Quadratic Equation

Form:
$$Ax^2 + Bx + C = 0$$
Roots:
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
Sum of Roots:
$$x_1 + x_2 = -\frac{B}{A}$$
Product of Roots:
$$x_1 \cdot x_2 = +\frac{C}{A}$$

Binomial Theorem

Form:
$$(x+y)^n$$
r^{th} term:
$$r^{th} = {}_nC_m \, x^{n-m} y^m$$
where: m=r-1

Progression

$$AM \cdot HM = (GM)^2$$

Arithmetic Progression:
$$d = a_2 - a_1 = a_3 - a_2$$
$$a_n = a_1 + (n-1)d$$
$$a_n = a_x + (n-x)d$$
$$S_n = \frac{n}{2}(a_1 + a_n)$$

Harmonic Progression:
- reciprocal of arithmetic progression

Geometric Progression:
$$r = a_2/a_1 = a_3/a_2$$
$$a_n = a_1 r^{n-1}$$
$$a_n = a_x r^{n-x}$$
$$S_n = a_1 \frac{1-r^n}{1-r}$$
$$S_\infty = \frac{a_1}{1-r}$$

Trigonometric Identities

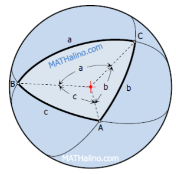
Squared Identities:
$$\sin^2 A + \cos^2 A = 1$$
$$1 + \tan^2 A = \sec^2 A$$
$$1 + \cot^2 A = \csc^2 A$$

Sum & Diff of Angles Identities:
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Identities:
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\cos 2A = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

# of diagonals:
$$d = \frac{n}{2}(n-3)$$

Spherical Trigonometry



Sine Law:
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin a}{\sin A}$$
Cosine Law for sides:
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
Cosine Law for angles:
$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
Spherical Polygon:
$$A_B = \frac{\pi R^2 E}{180°}$$
E = spherical excess
E = (A+B+C+D…) – (n-2)180°

Spherical Pyramid:
$$V = \frac{1}{3} A_B H = \frac{\pi R^3 E}{540°}$$

1 minute of arc = 1 nautical mile
1 nautical mile = 6080 feet
1 statute mile = 5280 feet
1 knot = 1 nautical mile per hour

n-sided Polygon

Interior Angle, γ:
$$\gamma = \frac{(n-2)180°}{n}$$
Deflection Angle, δ:
$$\delta = 180° - \gamma$$
Central Angle, β:
$$\beta = \frac{360°}{n}$$
$$Area = n \cdot A_{TRIANGLE}$$
$$Area = n \cdot \frac{1}{2} R^2 \sin\beta$$
$$Area = n \cdot \frac{1}{2} ah$$

Worded Problems Tips

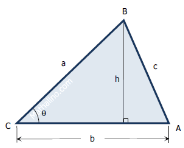
⊙ **Age Problems**
→ underline specific time conditions

⊙ **Motion Problems**
→ a = 0
→ s = vt

⊙ **Work Problems**
Case 1: Unequal rate
$$rate = \frac{work}{time}$$
Case 2: Equal rate
→ usually in project management
→ express given to **man-days** or **man-hours**

⊙ **Clock Problems**
$$\theta = \frac{11M - 60H}{2}$$
+ if M is ahead of H
- if M is behind of H

Triangle



$$A = \frac{1}{2} bh$$
$$A = \frac{1}{2} ab \sin C$$
$$A = \frac{1}{2} a^2 \frac{\sin B \sin C}{\sin A}$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$s = \frac{a+b+c}{2}$$

Common Quadrilateral

Square:
$$A = s^2$$
$$P = 4s$$
$$d = \sqrt{2}s$$
Rectangle:
$$A = bh$$
$$P = 2a + 2b$$
$$d = \sqrt{b^2 + h^2}$$
Parallelogram:
$$A = bh$$
$$A = ab \sin\theta$$
$$A = \frac{1}{2} d_1 d_2 \sin\theta$$
Rhombus:
$$A = ah$$
$$A = a^2 \sin\theta$$
$$A = \frac{1}{2} d_1 d_2$$

Polygon Names

3 - triangle
4 - quad/tetragon
5 - pentagon
6 - hexagon/sexagon
7 - septagon/heptagon
8 - octagon
9 - nonagon
10 - decagon
11 - undecagon/ monodecagon
12 - dodecagon/ bidecagon
13 - tridecagon
14 - quadridecagon
15 - quindecagon/ pentadecagon
16 - hexadecagon
17 - septadecagon
18 - octadecagon
19 - nonadecagon
20 - icosagon
21 - unicosagon
22 - do-icosagon
30 - tricontagon
31 - untricontagon
40 - tetradecagon
50 - quincontagon
60 - hexacontagon
100 - hectogon
1,000 - chilliagon
10,000 - myriagon
1,000,000 - megagon
∞ - aperio (circle)

Trapezoid

$$A = \frac{1}{2}(a+b)h$$
$$\frac{A_1}{A_2} = \frac{n}{m}\ ;\ w = \sqrt{\frac{ma^2 + nb^2}{m+n}}$$

Ellipse

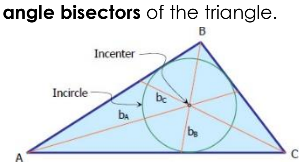
$$A = \pi ab \qquad C = 2\pi\sqrt{\frac{a^2+b^2}{2}}$$

Ex-circle – In-circle

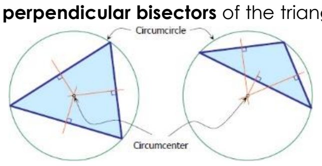
$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

Centers of Triangle

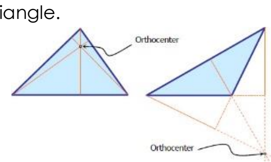
INCENTER
- the center of the inscribed circle **(incircle)** of the triangle & the point of intersection of the **angle bisectors** of the triangle.



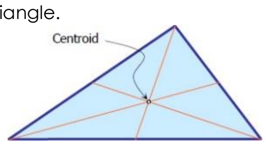
CIRCUMCENTER
- the center of the circumscribing circle (**circumcircle**) & the point of intersection of the **perpendicular bisectors** of the triangle.



ORTHOCENTER
- the point of intersection of the **altitudes** of the triangle.



CENTROID
- the point of intersection of the **medians** of the triangle.



EULER LINE
- the line that would pass through the **orthocenter**, **circumcenter**, and **centroid** of the triangle.

Triangle-Circle Relationship

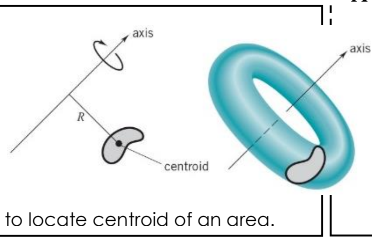
Circumscribing Circle:
$$A_T = \frac{abc}{4R}$$
$$diameter = \frac{opposite\ side}{sine\ of\ angle}$$
$$d = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
Inscribed Circle:
$$A_T = rs$$
Escribed Circle:
$$A_T = R_a(s-a)$$
$$A_T = R_b(s-b)$$
$$A_T = R_c(s-c)$$

General Quadrilateral

Cyclic Quadrilateral: (sum of opposite angles=180°)
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$
Ptolemy's Theorem is applicable:
$$ac + bd = d_1 d_2$$
$$s = \frac{a+b+c+d}{2}$$
Non-cyclic Quadrilateral:
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\frac{\varepsilon}{2}}$$

Pappus Theorem

Pappus Theorem 1:
$$SA = L \cdot 2\pi R$$
Pappus Theorem 2:
$$V = A \cdot 2\pi R$$
NOTE: It is also used to locate centroid of an area.



Prism or Cylinder

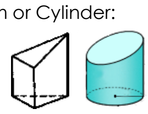
$$V = A_B H = A_X L$$
$$LA = P_B H = P_X L$$
A_B/P_B → Perimeter or Area of base
H → Height & L → slant height
A_X/P_X → Perimeter or Area of cross-section perpendicular to slant height

Pointed Solid

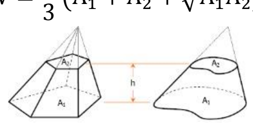
$$V = \frac{1}{3} A_B H$$
Right Circ. Cone
$$LA = \pi rL$$
Reg. Pyramid
$$LA = \frac{1}{2} P_B L$$

Special Solids

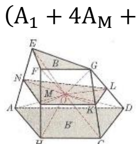
Truncated Prism or Cylinder:
$$V = A_B H_{ave}$$
$$LA = P_B H_{ave}$$



Frustum of Cone or Pyramid:
$$V = \frac{H}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

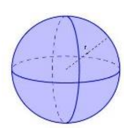


Prismatoid:
$$V = \frac{H}{6}(A_1 + 4A_M + A_2)$$

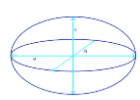


Spherical Solids

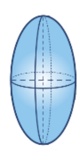
Sphere:
$$V = \frac{4}{3}\pi R^3$$
$$LA = 4\pi R^2$$



Spheroid:
$$V = \frac{4}{3}\pi abc$$
$$LA = 4\pi\left[\frac{a^2+b^2+c^2}{3}\right]$$

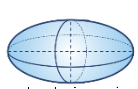


Prolate Spheroid:
$$V = \frac{4}{3}\pi abb$$
$$LA = 4\pi\left[\frac{a^2+b^2+b^2}{3}\right]$$



about major axis

Oblate Spheroid:
$$V = \frac{4}{3}\pi aab$$
$$LA = 4\pi\left[\frac{a^2+a^2+b^2}{3}\right]$$

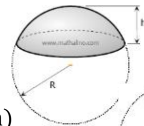


about minor axis

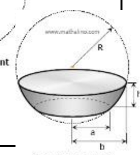
Spherical Lune:
$$\frac{A_{lune}}{\theta_{rad}} = \frac{4\pi R^2}{2\pi}$$
$$A_{lune} = 2\theta R^2$$
Spherical Wedge:
$$\frac{V_{wedge}}{\theta_{rad}} = \frac{\frac{4}{3}\pi R^3}{2\pi}$$
$$V_{wedge} = \frac{2}{3}\theta R^3$$

Spherical Zone:
$$A_{zone} = 2\pi Rh$$
Spherical Sector:
$$V = \frac{1}{3} A_{zone} R$$
$$V = \frac{2}{3}\pi R^2 h$$

Spherical Segment:
For one base:
$$V = \frac{1}{3}\pi h^2(3R - h)$$
For two bases:
$$V = \frac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$$



Spherical Segment Of One Base



Spherical Segment Of Two Bases

Archimedean Solids

- the only 13 polyhedra that are convex, have identical vertices, and their faces are regular polygons.

$$E = \frac{Nn}{2} \qquad V = \frac{Nn}{v}$$

where:
E → # of edges
V → # of vertices
N → # of faces
n → # of sides of each face
v → # of faces meeting at a vertex

Conic Sections

General Equation:
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Based on discriminant:
$B^2 - 4AC = 0$ ∴ parabola
$B^2 - 4AC < 0$ ∴ ellipse
$B^2 - 4AC > 0$ ∴ hyperbola

Based on eccentricity, e=f/d:
$e = 0$ ∴ circle
$e = 1$ ∴ parabola
$e < 1$ ∴ ellipse
$e > 1$ ∴ hyperbola

Circle

- the locus of point that moves such that its distance from a fixed point called the center is constant.

General Equation:
$x^2 + y^2 + Dx + Ey + F = 0$

Standard Equation:
$(x-h)^2 + (y-k)^2 = r^2$

Line Tangent to Conic Section

To find the equation of a line tangent to a conic section at a given point P(x_1, y_1):

In the equation of the conic equation, replace:

$x^2 \to xx_1$

$y^2 \to yy_1$

$x \to \frac{x + x_1}{2}$

$y \to \frac{y + y_1}{2}$

$xy \to \frac{xy_1 + yx_1}{2}$

Differential Calculus

Curvature:
$$k = \frac{y''}{[1 + (y')^2]^{\frac{3}{2}}}$$

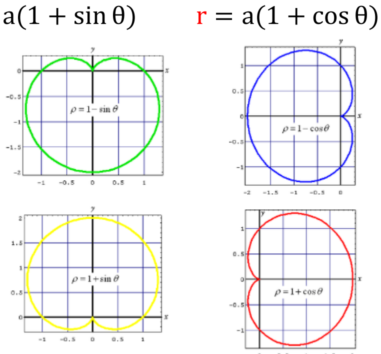
Radius of curvature:
$$\rho = \frac{[1 + (y')^2]^{\frac{3}{2}}}{y''}$$

Maxima & Minima (Critical Points):
$$\frac{dy}{dx} = y' = 0$$
(+) minima
(-) maxima

Point of inflection:
$$\frac{d^2y}{dx^2} = y'' = 0$$

Integral Calculus - The Cardioid

$A = 1.5\pi a^2$
$P = 8a$
$r = a(1 - \sin\theta)$ $\quad$ $r = a(1 - \cos\theta)$
$r = a(1 + \sin\theta)$ $\quad$ $r = a(1 + \cos\theta)$



Analytic Geometry

Slope-intercept form:
$y = mx + b$

Point-slope form:
$$m = \frac{y - y_1}{x - x_1}$$

Two-point form:
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_2}{x - x_2}$$

Point-slope form:
$$\frac{x}{a} + \frac{y}{b} = 1$$

Distance from a point to another point:
$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Distance from a point to a line:
$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

Distance of two parallel lines:
$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

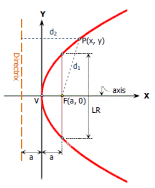
Angle between two lines:
$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Parabola

- the locus of point that moves such that it is always equidistant from a fixed point (focus) and a fixed line (directrix).

General Equation:
$y^2 + Dx + Ey + F = 0$
$x^2 + Dx + Ey + F = 0$

Standard Equation:
$(x-h)^2 = \pm 4a(y-k)$
$(y-k)^2 = \pm 4a(x-h)$



Elements:
Eccentricity, e:
$$e = \frac{d_f}{d_d} = 1$$

Length of latus rectum, LR:
$LR = 4a$

1 revolution
= 2π rad
= 360°
= 400 grads
= 6400 mills

Tetrahedron

$$H = a\sqrt{\frac{2}{3}}$$

$$SA = a^2\sqrt{3}$$

$$V = a^3 \frac{\sqrt{2}}{12}$$

Unit Circle

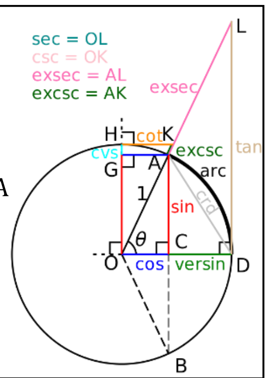
Versed sine:
$vers\ A = 1 - \cos A$

Versed cosine:
$covers\ A = 1 - \sin A$

Half versed sine:
$$hav\ A = \frac{1 - \cos A}{2}$$

Exsecant:
$exsec\ A = \sec A - 1$

sec = OL
csc = OK
exsec = AL
excsc = AK



⊙ Inflation:
$i_f = i + f + if$

⊙ Break-even analysis:
cost = revenue

⊙ Rate of return:
$$RR = \frac{\text{annual net profit}}{\text{capital}}$$

Annual net profit
= savings – expenses
– depreciation (sinking fund)

$$RP = \frac{1}{RR}$$

Depreciation

$BV_m = FC - D_m$

⊙ Straight-Line:
$$d = \frac{FC - SV}{n}$$
$D_m = d(m)$

CALTECH:
Mode 3 2

x (time)	y (BV)
0	FC
n	SV

⊙ Sinking Fund:
$$d = (FC - SV)\left[\frac{(1+i)^n - 1}{i}\right]^{-1}$$
$$D_m = d\left[\frac{(1+i)^m - 1}{i}\right]$$

where:
FC → first cost
SV → salvage cost
d → depreciation per year
n → economic life
m → any year before n
BV_m → book value after m years
D_m → total depreciation

⊙ Sum-of-the-Years-Digit (SYD):
$$d_m = (FC - SV)\left[\frac{n - m + 1}{\sum years}\right]$$
$$D_m = (FC - SV)\left[\frac{\sum_{n-m+1}^{n} x}{\sum_1^n x}\right]$$

CALTECH:
Mode 3 3

x (time)	y (BV)
0	FC
n	SV
n+1	SV

⊙ Declining Balance (Matheson):
$BV_m = FC(1 - k)^m$
$SV = FC(1 - k)^n$ k → obtained
$D_m = FC - BV_m$

CALTECH:
Mode 3 6

x (time)	y (BV)
0	FC
n	SV

⊙ Double Declining Balance:
$BV_m = FC(1 - k)^m$
$k = 2/n$ k → obtained
$D_m = FC - BV_m$

⊙ Service Output Method:
$$d = \frac{FC - SV}{Q_n}$$
$D = dQ_m$

where:
FC → first cost
SV → salvage cost
d → depreciation per year
Qn → qty produced during economic life
Qm → qty produced during up to m year
D_m → total depreciation

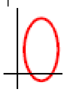
Ellipse

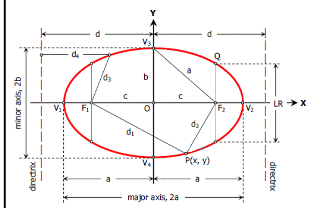
- the locus of point that moves such that the **sum** of its distances from two fixed points called the foci is constant.

General Equation:
$Ax^2 + Cy^2 + Dx + Ey + F = 0$

Standard Equation:
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$





Elements:

Location of foci, c:
$c^2 = a^2 - b^2$

Loc. of directrix, d:
$$d = \frac{a}{e}$$

Length of LR:
$$LR = \frac{2b^2}{a}$$

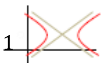
Eccentricity, e:
$$e = \frac{c}{a}$$

Hyperbola

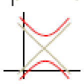
- the locus of point that moves such that the **difference** of its distances from two fixed points called the foci is constant.

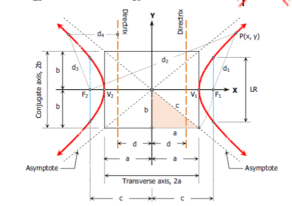
General Equation:
$Ax^2 - Cy^2 + Dx + Ey + F = 0$

Standard Equation:
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$





Elements:

Location of foci, c:
$c^2 = a^2 + b^2$

Same as ellipse:
Length of LR,
Loc. of directrix, d
Eccentricity, e

Eq'n of asymptote:
$y - k = \pm m(x - h)$

where:
m is (+) for upward asymptote;
m is (-) for downward
m = b/a if the transverse axis is horizontal;
m = a/b if the transverse axis is vertical

Engineering Economy

⊙ Simple Interest:
$I = Pin$
$F = P(1 + in)$

⊙ Compound Interest:
$F = P(1 + i)^n$
$$F = P\left(1 + \frac{r}{m}\right)^{mt}$$
$$ER = \frac{I}{P} = \left(1 - \frac{r}{m}\right)^m - 1$$

where:
F → future worth
P → principal or present worth
i → interest rate per interest period
r → nominal interest rate
n → no. of interest periods
m → no. of interest period per year
t → no. of years
ER → effective rate

⊙ Continuous Compounding Interest:
$F = Pe^{rt}$
$ER = e^r - 1$

⊙ Annuity:
$$F = A\left[\frac{(1+i)^{n'} - 1}{i}\right]$$
$$P = A\left[\frac{(1+i)^{n'} - 1}{i(1+i)^n}\right]$$

where:
F → future worth
P → principal or present worth
A → periodic payment
i → interest rate per payment
n → no. of interest periods
n' → no. of payments

⊙ Perpetuity:
$$P = \frac{A}{i} = F(1+i)^{-n}$$

⊙ Capitalized Cost:
$$C = FC + \frac{OM}{i} + \frac{RC - SV}{(1+i)^n - 1}$$
$AC = C \cdot i$
$$AC = FC \cdot i + OM + \frac{(RC - SV)i}{(1+i)^n - 1}$$

where:
C → capitalized cost
FC → first cost
OM → annual operation or maintenance cost
RC → replacement cost
SV → salvage cost
AC → annual cost

⊙ Single-payment-compound-amount factor:
$(F/P, i, n) = (1 + i)^n$

⊙ Single-payment-present-worth factor:
$(P/F, i, n) = (1 + i)^{-n}$

⊙ Equal-payment-series-compound-amount factor:
$$(F/A, i, n) = \left[\frac{(1+i)^{n'} - 1}{i}\right]$$

⊙ Equal-payment-sinking-fund factor:
$$(A/F, i, n) = \left[\frac{(1+i)^{n'} - 1}{i}\right]^{-1}$$

⊙ Equal-payment-series-present-worth factor:
$$(P/A, i, n) = \left[\frac{(1+i)^{n'} - 1}{i(1+i)^n}\right]$$

⊙ Equal-payment-series-capital-recovery factor:
$$(A/P, i, n) = \left[\frac{(1+i)^{n'} - 1}{i(1+i)^n}\right]^{-1}$$

Statistics

Measure of Natural Tendency

⊙ Mean, $\bar{x}$, μ → average
→ [Mode] [Stat] [1-var]
→ [Shift] [Mode] [▼] [Stat] Frequency? [on]
→ Input
→ [AC] [Shift] [1] [var] [$\bar{x}$]

⊙ Median, M_e → middle no.

$$M_e{}^{th} = \frac{n+1}{2}$$

$$M_e{}^{th} = \frac{1}{2}\left[\left(\frac{n}{2}\right) + \left(\frac{n}{2}+1\right)\right]$$

⊙ Mode, M_o → most frequent

Standard Deviation

⊙ Population standard deviation
→ [Mode] [Stat] [1-var]
→ [Shift] [Mode] [▼] [Stat] Frequency? [on]
→ Input
→ [AC] [Shift] [1] [var] [σx]

⊙ Sample standard deviation
→ [Mode] [Stat] [1-var]
→ [Shift] [Mode] [▼] [Stat] Frequency? [on]
→ Input
→ [AC] [Shift] [1] [var] [sx]

NOTE:
If not specified whether population/sample in a given problem, look for POPULATION.

Coefficient of Linear Correlation or Pearson's r

→ [Mode] [Stat] [A+Bx]
→ Input
→ [AC] [Shift] [1] [Reg] [r]

NOTE:
-1 ≤ r ≤ +1; otherwise erroneous

⊙ Population standard deviation

Variance

⊙ standard deviation = σ

⊙ variance = σ^2

⊙ relative variability = σ/x

Mean/Average Deviation

⊙ Mean/average value

$$mv = \frac{1}{b-a}\int_a^b f(x)dx$$

⊙ Mean value

$$RMS = \sqrt{\frac{1}{b-a}\int_a^b f(x)^2 dx}$$

Fractiles

⊙ Range
$= largest\ datum - smallest\ datum$

⊙ Coefficient of Range

$$= \frac{largest\ datum - smallest\ datum}{largest\ datum + smallest\ datum}$$

⊙ Quartiles

when n is even

$$Q_1 = \frac{1}{4}n \quad Q_2 = \frac{2}{4}n \quad Q_3 = \frac{3}{4}n$$

when n is odd

$$Q_1 = \frac{1}{4}(n+1)\ ;\ Q_1 = \frac{1}{4}(n+1)\ ;\ Q_1 = \frac{1}{4}(n+1)$$

⊙ Interquartile Range, IQR
$= largest\ quartile - smallest\ quartile$
$= Q_3 - Q_1$

⊙ Coefficient of IQR

$$= \frac{largest\ quartile - smallest\ quartile}{largest\ quartile + smallest\ quartile}$$

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

⊙ Quartile Deviation (semi-IQR) = **IQR/2**

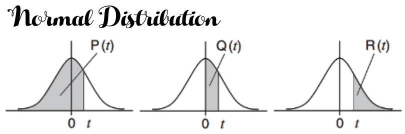
⊙ Outlier
→ extremely high or low data higher than or lower than the following limits:

$$Q_1 - 1.5IQR > x$$
$$Q_3 + 1.5IQR < x$$

⊙ Decile or Percentile

$$i_m = \frac{m}{10 \text{ or } 100}(n)$$

Normal Distribution



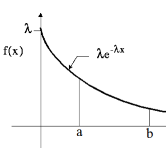
⊙ Z-score or standard score or variate

$$z = \frac{x-\mu}{\sigma}$$

x → no. of observations
μ → mean value, $\bar{x}$
σ → standard deviation

→ [Mode] [Stat]
→ [AC] [Shift] [1] [Distr]
left of z → [P(]
right of z → [R(]
bet. z & axis → [Q(]
→ Input

Exponential Distribution



$$P(x \geq a) = e^{-\lambda a}$$
$$P(x \leq a) = 1 - e^{-\lambda a}$$
$$P(a \leq x \leq b) = e^{-\lambda a} - e^{-\lambda b}$$

Transportation Engineering

Design of Horizontal Curve

⊙ Minimum radius of curvature

$$R = \frac{v^2}{g(e+f)}$$

R → minimum radius of curvature
e → superelevation
f → coeff. of side friction or skid resistance
v → design speed in m/s
g → 9.82 m/s²

⊙ Centrifugal ratio or impact factor

$$\text{Impact factor} = \frac{v^2}{gR}$$

R → minimum radius of curvature
v → design speed in m/s
g → 9.82 m/s²

Power to move a vehicle

$$P = vR$$

P → power needed to move vehicle in watts
v → velocity of vehicle in m/s
R → sum of diff. resistances in N

Design of Pavement

⊙ Rigid pavement without dowels

$$t = \sqrt{\frac{3W}{f}}$$

⊙ Rigid pavement with dowels

$$t = \sqrt{\frac{3W}{2f}} \quad \text{(at the edge)} \qquad t = \sqrt{\frac{3W}{4f}} \quad \text{(at the center)}$$

t → thickness of pavement
W → wheel load
f → allow tensile stress of concrete

⊙ Flexible pavement

$$t = \sqrt{\frac{W}{\pi f_1}} - r$$

f_1 → allow bearing pressure of subgrade
r → radius of circular area of contact between wheel load & pavement

⊙ Thickness of pavement in terms of expansion pressure

$$t = \frac{\text{expansion pressure}}{\text{pavement density}}$$

⊙ Stiffness factor of pavement

$$SF = \sqrt[3]{\frac{E_s}{E_p}}$$

E_s → modulus of elasticity of subgrade
E_P → modulus of elasticity of pavement

Traffic Accident Analysis

⊙ Accident rate for 100 million vehicles per miles of travel in a segment of a highway:

$$R = \frac{A\,(100{,}000{,}000)}{ADT \cdot N \cdot 365 \cdot L}$$

A → no. of accidents during period of analysis
ADT → average daily traffic
N → time period in years
L → length of segment in miles

⊙ Accident rate per million entering vehicles in an intersection:

$$R = \frac{A\,(1{,}000{,}000)}{ADT \cdot N \cdot 365}$$

A → no. of accidents during period of analysis
ADT → average daily traffic entering all legs
N → time period in years

⊙ Severity ratio, SR:

$$SR = \frac{f \cdot i}{f \cdot i \cdot p}$$

f → fatal
i → injury
p → property damage

⊙ Spacing mean speed, U_S:

$$U_s = \frac{\sum d}{\sum t} = \frac{n}{\sum\left(\frac{1}{U_1}\right)}$$

⊙ Time mean speed, U_t:

$$U_t = \frac{\sum \frac{d}{t}}{n} = \frac{\sum U_1}{n}$$

Σd → sum of distance traveled by all vehicles
Σt → sum of time traveled by all vehicles
ΣU_1 → sum of all spot speed
$1/\Sigma U_1$ → reciprocal of sum of all spot speed
n → no. of vehicles

⊙ Rate of flow:

$$q = kU_s$$

q → rate of flow in vehicles/hour
k → density in vehicles/km
U_S → space mean speed in kph

⊙ Minimum time headway (hrs)
= **1/q**

⊙ Spacing of vehicles (km)
= **1/k**

⊙ Peak hour factor (PHF)
= **q/q_{max}**

Discrete Probability Distributions

⊙ Binomial Probability Distribution

$$P(x) = C(n,x)\, p^x q^{n-x}$$

where:
p → success
q → failure

⊙ Geometric Probability Distribution

$$P(x) = p(q^{x-1})$$

⊙ Poisson Probability Distribution

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

Walli's Formula

$$\int_0^{\frac{\pi}{2}} \cos^m\theta \sin^n\theta\, d\theta = \frac{[(m-1)(m-3)(m-5)\dots(1 \text{ or } 2)][(n-1)(n-3)(n-5)\dots(1 \text{ or } 2)]}{(m+n)(m+n-2)(m+n-4)\dots(1 \text{ or } 2)} \cdot \alpha$$

NOTE:
α = π/2 for m and n are both even
α =1 otherwise

Fibonacci Numbers

$$a_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Tip to remember:

$$x^2 - x - 1 = 0$$

[Mode] [Eqn] [5]

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$

Period, Amplitude & Frequency

Period (T) → interval over which the graph of function repeats
Amplitude (A) → greatest distance of any point on the graph from a horizontal line which passes halfway between the maximum & minimum values of the function
Frequency (ω) → no. of repetitions/cycles per unit of time or **1/T**

Function	Period	Amplitude
y = A sin (Bx + C)	2π/B	A
y = A cos (Bx + C)	2π/B	A
y = A tan (Bx + C)	π/B	A

Measurement Corrections

Due to temperature:
(add/subtract); measured length

$$C = \alpha L(T_2 - T_1)$$

Due to pull:
(add/subtract); measured length

$$C = \frac{(P_2 - P_1)L}{EA}$$

Due to sag:
(subtract only); unsupported length

$$C = \frac{w^2L^3}{24P^2}$$

Due to slope:
(subtract only); measured length

$$C^2 = S^2 - h^2$$

Normal Tension:

$$P_N = \frac{0.204W\sqrt{AE}}{\sqrt{P_N - P}}$$

	measure	lay-out
too long	add	subtract
too short	subtract	add

too long

$$CD = MD\left(1 + \frac{e}{TL}\right)$$

too short

$$CD = MD\left(1 - \frac{e}{TL}\right)$$

Effect of Curvature & Refraction

$$h_{cr} = 0.067K^2$$

$$h = h_2 + \frac{D_2}{D_1 + D_2}(h_1 - h_2) - 0.067D_1D_2$$

Azimuth from South

Probable Errors

Probable Error (single):

$$E = 0.6745\sqrt{\frac{\Sigma(x - \bar{x})}{n - 1}}$$

Probable Error (mean):

$$E_m = \frac{E}{\sqrt{n}} = 0.6745\sqrt{\frac{\Sigma(x - \bar{x})}{n(n-1)}}$$

Proportionalities of weight, w:
E=error; d=distance; n=no. of trials

$$w \propto \frac{1}{E^2} \qquad w \propto \frac{1}{d} \qquad w \propto n$$

Stadia Measurement

Horizontal:

$$D = d + (f + c)$$

$$D = \left(\frac{f}{i}\right)s + C$$

$$D = Ks + C$$

Inclined:

$$D = Ks\cos\theta + C$$

$$H = D\cos\theta$$

$$V = D\sin\theta$$

Leveling

$$Elev_B = Elev_A + BS - FS$$

Inclined Upward:

$$error/setup = -e_{BS} + e_{FS}$$

Inclined Downward:

$$error/setup = +e_{BS} - e_{FS}$$

Total Error:

$$e_T = error/setup \cdot \text{no. of setups}$$

Reduction to Sea Level

$$\frac{CD}{R} = \frac{MD}{R + h}$$

Subtense Bar

$$D = \cot\frac{\theta}{2}$$

Double Meridian Distance Method (DMD)

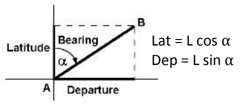
$$DMD_{first} = Dep_{first}$$

$$DMD_n = DMD_{n-1} + Dep_{n-1} + Dep_n$$

$$DMD_{last} = -Dep_{last}$$

$$2A = \Sigma(DMD \cdot Lat)$$

Area of Closed Traverse



Error of Closure:

$$= \sqrt{\Sigma L^2 + \Sigma D^2}$$

Relative Error/Precision:

$$= \frac{\text{Error of Closure}}{\text{Perimeter}}$$

Area of Irregular Boundaries

Trapezoidal Rule:

$$A = \frac{d}{2}[h_1 + h_n + 2\Sigma h]$$

Simpson's 1/3 Rule:

$$A = \frac{d}{3}[h_1 + h_n + 2\Sigma h_{odd} + 4\Sigma h_{even}]$$

Note: n must be odd

Double Parallel Distance Method (DPD)

$$DPD_{first} = Lat_{first}$$

$$DPD_n = DPD_{n-1} + Lat_{n-1} + Lat_n$$

$$DPD_{last} = -Lat_{last}$$

$$2A = \Sigma(DMD \cdot Dep)$$

Parabolic Curves

Symmetrical:

$$H = \frac{L}{8}(g_1 + g_2)$$

$$\frac{x^2}{y} = \frac{\left(\frac{L}{2}\right)^2}{H}L_1$$

Unsymmetrical:

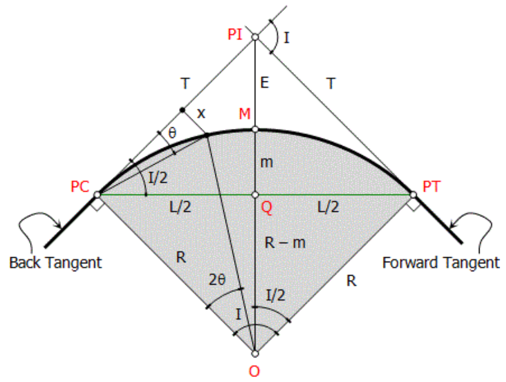
$$H = \frac{L_1L_2}{2(L_1 + L_2)}(g_1 + g_2)$$

$$g_3(L_1 + L_2) = g_1L_1 + g_2L_2$$

Note: Consider signs.

1 acre = 4047 m²

Simple, Compound & Reverse Curves



$$T = R\tan\frac{I}{2}$$

$$E = R\left[\sec\frac{I}{2} - 1\right]$$

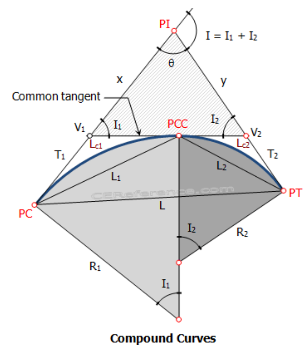
$$m = R\left[1 - \cos\frac{I}{2}\right]$$

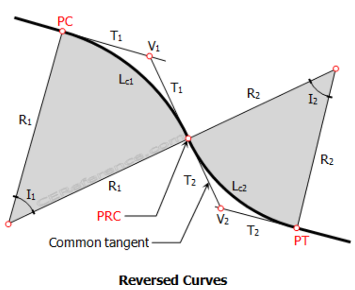
$$L = 2R\sin\frac{I}{2}$$

$$L_c = RI \cdot \frac{\pi}{180°}$$

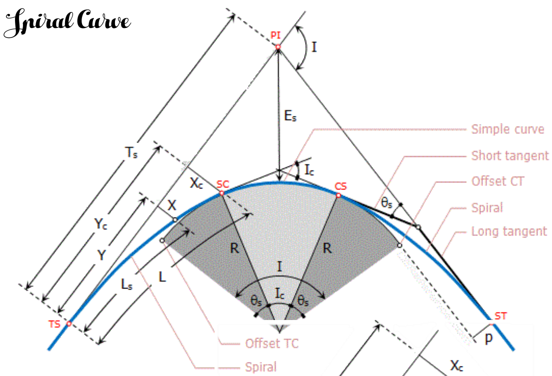
$$\frac{20}{D} = \frac{2\pi R}{360°}$$

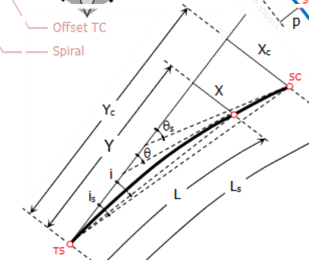
$$R = \frac{1145.916}{D}$$





Spiral Curve





$$\theta = \frac{L^2}{2RL_s} \cdot \frac{180°}{\pi}$$

$$i = \frac{\theta}{3}\ ;\ p = \frac{{L_s}^2}{24R}$$

$$x = \frac{L^3}{6RL_s}$$

$$Y = L - \frac{L^5}{40R^2{L_s}^2}$$

$$T_s = \frac{L_s}{2} + (R + p)\tan\frac{I}{2}$$

$$E_s = (R + p)\sec\frac{I}{2} - R$$

$$L_s = \frac{0.036k^3}{R}$$

$$e = \frac{0.0079k^2}{R}$$

$$\frac{D}{D_C} = \frac{L}{L_s}$$

LT → long tangent
ST → short tangent
R → radius of simple curve
L → length of spiral from TS to any point along the spiral
L_s → length of spiral
I → angle of intersection
I_c → angle of intersection of the simple curve
p → length of throw or the distance from tangent that the circular curve has been offset
x → offset distance (right angle distance) from tangent to any point on the spiral
x_c → offset distance (right angle distance) from tangent to SC
E_c → external distance of the simple curve
θ → spiral angle from tangent to any point on the spiral
$θ_s$ → spiral angle from tangent to SC
i → deflection angle from TS to any point on the spiral
i_s → deflection angle from TS to SC
y → distance from TS along the tangent to any point on the spiral

Earthworks

$$\frac{d_L}{\pm f_L} \quad \frac{0}{\pm f} \quad \frac{d_R}{\pm f_R}$$

$$A = \frac{f}{2}(d_L + d_R) + \frac{w}{4}(f_L + f_R)$$

Volume (End Area):

$$V_e = \frac{L}{2}(A_1 + A_2)$$

Volume (Prismoidal):

$$V_P = \frac{L}{6}(A_1 + 4A_m + A_2)$$

Prismoidal Correction:

$$C_P = \frac{L}{12}(c_1 - c_2)(d_1 - d_2)$$

$$V_P = V_e - C_p$$

Volume (Truncated):

$$V_T = A_{Base} \cdot H_{ave} = A\left(\frac{\Sigma h}{n}\right)$$

$$V_T = \frac{A}{n}(\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4)$$

Stopping Sight Distance

$$S = vt + \frac{v^2}{2g(f \pm G)}$$

$$a = g(f \pm G) \quad \text{(deceleration)}$$

$$t_b = \frac{v}{g(f \pm G)} \quad \text{(breaking time)}$$

$$Eff = \frac{f}{f_{ave}}(100)$$

v → speed in m/s
t → perception-reaction time
f → coefficient of friction
G → grade/slope of road

Parabolic Summit Curve

L > S

$$L = \frac{A(S)^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

L < S

$$L = 2(S) - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

L → length of summit curve
S → sight distance
h_1 → height of driver's eye
h_1 = 1.143 m or 3.75 ft
h_2 → height of object
h_2 = 0.15 m or 0.50 ft

Parabolic Sag Curve

L > S

$$L = \frac{A(S)^2}{122 + 3.5S}$$

L < S

$$L = 2(S) - \frac{122 + 3.5S}{A}$$

A → algebraic difference of grades, in percent
L → length of sag curve
S → sight distance

Underpass Sight Distance

L > S

$$L = \frac{A(S)^2}{800H} \qquad H = C - \frac{h_1 + h_2}{2}$$

L < S

$$L = 2(S) - \frac{800H}{A}$$

A → algebraic difference of grades, in percent
L → length of sag curve

$$L = \frac{A(K)^2}{395}$$

For passengers comfort, where K is speed in KPH

Horizontal Curve

L > S

$$R = \frac{S^2}{8M}$$

L < S

$$R = \frac{L(2S - L)}{8M}$$

L → length of horizontal curve
S → sight distance
R → radius of the curve
M → clearance from the centerline of the road

Properties of Fluids

$W = Mg$

$\gamma = \frac{W}{V}$; $\rho = \frac{M}{V}$

$\gamma = \rho g = \frac{pg}{RT}$

$s.v. = \frac{V}{M} = \frac{1}{\rho}$

$s.g. = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w}$

$E_B = -\frac{\Delta P}{\frac{\Delta V}{V}}$; $\beta = \frac{1}{E_B}$

$\mu = \tau \frac{dy}{dV} = \frac{FT}{L^2}$

$\upsilon = \frac{\mu}{\rho} = \frac{L^2}{T}$

$\sigma = \frac{pd}{4}$

$h = \frac{4\sigma\cos\theta}{\gamma d}$

Pressure

$p_{abs} = p_{gage} + p_{atm}$

$p = \gamma h$

$h_2 = \frac{s.g._1}{s.g._2} h_1$

$h_w = s.g._1 \, h_1$

Hydrostatic Forces

$e = \frac{I_g}{A\bar{y}}$ $e = \frac{\gamma I_g \sin\theta}{F}$

On plane surfaces:

$F = \gamma \bar{h} A$

On curved surfaces:

$F_h = \gamma \bar{h} A$

$F_v = \gamma V$

$F = \sqrt{{F_h}^2 + {F_v}^2}$

NOTE:
$\bar{h}$ = vertical distance from cg of submerged surface to liquid surface

$\bar{h} = \bar{y}$ (for vertical only)

Dams

$F_1 = \gamma A h_1 = \frac{1}{2}\gamma {h_1}^2$; $F_2 = \gamma A h_2 = \frac{1}{2}\gamma {h_2}^2$

$U_1 = \gamma h_2 B$; $U_2 = \frac{1}{2}(h_1 - h_2)\gamma B$

$RM = W_1(X_1) + W_2(X_2) + \ldots + W_n(X_n) + F_2\left(\frac{h_2}{3}\right)$

$OM = F_1\left(\frac{h}{3}\right) + U_1\left(\frac{1}{2}B\right) + U_2\left(\frac{2}{3}B\right)$

$R\bar{x} = RM - OM$

$FS_O = \frac{RM}{OM}$ & $FS_S = \frac{\mu R_y}{R_x}$

$e = \left|\frac{B}{2} - \bar{x}\right|$

$e < \frac{B}{6}$; $q = -\frac{R_y}{B}\left[1 \pm \frac{6e}{B}\right]$

$e > \frac{B}{6}$; $q = \frac{2R_y}{3\bar{x}}$

$e = \frac{B}{6}$; $q = -\frac{R_y}{B}$

$e = 0$; $q = \frac{2R_y}{B}$

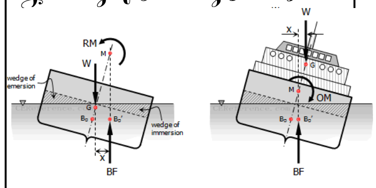
Stresses/Hoops

$S_t = \frac{pD}{2t}$

$s = \frac{2T}{pD}$

S_t = tensile stress
p = unit pressure
D = inside diameter
t = thickness of wall
s = spacing of hoops
T = tensile force

Stability of Floating Bodies



M is above G: Stable Position   M is below G: Unstable Position

MG = metacentric height

$MG = MB_O \pm GB_O$

Use (-) if G is above BO and (+) if G is below BO.
Note that M is always above BO.

$RM \text{ or } OM = Wx$
$= W(MG\sin\theta)$

$MB_O = \frac{B^2}{12D}\left[1 + \frac{\tan^2\theta}{2}\right]$

$MB_O = \frac{vs}{V_D \sin\theta} = \frac{I}{V_D}$

Buoyancy

$BF = W$

$BF = \gamma_w V_d$

$A_{bel} = \frac{sg_m}{sg_l} A_{tot}$

$V_{bel} = \frac{sg_m}{sg_l} V_{tot}$

Celerity (velocity of sound)

(rigid pipes)

$c = \sqrt{\frac{E_B}{\rho_w}}$

(non-rigid pipes)

$c = \sqrt{\frac{E_B}{\rho_w\left(1 + \frac{E_B D}{E t}\right)}}$

Relative Equilibrium of Fluids

Horizontal Motion:

$\tan\theta = \frac{a}{g}$

Inclined Motion:

$\tan\theta = \frac{a_h}{g \pm a_v}$

Vertical Motion:

$p = \gamma h\left(1 \pm \frac{a}{g}\right)$

Rotation:

$\tan\theta = \frac{\omega^2 x}{g}$

$y = \frac{\omega^2 x^2}{2g}$; $\frac{r^2}{h} = \frac{x^2}{y}$

$V = \frac{1}{2}\pi r^2 h$

$1 \text{ rpm} = \frac{\pi}{30} \text{ rad/sec}$

Bernoulli's Energy Theorem

z = elevation head; P/γ = pressure head; $v^2/2g$ = velocity head

$z_1 + \frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} + H.L.$

with pump:

$z_1 + \frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} + HA = z_2 + \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} + H.L.$

with turbine:

$z_1 + \frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} - HE = z_2 + \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} + H.L.$

$\text{efficiency} = \frac{\text{output}}{\text{input}}$; $HP = \frac{Q\gamma E}{746}$

Pump → Output & Turbine → Input

Major Losses in Pipes

Darcy Weisbach Eq'n:

$H.L. = f\frac{L}{D}\frac{v^2}{2g}$

$H.L. = \frac{0.0826 \, f \, L \, Q^2}{D^5}$

Manning's Formula:

$H.L. = \frac{10.29 \, n^2 \, L \, Q^2}{D^{16/3}}$

Hazen William's Formula:

$H.L. = \frac{10.64 \, L \, Q^{1.85}}{C^{1.85} \, D^{4.87}}$

Water Hammer

$\Delta P_{max} = \rho c v$

$t_c = \frac{2L}{c}$

A. TIME of closure:
↳ rapid/instantaneous

$\Delta P = \Delta P_{max}$

↳ Slow Closure

$\Delta P = \Delta P_{max}\left(\frac{t_c}{t_{actual}}\right)$

B. TYPE of closure:
↳ Partial Closure ($v_f \neq 0$)

$\Delta P = \rho c(v_i - v_f)$

↳ Total Closure ($v_f = 0$)

$\Delta P = \rho c v_i$

Series-Parallel Pipes

Series Connection:

$H.L._T = H.L._1 + H.L._2 + \ldots + H.L._n$

$Q_T = Q_1 = Q_2 = Q_n$

Parallel Connection:

$H.L._T = H.L._1 = H.L._2 = H.L._n$

$Q_T = Q_1 + Q_2 + \ldots + Q_n$

Fluid Flow

$Q = Av$

Q → discharge
→ flow rate
→ weight flux

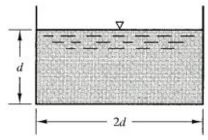
volume flow rate → m^3/s
weight flow rate → N/s
mass flow rate → kg/s

Most Efficient Sections

Rectangular:

$b = 2d$

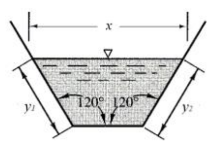
$R = \frac{d}{2}$



Trapezoidal:

$x = y_1 + y_2$

$R = \frac{d}{2}$

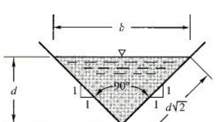


Triangular:

$b = 2d$

$A = d^2$

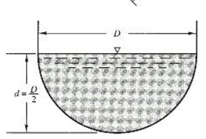
$\theta = 90°$



Semi-circular:

$d = r$ (full)

$R = \frac{r}{2}$



Circular:

Q_{max} if $d = 0.94D$

V_{max} if $d = 0.81D$

TRAPEZOIDAL:
For minimum seepage:

$b = 4d\tan\frac{\theta}{2}$

1 atm

= 101.325 **KPa**
= 2166 **psf**
= 14.7 **psi**
= 760 **mmHg**
= 29.9 **inHg**

Open Channel

Specific Energy:

$E = \frac{v^2}{2g} + d$

$v = C\sqrt{RS}$

Theoretically:

$C = \sqrt{\frac{8g}{f}}$

Manning Formula:

$C = \frac{1}{n}R^{1/6}$

Bazin Formula:

$C = \frac{87}{1 + \frac{m}{\sqrt{R}}}$

Kutter Formula:

$C = \frac{\frac{1}{n} + 23 + \frac{0.000155}{S}}{1 + \frac{n}{\sqrt{R}}\left(23 + \frac{0.000155}{S}\right)}$

If C is not given, use Manning's in V:

$v = \frac{1}{n}R^{2/3}S^{1/2}$

Constant Head Orifice

Without headloss:

$v = \sqrt{2gh}$

With headloss:

$v = C_v\sqrt{2gh}$

$Q = CA_o\sqrt{2gh}$

$C = C_c C_v$

$C_c = \frac{a}{A}$

$C_v = \frac{v}{v_t}$

$H.L. = \frac{v^2}{2g}\left[\frac{1}{{C_v}^2} - 1\right]$

$H.L. = \Delta H\left[1 - {C_v}^2\right]$

$y = \frac{x^2}{4{C_v}^2 h}$

Falling Head Orifice

Time to remove water from h_1 to h_2 with constant cross-section:

$t = \frac{2A_s}{CA_o\sqrt{2g}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$

Time to remove water from h_1 to h_2 with varying cross-section:

$t = \int_{h_2}^{h_1} \frac{A_s \, dh}{CA_o\sqrt{2gh}}$

Time in which water surfaces of two tanks will reach same elevation:

$t = \frac{2}{CA_o\sqrt{2g}}\frac{(A_{s1})(A_{s2})}{(A_{s1} + A_{s2})}\left(\sqrt{h_1} - \sqrt{h_2}\right)$

Hydrodynamics

Force on Curve Vane/Blade:

$\sum F_x = \rho Q(v_{2x} - v_{1x})$

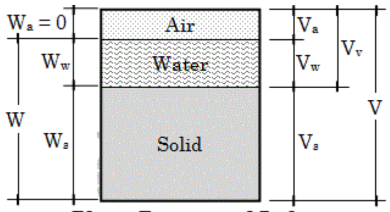
$\sum F_y = \rho Q(v_{2y} - v_{1y})$

Force on Pipe's Bend & Reducer:

(same as on Curve Vane/Blade)

Force on the Jet (at right angle):

$F = \rho Q v$



Phase Diagram of Soil

Volume

$$e = \frac{V_v}{V_s}$$

$$n = \frac{V_v}{V}$$

$$S = \frac{V_w}{V_v}$$

Weight

$$\omega = \frac{W_w}{W_s}$$

$0 < e < \infty$
$$e = \frac{n}{1-n}$$

$0 < n < 1$
$$n = \frac{e}{1+e}$$

$$Se = G_s\omega$$

$$\gamma = \frac{W}{V}$$

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_d = \frac{\gamma}{1+\omega}$$

Unit Weight:

$$\gamma = \frac{(G_s + G_s\omega)\gamma_w}{1+e}$$

$$\gamma = \frac{(G_s + Se)\gamma_w}{1+e}$$

When S=0:
$$\gamma_d = \frac{G_s\gamma_w}{1+e}$$

When S=100%:
$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e}$$

$$\gamma_{sub} = \gamma_{sat} - \gamma_w$$

$$\gamma_{sub} = \frac{(G_s - 1)\gamma_w}{1+e}$$

$$\gamma_{zav} = \frac{G_s\gamma_w}{1+G_s\omega}$$

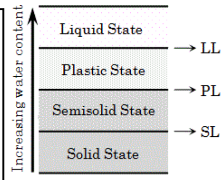
Specific Gravity of Solid:
$$G_s = \frac{\gamma_s}{\gamma_w}$$

Bulk Specific Gravity:
$$g = G_s(1-n)$$

Relative Compaction:
$$R = \frac{\gamma_d}{\gamma_{d\,max}}$$

Relative Density/ Density Index:
$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$D_r = \frac{\frac{1}{\gamma_{d\,min}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{d\,min}} - \frac{1}{\gamma_{d\,max}}}$$



Atterberg Limits

$$PI = LL - PL$$

$$LI = \frac{\omega - PL}{LL - PL}$$

$$SI = PL - SL$$

$$CI = \frac{LL - \omega}{LL - PI}$$

$$A_c = \frac{PI}{\mu}; \quad St = \frac{q_{u\,und}}{q_{u\,rem}}$$

μ = % passing 0.002mm

$$SL = \frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\gamma_w$$

$$SL = \frac{e}{G_s} \quad ; \quad SR = \frac{m_2}{V_2\gamma_w}$$

$$G_s = \frac{1}{\frac{1}{SR} - SL}$$

$$GI = (F-35)[0.2 + 0.005(LL-40)] + 0.01(F-15)(PI-10)$$

LI	State
LI < 0	Semisolid
0 < LI < 1	Plastic
LI > 1	Liquid

PI	Description
0	Non-plastic
1-5	Slightly plastic
5-10	Low plasticity
10-20	Medium plasticity
20-40	High plasticity
>40	Very High plastic

Ac	Class
Ac < 0.7	Inactive
0.7 < Ac < 1.2	Normal
Ac > 1.2	Active

Permeability

$$v = ki \; ; \; i = \frac{\Delta h}{L} \; ; \; v_s = \frac{v}{n}$$

$$Q = vA = kiA$$

Constant Head Test:
$$k = \frac{QL}{Aht}$$

Falling/Variable Head Test:
$$k = \frac{aL}{At}\ln\frac{h_1}{h_2}$$

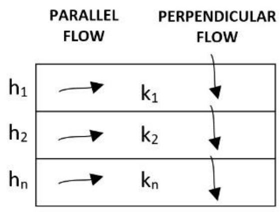
Hazen Formula
$$k = c \cdot D_{10}^2$$

Casagrande:
$$k = 1.4e^2 k_{0.85}$$

Kozeny-Carman:
$$k = C_1 \cdot \frac{e^2}{1+e}$$

Samarasinhe:
$$k = C_3 \cdot \frac{e^n}{1+e}$$

Stratified Soil



for Parallel flow:
$$k_{eq} = \frac{h_1k_1 + h_2k_2 + \ldots + h_nk_n}{H}$$

for Perpendicular flow:
$$k_{eq} = \frac{H}{\frac{h_1}{k_1} + \frac{h_2}{k_2} + \ldots + \frac{h_n}{k_n}}$$

Dr (%)	Description
0 – 20	Very Loose
20 – 40	Loose
40 – 70	Medium Dense
70 – 85	Dense
85 – 100	Very Dense

Pumping Test:

Unconfined:
$$k = \frac{Q\ln\frac{r_1}{r_2}}{\pi(h_1^2 - h_2^2)}$$

Confined:
$$k = \frac{Q\ln\frac{r_1}{r_2}}{2\pi t(h_1 - h_2)}$$

Sieve Analysis

Uniformity Coefficient:
$$Cu = \frac{D_{60}}{D_{10}}$$

Coeff. of Gradation or Curvature:
$$Cc = \frac{(D_{30})^2}{D_{60} \cdot D_{10}}$$

Sorting Coefficient:
$$So = \sqrt{\frac{D_{75}}{D_{25}}}$$

Suitability Number:
$$Sn = 1.7\sqrt{\frac{3}{(D_{50})^2} + \frac{1}{(D_{20})^2} + \frac{1}{(D_{10})^2}}$$

Stresses in Soil

Effective Stress/ Intergranular Stress:
$$p_E = p_T - p_W$$

Pore Water Pressure/ Neutral Stress:
$$p_W = \gamma_w h_w$$

Total Stress:
$$p_T = \gamma_1h_1 + \gamma_2h_2 + \ldots + \gamma_nh_n$$

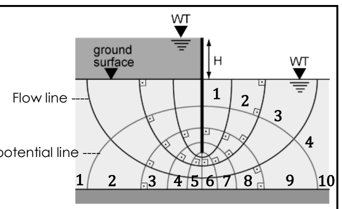
NOTE: Quick condition:
$$p_E = 0$$

Capillary Rise:
$$h_{cr} = \frac{C}{eD_{10}}$$

Flow Net / Seepage

Isotropic soil:
$$q = kH\frac{N_f}{N_d}$$

Non-Isotropic soil:
$$q = \sqrt{k_xk_z}H\frac{N_f}{N_d}$$



Nf → no. of flow channels [e.g. 4]
Nd → no. of potential drops [e.g. 10]

Compressibility of Soil

Compression Index, Cc:
$$C_c = 0.009(LL - 10\%)$$

$$C_c = \frac{e - e'}{\log\frac{\Delta P + P_o}{P_o}}$$

Swell Index, Cs:
$$C_s = \frac{1}{5}C_c$$

For normally consolidated clay:
$$S = \frac{e - e'}{1+e}H \quad \text{(for one layer only)}$$

$$S = \frac{C_cH}{1+e}\log\frac{\Delta P + P_o}{P_o}$$

With Pre-consolidation pressure, Pc:

when (ΔP+Po) < Pc:
$$S = \frac{C_sH}{1+e_o}\log\frac{\Delta P + P_o}{P_o}$$

when (ΔP+Po) > pc:
$$S = \frac{C_sH}{1+e}\log\frac{P_c}{P_o} + \frac{C_cH}{1+e}\log\frac{\Delta P + P_o}{P_c}$$

Over Consolidation Ratio (OCR):
$$OCR = \frac{p_c}{p_o}; \quad OCR = 1 \text{ (for normally consolidated soil)}$$

Coefficient of Compressibility:
$$a_v = \frac{\Delta e}{\Delta P}$$

Δe → change in void ratio
ΔP → change in pressure

Coefficient of Volume Compressibility:
$$m_v = \frac{\frac{\Delta e}{\Delta P}}{1 + e_{ave}}$$

Coefficient of Consolidation:
$$C_v = \frac{H_{dr}^2 T_v}{t}$$

Hdr → height of drainage path
→ thickness of layer if drained 1 side
→ half of thickness if drained both sides
Tv → factor from table
t → time consolidation

Coefficient of Permeability:
$$k = m_vC_v\gamma_w$$

Lateral Earth Pressure

AT REST:
$$k_o = 1 - \sin\emptyset$$

ACTIVE PRESSURE:
$$p_a = \frac{1}{2}k_a\gamma H^2 - 2cH\sqrt{k_a}$$

For Inclined:
$$k_a = \cos\beta\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset}}$$

For Horizontal:
$$k_a = \frac{1 - \sin\emptyset}{1 + \sin\emptyset}$$

If there is angle of friction α bet. wall and soil:
$$k_a = \frac{\cos^2\emptyset}{\cos\alpha\left[1 + \sqrt{\frac{\sin(\emptyset + \alpha)\sin\emptyset}{\cos\alpha}}\right]^2}$$

PASSIVE PRESSURE:
$$p_P = \frac{1}{2}k_P\gamma H^2 + 2cH\sqrt{k_P}$$

For Inclined:
$$k_P = \cos\beta\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset}}$$

For Horizontal:
$$k_P = \frac{1 + \sin\emptyset}{1 - \sin\emptyset}$$

If there is angle of friction α bet. wall and soil:
$$k_P = \frac{\cos^2\emptyset}{\cos\alpha\left[1 - \sqrt{\frac{\sin(\emptyset - \alpha)\sin\emptyset}{\cos\alpha}}\right]^2}$$

Shear Strength of Soil

θ → angle of failure in shear
Ø → angle of internal friction/shearing resistance
C → cohesion of soil

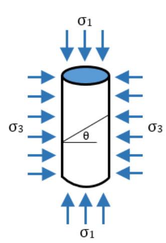
$$\theta = 45° + \frac{\emptyset}{2}$$

TRI-AXIAL TEST:

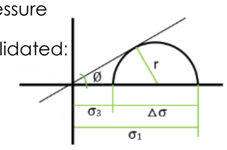
σ1 → maximum principal stress
→ axial stress

Δσ → additional pressure
→ deviator stress
→ plunger pressure

σ3 → minimum principal stress
→ confining pressure
→ lateral pressure
→ radial stress
→ cell pressure
→ chamber pressure

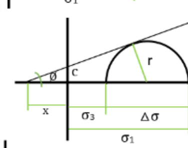


⊙ Normally consolidated:
$$\sin\emptyset = \frac{r}{\sigma_3 + r}$$

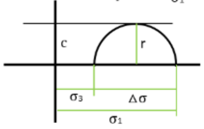


⊙ Cohesive soil:
$$\sin\emptyset = \frac{r}{x + \sigma_3 + r}$$

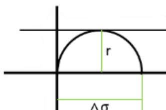
$$\tan\emptyset = \frac{c}{x}$$



⊙ Unconsolidated-undrained test:
$$c = r$$

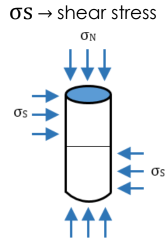


⊙ Unconfined compression test:
$$\sigma_3 = 0$$

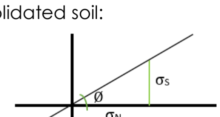


DIRECT SHEAR TEST:

σn → normal stress
σs → shear stress

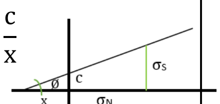


⊙ Normally consolidated soil:
$$\tan\emptyset = \frac{\sigma_S}{\sigma_N}$$



⊙ Cohesive soil:
$$\tan\emptyset = \frac{\sigma_S}{x + \sigma_N} = \frac{c}{x}$$

$$\sigma_S = c + \sigma_N\tan\emptyset$$



Terzaghi's Bearing Capacity (Shallow Foundations)

⊙ General Shear Failure
(dense sand & stiff clay)

Square Footing:
$$q_{ult} = 1.3cN_c + qN_q + 0.4\gamma BN_\gamma$$

Circular Footing:
$$q_{ult} = 1.3cN_c + qN_q + 0.3\gamma BN_\gamma$$

Strip Footing:
$$q_{ult} = cN_c + qN_q + 0.5\gamma BN_\gamma$$

⊙ Local Shear Failure
(loose sand & soft clay)

Square Footing:
$$q_{ult} = 1.3c'N_c' + qN_q' + 0.4\gamma BN_\gamma'$$

Circular Footing:
$$q_{ult} = 1.3c'N_c' + qN_q' + 0.3\gamma BN_\gamma'$$

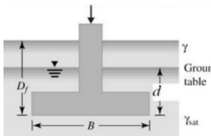
Strip Footing:
$$q_{ult} = c'N_c' + qN_q' + 0.5\gamma BN_\gamma'$$

⊙ Bearing Capacity Factor
$$N_q = \tan^2\left(45° + \frac{\emptyset}{2}\right)e^{\pi \tan \emptyset}$$
$$N_c = (N_q - 1)\cot \emptyset$$
$$N_\gamma = (N_q - 1)\tan 1.4\emptyset$$

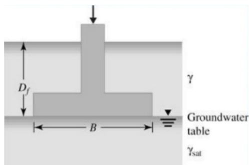
⊙ Parameters
q_{ult} → ultimate bearing capacity
q_u → unconfined compressive strength
c → cohesion of soil

$$c = \frac{q_u}{2}$$
$$q = \gamma D_f$$ (for no water table)
$$q_{allow} = \frac{q_{ult}}{FS} = \frac{P_{allow}}{A}$$
$$q_{net} = \frac{q_{ult} - q}{FS}$$

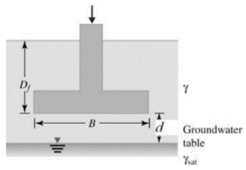
EFFECT OF WATER TABLE:



Case 1
$$q = \gamma(D_f - d) + \gamma' d$$
3rd term $\gamma = \gamma'$



Case 2
$$q = \gamma D_f$$
3rd term $\gamma = \gamma'$



Case 3
$$q = \gamma D_f$$
3rd term $\gamma = \gamma_{ave}$

for d ≤ B
$$\gamma_{ave} \cdot B = \gamma d + \gamma'(B - d)$$

for d > B
$$\gamma_{ave} = \gamma$$

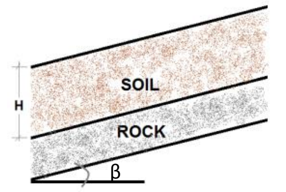
NOTE:
$$\gamma' = \gamma_{sub} = \gamma - \gamma_w$$

Soil Stability

⊙ Analysis of Infinite Slope

Factor of safety against sliding (without seepage)
$$FS = \frac{C}{\gamma H \sin\beta \cos\beta} + \frac{\tan\emptyset}{\tan\beta}$$

Factor of safety against sliding (with seepage)
$$FS = \frac{C}{\gamma_{sat} H \sin\beta \cos\beta} + \frac{\gamma'}{\gamma_{sat}}\frac{\tan\emptyset}{\tan\beta}$$

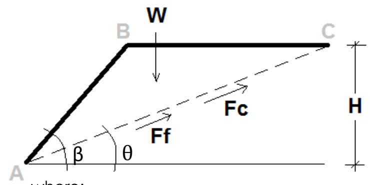


where:
C → cohesion
β → angle of backfill from horizontal
Ø → angle of internal friction
H → thickness of soil layer

⊙ Analysis of Finite Slope

Factor of safety against sliding
$$FS = \frac{F_f + F_c}{W \sin\theta}$$

Maximum height for critical equilibrium
(FS=1.0)
$$H_{cr} = \frac{4C}{\gamma}\left[\frac{\sin\beta\cos\emptyset}{1 - \cos(\beta - \emptyset)}\right]$$



where:
Ff → frictional force; Ff = μN
Fc → cohesive force
Fc = C x Area along trial failure plane
W → weight of soil above trial failure plane

Stability No.:
$$m = \frac{C}{\gamma H}$$

Stability Factor:
$$SF = \frac{1}{m}$$

$$\frac{H}{\tan\theta} - \frac{H}{\tan\beta} = BC$$

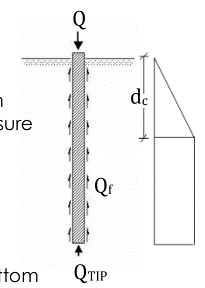
Capacity of Driven Piles (Deep Foundations)

⊙ Pile in Sand Layer
$$Q_f = PAk\mu$$
where:
P → perimeter of pile
A → area of pressure diagram
k → coefficient of lateral pressure
μ → coefficient of friction

$$Q_{tip} = p_e N_q A_{tip}$$
(AKA Qbearing)

where:
p_e → effective pressure at bottom
N_q → soil bearing factor
A_{tip} → Area of tip

$$Q_T = Q_f + Q_{tip}$$
$$Q_{des} = \frac{Q_T}{F.S.}$$



Critical depth, d_c:
Loose 10 (size of pile)
Dense 20 (size of pile)

⊙ Pile in Clay Layer
$$Q_f = CL\alpha P$$
where:
C → cohesion
L → length of pile
α → frictional factor
P → perimeter of pile

$$Q_{tip} = cN_c A_{tip}$$
(AKA Qbearing)

where:
c → cohesion
N_c → soil bearing factor
A_{tip} → Area of tip

$$Q_T = Q_f + Q_{tip}$$
$$Q_{des} = \frac{Q_T}{F.S.}$$

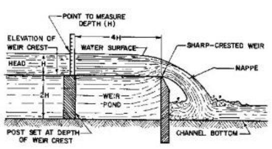
Group of Piles

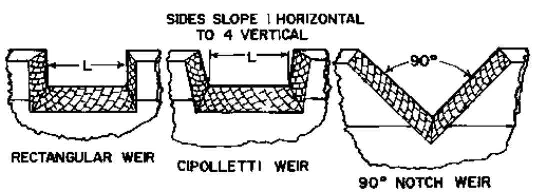
⊙ Group Efficiency (sand or clay)
$$Eff = \frac{Q_{des-group}}{Q_{des-indiv}}$$

⊙ Alternate Equation for Group Efficiency (sand only)
$$Eff = \frac{2(m + n - 2)s + 4d}{m\, n\, \pi\, D}$$

where:
m → no. of columns
n→ no. of rows
s → spacing of piles
D → diameter of pile

Weirs





⊙ **Rectangular**

Considering velocity of approach:
$$Q = \frac{2}{3}C\sqrt{2g}\,L\left[\left(H + \frac{v_a}{2g}\right)^{3/2} - \left(\frac{v_a}{2g}\right)^{3/2}\right]$$

Neglecting velocity of approach:
$$Q = \frac{2}{3}C\sqrt{2g}\;L\,H^{3/2}$$

Considering velocity of approach:
$$Q = m\,L\left[\left(H + \frac{v_a}{2g}\right)^{3/2} - \left(\frac{v_a}{2g}\right)^{3/2}\right]$$

Neglecting velocity of approach:
$$Q = m\,L\,H^{3/2}$$

Francis Formula *(when C and m is not given)*
Considering velocity of approach:
$$Q = 1.84\,L'\left[\left(H + \frac{v_a}{2g}\right)^{3/2} - \left(\frac{v_a}{2g}\right)^{3/2}\right]$$

Neglecting velocity of approach:
$$Q = 1.84\,L'\,H^{3/2}$$

NOTE:
L' = L for suppressed
L' = L – 0.1H for singly contracted
L' = L – 0.2H for doubly contracted

Time required to discharge:
$$t = \frac{2A_s}{mL}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

where:
W → channel width
L → weir length
Z → weir height
H → weir head

PARAMETERS:
C → coefficient of discharge
v_a → velocity of approach m/s
m → weir factor

⊙ **Triangular** (symmetrical only)
$$Q = \frac{8}{15}C\sqrt{2g}\,\tan\frac{\theta}{2}\,H^{5/2}$$
$$Q = m\,H^{5/2}$$

When θ=90°
$$Q = 1.4H^{5/2}$$

⊙ **Cipolletti** (symmetrical, slope 4V&1H)
θ = 75°57'50"
$$Q = 1.859\,L\,H^{3/2}$$

⊙ **with Dam:**
Neglecting velocity of approach:
$$Q = 1.71\,L\,H^{3/2}$$

Froude Number

$$N_F = \frac{v}{\sqrt{gd_m}}$$
where:
v → mean velocity (Q/A)
g → 9.81 m/s²
d_m → hydraulic depth (A/B)
B → width of liquid surface

$$N_F = \sqrt{\frac{Q^2 \cdot B_c}{A_c^{\,3} \cdot g}}$$
Take note that it is only derived from the critical depth equation.

Critical Flow $N_F = 1$
Subcritical Flow $N_F < 1$
Supercritical Flow $N_F > 1$

Reynold's Number

$$N_R = \frac{Dv}{\upsilon} = \frac{Dv\rho}{\mu}$$

Laminar Flow ($N_R \le 2000$)
$$hf = \frac{64}{N_R}$$

Turbulent Flow ($N_R > 2000$)
$$hf = f\frac{L}{D}\frac{v^2}{2g}$$
$$hf = \frac{0.0826\,f\,L\,Q^2}{D^5}$$

Boundary Shear Stress
$$\tau = \gamma RS$$

Boundary Shear Stress
(for circular pipes only)
$$\tau_o = \frac{f}{8}\rho v$$

Critical Depth

For all sections:
$$\frac{Q^2}{g} = \frac{A_c^{\,3}}{B_c}$$
where:
Q → flow rate m³/s
g → 9.81 m/s²
A_c → critical area
B_c → critical width

NOTE:
E is minimum for critical depth.

For rectangular sections **ONLY**:
$$d_c = \sqrt[3]{\frac{q^2}{g}} = \frac{2}{3}E_c$$
$$q = \frac{Q}{B}$$
$$E_c = \frac{v^2}{2g} + d_c$$
$$v_c = \sqrt{gd_c}$$

where:
q → flow rate or discharge per meter width
E_c → specific energy at critical condition
v_c → critical velocity

Hydraulic Jump

Height of the jump:
$$\Delta d = d_2 - d_1$$

Power Lost:
$$P = Q\gamma E$$

Length of the jump:
$$L = 220\,d_1 \tanh\frac{N_{F1} - 1}{22}$$

Solving for Q:

For all sections:
$$P_2 - P_1 = \frac{\gamma Q}{g}(v_1 - v_2)$$
$$P = \gamma \bar{h} A$$

For rectangular sections **ONLY**:
$$\frac{q^2}{g} = \frac{1}{2}(d_1 \cdot d_2)(d_1 + d_2)$$

Load Combinations

→ choose largest U in design

Basic Loads:

$$U = 1.4D + 1.7L$$

With Wind Load:

$$U = 0.75(1.4D + 1.7L + 1.7W)$$
$$U = 0.9D + 1.3W$$
$$U = 1.4D + 1.7L$$

With Earthquake Load:

$$U = 1.32D + 1.1f_1L + 1.1E$$
$$U = 0.99D + 1.1E$$

With Earth Pressure Load:

$$U = 1.4D + 1.7L + 1.7H$$
$$U = 0.9D$$
$$U = 1.4D + 1.7L$$

With Structural Effects:

$$U = 0.75(1.4D + 1.7L + 1.4T)$$
$$U = 1.4(D + T)$$

Strength Reduction Factors, Ø

(a) Flexure w/o axial load **0.90**
(b) Axial tension & axial tension w/ flexure **0.90**
(c) Axial comp. & axial comp. w/ flexure:
  (1) Spiral **0.75**
  (2) Tie **0.70**
(d) Shear & torsion **0.85**
(e) Bearing on concrete **0.70**

Design Conditions

Over-reinforced:
→ concrete fails first
→ fs < fy (USD)
→ Ms > Mc (WSD)

Under-reinforced:
→ steel fails first
→ fs > fy (USD)
→ Ms < Mc (WSD)

Balance Condition:
→ concrete & steel simultaneously fail
→ fs = fy (USD)
→ Ms = Mc (WSD)

Values

Choose Smaller Value/ Round-down
→ Moment Capacity
→
→

Choose Larger Value/ Round-up
→
→

Working Strength Design (WSD) or Alternate Strength Design (ASD)

Allowable Stresses (if not given):

⊙ Horizontal members (i.e. beam, slab, footing, etc.)
$f_c = 0.45\,f'_c$
$f_s = 0.50\,f_y$

⊙ Vertical members (i.e. column, wall, etc.)
$f_c = 0.25\,f'_c$
$f_s = 0.40\,f_y$

where:
f'c → compressive strength of concrete at 28 days
fy → axial strength of steel

Structural Grade	ASTM Gr.33 / PS Gr.230	**fy = 230 MPa**
Intermediate Grade	ASTM Gr.40 / PS Gr.275	**fy = 275 MPa**
High Carbon Grade	ASTM Gr.60 / PS Gr.415	**fy = 415 MPa**

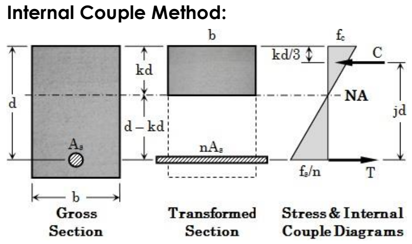
424.3.2 for fy = 275 MPa; fs ≤ 140 MPa
for fy = 415 MPa; fs ≤ 170 MPa

Modular Ratio, *n* (if not given):

$$n = \frac{E_{stronger}}{E_{weaker}} = \frac{E_{steel}}{E_{concrete}} = \frac{200{,}000}{4700\sqrt{f'_c}}$$

424.6.4 n must be taken as the nearest whole number & n ≥ 6
424.6.5 for doubly, use *n* for *tension* & use 2*n* for *compression*

Internal Couple Method:



Factor k:

$$k = \frac{n}{n + \frac{f_s}{f_c}}$$

Factor j:

$$j = 1 - \frac{1}{3}k$$

Moment Resistance Coefficient, R:

$$R = \tfrac{1}{2} f_c kj$$

Moment Capacity:

$$M_c = C \cdot jd = \tfrac{1}{2} f_c kdb \cdot jd = Rbd^2$$
$$M_s = T \cdot jd = A_s f_s \cdot jd$$

Provisions for Uncracked Section:

⊙ Solve for inertia of gross section, Ig.
⊙ Solve for cracking moment, Mcr.
⊙ Solve for actual moment, Ma:

$$M_a = \frac{wL^2}{8}$$ (for simply supported beam)

409.6.2.3. if Ma < Mcr, no crack; Ig = Ie
if Ma > Mcr, w/ crack; solve for Ie

⊙ Solve for inertia of cracked section:

$$I_{cr} = \frac{bx^3}{3} + nA_s(d - x)^2$$

⊙ Solve for effective moment of inertia, Ie:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr}$$

409.6.2.4. For simply supported, Ie = Ie (mid)
For cantilever, Ie = Ie (support)

$$I_e = \frac{I_{e\,mid} + I_{e\,support}}{2}$$

409.6.2.5. Factor for shrinkage & creep due to sustained loads:

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

time-dep factor, ξ:	
5 yrs +	2.0
12 mos	1.4
6 mos	1.0
3 mos	1.0

⊙ Solve for instantaneous deflection:

$$\delta_i = \frac{5wL^4}{384E_cI_e}$$ (for uniformly distributed load)

⊙ Solve for additional deflection:

$$\delta_{add} = \delta_{sus} \cdot \lambda$$
$$\delta_{add} = (\%\text{ of sustained load})\delta_i \cdot \lambda$$

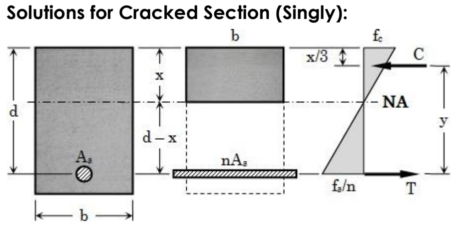
Say, 70% of load is sustained after n yrs.

$$\delta_{add} = 0.7\delta_i \cdot \lambda$$

⊙ Solve for final deflection:

$$\delta_{final} = \delta_i + \delta_{add}$$

Solutions for Cracked Section (Singly):



⊙ Location of neutral axis, NA:

$$A\bar{y}_{above\ NA} = A\bar{y}_{below\ NA}$$
$$bx\left(\frac{x}{2}\right) = nA_s(d - x)$$
$$x \rightarrow \text{obtained}$$

⊙ Solve transferred moment of inertia at NA:

$$I_{NA} = \frac{bx^3}{3} + nA_s(d - x)^2$$
$$I_{NA} \rightarrow \text{obtained}$$

⊙ Solve for Stresses or Resisted Moment:

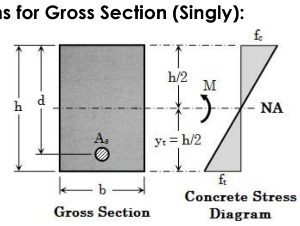
For concrete:

$$f_c = \frac{M_c \cdot x}{I_{NA}}$$

For tension steel:

$$\frac{f_s}{n} = \frac{M_s \cdot (d - x)}{I_{NA}}$$

Solutions for Gross Section (Singly):



⊙ Location of neutral axis, NA:

$$y_t = \frac{h}{2};\, y_t \rightarrow \text{obtained}$$

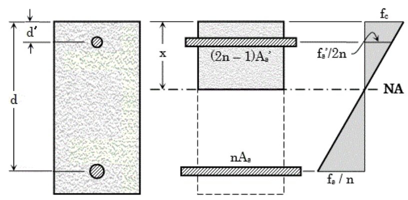
⊙ Solve moment of inertia of gross section at NA:

$$I_g = \frac{bx^3}{12}$$
$$I_g \rightarrow \text{obtained}$$

⊙ Solve for cracking moment:

$$f_r = 0.7\sqrt{f'_c} = \frac{M_{cr} \cdot y_t}{I_g}$$
$$M_{cr} \rightarrow \text{obtained}$$

Solutions for Cracked Section (Doubly):



⊙ Location of neutral axis, NA:

$$A\bar{y}_{above\ NA} = A\bar{y}_{below\ NA}$$
$$bx\left(\frac{x}{2}\right) + (2n - 1)A'_s(x - d') = nA_s(d - x)$$
$$x \rightarrow \text{obtained}$$

⊙ Solve transferred moment of inertia at NA:

$$I_{NA} = \frac{bx^3}{3} + (2n - 1)A'_s(x - d')^2 + nA_s(d - x)^2$$
$$I_{NA} \rightarrow \text{obtained}$$

⊙ Solve for Stresses or Resisted Moment:

For concrete:

$$f_c = \frac{M_c \cdot x}{I_{NA}}$$

For tension steel:

$$\frac{f_s}{n} = \frac{M_s \cdot (d - x)}{I_{NA}}$$

For comp. steel:

$$\frac{f'_s}{2n} = \frac{M'_s \cdot (x - d')}{I_{NA}}$$

Solutions for Uncracked Section (By Sir Erick):

⊙ Location of neutral axis, NA:

$$A\bar{y}_{above\ NA} = A\bar{y}_{below\ NA}$$
$$bx\left(\frac{x}{2}\right) = b(d - x)\left(\frac{d - x}{2}\right) + (n - 1)A_s(d - x)$$
$$x \rightarrow \text{obtained}$$

⊙ Solve transferred moment of inertia at NA:

$$I_{NA} = \frac{bx^3}{3} + \frac{b(d - x)^3}{3} + (n - 1)A_s(d - x)^2$$
$$I_{NA} \rightarrow \text{obtained}$$

⊙ Solve for Stresses or Resisted Moment:

For concrete:

$$f_c = \frac{M_c \cdot x}{I_{NA}}$$

For tension steel:

$$\frac{f_s}{n} = \frac{M_s \cdot (d - x)}{I_{NA}}$$

Ultimate Strength Design

⊙ Based in Strain Diagram:

$$\frac{\varepsilon_s}{d-c} = \frac{0.003}{c}$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right)$$

$$f_s = 600\left(\frac{d-c}{c}\right)$$

⊙ $a = \beta_1 c$

a → depth of compression block
c → distance bet. NA & extreme compression fiber

Provisions for β_1:

$$0.65 \le \beta_1 \le 0.85$$

* 1992 NSCP

$$\beta_1 = 0.85 - 0.008(f'_c - 30)$$

* 2001 NSCP

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 30)$$

* 2010 NSCP

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 28)$$

⊙ Ultimate Moment Capacity:

$$M_u = \emptyset M_n$$

$$M_u = \emptyset R_n b d^2$$

$$M_u = \emptyset f'_c b d^2\, \omega\left(1 - \frac{10}{17}\omega\right)$$

$$\omega = \rho \frac{f_y}{f'_c}$$

⊙ Coefficient of resistance, Rn:

$$R_n = f'_c \omega\left(1 - \frac{10}{17}\omega\right)$$

$$R_n = \frac{M_u}{\emptyset b d^2}$$

⊙ Steel reinforcement ratio, ρ:

$$\rho = \frac{A_s}{bd}$$

⊙ Combined ρ & Rn:

$$\rho = \frac{0.85f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right]$$

Steel Ratio

Steel ratio for balance condition:

$$\rho_b = \frac{0.85f'_c\beta_1 600}{f_y(600 + f_y)}$$

Maximum & Minimum steel ratio:

$$\rho_{max} = 0.75\rho_b$$

$$A_{s_{max}} = 0.75A_{s_b}$$

$$\rho_{min} = \frac{1.4}{f_y} \qquad \rho_{min} = \frac{\sqrt{f'_c}}{4f_y}$$

(choose larger between the 2)

Singly or Doubly?

Singly Reinforced Beam (SRB)

$\rho < \rho_{max}$ (rectangular only)

$A_s < A_{s_{max}}$ (any section)

Doubly Reinforced Beam (DRB)

$\rho > \rho_{max}$ (rectangular only)

$A_s > A_{s_{max}}$ (any section)

Minimum Concrete Covers:

20 mm → slab

40 mm → beam
→ column

75 mm → column footing
→ wall footing
→ retaining wall

Balance Condition for Doubly

$$\rho_{b_d} = \rho_{b_s} + \frac{A_s'}{bd}$$

$$\rho_{max_d} = 0.75\rho_{b_s} + \frac{A_s'}{bd}$$

$$A_{s_{max_d}} = \rho_{max_d} bd$$

Singly Reinforced Beam
INVESTIGATION

Computing M_U with given A_s:

(1st) Compute for a:

$$C = T$$

$$0.85f'_c ab = A_s f_s$$

(assume tension steel yields fs=fy)

$$0.85f'_c ab = A_s f_y$$

$$a \rightarrow \text{obtained}$$

(2nd) Check if assumption is correct:

$$a = \beta_1 c$$

$$c \rightarrow \text{obtained}$$

$$f_s = 600\left[\frac{d-c}{c}\right]$$

$$f_s \rightarrow \text{obtained}$$

If fs > fy, tension steel yields; correct a.
If fs < fy, tension steel does not yield; compute for new a.

(2nd-b) Recomputation:

$$C = T$$

$$0.85f'_c ab = A_s f_s$$

$$0.85f'_c\beta_1 cb = A_s \cdot 600\left[\frac{d-c}{c}\right]$$

$$c \rightarrow \text{obtained}$$

$$a = \beta_1 c$$

$$a \rightarrow \text{obtained}$$

(3rd) Solve for Moment Capacity:

$$M_u = \emptyset(C \text{ or } T)\left[d - \frac{a}{2}\right]$$

$$M_u = \emptyset(0.85f'_c ab)\left[d - \frac{a}{2}\right] \text{ or}$$

$$M_u = \emptyset(A_s f_s)\left[d - \frac{a}{2}\right]$$

Singly Reinforced Beam
DESIGN

Computing A_s with given W_D & W_L:

(1st) Compute ultimate moment, Mu:

$$W_U = 1.4W_D + 1.7W_L$$

$$M_U = \frac{W_U L^2}{8} \text{ (for simply supported)}$$

(2nd) Solve for coeff. of resistance, Rn:

$$R_n = \frac{M_U}{\emptyset b d^2}$$

(3rd) Solve for steel ratio, ρ:

$$\rho = \frac{0.85f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right]$$

Check:

$$\rho_{min} \le \rho \le \rho_{max}$$

If $\rho_{min} < \rho < \rho_{max}$, use ρ.
If $\rho_{min} > \rho$, use ρ_{min}.
If $\rho > \rho_{max}$, design doubly.

(4th) Solve for area of steel reinforcement, As and required no. of bars, N:

$$A_s = \rho bd$$

$$N = \frac{A_s}{A_b} = \frac{\rho bd}{\frac{\pi}{4}{d_b}^2}$$

Doubly Reinforced Beam
Investigation
if SRB or DRB:

(1st) Compute for a_b:

$$f_s = f_y = 600\left[\frac{d - c_b}{c_b}\right]$$

Thus,

$$c_b = \frac{600d}{600 + f_y}$$

$$c_b \rightarrow \text{obtained}$$

$$a_b = \beta_1 c_b$$

$$a_b \rightarrow \text{obtained}$$

(2nd) Solve for As_{max}:

$$C = T$$

$$0.85f'_c a_b b = A_{s_b} f_y$$

$$A_{s_b} \rightarrow \text{obtained}$$

$$A_{s_{max}} = 0.75A_{s_b}$$

(2nd) Solve for given As & compare:

If $A_s < A_{s_{max}}$
Solve the given beam using SRB Investigation procedure.

If $A_s > A_{s_{max}}$
Solve the given beam using DRB Investigation procedure.

Doubly Reinforced Beam
INVESTIGATION

Computing M_U with given A_s:

(1st) Compute for a:

$$C_c + C_s = T$$

$$0.85f'_c ab + A_s' f_s' = A_s f_s$$

(assume tension steel yields fs=fs'=fy)

$$0.85f'_c ab + A_s' f_y = A_s f_y$$

$$a \rightarrow \text{obtained}$$

(2nd) Check if assumption is correct:

$$a = \beta_1 c$$

$$c \rightarrow \text{obtained}$$

$$f_s = 600\left[\frac{d-c}{c}\right]$$

$$f_s \rightarrow \text{obtained}$$

If fs > fy, tension steel yields; correct a.
If fs < fy, tension steel does not yield; compute for new a.

$$f_s' = 600\left[\frac{c - d'}{c}\right]$$

$$f_s' \rightarrow \text{obtained}$$

If fs' > fy, compression steel yields; correct a.
If fs' < fy, compression steel does not yield; compute for new a.

(2nd-b) Recomputation:

$$C = T$$

$$0.85f'_c ab + A_s' f_s' = A_s f_s$$

NOTE: Use fs & fs' as

$$f_s = 600\left[\frac{d-c}{c}\right]$$

$$f_s' = 600\left[\frac{c-d'}{c}\right]$$

$$c \rightarrow \text{obtained}$$

$$a = \beta_1 c$$

$$a \rightarrow \text{obtained}$$

(3rd) Solve for Moment Capacity:

$$M_u = \emptyset C_c\left[d - \frac{a}{2}\right] + \emptyset C_c[d - d']$$

$$M_u = \emptyset(0.85f'_c ab)\left[d - \frac{a}{2}\right] + \emptyset(A_s' f_s')[d - d'] \text{ or}$$

$$M_u = \emptyset T\left[d - \frac{a}{2}\right]$$

$$M_u = \emptyset(A_s f_s)\left[d - \frac{a}{2}\right]$$

Doubly Reinforced Beam
DESIGN

Computing A_s with given Mu:

(1st) Solve for nominal M_1:

$$\rho_b = \frac{0.85f'_c\beta_1 600}{f_y(600 + f_y)}$$

$$\rho_{max} = 0.75\rho_b$$

$$A_{s1} = 0.75\rho_b \cdot bd$$

$$M_1 = (A_{s1} f_y)\left[d - \frac{a}{2}\right]$$

(2nd) Solve for nominal M_2:

$$M_2 = \frac{M_U}{\emptyset} - M_1$$

(3rd) Solve for As_2:

$$M_2 = (A_{s2} f_y)[d - d']$$

$$A_{s2} \rightarrow \text{obtained}$$

(4th) Solve for # of tension bars:

$$N = \frac{A_s}{A_b} = \frac{A_{s1} + A_{s2}}{\frac{\pi}{4}{d_b}^2}$$

(5th) Solve for fs':

$$f_s' = 600\left[\frac{c - d'}{c}\right]$$

If fs' > fy, compression steel yields;
As' = As2.
If fs' < fy, compression steel does not yield; **Use fs' to solve for As'.**

(6th) Solve for As':

$$A_s' f_s' = A_{s2} f_y$$

(7th) Solve for # of compression bars:

$$N = \frac{A_s}{A_b} = \frac{A_s'}{\frac{\pi}{4}{d_b}^2}$$

Design of Beam Stirrups

(1st) Solve for Vu:

$$\Sigma F_v = 0$$
$$V_u = R - w_u d$$
$$V_u = \frac{w_u L}{2} - w_u d$$

(2nd) Solve for Vc:

$$V_c = \frac{1}{6}\sqrt{f_c'}\,b_w d$$

(3rd) Solve for Vs:

$$V_u = \emptyset(V_c + V_s)$$
$$V_s \rightarrow \text{obtained}$$

(4th) Theoretical Spacing:

$$s = \frac{dA_v f_{y_n}}{V_s}$$

NSCP Provisions for max. stirrups spacing:

$$2V_c = \frac{1}{3}\sqrt{f_c'}\,b_w d$$

i. when Vs < 2Vc,

$$s_{max} = \frac{d}{2} \text{ or } 600\text{mm}$$

ii. when Vs > 2Vc,

$$s_{max} = \frac{d}{4} \text{ or } 300\text{mm}$$

iii. & not greater than to:

$$s_{max} = \frac{3A_v f_{y_n}}{b}$$

NOTE:

f_{y_n} → steel strength for shear reinforcement
A_v → area of shear reinforcement
n → no. of shear legs

$$A_v = \frac{\pi}{4} d^2 \cdot n$$

T-Beam

NSCP Provisions for effective flange width:

i. Interior Beam:

$$b_f = \frac{L}{4}$$
$$b_f = b_w + \frac{s_1}{2} + \frac{s_2}{2}$$
$$b_f = b_w + 8t_f$$

ii. exterior Beam:

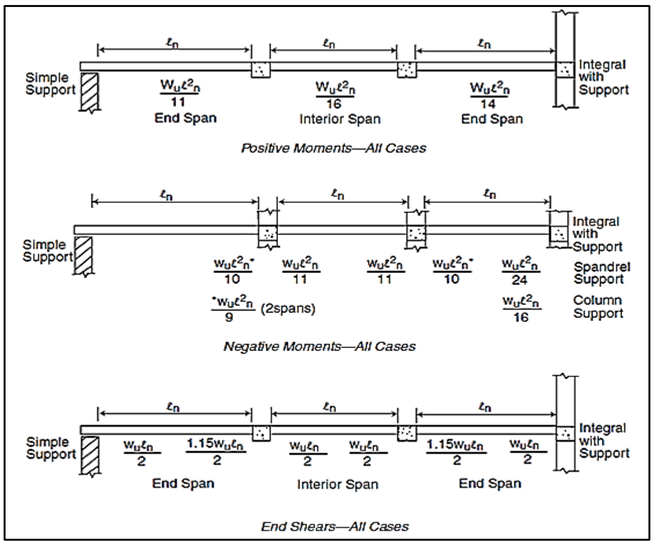
$$b_f = b_w + \frac{L}{12}$$
$$b_f = b_w + \frac{s_1}{2}$$
$$b_f = b_w + 6t_f$$

Thickness of One-way Slab & Beam

NSCP Provisions for minimum thickness:

	Canti-lever	Simple Support	One End	Both Ends
Slab	**L/10**	**L/20**	**L/24**	**L/28**
Beams	**L/8**	**L/16**	**L/18.5**	**L/21**

Factor: $\left[0.4 + \frac{f_y}{700}\right]$ $[1.65 - 0.0003\rho_c]$ (for lightweight concrete only)



Minimum Steel Ratio

For one-way bending:
k → steel ratio

i. fy = 275 MPa,
$$k = 0.0020$$
ii. fy = 415 MPa,
$$k = 0.0018$$
iii. fy > 415 MPa,
$$k = 0.0018\left[\frac{400}{f_y}\right]$$

For two-way bending:
ρ → steel ratio

$$\rho_{min} = \frac{1.4}{f_y} \quad \rho_{min} = \frac{\sqrt{f_c'}}{4f_y}$$

(choose larger between the 2)

Design of One-way Slab

(1st) Compute ultimate moment, Mu:

$$W_U = 1.4W_D + 1.7W_L$$
$$M_U = \frac{W_U L^2}{8}$$

(2nd) Solve for slab thickness, h:
See NSCP Provisions for minimum thickness.

(3rd) Solve for effective depth, d:

$$d = h - cc - \frac{d_b}{2}$$

(4th) Solve for a:

$$M_u = \emptyset(C)\left[d - \frac{a}{2}\right]$$
$$M_u = \emptyset(0.85f_c'ab)\left[d - \frac{a}{2}\right]$$
$$a \rightarrow \text{obtained}$$

(5th) Solve for As:

$$C = T$$
$$0.85f_c'ab = A_s f_y$$
$$A_s \rightarrow \text{obtained}$$

LONGITUDINAL OR MAIN BARS

(6th) Compute steel ratio, ρ:

$$\rho = \frac{A_s}{bd}$$

(7th) Check for minimum steel ratio:

$$\rho_{min} = \frac{1.4}{f_y} \;\&\; \rho_{min} = \frac{\sqrt{f_c'}}{4f_y}$$

If $\rho_{min} < \rho$, use ρ.
If $\rho_{min} > \rho$, use ρ_{min} & recompute As.

(8th) Determine # of req'd main bars:

$$N = \frac{A_s}{A_b} = \frac{A_s}{\frac{\pi}{4}{d_b}^2}$$

(9th) Determine spacing of main bars:

$$s = \frac{b}{N}$$

(10th) Check for max. spacing of main bars:

$$s_{max} = 3h \text{ or } 450\text{mm}$$

TEMPERATURE BARS/ SHRINKAGE BARS

(11th) Solve for As:

$$A_s = kb_{\perp}h$$

NSCP Provision for k:
i. fy = 275 MPa, **k = 0.0020**
ii. fy = 415 MPa, **k = 0.0018**
iii. fy > 415 MPa, **k = 0.0018 (400/fy)**

(12th) Determine # of req'd temp. bars:

$$N = \frac{A_s}{A_b} = \frac{A_s}{\frac{\pi}{4}{d_b}^2}$$

(13th) Determine spacing of temp. bars:

$$s = \frac{b}{N}$$

(14th) Check for max. spacing of temp. bars:

$$s_{max} = 5h \text{ or } 450\text{mm}$$

Design of Column

$$P = P_C + P_S$$
$$P = 0.85f_c'(A_g - A_{st}) + A_{st}f_y$$
$$\rho = \frac{A_{st}}{A_g}$$

Thus,

$$A_g = \frac{P}{0.85f_c'(1-\rho) + \rho f_y}$$
$$0.01A_g < A_{st} < 0.08A_g$$

TIED COLUMN

$$P_N = 0.8P$$
$$P_U = \emptyset 0.8P\,;\, \emptyset = 0.7$$
$$P_U = (0.7)(0.8)[0.85f_c'(A_g - A_{st}) + A_{st}f_y]$$

No. of main bars:

~~$N = \frac{A_{st}}{A_b}$~~

N is based on Pu.

Spacing of bars:

$$s = 16d_b$$
$$s = 48d_t$$
$$s = \text{least dimension}$$

NOTE: If spacing of main bars < 150mm, use 1 tie per set.

SPIRAL COLUMN

$$P_N = 0.85P$$
$$P_U = \emptyset 0.85P\,;\, \emptyset = 0.75$$
$$P_U = (0.75)(0.85)[0.85f_c'(A_g - A_{st}) + A_{st}f_y]$$
$$\rho_s = 0.45\frac{f_c'}{f_y}\left[\frac{A_g}{A_c} - 1\right] = \frac{\text{volume of spiral}}{\text{volume of core}}$$
$$s = \frac{\frac{\pi}{4}(d_{sp})^2 \cdot \pi(D_c - d_{sp})}{\frac{\pi}{4}(D_c)^2 \cdot \rho_s} = \frac{4A_{sp}}{D_c\rho_s}$$

Design of Footing

$$q_A = q_S + q_C + q_{sur} + q_E$$
$$q_E = \frac{P}{A_{ftg}} \quad ; \quad q_U = \frac{P_U}{A_{ftg}}$$

where:
q_A → allowable bearing pressure
q_S → soil pressure
q_C → concrete pressure
q_{sur} → surcharge
q_E → effective pressure
q_U → ultimate bearing pressure
Ø = 0.85

WIDE BEAM SHEAR

$$V_{U1} = q_U(B)(x)$$
$$V_{U1} \le \emptyset V_{wb} = \emptyset\frac{\sqrt{f_c'}}{6}Bd$$
$$\tau_{wb} = \frac{V_{U1}}{\emptyset Bd}$$
$$\tau_{wb(allw)} = \frac{\sqrt{f_c'}}{6}$$

PUNCHING/DIAGONAL TENSION SHEAR

$$V_{U2} = P_U - q_U(a+d)(b+d)$$
$$V_{U2} \le \emptyset V_{pc} = \emptyset\frac{\sqrt{f_c'}}{3}b_o d$$
$$\tau_{pc} = \frac{V_{U2}}{\emptyset b_o d}$$
$$\tau_{pc(allw)} = \frac{\sqrt{f_c'}}{3}$$

BENDING MOMENT

$$M_U = q_U(B)(x)\left(\frac{x}{2}\right)$$

** design of main bars and temperature bars – Same as slab.