

Subject : PLANE AND SPHERICAL TRIGONOMETRY  
Reviewer : ENGR. BABY HIDELEZA F. CASTILLO  
Date : JULY 21, 2018

---

### TOPIC 1: ANGLES

#### A. Definition:

- ✓ Plane angle - consists of two intersecting lines and intersecting point known as a vertex.

#### B. Classification of Angles:

- ✓ Acute angle - angle less than 90 degrees.
- ✓ Obtuse angle - angle more than 90° but less than 180°.
- ✓ Reflex angle - angle more than 180° but less than 360°.
- ✓ Related angle - an angle that differs from another by some multiple of 90°.
- ✓ Right angle - an angle equal to 90 degrees.
- ✓ Straight angle - an angle equal to 180°.
- ✓ Complementary angles - angles whose sum is 90°.
- ✓ Supplementary angles - angles whose sum is 180°

#### C. Measurement of Angles:

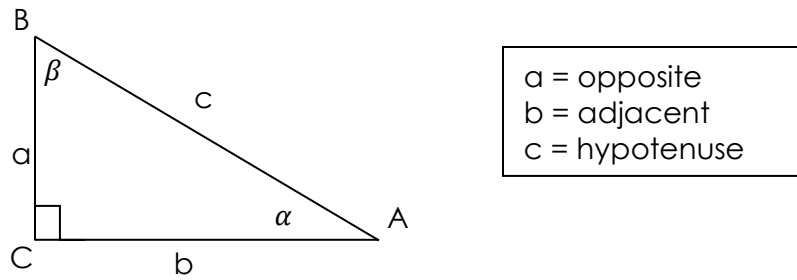
- ✓ 1 degree(°) = 60 minutes
- ✓ 1 minute = 60 seconds
- ✓ 90 deg = 100 grad
- ✓  $\pi$ rad = 180 deg
- ✓ 1 rev =  $2\pi$ rad
  - = 360°
  - = 400 grad
  - = 6400 mills
  - = 6400 gons

### TOPIC 2: RIGHT TRIANGLE

#### A. Pythagorean Theorem

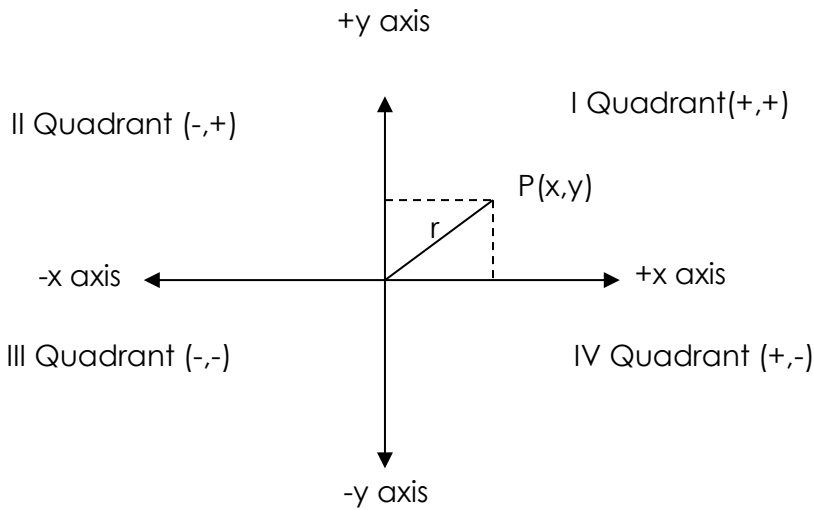
$$a^2 + b^2 = c^2$$

B. Six Trigonometric Functions



$\sin \alpha$	=	$\frac{\text{the length of the side opposite to } \alpha}{\text{the length of the hypotenuse}}$	$\frac{a}{c}$
$\cos \alpha$	=	$\frac{\text{the length of the side adjacent to } \alpha}{\text{the length of the hypotenuse}}$	$\frac{b}{c}$
$\tan \alpha$	=	$\frac{\text{length of the side opposite to } \alpha}{\text{length of the side adjacent to } \alpha}$	$\frac{a}{b}$
$\cot \alpha$	=	$\frac{\text{length of the side adjacent to } \alpha}{\text{length of the side opposite to } \alpha}$	$\frac{b}{a}$
$\sec \alpha$	=	$\frac{\text{the length of the hypotenuse}}{\text{length of the side adjacent to } \alpha}$	$\frac{c}{b}$
$\csc \alpha$	=	$\frac{\text{the length of the hypotenuse}}{\text{length of the side opposite to } \alpha}$	$\frac{c}{a}$

TOPIC 3: RECTANGULAR COORDINATE SYSTEM

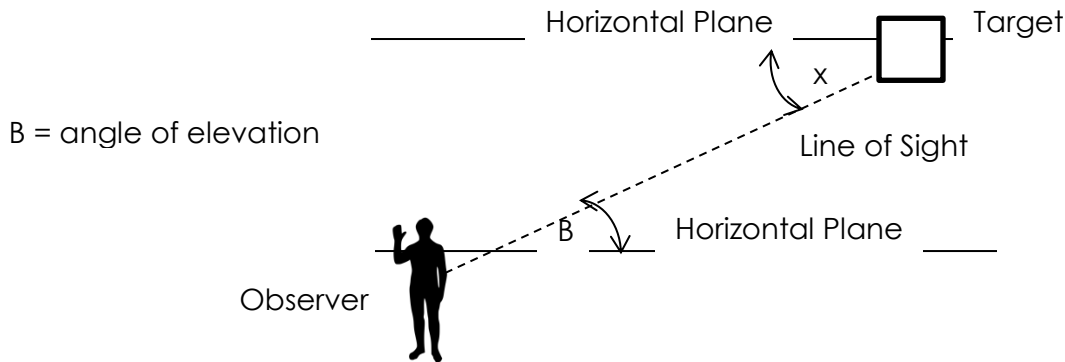


□ Sign of the functions by Quadrant

Function	I	II	III	IV
Sine	+	+	-	-
Cosine	+	-	-	+
Tangent	+	-	+	-

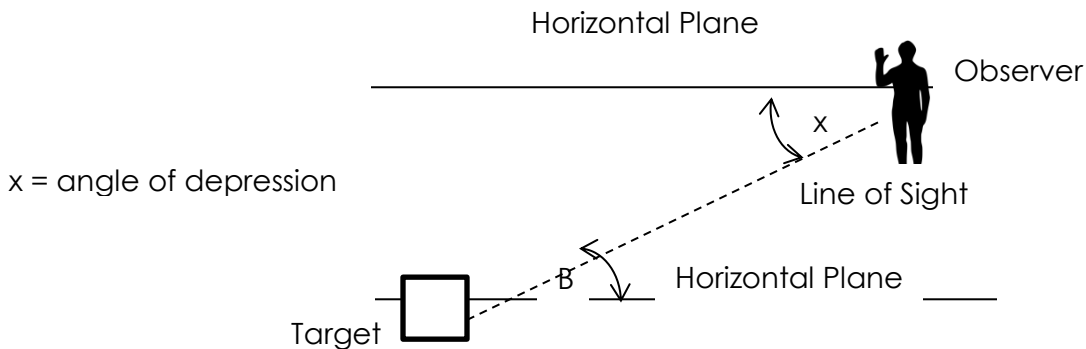
**TOPIC 4: ANGLE OF ELEVATION**

Angle of elevation is the angle above the horizontal plane of the observer.



**TOPIC 5: ANGLE OF DEPRESSION**

Angle of depression is the angle below the horizontal plane of the observer.

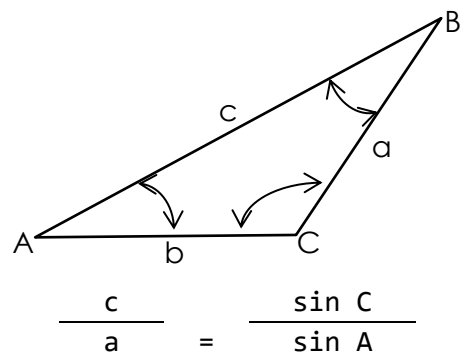


**TOPIC 6: OBLIQUE TRIANGLE**

A. Sine Law: Relates the sine of the angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{b} = \frac{\sin A}{\sin B} \quad \frac{c}{b} = \frac{\sin C}{\sin B}$$



$$A + B + C = 180$$

B. Cosine Law: Relates the cosine of the angle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Note:

1. In solving angles in a triangle given 3 sides, do not apply Sine Law, instead Cosine Law since the sine function cannot give an angle of more than 90 degrees.
2. The maximum angle of triangle will be at the opposite of the longest side.
3. The minimum angle of triangle will be at the opposite of the shortest side.

C. Tangent Law

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2} (A - B)}{\tan \frac{1}{2} (A + B)}$$

## TOPIC 7: TRIGONOMETRIC IDENTITIES

I. The Reciprocal Relation

1.  $\sin A = 1/\csc A$ ;  $\csc A = 1/\sin A$ ;  $\sin A \csc A = 1$
2.  $\cos A = 1/\sec A$ ;  $\sec A = 1/\cos A$ ;  $\cos A \sec A = 1$
3.  $\tan A = 1/\cot A$ ;  $\cot A = 1/\tan A$ ;  $\tan A \cot A = 1$

II. The Quotient Relation

1.  $\tan A = \sin A/\cos A$
2.  $\cot A = \cos A/\sin A$

III. The Pythagorean Relation

1.  $\sin^2 A + \cos^2 A = 1$
2.  $1 + \tan^2 A = \sec^2 A$
3.  $1 + \cot^2 A = \csc^2 A$

IV. Sum and Difference of Two Angles

1.  $\sin (A + B) = \sin A \cos B + \cos A \sin B$
2.  $\sin (A - B) = \sin A \cos B - \cos A \sin B$
3.  $\cos (A - B) = \cos A \cos B + \sin A \sin B$
4.  $\cos (A + B) = \cos A \cos B - \sin A \sin B$
5.  $\tan (A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$
6.  $\tan (A - B) = (\tan A - \tan B)/(1 + \tan A \tan B)$

V. Twice an Angle Formula

1.  $\sin 2A = 2 \sin A \cos A$
2.  $\cos 2A = \cos^2 A - \sin^2 A$ ;  $\cos 2A = 1 - 2 \sin^2 A$ ;  $\cos 2A = 2 \cos^2 A - 1$
3.  $\tan 2A = (2 \tan A)/(1 - \tan^2 A)$

VI. Half an Angle Formula

1.  $\sin^2(A/2) = (1 - \cos A)/2$
2.  $\cos^2(A/2) = (1 + \cos A)/2$
3.  $\tan^2(A/2) = (1 - \cos A)/(1 + \cos A)$

Also

$$\tan(A/2) = (1 - \cos A)/\sin A; \tan(A/2) = \sin A/(1 + \cos A)$$

## VII. Sum and Difference of Angles

1.  $\sin A + \sin B = 2 \sin [(A + B)/2] \cos [(A - B)/2]$
2.  $\sin A - \sin B = 2 \sin [(A - B)/2] \cos [(A + B)/2]$
3.  $\cos A + \cos B = 2 \cos [(A + B)/2] \cos [(A - B)/2]$
4.  $\cos A - \cos B = -2 \sin [(A + B)/2] \sin [(A - B)/2]$
5.  $\tan A + \tan B = \sin (A + B)/(\cos A \cos B)$
6.  $\tan A - \tan B = \sin (A - B)/(\cos A \cos B)$

## VIII. Sine and Cosine of nA

1.  $\sin nA = 2 \sin (n - 1) A \cos A - \sin (n - 2) A$
2.  $\cos nA = 2 \cos (n - 1) A \cos A - \cos (n - 2) A$

## IX. Product of Sine and Cosine as Sum

1.  $\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$
2.  $\cos A \cos B = \frac{1}{2} [\cos (A - B) + \cos (A + B)]$
3.  $\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

## TOPIC 8: INVERSE TRIGONOMETRIC FUNCTIONS

Let  $a =$  a positive number less than 1

$$m = \sqrt{1 - a^2} \quad ; \quad p = \sqrt{1 + a^2}$$

- |                                             |                                                |
|---------------------------------------------|------------------------------------------------|
| 1. $\text{Arc sin } a = \text{Arc cos } m$  | 3. $\text{Arc tan } a = \text{Arc sin } (a/p)$ |
| $\text{Arc sin } a = \text{Arc tan } (a/m)$ | $\text{Arc tan } a = \text{Arc cos } (1/p)$    |
| $\text{Arc sin } a = \text{Arc cot } (m/a)$ | $\text{Arc tan } a = \text{Arc cot } (1/a)$    |
| $\text{Arc sin } a = \text{Arc sec } (1/m)$ | $\text{Arc tan } a = \text{Arc sec } p$        |
| $\text{Arc sin } a = \text{Arc csc } (1/a)$ | $\text{Arc tan } a = \text{Arc csc } (p/a)$    |
| 2. $\text{Arc cos } a = \text{Arc sin } m$  | 4. $\text{Arc cot } a = \text{Arc tan } (1/a)$ |
| $\text{Arc cos } a = \text{Arc tan } (m/a)$ | 5. $\text{Arc sec } a = \text{Arc cos } (1/a)$ |
| $\text{Arc cos } a = \text{Arc cot } (a/m)$ | 6. $\text{Arc csc } a = \text{Arc sin } (1/a)$ |
| $\text{Arc cos } a = \text{Arc sec } (1/a)$ |                                                |
| $\text{Arc cos } a = \text{Arc csc } (1/m)$ |                                                |

## TOPIC 9: SPHERICAL TRIGONOMETRY

### A. Right Spherical Triangle

$$\bar{A} = 90 - A \quad ; \quad \bar{B} = 90 - B \quad ; \quad \bar{C} = 90 - c$$

### B. Napier's Rule

1. The sine of any middle part is equal to the product of the cosines of the opposite parts. (Sin-Coop)  
 $\sin \bar{A} = \cos \bar{B} \cos a$   
 $\cos A = \sin B \cos a$   
 $\sin \bar{c} = \cos a \cos b$   
 $\cos c = \cos a \cos b$
2. The sine of any middle part is equal to the product of the tangents of the adjacent parts. (Sin-Taad)

$$\begin{aligned}\sin \bar{A} &= \tan b \tan \bar{c} \\ \cos A &= \tan b \cot c \\ \sin \bar{c} &= \tan \bar{A} \tan B \\ \cos c &= \cot A \cot B\end{aligned}$$

C. Law of Sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

D. Law of Cosines

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos a \cos c + \sin a \sin c \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C\end{aligned}$$

E. Law of Cosines for Angles

$$\begin{aligned}\cos A &= -\cos B \cos C + \sin B \sin C \cos a \\ \cos B &= -\cos A \cos C + \sin A \sin C \cos b \\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c\end{aligned}$$

F. Area of Spherical Triangles and Polygons

$$A = \frac{\pi R^2 E}{180^\circ}$$

where:

R = radius of sphere

E = spherical excess

Value of E:

$$\text{With three known angles: } E = A + B + C - 180^\circ$$

**PROBLEMS**

- How many degrees is  $\frac{3}{4}$  pi radians?
 

a) $90^\circ$	c) $180^\circ$
b) $57.3^\circ$	d) $135^\circ$
- How many degrees is 4800 mils?
 

a) 270 deg	c) 180 deg
b) 90 deg	d) 215 deg
- The angular unit of  $\frac{1}{400}$  of the circumference of a circle is called:
 

a) grad	c) radian
b) mils	d) degree
- Which of the following is equivalent to  $90^\circ$ ?
 

a) 1600 mils	c) $\pi/2$ rad
b) 100 grads	d) all of these
- What is the measure of 2.25 revolutions counter-clockwise?
 

a) $-835^\circ$	c) $805^\circ$
b) $-810^\circ$	d) $810^\circ$













72.  $\text{Arctan}[2 \cos(\arcsin \frac{\sqrt{3}}{2})]$  is equal to:  
 a)  $\pi/3$  c)  $\pi/16$   
 b)  $\pi/4$  d)  $\pi/2$
73. Simplify  $4 \cos y \sin y (1 - 2 \sin^2 y)$   
 a)  $\tan 4y$  c)  $\cos 4y$   
 b)  $\sec 4y$  d)  $\sin 4y$
74. Solve for x in the given equation:  $\arctan(2x) + \arctan(x) = \pi/4$   
 a) 0.316 c) 0.281  
 b) 0.149 d) 0.421
75. Solve for x in the equation:  $\arctan(x+1) + \arctan(x-1) = \arctan(12)$   
 a) 1.50 c) 1.20  
 b) 1.34 d) 1.25
76. Find the value of A between 270 degrees and 360 degrees if  $2\sin^2 A - \sin A = 1$   
 a) 330 degrees c) 300 degrees  
 b) 340 degrees d) 320 degrees
77. Three forces 20 N, 30 N and 40 N are in equilibrium. Find the biggest angle formed by these forces.  
 a) 104.48 degrees c) 101.42 degrees  
 b) 102.36 degrees d) 103.21 degrees
78. A vertical pole 8 ft tall cast a shadow 5 ft long on level ground. Find the angle which the rays of the sun make with the horizontal.  
 a) 58 degrees c) 34 degrees  
 b) 60 degrees d) 25 degrees
79. From the top of building 100 m high the angles of depression of two cars due east of the observer are  $32^\circ 25'$  and  $58^\circ 53'$ , respectively. Find the distance between cars.  
 a) 96.54 m c) 92.3 m  
 b) 97.11 m d) 90.43 m
80. Solve the equation  $\cos^2 A = 1 - \cos^2 A$   
 a) 45 deg, 315 deg c) 45 deg, 135 deg  
 b) 45 deg, 125 deg d) 45 deg, 225 deg
81. What value of F satisfy the equation  $\tan(8F+1) = \cot(17)$  where all angles are in degrees?  
 a) 7 c) 8  
 b) 10 d) 9
82. What is the radius of the circle circumscribing an isosceles right triangle having an area of 162 sq. cm?  
 a) 12.73 c) 14.18  
 b) 13.52 d) 15.64
83. An isosceles right triangle has a perimeter of 17.071 units. Compute the area of the triangle in square units.  
 a) 11.5 c) 15.5  
 b) 10.5 d) 12.5



