



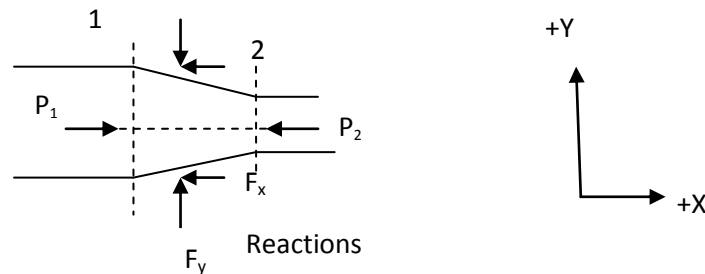
9 momentum-principle tutorial-solution

Civil Engineering (De La Salle University – Dasmariñas)

Tutorial 9

Application of Momentum Principle

1. A 60cm pipe is connected to a 30cm pipe by a standard reducer fitting. For the same flow of $0.9 \text{ m}^3/\text{s}$ of water and a pressure of 200Kpa, what force is exerted by the water on the reducer, neglecting any lost head?



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4} \times 0.6^2 = 0.2827 \text{ m}^2$

Diameter at section 2 (d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4} \times 0.3^2 = 0.07068 \text{ m}^2$

Discharge (Q) = $0.9 \text{ m}^3/\text{s}$

Velocity at section 1 (V_1) = $Q/A_1 = 0.9/0.2827 = 3.18 \text{ m/s}$

Velocity at section 2 (V_2) = $Q/A_2 = 0.9/0.07068 = 12.73 \text{ m/s}$

Pressure at section 1 (P_1) = 200Kpa = 200000 N/m^2

Force exerted by water on reducer = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{200000}{1000 \times 9.81} + \frac{3.18^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{12.73^2}{2 \times 9.81}$$

$$P_2 = 124030 \text{ N/m}^2$$

\sum Forces in X direction = Rate of change of momentum in X direction

$$(P_1 A_1 - P_2 A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 - P_2 A_2) - F_x = \rho Q (V_2 - V_1)$$

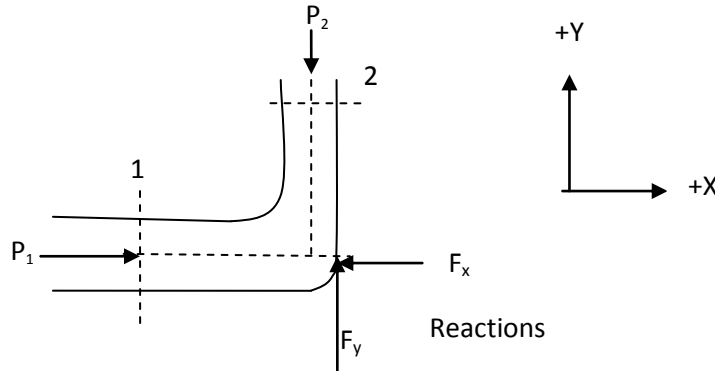
$$F_x = (P_1 A_1 - P_2 A_2) + \rho Q (V_1 - V_2)$$

$$= (200000 \times 0.2827 - 124030 \times 0.07068) + 1000 \times 0.9 (3.18 - 12.73) = 39178 \text{ N}$$

The forces in the Y-direction will balance each other and $F_y = 0$.

Hence the force exerted by water on reducer is 39178 N to the right.

2. A 500mm pipe carrying 0.8 m³/s of oil (sp gr 0.85) has a 90° bend in a horizontal plane. The loss of head in the bend is 1.1m of oil, and the pressure at the entrance is 290KPa. Determine the resultant force exerted by the oil on the bend.



Solution:

Diameter at section 1 and 2 (d) = 500mm = 0.5m

Area at section 1 and 2 ($A_1 = A_2$) = $\frac{\pi}{4} \times 0.5^2 = 0.19635\text{m}^2$

Discharge (Q) = 0.8 m³/s

Velocity at 1 and 2 ($V_1 = V_2$) = $Q/A_1 = 0.8/0.19635 = 4.07\text{m/s}$

Pressure at 1 (P_1) = 290KPa = 290000 Pa

Loss of head (h_L) = 1.1m

Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{290000}{0.85 \times 1000 \times 9.81} + \frac{4.07^2}{2 \times 9.81} = \frac{P_2}{0.85 \times 1000 \times 9.81} + \frac{4.07^2}{2 \times 9.81} + 1.1$$

$$P_2 = 280828 \text{ N/m}^2$$

\sum Forces in X direction = Rate of change of momentum in X direction

$$P_1 A_1 - F_x = \rho Q (V_{2x} - V_{1x})$$

$$P_1 A_1 - F_x = \rho Q (0 - V_1)$$

$$F_x = P_1 A_1 + \rho Q V_1$$

$$= 290000 \times 0.19635 + 0.85 \times 1000 \times 0.8 \times 4.07$$

$$= 59709 \text{ N}$$

\sum Forces in Y direction = Rate of change of momentum in Y direction

$$F_y - P_2 A_2 = \rho Q (V_{2y} - V_{1y})$$

$$F_y - P_2 A_2 = \rho Q (V_2 - 0)$$

$$F_y = P_2 A_2 + \rho Q V_2$$

$$= 280828 \times 0.19635 + 0.85 \times 1000 \times 0.8 \times 4.07$$

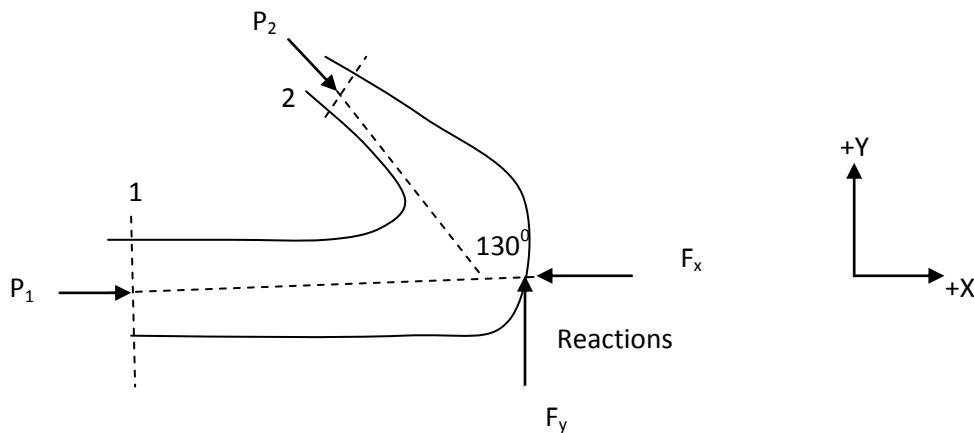
$$= 57908 \text{ N}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 83177 \text{ N}$$

Resultant force exerted by the water on the bend = 83177 N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{57908}{59709} = 44.1^\circ$$

3. The discharge of water through a 130° bend is 30 litres/s. The bend is lying in the horizontal plane and the diameters at the entrance and exit are 200mm and 100mm respectively. The pressure measured at the entrance is 100 kN/m², what is the magnitude and direction of the force exerted by the water on the bend?



Solution:

$$\text{Diameter at section 1 } (d_1) = 200 \text{ mm} = 0.2 \text{ m}$$

$$\text{Area at section 1 } (A_1) = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Diameter at section 2 } (d_2) = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{Area at section 2 } (A_2) = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

$$\text{Discharge } (Q) = 30 \text{ lps} = 0.03 \text{ m}^3/\text{s}$$

$$\text{Velocity at section 1 } (V_1) = Q/A_1 = 0.03/0.0314 = 0.95 \text{ m/s}$$

$$\text{Velocity at section 2 } (V_2) = Q/A_2 = 0.03/0.00785 = 3.82 \text{ m/s}$$

$$\theta = 180 - 130 = 50^\circ$$

$$\text{Pressure at 1 } (P_1) = 100 \text{ kPa} = 100000 \text{ Pa}$$

Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{100000}{1000 \times 9.81} + \frac{0.95^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{3.82^2}{2 \times 9.81}$$

$$P_2 = 93155 \text{ N/m}^2$$

Resultant force (F_R) = ?

Direction of resultant force = ?

Σ Forces in X direction = Rate of change of momentum in X direction

$$(P_1 A_1 + P_2 \cos\theta A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 + P_2 A_2 \cos\theta) - F_x = \rho Q (-V_2 \cos\theta - V_1)$$

$$F_x = (P_1 A_1 + P_2 A_2 \cos\theta) + \rho Q (V_1 + V_2 \cos\theta)$$

$$= (100000 \times 0.0314 + 93155 \times 0.00785 \cos 50) + 1000 \times 0.03 (0.95 + 3.82 \cos 50)$$

$$= 3712 \text{ N}$$

Σ Forces in Y direction = Rate of change of momentum in Y direction

$$F_y - P_2 \sin\theta A_2 = \rho Q (V_{2y} - V_{1y})$$

$$F_y - P_2 A_2 \sin\theta = \rho Q (V_2 \sin\theta - 0)$$

$$F_y = P_2 A_2 \sin\theta + \rho Q V_2 \sin\theta$$

$$= 93155 \times 0.00785 \sin 50 + 1000 \times 0.03 \times 3.82 \sin 50$$

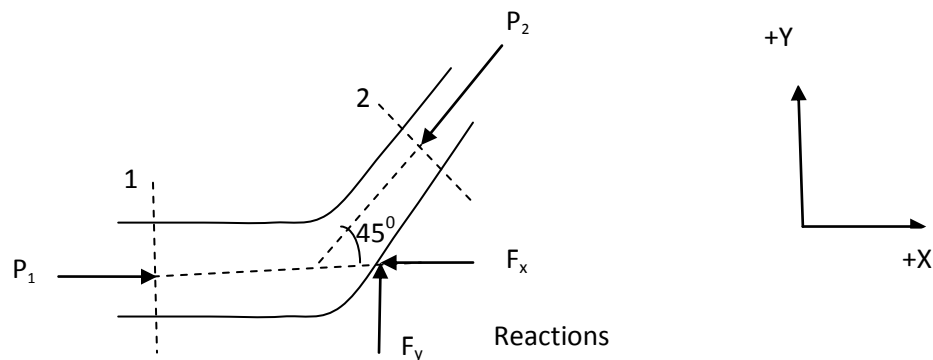
$$= 648 \text{ N}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 3768 \text{ N}$$

Resultant force exerted by the water on the bend = 3768 N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{648}{3712} = 10^\circ$$

4. A 45° pipe bend tapers from 600mm diameter at inlet to 300mm diameter at outlet. The pressure at inlet is 140 kN/m^2 and the rate of flow is $0.425 \text{ m}^3/\text{s}$. At outlet the pressure is 123 kN/m^2 gauge. Neglecting friction, calculate the resultant force exerted by the water on the bend in magnitude and direction. The bend lies in a horizontal plane.



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4} \times 0.6^2 = 0.2827\text{m}^2$

Diameter at section 2 (d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4} \times 0.3^2 = 0.07068\text{m}^2$

Discharge (Q) = 0.425 m³/s

Velocity at section 1 (V_1) = $Q/A_1 = 1.5$ m/s

Velocity at section 2 (V_2) = $Q/A_2 = 6.01$ m/s

Pressure at section 1 (P_1) = 140kN/m² = 140000N/m²

Pressure at section 2 (P_2) = 123kN/m² = 123000N/m²

Angle of bend (θ) = 45°

Resultant force (F_R) = ?

Direction of resultant force = ?

\sum Forces in X direction = Rate of change of momentum in X direction

$$(P_1 A_1 - P_2 \cos\theta A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 - P_2 A_2 \cos\theta) - F_x = \rho Q (V_2 \cos\theta - V_1)$$

$$F_x = (P_1 A_1 - P_2 A_2 \cos\theta) + \rho Q (V_1 - V_2 \cos\theta)$$

$$= (140000 \times 0.2827 - 123000 \times 0.07068 \cos 45) + 1000 \times 0.425 (1.5 - 6.01 \cos 45)$$

$$= 32262 \text{ N}$$

\sum Forces in Y direction = Rate of change of momentum in Y direction

$$F_y - P_2 \sin\theta A_2 = \rho Q (V_{2y} - V_{1y})$$

$$F_y - P_2 A_2 \sin\theta = \rho Q (V_2 \sin\theta - 0)$$

$$F_y = P_2 A_2 \sin\theta + \rho Q V_2 \sin\theta$$

$$= 123000 \times 0.07068 \sin 45 + 1000 \times 0.425 \times 6.01 \sin 45$$

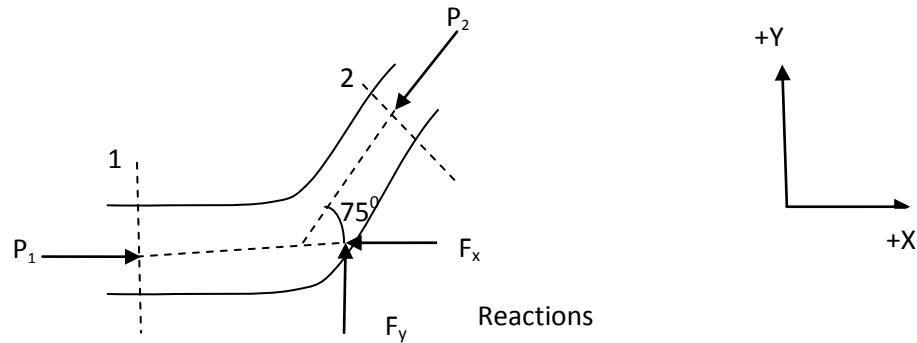
$$= 7953 \text{ N}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 33228 \text{ N}$$

Resultant force exerted by the water on the bend = 33228N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{7953}{32262} = 13.8^\circ$$

5. A 150mm diameter pipe on the horizontal plane carries water under the head of 16m of water with the velocity of 3.5 m/s. Find the direction and magnitude of the pipe bend, if the axis of the bend was turned with angle 75° . Assume no loss of energy at the pipe bend.



Solution:

Diameter ($d_1 = d_2$) = 150mm = 0.15m

Area at section 1 and 2 ($A_1 = A_2$) = $\frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$

Velocity ($V_1 = V_2$) = 3.5 m/s

Discharge (Q) = $A_1 V_1 = 0.01767 \times 3.5 = 0.0618 \text{ m}^3/\text{s}$

Pressure head = $\frac{P}{\rho g} = 16 \text{ m}$

$P = 156960 \text{ N/m}^2$

Pressure ($P = P_1 = P_2$) = 156960 N/m^2

Angle of bend (θ) = 75°

Resultant force (F_R) = ?

Direction of resultant force = ?

$\sum \text{Forces in X direction} = \text{Rate of change of momentum in X direction}$

$$(P_1 A_1 - P_2 \cos \theta A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 - P_2 A_2 \cos \theta) - F_x = \rho Q (V_2 \cos \theta - V_1)$$

$$F_x = (P_1 A_1 - P_2 A_2 \cos \theta) + \rho Q (V_1 - V_2 \cos \theta)$$

$$= (156960 \times 0.01767 - 156960 \times 0.01767 \cos 75^\circ) + 1000 \times 0.0618 (3.5 - 3.5 \cos 75^\circ)$$

$$= 2216 \text{ N}$$

$\sum \text{Forces in Y direction} = \text{Rate of change of momentum in Y direction}$

$$F_y - P_2 \sin \theta A_2 = \rho Q (V_{2y} - V_{1y})$$

$$F_y - P_2 A_2 \sin \theta = \rho Q (V_2 \sin \theta - 0)$$

$$F_y = P_2 A_2 \sin \theta + \rho Q V_2 \sin \theta$$

$$= 156960 \times 0.01767 \sin 75^\circ + 1000 \times 0.0618 \times 3.5 \sin 75^\circ$$

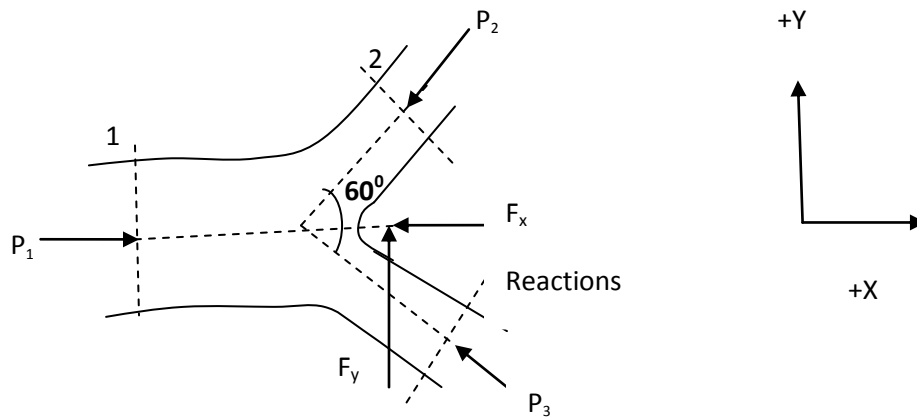
= 2888N

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 3640\text{N}$$

Resultant force exerted by the water on the bend = 3640N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{2888}{2216} = 52.5^\circ$$

6. A main pipe of diameter 500mm branches in two pipes of diameter 300mm each in the horizontal plane. Angle between the branches is 60° , which is symmetrical with respect to the main pipe. Flow discharge through the main pipe is $1.0 \text{ m}^3/\text{s}$, which is equally divided into the branch pipes. If the pressure intensity at the main pipe is 400KPa, find the magnitude and direction of resultant force in the bend. Assume no loss of energy due to branch junction and in pipe sections.



Diameter at section 1 (d_1) = 500mm = 0.5m

$$\text{Area at section 1 } (A_1) = \frac{\pi}{4} \times 0.5^2 = 0.19635\text{m}^2$$

Diameter at section 2 and 3 ($d_2 = d_3$) = 300mm = 0.3m

$$\text{Area at section 2 and 3 } (A_2 = A_3) = \frac{\pi}{4} \times 0.3^2 = 0.07068\text{m}^2$$

Angle of bend (θ) = 30° for pipe 2 and 3

Discharge through 1 (Q_1) = $1 \text{ m}^3/\text{s}$

Discharge through 2 and 3 ($Q_2 = Q_3$) = $Q_1/2 = 0.5 \text{ m}^3/\text{s}$

Pressure at 1 (P_1) = 400 KPa = $400000\text{N}/\text{m}^2$

Resultant force (F_R) = ?

Direction of resultant force = ?

Velocity at 1 (V_1) = $Q_1/A_1 = 5.09 \text{ m/s}$

Velocity at 2 (V_2) = $Q_2/A_2 = 7.07 \text{ m/s}$

Velocity at 3 (V_3) = $Q_3/A_3 = 7.07 \text{ m/s}$

Using Bernoulli's equation at 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{400000}{1000 \times 9.81} + \frac{5.09^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{7.07^2}{2 \times 9.81}$$

$$P_2 = 387962 \text{ N/m}^2$$

Using Bernoulli's equation at 1 and 3 ($Z_1=Z_3$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{400000}{1000 \times 9.81} + \frac{5.09^2}{2 \times 9.81} = \frac{P_3}{1000 \times 9.81} + \frac{7.07^2}{2 \times 9.81}$$

$$P_3 = 387962 \text{ N/m}^2$$

\sum Forces in X direction = Rate of change of momentum in X direction

$$\begin{aligned} (P_1 A_1 - P_2 \cos\theta A_2 - P_3 \cos\theta A_3) - F_x &= \rho [(Q_2 V_{2x} + Q_3 V_{3x}) - Q_1 V_{1x}] \\ (P_1 A_1 - P_2 A_2 \cos\theta - P_3 A_3 \cos\theta) - F_x &= \rho [(Q_2 V_2 \cos\theta + Q_3 V_3 \cos\theta) - Q_1 V_1] \\ F_x &= (P_1 A_1 - P_2 A_2 \cos\theta - P_3 A_3 \cos\theta) + \rho (Q_1 V_1 - Q_2 V_2 \cos\theta - Q_3 V_3 \cos\theta) \\ &= (400000 \times 0.19635 - 387962 \times 0.07068 \cos 30 - 387962 \times 0.07068 \cos 30) + 1000(1 \times 5.09 - \\ &0.5 \times 7.07 \cos 30 - 0.5 \times 7.07 \cos 30) \\ &= 30012 \text{ N} \end{aligned}$$

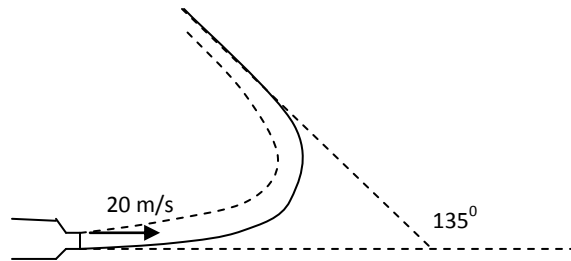
\sum Forces in Y direction = Rate of change of momentum in Y direction

$$\begin{aligned} F_y - P_2 \sin\theta A_2 + P_3 \sin\theta A_3 &= \rho [(Q_2 V_{2y} + Q_3 V_{3y}) - Q_1 V_{1y}] \\ F_y - P_2 A_2 \sin\theta + P_3 A_3 \sin\theta &= \rho [(Q_2 V_2 \sin\theta + Q_3 V_3 \sin\theta) - 0] \\ F_y &= (P_2 A_2 \sin\theta - P_3 A_3 \sin\theta) + \rho (Q_2 V_2 \sin\theta - Q_3 V_3 \sin\theta) \\ &= 387962 \times 0.07068 \sin 30 - 387962 \times 0.07068 \sin 30 + 1000 \times 0.5 \times 7.07 \sin 30 - 1000 \times 0.5 \times 7.07 \sin 30 \\ &= 0 \end{aligned}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 30012 \text{ N}$$

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{0}{30012} = 0^\circ$$

7. Determine the magnitude of resultant force and its direction on the vane shown in the figure below if a water jet of 50mm diameter and 20m/s velocity strikes the vane tangentially and deflects without friction.



Solution:

Velocity (V) = 20m/s

Diameter of jet (d) = 50mm = 0.05m

C/s of jet (A) = $\frac{\pi}{4} \times 0.05^2 = 0.001963 \text{ m}^2$

Discharge (Q) = AV = 0.001963 x 20 = 0.03927 m³/s

No friction: No loss of head

Pressure is atmospheric: so no pressure force

V is constant throughout.

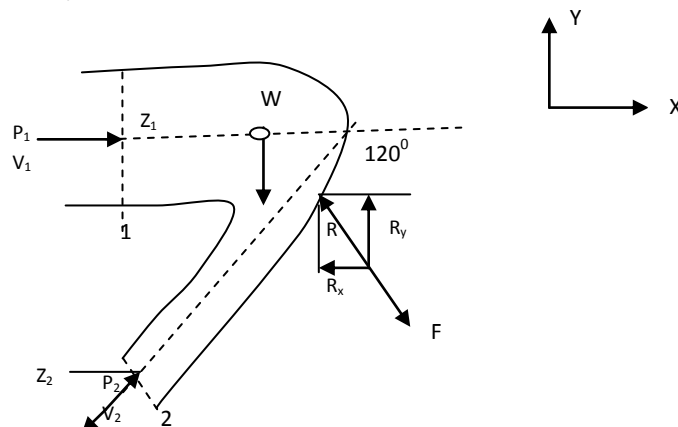
$$F_x = \rho Q(V_{1x} - V_{2x}) = 1000 \times 0.03927(20 - (-20 \cos 45)) = 1340.8 \text{ N}$$

$$F_y = \rho Q(V_{1y} - V_{2y}) = 1000 \times 0.03927(0 - 20 \sin 45) = 555.4 \text{ N}$$

$$\text{Resultant force} = \sqrt{F_x^2 + F_y^2} = \sqrt{1340.8^2 + 555.4^2} = 1451 \text{ N}$$

$$\text{Direction} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{555.4}{1340.8} = 22.5^\circ$$

8. The diameter of a pipe bend is 30cm at inlet and 15cm at outlet and the flow is turned through 120° in a vertical plane. The axis at inlet is horizontal and the center of the outlet section is 1.5m below the center of the inlet section. Total volume of water in the bend is 0.9m³. Neglecting friction, calculate the magnitude and direction of the force exerted on the bend by water flowing through it at 250lps and when the inlet pressure is 0.15N/mm².



Solution:

Diameter at section 1 (d_1) = 30cm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4} \times 0.3^2 = 0.07068\text{m}^2$

Diameter at section 2 (d_2) = 15cm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4} \times 0.15^2 = 0.01767\text{m}^2$

Discharge (Q) = 250 lps = 0.25 m³/s

Volume of water within control volume (Vol) = 0.9m³

Weight of water within control volume (W) = $\gamma_{\text{water}} \text{Vol} = 9810 \times 0.9 = 8829\text{N}$

Velocity at section 1 (V_1) = $Q/A_1 = 0.25/0.07068 = 3.54 \text{ m/s}$

Velocity at section 2 (V_2) = $Q/A_2 = 0.25/0.01767 = 14.15 \text{ m/s}$

$Z_2 = 0, Z_1 = 1.5\text{m}$

$\theta = 180 - 120 = 60^\circ$

Pressure at 1 (P_1) = $0.15\text{N/mm}^2 = 0.15 \times 10^6 \text{ N/m}^2$

Resultant force exerted by the water on the bend = ?

Applying Bernoulli's equation between 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{0.15 \times 10^6}{1000 \times 9.81} + \frac{3.54^2}{2 \times 9.81} + 1.5 = \frac{P_2}{1000 \times 9.81} + \frac{14.15^2}{2 \times 9.81} + 0$$
$$P_2 = 70870 \text{ N/m}^2$$

$\sum \text{Forces in X direction} = \text{Rate of change of momentum in X direction}$

$$(P_1 A_1 + P_2 \cos \theta A_2) - R_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 + P_2 A_2 \cos \theta) - R_x = \rho Q (-V_2 \cos \theta - V_1)$$

$$R_x = (P_1 A_1 + P_2 A_2 \cos \theta) + \rho Q (V_1 + V_2 \cos \theta)$$

$$= (0.15 \times 10^6 \times 0.07068 + 70870 \times 0.01767 \cos 60) + 1000 \times 0.25 (3.54 + 14.15 \cos 60)$$

$$= 13882 \text{ N}$$

$\sum \text{Forces in Y direction} = \text{Rate of change of momentum in Y direction}$

$$R_y - P_2 \sin \theta A_2 - W = \rho Q (V_{2y} - V_{1y})$$

$$R_y - P_2 A_2 \sin \theta - W = \rho Q (-V_2 \sin \theta - 0)$$

$$R_y = P_2 A_2 \sin \theta - \rho Q V_2 \sin \theta + W$$

$$= 70870 \times 0.01767 \sin 60 - 1000 \times 0.25 \times 14.15 \sin 60 + 8829$$

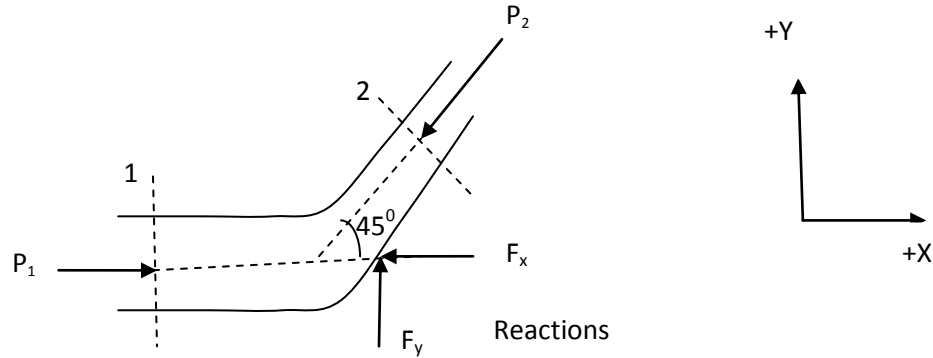
$$= 6850 \text{ N}$$

$$\text{Resultant force (R)} = \sqrt{R_x^2 + R_y^2} = 15481 \text{ N}$$

Resultant force exerted by the water on the bend = 15481N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{6850}{13882} = 26.3^\circ$$

9. A 45° reducing bend is connected in a pipe line carrying water. The diameter at inlet and outlet of the bend is 400mm and 200mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of the bend is 215.8KN/m^2 . The rate of flow of water is $0.5\text{m}^3/\text{s}$. The loss of head in the bend is 1.25m of oil of sp.gr. 0.85 .



Solution:

Diameter at section 1 (d_1) = 400mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4} \times 0.6^2 = 0.2826\text{m}^2$

Diameter at section 2 (d_2) = 200mm = 0.2m

Area at section 2 (A_2) = $\frac{\pi}{4} \times 0.2^2 = 0.0314\text{m}^2$

Discharge (Q) = $0.5\text{ m}^3/\text{s}$

Velocity at section 1 (V_1) = $Q/A_1 = 1.77\text{ m/s}$

Velocity at section 2 (V_2) = $Q/A_2 = 15.92\text{ m/s}$

Pressure at section 1 (P_1) = $215.8\text{KN/m}^2 = 215800\text{N/m}^2$

Loss of head = 1.25m of oil of sp gr 0.85 , $P = 0.85 \times 9810 \times 1.25\text{ N/m}^2$

Loss of head in terms of water (h_L) = $\frac{P}{\gamma} = \frac{0.85 \times 9810 \times 1.25}{9810} = 1.0625\text{m}$

Angle of bend (θ) = 45°

Resultant force (F_R) = ?

Direction of resultant force = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{215800}{1000 \times 9.81} + \frac{1.77^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{15.92^2}{2 \times 9.81} + 1.0625$$

$$P_2 = 86574\text{N/m}^2$$

\sum Forces in X direction = Rate of change of momentum in X direction

$$(P_1 A_1 - P_2 \cos \theta A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$\begin{aligned}
 (P_1 A_1 - P_2 A_2 \cos\theta) - F_x &= \rho Q (V_2 \cos\theta - V_1) \\
 F_x &= (P_1 A_1 - P_2 A_2 \cos\theta) + \rho Q (V_1 - V_2 \cos\theta) \\
 &= (215800 \times 0.1256 - 86574 \times 0.0314 \cos 45) + 1000 \times 0.5 (3.98 - 15.92 \cos 45) \\
 &= 21544 \text{ N}
 \end{aligned}$$

Σ Forces in Y direction = Rate of change of momentum in Y direction

$$\begin{aligned}
 F_y - P_2 \sin\theta A_2 &= \rho Q (V_{2y} - V_{1y}) \\
 F_y - P_2 A_2 \sin\theta &= \rho Q (V_2 \sin\theta - 0) \\
 F_y &= P_2 A_2 \sin\theta + \rho Q V_2 \sin\theta \\
 &= 86574 \times 0.0314 \sin 45 + 1000 \times 0.5 \times 15.92 \sin 45 \\
 &= 7551 \text{ N}
 \end{aligned}$$

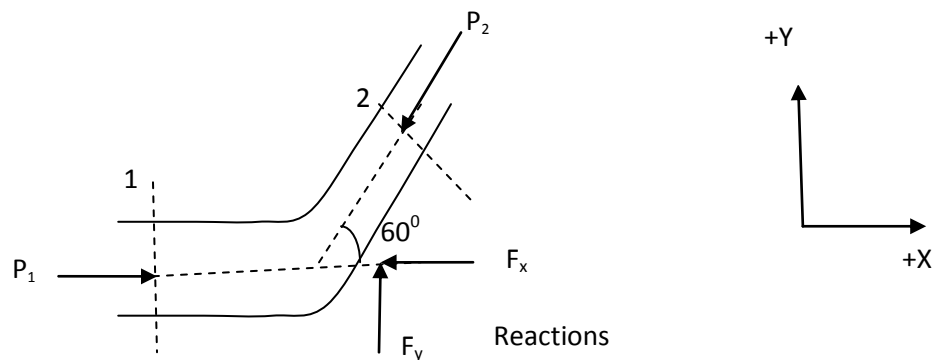
$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 22829 \text{ N}$$

Resultant force exerted by the water on the bend = 22829 N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{7551}{21544} = 19.3^\circ$$

10. The angle of a reducing bend is 60° . Its initial diameter is 300mm and final diameter is 150mm and is lifted in a pipeline carrying water at a rate of 330 lps. The pressure at the commencement of the bend is 3.1 bar. The friction loss in the pipe may be assumed as 10% of kinetic energy at the exit of the bend. Determine the force exerted by the reducing bend.

Solution:



Solution:

Diameter at section 1 (d_1) = 300mm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4} \times 0.3^2 = 0.07068 \text{ m}^2$

Diameter at section 2 (d_2) = 150mm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$

Discharge (Q) = 330lps = 0.33 m³/s

Velocity at section 1 (V₁) = Q/A₁ = 4.67 m/s

Velocity at section 2 (V₂) = Q/A₂ = 18.67 m/s

Pressure at section 1 (P₁) = 3.1 bar = 3.1x10⁵ N/m²

Loss of head (h_L) = 10% of velocity head at 2 = 0.1 $\frac{V_2^2}{2g} = 0.1 \frac{18.67^2}{2 \times 9.81} = 1.77\text{m}$

Angle of bend (θ) = 50°

Resultant force (F_R) = ?

Direction of resultant force = ?

Finding pressure at section 2 (P₂)

Using Bernoulli's equation at 1 and 2 (Z₁=Z₂)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{3.1 \times 10^5}{1000 \times 9.81} + \frac{4.67^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{18.67^2}{2 \times 9.81} + 1.77$$

$$P_2 = 129256 \text{ N/m}^2$$

\sum Forces in X direction = Rate of change of momentum in X direction

$$(P_1 A_1 - P_2 \cos \theta A_2) - F_x = \rho Q (V_{2x} - V_{1x})$$

$$(P_1 A_1 - P_2 A_2 \cos \theta) - F_x = \rho Q (V_2 \cos \theta - V_1)$$

$$F_x = (P_1 A_1 - P_2 A_2 \cos \theta) + \rho Q (V_1 - V_2 \cos \theta)$$

$$= (310000 \times 0.07068 - 129256 \times 0.01767 \cos 60) + 1000 \times 0.33 (4.67 - 18.67 \cos 60)$$

$$= 19229 \text{ N}$$

\sum Forces in Y direction = Rate of change of momentum in Y direction

$$F_y - P_2 \sin \theta A_2 = \rho Q (V_{2y} - V_{1y})$$

$$F_y - P_2 A_2 \sin \theta = \rho Q (V_2 \sin \theta - 0)$$

$$F_y = P_2 A_2 \sin \theta + \rho Q V_2 \sin \theta$$

$$= 129256 \times 0.01767 \sin 60 + 1000 \times 0.33 \times 18.67 \sin 60$$

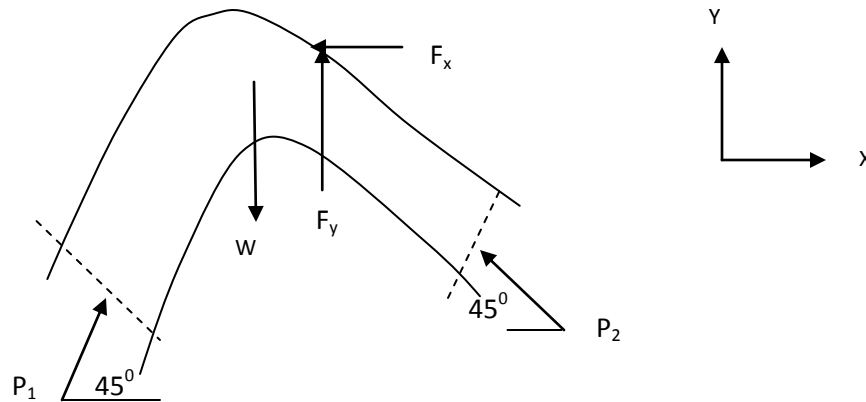
$$= 7314 \text{ N}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 20573 \text{ N}$$

Resultant force exerted by the water on the bend = 20573 N (to the right and downward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{7314}{19229} = 20.8^\circ$$

11. A 0.4m x 0.3m, 90° vertical bend carries 0.6 m³/s oil of sp gr 0.8 with a pressure of 120 Kpa at inlet to the bend. The volume of the bend is 0.1 m³. Find the magnitude and direction of the force on the bend. Neglect friction and assume both inlet and outlet sections to be at same horizontal level. Also assume that water enters the bend at 45° to the horizontal.



Solution:

Diameter at section 2 (d_1) = 0.4m

Area at section 2 (A_1) = $\frac{\pi}{4} \times 0.4^2 = 0.1256\text{m}^2$

Diameter at section 1 (d_2) = 0.3m

Area at section 1 (A_2) = $\frac{\pi}{4} \times 0.3^2 = 0.07068\text{m}^2$

Discharge (Q) = 0.6 m³/s

Weight of oil (W) = $\gamma_{oil} Vol = 0.8 \times 9810 \times 0.1 = 784.8\text{N}$

Velocity at section 1 (V_1) = $Q/A_1 = 4.8\text{ m/s}$

Velocity at section 2 (V_2) = $Q/A_2 = 8.5\text{ m/s}$

Pressure at section 1 (P_1) = 120 Kpa

$\theta = 45^\circ$

Resultant force (F_R) = ?

Direction of resultant force = ?

Finding pressure at section 2 (P_2)

Using Bernoulli's equation at 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{120000}{0.8 \times 1000 \times 9.81} + \frac{4.8^2}{2 \times 9.81} = \frac{P_2}{0.8 \times 1000 \times 9.81} + \frac{8.5^2}{2 \times 9.81}$$

$$P_2 = 100316\text{ N/m}^2$$

$\Sigma \text{ Forces in X direction} = \text{Rate of change of momentum in X direction}$

$$\begin{aligned}
 (P_1 \cos\theta A_1 - P_2 \cos\theta A_2) - F_x &= \rho Q (V_{2x} - V_{1x}) \\
 (P_1 A_1 \cos\theta - P_2 A_2 \cos\theta) - F_x &= \rho Q (V_2 \cos\theta - V_1 \cos\theta) \\
 F_x &= (P_1 A_1 \cos\theta - P_2 A_2 \cos\theta) + \rho Q (V_1 \cos\theta - V_2 \cos\theta) \\
 &= (120000 \times 0.1256 \cos 45 - 100316 \times 0.07068 \cos 45) + 0.8 \times 1000 \times 0.6 (4.8 \cos 45 - 8.5 \cos 45) \\
 &= 4388 \text{ N}
 \end{aligned}$$

Σ Forces in Y direction = Rate of change of momentum in Y direction

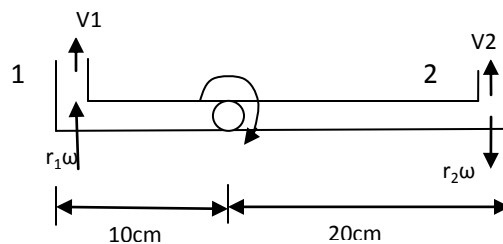
$$\begin{aligned}
 F_y + (P_1 \sin\theta A_1 + P_2 \sin\theta A_2) - W &= \rho Q (V_{2y} - V_{1y}) \\
 F_y + (P_1 A_1 \sin\theta + P_2 A_2 \sin\theta) - W &= \rho Q (V_2 \sin\theta - V_1 \sin\theta) \\
 F_y &= -(P_1 A_1 \sin\theta + P_2 A_2 \sin\theta) + W + \rho Q (V_2 \sin\theta - V_1 \sin\theta) \\
 &= -(120000 \times 0.1256 \sin 45 + 100316 \times 0.07068 \sin 45) + 784.8 + 0.8 \times 1000 \times 0.6 (4.8 \sin 45 - 8.5 \sin 45) \\
 &= -6115 \text{ N}
 \end{aligned}$$

$$\text{Resultant force } (F_R) = \sqrt{F_x^2 + F_y^2} = 7526 \text{ N}$$

Resultant force exerted by the water on the bend = 7526 N (to the right and upward)

$$\text{Direction of resultant force} = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{6115}{4388} = 54^\circ$$

12. The lawn sprinkler shown below has nozzles of 5mm diameter and carries a total discharge of 0.20 lps. Determine the angular speed of rotation of the sprinkler and torque required to hold the sprinkler stationary. Assume no friction at the pivot.



Solution:

Diameter of nozzle (d) = 5mm = 0.005m

Area of nozzle (A) = $\frac{\pi}{4} \times 0.005^2 = 1.963 \times 10^{-5} \text{ m}^2$

$r_1 = 10 \text{ cm} = 0.1 \text{ m}$

$r_2 = 20 \text{ cm} = 0.2 \text{ m}$

Total discharge = 0.2 lps

Discharge through each nozzle (Q) = 0.2/2 = 0.1 lps = 0.0001 m³/s

Relative velocity at outlet of each nozzle ($V_1 = V_2$) = Q/A = 5.09 m/s

For torque (T) = 0, Angular speed of rotation (ω) = ?

For $\omega = 0$, Torque (T) = ?

a. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Jet exerts force in opposite direction at nozzle 1 and 2 (downward direction).

Torque at 1: anticlockwise, torque at 2: clockwise.

As the torque arm for 2 is greater, the sprinkler will rotate clockwise if free to rotate.

Absolute velocity at 1 (V_{1a}) = $5.09 + r_1 \omega = 5.09 + 0.1 \omega$ (tangential velocity and relative velocity in the same direction)

Absolute velocity at 2 (V_{2a}) = $5.09 - r_2 \omega = 5.09 - 0.2 \omega$ (tangential velocity and relative velocity in opposite direction)

$$\text{Final moment of momentum} = \rho Q V_{2a} r_2 - \rho Q V_{1a} r_1 = 0$$

(Two torques in opposite direction, net torque = greater torque - smaller torque)

$$V_{1a} r_1 = V_{2a} r_2$$

$$(5.09 + 0.1 \omega) 0.1 = (5.09 - 0.2 \omega) 0.2$$

$$\omega = 10.18 \text{ rad/s}$$

$$\omega = \frac{2N\pi}{60}$$

$$N = 98 \text{ rpm}$$

b. For $\omega = 0$, velocities are V_1 and V_2 .

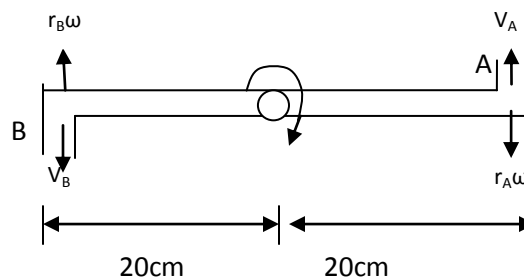
$$\text{Torque exerted by the water on sprinkler} = \rho Q V_2 r_2 - \rho Q V_1 r_1$$

$$= 1000 \times 0.0001 \times 5.09 \times 0.2 - 1000 \times 0.0001 \times 5.09 \times 0.1$$

$$= 0.0509 \text{ Nm}$$

Torque required to hold the sprinkler stationary = 0.0509 Nm

13. A lawn sprinkler shown in the figure has 0.8cm diameter nozzle at the end of a rotating arm and discharges water at the rate of 12m/s. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of the arm, if free to rotate.



Solution:

Diameter of nozzle (d) = 0.8cm = 0.008m

Area of nozzle (A) = $\frac{\pi}{4} \times 0.008^2 = 5.026 \times 10^{-5} \text{ m}^2$

$r_a = r_b = 20\text{cm} = 0.2\text{m}$

Relative velocity at A and B ($V = V_A = V_B$) = 12 m/s

Discharge through each nozzle (Q) = A V = 0.000603 m³/s

For angular velocity (ω) = 0, Torque required to hold the rotating arm stationary (T) = ?

For torque (T) = 0, constant speed of rotation of the arm (N) = ?

a. Jet exerts force in opposite direction at nozzle A and B (upward at A and downward at B).

Torque at A and B: both clockwise

(Two torques in same direction, net torque = sum of two torques)

For $\omega = 0$, velocities are V_A and V_B .

$$\begin{aligned}\text{Torque exerted by the water on sprinkler} &= \rho Q V_A r_A + \rho Q V_B r_B \\ &= 1000 \times 0.000603 \times 12 \times 0.2 + 1000 \times 0.000603 \times 12 \times 0.2 \\ &= 2.89 \text{ Nm}\end{aligned}$$

Torque required to hold the rotating arm stationary = 2.89 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = 12 - $r_a \omega$ = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

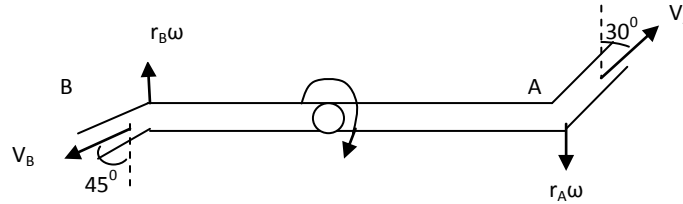
Absolute velocity at B (V_{2a}) = 12 - $r_b \omega$ = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

$$\begin{aligned}\text{Final moment of momentum} &= \rho Q V_{1a} r_a + \rho Q V_{2a} r_b = 0 \\ V_{1a} r_a &= - V_{2a} r_b \\ (12 - 0.2 \omega) 0.2 &= -(12 - 0.2 \omega) 0.2 \\ \omega &= 60 \text{ rad/s}\end{aligned}$$

$$\omega = \frac{2N\pi}{60}$$

N = 573 rpm

14. A lawn sprinkler with two nozzles 5mm in diameter each at 0.2m and 0.15m radii is connected across a tap capable of discharging 6 litres/min. The nozzles discharge water upwards and outwards from the plane of rotation. What torque will sprinkler exert on the hand if held stationary, and at what angular velocity will it rotate free?



Solution:

Diameter of nozzle (d) = 5mm = 0.005m

Area of nozzle (A) = $\frac{\pi}{4} \times 0.005^2 = 1.9635 \times 10^{-5} \text{ m}^2$

$r_A = 0.2\text{m}$, $r_B = 0.15\text{m}$

Assuming discharge to be equally divided,

Discharge ($Q_A = Q_B$) = 6/2 litres/min = 3/(1000x60) $\text{m}^3/\text{s} = 0.00005 \text{ m}^3/\text{s}$

Relative velocity at A (V_A) = $Q_A/A = 0.00005/1.9635 \times 10^{-5} = 2.54\text{m/s}$

Relative velocity at B (V_B) = $Q_B/A = 0.00005/1.9635 \times 10^{-5} = 2.54\text{m/s}$

Vertical component of relative velocity at A (V_{YA}) = $2.54 \cos 30 = 2.2\text{m/s}$

Vertical component of relative velocity at B (V_{YB}) = $2.54 \cos 45 = 1.8\text{m/s}$

For angular velocity (ω) = 0, Torque required to hold the rotating arm stationary (T) = ?

For torque (T) = 0, constant speed of rotation of the arm (N) = ?

a. Jet exerts force in opposite direction at nozzle A and B (downward at A and upward at B).

Torque at A and B: both clockwise

(Two torques in same direction, net torque = sum of two torques)

For $\omega = 0$, velocities are V_{YA} and V_{YB} .

$$\begin{aligned} \text{Torque exerted by the water on sprinkler} &= \rho Q V_{YA} r_A + \rho Q V_{YB} r_B \\ &= 1000 \times 0.00005 \times 2.2 \times 0.2 + 1000 \times 0.00005 \times 1.8 \times 0.15 \\ &= 0.0355 \text{ Nm} \end{aligned}$$

Torque required to hold the rotating arm stationary = 0.0355 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = $V_{YA} - r_A\omega = 2.2 - 0.2 \omega$ (tangential velocity and relative velocity in opposite direction)

Absolute velocity at B (V_{2a}) = $V_{YB} - r_B\omega = 1.8 - 0.15 \omega$ (tangential velocity and relative velocity in opposite direction)

Final moment of momentum = $\rho Q V_{1a} r_A + \rho Q V_{2a} r_B = 0$

$$V_{1a} r_a = - V_{2a} r_b$$

$$(2.2 - 0.2 \omega)0.2 = -(1.8 - 0.15 \omega)0.15$$

$$\omega = 11.36 \text{ rad/s}$$

$$\omega = \frac{2N\pi}{60}$$

$$N = 109 \text{ rpm}$$

15. A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18m/s. Calculate:
a) the force on the plate when it is stationary, b) the force on the plate when it moves in the same direction as the jet with a velocity of 6m/s, and c) the work done per sec and the efficiency in case (b).

Solution:

Diameter of jet (d) = 50mm = 0.05m

Area of jet (A) = $\frac{\pi}{4} \times 0.05^2 = 0.001963 \text{ m}^2$

Velocity of jet (V) = 18 m/s

a. Force exerted by the jet on the plate (F) = ρAV^2

$$= 1000 \times 0.001963 \times 18^2 = 636 \text{ N}$$

b. Velocity of plate (u) = 6m/s

Force exerted by the jet on the plate when the plate is moving (F_p) = $\rho A(V - u)^2$

$$= 1000 \times 0.001963 \times (18 - 6)^2 = 283 \text{ N}$$

c. Work done per/sec (W) = $F_p \times \text{distance/time} = F_p \times u = 283 \times 6 = 1698 \text{ J}$

Kinetic energy of jet (KE)/sec = $\frac{1}{2} (\rho AV)V^2 = \frac{1}{2} \rho AV^3 = \frac{1}{2} \times 1000 \times 0.001963 \times 18^3 = 5724 \text{ J}$

Efficiency = $W/KE = 1698/5724 = 0.3 = 30\%$

16. A jet of water 60 mm in diameter with a velocity of 15m/s strikes a flat plate inclined at an angle of 25° to the axis of the jet. Calculate the normal force exerted on the plate (a) when the plate is stationary, (b) when the plate is moving at 4.5 m/s in the direction of jet and (c) the work done per sec and the efficiency for case b.

Solution:

Diameter of jet (d) = 60mm = 0.06m

Area of jet (A) = $\frac{\pi}{4} \times 0.06^2 = 0.00283 \text{ m}^2$

Velocity of jet (V) = 15 m/s

Angle of inclination of the plate with the axis of jet (θ) = 25°

a. Normal force exerted on the plate (F) = $\rho AV^2 \sin\theta$

$$= 1000 \times 0.00283 \times 15^2 \sin 25 = 269 \text{ N}$$

b. Velocity of plate (u) = 4.5 m/s

Normal force exerted on the plate when the plate is moving (F_p) = $\rho A(V - u)^2 \sin\theta$

$$= 1000 \times 0.00283 \times (15 - 4.5)^2 \sin 25 = 132 \text{ N}$$

c. Work done per/sec (W) = $F_p \times \text{distance/time} = F_p \times u = 132 \times 4.5 = 594 \text{ J}$

Kinetic energy of jet (KE)/sec = $\frac{1}{2}(\rho AV)V^2 = \frac{1}{2}\rho AV^3 = \frac{1}{2} \times 1000 \times 0.00283 \times 15^3 = 4776 \text{ J}$

Efficiency = $W/KE = 594/4776 = 0.12 = 12\%$

17. A 75mm diameter jet of water having a velocity of 25m/s strikes a flat plate, the normal of which is inclined at 30° to the jet. Find the force normal to the surface of the plate and in the direction of the jet.

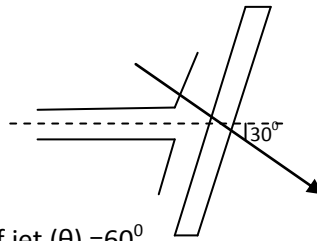
Solution:

Diameter of jet (d) = 75mm = 0.075m

Area of jet (A) = $\frac{\pi}{4} \times 0.075^2 = 0.00442 \text{ m}^2$

Velocity of jet (V) = 25 m/s

Angle made by normal with horizontal = 30°



Angle of inclination of the plate with the axis of jet (θ) = 60°

Normal force (F_n) = ?

Force in the direction of jet (F_x) = ?

$$F_n = \rho AV^2 \sin\theta = 1000 \times 0.00442 \times 25^2 \sin 60 = 2392 \text{ N}$$

$$F_x = F_n \cos 30 = 2392 \cos 30 = 2071 \text{ N}$$

18. A jet of 20mm in diameter moving with a velocity of 5m/s strikes a smooth plate, which is inclined at an angle of 20° to the horizontal. Compute the amount of flow on each side of the plate and the force exerted on the plate.

Solution:

Diameter of jet (d) = 20mm = 0.02m

Area of jet (A) = $\frac{\pi}{4} \times 0.02^2 = 0.000314 \text{ m}^2$

Velocity of jet (V) = 5 m/s

Angle of inclination of jet with horizontal (θ) = 20°

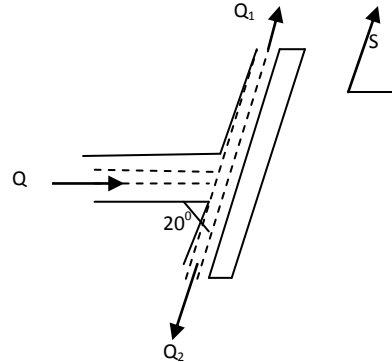
Q = Flow through jet before striking plate

Q_1 and Q_2 = Flow on upper and lower side of plate

A_1 and A_2 = Area of jet on upper and lower side of plate

$Q = A V = 0.000314 \times 5 = 0.00157 \text{ m}^3/\text{s}$

Force exerted on plate (F_x) = ?



Writing momentum equation in direction of S

$$\rho A_1 V^2 - \rho A_2 V^2 - \rho A V^2 \cos \theta = 0$$

With $Q = A V$, $Q_1 = A_1 V$, $Q_2 = A_2 V$

$$Q_1 - Q_2 = Q \cos \theta$$

From continuity, $Q_1 + Q_2 = Q$

Solving above two eq.,

$$Q_1 = \frac{Q}{2} (1 + \cos \theta) \text{ and } Q_2 = \frac{Q}{2} (1 - \cos \theta)$$

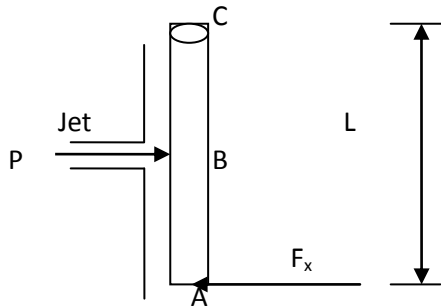
$$Q_1 = \frac{0.00157}{2} (1 + \cos 20) = 0.00152 \text{ m}^3/\text{s}$$

$$Q_2 = \frac{0.00157}{2} (1 - \cos 20) = 0.00005 \text{ m}^3/\text{s}$$

$$F_n = \rho A V^2 \sin \theta = 1000 \times 0.000314 \times 5^2 \sin 20 = 2.7 \text{ N}$$

$$F_x = F_n \cos(90 - 20) = 2.7 \cos 70 = 0.92 \text{ N}$$

19. A flat plate hinged about its top edge, is suspended vertically. It weighs 8KN. A jet of water, 50mm in diameter strikes the plate normally at its mid-point with a velocity of 50m/s. (a) Determine the horizontal force that should be applied to it at the bottom edge to keep it vertical. (b) Determine the angle of deflection where it stays in equilibrium, if it is allowed to rotate about the hinge.



Solution:

Weight (W) = 8kN = 8000N

Diameter of nozzle (d) = 50mm = 0.05m

Area of nozzle (A) = $\frac{\pi}{4} \times 0.05^2 = 0.001963\text{m}^2$

Velocity of jet (V) = 50m/s

$AB = BC = L/2$

a. Horizontal force at bottom (F_x) = ?

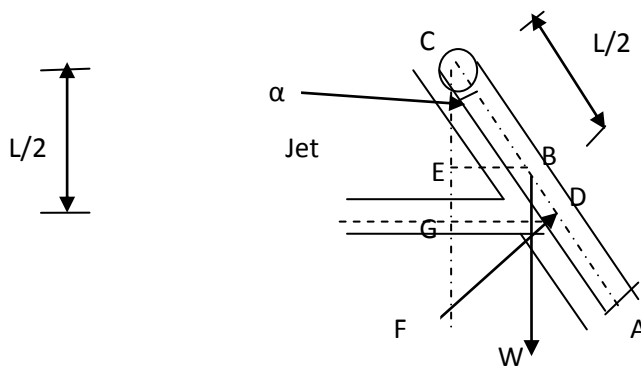
Force exerted by the jet on the plate (P) = $\rho AV^2 = 1000 \times 0.001963 \times 50^2 = 4907\text{N}$

Taking moment about hinge (C)

$P \cdot L/2 = F_x \cdot L$

$F_x = P/2 = 2453.5\text{ N}$

b. α = Angle of inclination of the plate with vertical



$CB = L/2$

As the jet strikes the plate at mid point and the plate deflects about hinge C, the vertical height from the center of the jet remains same as $L/2$ i.e. $GC = L/2$.

From $\triangle CGD$, Perpendicular distance from C to force $F = CD = CG / \cos\alpha = L/2 \sec\alpha$

From $\triangle CEB$, Perpendicular distance from C to weight $W = EB = CB \sin\alpha = L/2 \sin\alpha$

The axis of jet jet makes an angle $(90-\alpha)$ with the plate.

$$\begin{aligned} \text{Force exerted by the jet normal to the plate } (F) &= \rho AV^2 \sin(90 - \alpha) = \rho AV^2 \cos\alpha \\ &= 1000 \times 0.001963 \times 50^2 \cos\alpha = 4907 \cos\alpha \end{aligned}$$

Taking moments about hinge C

$$W \frac{L}{2} \sin\alpha = F \frac{L}{2} \sec\alpha$$

$$8000 \sin\alpha = 4907 \cos\alpha \sec\alpha$$

$$\alpha = 37.8^\circ$$

