SAMPLE PROBLEMS IN PROBABILITY

One can not go about without statistics.

"*Statistics are like bikinis. What they reveal is suggestive, but what they conceal is vital.*" – Aaron Levenstein

"*There are three kinds of lies: lies, damned lies, and statistics*" – Mark Twaine

An Approach to Solve the Questions

If sample was chosen through chance processes, we have to understand the notion of probability and sampling distribution.

To introduce probability….

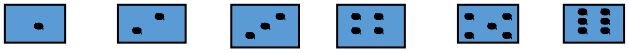
- ✓ Random experiment
- ✓ Sample space
- ✓ Event as subset of sample space
- ✓ Likelihood of an event to occur - probability of an event

EXAMPLE: Random Experiment

- Rolling a die and observing the number of dots on the upturned face
- Tossing a one-peso coin and observing the upturned face
- Measuring the height of a student enrolled this term

SAMPLE SPACE

Rolling a die and observing the number of dots on the upturned face

S={  }

S={1, 2, 3, 4, 5, 6}

EVENT

An event of observing odd-number of dots in a roll of a die

E_1 = { 1, 3, 5}

An event of observing even-number of dots in a roll of a die

E_2 = { 2, 4, 6}

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Computing Probability: A Priori

Example: If we select a card at random from a well-shuffled deck of cards then,
P(ace in a deck of cards) = 4/52

Computing Probability

Example:

- P(King or Spade)

  = P(King) + P(Spade) - P(King and Spade)

  $= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

- P(King or Queen) = P(King) + P(Queen)

  $= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$ =

Example:

SM Megamall has been the target of many shoplifters during March, but owing to increased security measures, 250 shoplifters were caught. Below is the summary of some characteristics of the shoplifters.

Sex	First-time offender	Repeat offender	Total
Male	85	60	145
Female	64	41	105
Total	149	101	250

Assuming that the shoplifter was chosen at random, what is:
(a) The probability that the shoplifter is female?
(b) The probability that the shoplifter is a female, given that he is a repeat offender?

a - $P(female) = \frac{105}{250} = 0.42$

b - $P(female|repeat) = \frac{41}{101} = 0.406$

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Example:

The weekly total long distance telephone charges of a travel agency follow a normal distribution with mean P2,750 and standard deviation P200.

(a) Find the probability that in a randomly selected week the total charges will be: (i) less than P3,000; (ii) more than P500; and (iii) between P2,250 and P2,950.

(b) Find the 75th percentile of the distribution. Interpret its meaning.

(c) Within what interval centered at the mean will the weekly total charges fall with 50% probability?

Let X be total long distance telephone charges

a (i) $P(X<3000)=P(Z<\frac{3000-2750}{200})=P(Z<1.25)=0.8944$

(ii) $P(X>500)=P(Z<\frac{500-2750}{200})=P(Z>-11.25)=0.9999$

(iii)
$$P(2250\le X\le 2950)=P\left(\frac{2250-2750}{200}\le Z\le\frac{2950-2750}{200}\right)$$
$$=P(-2.5<Z<1)=P(Z\le 1)-P(Z\le -2.5)$$
$$=0,841345-0.00621=0.835135$$

b. The 75th percentile is that particular weekly total long distance telephone charges such that 75% of all the other charges are below this weekly total long distance telephone charge. Hence,

$$P(X\le x)=0.75\Leftrightarrow P(Z\le z)=0.75\quad\Rightarrow z=0.6745$$
$$\Rightarrow 0.6745=\frac{x\text{-}2750}{200}\quad\Rightarrow x=0.6745\times 200+2750=2884.90$$

c.
$$P(x_1\le X\le x_2)=0.5\Leftrightarrow P(-z\le Z\le z)=0.5\Rightarrow P(Z\le z)=0.75\Rightarrow z=0.6745$$
$$\Rightarrow x_1=-0.6745\times 200+2750=2615.1;\quad x_2=0.6745\times 200+2750=2884.9$$

Example: How many ways can the six volumes of Harry Potter books be piled up so that the latest volume (the sixth) is always on top? How about when the last two volumes need to be grouped together but not necessarily in order.

Latest volume on top: Using the permutation rule, 5P5=120. This is because we treat the problem as to having only 5 items because the last should always be in place.

*Last two volumes grouped together: Using the permutation rule, (5P5)*2=120*2=240.*

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Example: In a certain city, the distribution of the first- and second-born children of two-child families by sex is shown below.

First-born	Second-born		Total
	Female	Male	
Female	540	512	1052
Male	492	456	948
Total	1032	968	2000

A family in this city with two children has a male first-born. What is the probability that its second-born is female?

$$P(\text{second child is female}\,|\,\text{first born is male}) = \frac{492}{948} = 0.519$$

Example:

Some descriptive statistics for a set of test scores are shown below. For this test, a certain student has a standardized score of $z = -1.2$. What was the raw score of this student?

DESCRIPTIVE STATISTICS

Variable	N	Mean	Median	StDev	SE Mean
Score	50	1045.7	1024.7	221.9	31.4

Variable	Minimum	Maximum	Q1	Q3
Score	628.9	1577.1	877.7	1219.5

$$\text{standard score} = \frac{\text{raw score-mean}}{\text{standard deviation}}$$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow X = Z\sigma + \mu$$

$$X = -1.2 \times 221.9 + 1045.7$$
$$= -266.28 + 1045.7$$
$$= 779.42$$

Example:

What is the probability that a cashier gives you a counterfeit coin that weighs more than 25?

$$P(X > 25) = P\left(Z > \frac{25-23}{5}\right) = P(Z > 0.4) = 1 - 0.6554 = 0.3446$$

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