5)^2$ – 28.05 = 50.5 m^2 (b)

15. A = 1/2$r^2\theta$ = 1/2$(15)^2(30^0)$ (Go to radian mode: ENTER THE ^{0} using SHIFT ANS 1) Ans. 58.9 cm^2

16. r = 2A/P = 2(65)/48 = 2.7083 (d)

17. A = 1/2(B1 + B2)h Let B1 = 4x B2 = 5x

36 = 1/2(4X + 5X)(2) X = 4 , B1 = 4(4) =16 and B2 = 5(4) = 20 (a)

18. s = (A + B + C + D)/2 A =

$$\sqrt{(S-A)(S-B)(S-C)(S-D) - ABCD\cos^2\theta}$$

s = (10 + 5 + 14.14 + 15) /2 = 22.07

Sum of opposite angles ÷ 2 = 225/2 = 112.5

A = 100 cm^2

CAL TECHNIQUE:

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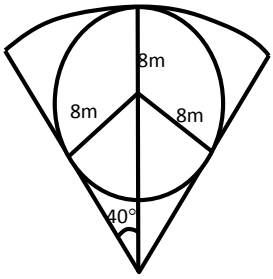
Input: $X = \frac{A+B+C+D}{2} : \sqrt{(X-A)(X-B)(X-C)(X-D) - ABCD\cos^2(Y/2)}$

A? 10 B? 5 C? 14.14 D? 15 Y? 225

Answer: 99.9915

19.

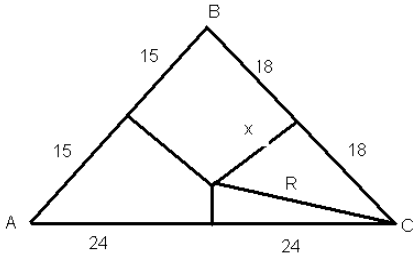


Radius of the sector = $8 + 8/\sin 40^0 = 20.4458$m

Area = $1/2(20.4458)^2 (80^0)$ (Radian Mode: To type 80^0 ENTER: 80 SHIFT Ans 1)

Ans. 291.84 cm^2

20. The point of intersection of the perpendicular bisectors is the radius of the circumscribed circle.



Use:

X = (A + B + C)/2 : D = $\sqrt{X(X-A)(X-B)(X-C)}$: (ABC)/(4D)

Note: D is the Area and the (ABC)/4D is the radius of the circumscribed circle.

A? 30 B? 36 C ? 48

Ans. D = 539.332 and R = 24.03

$x^2 + 18^2 = R^2 = 24.03^2$ x = 15.92 cm (b)

21. 11 sides. Number of diagonals =

n(n-3)/2 = 11(11-3)/2 = 44 (c)

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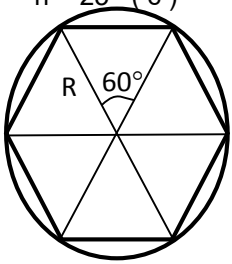
22. $n(n-3)/2 = 275$ $n = 25$ (c)

For 23-24



23. $\pi R^2 - 6(R^2 \sqrt{3} /4) = 15$ $R = 5.253$ cm

Area of hexagon = $6 (R^2 \sqrt{3} /4) = 71.7 \text{ cm}^2$ (a)

24. $\pi R^2 = 170$ $R = 7.356$ cm

$A = 6 R^2\sqrt{3} /4$

$6(7.356^2) \sqrt{3} / 4 = 147.55 \text{ cm}^2$ (b)

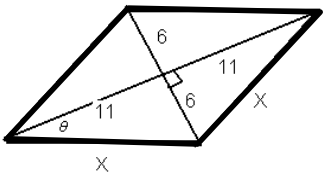
25. $E = A + B + C - 180 = 140 + 75 + 86 - 180 = 121^0$

Area = $\pi R^2 E /180 = \pi(4)^2(121)/180 = 33.89 \text{ m}^2$ (a)

26 – 27

26. Area of Rhombus = $1/2 \, d_1 \, d_2$

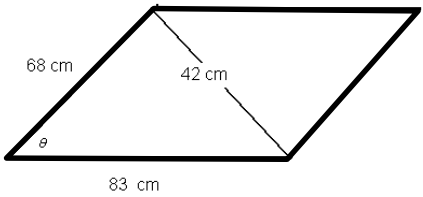
$132 = 1/2 (12) d_2$ $d_2 = 22$ cm (c)



$X^2 = 11^2 + 6^2$ $X = 12.53$

27. $\tan \theta = 6/11$ $\theta = 28.61^0$

28.



$42^2 = 68^2 + 83^2 - 2(68)(83)\cos \theta$

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$\theta = 30.27^0$, the other angle is $180 - 30.27^0 = 149.73^0$ (c)

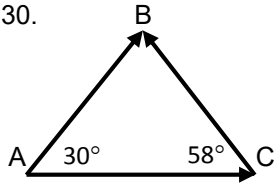
29. The other side = 102 – 24 – 32 = 46 cm

Use $X = \frac{A+B+C}{2} : \sqrt{X(X-A)(X-B)(X-C)}$ CALC

A ?24 B ?32 C? 46

Ans. 361.68 cm^2 (b)

30.



Assume AC =1 ,

Use Sin Law Technique

MODE 5 1

cos 30 cos 58 1

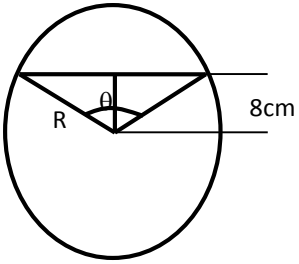
sin 30 - sin 58 0

AB = 0.84857 BC = 0.5

Then: 0.84857X + 0.5X + 1X = 400

X = 170.31 cm

31.



$\pi R^2 = 1018$ R = 18 cm , $\cos \theta/2 = 8/18$

$\theta = 127.22^0$

Area of the small segment $1/2R^2 (\theta - \sin \theta)$

$1/2(18)^2 (127.22^0 - \sin 127.22^0) = 230.7$

(Note: Go to Radian Mode, and input the 127.22^0 as 73.74 SHIFT Ans 1)

Area of the big segment = $\pi(18)^2 - 230.7 = 787.176$

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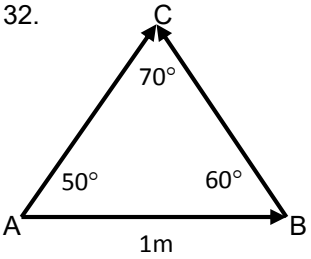
Ratio = 230.7 / 787.176 = 0.293 \

32.



Assume AB = 1 m then
USE SIN LAW TECHNIQUE:
MODE 5 1
cos 50 cos 60 1
sin 50 - sin 60 1
AC = 0.9216 BC = 0.815
The 1 X + 0.9216X + 0.815X = 271
X = 99.028
Shortest Distance = 0.815X = 0.815(99.028) = 80.71 m

33. n(n-3)/2 = 24(24-3)/2 = 252

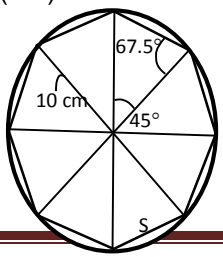
34. The easiest way to do this is trial and error.

USE $X = \frac{A+B+C}{2} : \sqrt{X(X-A)(X-B)(X-C)}$

Input A = 20 B = 28 C = 22 (1st Choice) Area = 218.57
A = 20 B = 28 C = 23 (2nd choice) Area = 227.125
Ans. 23

35. Sum of interior angles of a polygon = $(n-2)180^0$
Sum of its exterior angles = 360^0
$(n-2)180^0 = 360$ n = 4

36.



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Interior angle = 360/8 = 45^0

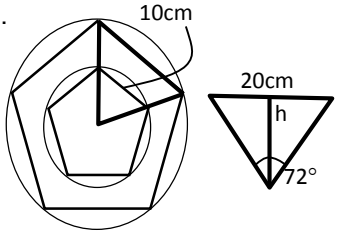
Using sine Law: S/sin 45^0 = 10/sin 67.5^0 S = 7.6537 cm

Area of one triangle =

1/2 (7.6537) (10 cos 22.5^0) = 35.355 cm^2

Area of octagon = 8(35.355) = 282.85 cm^2 (a)

37.



tan 36^0 = 10/h h = 13.7638 cm

for the small triangle: tan 36^0 = 5/h , h = 6.882 cm

Area of big pentagon =

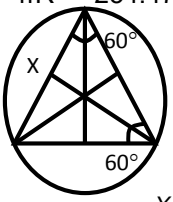
5(1/2)(20)(13.7638) = 688.19 cm^2

Area of small pentagon =

5(1/2)(10)(6.882) = 172.05 cm^2

Difference = 688.19 – 172.05 = 516.14 cm^2 (a)

38. πR^2 = 254.47; R = 9 cm

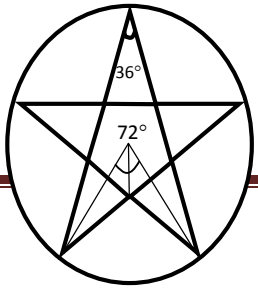


$R_{circumscibed} = \frac{X(X)(X)}{4(X^2\sqrt{3}/4)} = 0.57735\ X$

0.57735X = 9 ► X = 15.59 cm

Area of triangle = $X^2\sqrt{3}/4$ = $15.59^2\sqrt{3}/4$ = 105.22 m^2 Ans. (a)

39.



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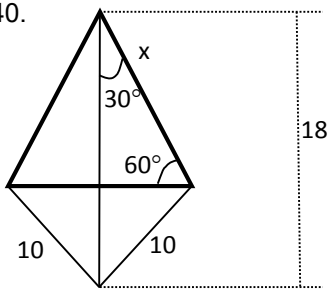
Interior angle (center) = $360/5 = 72^0$

Sum of interior angles of the vertices = $36(5) = 180^0$ (b)

40.



$10^2 = X^2 + 18^2 - 2(X)(18)\cos 30^0$ $\quad X = 19.95$ m

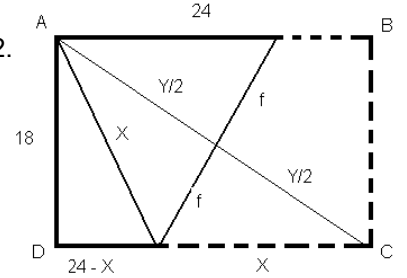
Or We can use trial and error.

$|x \angle 30 - 18| = 10$

Use Choice of $x = 19.95$ (Ans.)

41. $(n-2)180 \div n = (12-2)(180)/12 = 150^0$

42.



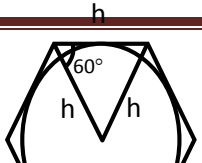
$Y^2 = 18^2 + 24^2$ $\quad Y = 30$ cm $\quad$ [Pol (18, 24) $\quad r = 30 = Y$]

$18^2 + (24-X)^2 = X^2$ $\quad X = 18.75$ cm

$(Y/2)^2 + f^2 = X^2$

$(30/2)^2 + f^2 = 18.75^2$ $\quad f = 11.25$

Length of the fold = $2f = 2(11.25) = 22.5$ cm



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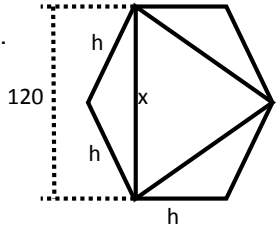
43.

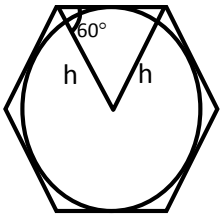
The radius of the circle is the altitude of the triangle shown.
$r = h \sin 60^0 = 0.866h$
$A = \pi r^2 = \pi(0.866h)^2 = 2.356h^2$

44.



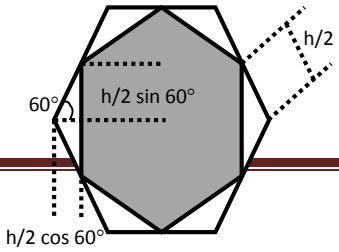
MODE 2:
$X = |1\angle 120 - 1| h = 1.732h$
Area of the inscribed equilateral triangle
$= X^2\sqrt{3}/4 = (1.732h)^2\sqrt{3}/4 = 1.3h^2$

h



The height of one triangle is the radius of the inscribed circle.
height = $h\cos 30^0 = 0.866h$
Area of the circle = $\pi(0.866h)^2 = 2.356h^2$

45.



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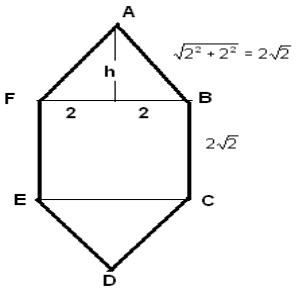
length of the inscribed hexagon = 0.433h(2)=0.866h

Area of the inscribed hexagon

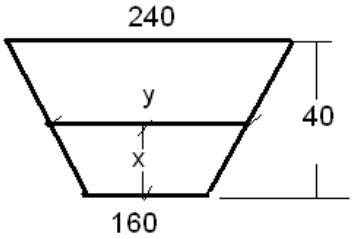
$=6(s^2 \sqrt{3} / 4) = 6 \sqrt{3} / 4 (0.866h)^2 = 1.948h^2$



$h = \sqrt{(2\sqrt{2})^2 - 2^2} = 2$ Area = 1/2(4 x 2) (2) + $2\sqrt{2}$ (4) = 19.314

47-48

Model the trapezoid.



y is a linear function of x

MODE 3 2

x y

0 160

40 240

y= 160 + 2x

$0\hat{x}$ = -80

(This is the distance below the 160 m side where the width = 0)

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MODE 3 3

Area is a parabolic function of the depth

x y

0 0

40 (240 + 160)/2 x 40 = 8000

-80 -.5(160x80) = -6400

4000$\widehat{x1}$ = 21.98 (distance of the dividing line from 160 m side.)

y = 160 + 2(21.98) = 203.96 (length of the dividing line).

PROBLEM SET 12

SOLID MENSURATION

1. If the edge of the cube is increased by 30%, by how much is the surface area increased? PAST CE BOARD

a. 60% b. 69%

c. 71% d. 30%

2. The volume of water in a spherical tank having a diameter of 4 m is 5.236 m^2 . What is the depth of water in the tank?

a. 1 b. 1.2

c. 1.3 d. 1.4

3. A circular cone having an altitude of 9 meter is divided into 2 segments having the same vertex. If the smaller altitude is 6 m, find the ratio of the volume of the small cone to the big cone.

a. 0.296 b. 0.331

c. 0.225 d. 0.411

4. The frustum of a regular pyramid has un upper base of 8 x 80 m and a lower base of 10 x 100 with an altitude of 5 m. Find the volume of the pyramid.

a. 7812.33 m^3 b. 4066.67 m^3

c. 5133.33 m^3 d. 7100.67 m^3

5. Find the volume of the cone to be constructed from a sector having a diameter of 72 cm and a central angle of 210^0.

a. 13503.66 b. 23503.22

c. 12112.44 d. 13503.44

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6. The bases of a right prism is a hexagon with one of each side equal to 6 cm. The bases are 12 cm apart. What is the volume of the right prism?

a. 1122.37 b. 1223.23
c. 1411.22 d. 1311.22

7. A trough having an equilateral triangle end sections has sides equal to 0.3 m and 6 meter long. Find the volume of water in the tank if the depth is one half the depth of the trough.

a. 0.0584 b. 0.0441
c. 0.3422 d. 0.4111

8. The bases of a right prism is a a regular hexagon with side 6 cm. If the volume of the right prism is 500 m^3, find the distance between the bases,

a. 5.39 m b. 5.35 m
c. 5.11 m d. 5.01 m

9. A solid cone has a height of 8 cm. It has a volume 4 times the smaller cone that could be cut from the same cone of the same axes. Find the height of the smaller cone.

a. 4.01 m b. 4.21 m
c. 5.04 m d. 5.12 m

10. A right circular cone has an altitude of 2e and a diameter of e. Compute the volume of the right circular cone.

a. $\pi e^3/6$ b. $\pi e^3/12$
c. $\pi e^3/8$ d. $\pi e^3/3$

11. A mixture compound from equal parts of two liquid, one white and the other black was placed in a hemispherical bowl. The total depth of two liquids is 3 cm. After standing for a short time the mixture separated, the white liquid settling below the black. If the thickness of the segment of the black liquid is 1 cm, find the radius of the bowl.

a. 4. 11 m b. 3.67 m
c. 3.33 m d. 3.33 m

12. A carpenter chisels hole of side 2 in trough a round post of radius 2 in, the axis of the hole intersecting that of the post at right angles. Find the volume of the wood cut out.

a. 331.2 cm^3 b. 250.8 cm^3

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c. 311.22 cm^3 d. 213.22 cm^3

13. The volume of the frustum of a triangular pyramid is 135 cu. m. The lower base is an equilateral triangle with an edge of 9 m. The upper base is 8 m above the lower base. What is the upper base edge in meters?

a. 1.911 m b. 2.311 m
c. 2.995 m d. 2.811 m

14. The base of a cylinder is a hexagon inscribed in a circle. If the difference in the circumference of the circle and the perimeter of a hexagon is 4 cm, find the volume of the cylinder if it has an altitude of 20 cm.

a. 10367.175 m^3 b. 10397.175 m^3
c. 10387.175 m^3 d. 12367.175 m^3

15. The axis of the cone makes an angle of 30^0 with the horizontal. If the length of the axis is 30 cm and its base radius is 20 cm, compute the volume of the curve.

a. 10982.8 cm^3 b. 10882.8 cm^3
c. 11882.8 cm^3 d. 10982.8 cm^3

16. The surface area of an open cylindrical tank is 68 m^2. If the diameter is 2/3 of its height, what is the height of the tank?

a. 5.275 m b. 5.475
c. 5.285 d. 5.175

17. A truncated prism with a horizontal square base have vertical edges of 7 cm, 7 cm , 10 cm and 10 cm respectively. If the volume of the prism is 4000 cu. cm, find the area of the base.

a. 413.588 cm^2 b. 479.588 cm^2
c. 460.588 cm^2 d. 470.588 cm^2

18. Compute the volume of a tetrahedron of side equal to 3 cm.

a. 3.34 cm^3 b. 3.18 cm^3
c. 4.21 cm^3 d. 5.11 cm^3

19. A through having an equilateral triangle and sections has sides equal to 0.4 m. If the through is full of water and is 8 m long, compute the volume of water in the through.

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a. 0.333 m^3 b. 0.554 m^3
c. 0.334 m^3 d. 0.814 m^3

20. A deep closed conical tank has a radius of 1.2 m at the top and a height of 4.8 m. It contains oil to a depth of 2.4 m. If the tank is the inverted position, determine the depth of oil at this position.

a. 0.21 m b. 0.31 m
c. 0.28 m d. 0.18 m

21. The volume of a tetrahedron is 12.3 m^3 . Find the surface area.

a. 38.39 m^3 b. 32.12 m^3
c. 41.12 m^3 d. 33.12 m^3

22. The lateral edge of a frustum of a regular pyramid is 1.8 m long. The lower base is 2.4 by 2.4 m while the upper base is 1 m by 1 m square. What is the volume of the frustum?

a. 4.9 m^3 b. 4.6 m^3
c. 4.1 m^3 d. 4.7 m^3

23. A right circular cone having a diameter of 1 m has a height of 60 cm. It is filled with salt to a height of 36 cm. Compute the volume of the salt.

a. 0.034 m^3 b. 0.012 m^3
c. 0.144 m^3 d. 0.061 m^3

24. The ratio of the volume to the lateral area of a right circular cone is 2:1. If the altitude is 15 cm, what is the ratio of the slant height to the radius?

a. 1.2 b. 1.5
c. 2.1 d. 2.4

Problems 25, 26, 27

Two identical closed conical tank contains equal amount of liquid. The first tank has a horizontal base at the bottom while that of the second tank is at the top. The liquid in the first tank stands 3 m deep. PAST CE
BOARD

25. What is the volume of the liquid in the 2nd tank?

a. 21.99 m^3 b. 22.12 m^3
c. 23.45 m^3 d. 18.99 m^3

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26. How deep is the liquid in the 2nd tank if its altitude is 6 m and the base radius is 2 m?

a. 6.11 m b. 5.74 m
c. 6.11 m d. 6.21 m

27. If the unit weight of the liquid is 9100 N/m^3, what is the weight of the liquid inside the tank in quintals?

a. 204 b. 202
c. 201 d. 205

Problems 28, 29, 30

The maximum area of the parabola inscribed in a right circular cone having a diameter of 24 cm is 207.8 cm^2. (PAST CE BOARD)

28. Compute the base of the parabola inscribed in the right circular cone.

a. 20.784 cm b. 21.221 cm
c. 24.221 cm d. 23.112 cm

29. Compute the height of the parabola of maximum area.

a. 15 cm b. 16 cm
c. 17 cm d. 18 cm

30. Compute the volume of the right circular cone.

a. 2412.74 cm^3 b. 2312.74 cm^3
c. 3412.74 cm^3 d. 2612.74 cm^3

31. A solid material in the form of a rectangular parallelepiped 4 in by 6 in by 8 in is painted blue. How many cubes will have 3 blue faces is the block is to cut to form cubes 1 in by 1 in by 1 in. CE Nov 2005

a. 4 b. 6
c. 8 d. 10

32. The volume of a solid truncated prism is 8200 cm^3. The base is rectangular with its length twice its width. The edges perpendicular to the base are 16 cm, 12 cm and 16 cm. Determine the width of the base in cm.

a. 18.11 cm b. 17.11 cm
c. 19.11 cm d. 21.11 cm

33. The corners of a cubical box touch the spherical shell that encloses it. If the volume of the box is 27000 cm^3, what is the volume of the space outside the box but inside the sphere?

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a. 46460.9 cm^3 b. 49460.9 cm^3
c. 47460.9 cm^3 d. 66460.9 cm^3

34. A spherical sector has a central angle of 60^0 and the radius of the sphere is 15 cm. Find the volume of the spherical sector. CE Board Nov 2007

a. 947.19 m^3 b. 747.19 m^3
c. 847.19 m^3 d. 647.19 m^3

35. A buoy is in the shape of a cone of height 2 cm and base diameter of 120 cm. If the depth submerged is 1.5 m, find the volume of the submerged portion. CE Board Nov 2007

a. 0.234 cm^3 b. 0.318 cm^3
c. 0.411 cm^3 d. 0.221 cm^3

36. How far from the vertex is the center of gravity of a tetrahedron if an edge is 50 cm?

a. 30.62 b. 31.22
c. 28.22 d. 35.21

37. What is the height of a circular cone of slant height $\sqrt{10x}$ and a base diameter of 2x?

a. x b. 2x
c. 3x d. 4x

38. The central angle of a spherical wedge is 40^0. Find its volume if its radius is 1 cm.

a. 0.4654 cm^3 b. 0.4354 cm^3
c. 0.4154 cm^3 d. 0.3654 cm^3

39. A sphere having a diameter of 30 cm is cut into 2 segments. The altitude of the 1^{st} segment is 6 cm. What is the ratio of the area of the second segment to that of the first?

a. 3: 1 b. 4: 1
c. 4.5:1 d. 5: 1

40. Given a sphere of diameter d, what is the % increase in its volume if the surface area is increased by 21%?

a. 35% b. 26%
c. 33% d. 41%

41. Two vertical conical tanks are joined together at the vertices by a pipe. Initially, the bigger tank is full of water. The pipe valve is open to

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allow the water to flow to the smaller tank until it is full. At this moment,

how deep is the water in the bigger tank. The bigger tank has a diameter of 6 ft and a height of 10 ft. The smaller tank has a diameter of 6 ft and a height of 8 ft. Neglect the volume of water in the pipeline.

a. 3.56 ft b. 5.84 ft
c. 3.33 ft d. 6.11 ft

42. A cubical container that measures 2 inches on a side is tightly packed with 8 marbles and is filled with water. All 8 marbles are in contact with the walls of the container and adjacent marbles. All of the marbles are of the same size. What is the volume of water in the container?

a. 3.811 in^3 b. 3.122 in^3
c. 3.343 in^3 d. 3.612 in^3

43. Find the volume common to 2 spheres of radii 6 cm and 9 cm , respectively if the distance between their centers is 10 cm.

a. 76.12 cm^3 b. 69.18 cm^3
c. 71.12 cm^3 d. 78.12 cm^3

44. Find the volume of a spherical cone in a sphere of radius 17 cm if the radius of its zone is 8 cm.

a. 1210.56 cm^3 b. 1410.56 cm^3
c. 1216.56 cm^3 d. 1230.56 cm^3

Problems 45, 46

An ocean buoy is in the form of a spherical cone. The slant side of a conical portion is 180 cm and the element of the cone is inclined 25^0 to the x axis.

45. Find the volume of the cone.

a. 1144363.8 cm^3 b. 1344363.8 cm^3
c. 1194363.8 cm^3 d. 1174363.8 cm^3

46. Find the total surface area of the cone.

a. 62,089.87 cm^2 b. 72,089.87 cm^2
c. 62,189.87 cm^2 d. 72,099.87 cm^2

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47. Find the volume of a triangular spherical pyramid whose base angles are 77^0, 102^0 and 121^0 on a sphere of radius 15 cm.

a. 2356.19 cm^3 b. 2656.19 cm^3

c. 3356.19 cm^3 d. 4356.19 cm^3

48. Into an inverted frustum of a pyramid, full of water a certain gold mass was immersed, Naturally, some of the water overflowed. Theh the mass of gold was removed and it was found out that the surface had

dropped 8 cm. If gold weighs 19.3 grams per cu m, what was the weight of the mass. Cross section at the top = 12 x 10, Cross section at the bottom = 6 x 5. Depth = 20 cm.

a. 14.57 kg b. 15.07 kg

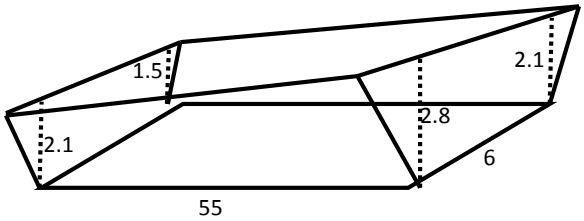
c. 12.45 kg d. 13.22 kg

49. The 3 dimensions of a rectangular parallelepiped are in the ratio of 2:3:5. If its volume is 810 cm^3, what is the smallest dimension.

a. 1 cm b. 2 cm

c. 3 cm d. 4 cm

The volume of the railroad cut is shown in the figure below. The base is a horizontal rectangle and at the ends are vertical. The slopes of the sides are 3: 5 (Vertical: Horizontal)



50. Find the area of the left section.

a. 16.833 m^2 b. 16.817

c. 21.223 d. 22.455

51. Find the area of the right section.

a. 25.886 m^2 b. 24.567

c. 31.223 d. 18.991

52. Find the Area of the midsection

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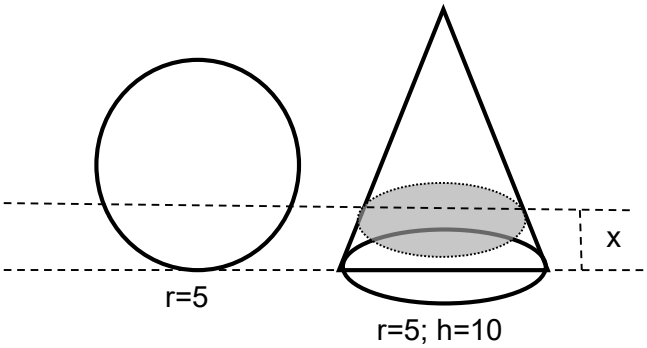
a. 22. 112 b. 21.141
c. 21.165 d. 22.281

53. Find the volume of the railroad cut.
a. 1, 166.2 m^2 b. 1,144.5
c. 1, 234.7 d. 1,188.3

A sphere of radius 5 cm and a right circular cone of base radius 5 cm stand on a plane.

54. Find the positon of the plane from the bottom that cuts the two solids in equal areas.



55. What is this Area?
a. 50.333 cm^2 b. 50.265
c. 53.221 d. 54.112

56. What is the Difference in Volume (from the bottom) of the two solids when the areas are equal.
a. 56. 55 cm^3 b. 73.31 cm^3
c. 89.12 cm^3 d. 45.56 cm^3

57. Find the depth (when the volumes are equal) from the bottom?

SOLUTION

1. $S = 6x^2$ x becomes 1.3x

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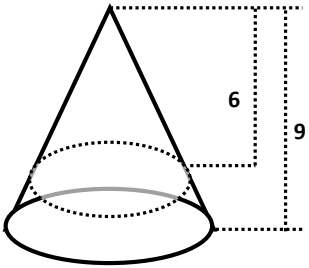
$S = 6(1.3x)^2 = 10.14x^2$

Increase = $(10.14 - 6)/6 = 69\%$

2. $V = \pi h^2/3\ (3R - h)$

$5.236 = \pi h^2/3\ (6 - h)$ $h = 1$

3.



$V = \pi R^2 h$ since R can be expressed as $R = kh$

Then $V = \pi(kh)^2(h)$

$k/\pi = Vh^3$

Thus $\frac{V_1}{V_2} = \frac{h_1^3}{h_2^3}$ $\frac{V_1}{V_2} = \frac{6^3}{9^3} = 0.2963$

4. $V = 1/3\ h(B_1 + B_2 + \sqrt{(B_1B_2)})$

$h = 5$ $B_1 = 8 \times 80 = 640$

$B_2 = 10 \times 100 = 1000$

$V = 4066.67\ m^3$

5. Area of the sector = lateral area of the resulting cone

slant height of the resulting cone = radius of the sector,

$1/2\ (36^2)\ (210^0) = 756\pi = \pi r (l) = \pi r(36)$

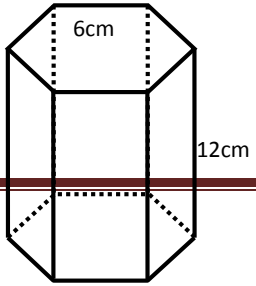
$r = 21$ cm

height of the cone $= \sqrt{(l^2 - r^2)} = \sqrt{(36^2 - 21^2)}$

$= 29.24$ cm

$V = 1/3\pi r^2 h = 1/3\ (\pi)(21^2)(29.24) = 13503.44$

6.



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V = Bh where B = base area

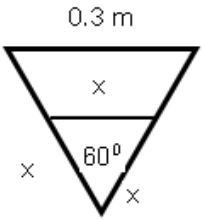
The hexagon has 6 equilateral triangles whose side is 6 cm.

Area of one triangle = $6^2\sqrt{3}/4 = 9\sqrt{3}$

Area of hexagon is $6 \times 9\sqrt{3} = 54\sqrt{3}$

V = $54\sqrt{3} \times 12 = 1122.37\ m^3$

7. Cross section:



altitude of triangle = 0.3 sin 60^0 = 02598 m

half of this altitude = 0.13 m

sin 60^0 = 0.13/ x x = 0.15 m

A = $x^2\sqrt{3}/4$ = 0.00974

Volume = Base Area x height

= 0.00974 x 5 = 0.05844 m^3

8. For a hexagon, the radius of the circumscribing circle is also 6 cm.

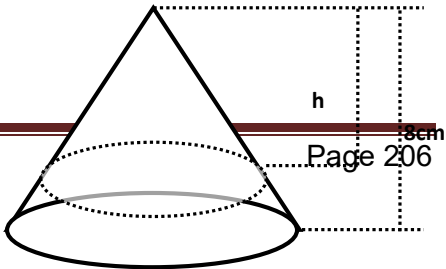
Area of any polygon = $\frac{nA_{circle}}{2\pi}\sin\frac{360^0}{n}$

$$= \frac{6(\pi)6^2}{2\pi}\sin\frac{360^0}{6} = 93.53$$

V = Bh

500 = 93.53 h; h = 5.345 cm

9. See Problem 3:



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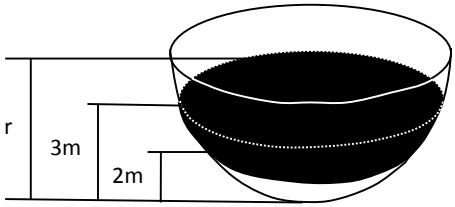
$$\frac{V}{4V} = \frac{h^3}{8^3} \; ; \quad h = 5.04 \text{ m}$$

10. $V = 1/3\pi r^2 h$

$= 1/3\pi\,(e/2)^2(2e) = \pi e^3/6$

11.



Total Volume $= \pi(3)^2/3\,(3r - 3)$

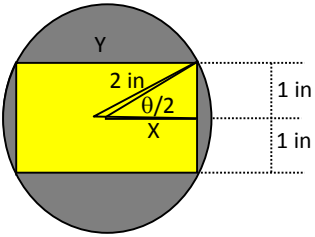
Volume of liquid below $= \pi 1^2/3\,(3r - 2)$

Volume of liquid above = Volume of liquid below

$\pi(3)^2/3\,(3r - 3) - \pi 2^2/3\,(3r - 2) = \pi 2^2/3\,(3r - 2)$

$r = 3.67$ m

12.



$\sin \theta/2 = 1/2 \quad \theta = 60^0$

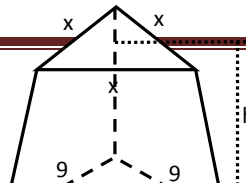
$Y = 2x = 2(2 \cos \theta/2) = 3.464$

Area of one shaded area $= 1/2r^2(\theta - \sin\theta)$ where θ is in rad.

$= 0.5(2^2)(60^0 - \sin 60^0) = 0.36234 \text{ in}^2$

Volume $= 2(0.36234)(2) + 2(3.464)(2) =$

$15.3054 \text{ in}^3 = (15.3054 \times 2.54^3) = 250.8 \text{ cm}^3$



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13.

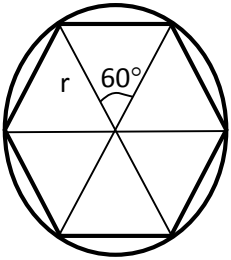
$V = 1/3h\ (\ B_1 + B_2 + \sqrt{(B_1B_2)}\ )$; $B_1 = 9^2\sqrt{3}/4 = 35.07$

$B_2 = x^2\sqrt{3}/4 = 0.433x^2$

$135 = 1/3(8)\ (\ 35.07 + 0.433x^2 + \sqrt{[(\ 35.07)(0.433x^2)]}\ )$

$x = 2.995$ m

14.



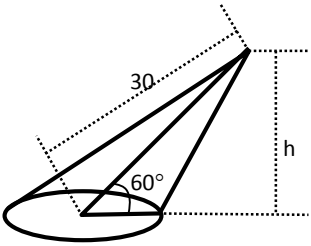
$2\pi r - 6r = 4$; $r = 14.125$ cm

Volume = Bh

$= 6(\ r^2\sqrt{3}/4)\ x\ 20$

$= 10367.175\ m^3$

15.

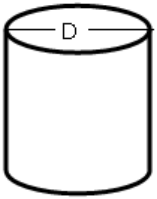


$V = 1/3Bh$

$= 1/3(\ \pi)(20^2)(\ 30 \sin 60^0)$

$= 10882.8\ cm^3$

16.



$S = \pi(D)H + \pi D^2/4$ $D = 2/3H$,

$68 = \pi(2/3H)H + \pi(\ 2/3H)^2/4$

$H = 5.275$

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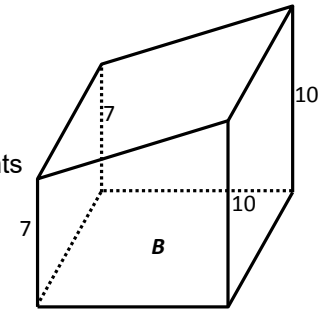
17.



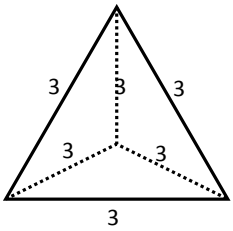
heights

V = B x average of the

4000 = B x (7 + 7 + 10 + 10)/4

B = 470.588 cm^2

18.

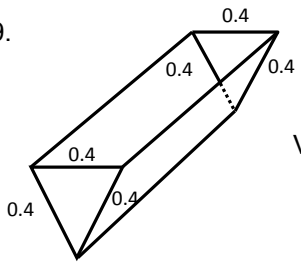


V = $\sqrt{2}/12\ e^3$

= $\sqrt{2}/12\ (3)^3$

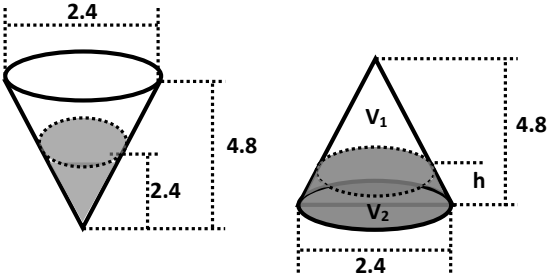
= 3.1819 cm^3

19.



V = Bh = $(0.4^2\sqrt{3}/4)(8)$ = 0.554 m^3

20.



Volume of water = $1/3\pi(0.6)^2(2.4)$ = 0.904779 m^3 = V_2

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$V_1 = 1/3\pi(1.2)^2(4.8) - 0.904779 = 6.333\ m^3$

$$\frac{V_1}{V_1+V_2} = \frac{(4.8-h)^3}{4.8^3} = \frac{6.333}{7.238} \qquad h = 0.209\ m$$

CALCULATOR TECHNIQUE:

For 1st cone: MODE 3 3

X	Y
0	0
4.8	$\pi(1.2)^2$
-4.8	$\pi(1.2)^2$

A = 0 B = C and C = 0.19635

Volume = $Ax + Bx^2/2 + Cx^3/3$

$= 0.19635x^3/3$

when (depth = x) = 2.4

V = 0.90478

For 2nd Cone: MODE 3 3

X	Y
0	$\pi(1.2)^2$
4.8	0
9.6	$\pi(1.2)^2$

A = 4.52389 B = -1.885 C = 0.19635

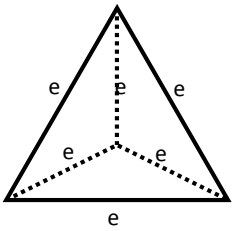
$Ax + Bx^2/2 + Cx^3/3 = 0.90478$

$4.52389x + -1.885x^2/2 + 0.19635x^3/3 = 0.90478$

Arrange and USE MODE 5 4

x = 0.209 Ans

21.



$V = e^3\sqrt{2}/12$

$12.3 = e^3\sqrt{2}/12$

$e = 4.708\ m$

$S = e^2\sqrt{3} = 38.39\ m^3$

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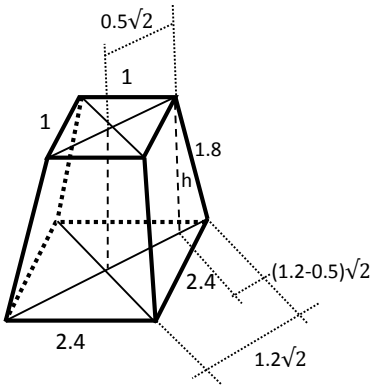
22.

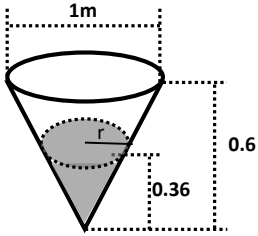


$h = \sqrt{1.8^2 - 0.99^2} = 1.503$ m

$V = h/3\ (B_1 + B_2 + \sqrt{(B_1 B_2)})$

$= 1.503/3(1^2 + 2.4^2 + (1)(2.4)) = 4.589\ m^3$

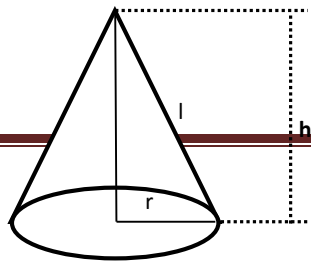
23.



r/ 0.36 = 0.5/0.6 r = 0.3

$V = 1/3\pi r^2 h = (1/3\pi)(0.3)^2(0.36) = 0.0339\ cm^3$

24.



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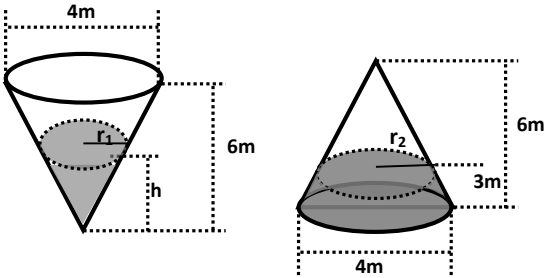
$$\frac{\pi rl}{1/3\pi r^2 h} = \frac{1}{2} \quad \frac{l}{r} = \frac{h}{6} = \frac{15}{6} = \frac{3}{2}$$

25,26,27



25. 3/r1 = 6/2 r1 = 1

V = π/3h($r_1^2 + r_1r_2 + r_2^2$)

= π/3(3) ($1^2 + 1(2) + 2^2$) = 21.99 m^2

26. r2/h = 2/6 r_2 = h/3

V = 1/3 πr_2^2h = 1/3π (h/3)^{2}h

21.99 = 1/3π (h/3)^{2}h

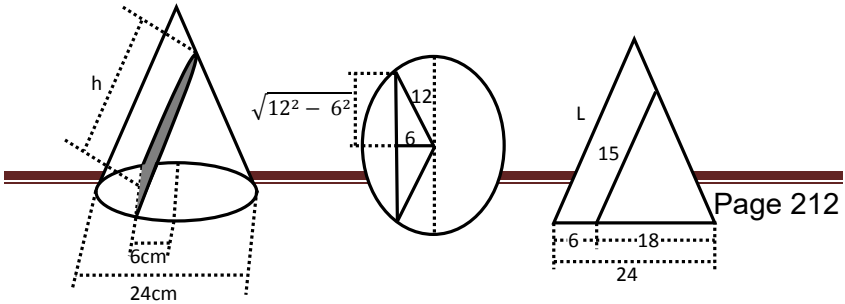
h = 5.739

27. 100 kg = 1 quintal

Weight = 21.99 x 9100 = 200109 N

W = mg m = W/9.81 = 20,398.47 kg

= 203.98 quintal



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For max area of inscribed parabola in a cone, the base of the parabola must pass at the midpoint of the radius.

28. Base of the parabola = 2 (10.392) = 20.784 cm

29. A = 2/3bh 208.7 = 2/3 (20.784) h

h = 15 cm

30. Let L = slant height of the cone

(Use similar triangles)

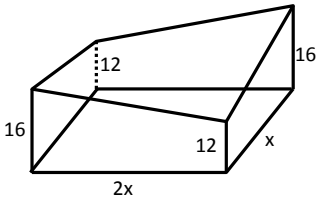
L/ 24 = 15/ (18) L = 20 cm

Height = 16 cm

$V = 1/3\pi R^2H = 1/3\pi(12)^2(16) = 2412.74\ cm^3$

31. The 8 corners of the cube will have 3 blue faces.

32.

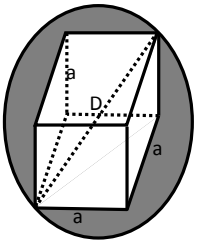


V = average of the heights x area

8200 = (16 + 16 + 12 + 12)/4 (2x)(x)

x = 17.113 cm

33.



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$27000 = a^3$ ► a = 30 cm

Diagonal of the cube = $\sqrt{3}a$ = diameter of the sphere

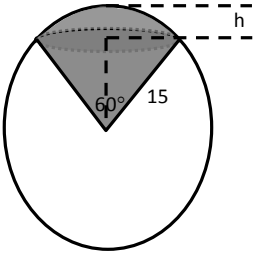
radius of the sphere = $\sqrt{3}(15) = 25.981$ cm

Volume of the sphere = $4/3\pi(25.981)^3 = 73460.89$ cm^3

Volume outside the box but inside the sphere =

$73460.89 - 27000 = 46460.9$ cm^3

34.



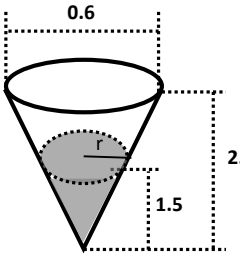
h = $15 - 15 \cos 30^0 = 2.01$ m

$V = 2/3\pi R^2 h$

$= 2/3\pi(15^2)(2.01)$

$= 947.19$ m^3

35.



r/ 1.5 = 0.6/ 2

r = 0.45 cm

$V = 1/3\pi(r^2)h$

$= 1/3\pi(0.45^2)(1.5)$

$= 0.318$ cm^3

36. The CG of a tetrahedron is 3/4 of the height from the base.

$V = 1/3\ B\ h$

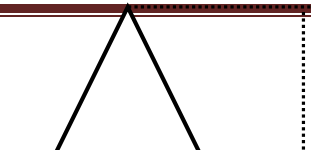
$B = 50^2\sqrt{3}/4 = 1082.53$

$V = \sqrt{2}/12\ e^3 = \sqrt{2}/12\ (50)^3 = 14731.39$ cm^3

$14731.39 = (1/3)\ 1082.53\ h$

h = 40.825 cm

CG is 3/4(40.825) = 30.62 cm



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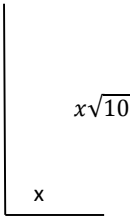
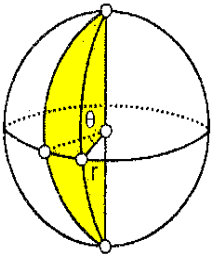
37.

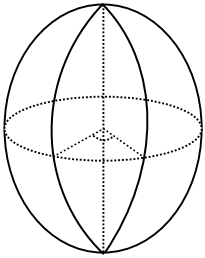
$h^2 = 10x^2 - x^2$

h = 3x



38.





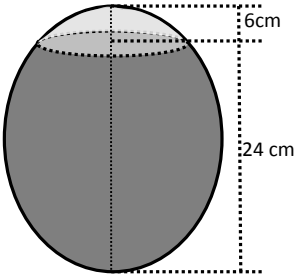
Volume of the sphere is $4/3\pi r^3$

Use ratio and proportion.

$4/3\pi(1)^3 / (360) = V/(40^0)$

$V = 0.4654\ cm^3$

39.



Area of a segment = $2\pi rh$

Ratio of area of segment

$= (2\pi r(24)) / (2\pi r(6))$

$= 4: 1$

40. Let radius of sphere = 1.

$V = 4/3(\pi)(1)^3 = 4\pi/3 = 4.189$

Surface Area = $4\pi r^2 = 4\pi(1)^2 = 4\pi$

Surface area becomes $4\pi(1.21) = 15.205$

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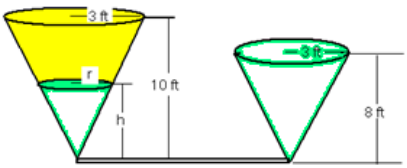
Then $15.205 = 4\pi x^2$ where x is the new radius.

$x = 1.1$ cm

New Volume = $4/3(\pi)(1.1)^3 = 5.575$

% increase in volume = $(5.575 - 4.189)/4.189 = 33\%$

41.



Volume of water in the bigger tank is $1/3\pi(3)^2(10) = 30\pi$

After water has flown from the big tank to the small tank.

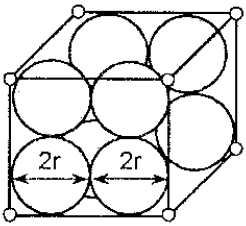
$1/3\pi(r^2)h + 1/3\pi(3)^2(8) = 30\pi$

$r/h = 3/10 \blacktriangleright r = 0.3h$

$1/3\pi(0.3h)^2h + 1/3\pi(3)^2(8) = 30\pi$

$h = 5.84$ ft

42.



side of cube = 2 inches

$4r = 2 \blacktriangleright r = 0.5$

radius of one marble = 0.5 in

Volume of water = volume of cube – 8 (volume of one sphere) = $2^3 - 8(4/3\pi(0.5)^3) = 3.811$ in^3

43.

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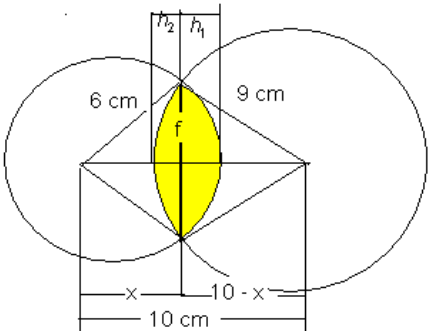


$$f^2 = 6^2 - x^2 = 9^2 - (10 - x)^2$$

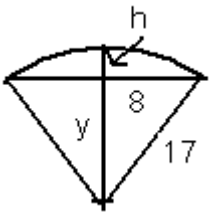
$x = 2.75$

$h_1 = 6 - x = 3.25$

$h_2 = 9 - (10 - x) = 1.75$

$V = 1/3\pi h_1^2 (3(6) - h_1) + 1/3\pi h_2^2(3(9) - h_2)$

$V = 243.24 \text{ cm}^3$



44.

$y^2 = 17^2 - 8^2$

$y = 15$

$y + h = 17$

$h = 2$

$Z = 2\pi Rh = 2\pi(17)(2) = 68\pi \text{ cm}^2$

$V = 1/3ZR = 1/3(68\pi)(17) = 1210.56 \text{ cm}^3$

45-46

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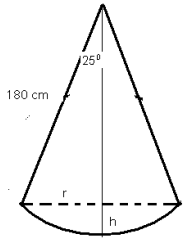


h = 180 – 180 cos 25^0 = 16.864 cm

Z = 2πRh = 2π(180)(16.864) = 19,072.73 cm^2

(46 Ans.)

V = 1/3ZR = 1/3(19,072.73)(180) = 1144363.8 cm^3

(45 Ans.)

r = 180 sin 25^0 = 76.071 cm

Surface of the cone = πr(180) = 43,017.14 cm^2

Total surface area = Z + 43,017.14 = 62,089.87 cm^2

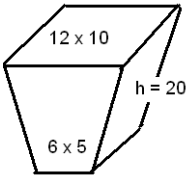
47.

E = 77 + 102 + 121 – 180 = 120^0

$V = \frac{\pi R^3 E}{540}$ R = 15 cm

V = 2356.19 cm^3

48.



Let us use the prismatoid technhique.

MODE 3 3

Input

X Y

0 6 x 5 = 30

20 12 x 10 = 120

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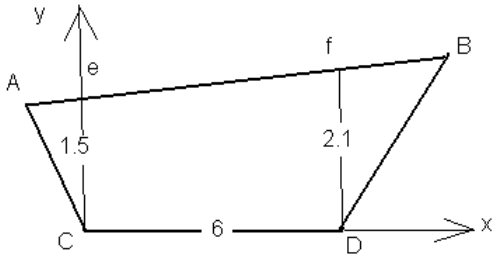
10 (12 + 6)/2 X (10 + 5)/ 2 = 67.5

A = 30 (SHIFT 1 5 1) B = 3 (SHIFT 1 5 2)
C = 0.075 (SHIFT 1 5 3)

Volume of water that overflows = $\int_{12}^{20} A + Bx + Cx^2 dx$ = 780.8 cu cm

= Volume of Gold
Weight of Gold = 780.8 x 19.3 = 15.07 kg

49.
Let 2x, 3x, 5x be the dimensions.
Then 2x(3x)(5x) = 810
All dimensions in meters.
50-51-52
Left side section



Coordinates e (0, 1.5) f(6 , 2.1)
C(0,0) D(6, 0)
Equation of AB MODE 5 1
0 1.5 1
6 2.1 1
-1/15 x + 2/3 y = 1
Equation of CA Side slope = 3/5 = 0.6
y = -0.6x or 0.6x + y = 0
Equation of DB MODE 5 1
6 0 1
7 0.6 1

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1/6x - 5/18y = 1

Coordinates of A:

MODE 5 1

-1/15 2/3 1

0.6 1 0

Int. (-15/7, 9/7)

Coordinates of B:

MODE 5 1

-1/15 2/3 1

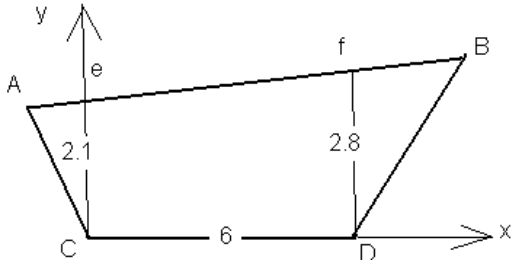
1/6 -5/18 1

Int. (10.2, 2.52)

Area of left section = 0.5 vector CB X vector DA

= 0.5 (10. 2 2.52) X (-15/7 – 6 9/7 – 0)

= 16.817



Right Section

Coordinates:

e(2.1, 0) f(6, 2.8) C(0, 0) D(6, 0)

Equation ef MODE 5 1

0 2.1 1

6 2.8 1

-1/18x + 10/21y = 1

Equation AC (side slope = 0.6)

y = -0.6x or 0.6x + y = 0

Equation DB

MODE 5 1

6 0 1

7 0.6 1

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1/6x - 5/18y = 1

Int. ef and CA

MODE 5 1

-1/18 10/21 1

0.6 1 0

A (-2.93, 1.758)

Int ef and DB

MODE 5 1

-1/18 10/21 1

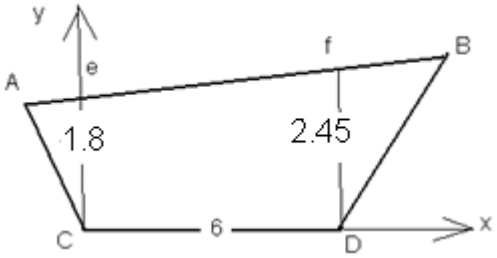
1/6 -5/18 1

B(11.793, 3.476)

Area of Right Section = 0.5 vector CB X vector DA

= 0.5 (11.793 3.476) X (-2.93 – 6 1.758)

= 25.886



For Area of the midsection

Coordinates

e(0,1.8) f(6, 2.45) D(6,0)

Equation ef MODE 5 1

0 1.8 1

6 2.45 1

Ans. – 13/216x + 5/9y = 1

Equation CA y = -0.6x

or 0.6x + y = 0

Intersection CA and AB :

MODE 5 1

A(-2.5412, 1.5247)

Equation DB (slope = 0.6)

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MODE 5 1
6 0 1
7 .6 1
1/6 x – 5/18y = 1
Int. of AB, DB
MODE 5 1
-13/216 5/9 1
1/6 -5/18 1
B(10.983, 2.99)
Area of Midsection = Vct CB X Vct DA
= .5 (10.983 2.99) X (-2.5412 – 6 1.5247)
= 21.141

Volume = 55/6(16.817 + 4(21.141) + 25.87) = 1,166.62 m^2

54-55-56
The Area of the sphere from height x can be modeled using the CALCU.
MODE 3 3
X Y
0 0
5 $\pi(5)^2$
10 0
A = 0 B = 31.4159 C = -31.146
Area = $31.4159x – 3.1416x^2$
For the Right Circular Cone
MODE 3 3
0 $\pi(5)^2$
10 0
20 $\pi(5)^2$
A = 78.584 B = -15.708 C = 0.7854
A = $78.584 – 15.708x + 0.7854x^2$

When the 2 A's are equal
$31.4159x – 3.1416x^2 = 78.54 -15.708+ 0.7854x^2$
x = 2 Ans (or x = 10)

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Area = $31.4159x - 3.1416x^2$ (when x = 2)
= $50.265\ m^2$

The Volume can be modeled by the equation:
$V = Ax + Bx^2/2 + Cx^3/3$
For the Sphere:
$V = 31.4159x^2/2 - 3.1416x^3/3$ when x = 2
= 54.45
For the Cone
$V = 78.54x - 15.708x^2/2 + 0.7854\ x^3/3$ when x = 2
= 127.76
Difference in Volume = 127.76 – 54.45 = $73.31\ m^3$

PROBLEM SET 13
ANALYTIC GEOMETRY (LINES)

1. Find the distance between the points A(4, -3) and B(-2, -5). PAST ECE BOARD
   a. 11.31 b. 9.54
   c. 10.11 d. 6.32
2. Find the distance between the points (5, -7) and (3, -6)?
   a. 2.236 b. 3.112
   c. 4.221 d. 8.221
3. If the distance between (3, y) and (8,7) is 13 is, they y is :
   a. 3 b. 19
   c. 12 d. 11
4. Determine the coordinates of the point which is 3/5 of the way from point (2, -5) to the point (-3,5). PAST ECE BOARD
   a. (-1,1) b. (-2,1)
   c. (-1,2) d. (1,-1)
5. The segment A(-1,4) to B(2, -2) is extended three times its own length. The terminal point is
   a. (11, -24) b. (-11,-20)
   c. (11,-18) d. (11,-20)
6. The slope of the line joining the points (3, -5) and (4, 10) is

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a. 15 b. -15
c. 12 d. 11

7. The slope of the line whose equation is
5x + 3y = 9 is
a. 5/3 b. -5/3
c. 3 d. -1/3

8. The slope of the line perpendicular to 4x – 5y = 2 is
a. -4/5 b. 4/5
c. -5/4 d. -2/5

9. What is the distance of the point (3, -7) to the line 4x + 6y – 5 = 0 ?
a. 5.67 b. 4.85
c. 3.33 d. 5.11

10. The two points on the line 2x – 3y + 4 = 0 which are at a distance 2 from the line 3x + 4y – 6 = 0
PAST CE BOARD
a. (-5,1) and (-5,2) b. (64, -44) and (4, -4)
c. (8,8) and (12,12) d. (44, -64) and (-4,4)

11. The distance from the point (2,1) to the line 4x – 3y + 5 = 0 is (PAST CE BOARD)
a. 1 b. 2
c.3 d. 4

12. The distance between the points 3x + y – 12 = 0 and 3x + y – 4 = 0. PAST EE BOARD
a. 2.53 b. 3.11
c. 1.45 d. 1.11

13. What is the slope of the line whose parametric equation is x = 3t + 7 and y = -4t – 5
a. 12/5 b. -4/3
c. 3/2 d. 2/3

14. Determine B such that 3x + 2y – 7 = 0 is perpendicular to 2x – By + 2 = 0 (PAST ECE BOARD)
a. 1 b. 2
c. 3 d. 4

15. What is the slope of the line whose parametric equation y = 4t + 6 and x = t + 1 (PAST CE BOARD)

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a. 1 b. 2
c. 3 d. 4

16. What is the area of the triangle whose vertices are A(2, -4) , B(1,6) and C (-8,3)? PAST CE BOARD
a. 33.5 b. 46.5
c. 52.1 d. 56.8

17. What is the area of the quadrilateral whose vertices in order are (0,0) , (3,1) , (2,4) and (1,2).
a. 5 b. 4
c. 2 d. 8

18. If the area of the triangle with vertices (5,2) , (x,4) and (0,-3) is 12.5, what is the value of x?
a. 11 b. 10
c. 12 d. 9

19. Find the equation of the line that passes through (4, 7) and (3, -5).
a. y = -41 + 12x b. y = 31 – 12x
c. y = -12 + 2x d. y = 12 + 7x

20. Find the equation of the line that passes trough the points (3, -8) and (4, -9) ?
a. X + Y + 5 = 0 b. X – Y – 5 = 0
c. X + Y + 6 = 0 d. X – Y – 6 = 0

21. Find the equation of the line that passes trough (3, -9) and slope = 3/2.
a.3x – 2y = 27 b. 2x – 3y = 27
c. 2Y = -27 - 3X d. 5Y = -27 + 3X

22. Find the equation of the line that passes trough (4, -9) and slope = -2.
a. 2X – Y = 1 b. 2X + Y = -1
c. Y = 2 - 2x d. Y = 3 + 2X

PAST CE BOARD (Poblems 23, 24, 25)
Line A has a slope of -4 and passes trough (-25,-10). Line B has x intercept of 10 and y intercept of 25.

23. The distance of line A to point (8, -3).
a. 35.85 b. 30.21
c. 33.71 d. 28.54

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24. The point of intersection of lines A and B.

a. (-80, 360) b.(-90,250)

c. (30, -420) d. (80, -440)

25. Determine the equation of a line perpendicular to line B and passing trough (-20,10).

a. 2x – 5y + 90 = 0 b. 5x – 3y + 40 = 0

c. 3x + 5y – 80 = 0 d. 2x – 5y -10 = 0

Problems (26, 27)

A triangle has the vertices A (1,3) , B(7,0) and C(4,6).

26. The centroid is at

a. (4, -1) b. (4,3)

c. (3, -1) d. (4, 8)

27. The orthocenter is at

a. (3,5) b. (3,4)

c. (-5,4) d. (4,-2)

28. The Euler line equation is at

a. x + y = 4 b. x + y = 6

c. x + y = 7 d. x – y = 5

29. Find the equation of the line that bisects the acute angle between the lines x – y -1 = 0 and 7x + y – 7 = 0

a. 4x + 5y = 1 b. 3x – y – 3 = 0

c. 3x + 2y = 2 d. 3x – 4y -7 = 0

30. Find the length of the line from point (5,3) to the y axis if the slope of the line is 3/2. PAST CE BOARD

a. 9.01 b. 8.12

c. 9.11 d. 9.22

31. One of the bisectors of the angles formed by the lines 4x + 3y – 4 = 0 and 5x + 12y – 60 = 0 has a positive x intercept. The intercept is

a. 31/7 b. 32/7 c. 33/7 d. 34/7

32. Find the equation of the line trough the intersection of the lines 7x – 13y + 46=0 and 19x + 11y – 41=0 and passing trough (3,1).

a. 21x + 7y = 70 b. 5x – 7y = 8

c. 31x – 34y = 59 d. 31x + 35y = 128

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Solution: 1 – 25

1. COMPLEX MODE:
   Let A = 4 – 3i B = -2 – 5i
   | A – B| = 6.32
2. A = 5 – 7i B = 3 – 6i
   |A –B| = 2.236
3. Using trial and error:
   A = 3 + 19i B = 8 + 7i
   | A – B| = 13
   Ans. y = 19
4. $x = x_1 + k(x_2 – x_1)$
   $= 2 + 3/5(-3 -2) = -1$
   $y = y_1 + k(y_2 – y_1)$
   $= -5 + 3/5(5 + 5) = 1$
   Ans. (-1,1)

   CAL TECHNIQUE: Let Vct A = (2 -5) Vct B = (-3 5)
   Vct C = Vct A + k(Vct B – Vct A)
   k = 3/5 Vct C = Vct A + (3/5)(VctB – Vct A)
   Vct C = (- 1 1) Coord. (-1 1)
5. A(-1,4) B(2,-2) C(a,b)
   Let AB = m; BC = 3m
   k = AB/(AC)
   = m/(m+ 3m) = 1/4
   2 = -1 + 1/4(a + 1) a = 11
   -2 = 4 + 1/4(b -4) b = -20
   Ans. (11, -20)

CAL TECHNIQUE:
ENTER: POL (2 –(-1), -2-4) =
The distance AB is stored to X and the angle with respect to the Y axis is stored to Y.
Thus: Distance AB = X and θ = Y
Since Distance from B to C = 3X .
ENTER Rec(3X, Y)
Result: (X = 9, Y = -18)
Coordinate of C = Coordinate of B + (9, -18)
= (2, -2) + (9, -18) = (11, -20)

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Another CAL TECHNIQUE USING VECTORS.

A(-1,4) to B(2, -2)

Vct A = (-1 4) Vct B = (2 -2) Vct C = ?

Then Vct B = Vct A + 1/4(Vct C – Vct A)

Vct C = 4(Vct B – Vct A) + Vct A

Vct C = (11 -20)

CAL TECHNIQUE USING COMPLEX NUMBERs.

Let A = -1 + 4i B = 2 – 2i C = ?

B = A + 1/4(C – A) or C = 4(B-A) + A

Store -1 + 4i to A and 2 – 2i to B.

C = 4(B-A) + A C = 11 – 20i (Ans. (11, -20))

6. slope = (-5 -10)/(3 -4) = 15

7. 5x + 3y = 9

For Ax + BY = C , slope = -A/B

A = 5, B = 3 slope = -5/3

8. 4x – 5y = 2

See Prob 7: A = 4 B = -5 slope = -A/B = 4/5

Then: slope perpendicular = -5/4

9. Use $d = \dfrac{Ah + Bk + C}{\sqrt{A^2 + B^2}}$

4x + 6y – 5 = 0 A = 4, B = 6, C = - 5

h = 3, k = - 7

d = 4.85

10. Use Trial and Error:

Test if the given points are on the line 2x + 3y + 4 = 0

The only points are (64,-44) and (4, -4).

because: 2x + 3y + 4 = 0 when x = 64 and y = - 44

or when x = 4, and y = - 4

Compute the distance from 2x + 3y + 4 = 0 to (64, -44)

Use $d = \dfrac{Ah + Bk + C}{\sqrt{A^2 + B^2}}$ where A = 2 , B = 3 , C = 4

h = 64, k = -44

and h = 4 , k = - 4

distance = 2. Ans. (64, -44), (4, -4)

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11. $d = \dfrac{Ah + Bk + C}{\sqrt{A^2 + B^2}}$

A = 4 B = -3 C = 5 h= 2, k = -1

d = 2

12. Get the distance of the two lines from (0,0) and subtract.

$$d = \frac{-12}{\sqrt{3^2 + 1^2}} - \frac{-4}{\sqrt{3^2 + 1^2}} = 2.53$$

(disregard sign)

13. x = 3t + 7, y = -4t – 5

Get two points on the line.

t = 0 (x,y) = (7, -5)

t = 1 (x,y) = (10, -9)

m = (-9 +5)/ (10 -7) = -4/3

14. slope of 3x + 2y – 7 = 0

3x + 2y – 7 = 0 slope = -3/2

2x – By + 2 = 0 slope = 2/B

(-3/2)(2/B) = -1 B = 3

15. Let t = 0 , then x = 1 , y = 6

t = 1, then x = 2 , y = 10

(1,6) (2,10) slope = (10-6)/(2-1) = 4

16.

$$\text{Area} = 1/2 \det\begin{bmatrix} 2 & -4 & 1 \\ 1 & 6 & 1 \\ -8 & 3 & 1 \end{bmatrix}$$ Ans. 46.5

17.

$$1/2 \begin{bmatrix} 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 4 & 2 & 0 \end{bmatrix}$$

Area = 1/2 (0 + 12 + 4 + 0 - 0 - 2 - 4 - 0) = 4

18. Use trial and error:

$$0.5 \det\begin{bmatrix} 5 & 2 & 1 \\ 12 & 4 & 1 \\ 0 & -3 & 1 \end{bmatrix} = -12.5$$

Ans. x = 5

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19. Y = A + Bx
Subtitute (4,7) and (3, -5)
7 = A + B(4)
-5 = A + B(3)
A = -41 B = 12
Equation: y = -41 + 12x or 12x – y = 41
Cal Technique: ENTER MODE 5 1
Input: 4 7 1
3 -5 1
Ans. X = 12/41 Y = -1/41
Equation: 12/41x – 1/41y = 1
or 12x – y = 41

20. Points are (3, -8) and (4, -9)
(See Cal Tech of Prob 19)
ENTER: MODE 5 1
Input: 3 -8 1
4 -9 1
Result: X = -1/5 Y = -1/5
Equation: -1/5 X – 1/5 Y = 1
X + Y = - 5 or X + Y + 5 = 0

21. slope = 3/2 passing (3, -9)
For Ax + By = C , slope = -A/B
Equation: 3x – 2y = C
ENTER: 3X – 2Y CALC X? 3 Y? - 9
Result: 3x – 2y = 27

22. Y = A + BX slope = -2 (4, -9)
See Soln of 21:
Equation: 2x + y = C
ENTER: 2X + Y CALC X? 4 Y? – 9
2X + Y = -1

23. Equation of line A: slope = -4 (-25, -10)
Then 4x + y = C
ENTER: 4X + Y CALC X? -25 Y? -10
Ans. 4x + y = -110 or 4x + y + 110 = 0
distance of line A to (8,-3)

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$d = \frac{Ah + Bk + C}{\sqrt{A^2 + B^2}}$ A = 4 B = 1 C = 110

h = 8 and k = -3

d = 33.71

24. Line A : 4x + y = -110 (1)

Line B: x/10 + y/25 =1 (2)

Solve 1 and 2 using MODE 5 1

Ans . x = -90 y = 250

25. slope of line B is -1/10 / 1/25 = -5/2

slope perpendicular to B is 2/5

slope = 2/5 (-20, -10)

Equation: 2x – 5y = C

ENTER: 2X – 5Y CALC X ? -20 Y? -10

Result: 2x – 5y = 10

26. Centroid = (1 + 7 + 4)/3 , (3 + 0 + 6)/ 3

= (4, 3)

27. The orthocenter is the intersection of the altitude.

A (1,3) , B(7,0) and C(4,6).

AB slope = (0 -3)/ (7-1) = -1/2

slope perpendicular = 2

Equation altitude from C to AB

ENTER: MODE 5 1

Input: 4 6 1

5 6+2 1

Result: X = 1 Y = -1/2

X – 1/2Y = 1 (1)

BC slope = (6 -0)/ (4 – 7) = -2

slope perpendicular = 1/2

Equation altitude from A to BC

ENTER MODE 5 1

Input: 1 3 1

2 3 + 1/2 1

Result: X =-1/5 Y = 2/5

-1/5x + 2/5y = 1 (2)

Solve equations (1) and (2) simultaneously using MODE 5 1

Ans. (3, 4)

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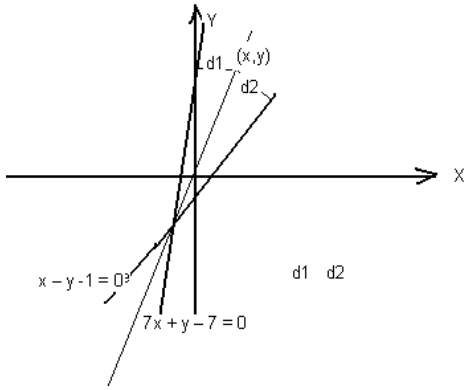
28. The Euler line is the line connecting the centroid and the orthocenter. (3,4) and (4,3)

MODE 5 1 :

Input: 3 4 1
4 3 1

Result: 1/7x + 1/7y =1 or x + y = 7

29. Let P(x,y) be the the point on the angular bisector.
Then P(x,y) is equidistant from the two given lines.



$$d = \frac{Ax + By + C}{\sqrt{A^2 + B^2}}$$

$d_1 = d_2$ (both above the lines)

$$\frac{1(x) - 1(y) - 1}{-\sqrt{1^2 + 1^2}} = \frac{7(x) + y - 7}{\sqrt{7^2 + 1^2}}$$

$3x – y – 3 = 0$

Note : the sign of B will also be the sign of the square root.

30. Equation of the line:

Equation of the line:

ENTER: MODE 5 1

INPUT: 5 3 1
6 3+3/2 1

Result: 1/3x – 2/9y = 1

When x = 0 , y = -4.5

Points (0,- 4.5) and (5,3)

Distance = | 0- 4.5 i – (5 + 3i) | = 9.01

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31. There are 2 bisectors:

$$\frac{4x+3y-4}{\sqrt{4^2+3^2}}=\frac{5x+12y-60}{\sqrt{5^2+12^2}} \quad \text{and} \quad \frac{4x+3y-4}{\sqrt{4^2+3^2}}=-\frac{5x+12y-60}{\sqrt{5^2+12^2}}$$

13(4x + 3y -4) = 5(5x + 12y – 60)
27x + 15y = -248 y = 0 x int = - 248/27
13(4x + 3y – 4) = -5(5x + 12y – 60)
77x + 99y = 352 y = 0 x int = 352/77 = 32/7
32. MODE 5 1
Solve 7x – 13y = - 46
19x + 11y = 41
x = 1/12 y = 43/12
MODE 5 1 Input:
3 1 1
1/12 43/12 1
x = 31/128 y = 35/128 31x + 35y = 128

PROBLEM SET 14
CIRCLES AND PARABOLA

Given: Problems 1 , 2
$x^2 + y^2 – 10x – 10y + 25 = 0$ PAST CE BOARD
1. Find the center
a.(5,-4) b. (5,5)
c. (6, -5) d. (4,-5)
2. Find the radius.
a. 5 b. 4
c. 2 d. 3
3. The shortest distance from A(3,8) to the circle
$x^2 + y^2 + 4x – 6y = 12$ is (PAST CE BOARD)
a. 2.1 b. 2.3
c. 2.3 d. 2.4
4. Find the coordinates of the center of the circle
$x^2 + y^2 – 2x – 4y – 31 = 0$?
a. (-1,-1) b. (-2,-2)
c. (1,2) d. (2,1)
5. Find the radius of the circle in problem 4
a. 4 b. 5

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c. 6 d. 7

6. What is the area of the circle $4x^2 + 4y^2 – 7x + 6y - 35 = 0$? PAST CE BOARD

a. 31.66 b. 67.22

c. 144.22 d. 161.22

7. What is the area of the circle $x^2 + y^2 + 8x + 4y – 61 = 0$? PAST CE BOARD

a. 233.22 b. 254.47

c. 311.22 d. 221.33

8. Find the area bounded by the curve $x^2 -10x + y^2 + 10y + 25 = 0$? CE MAY 2004

a. 56.77 b. 78.54

c. 65.55 d. 89.11

9. What is the equation of the radical axis of the circles $x^2+ y^2 -18x - 14y + 121 = 0$ and $x^2 + y^2 – 6x + 6y = 14$

a. $12x + 20y =135$ b. $12x – 20y = 135$

c. $13x – 20y = 115$ d. $15x + 20y = 122$

CE BOARD Nov 2006 Problems 10, 11

A circle has the equation $x^2 + y^2 – 6x + 12y + 9 = 0$

10. Find the radius

a. 5 b. 4

c. 6 d. 7

11. Find the center.

a.(4,-7) b. (3, -6)

c. (9.-1) d. (5,-8)

12. What is the distance from the center of the circle to the line $y = 2x + 10$?

a. 8.87 b. 9.84

c. 9.11 d. 8.12

13. What is the equation of the radical axis of the circles?

$x^2 + y^2 =1$ and $x^2 + y^2 – 6x + 6y + 11 = 0$

a. $x – y = 2$ b. $x + y = 2$

c. $x + 2y = 4$ d. $2x – y = 1$

13a, 13b, 13c

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A circle is tangent to the line $x + y = 4$ at $(-3, 7)$ and its center lies on the y axis.

13a. Find the coordinate of the center of the circle

a. (0, 12) b. (0, 10)

c. (0, 5) d. (0, 6)

13b. The radius of the circle is

a. 4.24 b. 5.33

c. 4.11 d. 4.07

14. The focus of the parabola $y^2 = 16x$ is at?

PAST CE BOARD

a. (4,0) b. (0,4)

c. (3,0) d. (0,3)

15. What is the length of the latus rectum of the curve $x^2 = 20y$?

PAST CE BOARD

a. 10 b. 5

c. 20 d. 25

Given the parabola $4y = x^2 – 6x + 21$

Problems 16, 17, 18

16. The vertex is at

a. (2,3) b. (-3,3)

c. (3,3) d. (4,-4)

17. The length of the latus rectum is

a. 6 b. 4

c. 3 d. 5

18. The focus is at

a. (3,5) b. (3,4)

c. (4,4) d. (5,6)

Given the parabola $4y^2 – 4x + 7y + 12 = 0$

Problems 19, 20, 21

19. The vertex is at

a. (143/64, -7/8) b. (123/64, 7/8)

c. (123/8, -7/64) c. (111/64, -9/64)

20. The length of the latus rectum is

a. 1 b. 1/2

c. 1/4 d. 1/8

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21. The focus is at
    a. (159/64, -7/8) b. (123/64, -5/8)
    c. (123/64, 3/8) d. (114/64, -8/63)
22. The equation of the axis of symmetry of $y= 2x^2 – 7x + 5$ is (PAST ECE BOARD)
    a. $2x + 7 = 0$ b. $4x – 7 = 0$
    c. $4x + 7 = 0$ d. $3x – 7 = 0$
23. The focus of the parabola $y^2 + 4x – 4y – 8 = 0$ is at PAST (EE BOARD)
    a. (1,2) b. (2,3)
    c. (2,2) d.(3,3)
24. From problem 23, what is the equation of the directrix?
    a. $x = 2$ b. $x = 1$
    c. $x = 0$ d. $x = 1.5$

A circle passes trough (2,3), (6,1) and (4,-3)
Problems 25, 26
25. The center of this circle is
    a. (1,4) b. (3,0)
    c. (1, -1) d. (4,2)
26. The radius of this circle is
    a. 3.162 b. 3.162
    c. 5.262 d. 1.112
27. Find the center of the circle that circumscribes the triangle formed by the lines $y = 0$, $y = x$ and $2x + 3y = 10$
    a. (3,0) b. (4,0)
    c. (5/2,0) d. (-4.5,0)

Given a parabola with horizontal axis that passes trough (0,0), (-8, 3) and (4, 9). Problems 28, 29, 30
28. Find the equation of the parabola.
a. $27x = -114y + 14y^2$ b. $37x = 114y + 14y^2$
c. $17x = -114y - 14y^2$ d. $27x = -119y + 14y^2$
29. Find the vertex of this parabola.
a. (-361/42, 57/14) b. (-361/12, 57/6)
c. (361/42, 57/5) d. (-367/22, 51/14)
30. Find the equation of its directrix.

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a. x = -9.31 b. x = -8.65
c. x = 9.11 d. x = 9.22

31. Find the length of the common chord to the parabolas $y^2 = 2x + 4y + 6$ and $y^2 = 3x + 3y + 1$
a. 12.45 b. 16.12
c. 13.34 d. 16.21

32. Find the equation of the parabola whose focus is at (1,0) and vertex is at (2,0).
a. $y^2 + 4x – 8 = 0$ b. $y^2 + 2x – 8 = 0$
c. $y^2 + 6x – 8 = 0$ d. $y^2 + 4x + 2 = 0$

33. When the load is uniformly distributed horizontally, the cable of a suspension bridge hangs in a parabolic arc. If the bridge is 300 ft long , the towers 60 ft high and the cable 20 ft above the roadbed at the center, find the distance from the roadbed 50 ft from the center.
a. 31.22 b. 24.44
c. 28.21 d. 32.11

34. Find the equation of the parabola with horizontal axis which crosses the x axis at x = 2 and the y axis at y = -1 and y = 7.
a. $y^2 + 2x -12y – 14 = 0$ b. $2y^2 + 2x -12y – 14 = 0$
c. $2y^2 + 7x -12y – 14 = 0$ d. $2y^2 + 7x -12y – 11 = 0$

35. Find the distance from the point (6,4) to the circle $x^2 + 16x + y^2 + 16y + 64 = 0$
a. 4.47 b. 5.43
c. 7.21 d. 3.45

36. Determine the length of the latus rectum of the parabola $x^2 – 6x – 12y – 51 = 0$ CE BOARD May 2006
a. 11 b. 10
c. 6 d. 12

37. There is a fixed circle having a radius of 6 with center (10,12). Find the equation of the curve connecting the centers of all circles tangent to the fixed circle and the x axis. PAST CE BOARD
a. $x^2 – 10x – 36y + 208 = 0$ c. $x^2 – 30x – 36y + 208 = 0$
b. $x^2 – 20x – 36y + 201 = 0$ d. $x^2 – 20x – 36y + 208 = 0$

Problems 38, 39, 40
A parabola having an axis parallel to the y axis passes trough the points A, B and C as follows

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A (1,1)
B (2,2)
C (-1,5)

38. The equation of the parabola is

a. $Y = 2 - 2x + x^2$ b. $Y = 3 - 3x + x^2$
c. $Y = 2 - 2x + 2x^2$ d. $Y = -2 - 2x + x^2$

39. The length of the latus rectum of the parabola.

a. 2 b. 1
c.3 d. 4

40. The vertex is at

a. (1,1) b. (-1,1)
c.(2,1) d. (-2,-2)

Problems 41, 42, 43

A parabola has its axis parallel to the x axis and passes trough (5,4), (11,-2) and (21, -4).

41. The equation of the parabola is

a. $X = 5 - 2y + 1/2y^2$ b. $X = 5 - y + 1/2y^2$
c. $X = 15 - 2y + y^2$ d. $X = 5 - 2y + 2y^2$

42. The length of the latus rectum is

a. 2 b. 1/2
c. 3 d. 4

43. The equation of the directrix is :

a. $3x - 2 = 0$ b. $2x - 5 = 0$
c. $5x + 2 = 0$ d. $7x - 4 = 0$

1. Note: For a circle A

SOLUTION: A A B B

A = -10 B = 1 $x^2 + y^2 - 10x - 10y + 25 = 0$

--B/(2A) = 5

same for y:

Center (5,5)

2. Substitute x =5 and y = 5 in

$x^2 + y^2 - 10x - 10y + 25$

Ans. -25

radius = $\sqrt{-25}$ = 5

3. Get the center:

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A A B B

$x^2 + y^2 + 4x - 6y = 12$

A = 1 B = 4

-B/(2A) = -2 (for x)

A = 1 B = - 6

-B/(2A) = 3 (for y)

Center is (-2, 3)

Get the radius:

Substitute x = -2 and y = 3 in $x^2 + y^2 + 4x - 6y - 12$

Answer: -25

radius = $\sqrt{-25}$ = 5

Distance from center to (3,8)

| -2 + 3i – (3 + 8i) | = 7.07

shortest distance from (3,8) to circle = 7.07 – 5 = 2.07

or 2.1

4. A A B B

$x^2 + y^2 - 2x - 4y - 31 = 0$?

A = 1 B = -2

-B/(2A) = 1

A = 1 B = - 4

-B/(2A) = 2

Center (1,2)

5. Substitute x = 1 and y = 2 in $x^2 + y^2 - 2x - 4y - 31$

Ans. -36

r = $\sqrt{-36}$ = 6

6. A A B B

$4x^2 + 4y^2 - 7x + 6y - 35 = 0$?

A = 4 B = - 7

-B/(2A) = 7/8

A = 4 B = 6

-B/(2A) = -3/4

Substitute x = 7/8 and y = -3/4 in

$4x^2 + 4y^2 - 7x + 6y - 35$ = -645/16

4Area = 645/16 π = 126.64

Area = 31.66

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A A B B

7. $x^2 + y^2 + 8x + 4y – 61 = 0$?

A = 1 B = 8

-B/(2A) = - 4

A = 4 B = 4

-B/(2A) = -2

Let x = -4 and y = - 2

$x^2 + y^2 + 8x + 4y – 61 = -81$

Area = 81π = 254.45

8.

A B A B

x^2 -10x + y^2 + 10y + 25 = 0?

A = 1 B = -10

-B/(2A) = 5

A = 1 B = 10

-B/(2A) = -5

LET x = 5 and y = -5

x^2 -10x + y^2 + 10y + 25 = -25

Area = $\pi(25)$ = 78.54

9.

x^2+ y^2 -18x -14y = - 121 and $x^2 + y^2 – 6x + 6y = 14$

SUBTRACT the two equations.

-18x -14y – (-6x + 6y) = -121 - 14

-12x -20y = -135

12x + 20y =135

10. A B A B

$x^2 + y^2 – 6x + 12y + 9$

A = 1 B = - 6

-B/(2A) = 3

A = 12 B = 1

-B/(2A) = -6

x = 3 , y = - 6

$x^2 + y^2 – 6x + 12y + 9$ = -36

radius = $\sqrt{(-36)}$ = 6

11. center (3, -6)

12. y = 2x + 10 2x – y + 10 = 0

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A = 2 B = -1 C = 10 h = 3, k = - 6

$$d = \frac{Ah + Bk + C}{\sqrt{A^2 + B^2}} = 9.84$$

13.

Subtract the 2 equations.

$x^2 + y^2 = 1$ and $x^2 + y^2 - 6x + 6y = -11$

Ans. $6x - 6y = 12$ or $x - y = 2$

13a, 13b

The circle is tangent to $x + y = 4$ at (-3, 7)

Equation of the normal line (slope of tangent = -1, slope of normal = 1)

MODE 5 1

-3 7 1

-2 7+ 1 1

Equation of Normal Line:

$-1/10x + 1/10y = 1$ or $- x + y = 10$

when x = 0, y = 10

Center (0, 10)

Distance from (-3, 7) to (0, 10)

MODE 2: $| -3 + 7i - (0 + 10i) | = 3\sqrt{2} = 4.24$

14. Latus rectum = 16 $4a = 16$

a = 4 Parabola opens horizontal to the right.

Focus is at (0, 0 + 4) = (0,4)

15. Parabola opens upward V(0,0)

Latus Rectum is 20.

16.

A B C

Rewrite as $y = x^2/4 - 6/4x + 21/4$

A = 1/4 B = -6/64

$-B/(2A) = 3$

Let x = 3

$x^2/4 - 6/4x + 21/4 = 3$

Vertex is at (3,3)

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Given the parabola $4y = x^2 - 6x + 21$

16. The vertex is at

17. The length of the latus rectum is

Rewrite as $y = x^2/4 - 6/4x + 21/4$

Latus rectum = reciprocal of the coefficient of x^2

$= 4$

18. The focus is at

$4a = 4$, $a = 1$

Parabola is vertical. Vertex is at (3,3)

Focus is at (3, 3 + 1) = (3,4)

19. Rewrite as:

$x = 4y^2/4 + 7y/4 + 12/4$

A B

$x = y^2 + 7y/4 + 3$

$A = 1$ $B = 7/4$

$-B/(2A) = -7/8$

Let $y = -7/8$

$y^2 + 7y/4 + 3 = 143/64$

Vertex is at (143/64, -7/8)

20. Length of latus rectum = reciprocal of coeff of y^2

Latus Rectum = 1/4

21. Focus

(143/64 + 1/4, -7/8) or (159/64, -7/8)

22.

A B C

$y = 2x^2 - 7x + 5$

$A = 2$ $B = -7$

$-B/(2A) = 7/4$

$x = 7/4$ is the axis of symmetry.

or $4x - 7 = 0$

23. Rewrite as:

A B

$4x = - y^2 + 4y + 8$

$x = -y^2/4 + y + 2$

Latus Rectum is 4 $4a = 4$, $a = 1$

Opens to the left.

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To get the vertex.
A = -1/4 B = 1
-B/(2A) = 2
y = 2
$-y^2/4 + y + 2 = 3$
Vertex is at (3,2)
Focus is at (3 – 1,2) = (2,2)

24. Vertex is at (3,2) Directrix is a vertical line. Parabola opens to the left. directrix is x = 3 + 1 or x = 4

25. Equation of the circle
$x^2 + y^2 + Dx + Ey + F = 0$
$Dx + Ey + F = -x^2 – y^2$ (1)
substitute the points (2,3), (6,1) and (4,-3) in
$2D + 3E + F = -2^2 - 3^2$
$6D + 1E + F = -6^2 -1^2$
$4D – 3E + F = -4^2 –(-3)^2$
USE MODE 5 2
D = -6 E = 0 F = -1
Equation of the circle is:
$x^2 + y^2 -6x -1 = 0$
Center:
A B A B
$x^2 -6x + y^2 + 0y - 1 = 0$
A = 1 B = -6
-B/(2A) = 3
The center is at (3,0)

26. Let x = 3 and y = 0
$x^2 + y^2 -6x -1 = -10$
radius = $\sqrt{10}$ = 3.162

27. Find the vertices of the triangle.
y = 0
y = x Intersection (0,0)
y = x, 2x + 3y = 10
x – y = 0
2x + 3y = 10
Intersection: (2,2)

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y = 0 and 2x + 3y = 10
Intersection: (5, 0)
The vertices of the triangle are A(0,0), B(2,2) and C(5,0)
Use: $Dx + Ey + F = -x^2 - y^2$
$D(0) + E(0) + F = 0$ (0,0)
$F = 0$
$2D + 2E = -2^2 - 2^2$ (2,2)
$2D + 2E = -8$
$D(5) + E(0) = -5^2 - 0^2$
$D = -5$
and $E = 1$
The equation of the circle is
A A B
$x^2 + y^2 - 5x + 1 = 0$
A = 1 B = - 5
-B/(2A) = 5/2
The center is at (5/2, 0)
28. ENTER: MODE 3 3
Input the given coordinates.
X Y
0 0
3 -8
9 4
(Note the x and y positions are interchanged because we are looking for the parabola of the form $x = A + By + Cy^2$
A (SHIFT 1 5 1) A = 0
B (SHIFT 1 5 2) B = -38/9
C (SHIFT 1 5 3) C = 14/27
The equation of the parabola is $x = -38/9y + 14/27y^2$
or $27x = -114y + 14y^2$
29.
B A
$x = -114/27y + 14/27y^2$
A = 14/27 B = -114/27
-B/(2A) = 57/14
Let y = 57/14

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$-114/27y + 14/27y^2 = -361/42$
The vertex is at $(-361/42, 57/14)$
30.
Latus rectum = $27/114 = 4a$ $a = 9/152$
Parabola opens to the left.
Directrix is $x = -361/42 – 9/152 = -8.65$
31.
Find the intersection:
$2x + 4y + 6 = 3x + 3y + 1$
$-x + y + 5 = 0$
$y = x – 5$
Substitute this in $y^2 = 2x + 4y + 6$
$(x -5)^2 = 2x + 4(x +5) + 6$
$x^2 -10x + 25 = 2x + 4x + 20 + 6$
$x^2 -16x -1 = 0$
Use mode 5 3.
The roots are $x = 8 + \sqrt{65}$ and $x = 8 - \sqrt{65}$
and $y = x – 5$, $y = 3 + \sqrt{65}$ and $3 - \sqrt{65}$
The endpoints of the chords are $(8 + \sqrt{65}, 3 + \sqrt{65})$
and $(8 - \sqrt{65}, 3 - \sqrt{65})$
Let $A = 8 + \sqrt{65} + (3 + \sqrt{65})i$
$B = 8 - \sqrt{65} - (3 - \sqrt{65})i$
$|A – B| = 16.12$
32.
Distance from vertex to focus is a.
$a = 2 – 1 = 1$
$4a = 4$
The parabola is horizontal opening to the left.
$(y – k)^2 = -4a (x – h)$
$(y – 0)^2 = -4(x – 2)$
$y^2 = -4x + 8$ or $y^2 + 4x – 8 = 0$
33.

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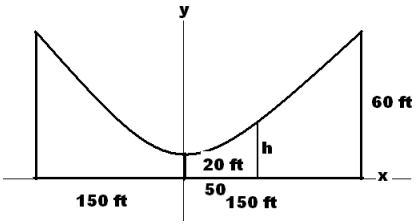

Get 3 coordinates.
ENTER: MODE 3 3
X Y
0 20
150 60
-150 60
ENTER: 50 SHIFT 1 5 6 =
Ans. $50\hat{y}$ = 24.444

34. The three coordinates are
(2, 0) , (0, -1) and (0, 7)
ENTER: MODE 3 3
Input
X Y
0 2
-1 0
7 0
Note: the coordinates are reversed because the axis is horizontal.
$X = A + BY + CY^2$
SHIFT 1 5 1 or 1 7 1 A = 2
SHIFT 1 5 2 or 1 7 2 B = 12/7
SHIFT 1 5 3 or 1 7 3 C = - 2/7
$X = 2 + 12/7y – 2/7y^2$
$7x = 14 + 12y – 2y^2$ Ans. $2y^2 + 7x - 12y – 14 = 0$
35.
A B A B
$x^2 + 16x + y^2 + 16y + 64$
A = 1 B = 16
-B/(2A) = -8

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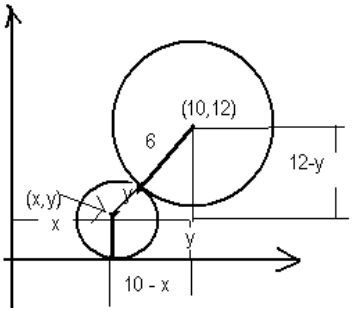
Center is at (- 8, -8)
Let x = -8 , y = - 8
$x^2 + 16x + y^2 + 16y + 64 = -64$
radius = $\sqrt{64} = 8$
distance = $| 6 + 4i - (8 + 8i) | = 4.47$
36. Rewrite as $12y = x^2 - 6x - 51$
$y = x^2/12 - 6x/12 - 51/12$
Latus Rectum is the reciprocal of the coefficient of x^2
$= 12$.

37.



$(6 + y)^2 = (10-x)^2 + (12-y)^2$
$36 + 12y + y^2 = 100 - 20x + x^2 + 144 - 24y + y^2$
$x^2 - 20x - 36y + 208 = 0$
38.
ENTER: MODE 3 3
X Y
1 1
2 2
-1 5
A = 2 B = -2 C = 1
The equation is $Y = 2 - 2x + x^2$
39.
Length of the latus rectum is reciprocal of coefficient of $x^2 = 1$.
40. $Y = x^2 - 2x + 2$

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A = 1 B = -2
-B/(2A) = 1
Let x = 1
$x^2 – 2x + 2 = 1$
Vertex (1,1)

41. ENTER: MODE 3 3

X	Y
4	5
-2	11
-4	21

A = 5 B = -2 C = 1/2
$X = 5 – 2y + 1/2y^2$

42. Length of Latus Rectum = reciprocal of coefficient of y^2
= 1/(1/2) = 2

43.
A B
$X = 1/2y^2 – 2y + 5$
A = 1/2 B = - 2
-B/(2A) = 2
Let y = 2
$1/2y^2 – 2y + 5 = 3$
The b vertex is at (3, 2)
The parabola opens to the right.
Length of Latus Rectum = 4a = 2 , a = 1/2
Directrix: x = 3 – a = 3 - 1/2
x = 2.5 or 2x – 5 = 0

PROBLEM SET 15
ELLIPSE AND HYPERBOLA

CE BOARD Nov 2006
The equation of the ellipse is
$16x^2 + 25y^2 -128x -150y + 381 = 0$
1. Find the coordinates of the center of the ellipse.

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a. (3, 5) b. (4,3)
c. (-2,-3) d. (3, -2)

2. Find the length of the minor axis.
a. 2 b. 3
c. 4 d. 5

3. Find the distance between the foci.
a. 1 b. 2
c. 3 d. 4

4. PAST CE BOARD Compute the length of the latus rectum of the ellipse $x^2 + 2y^2 + 4x + 4y + 4 = 0$
a. 2.21 b. 1.41
c. 1.21 d. 1.31

5. PAST CE BOARD
Find the ratio of the minor axis to the major axis of the ellipse $9x^2 + 16y^2 = 144$
a. 4/3 c. 1/2
c. 3/8 d. 3/4

PAST CE BOARD PROBLEMS 6, 7, 8, 9, 10, 11, 12
Given the ellipse $4x^2 + 9y^2 – 64x + 54y + 301 = 0$

6. Determine the center
a. (8,1) b. (8,-3)
c. (4,1) d. (3,7)

7. Determine the length of the major axis
a. 2 b. 3
c. 2 d. 4

8. Determine the length of the latus rectum.
a. 4/3 b. 8/3
c. 10/3 d. 13/3

9. The eccentricity is
a. 0.745 b. 0.671
c. 0.867 d. 0.541

10. One of the directrix is
a. x = 13.11 b. x = 12.02
c. x = 9.01 d. x = 7.21

11. Find the area
a. 18.85 b. 19.1

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c. 11.34 d. 17.21

12. Find the circumference

a. 16.02 b. 12.23

c. 11.22 d. 14.11

13. The perimeter of an ellipse is 28.448 cm. The major axis is 10 cm long and lies on the x axis with its center on the origin. Determine the equation of the ellipse. PAST CE BOARD.

a. $16x^2 + 25y^2 = 400$ b. $25x^2 + 16y^2 = 400$

c. $32x^2 + 25y^2 = 600$ d. $25x^2 + 32y^2 = 600$

The equation of the ellipse is given by $16x^2 + 36y^2 = 576$

CE BOARD Nov 2004 Problems 14, 15, 16

14. Compute the equation of the polar of the point (4,-6) with respect to the ellipse.

a. $6x - 27y = 72$ b. $6x + 27y = 72$

c. $3x - 27y = 72$ d. $6x - 3y = 72$

15. Find the second eccentricity.

a. 1.221 b. 1.118

c. 1.311 d. 1.411

16. Find the length of the diameter of the ellipse $x^2 + 3y^2 + 2x - 6y = 0$ which has a slope of 1

a. 1.45 b. 2.83

c. 1.55 d. 1.44

17. Find the distance between the vertices of the ellipse having the equation $64x^2 + 25y^2 + 768x - 200y + 1104 = 0$

CE BOARD MAY 2002

a. 8 b. 12

c. 14 d. 16

18. What is the eccentricity of the curve $9x^2 + 25y^2 - 144x + 200y + 751 = 0$?

a. 0.6 b.0.7

c. 0.8 d. 0.9

Given the hyperbola $9x^2 - 4y^2 - 36x + 8y - 4 = 0$.

Problems 19, 20, 21, 22

19. Locate the center.

a. (2,-1) b. (2,1)

c. (2,-2) d. (4, -1)

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20. Find the eccentricity

a. 1.08 b. 1.11

c. 1.22 d. 1.80

21. Find the latus rectum.

a. 9 b. 12

c. 14 d. 15

22. One of the vertices is at

a. (0,1) b. (-1,0)

c. (-1,-1) d. (2, 0)

Given the hyperbola $9x^2 - 16y^2 + 18x + 64y - 19 = 0$

Problems 23, 24, 25, 26

23. Find the center.

a. (1,3) b. (-1,2)

c.(3, 1) d. (1,4)

24. The eccentricity is

a. 1.338 b.1.231

c. 1.411 d. 1.111

25. One of the asymptotes

a. $8y + 10x + 25 = 0$ b. $8y – 4x + 25 = 0$

c. $8y – 9x – 25 = 0$ d. $9x + 8y – 24 = 0$

26. The latus rectum is

a. 1.22 b. 2.37

c. 2.11 d. 4.11

27. Find the equation of the locus of a point which moves so that its distance from the point (2,0) is 2/3 of its distance from the line y = 5.

a. $9x^2 + 5y^2 - 36x + 40y – 64 = 0$

b. $9x^2 + 5y^2 - 16x + 40y – 64 = 0$

c. $9x^2 + 2y^2 - 36x + 40y – 64 = 0$

d. $9x^2 + 5y^2 - 36x - 40y – 64 = 0$

28. Find the equation of the locus of points which moves so that its distance from the point (4,2) is twice its distance from the line x = 1.

a. $3x^2 – y^2 + 4y – 16 = 0$

b. $3x^2 – 2y^2 + 4y – 16 = 0$

c. $3x^2 – y^2 + 3y – 16 = 0$

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d. $3x^2 – y^2 + 4y – 12 = 0$

29. What is the area of $9x^2 + 25y^2 – 225 = 0$?

PAST CE BOARD

a. 47.1 b. 50.2

c. 63.8 d. 72.3

30. Point P(x,y) moves with a distance from point (0,1) one-half of its distance from the line y = 4. The equation of its locus is (PAST ECE BOARD)

a. $2x^2 – 4y^2 = 5$ b. $4x^2 + 3y^2 = 12$

c. $2x^2 + 5y^3 = 3$ d. $x^2 + 2y^2 = 4$

A hyperbola has the equation $x^2 -8x – 4y^2 + 64y = 256$

Problems 31, 32, 33 CE BOARD NOV 2007

31. Determine the center

a. (4,8) b. (4, -3)

c. (3,3) d. (-4,8)

32. The distance between the foci.

a. 8.944 b. 6.112

c 3.444 d. 9.112

33. The distance between the vertices.

a. 4 b. 6

c. 12 d. 8

Answers.

1.

A A B B

$16x^2 + 25y^2 -128x -150y + 381 = 0$

$A = 16 \quad B = -128$

$-B/(2A) = 4$

$A = 25 \quad B = -150$

2. Let $x = 4$ and $y = 3$

$16x^2 + 25y^2 -128x -150y + 381 = -100$

$a^2 = 100/16 \quad a = 5/2$

$b^2 = 100/25 \quad b = 2$

Length of minor axis = $2b = 2(2) = 4$

3. $a^2 = b^2 + c^2$

$(5/2)^2 = 2^2 + c^2$

$c = 3/2$

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distance between foci = 2c = 2(3/2) = 3

4.

A A B B

$x^2 + 2y^2 + 4x + 4y + 4 = 0$

A = 1 B = 4

-B/(2A) = -2

A = 2 B = 4

-B/(2A) = -1

Center is at (-2, -1)

X = -2 Y = -1

$x^2 + 2y^2 + 4x + 4y + 4 = -2$

$a^2 = 2/1$ $a = \sqrt{2}$

$b^2 = 2/2$ $b^2 = 1$

Latus RECTUM = $2b^2/a = 2(1)/\sqrt{2} = 1.414$

5. $a^2 = 144/9$ $a = 4$

$b^2 = 144/16$ $b = 3$

b/a = 3/4

6.

A A B B

$4x^2 + 9y^2 – 64x + 54y + 301$

A = 4 B = -64

-B/(2A) = 8

A = 9 B = 54

-B/(2A) = -3

Center is at (8, -3)

7. X = 8 Y = - 3

$4x^2 + 9y^2 – 64x + 54y + 301 = -36$

$a^2 = 36/4$ $a = 3$

$b^2 = 36/9$ $b = 2$

major axis = 2a = 2(3) = 6

8. Latus Rectum = $2b^2/a = 2(2)^2/ 3 = 8/3$

9. $a^2 = b^2 + c^2$

$3^2 = 2^2 + c^2$

$c = \sqrt{5}$

$e = c/a = \sqrt{5}/ 3 = 0.745$

10. $d= a/e = 3/(\sqrt{5}/ 3) = 9/\sqrt{5}$

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The directrices are $x = 8 + 9/\sqrt{5}$ and $x = 8 - 9/\sqrt{5}$

$x = 12.02$

11. Area = $\pi ab = \pi(3)(2) = 6\pi$

12. $C = 2\pi\sqrt{\frac{a^2+b^2}{2}}$

$a = 3 \quad b = 2$

$C = 16.02$

13. $C = 2\pi\sqrt{\frac{a^2+b^2}{2}}$

$C = 28.448 \quad 2a = 10 \quad a = 5, \; b = 4$

Equation:

$x^2/a^2 + y^2/b^2 = 1$

$x^2/5^2 + y^2/4^2 = 1$

$16x^2 + 25y^2 = 400$

14.

replace x^2 by xx_1 and y^2 by yy_1 where $(x_1, y_1) = (4, -6)$.

$16xx_1 + 36yy_1 = 576$

$16x(4) + 36y(-6) = 576$

$6x - 27y = 72$

15.

$a^2 = 576/16 \quad a = 6$

$b^2 = 576/36 \quad b = 4$

$a^2 = b^2 + c^2 \quad c = 2\sqrt{5}$

second eccentricity = $c/b = 2\sqrt{5} / 4 = 1.118$

16.

A A B B

$x^2 + 3y^2 + 2x - 6y = 0$

$A = 1 \quad B = 2$

$-B/(2A) = -1$

$A = 3 \quad B = -6$

$-B/(2A) = 1$

Center is at (-1,1)

Equation of the line: slope = 1

$Y = A + Bx$

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1 = A + 1(-1)
A = 2
Y = 2 + X
Substitute this in $x^2 + 3y^2 + 2x – 6y = 0$
$x^2 + 3(2 + x)^2 + 2x – 6 (2+x) = 0$
x = 0 , y = 2
Distance from (0,2) to (-1,1) is
| 0 + 2i – (-1 + i) | = $\sqrt{2}$
Diameter = $2\sqrt{2} = 2.828$
17.
A A B B
$64x^2 + 25y^2 + 768x -200y + 1104 = 0$
A = 64 B = 768
-B/(2A) = -6
A = 25 B = -200
-B/(2A) = 4
Center is at (-6,4)
Let X = -6 , Y = 4
$64x^2 + 25y^2 + 768x -200y + 1104 = -1600$
$a^2 = 1600/25$ a = 8
$b^2 = 1600/64$ b = 5
Distance between vertices is 2a = 16.
18.
A A B B
$9x^2 + 25y^2 -144x + 200y + 751 = 0$?
A = 9 B = -144
-B/(2A) = 8
A = 25 B = 200
-B/(2A) = -4
Center is at (8, -4)
Let x = 8 , y = - 4
$9x^2 + 25y^2 -144x + 200y + 751 = -225$
$a^2 = 225/9$ a = 5
$b^2 = 225/25$ b = 3
$a^2 = b^2 + c^2$ c = 4
e = c/a = 4/5 = 0.8

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19.

A A B B

$9x^2 – 4y^2 – 36x + 8y – 4 = 0$

A = 9 B = - 36

-B/(2A) = 2

A = -4 B = 8

-B/(2A) = 1

Center is at (2,1)

20.

Let X = 2 Y = 1

$9x^2 – 4y^2 – 36x + 8y – 4 = -36$

$a^2 = 36/9$ a = 2

$b^2 = 36/4$ b = 3

$c^2 = a^2 + b^2$

$= 2^2 + 3^2$

$c = \sqrt{13}$

$e = \sqrt{13}/2 = 1.803$

21. Latus Rectum = $2b^2/a = 2(3^2)/2 = 9$

22. The vertices are at (2 + a, 1) and (2 – a, 1)

or (4,1) and (0, 1)

23.

A A B B

$9x^2 -16y^2 + 18x + 64y -19 = 0$

A = 9 B = 18

-B/(2A) = -1

A = -16 B = 64

-B/(2A) = 2

The center is at (-1,2)

24.

LET x = -1 y = 2

$9x^2 -16y^2 + 18x + 64y -19 = 36$

(If the right side is positive , the hyperbola is vertical)

$a^2 = 36/16$ (coefficient of y) a = 3/2

$b^2 = 36/9$ b = 4/3

$c^2 = a^2 + b^2$ $c^2 = (3/2)^2 + (4/3)^2$

$c = \sqrt{145}/16$

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e = c/a = (√145 /16)/ (3/2) = 1.338

25.

$y - k = \pm a/b\ (x - h)$

Note: the hyperbola is vertical

$y - 2 = \pm (3/2)/(4/3)\ (x + 1)$

$y - 2 = \pm 9/8(x + 1)$

$8y - 16 = 9x + 9$ (+ part)

$8y - 9x - 25 = 0$

26. Latus rectum $= 2b^2/a$

$= 2(4/3)^2/(3/2) = 2.37$

27. Let us use trial and error.

Get a point from the 1st choice

$9x^2 + 5y^2 - 36x + 40y - 64 = 0$

when $x = 0$ $5y^2 + 40y - 64 = 0$, $y = 1.366, -9.367$

The point is (0, 1.366)

distance of (2,0) and (0, 1.366) =

$| 2 + 0i - (0 + 1.3666i) = 2.422$

Distance from (0, 1.366) to $y = 5$ is

$5 - 1.366 = 3.634$

Is $2.422 = 2/3(3.634)$? Yes!

The answer is $9x^2 + 5y^2 - 36x + 40y - 64 = 0$

28. We use trial and error. Let us use the 1st choice.

$3x^2 - y^2 + 4y - 16 = 0$

When $x = 0$

$-y^2 + 4y - 16 = 0$ y are imaginary (not possible)

When $y = 0$, $3x^2 - 16 = 0$ $x = 2.309, -2.309$

The point is (2.309, 0)

Distance of (2.309, 0) to (4,2)

$= | 2.309 + 0i - (4 + 2i) | = 2.619$

Distance of (2.309, 0) to $x = 1$ is $2.309 - 1 = 1.309$

Is $2.619 = 2(1.309)$? Yes!

Answer is $3x^2 - y^2 + 4y - 16 = 0$

29. $b^2 = 225/25$ $b = 3$

$a^2 = 225/9$ $a = 5$

Area = πab = π(5)(3) = 47.1

30. Let us get a point from $2x^2 - 4y^2 = 5$

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When y = 0 , x = 1.581 or – 1.581
A point is (1.581, 0)
Distance from (1.581, 0) to (0, 1) is
| 1.581 + 0i – (0 + 1i)| = 1.871
Distance of (1.581, 0) to y = 4 is 4
Is 1.871 = 1/2 (4) Wrong!
Let us use $4x^2 + 3y^2 = 12$
If y = 0 , x = $\sqrt{3}$, -$\sqrt{3}$
A point is ($\sqrt{3}$, 0)
Distance of ($\sqrt{3}$, 0) to (0,1) is
| $\sqrt{3}$ + 0i - (0 + 1i) | = 2
Distance of($\sqrt{3}$, 0) to line y = 4 is 4
Is 2 = 1/2(4) ? Yes.
Ans. b. $4x^2 + 3y^2 = 12$
31.
A B A B
x^2 -8x – $4y^2$ + 64y = 256
B = -8 A = 1
-B/(2A) = 4
A = -4 B = 64
-B/(2A) = 8
Center is at (4,8)
32.
Let X = 4 , Y = 8
x^2 -8x – $4y^2$ + 64y - 256 = -16
a^2 = 16/1 a = 4
b^2 = 64/16 b = 2
c^2 = a^2 + b^2
c = 2$\sqrt{5}$
Distance between foci = 2c = 4$\sqrt{5}$ = 8.944
33. Distance between vetices = 2a = 2(4) = 8

PROBLEM SET 16

(ROTATION OF AXES/ LINES TANGENT TO CONICS)

For a curve $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
To remove the xy term: rotate the axes by θ

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where $\tan 2\theta = \frac{B}{A-C}$ and apply the coordinate transformation :

x = x' cos θ - y'sin θ

y = x' sin θ + y'cos θ

For the hyperbola xy = 8 CE BOARD May 2005

Problems 1, 2, 3

1. Determine the distance between the vertices

a. 4 b. 6

c. 8 d. 12

2. Compute the length of the conjugate axes.

a. 2 b. 4

c. 6 d. 8

3. Compute the eccentricity

a. 1.23 b. 1.66

c. 1.41 d. 1.52

4. Given the conic $24xy - 7y^2 = 144$, what is the eccentricity?

a. 3/5 b. 4/5

c. 5/4 d. 7/4

5. Given the conic $13x^2 + 10xy + 13y^2 + 6x - 42y - 27 = 0$

Locate the center.

a. (1,-1) b. (-1,2)

c. (2,1) d. (-3,1)

6. Find the equation of the line tangent to the hyperbola $4xy + y^2 - 6y = 15$ at the point (2,3)

a. 3x + 2y = 12 b. 4x – 3y = 1

c. 3x – 3y = 5 d. 5x + 4y = 2

7. Find the equation of the line tangent to

$x^2 - 6xy + 9y^2 - 4x + 7 = 0$ at (2,1)

a. 2x + y = 5 b. 3x + 2y = 7

c. x – y = 1 d. x -3y = 4

8. Find the equation of the line tangent to the conic

$4x^2 - 5xy + 2y^2 + 3x - 2y = 0$ and passes trough point (2,3).

a. x – y + 1 = 0 b. x + y - 5 = 0

c. 2x + 3y - 7 = 0 d. 3x + 4y + 1 = 0

9. Find the equation of the tangent line to the conic

$3x^2 - 3xy + 4x + y - 3 = 0$ at (-1,1).

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a. $3x - 4y + 3 = 0$ b. $5x - 4y + 9 = 0$

c. $3x + 5y - 2 = 0$ d. $3x + 2y - 5 = 0$

10. Find the equation of the tangent line to the conic $5y^2 + 4x - 2y - 3 = 0$ at the point (2,0).

a. $x + 2y = 2$ b. $3x + y = 1$

c. $2x - y = 1$ d. $4x + y = 8$

11. Find the area of the ellipse $5x^2 - 4xy + 8y^2 - 36 = 0$

a. 4π b. 3π c. 6π d. 8π

12- 13 – 14 - 15

Given $4x^2 - 24xy + 11y^2 - 24x + 32y + 40 = 0$

12. The equation is

a.ellipse b. parabola c. hyperbola c. circle

13. The slope of the rotated x axis is

a. 1/2 b. 3/4 c. 2/3 d. 6/7

14. The eccentricity of the conic is

a. 1.414 b. 2.828 c. 2.236 d. 3.000

15. The coordinate of the vertex of $\sqrt{x} + \sqrt{y} = 1$?

a. (1/4, 1/4) b. (0 , 1) c. (1, 0) d. (1/16, 9/16)

Solutiion:

1. $\tan 2\theta = \dfrac{1}{0-0}$ $2\theta = 90^0$ $\theta = 45^0$

$\cos\theta = 1/\sqrt{2}$ $\sin\theta = 1/\sqrt{2}$

$x = x' \cos\theta - y'\sin\theta$

$y = x' \sin\theta + y'\cos\theta$

$x = x'(1/\sqrt{2}) - y'(1/\sqrt{2}) = 1/\sqrt{2}\,(x' - y')$

$y = x'(1/\sqrt{2}) + y'(1/\sqrt{2}) = 1/\sqrt{2}\,(x' + y')$

$xy = 8$

$[1/\sqrt{2}\,(x' - y')][1/\sqrt{2}\,(x' + y')] = 8$

$1/2\,(x'^2 - y'^2) = 8$

$x'^2 - y'^2 = 16$

$x'^2/4^2 - y'^2/4^2 = 1$ $a = 4$ $b = 4$

Distance between vertices = $2a = 2(4) = 8$

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CAL TECHNIQUE:
Get a point from xy = 8. When x = 1, y = 8
Pol(1,8) r = 8.062257 θ = 82.874
This is stored into (X,Y)
ENTER: Rec(X, Y – 45)
Then ENTER: $X^2 – Y^2$
Result: $X^2 – Y^2 = 16$
Then: $X^2/4^2 – Y^2/4^2 = 1$ a = 4 , b = 4
2. Length of the conjugate axes = 2a = 8
3. $c^2 = a^2 + b^2$
$c^2 = 4^2 + 4^2$ $c = 4\sqrt{2}$
$e = c/a = (4\sqrt{2})/4 = 1.414$
4. $\tan 2\theta = \frac{B}{A-C}$

$$\tan 2\theta = \frac{24}{0-(-7)} = \frac{24}{7}$$

$\theta = \tan^{-1}(24/7)/2$ Store this to X, then get cos X and sin X.
$\cos\theta = 4/5$ $\sin\theta = 3/5$
Transfomation:
$x = x'\cos\theta - y'\sin\theta$
$= x'(4/5) – y'(3/5)$
$= 1/5(4x' – 3y')$
$y = x'\sin\theta + y'\cos\theta$
$= x'(3/5) + y'(4/5)$
$= 1/5(3x' + 4y')$
$24xy – 7y^2 = 144$
$24(1/25)(4x' – 3y')(3x' + 4y') – 7(1/5)^2 (3x' + 4y')^2 = 144$
$24(12x'^2 + 7x'y' - 12y'^2) – 7(9x'^2 + 24x'y' + 16y'^2) = 3600$
$225x'^2 - 400y'^2 = 3600$
$x'^2/4^2 - y'^2/3^2 = 1$
a = 4 b = 3
$c^2 = a^2 + b^2$
$= 4^2 + 3^2$
c = 5 e = c/a = 5/4
CAL TECHNIQUE:
$\theta = 1/2 \tan^{-1}(24/7)$ Store to A

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A = 0 B = 24 C = - 7
$B^2 – 4AC = 24^2 – 4(0)(-7) > 0$ Hyperbola
We are expecting $x^2/a^2 – y^2/b^2 = 1$
Get 2 points from $24xy – 7y^2 = 144$
If y = 1 $24x(1) – 7(1)^2 = 144$ x = 151/24
If y = 2 $24x(2) – 7(2)^2 = 144$ x = 43/12
Enter: Pol(151/24,1) Then Rec(X, Y-A)
Result: x = 5.6333 and y = -2.975
ENTER: Pol(43/12,2) Then Rec(X, Y-A)
Result: x = 4.06667 and y = -0.55
These Points must satisfy
$x^2/a^2 – y^2/b^2 = 1$
$5.6333^2/a^2 – (-2.975)^2/b^2 = 1$
$4.06667^2/a^2 – (-0.55)^2/b^2 = 1$ USE MODE 5 1
$1/a^2 = 0.0624$ $1/b^2 = 0.1111$
a = 4 b = 3
The equation is $x'^2/4^2 – y'^2/3^2 = 1$

5.

A = 13 B = 10 C = 13
$\tan 2\theta = B/(A –C) = 10/(13 – 13)$
$2\theta = 90^0$
$\theta = 45^0$
$\cos \theta = 1/\sqrt{2}$ $\sin \theta = 1/\sqrt{2}$
$x = x' (1/\sqrt{2}) - y'(1/\sqrt{2}) = 1/\sqrt{2} (x' – y')$
$y = x'(1/\sqrt{2}) + y'(1/\sqrt{2}) = 1/\sqrt{2} (x' + y')$
$13/2 [(x' – y')]^2 + 10/2 (x' + y')(x' – y')] + 13/2(x' + y')^2$
$+ 6/\sqrt{2} (x' – y') – 42/\sqrt{2} (x' + y') – 27 = 0$
$6.5 (x'^2 – 2x'y' + y'^2) + 5(x'^2 – y'^2) + 6.5(x'^2 + 2x'y' + y'^2)$
$+ 3\sqrt{2} (x' – y') - 21\sqrt{2} (x' + y') – 27 = 0$
$18x'^2 + 8y'^2 -18\sqrt{2} x' - 24\sqrt{2} y' – 2 7= 0$
The conic is an ellipse.
Center from the new axes.
A A B B
$18x'^2 + 8y'^2 -18\sqrt{2} x' - 18\sqrt{2} y' – 2 7= 0$
A = 18 B = $-18\sqrt{2}$
$-B/(2A) = \sqrt{2}/2$

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A = 8 B = - 24√2
-B/(2A) = 3√2 / 2
The point from the new axes is
(√ 2 / 2, 3√2 / 2) = (x', y')
But: x = 1/√2 (x' – y')
= 1/√2 (√2 /2 - 3√2 / 2) = -1
y = 1/√2 (x' + y')
= 1/√2 (√2 /2 + 3√2 / 2) = 2
Ans. (-1, 2)
6. Test if x = 2 and y = 3 is in the conic.
Substitute x = 2 and y = 3 in $4xy + y^2 – 6y$
$4xy + y^2 – 6y = 15$ (ok)
For these problems: Replace x^2 by xx_1 , y^2 by yy_1,
xy by $1/2 (x_1y + y_1x)$, x by $1/2 (x + x_1)$ and
y by $1/2 (y + y_1)$ or
Replace x^2 by 2x , y^2 by 3y , xy by1/2(2y + 3x),
x by 1/2 (x + 2) , y by 1/2(y + 3)
4/2(2y +3x) + 3y – 6/2 (y + 3) = 15
4y + 6x = 24 or 2y + 3x = 12 Ans.
7. Is the point (2,1) in the conic?
Let x =2 , y= 1
$x^2 – 6xy + 9y^2 – 4x + 7 = 0$ (YES)
Replace x^2 by 2x , y^2 by 1y , xy by 1/2 (2y + 1x),
x by 1/2(x + 2) and y by 1/2 (y + 1).
2x – 3(2y + x) + 9(y) – 2(x + 2) + 1 = 0
-3x + 3y + 1 = 0
x – y – 1 = 0
8.
Test if the point 2,3 is on the conic.
x = 2 , y = 3
$4x^2 – 5xy + 2y^2 + 3x – 2y = 4$
(2,3) is not in the conic.
Get the equation of the polar to the curve or the chord of contact.
Replace x^2 by 2x , y^2 by 3y , xy by 1/2(3x + 2y)
x by 1/2(x + 2) and y by 1/2(y + 3)
4(2x) -5/2(3x + 2y) + 2(3y) + 3/2 (x + 2) – 2/2 (y +3) = 4

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$2x – 7.5x – 5y + 6y + 1.5x + 3 – y – 3 = 4$

$x = 0$ (This is the chord of contact)

The intersection of $4x^2 – 5xy + 2y^2 + 3x – 2y = 0$

and $x = 0$

$2y^2 – 2y = 0$

$y = 0$ and $y = 1$

or (0,0) and (0,1).

There are 2 tangent lines.

(0,0) and (2,3)

$m = 3/2$ $y = 3/2x$

(0,1) and (2,3)

$m = (3 - 1) / (2 – 0)$

$m = 1$: Equation : $y - 1 = 1(x - 0)$

or $x – y + 1 = 0$

For (0,-1) and (2,3) $m = 2/4 = 1/2$

$y – (-1) = 1/2(x – 0)$ or $y = 2 + 1/2x$

9.

Check if the point (-1,1) is on the conic.

$x = -1, y = 1$

$3x^2 – 3xy + 4x + y – 3 = 0$ (OK)

Replace x^2 by $–x$

y^2 by y

xy by $1/2 (x(1) + y(-1))$

x by $1/2 (x -1)$

y by $1/2(y + 1)$

$3(-x) – 3/2 (x – y) + 4/2 (x -1) + 1/2(y +1) – 3 = 0$

$-3x – 1.5x + 2x + 1.5y – 2 + 0.5y + 0.5 – 3 = 0$

$-2.5x + 2y -4.5 = 0$

$5x -4y + 9 = 0$

10.

Check if the point (2,0) is on the conic.

$x = 2, y = 0$

$5y^2 + 4x – 2y – 3 = 5$

(2, 0) is not on the conic.

Get the equation of the polar or the chord of contact.

Replace y^2 by $0y$, x by $1/2 (x + 2)$, y by $1/2 (y + 0)$

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$0 + 2(x + 2) – y – 3 = 0$

$2x + 4 – y – 3 = 0$

$2x – y = -1$

$y = 2x + 1$

Substitute this in

$5y^2 + 4x – 2y – 3 = 0$

$5(2x + 1)^2 + 4x – 2(2x + 1) – 3 = 0$

$5(4x^2 + 4x + 1) + 4x – 4x – 2 – 3 = 0$

$20x^2 + 20x = 0$

$x = 0$ and $x = -1$

If $x = 0$, $y = 1$ (0,1)

If $x = -1$, $y = -1$ (-1,-1)

Equation of the tangent line

(0,1) , (2, 0) $m = - 1/2$

$y - 0 = -1/2 (X – 2)$

$-2Y = X – 2$

$X + 2Y = 2$ Ans.

The other tangent line:

$x – 3y = 2$ from (2,3) and (-1,-1)

11. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

STORE the following values in the calculator.

$A = 5$ $B = -4$ $C = 8$ $D = 0$ $E = 0$ $F = -36$

$B^2 – 4AC = -144 < 0$ ELLIPSE

$M = (C –A)/ B$ and $M^2 + (M^2 + 1)^{1/2} = 1/2$ (slope of the rotated x axis) and $-2 /1$ is the slope of the rotated y axis.

Compute

Points on the rotated x and y axes are :

$(x_1, y_1) = (2, 1)$ and $(x_2 , y_2) = (-1, 2)$

$$\frac{Ax_1^2 + Bx_1y_1 + Cy_1^2}{x_1^2 + y_1^2} = 4 \qquad \frac{Ax_2^2 + Bx_2y_2 + Cy_2^2}{x_2^2 + y_2^2} = 9$$

The equation with no xy term is $A'x^2 + B'y^2 = 36$

$4x'^2 + 9y'^2 = 36$ or $x'^2 / 3^2 + y'^2 /2^2 = 1$

Area = $\pi ab = \pi(3)(2) = 6\pi$

12 – 13 – 14

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STORE:

A = 4 B = -24 C = 11 D = -24 E = 32 F = 40

$B^2 - 4AC = 400 > 0$ (hyperbola)

Compute $A'x'^2 + B'y'^2 + C'x' + D'y' + F = 0$

$M = \dfrac{C - A}{B}$ $M + \sqrt{M^2 + 1}$ = 3/4 (SLOPE of the rotated x axis)

(-4/3 is the slope of the rotated y axis)

(x_1, y_1) = (4, 3) (x_2, y_2) = (-3 , 4)

$A' = \dfrac{Ax_1^2 + Bx_1y_1 + Cy_1^2}{x_1^2 + y_1^2} = -5$

$B' = \dfrac{Ax_2^2 + Bx_2y_2 + Cy_2^2}{x_2^2 + y_2^2} = 20$

$C' = \dfrac{Dx_1 + Ey_1}{\sqrt{x_1^2 + y_1^2}} = 0$ $D' = \dfrac{Dx_2 + Ey_2}{\sqrt{x_2^2 + y_2^2}} = 40$

The equation is $-5x'^2 + 20y'^2 + 40y' + 40 = 0$

$-5x'^2 + 20(y'^2 + 2y' + 1) = -40 + 20$

$-5x'^2 + 20(y' + 1)^2 = -20$

$(y' + 1)^2 / 1^2 - x'^2 / 2^2 = 1$

a = 1 b = 2 $c = \sqrt{a^2 + b^2} = \sqrt{5}$

and e = c/a $= \sqrt{5} = 2.236$

15. $\sqrt{y} = 1 - \sqrt{x}$

$y = (1 - \sqrt{x})^2 = 1 - 2\sqrt{x} + x$

$2\sqrt{x} = 1 + x - y$

$4x = (1 + x - y)^2$

$4x = 1 + x^2 + y^2 + 2x - 2xy - 2y$

$x^2 - 2xy + y^2 - 2x - 2y + 1 = 0$

A =1 B = -2 C = 1 D = -2 E = -2 F = 1

$B^2 - 4AC = 0$ (PARABOLA)

M = (C –A)/B $M + \sqrt{M^2 + 1} = 1$

(slope of the rotated x axis)

(x_1, y_1) = (1, 1) is the point on the rotated x axis.

(x_2, y_2) = (-1 , 1) is the point on the rotated y axis.

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$$A' = \frac{Ax_1^2 + Bx_1y_1 + Cy_1^2}{x_1^2 + y_1^2} = 0$$

$$B' = \frac{Ax_2^2 + Bx_2y_2 + Cy_2^2}{x_2^2 + y_2^2} = 2$$

$$C' = \frac{Dx_1 + Ey_1}{\sqrt{x_1^2 + y_1^2}} = -2\sqrt{2}$$

$$D' = \frac{Dx_2 + Ey_2}{\sqrt{x_2^2 + y_2^2}} = 0$$

Equation of the parabola:

$0x'^2 + 2y'^2 - 2\sqrt{2}\,x' + 0y' + 1 = 0$

$y'^2 - \sqrt{2}\,x' + 1/2 = 0$

$y'^2 = \sqrt{2}\left(x' - \frac{1}{2\sqrt{2}}\right)$

V($\frac{1}{2\sqrt{2}}$, 0) in the rotated x', y' axes

Since it is rotated 45^0 (slope is 1)

$x = \frac{1}{2\sqrt{2}} \cos 45^0 = 1/4$ $\quad y = \frac{1}{2\sqrt{2}} \sin 45 = 1/4$

(x,y) = (1/4, 1/4)

PROBLEM SET 17
SOLID ANALYTIC GEOMETRY

1. Find the distance between A (3,4, -6) and B (4, 5, -7) .
PAST CE BOARD

a. 1.234 b. 1.732
c. 1.812 d. 2.112

2. Find the distance between the points A(4, 8, -1) and B (9, 1, 2)?

a. 8.22 b. 9.11
c. 10.12 d. 11.22

3. The distance between the points A(2, 10, 4) and B (8, 3, z) is 9.434. What is thevalue of z

a. 5 b. 6
c. 1 d. 9

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4. PAST CE BOARD
If DE = 5.745 cm and the coordinates of D are (2,5,4) and (x, 3,6) respectively. What is the value of x ?

a. 7 b. 9
c. 4 d. 8

5. Find the distance from the plane $7x – 6y + 6z + 8 = 0$ and the point (3, 2, -1).

a. 2 b. 1
c. 3 d. 4

6. Two faces of a cube lie in the planes $2x – y + 2z – 3= 0$ and $6x – 3y + 6z + 8 = 0$. Find the volume of the cube.

a. 4411/125 b. 4913/729
c. 245/343 d. 111/71

7. What is the radius of the sphere
$x^2 + y^2 + z^2 + 8x – 2y + 1 = 0$? PAST CE BOARD

a. 5 b. 4
c. 2 d. 3

8. What is the volume of the sphere whose equation is
$4x^2 + 7x + 4y^2 – 3y + 4z^2 + 2z -8 = 0$?

a. 21.426 b. 23.451
c. 28.221 d. 29.332

Problems 9, 10
Given the points A(2, 1, -3) , B(-8, 1, 4) and C(3, -1,6).
9. Find the equation of the plane determined by the points A, B and C.
A(2,1,-3) , B(-8,1,4), C(3,-1,6)
a. $14x + 97y + 20z – 65 = 0$ b. $12x + 97y + 20z – 65 = 0$
c. $14x + 92y + 20z – 65 = 0$ d. $11x + 97y + 23z – 65 = 0$
10. Find the area of the triangle ABC.
a. 75.03 b. 50.01
c. 34.11 d. 78.08

Given the points A(5, -2,7) and B(3, 2, 8)
Problems 11, 12, 13, 14
11. The parametric equation of AB is
a. $x = 3 – 2t$, $y = 2 + 4t$, $z = 8 + t$
b. $x = 4 -4t$, $y = 7 + 2t$, $z = 12 – 4t$
c. $x = 3 – 2t$, $y = 12 -3t$, $z = 11 + 2t$

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d. x = 4 + 2t, y = 11 + 2t, z = -19 + 3t

12. The point where AB pierces the xy plane is

a. (15, -23, 0) b. (19, -30, 0)

c. (12, 21, 0) d. (-11,23, 0)

13. The direction number of AB is

a. < - 2 4 1 > b. < 2 1 7 >

c. < 1 2 3> d. < 8 1 -1 >

14. The direction cosine of AB is

a. < -0.436 0.8728 0.2182 >

b. < -0.476 0.8912 0.2456 >

c. < 0.436 0.8999 0.2188 >

d. < -0.496 0.8798 0.2182 >

15. Find the distance between the lines

$\frac{x}{6} = \frac{y}{-2} = \frac{z-6}{1}$ and $\frac{x-7}{6} = \frac{y}{-2} = \frac{z+1}{1}$

a. 2 b. 3

c. 4 d. 5

16. Find the angles between between the lines whose direction numbers are < 1 2 2 > and < 3 4 -12 >

a. 178.1^0 b. 121.1^0

c. 111.1^0 d. 109.47^0

17. Find the distance between the lines

$\frac{x}{1} = \frac{y}{2} = \frac{z-6}{3}$ and $\frac{x}{3} = \frac{y}{2} = \frac{z}{1}$

a. 3.112 b. 2.449

c. 4.112 d. 1.441

18. A point has a coordinate (2, 3, -6). Find the angle tat the line joining the point and the origin makes with the z axis.

a. 149^0 b. 111^0

c. 122^0 d. 104^0

19. Find the area of the triangle that the coordinate planes cut from the plane x + 4y + 8z = 16.

a. 12 b. 24

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c. 36 d. 48

20. From problem 19. Find the volume of the tetrahedron formed.

a. 22.33 b. 21.33

c. 11.22 d. 18.22

21. Find the angle between the planes $2x – y – 2z + 5 = 0$ and $4x + 8y + z – 3 = 0$. CE BOARD MAY 2010

a. 85.75^0 b. 81.23^0 c. 67.21^0 d. 31.23^0

22. A parametric equation of the line

$5x – 2y + 3z – 13 = 0$, $3x – y + z – 7 = 0$ is

a. $x = 1 + t$, $y = -4 + 4t$, $z = t$

b. $x = 1 – t$, $y = -4 + 3t$, $z = 2t$

c. $x = 2 + t$, $y = -3 – 5t$, $z = t$

d. $x = 1 -2t$, $y = 3 + 4t$, $z = 2t$

23. Find the equation of the plane which is perpendicular to the plane $x + 3y + 2z + 7 = 0$ and which passes trough R(0,1,2) and S(-2, -1, 1).

a. $3x – y + z = 1$ b. $2x + y – 3z = -5$

c. $x – 3y + 4z = 5$ d. $3x + 4y + z = 4$

24. Find the equation of the plane which is tangent to the sphere $x^2 + y^2 + z^2 – 10x + 4y – 6z – 187 = 0$ at the point (-9, 3, 1).

a. $13x + 2y – 4z = -115$ b. $-14x + 5y – 2z = 139$

c. $12x + 5y + z = -92$ d. $2x + 5y – 8z = 21$

25-26-27

Given A(3,2, -5), B(2, -3, 8).

25. A point C is selected between AB such that AC = 3/5 AB, Find the cooodinate of C.

a. (23/5, 1/5, -2/5) b. (21/5, 1/5, -1/5)

c. (3/5, -14/5, 23/5) d. (12/5 c -1 2.8)

26. AB is extended to C such that AC = 5AB, find the coordinate of C.

a. (-3 21 73) b. (- 3 -28 73)

c. (2 1 - 5) d. (1 , 5, - 21)

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27. Find the distance between line AB and C(4, 7, 12).
a. 8.62 b. 4.45 c. 3.44 d. 11.22

28-29-30
Given the planes 4x + 7y – 3z = 21 and 3x + 3y – 7z = 2.

28. Find the parametric equation of the intersection of the planes.

a. x = -49/9 – 20t, y = 55/9t + 38t, z = - 9t
b. x = 31/3 + 40t , y = 21/6 – 21t, z = 21t
c. x = 9 – 2t, y = 21 – 4/3t , z = 1 + 2t
d. x = -49/9 – 40t, y = 55/9 + 19t, z = -9t

29. Find the coordinate where this line intersect the xy plane.
a. (-21.9, 12/7) b. (-49/9, 55/9)
c. (31/9, 21/9) d. (43,9, 21/9)

30. Find the angle that this line makes with the z xis.
a. 106.7^0 b. 101.5^0 c. 111.21^0 d. 89.21^0

Answers/ Solution

1. Input VctA = < 3 4 -6 > VctB = < 4 5 -7>
Abs(VctA – VctB) = 1.732

2. Vct A = < 4 8 -1> Vct B = < 9 1 2 >
abs(Vct – VctB) = 9.11

3. Let Vct A = < 2 10 4 >
Use Trial and Error: Let Vct B = < 8 3 6 >
Abs(VctA – VctB) = 9.434 Ans. z = 6

4. Use trial and error.
Let VctA = < 2 5 4 >
VctB = < 7 3 6 >
Abs(VctA – VctB) = 5.74456
Ans. x = 7

5. $d = \dfrac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}$

A = 7 B = -6 C = 6 D = 8
$x_1 = 3$, $y_1 = 2$, $z_1 = -1$

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d = 1

6. Get a point on one of the plane and get the distance from this point to the other plane. This will be the side of the cube.

$2x - y + 2z - 3 = 0$, Let $x = 0$, $y = 0$,

then $2z - 3 = 0$, $z = 3/2$

The point is (0,0, 3/2)

$$d = \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}$$

$(x_1, y_1, z_1) = (0, 0, 3/2)$

A = 6 B = -3 C = 6 and D = 8

d= 17/9

$V = d^3 = 4913/729$

7. A A A B B

$x^2 + y^2 + z^2 + 8x - 2y + 1 = 0$

A = 1 B = 8

-B/(2A) = -4

A = 1 B = -2

-B/(2A) = 1

So the center is at (-4, 1, 0)

Substitute x = -4, y = 1 and z = 0

$x^2 + y^2 + z^2 + 8x - 2y + 1 = -16$

radius = $\sqrt{16} = 4$

8.

A B A B A B

$4x^2 + 7x + 4y^2 - 3y + 4z^2 + 2z - 8 = 0$

A = 4 B = 7

-B/(2A) = -7/8

A = 4 B = -3

-B/(2A) = 3/8

A = 4 B = 2

-B/(2A) = -1/4

The center is at (-7/8, 3/8, -1/4)

Let x = -7/8 y = 3/8 z = -1/4

$4x^2 + 7x + 4y^2 - 3y + 4z^2 + 2z - 8 = -95/8$

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$r = \sqrt{\frac{95/8}{4}}$ r = 1.723

V = 4/3 π r^3 = 21.426 cubic units

9. Let VctA = < 2 1 -3 >
VctB = < -8 1 4>
VctC = < 3 -1 6>
The direction number of the plane is
(VctA - VctB) x (VctA – Vct C) = < 14 97 20 >
The equation of the plane is Ax + By + Cz + D = 0
14x + 97y + 20z + D = 0
Substitute a point say x = 2, y = 1 and z = - 3
then D = -65
The equation of the plane is
14x + 97y + 20z – 65 = 0

10. Area of the triangle is
0.5 Abs ((VctA - VctB) x (VctA – Vct C))
= 50.01 sq units

11. Direction Number of line AB = (3 – 5 2 + 2 8 – 7)
= (-2 4 1)
Parametric Equation of line AB is :
x = 3 - 2t y = 2 + 4t z = 8 + t Ans. (a)

12. From the parametric equation: z = 0
0= 7 + t , t = - 8
Thus: x = 3 – 2(-8) = 19
and y = 2 + 4(-8) = -30
Answer: (19, -30, 0) Ans. (b)

13. Direction Number AB = < -2 4 1 >

14. Let Vct A = < -2 4 1 >
Direction Cosine of AB
= VctA÷ Abs(VctA) = < -0.436 0.8728 0.2182 >

15.
Direction Number of Both lines = < 6 - 2 1 >
A point from the first line is (0, 0, 6)
A point from the second line is (7 , -0, -1)

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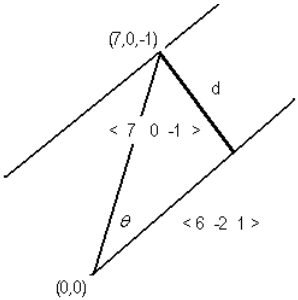

Let A = < 7 0 -1 > and B = < 6 -2 1 >

A x B = ABsin θ

Asinθ = (A x B) ÷ B = distance

VctA = < 7 0 -1 > and Vct B = < 6 -2 1 >

d = abs [(Vct A x Vct B) ÷ abs (VctB)]

d = 3

16. Let VctA = < 1 2 2 > and VctB = < 3 4 -12 >

Then VctA dot Vct B = abs(VctA) abs(VctB) cos θ

cos θ = VctA dot Vct B ÷ (abs(VctA) abs(VctB))

= - 1/3

θ = 109.47^0

17. The lines are not parallel.

Direction Number of the 1st line = < 1 2 3 > = VctA

Direction Number of the 2nd line = < 3 2 1 > = VctB

Direction Number of the line that is perpendicular to both lines is VctA x VctB

A point of 1st line is (0, 0, 6)

A point on the 2nd line is (0 , 0, 0)

Let VctC = < 0 0 6 >

The distance between the two lines is the scalar projection of vector C on VctA x VctB

distance = Vct C dot [(VctA x VctB)÷ Abs(VctA xVctB)]

= 2.449

18. Let VctA = < 2 3 -6>

VctA/Abs(VctA) = < -0.2857 0.4285 -0.8571>

Then cos θ_z = -0.8571

θ_z = 149^0

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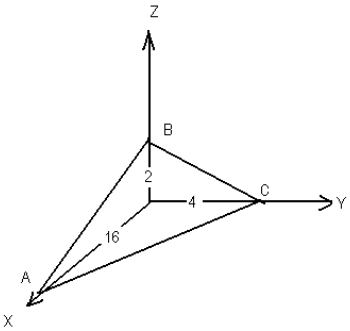
19. When x = 0, y = 0 , z = 2 (0,0,2)
When x = 0, z = 0, y = 4 (0,4,0)
When y = 0, z = 0, x = 16 (16,0,0)



$AB^2 = 16^2 + 2^2$ AB = $2\sqrt{65}$ = A
$BC^2 = 2^2 + 4^2$ BC = $2\sqrt{5}$ = B
$AC^2 = 16^2 + 4^2$ AC = $4\sqrt{17}$ = C

Use Heron's Formula:

s = $\frac{A+B+C}{2}$ and A = $\sqrt{S(S-A)(S-B)(S-C)}$

A = 36

20. V = 1/3Bh = 1/3 (1/2 x 4 x 16) (2) = 21.33 cubic units

21. STORE:

Vct A = (2 -1 -2) Vct B = (4 8 1)

cos θ = (Vct A dot Vct B)/ (abs(VctA) abs(VctB))

θ = 85.75^0

22. Compute the Direction Number of the Line

Vct A = (5 - 2 3) Vct B = (3 -1 1)

VctA X Vct B = (1 4 1)

Get a point on the line.

Set z = 0

5x – 2y = 13

3x – y = 7

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MODE 5 1 x = 1 y = - 4

Parametric Equation of the line

$$\frac{x-1}{1} = \frac{y+4}{4} = \frac{z}{1} = t$$

x = 1 + t, y = -4 + 4t , z = t

23. Direction number of x + 3y + 2z + 7 = 0 is (1 3 2)

line vector on the unknown plane : (-2 -0 -1 -1 1-2)

or (-2 -2 -1)

Vector perpendicular to x + 3y + 2z + 7 = 0 and the unknown plane is the the direction number of the unknown plane.

Vct A =(1 3 2) Vct B = (- 2 2 1)

Vct A x Vct B = (1 -3 4)

Equation of the plane: x – 3y + 4z = C

Substitute x = 0, y = 1 , z = 2

0 – 3(1) + 4(2) = c

c = 5

Equation x – 3y + 4z = 5

24. Get the center of the sphere.

x^2 -10x + 25 + y^2 + 4y + 4 + z^2 – 6z + 9 = 187 + 25 + 4 + 9

$(x – 5)^2 + (y + 2)^2 + (z – 3)^2 = 15^2$

C(5, -2, 3)

Direction Number of the plane = (-9 – 5 3 + 2 1 – 3)

= (-14 5 - 2)

Equation of the plane: -14x + 5y – 2z = C

Substitute x = -9 y = 3 z = 1

-14(-9) + 5(3) – 2(1) = 139

-14x + 5y – 2z = 139

25. A= (3 2 -5) B = (2, -3, 8)

Then C = A + (3/5)(B – A) =

= (3 2 -5) + (3/5)[(2 – 3 8) – (3 2 - 5)]

= (2.4 -1 2.8)

26.

A B C

x 5x

m = x/6x = 1/6

B = A + 1/6(C – A)

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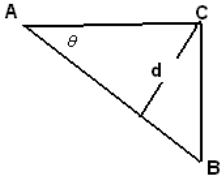
6B = 6A + C - A
C = 6B – 5A = 6(2 -3 8) - 5(3 2 - 5)
C = (-3 - 28 73)
27. Vct AB = (-1 -5 13) AC = (4 7 12) - (3 2 - 5)
= (1 5 17)



AC x AB = |AC||AB| sin θ
d = AC sin θ = abs(AC x AB / abs(AB))
d = 8.62

28. The Direction number of the plane is
(4 7 -3) x (3 3 - 7) = (-40 19 - 9)
Get a point on the plane.
Set z = 0
Then 4x + 7y = 21
3x + 3y = 2
MODE 5 1 x = -49/9 y = 55/9
Parametric equation:
x = -49/9 – 40t, y = 55/9 + 19t, z = 0 – 9t
(Note: This is not unique)

29. From 28. Parametric Equation
x = -49/9 – 40t , y = 55/9 + 19t , z = -9t
Set z = 0, then t = 0
Thus x = -49/9 and y = 55/9

30. Direction Number = (-40 19 - 9)
Direction Cosines = (-40 19 – 9) / $\sqrt{40^2+19^2+9^2}$
NOTE: Store: Vct A = (- 40 19 – 9)
Direction Cosine =
(VctA(Abs(Vct A)) = (-0.885 0.4204 -0.1992)

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Then: $\cos \theta_x = -0.1992 \quad \theta_x = 101.5^0$

PROBLEM SET 18
POLAR COORDINATES , CYLINDRICAL COORDINATES, SPHERICAL COORDINATES, PARAMETRIC EQUATIONS and POLAR CURVES

1. Find the polar coordinate of (3, -6).

a. (6.708, -63.43^0) b. (6.718, -43.43^0)

c. (6.728, -67.43^0) d. (4.708, -67.43^0)

2. Find the rectangular coordinate of (100, 120^0).

a. (50, 86.6) b. (-50, 86.6)

c. (-80, -86.6) d. (-50, -86.6)

3. The coordinate axes is rotated 70^0clockwise. What is the new coordinate of the the point (5, 8) with respect to the new axes?

a. (-5.81, 7.43 b. (-3.54, 7.63)

c. (-5.54 8.63) d. (-5.55, 6.67)

4. The new coordinate is (7, -9) when the the coordinate axes is rotated 45^0 counterclockwise. What is the old coordinate?

a. (-1.41, -11.31) b. (-1.21, -11.31)

c. (-1.41, -13.31) d. (-1.31, -11.31)

5. The coordinate (4,3) becomes (2.733, 4.187) when the coordinate axes is rotated at θ counterclockwise. What is θ ?

a. 30^0 b. 10^0

c. 20^0 d. 15^0

6. What is the polar coordinate on the point on a circle $x^2 + y^2 = 25$ when $x = 3$ in the second quadrant?

a. (5, -63.13^0) b. (5, -53.13^0)

c. (5, -23.13^0) d. (5, -43.13^0)

7. $y = x^2$ in polar coordinate sytem is

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a. $\cos\ \theta\ =\ r \sin\ \theta$ b. $\tan\ \theta\ =\ r \cos^2\ \theta$
c. $\sin\ \theta\ =\ r \cos^2\ \theta$ d. $\cot\ \theta\ =\ r \cos^2\ \theta$

8. $y = 4x + 1$ in polar coordinate system is
a. $r(\sin\ \theta - 4 \cos\ \theta) = 1$
b. $r(\cos\ \theta - 4 \sin\ \theta) = 1$
c. $r(\sin\ \theta - 4 \sec\ \theta) = 1$
d. $r(4\sin\ \theta - \cos\ \theta) = 1$

9. $x^2 + y^2 + 8x = 0$ in polar coordinate system is
a. $r = -8\sin\ \theta$ b. $r = -8 \cos\ \theta$
c. $r = -4 \sin\ \theta$ d. $r = -4 \cos\ \theta$
10. $x^2 - y^2 = 1$ in polar coordinate is
a. $r = \cos 2\theta$ b. $r = \tan 2\theta$
c. $r = \csc 2\theta$ d. $r = \cot 2\theta$
11. $r(4\sin\ \theta - \cos\ \theta) = 1$ in Cartesian form is
a. $4x + y = 1$ b. $4y - x = 1$
c. $3x + y = 1$ d. $2x + y = 2$

12. $r = 3 \cos\ \theta$ in Cartesian form is
a. $x^2 + y^2 = 2x$ b. $x^2 + y^2 = 3x$
c. $x^2 + y^2 = 3x/2$ d. $x^2 + y^2 = x$

13. $r (1 + \cos\ \theta) = 1$ in Cartesian Form is
a. $y^2 + 2x = 1$ b. $y + 2x^2 = 1$
c. $y^2 + 4x = 1$ d. $y^2 + x = 2$
14. (3, 4, 5) in Cylidrical Coordinate System is
a. $(5, 53.13^0, 5)$ b. $(5, 63.13^0, 5)$
c. $(4, 53.13^0, 5)$ d. $(5, 53.13^0, 6)$
15. (-4, -5, 10) in Cylindrical Coordinate System is
a. $(6.4, -128.65^0, 12)$ b. $(5.4, -128.65^0, 10)$
c. $(5.4, -128.65^0, 10)$ d. $(6.4, -123.65^0, 10)$
16. $(20, 40^0 , 5)$ in Orthogonal System is
a. (15.32, 14.855, 5) b. (15.32, 12.855, 5)
c. (13.32, 12.855, 5) d. (15.32, 15.855, 5)

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17. (19, 120^0, -6) in Orthogonal System is
a. (-9.5, 12.45, -6) b. (-9.1, 16.45, -6)
c. (-9.5, 23 .45, -6) d. (-9.5, 16.45, -6)
18. (3, 4, 5) in Spherical Coordinate System is
a. (7.07, 53.13^0, 45^0) b. (7.07, 53.13^0, 35^0)
c. (6.07, 53.13^0, 45^0) d. (4.07, 53.13^0, 45^0)
19. (-3, 8, 1) in Spherical Coordinate System is
a. (8.602, 110.556^0, 83.34^0) b. (8.602, 120.556^0, 63.34^0)
c. (6.602, 110.556^0, 83.34^0) d. (8.602, 130.556^0, 93.34^0)
20. (12, 30^0, 20^0) in Orthogonal System is
a. (4.554, 2.052, 11.276) b. (4.554, 2.052, 12.276)
c. (3.554, 2.052, 11.276) d. (3.554, 2.052, 11.276)
21. (14, 120^0 , 50^0) in Orthogonal Sytem is
a. (-3.547, 5.9704, 9) b. (-3.447, 5.2704, 9)
c. (-3.447, 6.9704, 9) d. (-5.632, 9.287, 9)
22. The parametric equation of $y^2 = 4x$ is
a. $x = t^2$, $y = 2t$ b. $x = t^2$, $y = 4t$
c. $x = t$, $y^2 = 8t$ d. $x = t$, $y^2 = 16t$
23. The parametric equation of $(x - 7)^2 + (y - 4)^2 = 1$ is
a. $x = 7 + \cos t$, $y = 4 + \sin t$ b. $x = 7 + \cos t$, $y = 4 - \sin t$
c. $x = 7 - \cos t$, $y = 4 - \sin t$ d. $x = -7 + \cos t$, $y = -4 + \sin t$

24. Trasform the parametric equation
$x = \sqrt{5} \sin t$, $y = 3 \cos t$, $z = t$ into general equation
a. $9x^2 + 5y^2 = 45$ b. $10x^2 + 9y^2 = 45$
c. $9x^2 + 4y^2 = 45$ d. $12x^2 + 5y^2 = 45$

25. Trasform $x = t + 3$, $y = t^2/4$ into a rectangular equation.
a. $6y = (x-3)^2$ b. $3y = (x-3)^2$
c. $2y = (x-3)^2$ d. $4y = (x-3)^2$
26. Transform $x = 3 - t$, $y = t^2 - 2$ into rectangular equation.
a. $y = 7 - 4x + x^2$ b. $y = 7 - 6x + x^2$
c. $y = 3 - 4x + x^2$ d. $y = 5 - 6x + x^2$
27. Trasform $x = \ln t$, $y = t^2 - 1$ into a rectangular equation.
a. $y = e^{2x} - 1$ b. $y = 3e^{2x} - 1$
c. $y = e^{2x} + 1$ d. $y = -e^{2x} + 1$

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28. The equation $r = 2$ is
a. circle with radios 2 , center on the y axis
b. circle with radius 1, center at the origin
c. spiral
d. circle of diameter 2, center at the origin.

29. The polar curve $r = 4 - 4 \cos \theta$ is
a. cardiod b. limacon
c. ellipse d. lemniscates

30. The polar curve $r = \sec \theta$ is
a. parabola b. line
c. ellipse d. hyperboa

31. The polar curve $r = 3 - 2 \cos \theta$ is a
a. limacon with a dent b. limacon with an inner loop
c. cardiod d. 3 leaf rose

32. The polar curve $r^2 = \cos 2\theta$ is a
a. lemniscate b. 4 leaf rose
c. spiral d. limacon

33. The polar curve $r = \sin 3\theta$ is a
a. 3 leaf rose b. 6 leaf rose
c. 9 leaf rose d. ellipse

34. The polar curve $r = \cos 4\theta$ is
a. 4 leaf rose b. 8 leaf rose
c. 2 leaf rose d. 16 leaf rose

35. The spiral curve $r = 4\theta$ when $\theta = 49^0$ has abscissa of
a. 1.21 b.1.03
c. 2.04 d. 1.56

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Answers

1. Pol(3, -6) = (6.708, -63.43^0)
2. Rect(100,120) = (-50, 86.6)
3. Pol(5,8) = (9.434, 58^0)

New $\theta = 58 + 70 = 128^0$

Rect(9.434, 126) = (-5.81, 7.43)

4. Pol(7, -9) = (11.4, -52.12^0)

Old angle: $-52.12^0 - 45^0 = -97.12^0$

Rect(11.4, -97.12^0) = (-1.41, -11.31)

5. Pol(4,3) = (5, 36.87^0)

Pol(2.733, 4.187) = (5, 56.87^0)

Angle = $56.87^0 - 36.87^0 = 20^0$

6.

$x = 3 \quad 3^2 + y^2 = 25$, $y = -4$ (2nd quadrant)

Pol(3, -4) = (5, -53.13^0)

7. $x = r \cos\theta$, $y = r \sin\theta$

$r \sin\theta = (r \cos\theta)^2$

$r \sin\theta = r^2 \cos^2\theta$

$\sin\theta = r \cos^2\theta$

8.

$x = r \cos\theta \quad y = r \sin\theta$

$r \sin\theta = 4 r \cos\theta + 1$

$r(\sin\theta - 4 \cos\theta) = 1$

9. $x = r \cos\theta \quad y = r \sin\theta$

$(r\cos\theta)^2 + (r \sin\theta)^2 + 8(r \cos\theta) = 0$

$r^2 (\cos^2\theta + \sin^2\theta) + 8r \cos\theta = 0$

$r^2 + 8r\cos\theta = 0$

$r + 8\cos\theta$

$r = -8\cos\theta$

10.

$x = r\cos\theta \quad y = r \sin\theta$

$(r \cos\theta)^2 - (r \sin\theta)^2 = 1$

$r^2 (\cos^2\theta - \sin^2\theta) = 1$

$r \cos 2\theta = 1$

$r = \sec 2\theta$

11.

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$\sin\theta = y/r \quad \cos\theta = x/r$

$r(4y/r - x/r) = 1$

$4y - x = 1$

12. $r = 3x/r$

$r^2 = 3x$

$x^2 + y^2 = 3x$

13. $r(1 + x/r) = 1$

$r + x = 1$

$r = 1 - x$

$r^2 = (1-x)^2$

$x^2 + y^2 = 1 - 2x + x^2$

$y^2 + 2x = 1$

14.

Pol(3,4) = $(5, 53.13^0)$

Clindrical Coordinate = $(5, 53.13^0, 5)$

15. Pol(-4, -5) = $(6.4, -128.65^0)$

Cylindrical Coordinate = $(6.4, -128.65^0, 10)$

16. Rect(20, 40^0) = (15.32, 12.855)

Orthogonal System = (15.32, 12.855, 5)

17. Rect (19, 120^0) = (-9.5, 16.45)

Orthogonal System = (-9.5, 16.45, -6)

18.

Pol(3,4) $r = 5 \quad \theta = 53.13^0$

This is stored to X and Y.

Pol(5, X) $r = 7.07 \quad \theta = 45^0$

Spherical Coordinate = $(7.07, 53.13^0, 45^0)$

19. See Solution 18

Pol(-3, 8) $r = 8.544 \quad \theta = 110.55^0$

Pol(1, X) $r = 8.602 \quad \theta = 83.324^0$

Spherical Coordinate $(8.602, 110.55^0, 83.324^0)$

20.

Rec(12, 20) $X = 11.276 \quad Y = 4.104$

Rec(Y, 30) $X = 3.554 \quad Y = 2.052$

Orthogonal Coordinate (3.554, 2.052, 11.276)

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21.
REC(14, 50) X = 8.999 Y = 10.724
REC(Y, 120) X = -5.362 Y = 9.287
Spehrical Coordinate: (-5.362, 9.287, 9)
22. Let $x = t^2$, $y^2 = 4t^2$ or $y = 2t$
23. $x - 7 = \cos t$ or $x = 7 + \cos t$
$y - 4 = \sin t$ or $y = 4 + \sin t$
Parametric Equation:
$x = 7 + \cos t, y = 4 + \sin t$
24. $\sin t = x/\sqrt{5}$ $\cos t = y/3$
$\sin^2 t + \cos^2 t = 1$
$(x/\sqrt{5})^2 + (y/3)^2 = 1$
$x^2/5 + y^2/9 = 1$
$9x^2 + 5y^2 = 45$
25. $t = x - 3$
$y = (x-3)^2/4$
$4y = (x-3)^2$

26.
$t = 3 - x$
$y = (3 - x)^2 - 2$
$y = 9 - 6x + x^2 - 2$
$y = 7 - 6x + x^2$
27. $x = \ln t$
$t = e^x$
$y = t^2 - 1$
$y = e^{2x} - 1$
28. a
29. b
30. line
31. a
32. a
33. a
34. b
35. $r = 4(49^0) = 3.421$ rec (3.421, 49) $x = 1.028$

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PROBLEM SET 19
LIMITS

1. Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ PAST CE BOARD

a. 0 b. 1/2
c. 2 d. -1/2

2. $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$ PAST ECE BOARD

a. $e^{2\pi}$ b. $e^{2/\pi}$
0. d. DNE

3. $\lim_{x\to 0} \sin(1/x)$ PAST CE BOARD

a. 0 b. -1
d. 1/2 d. DNE

4. $\lim_{x\to 0} \frac{\tan 2x - 2\sin x}{x^3}$ PAST CE BOARD

a. 0 b. 3
c. infinity d. DNE

5. $\lim_{x\to 1} \frac{x^2-1}{x^2+3x-4}$ PAST CE BOARD

a. 1/5 b. 2/5
c. 3/5 d. 4/5

6. $\lim_{x\to 4} \frac{x-4}{x^2-x-12}$ PAST ECE BOARD

a. 1/4 b. 1/3
c. 1/5 d. 1/7

7. $\lim_{x\to \infty} \frac{x+4}{x-4}$ PAST ME BOARD

a. 0 b. 1
c. 2 d. DNE

8. $\lim_{x\to \infty} \frac{3x^4-2x^3+7}{5x^3+x-3}$

a. 0 b. ∞
c. -1 d. DNE

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9. $\lim_{x\to\infty}(e^x + x)^{1/x}$

a. 2 b. e
c. 3 d. 4

10. $\lim_{x\to\pi/2}(\tan x \tan 2x)$

a. 1 b. 2
c. -2 d. 3

11. $\lim_{x\to 0}\dfrac{e^x - e^{-x}}{\tan x}$

a. 1 b. -1
c. 2 d. -2

12. $\lim_{x\to\infty}\dfrac{x}{\sqrt{1+x^2}}$

a. 1 b. 0
c. -1 d. 2

13. $\lim_{x\to 2}\dfrac{x^3 - x^2 - 4}{x^2 - 4}$

a. 1 b. 2
c. 3 d. 4

14. $\lim_{x\to\infty}(e^x - x)$

a. ∞ b. 0
c. 1 d. DNE

15 . $\lim_{x\to 0}(1-\sin x)^{1/x}$

a. 1/e b. 2/e
c. e d. $1/e^2$

16. $\lim_{x\to 3}\dfrac{3}{x-3}$

a. 0 b. DNE
c. ∞ d. - ∞

17. $\lim_{x\to\infty}\cos(\dfrac{2}{x})^{x^2}$

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a. e^{-2} b. e
c. 0 d. 1

18. $\lim_{x\to 0} (1 + ax)^{b/x}$
a. $e^{a/b}$ b. $e^{b/a}$
c. e^{ab} d. $e^{2b/a}$

29. $\lim_{x\to 0} (1/x - 1/\sin x)$
a. DNE b. 0
c. ∞ d. - ∞

SHIFT TO RAD MODE :
Substitute x = - 0.0001 and x = 0.0001 to the given.
(1/x – 1/sin x) when x = -0.00001 is 1.67 x 10^{-5}
when x = 0.00001 , -1.67 x 10^{-6}

21.

Solution:

1. Substitute x = -0.00001 and x = + 0.00001 in $\frac{1-\cos x}{x^2}$. (Go to radian mode)
Answer. 1/2
2. Subtitute x = 0.9999 and x = 1.0001
Answers are 1.89 and 1.8901
Ans. $e^{2/\pi}$
3. Radian Mode:
Let x = -0.00001 and x = 0.00001
Values: -0.0357 and 0.0357
The values different. The limit does not exist.

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4. Let x = 0.00001 and x = -0.00001
Ans. 3 and 3 Ans. 3

5. Let x = 0.9999 and 1.00001
The values are close to 0.4
Ans. 2/5

6. Let x = 3.999 and x = 4.001
Values are close to 0.142857 or 1/7

7. Disregard the constants.
x/x = 1 Ans. 1

8. Just consider the highest exponents in the numerator and denominator.

$$\frac{3x^4}{5x^3} = 0.6x \quad \text{Ans. } 0.6(\infty) = \infty$$

9. substitute x = 100
(x + $e^x)^{1/X}$ CALC 100= 2.71828 = e

10. Let x = π/2 - 0.00001 and x = π/2 + 0.00001
Go to radian mode.
ENTER: $(\tan x \tan 2x$ CALC π/2 - 0.00001
AND $(\tan x \tan 2x$ CALC π/2 + 0.00001
Values are -2 and -2
Ans. -2

11. Let x = -0.00001 and x = 0.00001
Values are 2 and 2
Ans. 2

12. Disregard 1. $\frac{x}{\sqrt{x^2}} = 1$

13. Let x = 1.9999 and x = 2.00001
Values are close to 2.
Ans. 2

14. e^x grows faster than x as x becomes bigger.
Ans. ∞

15. Let x = -0.00001 and x = 0.00001
Values are close to 0.3678776 or 1/e

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16. Substitute x = 2.999 and 3.001
The values of 3/(x- 3) are -3000 and 3000
Values are different. Limit does not exists

17. Shift to RAD MODE
ENTER: $\cos(\frac{2}{x})^{x^2}$, substitute x = 100
Value = 0.13532 Close to e^{-2}

18.
Assume a = 2 and b = 3
$(1 + 2x)^{3/x}$ let x = - 0.0001
Value = 403.67
If x = + 0.0001
Value = 4013. 17
Values are close, there is a limit
a = 2 , b = 3 from 1st choice $e^{a/b}$ = 1.95 (wrong)
from choice c, $e^{ab} = e^{2(3)}$ = 403.43 (Anwer) e^{ab}

PROBLEM SET 22
DERIVATIVES, SLOPES, TANGENT LINES, NORMAL LINES, RADIUS OF CURVATURE , VELOCITY AND ACCELERATION

1. Find the derivative of $x = 3y^4 + 7y^3 + 1$ with respect to x. PAST CE BOARD

a. $\frac{1}{12y^3 + 21y^2}$ b. $\frac{1}{6y^3 + 21y^2}$

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c. $\frac{1}{12y^3+18y^2}$ d. $\frac{1}{13y^3+21y^2}$

2. Find the derivative of $y = 3^{3x}$ PAST CE BOARD

a. $3^{3x+1} \ln 3$ b. $3^{3x+1} \ln (3x)$

c. $3^{3x} \ln 3$ d. $3^{3x+2} \ln (3x)$

3. What is the derivative of $2 \cos (2 + x^3)$ with respect to x? PAST CE BOARD

a. $-6x^2\sin(2 + x^3)$ b. $6x^2\sin(2 + x^3)$

c. $3x^2\sin(2 + x^3)$ d.- $3x^2\sin(2 + x^3)$

4. Find y' in the curve $y = (1 – 2x)^3$ at the point (1,-1). PAST CE BOARD

a. -6 b. 6

c. -3 d. 3

5. What is the slope of the curve when $y = 2 \sin x$ when $x = 1^0$?

a. 1 b. 2

c. 3 d. 4

6. Detemine the slope of the curve $x^2 + y^2 – 6x + 10y + 5 = 0$ at the point (1,0). PAST CE BOARD

a. 1/5 b. 2/5

c. 3/5 d. 4/5

7. Determine the velocity when t = 4 given the equation $D = 20t + \frac{5}{t+1}$ where D is in meters and t is in seconds. PAST CE BOARD

a. 19.8 m/s b. 12.6 m/s

c. 13.4 m/s d. 14.5 m/s

8. Find the slope of the curve $y = 64(4 + x)^{1/2}$ at the point (0,12). PAST CE BOARD

a. 1 b. 1.5

c. 2 d. 2.5

9. Find the slope of the curve whose parametric equation is given by $y = t^3 + t^2 + 1/t$ and $x = t^4 + 3t^2$ when t = 1/2.

a. -9/14 b. 3/14

c. 4/17 d. 8/17

10. Find the derivative with respect to x .

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CE BOARD MAY 2006

y = $\frac{1}{1-u}$ $u = \frac{1}{1+x}$

a. -1/x b. $1/x^2$

c. $-1/x^2$ d. 2/x

11. Find the slope of the curve $r = \sin^2 \theta$ at $\theta = \pi/3$?

CE BOARD Nov 2007

a. 4.5 b. -5.2

c. 2.33 d. -4.5

12. Find the slope of $r = \tan \theta$ when $\theta = \pi/4$

CE BOARD NOV 2007

a. 1 b. 2

c. 3 d. 4

13. If $y = 4 \cos x + \sin 2x$, what is the slope of the curve when x = 2.

PAST ECE BOARD

a. -2.21 b. -3.25

c. -4.94 d. -2.21

14. Find the slope of the ellipse $x^2 + 4y^2 - 10x - 16y + 5 = 0$ at the point where $y = 2 + \sqrt{8}$ and x = 7.

a. -0.1463 b. -0.1538

c. -0.1654 d. -0.1768

15. Find dy/dx $(xy)^x = e$ PAST ME BOARD

a. $\frac{-y(1-\ln xy)}{x^2}$ b. $\frac{-y(1+\ln xy)}{x}$

c. 0 d. y/x

16. Given the curve $y^2 = 5x - 1$ at the point (1, -2) , find the equation of the tangent line to the curve. PAST CE BOARD

a. 5x + 4y + 3 = 0 b. 3x + 4y + 5 = 0

c. 2x + 3y + 4 = 0 d. 5x + 3y + 1 = 0

17. From problem 16, find the equation of the normal line at the point (1, -2)

a. 2x -5y -12 = 0 b. 4x -5y -14 = 0

c. 4x -8y -20 = 0 d. 4x -y -14 = 0

18. Find the radius of curvature of the ellipse $3x^2 + y^2 = 12$ at the point (1,3). PAST CE/ME BOARD

a. 3.122 b. 2.121

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c. 5.112 d. 4.221

19. Find the center of curvature of problem 18.

a. (-0.5, 1.5) b. (-1.5, 1)

c. (-0.5, 2.5) d. (0.5, 1.5)

20. Find the radius of curvature at any point of the curve $y + \ln \cos x = 0$ PAST ECE/CE BOARD

a. cos x b. sec x

c. 1.211 d. 1

21. Find the slope of the curve $x^2 + 4xy + y^2 + 3x - 5y - 6 = 0$ at the point on the curve (1,2).

a. 16/3 b. -13/3

c. -5 d. -11/3

22. Find the angle between the radius vector and the tangent line of the curve $r = a \sec 2\theta$ and $\theta = \pi/8$. CE BOARD MAY 2008

a. 33.21^0 b. 26.56^0

c. 14.12^0 d. 11.23^0

23. Find the angle between the radius vector and the tangent line to the curve $r = \theta^2$ at $\theta = \pi$. CE BOARD MAY 2008

a. 57.5^0 b. 34.2^0

d. 45.2^0 d. 33.1^0

Problems 24, 25

At time $0 \le t \le 2\pi$, the position of the particle moving along a path in the xy plane is given by the parametric equation $x = e^t \sin t$ and $y = e^t \cos t$.

24. Find the slope of the path of the particle at time t = 0.

a. 1 b. -1

c. 2 d. -2

25. What is the speed of the particle when t = 1.

a. 3.84 b. 1.22

c. 2.45 d. 4.33

26. Find the radius of curvature of $r = \tan \theta$ when $\theta = 3\pi/4$.

a. 2.11 b. 2.24

c. 3.11 d. 4.11

Problems 27, 28

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A particle moves in accordance with the equation $r = t^2 + t$ and $\theta = 1/6t^3$ where t denotes the time in seconds, θ is in radians and r is in meters.

27. Find the position in rectangular coordinate when t = 2 sec.
    a. (1.41, 5.83) b. (3.11, 2.34)
    c. (3.23, 4.56) d. (4.11, 6.12)
28. Find the speed when t = 2 sec.
    a. 12 m/s b. 5 m/s
    c. 11 m/s d. 13 m/s

Problems 29, 30

A particle moves along the curve $r = 5 + 4 \sin \theta$ with a constant angular speed $d\theta/dt$ of 2 radians per second where θ is in meters.

29. Find the speed of the particle when $\theta = \pi/3$.
    a. 12.3 m/s b. 17.4 m/s
    c. 11.2 m/s d. 12.1 m/s
30. Find the acceleration of the particle when $\theta = \pi/3$.
    a. $50.3 m/s^2$ b. $49.1 m/s^2$
    c. $34.56 m/s^2$ d. $11.21 m/s^2$
31. What is the equation of the tangent line to the curve $9x^2 + 25y^2 - 225 = 0$ at (0,3)? PAST CE BOARD
    a. $y - 1 = 0$ b. $x + 2 = 0$
    c. $x + 3 = 0$ d. $y - 3 = 0$
32. Find the equation normal to the curve $x^2 = 16y$ at (4,1) PAST CE BOARD
    a. $3x + y = 17$ b. $4x - y = 15$
    c. $2x + y = 9$ d. $x + y = 5$
33. Find the derivative with respect to x of $\frac{(x+1)^3}{x}$.

PAST CE BOARD

a. $\frac{2x(x+1)^2}{x^2} - \frac{(x+1)^3}{x^2}$ b. $\frac{x(x+1)^2}{x^2} - \frac{2(x+1)^3}{x^2}$

c. $\frac{3x(x+1)^2}{x^2} - \frac{3(x+1)^3}{x^2}$ d. $\frac{3x(x+1)^2}{x^2} - \frac{(x+1)^3}{x^2}$

34. Find the radius of curvature of $y^2 = 4x$ at the point (4,4)
    a. 34.12 b. 22.36

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c. 11.23 d. 33.12

35. Find the slope of the curve $x = 6y^3 - 2y^2$ at the point (6,3). PAST CE BOARD

a. 1/110 b. 1/150

c. 1/120 d. 1/300

36. Find the slope of the curve $y = 6(4 + x)^{1/2}$ at the point (0,12).

a. 2 b. 1.5

c. 2 d. 3

37. At what point does the curve $x^3 - 9x - y = 0$ have a slope of 18?

a. (4, 28) b. (3, 0)

c. (1, -8) d. (2, 1)

38. Find the curvature of the curve $y^2 = 16x$ at the point (4,8). PAST CE BOARD

a. 0.0442 b. 0.0331

c. 0.1221 d. 0.0236

Problems 39, 40

A particle moves along the path whose parametric equation are $x = t^3$ and $y = 12t^2$. (PAST CE BOARD)

39. What is the velocity when t = 2 sec?

a. 45.67 b. 49.48

c. 48.12 d. 44.21

40. What is the acceleration when t = 2 sec?

a. 26.83 b. 32.12

c. 33.43 d. 41.22

41. Find the second derivative of $y = x(x+1)^3$ when $x = 1$.

a. 24 b. 34

c. 36 d. 44

Given the ellipse $25x^2 + 4y^2 = 100$. (PAST CE BOARD)

Problems 42, 43

42. Find the equation of the diameter of the ellipse which bisects chords having a slope of 1/2.

a. $25x + 2y = 0$ b. $50x - 3y = 0$

c. $25x + 4y = 0$ d. $30x - 5y = 0$

43. What is the slope of the curve at the point (1.2, 4)?

a. -1.212 b. -1.341

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c. -1.875 d. -3.232

44. Determine the slope of the tangent to the curve y = 2ln x at the point where x = 1.

a. 1 b. 3

c. 2 d. 4

45. What is the 2nd derivative of y = x sin x when x = 1^0?

a. 1 b. 2

c. 3 d. 4

Solution

1 = $12y^3y' + 21y^2y'$

$y' = \dfrac{1}{12y^3 + 21y^2}$

2. $y' = 3^{3x} \ln(3)\ d/dx\ (3x)$

$= 3^{3x}(3) \ln 3$

$= 3^{3x+1} \ln 3$

3. $y = 2\cos(2 + x^3)$

$y' = 2(-\sin(2 + x^3))\, 3x^2$

$= -6x^2 \sin(2 + x^3)$

4. $\dfrac{d}{dx}(1-2x)^3 \,|_{x=1}$ = -6

5. Go to radian mode:

ENTER: $\dfrac{d}{dx}(2\sin x) \,|\, x = 1^0$

Ans. 2

6. $x^2 + y^2 - 6x + 10y + 5 = 0$

$2x + 2yy' - 6 + 10y' = 0$

Let X = D , write as

2x + 2yD – 6 + 10yD= 0,D and enter SHIFT CALC

Substitute x = 1 and y = 0 and solve for y'

Ans. 2/5

7. $\dfrac{d}{dx}(20x + \dfrac{5}{x+1}) \,|_{x=4}$ =19.8 m/s

8. $\dfrac{d}{dx}(6(4+x)^{1/2}) \,|_{x=0}$ = 1.5

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9. $dy/dt = d/dt(t^3 + t^2 + 1/t)$ when t is 1/2 = -2.25
$dx/dt = d/dt(t^4 + 3t^2)$ when t is 1/2 = 7/2
slope = dy/dt / dx/dt = -2.25/3.5 = -9/14

10. $y = \dfrac{1}{1-\dfrac{1}{1+x}} = \dfrac{x+1}{(x+1)-1} = \dfrac{x+1}{x}$

$dy/dx = \dfrac{x-(x+1)}{x^2} = \dfrac{-1}{x^2}$

11. $x = r \cos \theta$
$x = \sin^2 \theta \cos \theta$
$dx/d\theta$ when $\theta = \pi/6$ is -0.2165
$y = r \sin \theta$
$y = \sin^2\theta \sin \theta$
$y = \sin^3\theta$
$dy/d\theta$ when $\theta = \pi/3$ is 1.125
slope = $dy/dx = dy/d\theta / dx/d\theta$ = 1.125/(-0.2165) = -5.2

12. $x = r \cos \theta$
$x = \tan \theta \cos \theta$
$dx/d\theta$ when $\theta = \pi/4$ is 0.707107
$y = r \sin \theta$
$y = \tan \theta \sin \theta$
$dy/d\theta$ when θ = 2.12132
slope = $dy/d\theta / dx/d\theta$ = 2.12132 / 0.707107 = 3

13. $d/dx(4 \cos x + \sin 2x)$ when x = 2 is -4.94

14.
$x^2 + 4y^2 - 10x - 16y + 5 = 0$
$2x + 8yy' - 10 - 16y' = 0$
Let $y' = D$
Input:
$2x + 8yD - 10 - 16D = 0$, D
and use SHIFT CALC
x = 7 and $y = 2 + \sqrt{8}$
D = -0.1768

15.
$(xy)^x = e$

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x ln (x y) = ln e
x ln (xy) = 1
x(ln x + ln y) = 1
xln x + xln y = 1
x(1/x) + lnx + x(1/y)y' + ln y = 0
1 + ln x + xy'/y + ln y = 0
xy'/y = -1 – ln x – ln y
xy'/y = -1 – ln xy

$$y' = \frac{-y(1+\ln xy)}{x}$$

16. $y^2 = 5x - 1$
Is (1, -2) on $y^2 = 5x - 1$
$(-2)^2 = 5(1) - 1$ OK

$y = -\sqrt{5x-1}$ (since y = -2 is negative)

d/dx $\sqrt{5x-1}$ when x =1 is -5/4 (slope of tangent line)
MODE 5 1
1 -2 1
2 -2-5/4 1
-5/3 x – 4/3y = 1
5x + 4y + 3 = 0

17. slope of tangent = -5/4,
slope of normal = 4/5
y = A + Bx
-2 = A + (4/5)(1)
A = -14/5
Thus y = -14/5 + 4/5x (Equation of Normal line)
5y = -14 + 4x
4x -5y -14 = 0

18. $3x^2 + y^2 = 12$

$y = \sqrt{12-3x^2}$

$y' = 0.5(12-3x^2)^{-0.5}(-6x)$

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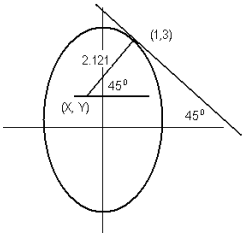
Radius of curvature : $\rho = \dfrac{(1+(y')^2)^{3/2}}{y''}$

y' = d/dx($\sqrt{12-3x^2}$) when x = 1 is -1

y'' = d/dx (0.5(12-3x^2)$^{-0.5}$(-6x)) when x = 1 is -1.33333

radius of curvature = $\rho = \dfrac{[1+(-1)^2]^{3/2}}{-1.33333}$ = - 2.121

19.



Radius of Curvature is negative (concave downward)

SLOPE = -1 , (45^0 sloping to the left)

X = 1 - 2.121 cos 45^0 = -0.5

Y = 3 – 2.121 sin 45^0 = 1.5

Center of Curvature (-0.5, 1.5)

20. $\rho = \dfrac{(1+(y')^2)^{3/2}}{y''}$

y = -ln cos x

y' = -1/cosx (-sin x) = tan x

y'' = sec^2 x

$$\rho = \frac{(1+(\tan x)^2)^{3/2}}{\sec^2 x}$$

$$\rho = \frac{\sec^3 x}{\sec^2 x} = \sec x$$

21.

Differentiate:

2x + 4(xy' + y) + 2yy' + 3 – 5y' = 0

Let y' = D

2x + 4(xD + y) + 2yD+ 3 – 5D = 0, D

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then ENTER SHIFT CALC x = 1 and y = 2
D = -13/3
SLOPE = -13/3
22. The angle between the radius vector and the tangent line is

$$\tan\Psi = \frac{r}{\frac{dr}{d\theta}}$$

Let a = 1
sec 2θ when θ = π/8 is $\sqrt{2}$
d/dθ (sec 2θ) when θ = π/8 is 2.8283

$$\tan\Psi = \frac{\sqrt{2}}{2.8283} \quad \Psi = 26.56^0$$

23.

$$\tan\Psi = \frac{r}{dr/d\theta}$$

$$\tan\Psi = \frac{\pi^2}{d/d\theta(\theta^2)\,|\,\theta=\pi} = \tan\Psi = \frac{\pi^2}{2\pi}$$

$$\Psi = 57.5^0$$

24.
dy/dt = d/dt(e^tcost) when t = π/2 is -4.8105
dx/dt = d/dt(e^tsin t) when t = π/2 is 4.8105
dy/dx = dy/dt / dx/dt = - 1

25. dy/dt = d/dt(e^tcos t) when t = 1 is -0.81866
dx/dt = d/dt (e^tsin t) when t = 1 is 3.756
when t= 1, speed is

$$v = \sqrt{(dy/dt)^2 + (dx/dt)^2} = \sqrt{(-.81866)^2 + (3.756)^2}$$

= 3.84
26.

$$\text{Radius of Curvature} = \frac{(r^2 + r'^2)^{3/2}}{r^2 + 2r'^2 - rr''}$$

r = tan θ r(3π/4) = -1 = A
r'(3π/4) = d/dθ(tan θ) when θ = 3π/4

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r'(3π/4) = 2 =B
r''(3π/4) = d/dθ ($\sec^2\theta$) when θ = 3π/4
r''(3π/4) = - 4 = C

$$\rho = \frac{(A^2 + B^2)^{3/2}}{A^2 + 2B^2 - AC} = 2.236$$

27. r = t^2 + t r(2) = 6 m
θ = 1/6 t^3 θ(2) = 4/3 m
Rect(6, 4/3) (Go to rad mode)
= (1.411, 5.83)

28. r when t = 2 is 6 m
dr/dt when t = 2 = d/dt(t^2 + t) when t = 2 is 5 m/s
dθ/dt when t = 2 is d/dt(1/6 t^3) when t = 2 is 2 m/s
v_r = 5
v_θ = r dθ/dt = 6(2) = 12 m/s
v = Abs< 5 12 > = 13 m/s

29. r when θ = π/3 is 8.464
v_r = dr/dt = d/dt(5 + 4 sin θ) when θ = π/3
=[d/dθ(5 + 4sinθ)|θ= π/3] dθ/dt
= 2(2)
v_r = 4 (radial speed)
v_θ = r dθ/dt = 8.464(2)
16.928 (transverse speed)
v = Abs< 4 16.928 > = 17. 4

30. a_r = r'' – r(θ')^2
r'' = [d/dθ(4cos θ x 2) | θ = π/3] dθ/dt
r'' = -13.85
a_r = -13.856 – 8.464(2)^2 = -47.712
θ'' = 0 since θ' = constant
a_θ = r θ'' + 2r'θ'
= 0 + 2(4)(2) = 16
a = Abs < -47.712 16 > = 50.3

31. Get the slope:
$9x^2 + 25y^2 – 225 = 0$
18x + 50yy' = 0
18(0) + 50(3)y' = 0

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y' = 0 (slope) The line is horizontal.
Equation. y = 3 of y – 3 = 0

32.

$y = x^2/16$

d/dx ($x^2/16$) | x = 4 is 1/2 (Tangent)

slope normal = -2

Y = A + BX

1 = A + (-2) (4)

A = 9

Equation Normal:

Y = 9 -2X

2X + Y = 9

33. Use quotient rule:

$$\frac{d}{dx}\frac{(x+1)^3}{x} = \frac{x(3)(x+1)^2 - (x+1)^3(1)}{x^2}$$

$$= \frac{3x(x+1)^2}{x^2} - \frac{(x+1)^3}{x^2}$$

The calculator trick for this one.

Differentiate $\frac{(x+1)^3}{x}$ when x = 2 (any value not = zero will do)

which is 6.75. Then substitute in the choices. The only function choice that will give 6.75 is

$$\frac{3x(x+1)^2}{x^2} - \frac{(x+1)^3}{x^2}.$$

34.

$$\rho = \frac{(1+(y')^2)^{3/2}}{y''}$$

$$y = \sqrt{4x} = 2\sqrt{x}$$

y' = d/dx ($2\sqrt{x}$) | x = 4 is 0.5 = A

y'' = d/dx ($2(1/2)\, x^{-1/2}$)| x = 4 is -0.0625 = B

Then $\rho = \frac{(1+A^2)^{3/2}}{B}$ = -22.36

The sign indicates that the curve is concave downward.

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35. dy/dx at x = 6 is ?
dx/dy = d/dy($6y^3 - 2y^2$)| y = 3
dx/dy = 150
dy/dx = 1/150
36. y' = d/dx(6(4+ x)$^{1/2}$) | x = 0 is 1.5
37.
$x^3 - 9x - y = 0$
$3x^2 - 9 - y' = 0$
$3x^2 - 9 - 18 = 0$
x = 3 and - 3
If x = 3 , y = $x^3 - 9x$ = 0
If x = -3, y = $x^3 - 9x$ = 0
The point are (3, 0) and (-3, 0)
38.

$$K = \frac{y''}{(1+(y')^2)^{3/2}}$$

$y = \sqrt{16x}$ = $4\sqrt{x}$
y' = d/dx ($4\sqrt{x}$) | x = 4 is 1 = A
y'' = d/dx (4(1/2) $x^{-1/2}$) | x = 4 is -0.125 = B

$$K = \frac{B}{(1+A^2)^{3/2}} = -0.0442$$

39.
v_x = dx/dt = d/dt(t^3)| t = 2 is 12
v_y = dy/dt = d/dt($12t^2$) | t= 2 is 48
v = Abs < 12 48 > = 49.48

40.
a_x = d/dt($3t^2$) | t = 2 is 12
a_y = d/dt(24t) | t = 2 is 24
a = Abs < 12 24 > = 26.83
41.
y'(1) = d/dx (x(x +1)^3)| x = 1 is 20
y(1+0.00001) = d/dx (x(x +1)^3) | x = 1.00001 is

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$$2^{nd}\text{ derivative} = \frac{y'(1.00001) - y'(1)}{0.00001} = \frac{20.00036}{0.00001} - \frac{20}{0.00001}$$

$$= 36$$

42.

$25x^2 + 4y^2 = 100$

$50x + 8yy' = 0$

$50x + 8y(1/2) = 0$

$50x + 4y = 0$

$25x + 2y = 0$

43.

$4y^2 = 100 - 25x^2$

$y^2 = 25 - x^2/4$

$y = \sqrt{25 - 6.25x^2}$

$y' = d/dx(\sqrt{25 - 6.25x^2}) \mid x = 1.2$ is -1.875

44.

$d/dx(2\ln x) \mid x = 1$ is 2

45.

$y'(1^0) = d/dx(x \sin x) \mid 1^0$ is 0.034903

$y'(1^0 + 0.00001) = d/dx(x \sin x) \mid 1^0 + 0.00001$

$= 0.03492303$

$y''(1^0) = (0.03492303 - 0.034903) / 0.00001$

$= 2$

PROBLEM SET 21
MAXIMA MINIMA, RELATED RATES, ERRORS

1. A car starting 12:00 Noon travels west at a speed of 30 kph. Another car starting from the same point at 2:00 PM travels north at 45 kph. Find how fast are the two cars separating at 4:00 PM? PAST CE BOARD

a. 57 kph b. 51 kph

c. 81 kph d. 44 kph

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2. Car A moves due east at 30 kph at the same instant Car B is moving S 30^0 E with a speed of 60 kph. The distance from A to B is 30 km. Find how fast is the speed between them are separating after 1 hr. PAST CE BOARD

a. 40 kph b. 35 kph
c. 45 kph d. 50 kph

3. Determine the diameter of a closed cylindrical tank having a volume of 11.3 cm^3 to obtain a minimum surface area. PAST CE BOARD

a. 2.561 b. 2.112
c. 2.432 d. 3.122

4. A closed cylindrical tank has a capacity of 576.53 m^3. Find the minimum surface area of the tank in m^2.
PAST CE BOARD

a. 378.2 b. 331.2
b. 383.4 d. 312.12

5. With only 381.7 m^2 of materials, closed cylindrical tank of maximum volume is to be made. What should be the height of the tank in m?

a. 9 b. 10
c. 11 d. 12

6. A rectangular box with a square base and open at the btop is to have a capacity of 16823 cm^3. Find the height of the box that requires minimum amount of material in cm.

a. 13.24 b. 16.14
c. 21.23 d. 11.22

7. The cost of a certain product is defined as follows.

$C = 50t^2 – 200t + 10000$ where t is in years.

Find the maximum value from year 1995 to 2002.
PAST CE BOARD

a. 11,600 b. 9800
c. 9850 d. 11,050

8. A revolving searchlight in a lighthouse 2 km offshore is following a car traveling slowly along the shore. When the car is 1 km from the point on the shore that is closest to the lighthouse, the searchlight is

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turning at the rate of 0.25 rev/hr. How fast in kph is the car traveling at this moment?

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a. 2.98 b. 3.93
c. 4.11 d. 6.12

9. A rectangle is inscribed in an a ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$.

What is the maximum possible area of the rectangle?

a. 10 b. 13
c. 12 d. 36

PAST CE BOARD

10. What is the nearest distance from the external point (4,2) to the curve $y^2 = 8x$. PAST CE BOARD

a. 3.12 b. 2.34
c. 2.83 d. 4.11

11. The sum of 2 numbers is K. The product of one by the cube of the other is to be a maximum. What is one of the number. PAST CE BOARD

a. K/4 b. K/2
c. 2K/3 d. K/8

12. At 4:00 PM boat A left in the direction N 45^0 E . At 4:30 PM boat B left the same pier in the direction S 30^0 E at 32 mph. How fast were they separating at 5:00 PM in mph? PAST CE BOARD

a. 34.23 b. 38.21
c. 44.61 d. 54.33

13. The sides of the triangle ABC are AB = 7 cm, BC =5 cm and CA = 9 cm. Determine the width of the largest rectangle that can be inscribed in it such that the longer side of the rectangle is on the 9 cm side of the rectangle. PAST CE BOARD.

a. 2.12 m b. 1.94 m
c. 1.22 m d. 1.44 m

A right circular cylinder is inscribed in a right circular cone of radius 6 cm. PAST CE BOARD

14. Find the radius of the cylinder if its volume is maximum.

a. 3 b. 4
c. 5 d. 2.5

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15. Find the maximum volume if the altitude of the cone is 12 cm.

a. 201.06 cm^3 b. 211.23 cm^3

c. 251.23 cm^3 d. 134.22 cm^3

16. Find the radius of the cylinder if its lateral area is a minimum.

a. 4 b. 3

c. 2 d. 3.5

Problems 17, 18, 19

It is estimated that between the hrs of noon and 7 PM, the speed of a highway traffic flowing past the intersection of EDSA and Ortigas Avenue is approximately

$S = t^3 - 9t^2 + 15t + 45$ kph where 't' is the number of hrs past noon. (PAST CE BOARD)

17. At what time between noon and 7 PM is the traffic moving the fastest?

a. 1 PM b. 2 PM

c. 3 PM d. 4 PM

18. At what time between noon and 7 PM is the traffic moving the slowest.

a. 2 PM b. 3 PM

c. 4 pm d. 5 PM

19. What is the slowest speed if it moving at this time.

a. 21 kph b. 19 kph

d. 20 kph d. 22 kph

20. A triangular corner lot has perpendicular sides of lengths 120 m and 160 m resepectively. Find the area and perimeter of the largest rectangle that can be constructed on the lot with sides parallel to the street? PAST CE BOARD

a. 4000 m^2, 200 m b. 4800 m^2 , 280 m

c. 3600 m^2 , 220 m d. 4200 m^2 , 240 m

21. The total cost of producing a gift item is

$C = 60x^2 - 0.01x^3$ where x is the number of units produced. Determine the value of x to make the average unit cost a minimum.

a. 2000 b. 3000

c. 4000 d. 5000

22. The length of the sides of the triangle are 4.25 cm, 9.61 cm and 8.62 cm. A rectangle is inscribed in it such that the longer side is on

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the 9.61 cm side of the triangle. What is the length of the rectangle if its area is maximum?

a. 4.52 b. 4.81
c. 5.22 d. 5.01

23. A cable is to be run from a power plant on one side of the river 900 km wide to a factory on the other side 3 km upstream. The cost of running the cable overland is $4 per meter while the cost underwater is $ 5 per meter. Find the value of x for the most economical cost. PAST CE BOARD

a. 1400 m b. 1800 m
c. 1200 m d. 1500 m

24. Water is flowing into a frustum of a cone at a rate of 100 liters per minute. The upper radius of the frustum of the cone is 1.5 m while the lower radius is 1 meter with a height of 2 m. If the water rises in the tank at the rate of 0.04916 cm/s, find the depth of water. PAST CE BOARD

a. 13.5 cm b. 15.5 cm
c. 12.23 cm d. 12.07 cm

25. Water is flowing in a conical cistern at the rate of 8 m^3/min. If the height of the inverted cone is 12 m and the radius of its circular opening is 6 m. How fast is the water rising when the water is 4 m in depth. PAST ECE/ME BOARD.

a. 0.36 m/min b. 0.64 m/min
c. 0.44 m/min d. 0.56 m/min

26. A man is walking at the rate of 4 m/s across a bridge 30 m above a river. A boat traveling 12 m/s at right angle to the roadway of the bridge pass directly beneath her. How fast is the distance between the boat and the woman increasing 5 sec later.

a. 13.45 m/s b. 11.43 m/s
c. 12.11 m/s d. 14. 22 m/s

27. The sides of a triangle are 6 m and 9 m respectively. If the included angle is changing at the rate of 2 rad/s, at what rate is the 3rd side changing when the included angle is 60^0?

a. 11.78 m/s b. 12.33 m/s
c. 10.23 m/s d. 8.91 m/s

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28. A ballon leaving the ground 18 m from the observer rises 3 m/s. How fast is the angle of elevation of the line of sight increasing after 8 seconds.

a. 0.06 rad/s b. 0.05 rad/s
c. 0.07 rad/s c. 0.08 rad/s

29. Find the minimum volume of a rignt circular cylinder that can be inscribed in a sphere of radius 100 cm.

a. 2418399.152 cm^3 b. 2438399.152 cm^3
c. 3418399.152 cm^3 d. 7418399.152 cm^3

30. Find the area of the largest rectangle with sides parallel to the coordinate axes which can be inscribed in the area bounded by the two parabolas $y = 26 – x^2$ and $y = x^2 + 2$.

a. 32 b. 64
c. 128 d. 72

31. Find the area of the largest isosceles triangle that can be inscribed in a circle of radius 6 inches.

a. 51.32 b. 46.76
c. 32.21 d. 41.22

32. A statue 3 m high is standing on a base 4 m high. If an observer's eye is 1.5 m above the ground. How far should he stand from the base in order that the angle subtended by the statue is maximum. PAST ECE BOARD

a. 3.71 m b. 4.33 m
c. 2.11 m d. 1.81 m

33. Water is running in a hemispherical bowl having a radius of 10 cm at a constant rate of 3 cm^3.min. When the water is x cm deep, the water level is rising at the rate of 0.0149 cm/min. What is the value of x? PAST CE BOARD.

a. 3 b. 2
c. 4 d. 5

34. Find the equation of the line tangent to the curve $y = x^3 – 6x^2 + 5x + 2$ at its point of inflection.

a. Y = 10 – 7x b. Y = 20 – 7x
c. Y = 10 + 7x d. Y = 11 + 7x

35. Determine a, b and c so that the curve $y = ax^3 + bx^2 + cx$ will have a slope of 4 at its point of inflection (-1,-5).

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a. a = 2 b = 1 c = 7 b. a = 1 b = 3 c = 7
c. a = -2 b = -3 c = 1 d. a = 4 b = 1 c = 2

36. Determine a, b, c and d so that the curve $y = ax^3 + bx^2 + cx + d$ will have a critical point at the origin and a point of inflection at (2,4).

a. a = -1/4 , b = 3/2 , c = 0 , d = 0
b. a = 1/2, b = -1 , c = 2 , d = 1
c. a = -1, b = 2 , c = -3 , d= 0
d. a = 1, b = -1, c = 2 , d = -1

37. Find the minimum length of a ladder that will reach a building and rest on a wall 1.5 m high which is 1.2 m from a building.

a. 3.4 m b. 3.8 m
c. 4.1 m d. 4.4 m

38. An object is projected vertically upward. If the distance traveled by an object is expressed as $h = 100t – 16.1t^2$. What is the velocity of the object after 2 seconds?

a. 33.2 m/s b. 35.6 m/s
c. 33.2 m/s d. 32.8 m/s

39. Find acute angle of intersection of the parabolas
$y = x^2 - 2x - 3$ and $y = -x^2 + 6x - 3$. PAST CE BOARD

a. 36.03^0 b. 41.21^0
c. 44.12^0 d. 11.8^0

40. Find the acute angle of intersection between y = sin x and y = cos x at the first point of intersection in the 1st quadrant.

a. 67.12^0 b. 39.34^0
c. 70.53^0 d. 41.22^0

41. The equation $y = ax^3 + bx^2 + cx + d$ has a critical point at (-1,4) and a point of inflection at (2, -50). Find the coefficients a, b, c and d.

a. a = 1 c = -15 d= -4 and b = -6
b. a = 1 c = -11 d= -1 and b = -6
c. a = -1 c = -15 d= 4 and b = -6
d. a = 2 c = -10 d= -4 and b = -6

42. If P 20,000 will watch a concert at an admission price of P 100, and if for evert P 5 added to the price, 500 fewer people will attend, what is maximum gate receipt?

a. 2, 550,000 b. 2, 250,000

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c. 2, 270,000 d. 2, 150,000

43. The cost of fuel in a locomotive is proportional to the square of the speed and is $25 per hour for a speed of 25 miles per hour. Other cost amount to $100 per hr regardless of the speed. What is the speed which will make the cost per mile a minimum?

a. 40 mph b. 30 mph

c. 50 mph d. 76 mph

44. A ferriswheel has a radius of 10 m. Its center is 12 m above the ground. When the passenger is 17 m above the ground, he is moving at the rate of 1.81 m/s. What is the speed of rotation of the wheel in rpm? CE BOARD Nov 2005

1. 3 b. 5

c. 4 d. 2

45. A particle moves according to the parametric equations $y = 2t^2$ and $x = t^3$ where x and y are the displacements in meters and time is in seconds. Determine the acceleration of the body after 3 seconds. PAST CE BOARD.

a. 18.44 m/s^2 b. 12.44 m/s^2

c. 28.44 m/s^2 d. 11.22 m/s^2

46. Find the acute angle of interection of the curves $x + xy = 1$ and $y^3 = (x+1)^2$. CE BOARD Nov 2008

a. 34.6^0 b. 70.4^0

c. 78.1^0 d. 81.2^0

47. A particle moves on the x axis so that its position at any time is given by $x = 2te^{-t}$. Find the acceleration of the particle at $t = 0$.

a. -2 b. 3

c. -4 d. -1

48. Consider the curve $y^2 = 4 + x$ and chord AB joining points A(-4,0) and B(0,2) on the curve. Find the x coordinate of the point on the curve where the tangent line is parallel to chord AB.

a. 1 b. 2

c. -1 d. -3

49. Two posts one is 10 m high and the other 15 m high stand 30 m apart. They are to be stayed by transmission wires attached to a single stake at ground level, the wires running to the top of the posts. Where should the stake be placed to use the least amount of wire.

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a. 15 m b. 11 m
c. 12 m d. 14 m

50. The parametric equation of a moving particle is $x = t\cos 3t$, $y = t^4e^t$ and x, y are in meters and time t is in seconds. Find the acceleration of the particle when time t = 1 sec. Find the acceleration of the particle.

a. 43.21 m/s^2 b. 57.65 m/s^2
c. 44.12 m/s^2 d. 71.22 m/s^2

51. Find the minmum length of the line segment intercepted between the positive coordinate axes is a minimum that passes trough (3,4)
PAST CE BOARD

a. 9.56 b. 9.87
c. 10.12 d. 11.21

52. What is the mimumum vertical distance between the parabolas $x = y^2$ and $y = 1 + x^2$

a. 0.671 b. 0.5275
c. 0.5333 d. 0.5771

A gutter with trapezoidal cross section is to be made from a long strip of metal 22 cm wide by bending up the edges. If the base is to be 14 cm wide,

53. What width across the top gives the greatest carrying volume?

a. 18 cm b. 16 cm
c. 17 cm d. 19 cm

54. What is the greatest carrying volume?

a. 58.1 cu cm b. 58.3 cu cm
c. 59.1 cu cm d. 60.1 cu cm

55. A circular cone was carved out of a sphere of radius 10 cm. What is the least amount of material wasted.

a. 2947.7 cm^3 b. 3411.4 cm^3
c. 4422.1 cm^3 d. 2666.1 cm^3

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56. Water is flowing in a conical reservoir 40 ft deep and 16 feet across the top at the rate of 2 ft^3/min. Find how fast the surface is rising when the water is 4 ft deep.

a. 0.85 ft/min b. 0.95 ft/min
c. 0.83 ft/min d. 0.66 ft/min

57. Suppose the rectangular prism's dimension were determined. The length of the base was 15 in with a possible error of $\mp$ 0.25 in. The width is 10 inches with a possible error of $\mp$ 0.1 in. The height is 24 in with a possible error of $\mp$ 0.3 in. What is the maximum error on the volume of the prism?

a. 134 in^3 b. 151 in^3
c. 141 in^3 d. 167 in^3

58. The sides of a triangle are measured to be 30 m and 45 m and to include a 30^0 angle. The possible error in measuring the 1^{st} side is 0.15 m and in the other is 0.2 m while the maximum possible error in measuring the angle is 30'. Find the largest possible error in the computed area (using differentials)

a. 8.335 m^2 b. 8.289 m^2
c. 9.122 m^2 d. 10.332 m^2

59-60

The measurement of the side of the cube is 6 cm long. If this length has an error of 1/12 cm. Find approximately (using differentials) the greatest error in

59. The Volume

a. 9 cm^3 b. 10 cm^3
c. 11 cm^3 d. 12 cm^3

60. The surface Area

a. 4 cm^2 b. 5 cm^2
c. 6 cm^2 d. 7 cm^2

61-62

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In the 1st quadrant of the curve $y = 9 – x^2$, a tangent line is drawn. The tangent line intersects the coordinate axes at points U and V.

61. Find the minimum length of UV.

a. 10.23 b. 10.76
c. 11.18 d. 9.33

62. Find the coordinates of the point of tangency.

a. (1,8) b. (2, 5)
c. (1.2, 7.56) d. (0.5, 8.75)

63. Water is flowing into a paraboloid of revolution of height 3 m and radius 1 m at the top at 0.3 cu m per min. At what rate is the depth increasing when water is 0.7 m.

a. 0.205 m/s b. 0.409 m/s
c. 0.112 m/s d. 0.331 m/s

64. A sector of central angle θ is removed from a circular sheet of tin
(radius = 100 cm) and the remaining material is made into a right circular cone. Find the value of θ so that the volume of the cone will be maximum.

a. 68.07 deg b. 67.32 deg
c. 66.06 deg d. 58.44 deg

The upper end of the ladder 5 m long leans against a vertical wall. Suppose the foot of the ladder slips away from the wall at the rate of 0.1 m/min

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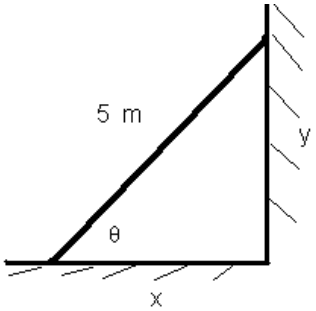

65. How fast is the top of the ladder descending when its foot is 3 m from the wall?

a. 0.986 m/min b. 0.075 m/min
c. 0.331 m/min d. 0.044 m/min

66. When will the top and bottom of the ladder move at the same rate?

a. x = 3.45 m b. x = 4.3 m
c. x = 3.54 m d. x = 3.67 m

67. When is the top of the ladder descending at the rate of 0.15 m/min?

a. x = 4.21 m b. x = 5.11 m
c. x = 4.16 m d. x = 3.86 m

68. How fast is the angle θ at the foot of the ladder decreasing when the foot is 3 m from the wall?

a. 0.025 rad/min b. 0.011 rad/min
c. 0.112 rad/min d. 0.442 rad/min

Answers.

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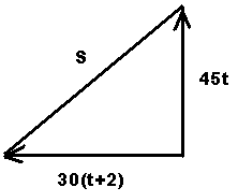
1.



Let t = time of 1st car after 12:00

t + 2 = time of the 2nd car.

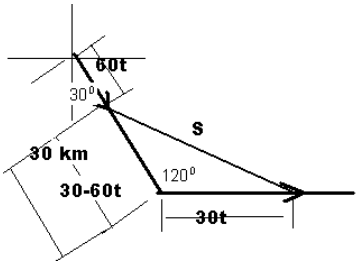
$S = \sqrt{(45t)^2 + [30(t+2)^2]}$

at 4: 00 PM t = 2

dS/dt when t = 2

d/dt $\sqrt{(45t)^2 + [30(t+2)^2]}$ | t = 2 is 51 kph

2.



$S = \sqrt{(30-60t)^2 + (30t)^2 - 2(30-60t)(30t)\cos 120^0}$

dS/dt when t = 1

= d/dt $\sqrt{(30-60t)^2 + (30t)^2 - 2(30-60t)(30t)\cos 120^0}$ | t = 1

= 45 kph

3.

$r = \sqrt[3]{\frac{V}{2\pi}}$ (radius of a closed cylinder of minimum area)

V = 11.3 r = 1.216 cm

d = 2(1.216) = 2.432 cm

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4.

$$r = \sqrt[3]{\frac{V}{2\pi}} \quad V = 576.53$$

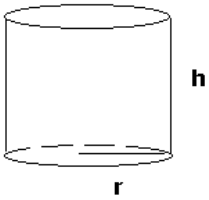
r = 4.51 cm

$V = \pi r^2 h = 576.53$

h = 9.02 m

$S = 2\pi r^2 + 2\pi rh = 383.4$ m

5.



This is also a problem of minimum area , max volume.

h is always equal to 2r

h = 2r

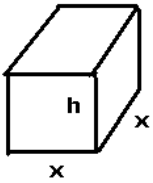
$381.7 = 2\pi r^2 + 2\pi r\, h$

r = h/2

$381.7 = 2\pi(h/2)^2 + \pi(h)(h)$

h = 9

6.



Then $16823 = x^2 y$

$y = 16823/x^2$

$V = x^2 + 4hx$

$S = 4hx + x^2$

$S = 4(16823/x^2)\, x + x^2$

$S = 67292/x + x^2$

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$ds/dx = -67292/x^2 + 2x = 0$

$x = 32.28$

and $y = 16823/x^2 = 16.14$ cm

7. Use the table menu of your calculator.

ENTER: MODE 7

Input $f(x) = 50x^2 – 200x + 10000$

START = 1

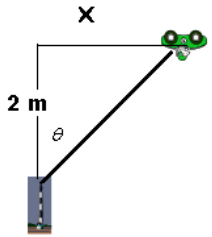
END = 7

STEP = 1

The maximum value will be at x = 7

Ans. 11050

8.



0.25 rev/hr = 0.25 x 2π = $\pi/2$ rad/hr

$\tan\theta = \frac{x}{2}$ $\theta = \tan^{-1}(0.5x)$

$d\theta/dt = d/dt (\tan^{-1}(0.5x))$

$= d/d\theta(\tan^{-1}(0.5x))| x = 1 \; dx/dt$

$\pi/2 = (2/5)\, dx/dt$

$dx/dt = 3.93$ kph

9.

a = 3 and b = 2

et x = length y = width

For maximum area:

$x = a\sqrt{2}$ and $y = b\sqrt{2}$

A = xy where

$A = (3\sqrt{2})(2\sqrt{2}) = 12$

10.

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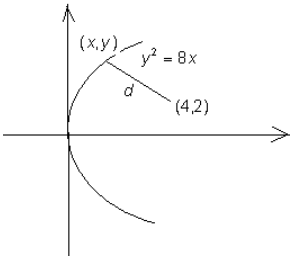

$$d^2 = (x-4)^2 + (y-2)^2$$

$$x = \frac{y^2}{8}$$

$d^2 = (y^2/8 - 4)^2 + (y-2)^2$

$2dd' = 2(y^2/8 -4)(2y)(1/8) + 2(y-2) = 0$

$y = 4$

$x = y^2/8 = 2$

The point is (2, 4)

Distance between (2,4) and (4, 2)

is Abs(-2 2) = 2.828

11.

Let x = one number

K – x is one of the number

$P = (K-x)(x)^3 = 0$

$P' = (K-x)(3x^2) + x^3(-1) = 0$

$(K-x)(3) - x = 0$

$3K = 4x$

$x = 3K/4$ and the other number is K/4

12. Let t = time after 4 PM

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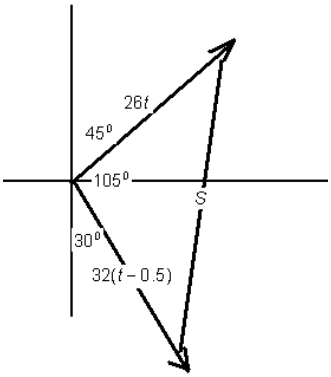
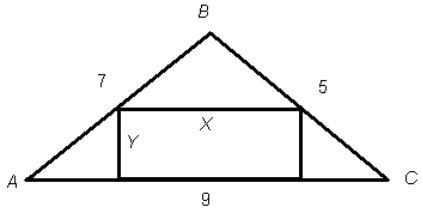


$S = \sqrt{(26t)^2 + (32(t-0.5))^2 - 2(26t)(32)(t-0.5)\cos 105^0}$

dS/dt when t= 1 hr (5 PM) is 44.6088 mph

13.



For the inscribed rectangle of maximum area, the height y must be half the altitude of the circumscribed triangle.

Input $\cos^{-1}\dfrac{A^2+B^2-C^2}{2AB}$ CALC? A = 7, B = 9, C = 5 =

for angle A: A = 7 , B = 9 , C = 5

angle A = 33.56^0

height of triangle = 7 sin A = 3.87

Then y = 3.87 /2 = 1.935 m

14.

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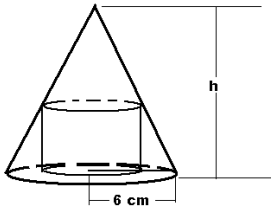

For a cylinder of maximum volume inscribed in a cone,

radius = 2/3 of the height of the circumscribing cone.

= (2/3)6 = 4 cm

15. For a cylinder of maximum volume inscribed in a cone,

height = 1/3 of the height of circumscribing cone.

h = 1/3(12) = 4 cm and r = 4 cm from prob 14.

Then $V = \pi r^2 h$

$= \pi(4)^2(4) = 201.06\ cm^3$

16.

For maximum lateral area of a cylinder inscribed in a cone,

radius = 1/2 of the radius of the circumscribing cone.

radius = 1/2(6) = 3 m

17 – 18 – 19

Note: t = 1 1 PM

t = 2 2 PM

t = 3 3 PM

ENTER: MODE 7

Input: $t^3 - 9t^2 + 15t + 45$ (replace t by X)

START = 1

END = 6

STEP = 1

DISPLAY:

X	Y
1	52
2	47
3	36
4	25
5	20
6	27

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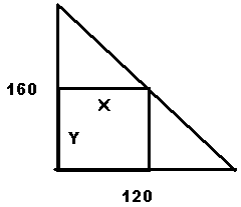
17. Traffic is fastest at t = 1 1: 00 PM
18. Traffic is slowest at t = 5 5:00 PM
19. Slowest speed = 20 kph

20.

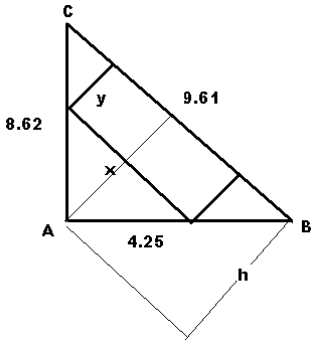


A = xy y/ (120 –x) = 160/120 y = 4/3(120-x)
A = x(4/3)(120 –x)
A = 4/3 (120 – x^2) dA/dx = 0 120 – 2x = 0 x = 60
and y = 4/3 (120 -60) = 80

Note(For maximum area of rectangle inscribed in a triangle.
x = 1/2 (160) = 80
y = 1/2 (120) = 60
A = 80 x 60 = 4800 m^2
Perimeter = 2x + 2y = 280 m

21.
C = $60x^2 – 0.01x^3$
$C/x = 60x – 0.01x^2$
$d(C/x)/dx = 60 – 2(0.01)x = 0$
x = 3000

22.



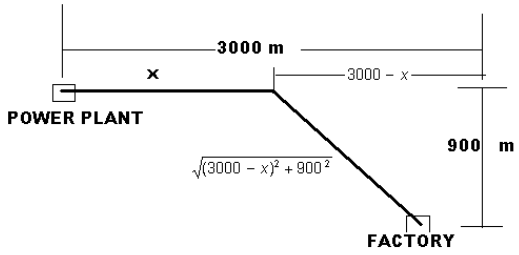
The sides 4.25 , 8.62 and 9.61 form a right triangle
since Abs(4.25 8.62) = 9.61
tan B = 8.62/ 4.26 B = 63.7^0
h = 4.26 sin 63.7 = 3.82 m
For maximum inscribed rectangle, x = 1/2h
y = 1/2(3.82) = 1.91 m
Use similar triangles.

$$\frac{h-y}{x}=\frac{h}{9.61} \quad , \ y= 1.91 \ , \ h = 3.82$$

x = 4.805 m

Or USE MODE 3 2 (Triangle CBA)
x y
0 9.61
3.83 0
(3.83/2) $\hat{Y}$ = 4.805 (This is also half the hypotenuse)

23.



Cost of the cable = 4x + $5\sqrt{(3000-x)^2+900^2}$

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Just use trial and error. The derivative of the cost must be zero for most economical cost.

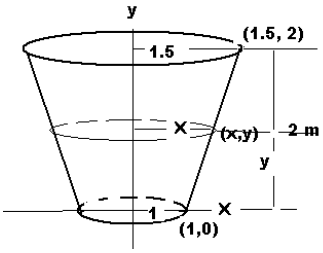
$d/dx(4x + 5\sqrt{(3000 - x)^2 + 900^2}) \mid x = 1800$ is 0.

Answer: 1800

24.

100 liters per min = 100/1000 = 0.1 m^3/ min

= 1/600 m^3/sec dy/dt = 0.0004916 m/s



Volume of water in the tank

= 1/3πy [$x^2 + (1)^2 + 1(x)$]

Find the relationship between x and y.

The coordinates are (1.5,2) and (1,0)

MODE 3 2

x y

1 0

1.5 2

A = -4 B = 4 y = -4 + 4x or x = 1 + 0.25y

V = 1/3πy [$(1 + 0.25y)^2 + 1 + 1(1 + 0.25y)$]

Let f(y) = 1/3πy [$(1 + 0.25y)^2 + 1 + 1(1 + 0.25y)$]

Then dV/dt = d/dt f(y)

dV/dt = d/dy f(y) dy/dt

d/dy f(y) = dV/dt / dy/dt

= 1/600 / 0.0004916

= 3.39

Using trial and error:

d/dy(f(y) when y = 0.155 is 3.39 (Ok)

Ans. 0.155 m or 15.5 cm

CALCULATOR TECHNIQUE:

MODE 3 3

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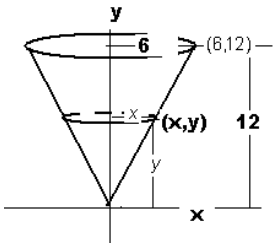
X Y
0 $\pi(1)^2$ (base)
2 $\pi(1.5)^2$ (top)
1 $\pi(1.25)^2$ (at the middle)
$dV/dt = x\hat{y}\, dy/dt$ $dV/dt = 1/60\ m^3/s$ $dy/dt = 0.0004916$ m/s
Using Trial and Error:
$x\hat{y} = 1/60 / 0.0004916 = 33.9$
$0.155\hat{y} = 3.39$ (Ans.)
To get exact values (No Trial and Error)
or get the value of A (A = 3.1416) ,
get the value of B (B = 1.5708)
get the value of C (C = 0.19634)
$dV/dt = (A + Bx + Cx^2)\, dx/dt$ (Note dx/dt is actually dy/dt)
$1/60 = (A + Bx + Cx^2) (0.0004916)$
$A + Bx + Cx^2 = 1/600 / 0.0004916 =$
$Cx^2 + Bx + A - 1/600/ (0.0004916) = 0$
MODE 5 3
x = 0.155 (discard x = - 8.15)

25.



$V = 1/3\pi x^2y$
$x/y = 6/12 \quad x = 0.5y$
$V = 1/3\pi(0.5y)^2y$
$dV/dt = d/dy\ (1/3\pi(0.5y)^2y)\, dy/dt$
$8 = d/dy(1/3\pi(0.5y)^2y) \mid y = 4 \quad dy/dt$
$8 = 4\pi\ dy/dt$
$dy/dt = 0.6366$ m/min
CALCULATOR TECHNIQUE:

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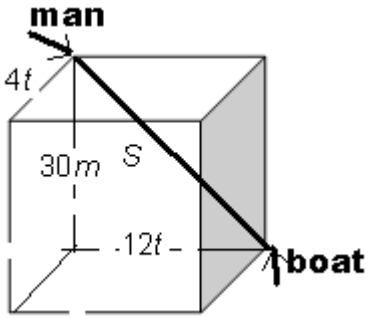
MODE 3 3
x y
0 0
12 $\pi(6)^2$
-12 $\pi(6)^2$
when height = 4
dV/dt = $4\hat{y}$ dy/dt
8 = 12.56637 dy/dt
dy/dt = 0.6366 m/min

26.

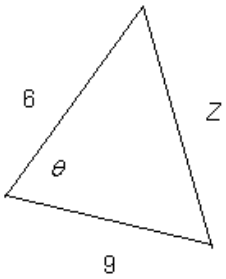


$S = \sqrt{(4t)^2 + (12t)^2 + 30^2}$

d/dt S = d/dt $\sqrt{(4t)^2 + (12t)^2 + 30^2}$ | t= 5

= 11.43 m/s

27.



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$$Z = \sqrt{6^2 + 9^2 - 2(6)(9)\cos\theta}$$

dZ/dt = d/dt $\sqrt{6^2 + 9^2 - 2(6)(9)\cos\theta}$

= d/dθ $\sqrt{6^2 + 9^2 - 2(6)(9)\cos\theta}$ dθ/dt

dZ/dt when θ = 60^0 or π/3

dZ/dt = d/dθ $\sqrt{6^2 + 9^2 - 2(6)(9)\cos\theta}$ | θ = π/3 (2)

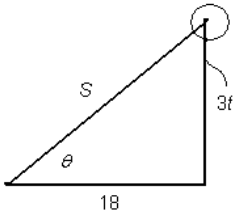
= 5.89188(2)

= 11.78 m/s

(Note: The calculator must be in radian mode)

Just replace θ by x.

28.



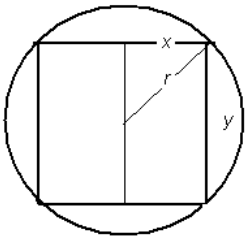
$$\tan\theta = \frac{3t}{18} \qquad \theta = \tan^{-1}(3t/18)$$

dθ/dt = d/dt $\tan^{-1}(3t/18)$ | t = 8

Note: Go to RAD MODE

dθ/dt = 0.06 rad/s

29.



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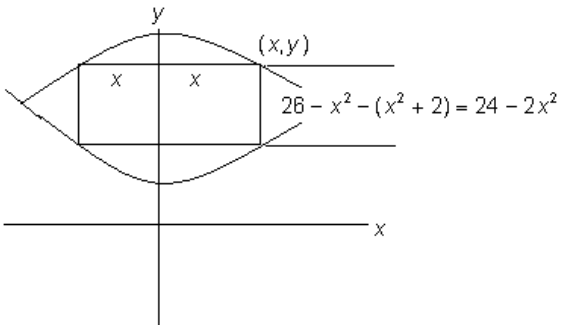
For a minimum volume of a cylinder inscribed in a sphere,
Volume = $1/\sqrt{3}$ of the Volume of the sphere.

$V = 1/\sqrt{3}\ (\ 4/3\pi(100)^3\ )$

$= 2418399.152\ cm^3$

30.



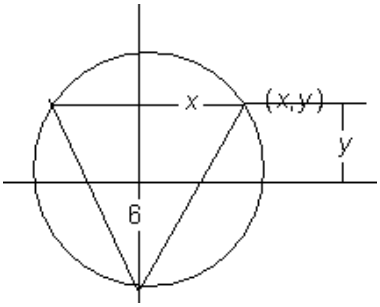
$A = 2x(24-2x^2) = 48x - 4x^3$

$dA/dx = 48 - 12x^2 = 0$

$x = 2$

$A = 48(2) - 4(2)^3 = 64$

31.



$A = 1/2\ (\ 6 + y)(2x\ ) = x(\ 6 + y)$

$x^2 + y^2 = 36$

$y = \sqrt{36 - x^2}$

$A = x\ (\ 6 + \sqrt{36 - x^2}\ )$ (1)

$A = 6x + x\sqrt{36 - x^2}$

This is difficult to differentiate.

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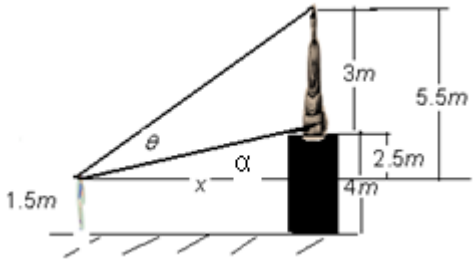
Use table menu.
ENTER MODE 7 and enter:

f(x) = $6x + x\sqrt{36 - x^2}$
START = 0
END = 6 (This is the max possible value of x)
STEP = 0.5
PARTIAL DISPLAY:
4.5 44.848
5 46.583
5.5 46.188

The maximum value lies between x = 4.5 and x = 5.5.
START = 4.5
END = 5.5
STEP = 0.1
Maximum value is at x= 5.2 with f(x) = 46.756
Ans. 46.76

32.



tan α = 2.5/x tan (θ + α) = 5.5 /x
α = $\tan^{-1}$ (2.5/x) θ + α = $\tan^{-1}$ (5.5 /x)
θ = $\tan^{-1}$ (5.5 /x) - $\tan^{-1}$ (2.5/x)
USE MODE 7
f(x) = $\tan^{-1}$ (5.5 /x) - $\tan^{-1}$ (2.5/x)
START: 0
END: 10
STEP: 1
DISPLAY:
x F(x)

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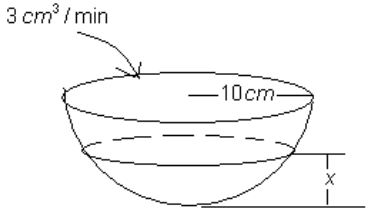
1 11.496
2 18.676
3 21.583
4 21.967 Maximum Angle
5 21.161

START: 3
END: 5
STEP: 0.1
3.6 22.015
3.7 22.024
3.8 22.018
START: 3.6
END: 3.8
STEP: 0.01
MAX 3.71 22.024
Distance = 3.71 and angle = 22.024^0
(Note: For this type of problem)
The value of x for maximum value of θ = $\sqrt{2.5 \times 5.5}$
= 3.7081

33.



$V = 1/3\pi x^2(3(10) – x)$
$V = \pi/3 (30x^2 – x^3)$
when x = ? $dV/dt = 3$ $dx/dt = 0.0149$
$dV/dt = \pi/3 (60x – 3x^2) dx/dt$
$3 = \pi/3 (60x – 3x^2)(0.0149)$
x = 4

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MODE 3 3
x y
0 0
10 $\pi(10)^2$
20 0
A= 0 B = 62.83185 C = -3.1416
$dV/dt = (Bx + Cx^2) dx/dt$
$3/0.0149 = (Bx + Cx^2)$
$Cx^2 + Bx - 3/0.0149 = 0$
USE MODE 5 3
x = 4 (16 is discarded)

34.
Point of Inflection: y'' = 0
$y(x) = x^3 – 6x^2 + 5x + 2$ (1)
$y'(x) = 3x^2 -12x + 5$ (2)
$y''(x) = 6x -12 = 0$
x = 2
From 1: y(2) = -4 (The point is (2, -4))
From 2: y'(2) = -7 (this is the slope)
Y = A + Bx
-4 = A + (-7)(2)
A = 10
Y = 10 – 7x

35.
$y = ax^3 + bx^2 + cx$
$-5 = a(-1)^3 + b(-1)^2 + c(-1)$
-5 = -a + b - c (1)
$y' = 3ax^2 + 2bx + c$
$4 = 3a(-1)^2 + 2b(-1) + c$
3a – 2b + c = 4 (2)
y'' = 6ax + 2b (-1,5) is a point of inflection.
0 = 6a(-1) + 2b
-6a + 2b = 0 (3)
Solve equations 1, 2 and 3 using MODE 5 2

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a = 1 b = 3 c = 7

36.

$y = ax^3 + bx^2 + cx + d$

at (0,0)

$0 = a(0)^3 + b(0)^2 + c(0) + d$

$d = 0$

at (2,4)

$4 = a(2)^3 + b(2)^2 + c(2) + d$

$4 = 8a + 4b + 2c$ since $d = 0$ (1)

$y' = 3ax^2 + 2bx + c$

Critical point is at (0,0) , $y' = 0$

$4 = 0 + 0 + c$

$c = 0$

$y'' = 6ax + 2b$ (Point of inflection is at $x = 2$)

$0 = 6a(2) + 2b$

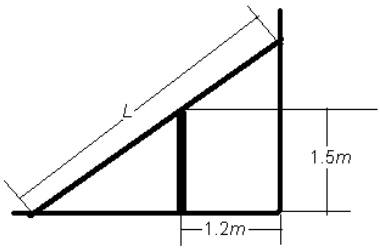
$12a + 2b = 0$ (1)

From 1, $8a + 4b = 4$ (2) since $c = 0$

Solve equations 1 and 2.

$a = -1/4$, $b = 3/2$

37.



For the minimum length of ladder :

$$L^{2/3} = 1.2^{2/3} + 1.5^{2/3}$$

L = 3.8 m

38.

velocity after 2 seconds = $d/dt(100t - 16.1t^2)| t = 2$

= 35.6 m/s

39. Find the intersection.

$x^2 - 2x - 3 = - x^2 - 6x - 3$

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$2x^2+4x = 0$

$x = 0$ and $x = 4$

$y = -3$ and $y = 5$

Point of intersection (0,-3) and (4,5)

Get the slope of $y = x^2 – 2x – 3$ when $x = 0$

$d/dx(x^2 – 2x – 3)| x = 0$ is -2

Get the slope of $y = - x^2 – 6x – 3$

$d/dx(- x^2 + 6x – 3)| x = -2$ is 6

Angle of intersection = $\tan^{-1}(6) – \tan^{-1}(-2) = 143.97^0$

The acute angle is $180 – 143.973 = 36.02^0$

(You can also use the other point (4,5))

40. Solve for the intersection.

$\sin x = \cos x$

$x = \pi/4$

$d/dx \sin x | x = \pi/4$ is 0.7071

$d/dx \cos x| x = \pi/4$ is -0.7071

$\tan^{-1}(0.7071) – \tan^{-1}(-0.7071) = 70.53^0$

41.

The curve passes trough (-1,4) and (2, -50).

$y = ax^3 + bx^2 + cx + d$

$4 = a(-1)^3 + b(-1)^2 + c(-1) + d$

$4 = -a + b – c + d$ (1)

$-50 = a(2)^3 + b(2)^2 + c(2) + d$

$-50 = 8a + 4b + 2c + d$ (2)

$y' = 3ax^2 + 2bx + c$

$0 = 3a(-1)^2 + 2b(-1) + c$ (Critical point: $y' = 0$)

$0 = 3a – 2b + c$ (3)

$y'' = 6ax + 2b$

$0 = 6a(2) + 2b$ (4) (Inflection point, $y'' = 0$)

The equations are :

$-a + b – c + d = 4$ (1)

$8a + 4b + 2c + d = -50$ (2)

$3a – 2b + c = 0$ (3)

$12a +2b = 0$ (4)

From (4). $b = -6a$

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Substitute this to all the equations to reduce the system to 3 unknowns.

-a - 6a – c + d = 4

-7a – c + d = 4 (new 1)

8a + 4(-6a) + 2c + d = -50

-16a + 2c + d = -50 (new 2)

3a – 2(-6a) + c = 0

15a + c + 0d = 0 (new 3)

Solve new 1, new 2, new 3.

a = 1 c = -15 d= -4 and b = -6

42.

Price	People that will attend
100	20000
105	19500

ENTER: MODE 3 2

Input the given data.

X	Y
100	20000
105	19500

A = 30000 B = -100

Price in terms of attendance is 30000 – 100x

Gate Receipt = x(30000 -100x)

d/dx(x(30000-100x)) = 30000 – 200x = 0

x = 150

Gate receipt = 150(30000-100(150)) = 2, 250,000

43.

Let C = cost of fuel per hr

Then $C = kv^2$

$C = 2\ 5$, v = 25

$25 = k(25)^2$

k = 1/25

$C = v^2/25$

Total Cost per hr $= v^2/25 + 100$

Cost per mile = $\frac{cost\ per\ hr}{speed\ in\ miles/hr}$

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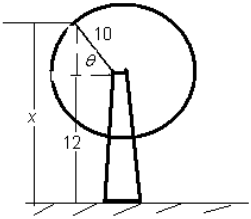
$$X = \frac{v^2/25 + 100}{v} \qquad X = \frac{v}{25} + \frac{100}{v}$$

dX/dv must be zero.

$dX/dv = 1/25 - 100/v^2 = 0$

$v = 50$ mph

44.



The height at any angle θ is

$x = 12 + 10\sin\theta$

When $x = 17$, $\theta = 30^0$

$dx/dt = d/dt (12 + 10 \sin\theta)$

$= d/d\theta (12 + 10 \sin\theta) d\theta/dt$

$1.81 = [d/d\theta (12 + 10 \sin\theta)|\theta = 30^0] d\theta/dt$

Note: Go to radian mode.

30 must be typed 30^0 (30 SHIFT Ans 1)

$1.81 = 8.66\, d\theta/dt$

$d\theta/dt = 0.209$ rad/s

0.209 rad /s x $60/(2\pi)$ min/rev = 2 RPM

45.

$ay = d/dt(4t) \mid t = 3$ is 4

$ax = d/dt(3t^2)| t = 3$ is 18

$a = Abs< 4 \ 18 > = 18.44$ m/s^2

46. Find the intersection.

$x + xy = 1 \qquad xy = 1 - x$

$y = (1 - x)/x$ and $y = (x + 1)^{2/3}$

$(1-x)/x = (x + 1)^{2/3}$

$(1-x) = x(x + 1)^{2/3}$

USE SHIFT CALC.

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x = 0.4396

d/dx ((1 – x)/x | x = 0.4396 is -5.175

d/dx((x + 1)$^{2/3}$) | x = 0.4396 is 0.59

$\tan^{-1}$(0.59) – $\tan^{-1}$(-5.175) = 109.6^0

The acute angle is $180 – 109.6^0 = 70.4^0$

47. d/dt($2te^{-t}$) when t = 0 is 2

d/dt($2te^{-t}$) when t = 0.000001 is 1.999996

a = (1.999996 – 2)/ (0.000001) = - 4

48.

slope AB = (2 - 0)/ (0 + 4) = 1/2

For $y^2 = 4 + x$

2yy' = 1 but y' = 1/2

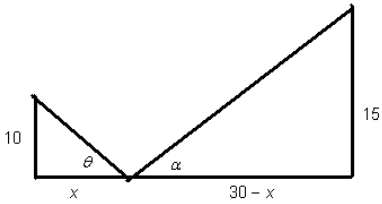
2(y)(1/2) = 1

y = 1 then

$1^2 = 4 + x$

x = -3

49.



For least amount of wire, θ = α

$$\frac{10}{x} = \frac{15}{30 - x}$$

x = 12

50.

For a_x d/dt(tcos 3t)| t = 1 is -.4133

d/dt(tcos 3t) | t = 1.000001 is -1.413344

a_x = (-1.413344 + 0.4133)/0.000001 = 8.063 m/s^2

For a_y

d/dt(t^4e^t): t = 1 is 13.591

d/dt(t^4e^t)| t = 1.000001 is 13.5914662

a_y = (13.5914662 – 13.591)/ 0.000001 is 57.08

a = Abs< 8.063 57.08 > = 57.65 m/s^2

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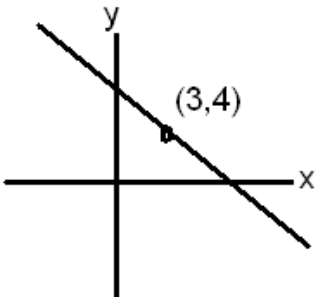
51.



Equation of the line: $y = A + Bx$

$4 = A + 3B$

$A = 4 - 3B$

$y = 4 - 3B + Bx$

x intercept $y = 0$, $4 - 3B + Bx = 0$

$x = (3B-4)/B = 3 - 4/B$

y intercept $x = 0$ $y = 4 - 3B$

$L = \sqrt{(x^2 + y^2)} = \sqrt{[(3 - 4/B)^2 + (4 - 3B)^2]}$

Let $B = x$ USE MODE 7

$f(x) = \sqrt{[(3 - 4/x)^2 + (4 - 3x)^2]}$

START $X = -2$ END $x = 0$ STEP .1 (SLOPE IS NEGATIVE)

Min length is 9.8656 (happens between $x = -1.2$ and $x = -1$

START $X = -1.2$ END = -1 STEP 0.01

Min Length = 9.8656 at $B = -1.1$

52.

Minimum Vertical Distance = $y_2 - y_1$

$= 1 + x^2 - \sqrt{x}$

USE MODE 7

$f(x) = 1 + x^2 - \sqrt{x}$

START $x = 0$ END = 1 STEP 0.1

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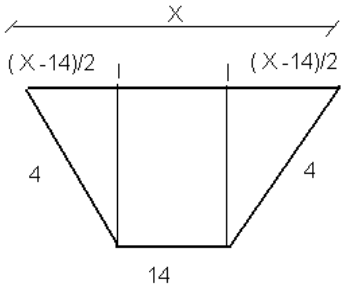
Min distance x = 0.4 , happened between x = 0.3 , x =0.5
START X = 0.3 END = 0.5 STEP 0.01
Min distance happens when x = 0.4 Distance = 0.5275

53-54



Altitude of the trapezoid = $\sqrt{(4^2)-(\frac{(x-14)}{2})^2}$

Area of the trapezoid = (X + 14)/2 $\sqrt{(4^2)-(\frac{(x-14)}{2})^2}$

MODE 7

f(x) = (X + 14)/2 $\sqrt{(4^2)-(\frac{(x-14)}{2})^2}$

START: 14 (Note x cannot be less than 14)
END: 22
STEP : 0.5
DISPLAY:

X	F(X)
14	56
14.5	56.888
15	57.545
15.5	57.953
16	**58.094**
16.5	57.944
17	57.475

Max Volume = 58.094 , when x = 16
START: 15.5

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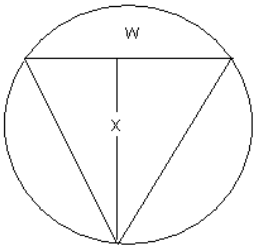
END: 16.5
STEP: 0.1
F(x)
15.9 58.088
16 58.094 Width at the top with Max Volume
16.1 58.088

55.



Let w = diameter of the cone and x = height.

Then w = $2\sqrt{20x - x^2}$ (Width = $2\sqrt{}$ (Dx – $x^{2)}$)
where D = diameter of the circle

Volume of the cone = 1/3 π $W^2/4$ x

= 1/3 π ($2\sqrt{20x - x^2}$)^2/4 x
= 1/3 π (4 (20x - x^2) /4) x
= 1/3π ($20x^2$ - x^3)

MODE 7
f(x) = 1/3π ($20x^2$ - x^3)
START: 1
END: 20
STEP: 1
Partial Display:
X F(X)
12 1206.3

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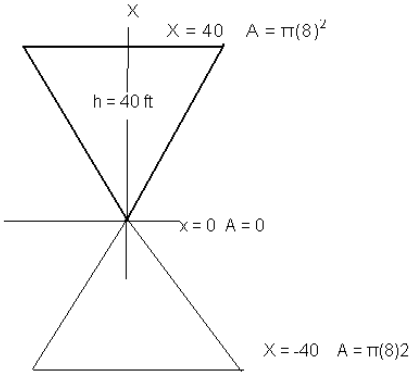
13 1238.8
14 1231. 5
START: 12
END: 14
STEP: 0.1
Max Volume:
X F(X)
13.3 1241.0997 or 1241.1
Material Wasted = $4/3\pi$ (10)^3 - 1241.1 =

56.



When water is 4ft deep
dV/dt = $4\hat{y}$ dy/dt where dy/dt = the speed, water surface is rising.
MODE 3 3
X Y
0 0
40 π(8)^2
-40 π(8)^2
$4\hat{y}$ = 2.0106193 (4 SHIFT 1 5 6)
2 = 2.0106193 dy/dt dy/dt = 0.95 ft/min

57.
V = xyz

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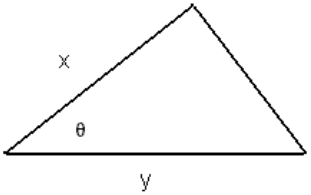
dV = $\partial V / \partial x dx + \partial V / \partial y dy + \partial V / \partial z dz$

= yz (0.25) + xz (0.1) + xy (0.3)

x = 15, y = 10, z = 24

= 141 in^3

58.



A = 0.5 x y sin θ

d A = $\partial V / \partial x dx + \partial V / \partial y dy + \partial V / \theta d\theta$

= 0.5 y sin θ dx + 0.5 x sin θ dy + 0.5 xy cos θ dz

= 8.289 m^2

59-60

$V = s^3$

$dV = 3s^2 ds$

$dV = 3(6)^2 (1/12) = 9\ cm^3$

$S = 6s^2$

$dS = 12\ s\ ds = 12\ (6)\ (\ 1/12) = 6\ cm^2$

USING CAL TECHNIQUE:

59. $d/dx\ (x^3)$ x= 6 x 1/12 = 9

60. $d/dx\ (6x^2)$ x = 6 x 1/12 = 6

61-62

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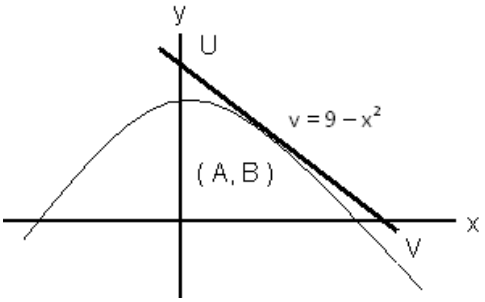

$y = 9 - x^2$ $y' = -2x$
slope at (A, B) = $-2A$
Relationship between A and B: $B = 9 - A^2$
Equation of line UV :
$y - B = -2A (x - A)$
When $x = 0$, $y = B + 2A^2$ $(0, B + 2A^2)$
when $y = 0$, $0 - B = -2A (x - A)$ $x = B/(2A) + A$
$(0.5 B/A + A, 0)$

Distance UV = $\sqrt{(0.5B/A + A)^2 + (B + 2A^2)^2}$

$B = 9 - A^2$

$$D = \sqrt{(0.5\frac{9 - A^2}{A} + A)^2 + (9 - A^2 + 2A^2)^2}$$

USE MODE 7, Replace A by X.
START: 0
END: 3 (max possible value is less than 3)
STEP: 0.5

x	f(x)
0	error
0.5	13.081
1	**11.18**
1.5	11.858

……..
The minimum distance = 11.18 with $x = 1$ and $y = 9 - x^2 = 8$

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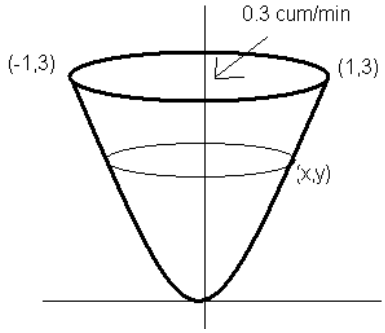
63.



Get the relationship between x and y
MODE 3 3
X Y
0 0
1 3
-1 3
A = 0 B = 0 C = 3
Thus: $y = 3x^2$ or $x^2 = y/3$
Volume = ½ π x^2 y
Then: V = ½ π(y/3)(y)

$$dV/dt = 1/2 d/dt (\pi(y/3)y)) = 1/2 d/dy (\pi(y/3)y))_{y=0.7} \, dy/dt$$

$$0.3 = 0.733 \, dy/dt$$

$$dy/dt = 0.41 \text{ m/s}$$

64.
Let x = radius of the cone
Then $2\pi(100) - 100\theta = 2\pi x$

$$x = 100(1 - \theta/(2\pi))$$

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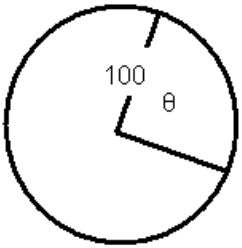

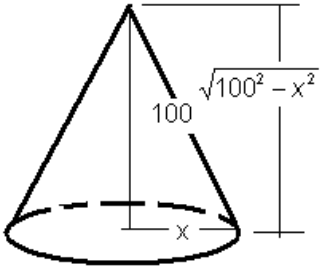

Volume of Cone = 1/3π (x^2) $\sqrt{100^2 - x^2}$

USE MODE 7

ENTER: f(x) = 1/3π (x^2) $\sqrt{100^2 - x^2}$

START? 0

END? 100

STEP? 10

Biggest Value happens between x = 80

ENTER: AC

START? 70

END? 90

STEP: 1

Biggest value happens between 81 and 83

START: 81

END: 83

STEP: 0.1

Biggest Value happens when x= 81. 6

START: 81. 5

END: 81.7

STEP: 0.01

Biggest Value happens when x = 81.65

Then

x = 100 (1- θ/(2π))

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81.65 = 100 (1 - θ/(2π))
θ = 1.153 or 66.06^0

65-66-67-68

$X^2 + Y^2 = 5^2$
Then Y = $\sqrt{25 - x^2}$
dx/dt = 0.1 m/min
Then dy/dt = d/dt $\sqrt{25 - x^2}$
= d/dx $\sqrt{25 - x^2}$ $_{x=3}$ dx/dt = -0.75(0.1)

65. = -0.075 m/min

66. dy/dt = - dx/dt (y is going down)
dy/dt = d/dx($\sqrt{25 - x^2}$) dx/dt
Since dy/dt = -dx/dt
Then d/dx ($\sqrt{25 - x^2}$)x= x? = -1
Using trial and error (Use the choices)
x = 3.536

67. When is the top of the ladder descending at 0.15 m/min?
dy/dt = d/dx($\sqrt{25 - x^2}$) dx/dt
dy/dt = -0.15
dx/dt = 0.1
-0.15 = d/dx($\sqrt{25 - x^2}$) when x = ? (0.1)
Using Trial and Error (Use the choices)
x = 4.16

68. Tan θ = y/x

$$\tan\theta = \frac{\sqrt{25 - x^2}}{x}$$

and $$\theta = \tan^{-1}\frac{\sqrt{25 - x^2}}{x}$$

$$\theta = \tan^{-1}\frac{\sqrt{25 - x^2}}{x}$$

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$d\theta/dt = d/dt \tan^{-1} \frac{\sqrt{25-x^2}}{x}$

$= d/dx \tan^{-1} \frac{\sqrt{25-x^2}}{x}\Big|_{x=3} dx/dt$

(Rad Mode)

$= -1/4(0.1) = -0.025$ rad/min

PROBLEM SET 22
INTEGRATION, AREA, VOLUME, SURFACE AREA, LENGTH OF AN ARC, CENTROID

1. Evaluate $\int (3x-1)^3 dx$ PAST ECE BOARD

a. $\frac{1}{9}(3x-1)^4 + c$ b. $(3x-1)^4 + c$

c. $\frac{1}{3}(3x-1)^4 + c$ c. $\frac{1}{12}(3x-1)^4 + c$

2. Evaluate $\int \cos 2x e^{\sin 2x} dx$ PAST CE BOARD

a. 1/2 $e^{\sin 2x}$ + c b. 2 $e^{\sin 2x}$ + c

c. 1/2 $e^{\cos 2x}$ + c d. 1/2 $e^{\cos 2x}$ + c

3. $\int \cos^2 y dy$ PAST CE BOARD

a. 1/2y + 1/4sin2y + c b. 1/4y + 1/2sin2y + c

c. 1/2y + 1/2sin2y + c d. 1/9y + 1/2sin2y + c

4. $\int \sqrt{1-\cos x} dx$

$2 \sin^2 x = 1 - \cos x$

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a. $\sqrt{2}(-\cos x/2)+c$ b. $1/\sqrt{2}(-\cos x/2)+c$

c. $2\sqrt{2}(-\cos x/2)+c$ d. $4\sqrt{2}(-\cos x/2)+c$

5. $\int \frac{e^x-1}{e^x+1}dx$

a. ln [(e^{-x} +1)(e^x -1)] + c b. ln [(e^x +1)(e^{-x} +1)] + c

c. ln [(e^x +1)(2e^{-x} -1)] + c d. ln [(e^x -1)(e^{-x} -1)] + c

Problems 6, 7, 8

CE BOARD May 2008

The motion of a particle in space from t = 0 to t = 10 sec is defined by: distance is in meters.

$a_x = 8t \quad a_y = 2 - 0.3t \quad a_z = 5$

6. What is the velocity after 2 seconds ?

a. 67.33 m/s b. 64.23 m/s

c. 87.12 m/s d. 98.12 m/s

7. What is the acceleration of the particle after 10 sec?

a. 9.5 m/s^2 b. 8.1 m/s^2

c. 11.2 m/s^2 d. 10.8 m/s^2

8. What is the total distance traveled after 10 sec.

a. 361.22 m b. 981.11 m

c. 287.71 m d. 298.11 m

9. Find the area bounded by the curve $x^2 = 16y$ and the x axis from x = 1 to x = 3.

a. 0.5417 b. 0.2212

c. 0.5417 d. 0.8112

Problems 10, 11, 12

An area is bounded by the curve y = sin x , the x axis and the lines x = 0 and x = π.

10. What is the area bounded by the curves?

a. 3 b. 2

c.1 d. 4

11. How far is the centroid of the region above the x axis?

a. 0.3926 b. 0.4412

c. 0.8122 d. 0.7611

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12. What is the volume generated when the region is revolved about the x axis?

a. 3.45 b. 5.67
c. 4.93 d. 5.11

13. Evaluate $\int_0^a \frac{x^2}{(a^2+x^2)^2}dx$

a. $\frac{\pi-2}{4a}$ b. $\frac{\pi-2}{a}$
c. $\frac{\pi-2}{2a}$ d. $\frac{\pi-2}{8a}$

14. The circle $x^2 + y^2 = 36$ is revolved about the line y = 4. What is the volume generated? PAST CE BOARD

a. 9812.33 b. 8122.23
c. 9948.56 d. 1221.23

15. The area bounded by the curve $y^2 = 12x$ and the line x = 3 is revolved about the line x =3. What is the volume generated? PAST CE BOARD

a. 191.23 b. 123.45
c. 180.96 d. 143.33

16. What is the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$? PAST CE BOARD

a. 5.33 b. 4.33
c. 8.22 d. 9.11

17. Evaluate $\int_0^2 \int_0^y 3x^2 + 9y^2 dxdx$ PAST CE BOARD

a. 20 b. 30
c. 40 d. 50

18. Find the area bounded by the curve $r^2 = a^2 \cos 2\theta$

a. $0.4a^2$ b. $0.3a^2$
c. $0.5a^2$ d. $1a^2$

19. Find the area enclosed by the curve $x^2 + 8y + 16 = 0$, the x axis, the y axis and the line $x - 4 = 0$. PAST CE BOARD

a. 10.67 b. 11.22
c. 13.22 d. 14.11

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20. The area bounded by the curve $y = 2x^{1/2}$ the line y = 6 and the x axis is to be revolved about the line y = 6. Determine the cenntroid of the volume generated. PAST CE BOARD

a. 1.5 b. 1.6
c. 1.7 d. 1.8

21. Find the length of the arc of the circle $x^2 + y^2 = 64$, bounded by x = -1 and x= -3 in the 2nd quadrant. PAST CE BOARD

a. 2.07 b. 3.01
c. 1.98 d. 1.88

22. Find the length of the arc whose parametric equation is $x = t^3$ and $y = 2t^2$ where $0 \le t \le 2$.

a. 11.67 b. 11.52
c. 12.43 d. 14.33

23. Find the radius of curvature of the vector equation R(t) = $2ti + (t^2 - 1)j$ when t = 1

a. 0.1768 b. 0.2122
c. 0.1222 d. 0.5612

Problems 24, 25

A particle moves along the x axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position x(t) is 5 for t = 2.

24. For what value of t, $0 \le t \le 3$ is the partilcle's instantaneous velocity the same as its average velocity on the closed interval [0, 3]

a. 1.23 b. 1.44
c. 1.79 d. 1.33

25. Find the total distance traveled by the particle from time t = 0 time t = 3.

a. 17 b. 18
c. 19 d. 20

Problems 26, 27

R is the region enclosed by the graphs $y = \ln(x^2 + 1)$ and $y = \cos x$.

26. What is the area of the region R.

a. 0.584 b. 0.332
c. 0.122 d. 0.612

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27. The base of the solid is a region R. Each cross section of the solid perpendicular to the x axis is an equilateral triangle. What is the volume of the solid?

a. 0.891 b. 0.198
c. 0.223 d. 0.441

Problems 28, 29

Let R be the region in the 1st quadrant eclosed by the graphs $y = e^{-x^2}$, y = 1 – cos x and the y axis.

28. Find the area of the region.

a. 0.78 b. 0.56
c. 0.59 d. 0.44

29. Find the volume when the region is revolved about the x axis.

a. 1.75 c. 1.81
c. 1.22 d. 1.33

30. A cubic polynomial function is defined by f(x) = $4x^3 + ax^2 + bx + c$. The function f has a local minimum at x = -1 , and the graph has a point of inflection at x= -2. If $\int_0^1 f(x)dx = 32$, what is the value of c?

a. 2 b. 3
c. 4 d. 5

31. What is the total length of the curve r = 4 sin θ

a. 2π b. 3π
c. 4π d. 5π

32. Find the perimeter of the cardiod r = 4(1 – sin θ)
PAST CE BOARD

a. 24 b. 32
c. 34 d. 40

33. What is the area bounded by the curve y = 6 cos x from the x axis to x = π/6 to x = 2π.

a. 22 b. 21
c. 31 d. 19

34. Find the surface area generated by rotating the 1st quadrant portion of the curve $x^2 = 16 – 8y$ about the y axis. PAST CE BOARD

a. 67.11 b. 61.27

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c. 71.23 d. 81.33

35. X years from now, one investment plan will be generating a profit at the rate of $50 + x^2$ thousand pesos per year, while a second plan will be generating at the rate of $(200 + 5x)$ thounsand pesos per year. Find the total net excess profit if you invest in the 2nd plan instead of the second plan up to the time the two plans yield equal profit. PAST CE BOARD

a. P 1687.50 b. P 1211.23
c. P 1451.61 d. P 1811.33

Problems 36, 37, 38, 39

Given the equation of the curve $y = 0.1(1600 – x^2)$. CE BOARD Nov 2004

36. Find the area bounded by the curve and the x axis.

a. 8533.33 b. 8912.33
c. 8921.44 d. 1892.44

37. Find the moment of Inertia with respect to the x axis.
Moment of rectangle with respect to the base = $1/3bh^3$

a. 49.932×10^6 units^4 b. 59.932×10^6 units^4
c. 39.932×10^6 units^4 d. 89.932×10^6 units^4

38. Find the moment of inertia with respect to the y axis.

a. 2.73×10^6 b. 4.73×10^6
c. 6.73×10^6 d. 1.73×10^6

39. Find the radius of gyration with respect to the y axis.

a. 12.56 b. 17.89
c. 11.22 d. 13.44

Problems 40, 41, 42

Given: Area bounded by the curve $y = xe^{-x}$, the x axis and the maximum ordinate.

40. Find the area.

a. 0.345 b. 0.264
c. 0.441 d. 0.133

41. Find the volume generated if the area is revolved about the x axis.

a. 0.611 b. 0.312
c. 0.254 d. 0.761

42. Find the moment of inertia with respect to the x axis.

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a. 8.71×10^{-3} b. 6.71×10^{-3}
c. 3.71×10^{-3} d. 3.71×10^{-3}

43. Find the area which is inside the circle $r = 3\cos\theta$ and outside the cardiod $r = 1 + \cos\theta$

a. π b. $\pi/2$
c. $\pi/6$ d. $\pi/8$

44. Find the length of the curve $x = 2(2t+3)^{3/2}$ and $y = 3(t+1)^2$ from $t = -1$ to $t = 3$.

a. 76 b. 78
c. 72 d. 80

45. A hemispherical tank of radius 6 m is filled with water to a depth of 4 m Find the work done in pumping the water to the top of the tank.

a. 7889.67 kN m b. 7122.23 kN m
c. 8111.33 kN m d. 8911.33 kN m

46. A tank is filled with water has the form of a paraboloid of revolution whose axis is vertical. If the depth of the tank is 12 ft and the diameter of the top is 8 ft, find the work done in pumping the water to the top of the tank.

a. 37.54 tons m b. 43.12 tons m
c. 51.22 tons m d. 71.22 tons m

47. Find the area between the curves $y = 2^x$ and the chord joining the points (0,1) and (2,4).

a. 0.91 b.0.67
c. 0.45 d. 0.56

48. Find the area bounded by the curve $y = 4 \sin x$ and the lines $x = \pi/3$ and $x = \pi$ and the x axis.

a. 6 b. 8
c. 10 d. 12

49. Find the moment of inertioa with respect to the y axis of the curve $x^2 = 8y$ bounded by the line $y = 2$ and $x = 0$. PAST CE BOARD

a. 17.07 b. 11.23
c. 13.08 d. 19.92

50. An object is moving so that its speed after t minutes is $3 + 2t + 6t^2$ m/min. It travels 10 m in the 1st minute. How far does it travel in the 1st 2 minutes? PAST CE BOARD

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a. 10 b. 20
c. 30 d. 40

Problems 51, 52, 53

A curve has an equation $y = 2\sqrt{x}$ which intersects the line $x = y$.

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51. Find the area between the curve and the line.
a. 2.67 b. 3.33
c. 8.11 d. 5.11

52. What is the volume generated by revolving the area about the x axis?
a. 31π/3 b. 21π/3
c. 14π/3 d. 32π/3

53. Find the distance of the centroid of the common area from the x axis?
a. 1.5 b. 2
c. 2.5 d. 3

54. Find the area bounded by the parabola $x^2 - 4x + 12y - 20 = 0$ and the x axis between
$x = 1$ and $x = 2$.
a. 1.97 b. 1.22
c. 1.62 d. 1.88

55. Find the length of the arc $y = x\sqrt{x}$ from $x = 0$ to $x = 4/3$.
a. 3.1 b. 2.07
c. 1.11 d. 3.41

56. A variable square whose plane is perpendicular to the x axis has one vertex on the x axis and the opposite vertex on the curve $xy = 4$. Find the volume of solid generated as the square moves from $x = 1$ to $x = 4$.
a. 6 b. 12
c. 18 d. 20

57. A solid has an elliptical base with major axis of 18 inches and a minor axis of 12 inches. Find the volume of solid if every section perpendicular to the major axis is an equilateral triangle.
a. 767.23 b. 748.24
c. 812.33 d. 612.34

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58. Find the volume if the 1st octant , bounded by the surface x = 1 and $x^2 = y + 2z$. CE BOARD May 2004

a. 0.03 b. 0.04

c. 0.05 d. 0.06

59. What is the length of one arch of the curve having the following parametric equation. PAST CE BOARD

$x = 2\theta - 2 \sin \theta$

$y = 2 - 2 \cos \theta$

a. 24 b. 16

c. 18 d. 12

60. The axes of two right circular cylinders of radius 4 cm intersect at right angles. Find the volume common to the cylinders.

a. 1013/3 b. 1024/3

c. 2314/3 d. 7182/3

Answers

1. Let u $3x - 1$ $du = 3dx$

$$\int (3x-1)^3 dx = \frac{1}{12}(3x-1)^4 + c$$

2. Let u = $2\sin 2x$ $du = 4\cos 2x\, dx$

$$\int \cos 2x e^{\sin 2x} dx = \int e^u \frac{du}{2}$$

$= 1/2\, e^u + c$

$= 1/2\, e^{\sin 2x} + c$

3. $\cos^2 y = 1/2 + 1/2\cos 2y$

$$\int \cos^2 y dy = \int (1/2 + 1/2\cos 2y) dy$$

$= 1/2y + 1/4\sin 2y + c$

4. $\sqrt{1-\cos x} = \sqrt{2\sin^2 \frac{x}{2}} = \sin x/2$

$$\int \sqrt{1-\cos x} dx = \int \sqrt{2} \sin \frac{x}{2} dx = 2\sqrt{2}(-\cos x/2) + c$$

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5. $\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{dx}{e^x + 1}$

$= \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^{-x}} dx$

$= \ln(e^x + 1) + \ln(1 + e^{-x})$

$= \ln[(e^x + 1)(e^{-x} + 1)] + c$

6. $v_x = \int_0^{10} 0.8t dt \qquad v_y = \int_0^{10} (2 - 0.3t) dt \qquad v_z = \int_0^{10} 5 dt$

$v_x = 40 \qquad v_y = 5 \qquad v_z = 50$

v = abs< 40 5 50 >

v = 64.226 m/s

7. a = < .8(10) 2 – 0.3(10) 5 >

= < 8 -1 5 >

Abs< 8 -1 5 > 9.4868 m/s^2

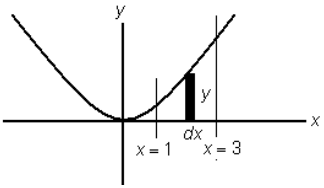
8. $S_x = \int_0^{10} 0.4t^2 dt = 133.33$

$S_y = \int_0^{10} (2t - 0.3t^2/2) dt = 50$

$S_z = \int_0^{10} 5t dt = 250$

S = Abs< 133.33 50 250 > = 287.71 m

9.



Area = $\int_1^3 y dx = \int_1^3 \frac{x^2}{16} dx$ = 0.5417 sq units

10.

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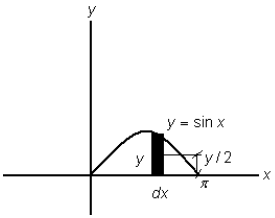


$$A = \int_0^{\pi} \sin x dx \;=\; 2$$

11.

$$\bar{y} = \frac{\int_0^{\pi} (y/2)\sin x dx}{2} = \frac{\int_0^{\pi} (\sin x/2)\sin x dx}{2}$$

$= \; 0.3926$

12. Using Pappus Theorem:

$$V = 2\pi(\,A)\;\bar{y}$$

$= \; 4.9335$ sq units

13.

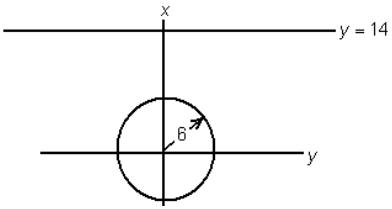
This is a difficult integral. Just use trial and error.

Let a = 1 , then $\int_0^1 \frac{x^2}{(1^2 + x^2)^2} dx \;=\; 0.1427$

This will just fit to choice $\frac{\pi - 2}{8}$ where a = 1

Ans. (d)

14.



Using Pappus Theorem: $V = 2\pi\,(\,\pi(6)^2\,)(14)$

$= \; 9948.56$ cubic units

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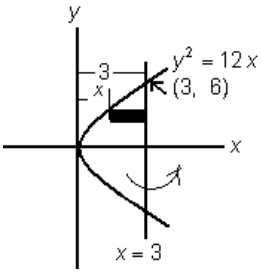
15.



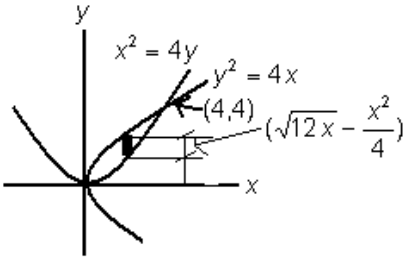
$$V = \pi\int(3-x)^2 dy = \pi\int_{-6}^{6}(3-\frac{y^2}{12})^2 \quad \text{dx= } 180.96$$

Note: The centroid of the parabolic segment is at 2/5 of the height from the base.

$V = 2\pi A\, y_c = 2\pi(\ 2/3)(\ 12 \text{ x } 3\ )\ (\ 2/5\ (3)\ )$

$= \ 180.96$ cubic units

16.



Point of intersection : Solve simultaneously:

$$\sqrt{4x} = \frac{x^2}{4} \qquad x = 0 \text{ and } x = 4$$

$y = 0$ and $y = 4$

Point of Intersection (0,0) and (4,4)

$$A = \int_0^4 \sqrt{4x} - \frac{x^2}{4} dx$$

A = 5.333 sq units

17.

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$$\int_0^2 \int_0^{2y} 3x^2 + 9y^2 dxdy =$$

$$\int_0^2 (x^3 + 9y^2 x)\Big|_0^y dy = \int_0^2 (y^3 + 9y^3) dy = 40$$

18. (This a a lemniscatE)

Let a = 1 USE MODE 7

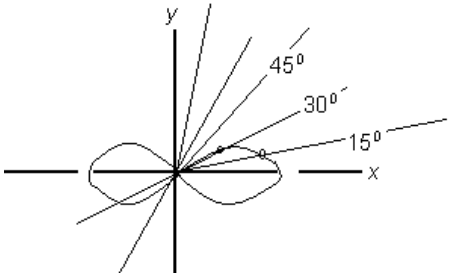
Input f(x) = $\sqrt{\cos 2x}$

START = 0 END = 360^0 STEP = 15^0

X	F(X)
0	1
15	0.9306
30	0.7071
45	0
60	ERROR
75	ERROR
90	ERROR
….	(Error means there is no curve there)
150	0.7071
165	0.9306
180	1

….

Plot this points:



$$A = 2\int_0^{\pi/4} \frac{1}{2} r^2 d\theta = 2\int_0^{\pi/4} \frac{1}{2}(\cos 2\theta) d\theta$$

Switch to Rad Mode.

A = 1

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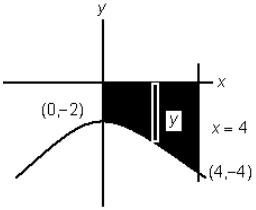
Area = $1a^2$

19.

$x^2 = -8(y - 2)$

V(0,2)



$8y = -x^2 - 16$

$y = -x^2/8 - 2$

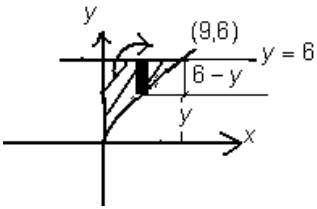
$A = \int_0^4 \left| \frac{-x^2}{8} - 2 \right| dx$ = 10.667 sq units

Note: To enter | | , enter SHIFT hyp

20.

$y = 2x^{1/2}$ is $y^2 = 4x$ (Parabola)

when y = 6 , $6^2 = 4x$, x = 9



$V = \int_0^9 \pi(6-y)^2 dx = \int_0^9 \pi(6-2\sqrt{x})^2 dx$

= 169.646 cubic units

$V\bar{x} = \int x dV$

$169.646\bar{x} = \int_0^9 x[\pi(6-2\sqrt{x})]^2 dx$

$169.646\ \bar{x} = 305.3628$

$\bar{x} = 1.8$

21.

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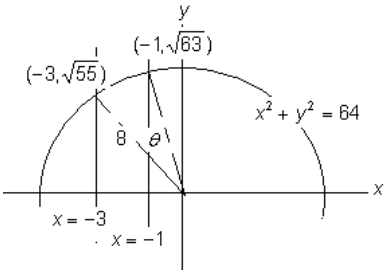

Switch to Radiam Mode:

Pol(-3, √55) = (8, 1.95519)

Pol(-1, √63) = (8, 1.696124)

Note: Pol is SHIFT +

S = rθ = 8(1.95519 – 1.696124)

= 2.0725

22.

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

dx/dt = 3t² dy/dt = 4t

$$L = \int_0^2 \sqrt{(3t^2)^2 + (4t)^2}\, dt = 11.517$$

23.

Radius of Curvature for a curve R = xi + yj where x and y are functions of t is:

$$R = \frac{x'(y'') - y'(x'')}{(x'^2 + y'^2)^{3/2}}$$

x = 2t x(1) = 2

x' = 2 x'(2) = 2

x'' = 0 x''(0) = 0

y = t² -1 y(1) = 0

y' =2t y'(1) = 2

y'' = 2 y''(2) = 2

Substitute in the formula:

R = 0.1768

24.

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$x(t) = \int v(t)dt = \int 3t^2 - 2t - 1dt + c$

$x(t) = t^3 - t^2 - t + c$

$x(2) = 5$

$5 = 2^3 - 2^2 - 2 + c$

$c = 3$

$x(t) = t^3 - t^2 - t + 3$

$x(3) = 18 \quad x(0) = 3$

Average velocity at [0 3] = $\frac{x(3) - x(0)}{3 - 0} = \frac{18 - 3}{3} = 5$

$v(t) = 3t^2 - 2t - 1$ (instantaneous velocity)

Then $3t^2 - 2t - 1 = 5$

$3t^2 - 2t - 6 = 0$

$t = 1.786$ sec

25.

Total distance $S = \int_0^3 |3t^2 - 2t - 1| \, dt$

$S = 17$

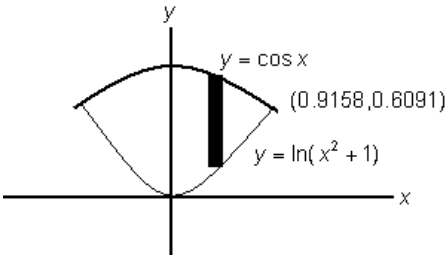
26.

Solve for the intersection:

$\ln(x^2 + 1) = \cos x$

(Go to rad mode and use SHIFT CALC)

$x = 0.9158$



$A = \int_0^{0.9158} \cos x - \ln(x^2 + 1) dx$

$= 0.584$

27.

The base of the triangle is always $\cos x - \ln(x^2 + 1)$

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The area of an equilateral triangle is $s^2\sqrt{3}/4$

Then $V = 2\sqrt{3}\int_0^{0.9158}[(\cos(x) - \ln(x^2+1)]^2/4$

= 0.19834

28.

Solve for the intersection.

$$e^{-x^2} = 1 - \cos x$$

Use SHIFT CALC, (Switch to Radian Mode)

x = 0.941944

and y = 0.41178

Intersection: (0.941944, 0.41178)

Use MODE 7 to plot some points of both curves.

For e^{-x^2} START = 0 END = 1 STEP = 0.1

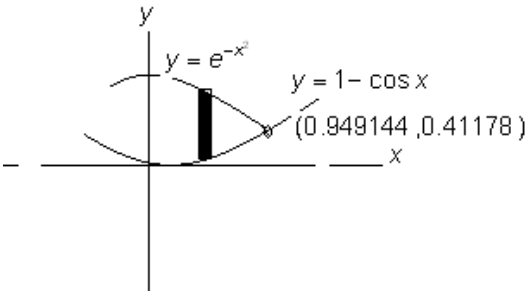
x	f(x)
0	1
0.1	0.99
0.2	0.9607
0.3	0.9139

……

For 1 – cos x START = 0 END = 1 STEP = 0.1

x	f(x)
0	0
0.1	4.99×10^{-3}
0.2	0.0199

……..



$$A = \int_0^{0.949144}(e^{-x^2} - (1 - \cos x))dx$$

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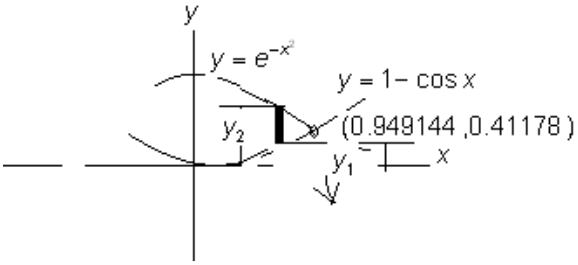
A = 0.5909

29.



$$V = \pi \int_0^{0.949144} (y_2^{\,2} - y_1^{\,2})dx = 1.7465$$

where $y_2 = e^{-x^2}$ and $y_1 = 1 - \cos x$

30.

$f(x) = 4x^3 + ax^2 + bx + c$

$f'(x) = 12x^2 + 2ax + b$

$0 = 12(-1)^2 + 2(a)(-1) + b$ (minimum at x = -1)

$-2a + b = -12$ (1)

$f''(x) = 24x + 2a$

$0 = 24(-2) + 2a$ (Inflection at x = -2)

$a = 24$ m substitute in eq 1, $b = 36$

$$\int_0^1 f(x)dx = 32$$

$$\int 4x^3 + 24x^2 + 36x + c = 32$$

$$x^4 + 8x^3 + 18x^2 + cx \Big|_0^1 = 32$$

$1^4 + 8(1)^3 + 18(1)^2 + c(1) = 32$

$c = 5$

31. Convert $r = 4 \sin \theta$ to rectangular form.

$\sin \theta = y/r$

$r = 4y/r$

$r^2 = 4y$

$x^2 + y^2 = 4y$

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$x^2 + y^2 - 4y = 0$

$x^2 + (y-2)^2 = 2^2$

radius = 2 $A = \pi(2)^2 = 4\pi$

32. The Curve is a CARIOD.

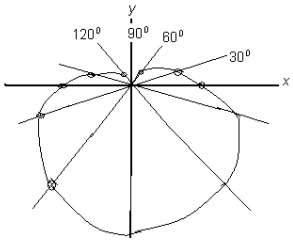
USE MODE 7 to plot the curve .

START = 0 END = 360 , STEP = 30

(Use Degree Mode)

X	F(X)
0	4
30	3
60	0.5358
90	0
120	0.5358
150	2

…….



$$L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{(4 - 4\sin\theta)^2 + (-4\cos\theta)^2}\, d\theta$$

L = 32

33.

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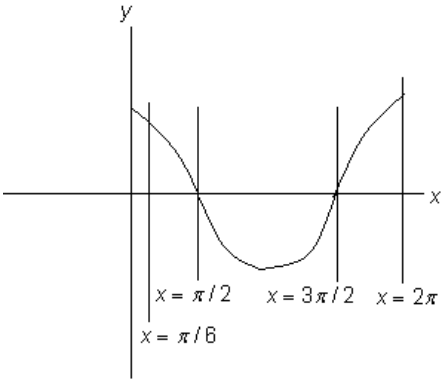


Divide the integration to 3 integrals.

$$A = \left| \int_{\pi/6}^{\pi/2} 6\cos x dx \right| + \left| \int_{\pi/2}^{3\pi/2} 6\cos x \right| + \left| \int_{3\pi/2}^{2\pi} 6\cos x \right| =$$

= 21

CAL TECHNIQUE:

Total Area = $\int_{\pi/6}^{2\pi} |6\cos x| dx$

SWITCH to LINE MODE (SHIFT MODE 2)

ENTER: ∫ abs(6 cos x) , π/6, 2π, 0.01)

Answer: 21 (Closest. The answer is not exact)

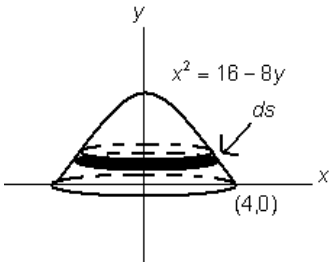
34.

$x^2 = 16 - 8y$

$x^2 = -8(y - 2)$

V(0,2)

opens downward.



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$ds = 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

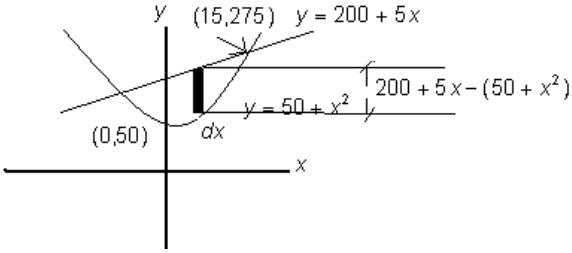
$x^2 = 16 - 8y$

$2x = -8y'$

$y' = -x/4$

$s = 2\pi\int_0^4 x\sqrt{1+(x/4)^2}\,dx = 61.27$

35.



The two plans will yield equal profit when

$200 + 5x = 50 + x^2$

$x^2 - 5x - 150 = 0$

$x = 15$ (disregard the negative root)

Total net excess profit is area bounded by the curves from x= 0 to x = 15.

Total net excess profit = $\int_0^{15} 200 + 5x - (50 + x^2)dx$

= P 1687.50

36.

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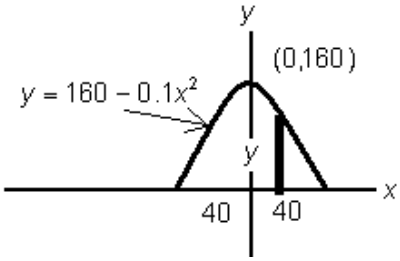

A = 2/3 (80 x 160) = 8533.33 sq units

37.

$$I_x = \int_{-40}^{40} \frac{dx(y)^3}{3} = \int_{-40}^{40} \frac{(160-0.1x^2)^3}{3} dx$$

= 49.932 x 10^6 units^4

38.

$$I_y = \int x^2(ydx) = \int_{-40}^{40} x^2(160-0.1x^2)dx$$

2.73 x 10^6 units ^{4}

39.

$$k_y = \sqrt{\frac{I_y}{A}} = \quad k_y = \sqrt{\frac{2.73x10^6}{8533.3}} = 17.89$$

40.

Maximum ordinate.

$y = xe^{-x}$

$y' = x(-e^{-x}) + e^{-x} = 0$

$x = 1$, maximum ordinate $= 1e^{-1} = 1/e$

Plot the curve:

ENTER: MODE 7 $f(x) = xe^{-x}$ START = 0 END = 1

STEP = 0.1

x	f(x)
0	0
0.1	0.0904
0.2	0.1637

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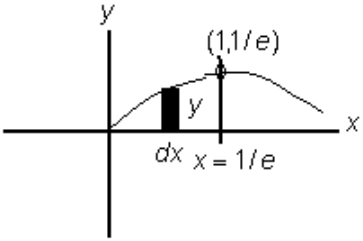
0.3 0.2222
0.4 0.2681
….
1 0.3678



$$A = \int_0^1 xe^{-x}dx = 0.2642$$

41.

$$V = \int_0^1 \pi y^2 dx = \int_0^1 \pi (xe^{-x})^2 dx$$

V = 0.254

42.

Moment of Inertia of rectangle with the base = $bh^3/3$

$$I_x = \int_0^1 dx \frac{y^3}{3} = \int_0^1 \frac{(xe^{-x})^3}{3} dx = 8.71 \times 10^{-3}$$

43.

Point of intersection:

$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

$\theta = 60^0$ or $\pi/3$

and $r = 1 + \cos \pi/3 = 3/2$

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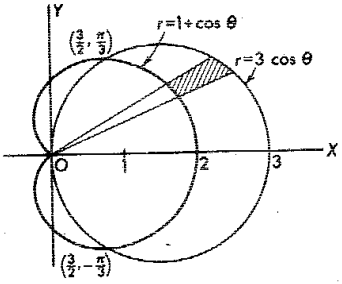


1/2 $A = 1/2\int(r_2^{\,2} - r_1^{\,2}) = 0.5\int_0^{\pi/3}(3\cos\theta)^2 - (1+\cos\theta)^2 d\theta$

1/2A =π/2

A = π

44.

$L = \int\sqrt{x'^2+y'^2}dt$ where x' = dx/dt and y' = dy/dt

$x = 2(2t+3)^{3/2}$

$x' = (2)(3/2)(2t+3)^{1/2}\ (2) = 6(2t+3)^{1/2}$

$y = 3(t+1)^2$

$y' = 6(t+1)$

L = $L = \int_{-1}^{3}\sqrt{x'^2+y'^2}dt = \int_{-1}^{3}\sqrt{36(2t+3)+36(t+1)^2}dt$ =72

45.

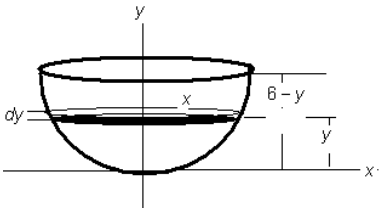
dW = (9810) π(x²)dy (6 – y)

$x^2 + (y-6)^2 = 36$

$x^2 = 36 - (y-6)^2 = 36 - (y^2 - 12y + 36)$

$x^2 = 12y - y^2$

$W = \int_0^4 9.81(\pi)(12y - y^2)(6-y)dy$ = 7889.67 knM



46.

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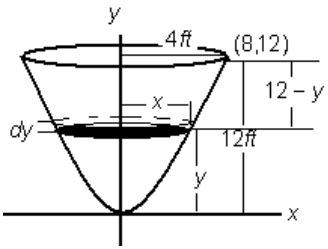

$x^2 = cy$

$4^2 = c(12)$ $c = 4/3$

$x^2 = 4/3\ y$

$dW = 62.4\ \pi x^2 dy\ (12 - y)$

$$W = \int_0^{12} 62.4\pi(4/3y)(12-y)dy$$

= 75277.59 ft lbs or 37.54 ft tons

(divide by 2000)

47.

Get the equation of the line.

ENTER: MODE 3 2

Input

X	Y
0	1
2	4

ENTER: AC

A = 1 B = 3/2

y = 1 + 3/2 x

By inspection, the points (0,1) and (2,4) are both on the curves thus, these points are intersection points.

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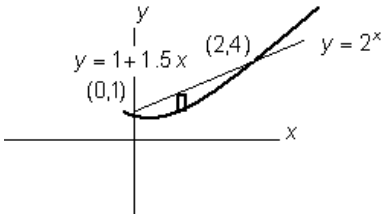
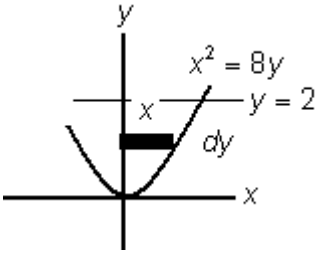


$$A = \int_0^2 2^x - (1+1.5x)dx = 0.6719$$

48.

$$A = \int_{\pi/3}^{\pi} 4\sin x dx = 6$$

49.



$$I = \int \frac{dy(x)^3}{3} = \int_0^2 \frac{dy(\sqrt{8y})^3}{3} = 17.07$$

50.

ds/dt = v

$ds/dt = 3 + 2t + 6t^2$

$ds = (3 + 2t + t^2)\,dt$

t = 1 s = 10

t = 2 s = ?

$$\int_{10}^{s} ds = \int_1^2 (3+2t+6t^2)dt$$

s – 10 = 20

s = 30

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51. Find the area between the curve and the line.

Intersection:

$y^2 = 2x$ and $x = y$

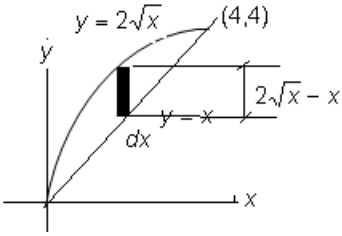
$y^2 = 2y$

$y^2 - 2y = 0$

$y(y - 2) = 0$

$y = 0$ and $y = 2$

$x = 0$ and $x = 2$



$$A = \int_0^4 2\sqrt{x} - x\,dx = 2.667$$

Alternative solution (Subtract the parabola with the triangle)

A = 2/3(4)(4) - 1/2(4 x 4) = 2.667

52. What is the volume generated by revolving the area about the x axis?

$$V = \pi \int_0^4 [(2\sqrt{x})^2 - x^2]dx = \frac{32\pi}{3}$$

53. Find the distance of the centroid of the common area from the x axis?

Using Pappus Theorem:

$V = 2\pi A \bar{y}$

$32\pi/3 = 2\pi (2.667) \bar{y}$

$\bar{y} = 2$

54.

CE BOARD May 2007

Get the vertex.

Rewrite as:

$12y = -x^2 + 4x + 20$

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$y = -x^2/12 + 4/12x + 20/12$

$A = -1/12 \quad B = 4/12$

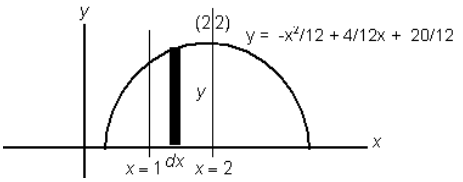
$-B/(2A) = 2$

Let $x = 2$

$-x^2/12 + 4/12x + 20/12 = 2$

Vertex (2, 2)

Opening downward , with Latus rectum = 12.



$$A = \int_1^2 \frac{-x^2 + 4x + 20}{12} dx = 1.9722$$

55.

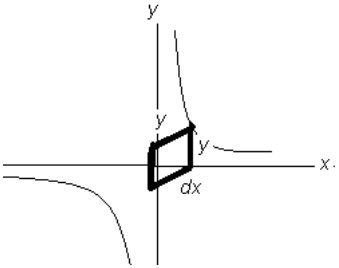
$y' = 3/2\, x^{1/2}$ and $(y')^2 = 9/4x$

$$\sqrt{1+(y')^2} = \sqrt{1+9/4x}$$

$$L = \int_0^{4/3} \sqrt{1+(y')^2}\,dx = \int_0^{4/3} \sqrt{1+(9/4)x}\,dx$$

L = 2.074

56.



$xy = 4 \quad y = 4/x$

$$V = \int_1^4 y^2 dx = \int_1^4 (4/x)^2 dx$$

V = 12

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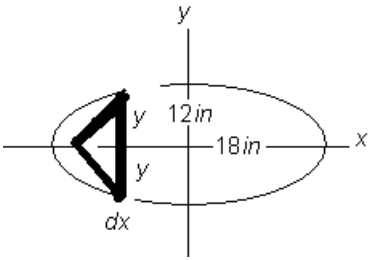
57.



Area of a triangle is $s^2\sqrt{3}/4$

$$V = \int \frac{(2y)^2\sqrt{3}}{4}dx = \int_{-18}^{18} \frac{4y^2\sqrt{3}}{4}dx \qquad (1)$$

$x^2/9^2 + y^2/6^2 = 1$

$y^2 = 6^2(1 - x^2/9^2)$

then: $V = \int_{-9}^{9} 6^2(1-x^2/9^2)\sqrt{3}dx =$

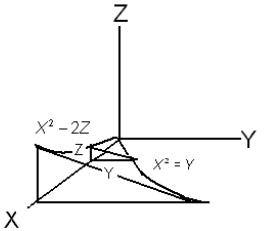
V = 748.24 cu units

58.

$x^2 = y + 2z$

trace in the xy plane, set z = 0, $x^2 = y$

trace in the xz plane, set y = 0, $x^2 = 2z$



dV = 1/2zy dx

$$V = \int_0^1 1/2zydx = \int_0^1 (1/2)(x^2/2)(x^2)dx$$

V = 0.05

59.

Find some points on $x = 2\theta - 2\sin\theta$

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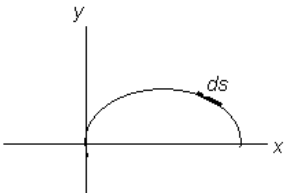
USE MODE 7 , f(x) = 2x – 2 sin x (rad mode)
START = 0 , END = 2π , STEP = π/2
SAME ALSO for y = 2 – 2cos θ
f(x) = 2- 2 cos x START = 0 , END = 2π,
STEP = π/2

θ	X	Y
0	0	0
π/2	1.415	2
π	6.28	4
3π/2	11.424	2
2π	12.566	0

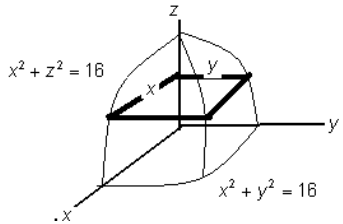


$$ds = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

x = 2θ- 2 sin θ
dx/dθ = 2 – 2cosθ
y = 2 – 2 cos θ
dy/dθ = 2 sin θ

$$s = \int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta \quad = 16$$

60.



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The Cross section is always a square.

1/8 V = $\int xydz$

1/8V = $\int_0^4 \sqrt{16-z^2}\sqrt{16-z^2}dz$

V = 1024/3

PROBLEM SET 23
DIFFERENTIAL EQUATIONS

1. The solution of the D.E.
$y' = xe^{3x}$ $y(0) = 0$ is

a. $y = (3x+1)e^{3x} - 1$ b. $y = (3x-1)e^{3x} - 1$
c. $y = (3x+1)e^{3x} - 2$ d. $y = (3x-1)e^{3x} + 1$

2. The general solution of $(y + 2)dx + (x - 2)dy = 0$
a. $xy - 2y + 2x = c$ b. $xy - 2y - 2x = c$
c. $xy - 2y + 4x = c$ d. $xy + 2y + 2x = c$

3. The general solution of the D.E.
$(x + y)dx + (x-y)dy = 0$.
a. $x^2 - 2xy - y^2 = c'$ b. $x^2 - 2xy + y^2 = c'$
c. $x^2 - 4xy - y^2 = c$ d. $x^2 + 2xy - y^2 = c'$

4. The general solution of $(2x + 3y)dx + (3x - 4y)dy = 0$ is
a. $x^2 + 3xy - 2y^2 = c$ b. $2x^2 + 3xy - 2y^2 = c$
c. $3x^2 + 2xy - 2y^2 = c$ d. $x^2 + 3xy + 2y^2 = c$

5. The general solution of the D.E. $dy/dx + y \cot x = 1$ is
a. $y = -\cot x + c \sin x$ b. $y = -\cot x + c \cos x$
c. $y = -\tan x + c \csc x$ d. $y = -\cot x + c \csc x$

6. A body moves such that its acceleration is given by $a = 2 + 12t$ where t is in seconds and 'a' is in m/s^2. If the body starts from rest and its velocity after 1 sec is 2 m/s, what is the distance traveled after 4 sec? (PAST CE BOARD)
a. 100 m b. 120 m
c. 124 m d. 140 m

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7. Find the orthogonal trajectories of the family of curves

$y^2 = cx$

a. $y^2 + 2x^2 = c'$ b. $y^2 + x^2 = c'$

c. $y^2 - 2x^2 = c'$ d. $y^2 + 4x^2 = c'$

Problems 8, 9

A ball is dropped from rest in a liquid and the acceleration of the ball as a function of velocity is given by

$a = 32 - 0.25v$ where a is in m/s^2 and v is in m/s (Similar to CE BOARD Nov 2009)

8. What is the the velocity of the ball after 2 seconds?

a. 45.21 m/s b. 50.4 m/s

c. 71.2 m/s d. 67.2 m/s

9. What is the distance the ball had traveled after 2 seconds?

a. 35.67 m b. 54.64 m

c. 36.12 m d. 41.23 m

Problems 10, 11

A body weighing 64 lb is dropped from a certain height with an initial velocity of 10 ft/s. Assume that air resistance is proportional to the velocity of the body. If the limiting velocity is known to be 128 ft/s, find the position of the body after 5 seconds. $g = 32\ ft/s^2$.

10. What is the velocity after 3 seconds?

a. 81.23 ft/s b. 72.26 ft/s

c. 84.56 ft/s d. 41.33 ft/s

11. What is the position after 3 seconds.

a. 121.34 m b. 134.95 m

c. 111.21 m d. 181.23 m

12. Radium decompose at the rate proportional to the amount present. If the half life is 1600 years, find the percenrtage remaining after at the end of 200 years.

a. 95.5% b. 91.7%

c. 81.2% d. 79.1%

The rate at which the body cools is proportional to the difference in temperature between the body and the surrounding atmosphere. If body in air at 20^0 cools from 100^0C to 70^0 C in 3 minutes.PAST EE, ECE BOARD EXAM

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13.-a Express the temprerature of the body at any time t.

a. $T = 20 + 80e^{-0.197t}$ b. $T = 20 + 80e^{-0.1567t}$

c. $T = 20 - 80\ e^{0.567t}$ c. $T = 20 - 80\ e^{0.1567t}$

13-b What is the temperature after 5 minutes?

a. 56.55 deg C b. 59.33 deg C

c. 67.33 deg C d. 66.22 deg C

13-c. When will the temperature be 65^0C?

a. 3.33 min b. 3.67 min

c. 3.45 min d. 3.91 min

14. The population of a community is known to increase at the rate proportional to the number of people present at any time t. If the population has doubled in 5 years, how long will it take to triple?

a. 6.73 years b. 7.92 years

c. 6.94 years d. 8.13 years

15. A tank contains 50 gal of water. Brine containing 2 pounds per gal flows into the tank at the rate of 2 gal/min and the mixture kept uniform by stirring runs out at the same rate. How long will it take before the quantity of salt in the tank is 50 lbs.

a. 12.34 min b. 17.33 min

c. 11.34 min d. 41.22 min

16. Find the differential equation of the family of lines passing trough the origin. PAST CE BOARD

a. xdx + ydy = 0 b. xdx – ydy = 0

c. xdy – ydx = 0 c. xdy + ydx = 0

17. Find the differential equation of the family of parabolas having their vertices at the origin and foci on the x axis PAST CE BOARD.

a. ydx – 2xdy = 0 b. 2ydx – xdy = 0

c. ydx – xdy = 0 d. 3ydx – 2xdy = 0

18. Given the simulateneous differential equations

2 dx/dt – 3dy/dt = y – x + k

3 dx/dt + 2dy/dt = x – cos t

Find dy/dt. PAST ME BOARD

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a. $\frac{5x-2\cos t-3y-3k}{13}$ b. $\frac{5x-4\cos t-3y-3k}{11}$

c. $=\frac{-5x-2\cos t-3y-3k}{13}$ d. $=\frac{5x-2\cos t-3y+4k}{13}$

19. Find the general soluition of y'' – 3y' + 2y = 0

a. $y=c_1e^{2x}+c_2e^{x}$ b. $y=c_1e^{3x}+c_2e^{x}$

c. $y=c_1e^{2x}+c_2e^{-x}$ d. $y=c_1e^{-2x}+c_2e^{x}$

20. Solve y'' – 2y' – y = 0

a. $y=c_1e^{(-1+\sqrt{2})x}+c_2e^{(-1-\sqrt{2})x}$

b. $y=c_1e^{(1+\sqrt{2})x}+c_2e^{(1-\sqrt{2})x}$

c. y = cos √2 x + c_2 sin √2 x

d. y = e^x (c_1 cos √2 x + c_2 sin √2 x)

21. Solve Y'' – 4Y' + 5Y = 0

a. $y=e^{2x}(c_1\cos x+c_2\sin x)$ b. $y=e^{-2x}(c_1\cos x+c_2\sin x)$

c. $y=e^{2x}(c_1\cos 2x+c_2\sin 2x)$ d. $y=e^{2x}(c_1\cos 3x+c_2\sin 3x)$

22. Let f be a function such that f''(x) = 6x + 8.
What is the value of f(x) if the graph of f is tangent to the line 3x – y = 2 at the point (0, -2).

a. f(x) = x^3 + 6x^2 + 3x - 2 b. f(x) = x^3 + 4x^2 + 3x - 4

c. f(x) = x^3 + 4x^2 + 3x - 2 d. f(x) = x^3 + 2x^2 + 5x - 2

23. At any time t >= 0, the velocity of the particle traveling along the x axis is given by the D.E.

$\frac{dx}{dt}-10x=60e^{4t}$. Find the general solution of the D.E.

a. $x=ce^{-10t}-10e^{-4t}$ b. $x=ce^{12t}-10e^{4t}$

c. $x=ce^{10t}-10e^{-4t}$ d. $x=ce^{10t}-10e^{4t}$

Problems 24, 25, 26

A particle moves along the curve defined by the equation y = x^3 – 3x. The x coordinate of the particle x(t) statisfies the equation

$\frac{dx}{dt}=\frac{1}{\sqrt{2t+1}}$ for $t\geq 0$ with x(0) = -4.

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24. Find the value of x when t = 4.
a. 2 b. -2 c. 3 d. -3
25. Find the value y when t = 4
a. 1 b. 2
c. 3 d. 4
26. Find the speed of the particle when t = 4.
a. 3.018 b. 2.112
c. 4.112 d. 3.111
27. A tank is initially filled with 100 gal of salt containing 1 lb of salt per gallon. Fresh brine containing 2 lb of salt per gallon runs into the tank at the rate of 5 gal per minute and the mixture assumed to be kept uniform by stirring, runs out at the same rate. When will the amount of the tank be 150 lb?
a. 12.33 min b. 11.44 min
c. 13.86 min d. 12.22 min

28. Find the solution of the D.E. $dy/dx + 3y/x = x^2$
a. $Y = x^3/6 + C/x^3$ b. $Y = x^3/12 + C/x^3$
c. $Y = x^3/9 + C/x^3$ d. $Y = x^3/6 + C/x^2$

A motorboat and its load weigh 2,150 N. Assume that the propeller force is constant and numerically equal to 6.7v Newton, where v is velolcity in m/s and boat starts from rest. PAST EE/ CE BOARD

29. Determine the speed of the boat at the end of 10 sec.
a. 4.78 m/s b. 4.32 m/s
c. 5.11 m/s d. 4.56 m/s

30. Determine the distance traveled at the end of 10 sec.
a. 23.56 m b. 22.67 m
c. 32.45 m d. 41.33 m

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Answers.

1.

$dy = xe^{3x}\,dx$

$y = \int xe^{3x}dx + c$

$y = (3x+1)e^{3x} + c$

when $x = 0$, $y = 0$

$0 = (3(0)+1)e^{0} + c$

$c = -1$

$y = (3x+1)e^{3x} - 1$

2.

$(y+2)dx + (x-2)dy = 0$

$\frac{dx}{x-2} + \frac{dy}{y+2} = 0$ Integrate.

$\ln(x-2) + \ln(y+2) = \ln c$

$\ln(x-2)(y+2) = \ln c$

$(x-2)(y+2) = c$

$xy - 2y + 2x - 4 = c$ or $xy - 2y + 2x = c$

3.

Let $M = 2x + 3y$ $N = 3x - 4y$

$\frac{\partial M}{\partial y} = 3$ $\frac{\partial N}{\partial x} = 3$ The D.E. is exact.

$\int_0^x (2t+3y)dt + \int_0^y (3(0)-4t)dt = c$

$t^2 + 3ty \Big|_0^x + -2t^2 \Big|_0^y = c$

$x^2 + 3xy - 2y^2 = c$

4.

The D.E. is homogeneous.

Let $y = vx$ $dy = vdx + x\,dv$

$(x + vx)dx + (x - vx)(vdx + xdv) = 0$

$x(1+v)\,dx + x(1-v)(vdx + x\,dv) = 0$

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(1 + v)dx + (1 – v) (vdx + x dv) = 0
dx(1 + v + v – v^2) + (1 – v) x dv = 0
dx(1 + 2v – v^2) + (1 – v) x dv = 0

$$\frac{dx}{x}+\frac{(1-v)dv}{1+2v-v^2}=0$$ Integrate:

ln x + 1/2 ln (1 + 2v – v^2) = ln c
2ln x + ln (1 + 2v – v^2) = ln c^2
ln x^2 + ln (1 + 2v – v^2) = ln c^2
v = y/x
ln (x^2 [1 + 2y/x – y^2/x^2) = ln c^2
x^2 + 2xy – y^2 = c'

5.

This is a linear D.E. of the form dy/dx + yP(x) = Q(x)
The solution is:

$$ye^{\int P(x)dx}=\int Q(x)^{\int P(x)dx}dx+c$$

$$e^{\int P(x)dx}=e^{\int \cot xdx}=e^{\ln \sin x}=\sin x$$

$$y\sin x = \int \sin x dx + c$$

y sin x = - cos x + c
y = -cotx + c csc x

7. The orthogonal trajectory equation is:

f(x,y, - 1/y') = 0
y^2 = cx Differentiate:
2yy' = c
Eliminate c.

$$\frac{y^2}{2yy'}=\frac{cx}{c}$$ y = 2xy' y' = y/(2x)

-1/y' = -(2x)/y
Solve:

y' = -(2x)/y $$\frac{dy}{dx}=-\frac{2x}{y}$$

ydy = -2xdx Integrate:
$y^2/2$ = -x^2 + c

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$y^2 + 2x^2 = c'$

6.

$dv/dt = 2 + 12t$

$dv = (2 + 12t)\,dt$ Integrate.

$v = 2t + 6t^2 + c$

when $t = 1$ $v = 2$

$2 = 2 + 6 + c$

$c = -6$

$v = 2t + 6t^2 - 6$

$ds/dt = 2t + 6t^2 - 6$

$$s = \int_0^4 (2t + 6t^2 - 6)dt$$

s = 120 m

8.

$a = dv/dt = 32 - 0.25v$

$$\int_0^v \frac{dv}{32 - 0.25v} = \int_0^2 dt = 2$$

Using Trial and Error (Use the choices)

Ans. v = 50.4 m/s

9.

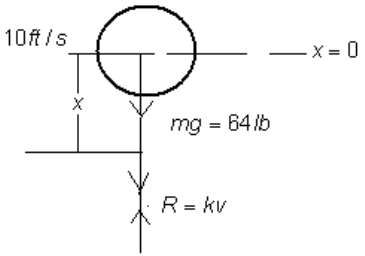
$a = vdv/ds$ $vdv/ds = 32 - 0.25v$

From 8,

$$\int_0^{50.4} \frac{vdv}{32 - 0.25v} = \int_0^s ds = s$$

s = 54.64 m

10.



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From Newton's Law of Motion:

(64/32)dv/dt = 64 - kv

Limiting velocity = 128 ft/s , dv/dt here is 0.

0 = 64 – k(128)

k = 0.5

Thus:

2dv/dt = 64 – 0.5v

dv/dt = 32 – 0.25v

$$\frac{dv}{32-0.25v} = dt$$

when t = 0, v = 10

t = 3, v = ?

$$\int_{10}^{v} \frac{dv}{32-0.25v} = \int_{0}^{t} dt = \int_{0}^{3} dt = 3$$

Using trial and error. (Use the choices)

v = 72.26 ft/s

11. vdv/ds = 64 – 0.5 v

$$\int_{10}^{72.26} \frac{vdv}{32-0.25v} = \int_{0}^{s} ds = s$$

s = 134.95 m

12.

ENTER: MODE 3 5

0 1

1600 1/2

Then: $200\hat{y}$ = 0.917 or 91.7%

13-a, 13-b , 13-c

ENTER: MODE 3 5

X Y

0 100 – 20 = 80

3 70 – 20 = 50

ENTER: AC

A = 80 (SHIFT 1 5 1) B = -0.1567 (SHIFT 1 5 2)

T – 20 = 80 $e^{-0.1567t}$

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13. $T = 20 + 80e^{-0.1567t}$

14. After 5 min T- 20 = $5\hat{Y}$ (5 SHIFT 1 5 5)
T = 56.55
15. (65- 20) $\hat{x}$ = 3.673 min

14.
ENTER: MODE 3 5
Input the given coordinates.

X	Y
0	1
5	2

Then 3 $\hat{X}$ = 7.92 years
15.
Let x = amount of salt in the tank at any time
dx/dt = rate of entrance of salt - rate of exit of salt
= 2 lb/gal x 2 gal/min - $\frac{x\ lbs}{50gal}(2)\frac{gal}{\min}$

dx/dt = 4 – x/25 or $\frac{dx}{4-0.04x} = dt$

$\int_0^{50} \frac{dx}{4-x/25} = \int_0^t dt$ t = 17.33 min

16.
Equation of Line : Y = A + Bx , since it x = 0 , y = 0
then A = 0. Thus: y = Bx , But B = y'
Then: y = y'x
y = dy/dx x or xdy – ydx = 0

17.
Since the foci are on the x axis, the family of parabolas are horizontal.
$y^2 = cx$ (Vertex at the origin)
2yy' = c (Divide the 2 equations.

$\frac{y^2}{2yy'} = x$ or y = 2xy'

y = 2x dy/dx
ydx – 2xdy = 0

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18.

Use Cramers Rule:

$$D = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 13$$

$$dy/dt = \frac{\begin{vmatrix} 2 & y - x + k \\ 3 & x - \cos t \end{vmatrix}}{13} = \frac{2(x - \cos t) - 3(y - x + k)}{13}$$

$$= \frac{5x - 2\cos t - 3y - 3k}{13}$$

19.

Replace y'' by m^2, y' by m and 2y by 2

Auxillary Equation: $m^2 - 3m + 2 = 0$

$m = 2$ and $m = 1$

General Solution: $y = c_1 e^{2x} + c_2 e^{x}$

20.

Auxillary Equation:

$m^2 - 2m - 1 = 0$

$m = 1 + \sqrt{2}$ and $m = 1 - \sqrt{2}$

$$y = c_1 e^{(1+\sqrt{2})x} + c_2 e^{(1-\sqrt{2})x}$$

21.

Auxillary Equation:

$m^2 - 4m + 5 = 0$

$m = 2 + i, 2 - i$

General Solution: $y = e^{2x}(c_1 \cos x + c_2 \sin x)$

22.

$f''(x) = 6x + 8$

$f'(x) = \int (6x + 8)dx + c$

$f'(x) = 3x^2 + 8x + c$

Since the graph is tangent to the line $3x - y = 2$ at (0,-2):

Get the slope of the line. $3 - y' = 0$, $y' = 3$

$3 = 3(0)^2 + 8(0) + c$, $c = 3$

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f'(x) = $3x^2 + 8x + 3$

f(x) = $\int 3x^2 + 8x + 3dx$ + c'

f(x) = $x^3 + 4x^2 + 3x + c'$

at x = 0, y = - 2

-2 = 0 + 0 + 0 + c'

c' = -2

f(x) = $x^3 + 4x^2 + 3x - 2$

23.

This is a linear D.E.

Solution is:

$$xe^{\int -10dt} = \int e^{\int -10dt} 60e^{4t} dt + c$$

$$xe^{-10t} = \int e^{-10t} 60e^{4t} dt + c$$

$$xe^{-10t} = 60\int e^{-6t} dt + c$$

$$xe^{-10t} = -10e^{-6t} + c$$

$$x = ce^{10t} - 10e^{4t}$$

24.

$$\int_{-4}^{x} dx = \int_0^4 \frac{1}{\sqrt{2t+1}} dt$$

x + 4 = 2 , x = - 2

25.

$y = x^3 - 3x$

$dy/dt = (3x^2 - 3)\, dx/dt$

x = -2 , $dx/dt = \dfrac{1}{\sqrt{2t+1}}$

$$\frac{dx}{dt} = \frac{1}{\sqrt{2(4)+1}} = 1/3$$

and dy/dt = $(3(-2)^2 - 3)(1/3) = 3$

26.

speed = Abs< 3 1/3 > = 3.018

27.

Initial amount of salt = 100 lb

Let x = amount of salt in the tank at any time

dx/dt = rate of entrance of salt – rate of exit.

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dx/dt = 2 lb/gal (5 gal/min) - $\frac{x(5)}{100}$

dx/dt = 10 – x/20 when t = 0, x = 100 lb

dx/dt + x/20 = 10

THE D.E. is a linear D.E. whose solution is

x *IF* = $\int IF \cdot Q(t)dt + C$

where IF = $e^{\int 1/20 dt}$ = $e^{t/20}$

x $e^{t/20}$ = $\int e^{t/20}(10)dt = 200e^{t/20} + c$

x = 200 +c $e^{-t/20}$

when t =0 , x = 100

100 = 200 + c , c = -100

x = 200 – 100 $e^{-t/20}$

when x = 150

150 = 200 – 100$e^{-t/20}$

100$e^{-t/20}$ = 50

$e^{-t/20}$ = 0.5

t = -20 ln(0.5) = 13.863 min

CALCULATOR TECHNIQUE

dx/dt = 10 – x/20 when t = 0, x = 100 lb

$$\frac{dx}{10 - 0.05x} = dt$$

$$\int_{100}^{150} \frac{dx}{10 - 0.05x} = \int_0^t dt$$

t = 13.863 min

28.

The D.E. is a linear ODE

The solution of the ODE dy/dx + yP(x) = Q(x) is

Y *IF* = $\int IF \cdot Q(x)dx + C$

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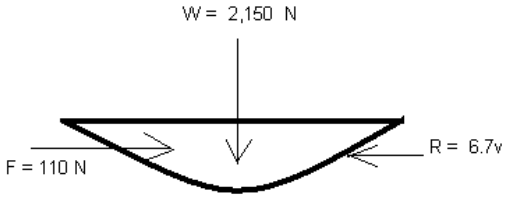
where IF = $e^{\int P(x)dx}$ P(x) = $e^{\int 3/x dx} = e^{3\ln x} = x^3$

$Yx^3 = \int x^3(x^2)dx + c$

$Yx^3 = x^6/6 + C$

$Y = x^3/6 + C/x^3$

29-30



From Newtons Law: F = ma

110 – 6.7v = 2150/9.81 dv/dt

219.164 dv/ (110 – 6.7v) = dt

at the end of 10 sec

$$\int_0^v \frac{219.164}{110-6.7v} = \int_0^{10} dt = 10$$

Using Trial and Error: (Use the choices)

v = 4.32 m/s

To get the distance after 10 sec.

a = vdv/ds = (110- 6.7v)/ 219.164

219.164 v d v /(110 -6.7 v = ds

$$\int_0^{4.32} \frac{219.164vdv}{110-6.7v} = \int_0^s ds = \text{ s}$$

Using Calculator: s = 22.67 m

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PROBLEM SET 24
Vector Calculus

1. A point moves to the curve $y = x^2$, starts at the origin at t= 0 and moves to the right. The distance of the point from the y axis is proportional to the time. At t = 1, the point is at (2,4). The vector that describes the motion is

a. $f(t) = 2ti + 4t^2j$ b. $f(t) = ti + 4t^2j$
c. $f(t) = 2ti + 2t^2j$ d. $f(t) = 2ti + 6t^2j$

2. A point moves on the curve $y = x^3$, starting at the origin at time t = 2, moving to the right and reaching (2,8) at time t = 4. Suppose the slope of the line trough the particle tangent to the path of the particle is proportional to the cube of the elapsed time from the start of motion, what is the vector field that describes the motion?

a. $f(t) = \frac{\sqrt{2}}{2}(t-2)^{3/2}i + \frac{\sqrt{2}}{4}(t-2)^{3/2}j$

b. $f(t) = \frac{\sqrt{2}}{2}(t-2)^{3/2}i + \frac{\sqrt{2}}{2}(t-2)^{3/2}j$

c. $f(t) = \frac{\sqrt{2}}{8}(t-2)^{3/2}i + \frac{\sqrt{2}}{4}(t-2)^{3/2}j$

d. $f(t) = \frac{\sqrt{2}}{16}(t-2)^{3/2}i + \frac{\sqrt{2}}{4}(t-2)^{3/2}j$

3. Find the unit tangent vector to the curve represented by $r(t) = \cos t\, i + \sin t\, j + k$ at the point where $t = \pi/2$

a. –i b. 1 + 2i
c. 2i –j + k d. 0

Problems 4, 5, 6, 7, 8, 9

The motion of the particle is defined by the position vector $r = 5ti + 3t^2j + 1/3t^3 k$ where r is in m and t is in seconds.

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At the instant when t = 2 sec,

4. What is the magnitude of the velocity?

a. 12.3 m/s b. 13.6 m/s

c. 14.3 m/s d. 11.2 m/s

5. What is the tangential acceleration?

a. 6.47 m/s^2 b. 4.45 m/s^2

c. 11.23 m/s^2 d. 10.12 m/s^2

6. What is the normal acceleration?

a. 2.23 m/s^2 b. 4.33 m/s^2

c. 6.12 m/s^2 d. 3.34 m/s^2

7. What is the radius of curvature?

a. 45.34 m b. 58.13 m

c. 11.22 m d. 87.12 m

8. What is the unit tangent vector?

a. < 0.36764 0.8823 0.29412 >

b. < 0.36764 0.9823 0.69412 >

c. < 0.39764 0.1823 0.29412 >

d. < 0.36764 -0.8423 0.23412 >

9. What is the unit normal vector

a. < -0.747 0.0416 0.359 >

b. < -0.747 0.2916 -0.959 >

c. < -0.747 0.0916 0.659 >

d. < -0.747 0.0916 0.659 >

10. A particle starting from rest moves with acceleration vector $a = 4ti - 3t^2j + 6k$ meters. What is the principal radius of curvature when t = 2.

a. 30 m b. 40 m

c. 50 m d. 70 m

11. What is the angle between the tangents to the curve $x = t$, $y = t^2$, $z = t^4$ at the points where t = 1 and t = 2?

a. 24.56^0 b. 22.09^0

c. 11.34^0 d. 18.12^0

Problems 12, 13, 14

The position vector of a particle p is given by $r(t) = t^2i + (2t-1)j + (t - t^2)k$. At t = 2,

12. What is the speed of P

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a. 5.385 b. 4.123
c. 5.234 d. 6.721

13. What is the acceleration of P
a. 2i – 2k b. 2j + 2k
c. 2i – 3j + 4k c. i + j + k

14. What is the angle between velocity and acceleration of P when t = 2?
a. 34.5^0 b. 23.2^0
c. 31.2^0 d. 41.2^0

15. What is the gradient of the function
F = $\phi(x,y,z) = xy^2 + yz^3$ at the point (2, -1,1)?
a. i - 3j – 3k b. 1 + 2i – 4j
c. -i - 3j – 3k d. i - 4j – 3k

16. The vector that is normal to the surface $xy^3z^2 = 4$ at the point (-1, -1, 2) is
a. -4i -12j + 4k b. -4i +12j + 2k
c. -4i -12j -3k d. -2i -12j + 4k

17. What is the divergence of the Vector Field $F = y^2i + 2x^2zj – xyz\ k$ at (1, 1, 4)
a. 1 b. –1
c. 0 d. 3

18. What is the divergence of the vector Field
V = xyz i + $3x^2y$ j + $(xz^2 – y^2z)$ k at (1,3,4)?
a. 13 b. 14 c. 15 d. 16

19. What is the curl of the vector field
V = $y^2i + 2x^2z – xyz\ k$
a. $(- xz – 2x^2)$ i + yz j + (4xz – 2y) k
b. $(- xz – 2y^2)$ i + xz j + (4xz – 2y) k
c. $(- xz – 2x^2)$ i + 3yz j + (4yz – 2y) k
d. $(- 4xz – 2x^2)$ i + 3yz j + (4xz – 2y) k

20. What is the divergence and curl of the vector field
V = $(x^2 + yz)i + (y^2 + xz)j + (z^2 + xy)k$?
a. 2x + 2y + 2z, -i
b. x + y + z, 0
c. 3x – 4y + z, i

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d. x + y + z, 0

21. Find the equation of the tangent plane to the surface $x^2 + 4y^2 – 4z = 0$ at (2,2,5).

a. x + 4y – z = 5

b. 4x – 3y + 5z = 27

c. 4x – 3y + 9z = 47

d. 3x – 4y + z = 7

22. Find the parametric equation of the normal line of the surface $z = xy^2$ at (2,1,2) ?

a. x = 2 – t, y = 1 – 4t , z = 2 + t

b. x = 2 + t, y = t, z = 1 + t

c. x = t , y = t – 1 , z = t – 1

d. x = 4t + 2, y = 2 – 3t, z = t

Problems 23, 24, 25

Find the work done in moving the particle along the arc C if the motion is caused by the force Field F. Assume that arc is measured in meters and force is in Newtons.

(QUESTIONS SIMILAR TO CE BOARD NOV 2009)

23. Force Field = $2xyi + (x^2 + y^2)j$, C is the line segment from the origin to the point (1,1).

a. 3/4 J b. 4/3 J

d. 5/3 J d. 8/3 J

24. Force Field = $2x^2y\, i + (x^2 + 3y)j$

a. 1568/15 J b. 2134/123 J

c. 1123/15 J d. 4134/115 J

25. Force Field = $xyz\, i + e^y j + (x + z)k$

C: $R(t) = 3ti + t^2j + 2tk \quad 0 <= t <= 3$

a. 9123.45 J b. 9021.8 J

c. 8133.41 J c. 7114.5 J

26. Find the length of the arc of the curve whose parametric equation is given by $x = t^2$, $y = t^3$ $z = t^4$ from t = 0 to t = 1.

a. 1.234 b. 1.792

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c. 1.556 d. 1.443

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Answers.

1.

Let f(t) = xi + yj

distance from the y axis is x(t) = kt

then: $y(t) = k^2t^2$

And $f(t) = kti + k^2t^2j$

$f(1) = ki + k^2j = 2 + 4i$

then k = 2

and $f(t) = 2ti + 4t^2j$

2.

The vector field is f(t) = xi + yj

Let $y = x^3$

$y' = \text{slope} = 3x^2 = k(t-2)^3$

then $x^2 = k/3(t-2)^3$

$x = \sqrt{\frac{k}{3}}(t-2)^{3/2}$ and $y = x^3 = (\sqrt{\frac{k}{3}}(t-2)^{3/2})^3$

when t = 4, x = 2 and y = 8

Thus $2 = \sqrt{\frac{k}{3}}\ (4-2)^{3/2}$

$$\sqrt{\frac{k}{3}} = \frac{2}{2^{3/2}} \qquad k = 3/2$$

Then $f(t) = xi + yj = \sqrt{\frac{k}{3}}(t-2)^{3/2}i + (\sqrt{\frac{k}{3}}(t-2)^{3/2})^3 j$

$$f(t) = \frac{\sqrt{2}}{2}(t-2)^{3/2}i + \frac{\sqrt{2}}{4}(t-2)^{3/2}j$$

3.

For unit Tangent:

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T = $\frac{r'(t)}{|r'(t)|}$ when t = 2

r = cos t i + sin t j + k

r' = -sin t i + cost t j

r'(π/2) = -1 i + 0j = -1i

Abs< -1 0 > = 1

Thus unit tangent = -1i/1 = -i

4.

r'(t) = 5i + 6tj + t^2k

v = r'(2) = 5i + 12j + 4k

velocity = Abs< 5 12 4 > = 13.6 m/s

5.

a = r''(t) = 0i + 6j + 2tk

a = r''(2) = 0i + 6j + 4k

acceleration when t = 2 is Abs < 0 6 4 > = 7.21 m/s^2

Unit tangent = $\frac{r'(t)}{|r'(t)|}$ = $\frac{5i+12j+4k}{13.6}$

=< 0.36764 0.8823 0.29412 >

Tangential Acceleration =

(0i + 6j + 4k) dot < 0.36764 0.8823 0.29412 >

= 6.47 m/s^2

6.

a_t^2 + a_n^2 = a^2

6.47^2 + a_n^2 = 7.21^2

a_n = 3.182 m/s^2

7.

$a_n = \frac{v^2}{\rho}$ 3.182 = $\frac{13.6^2}{\rho}$

ρ = 58.13 m

8. From (6)

Unit tangent vector = < 0.36764 0.8823 0.29412 >

9.

a = a_t T + a_n N

a = < 0 6 4 >

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a_t = 6.47

a_n = 3.182

T = < 0.36764 0.8823 0.29412 >

N = ?

< 0 6 4 > = 6.47 < 0.36764 0.8823 0.29412 >
+ 3.182 < N >

Let VctA = < 0 6 4 >

VctB = < < 0.36764 0.8823 0.29412 >

Then:

VctA = 6.47 Vct B + 3.182 N

Then: $N = \frac{VctA - 6.47 VctB}{3.182}$

N = < -0.747 0.0916 0.659 >

10.

acceleration when t = 2 is a = 8i – 12j + 6k

acceleration = Abs < 8 -12 6 > = 15.62

$a_n = \frac{v^2}{\rho}$

a = dv/dt

dv = adt

dv =($4ti - 3t^2j + 6k$) dt

$v = \int (4ti - 3t^2 j + 6k)dt + c$

$v = 2t^2i - t^3j + 6tk + c$

when t = 0 , v =0 c = 0

Then: $v = 2t^2i - t^3j + 6tk$

When t = 2, v = 8i – 8j + 12 k

Abs< v > = Abs < 8 -8 12 > = 16.492

Tangent Vector = < 8 - 8 12 >

Unit Tangent = < 8 -8 12 > / 16.492

= < 0.485 -0.485 0.7276 >

Then tangential acceleration

a_t = <8i – 12j + 6k > dot < 0.485 -0.485 0.7276 >

= 14.0656

$a_n^2 = a^2 - a_t^2$

$= 15.62^2 - 14.0656^2$

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$= 6.793 \text{ m/s}^2$

$a_n = \frac{V^2}{\rho}$

$6.793 = 16.492^2 / \rho$

$\rho = 40$ m

11.

The tangent vector when t = 1.

$V(t) = ti + t^2j + t^4k$

$V'(t) = 1i + 2tj + 4t^3k$

$V'(1) = 1i + 2i + 4k$

The tangent Vector when t = 2

$V'(2) = 1i + 4j + 4(2)^3k$

$= 1i + 4j + 32k$

Let VctA = < 1 2 4 > and VctB = < 1 4 32 >

VctA dot VctB = Abs(VctA) Abs(Vctb) cos θ

$\cos \theta = 22.09^0$

12.

Speed of P.

dr/dt = 2ti + 2j + (1 -2t) k

when t = 2.

dr/dt = 4i + 2j –3 k

speed = Abs < 4 2 -3 > = 5.385

13.

a = r''(t) = 2i – 2k

14.

The velocity vector = < 4 2 -3 > = VctA

The acceleration vector = < 2 0 -2 > = VctB

VctA dot Vct B = Abs(VctA)(VctB) cos θ

$\theta = 23.2^0$

15.

(Fx means partial derivative with respect to x)

$Fx = y^2$

$Fy = 2xy + z^3$

$Fz = 3z^2y$

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then $\nabla\phi =$ y^2 i + $(2xy + z^3)$j + $3z^2y$ k

at point (2, -1,1) $\nabla\phi =$ i - 3j – 3k

16.

Let $F = xy^3z^2 - 4$

(Note Fx means partial derivative with x)

Then $Fx = y^3z^2$ $Fy = 3xy^2 z^2$ and $Fz = 2xy^3z$

Then grad(F) = $\nabla F = y^3z^2$ i + $3xy^2$ z j + $2xy^3z$ k at the point (x = -1, y = -1 and z = 2)

∇F = -4i -12j + 4k

17.

Divergence = $F_x + F_y + F_z$

= 0 + 0 + - xy = - xy

at (1, 1, 4) Divergence = - 1(1) = -1

18.

V = xyz i + $3x^2y$ j + $(xz^2 - y^2z)$ k

Divergence F = Fx + Fy + Fz = yz + $3x^2$ + $(2xz - y^2)$

= 12 + 3 -1 = 14

19.

(Note: F_1x is the partial derivative of F_1 with respect to x)

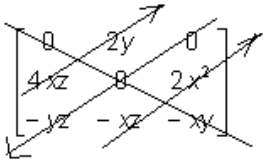
$F_1 = y^2$ $F_2 = 2x^2z$ F_3 = -xyz

F_1x = 0 $F_2x = 4xz$ F_3x = -yz

$F_1y = 2y$ $F_2y = 0$ F_3y = -xz

$F_1z = 0$ $F_2z = 2x^2$ F_3 z = -xy

Jacobi Matrix:



(Note) The trace (sum of the elements in the principal diagonal)

= -xy

and The curl (difference in the elements located symmetrically with respect to the principal diagonal in the direction shown)

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Curl = $(- xz – 2x^2) i + yz j + (4xz – 2y) k$

20.

$F1 = x^2 + yz$ $F2 = y^2 + xz$ $F3 = z^2 + xy$

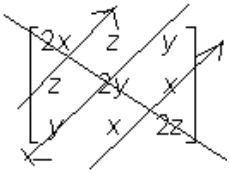
F1x = 2x F2x = z F3x = y

F1y = z F2y = 2y F3y = x

F1z = y F2z = x F3z = 2z

Then:

Jacobi Matrix =



Divergence = 2x + 2y + 2z

Curl = (x – x)i + (y – y)j + (z – z)k

= 0

21.

Let $F(x,y,z) = x^2 + 4y^2 – 4z$

Gradient F = Fx i+ Fyj+ Fz k is the direction number of the normal line to the curve at (2,2,5)

Fx =2x Fy = 8y and Fz = -4

Gradient F = 2xi + 8yj – 4k

at (2,2,5) Gradient F = 4i + 16j – 4k

Equation of the Plane:

4 (x – 2) + 16(y – 2) - 4(z – 5) = 0

4x + 16y – 4z = 20

or x + 4y – z = 5

22.

$F = z – xy^2$

Fx = $-y^2$ Fy = -2xy Fz = 1

Gradient F = $-y^2$i – 2xyj + k

at (2,1,2) Gradient F = -i – 4j + k

Direction Number of the Line = < -1 -4 1 >

$\frac{x-2}{-1} = \frac{y-1}{-4} = \frac{z-2}{1}$ = t

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Then $x = 2 - t$, $y = 1 - 4t$, and $z = 2 + t$

23.

$W = \int_A^B F \cdot dR$

Get the Parametric Equation of R.

(0,0) to (1,1) $x = t$, $y = t$ $t = [0\ \ 1]$

$R = xi + yj$

$R = ti + tj$ $dR = (1i + 1j)dt$

$F = 2xyi + (x^2 + y^2)j$

$= 2t(t)i + (t^2 + t^2)j$

$= 2t^2i + 2t^2j$

F dot dR = $(2t^2i + 2t^2j)$ dot $(1i + 1j)$

$= 2t^2 + 2t^2 = 4t^2$

$W = \int_0^1 4t^2 dt$ = 4/3 Joulles

24.

C is the arc of the parabola $y = 3x^2 + 2x + 4$ from (0,4) to (1,9).

$R = xi + yj$

Transform C into R .

$y = 3x^2 + 2x + 4$

Let $x = t$

$y = 3t^2 + 2t + 4$

$R = ti + (3t^2 + 2t + 4)j$ $0 < t < 1$

$dR = (1i + (6t + 2)j)dt$

$F = 2t^2(3t^2 + 2t + 4)\,i + [t^2 + 3(3t^2 + 2t + 4)]\,j$

F dot dR = $2t^2(3t^2 + 2t + 4)1 + [t^2 + 3(3t^2 + 2t + 4)](6t + 2)\ dt$

W =

$\int_0^1 2t^2(3t^2 + 2t + 4)1 + [t^2 + 3(3t^2 + 2t + 4)](6t + 2)\ dt$

= 1568/15 J

25. $R(t) = 3ti + t^2j + 2tk$

$R'(t) = 3i + 2tj + 2k$

Since $R = xi + yj + zk$ $= 3ti + t^2j + 2tk$

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then: $x = 3t$, $y = t^2$ and $z = 2t$

Force Field $= (3t)(t^2)(2t)i + e^{t^2} j + (3t + 2t) k$

$= 6t^4 i + e^{t^2} j + (5t) k$

F dot dR $= [6t^4 i + e^{t^2} j + (5t) k]$ dot $[3i + 2tj + 2k]\ dt$

$= 18t^4 + 2te^{t^2} + 10t\ dt$

$$W = \int_0^3 (18t^4 + 2te^{t^2} + 10t)\ dt = \mathbf{9021.8\ Joulles}$$

26.

$$\text{Length} = \int_0^t \sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2}\, dt$$

$$= \int_0^t \sqrt{(2t)^2 + (3t^2)^2 + (4t^3)^2}\, dt$$

USE CALCULATOR:

= 1.792 units

PROBLEM SET 25
LAPLACE TRANSFORMS and SERIES

1. Find the Laplace Transform of $f(t) = t^3$

a. $3/s^4$ b. $6/s^4$

c. $12/s^4$ c. $8/s^4$

2. Find the Laplace Transform of cos 5t

a. $s/(s^2 + 5)$ b. $s/(s^2 + 25)$

c. $1/(s^2 + 5)$ d. $1/(s^2 + 25)$

3. Find the Laplace Transform of e^{3t} cos 2t

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a. $(s – 3)/((s -3)^2 + 4)$ b. $((s + 3)/((s + 3)^2 + 4)$
c. $3/(s^2 + 4)$ d. $3s/(s^2 + 4)$

4. Find the Inverse Laplace Transform of $3/(s^2 – 16)$
a. 3 sinh 4t b. ¾ sinh 4t
c. 4 sinh 3t c. 4/3 sinh 3t

5. Find the Laplace Transform of tsint
a. $2s/(s^2 + 1)^2$ b. $s/(s^2 + 1)^2$
c. $1/(s^2 + 1)^2$ d. $2/(s^2 + 1)^2$

6. Inverse Laplace of $3/s^5$
a. $1/6\ t^4$ b. $1/8t^4$
c. $1/3t^4$ d. $1/12t^4$

7. Inverse Laplace Transform of $\frac{1}{s(s+3)}$
a. $1/3 – 1/3e^{-3t}$ b. $1/3 + 1/3e^{3t}$
c. $1/5 + 1/5e^{3t}$ d. $3 – 3\ e^{-3t}$

8. $1 - x + x^2/2! - x^3/3! + \ldots$ is the same as.
a. e^{-x} b. e^{2x}
c. ln x d. e^x

9. $x - x^3/3! + x^5/5! - x^7/7! + \ldots$ is
a. cos x b. sin x
c. tanx d. sec x

10. $1 - x^2/2! + x^4/4! – x^6/6! + \ldots$
a. cos x b. sin x
c. tanx d. sec x

11. What is the 4^{th} derivative of $1/(1 + x^2)$ when x = 0?
a. 24 b. 0
c. 12 d. -2
.

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1. $L(t^n) = n!/s^{n+1} = 3!/s^3 = 6/s^4$
2. $L(\cos at) = s/(s^2 + a^2) = s/(s^2 + 25)$
3. $L(e^{3t}\cos 2t) = L(\cos 2t)_{s = s-3} = (s-3)/[(s-3)^2 + 4$
4. $L^{-1}(3/(s^2 - 16)) = \frac{3}{4}\sinh 4t$
5. $L(t\sin t) = -\frac{d}{ds}(L\sin t) = -\frac{d}{ds}(1/(s^2+1)) = 2s/(s^2+1)^2$
6. $L^{-1}(3/s^5) = 3L^{-1}(1/s^5) = 3/4!\, t^4 = 1/8t^4$
7. $L^{-1}\dfrac{1}{s(s+3)} = L^{-1}\,A/s + L^{-1}\,B/(s+3) = 1/3 - 1/3e^{-3t}$

$1 = A(s+3) + Bs$

$s = 0 \quad A = 1/3$

$s = -3 \quad B = -1/3$

8. $e^x = 1 + x + x^2/2! + x^3/3! + \ldots.$
   Then $e^{-x} = 1 - x + x^2/2! + x^3/3! + \ldots.$ Ans.
9. sin x
10. cos x
11.

Divide $1/(1 + x^2)$

$1/(1+x^2) = 1 - x^2 + x^4 - x^6 + x^8 + \ldots$

1st derivative $-2x + 4x^3 - 6x^5 + 8x^7$ $x = 0$, deriv= 0

2nd derivative $-2 + 12x^2 - 30x^4 + 56x^6$ $x = 0$ deriv = 0

3rd derivative $0 + 24x - 120x^3 + 336x^5$ $x = 0$ deriv =

4th derive $24 + \ldots..$ Ans. 24

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