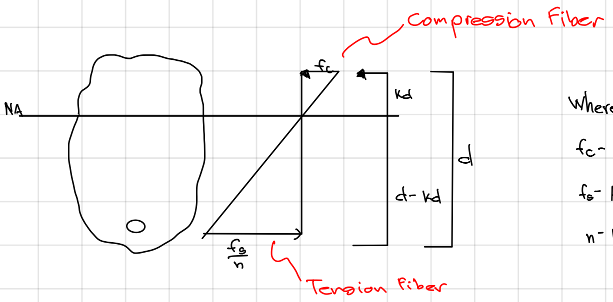
* Working Strength Design (WSD)



Where:

f_c - Allowable Stress of Concrete

f_s - Allowable Stress of Steel

n - Modular Ratio

- Allowable Stress:

1.) Horizontal Members (Beams, Slabs, Footing, etc)

$f_s = 0.45 f'_c$ Compression strength of concrete @ 28days

$f_s = 0.5 f_y$ (for slab reinforcement, $d_b \leq 10$ mm, $f_y \leq 200$ MPa)

$f_s = 140$ MPa (for $f_y = 280$ MPa)

$f_s = 170$ MPa (for $f_y = 420$ MPa)

2.) Vertical Member (Columns, Walls, etc)

$f_c = 0.25 f'_c$

$f_s = 0.4 f_y$

429.4.2 Tensile stress in reinforcement f_s shall not exceed the following:

1. Grade 280 reinforcement .... 140 MPa
2. Grade 420 reinforcement or greater and welded wire fabric (plain or deformed) .... 170 MPa
3. For flexural reinforcement, $\phi 10$ mm or less, in one-way slabs of not more than 4 m span but not greater than 200 MPa .... $0.50 f_y$

429.6.2 Stress-strain relationship of concrete is a straight line under service loads within permissible service load stresses.

429.6.3 In reinforced concrete members, concrete resists no tension.

$S = 200 \times 10^3$ MPa

modulus of elasticity

429.6.4 It shall be permitted to take the modular ratio, $n = E_s/E_c$, as the nearest whole number (but not less than 6). Except in calculations for deflections, value of n for lightweight concrete shall be assumed to be the same as for normal weight concrete of the same strength.

for tention bar

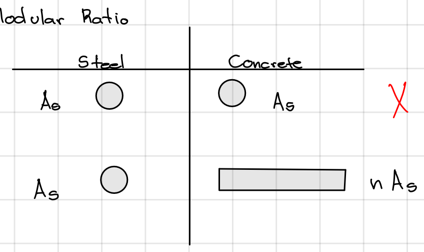
for compression bar

429.6.5 In doubly reinforced flexural members, an effective modular ratio of $2E_s/E_c$ shall be used to transform compression reinforcement for stress computations. Compressive stress in such reinforcement shall not exceed permissible tensile stress.

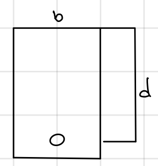
- Solution / Analysis

1. ΣAy above NA $= \Sigma Ay$ below NA
2. $I_T = \Sigma I_{NA} + Ad^2$
3. $F_B = \frac{Mc}{I}$

Modular Ratio



Sit 1



(S1) For the A of reinforcement

$A_S = \frac{\pi}{4} d_B{}^2 N$

$= \frac{\pi}{4}(32)^2(5)$

$= 1280\pi \ mm^2$

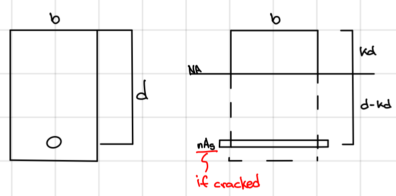
For steel ratio

$\rho = \frac{A_S}{bwd}$

$= \frac{1280\pi}{300(580)}$

$\rho = 0.02311$

Composite $\xrightarrow[\text{Transform section}]{\text{Cracked}}$ Homogenous Beam



$K = -np + \sqrt{n^2p^2 + 2np}$ ~ for rectangular beam only

$= -9(0.02311) + \sqrt{9^2(0.02311)^2 + 2(9)(0.02311)}$

$k = 0.46969(580)$

$kd = 272.418 \ mm$

Derive of k formula

$\Sigma Ay_{\text{above NA}} = \Sigma Ay_{\text{below NA}}$

$b\,kd\left(\frac{kd}{2}\right) = nA_s$

$\frac{bk^2d^2}{2} = nA_s d(1-k)$

$k^2 = \frac{2nA_s d}{bd^2}(1-k)$

$k^2 = \frac{2nA_s}{bd}(1-k)$

$k^2 = 2n\rho(1-k)$

$k^2 = 2n\rho - 2n\rho k$

$k^2 + 2n\rho k - 2n\rho = 0$

$k = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{-(2n\rho) \pm \sqrt{(2n\rho)^2 - 4(1)(-2\rho)}}{2(1)}$

$= \frac{-2n\rho \pm \sqrt{4n^2\rho^2 + 4(2n\rho)}}{2}$

$k = -n\rho + \sqrt{n^2\rho^2 + 2n\rho}$ //

(S2)

$\Sigma Ay_{\text{above NA}} = \Sigma Ay_{\text{below NA}}$

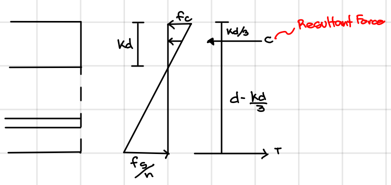
$b(kd)\left(\frac{kd}{2}\right) = nA_s(d-kd)$

$300\,kd\left(\frac{kd}{2}\right) = 9(1280\pi)(580-kd)$

$kd = 272.418 \ mm$

$K = \frac{272.418}{2} =$

2. Moment Capacity of the beam



(S1)

$j = 1 - \frac{k}{3}$

$j = 1 - \frac{0.46969}{3}$

$j = 0.84344$

Based on Concrete Strength

$M_c = \frac{1}{2} f_c (kd)(b)\left(d - \frac{kd}{3}\right)$

$= \frac{1}{2} f_c (kd^2)(b)\left(1 - \frac{k}{3}\right)$

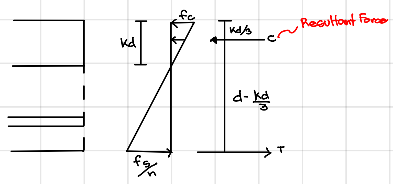
$= \frac{1}{2} f_c (kd^2)(b)(j)$

$= \frac{1}{2} f_c\, bd^2\, kj$

$= \frac{1}{2}\left[0.45(24)\right](300)(580)^2(0.46969)(0.84344)$

$= 215.8898 \ kN\cdot m$

Base on Steel Strength:



$\sigma = \frac{Py}{A}$

$P = \sigma A$

$M_s = nA_s\left(\frac{f_s}{n}\right)\left(d - \frac{kd}{3}\right)$

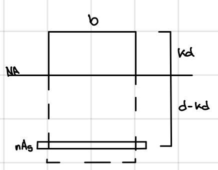
$= A_s f_s d\left(1 - \frac{k}{3}\right)$

$= A_s f_s jd$

$= 1280\pi(130)(0.84344)(580)$

$= 255.732 \ kN\cdot m$

(S2)



$I_T = \Sigma I_{NA} + Ad^2$

modular ratio, Moment of Inertia circle, no. of steel, Ad²

$= \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2 + (n)\left(\frac{\pi}{64} d^4\right)(N) + \underbrace{nA_s(d-kd)^2}_{Ad^2}$

$= \frac{b(kd)^3}{12} + b(kd)\left(\frac{kd}{2}\right)^2 + (n)\left(\frac{\pi}{64} d_B{}^4\right)(N) + nA_s(d-kd)^2$

Note: According to code do not include the I_{NA} of the reinforcement!

$(n)\left(\frac{\pi d_B^4}{64}\right)(N)$

$= \frac{b(kd)^3}{12} + \frac{b(kd)^3}{4} + nA_s(d-kd)^2$ → Multiply 2nd term by $\frac{3}{3}$ to transform to similar fraction

$= \frac{b(kd)^3}{3} + nA_s(d-kd)^2$

$= \frac{300(272.418)^3}{3} + 9(1280\pi)(580-272.418)^2$

$= 5445.582 \times 10^6 \ mm^4$

Based on Concrete Strength

$f_c = \frac{M_c(kd)}{I_T}$

$0.45 f'_c = \frac{M_c(kd)}{I_T}$

$0.45(24) = \frac{M_c(272.418)}{5445.582 \times 10^6}$

$M_c = 215.8898 \ kN\cdot m$

Based on Steel Strength

$\frac{f_s}{n} = \frac{M_s(d-kd)}{I_T}$

$\frac{130}{9} = \frac{M_s(580-272.418)}{5445.582 \times 10^6}$

$M_s = 255.732 \ kN\cdot m$

3.) Condition of the Beam

(a.) Over Reinforced

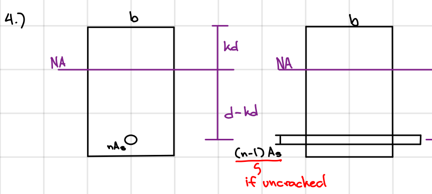
b) Under Reinforced

c) Balanced Condition

d.) Cannot be Determined

Over Reinforced	Under Reinforced	Balanced Condition
• Concrete fails first	• Steel fails first	• Concrete & Steel fail simultaneously
• $M_c < M_s$ (WSD)	• $M_c > M_s$ (WSD)	• $M_c = M_s$ (WSD)
• $f_s < f_y$ (USD)	• $f_s > f_y$ (USD)	• $f_s = f_y$ (USD)

4.)



$A_s = \frac{\pi}{4} d_B^2 N$

$A_s = \frac{\pi}{4}(28)^2(4)$

$= 784\pi \text{ mm}^2$

S_2

$\Sigma A_{above\ NA} = \Sigma A_{below\ NA}$

$b(kd)\left(\frac{kd}{2}\right) = b(h-kd)\left(\frac{h-kd}{2}\right) + nA_s(d-kd) - A_s(d-kd)$

$\frac{1}{2} b(kd)^2 = \frac{1}{2} b(h-kd)2 + (n-1)A_s(d-kd)$

$\frac{1}{2}(400)(kd)^2 = \frac{1}{2}(400)(600-kd)^2$

$kd = 313.405 \text{ mm}$

Moment of Inertia

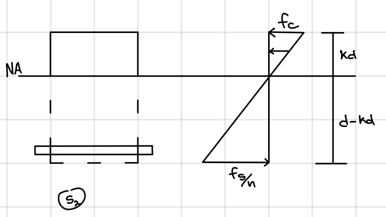
$I_T = \frac{b(kd)^3}{3} + \frac{b(h-kd)^3}{3} + (n-1)A_s(d-kd)^2$

$I_T = \frac{400(313.405)^3}{3} + \frac{400(600-313.405)^3}{3} + (8-1)A_s(500-313.405)^2$

$I_T = 7.843 \times 10^9 \text{ mm}^4$

Sit 2

5. Calculate the Reinforced steel area



(S_2)

By Similar Triangles

$\frac{kd}{f_c} = \frac{d-kd}{f_s/n}$

$\frac{kd}{7} = \frac{600-kd}{\frac{130}{10}}$

$kd = 210 \text{ mm}$

$\Sigma Ay_{above\ NA} = \Sigma Ax_{below\ NA}$

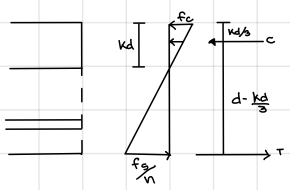
$b(kd)\left(\frac{kd}{2}\right) = nA_s(d-kd)$

$\frac{1}{2} b(kd)^2 = nA_s(d-kd)$

$\frac{1}{2}(350)(210)^2 = 10(A_s)(600-210)$

$A_s = 1{,}978.846 \text{ mm}^2$

6. Compressive Strength of Concrete



$\sigma = \frac{P}{A}$

$P = \sigma A$ for uniform stress only

(S_1)

Force = Vol. of pres. diagram

$C = \frac{1}{2} f_c (kd)(b)$

$C = \frac{1}{2}(7)(210)(350)$

$C = 257{,}250 \text{ N}$

(S_2)

$\Sigma F_H = 0$

$C = T$

$C = nA_s\left(\frac{f_s}{n}\right)$

$C = A_s f_s$

$C = 1{,}978.846(130)$

$C = 257{,}250 \text{ N}$

$C = T$

7. Applied Bending Moment

$I_T = \Sigma I_{NA} + Ad^2$

$I_T = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$

$I_T = \frac{350(210)^3}{3} + 10(1{,}978.846)(600-210)^2$

$I_T = 4\,090.275 \times 10^6 \text{ mm}^4$

Based on Concrete Strength

$f_c = \frac{M_c(kd)}{I_T}$

$7 = \frac{M_c(210)}{4{,}090.275 \times 10^6}$

$M_c = 136.343 \text{ kN}\cdot\text{m}$

Based on Steel Stress

$\frac{f_s}{n} = \frac{M_s(d-kd)}{I_T}$

$\frac{130}{10} = \frac{M_s(600-210)}{4{,}090.275 \times 10^6}$

$M_s = M_c$

(S_2)

$M = (C \text{ or } T)\left(d - \frac{kd}{3}\right)$

$M = 257{,}250\left(600 - \frac{210}{3}\right)$

$M = 136.343 \text{ kN}\cdot\text{m}$

419.2.2 Modulus of Elasticity

419.2.2.1 Modulus of elasticity, E_c, for concrete shall be permitted to be calculated as (a) or (b):

a. For values of w_c between 1440 and 2560 kg/m^3

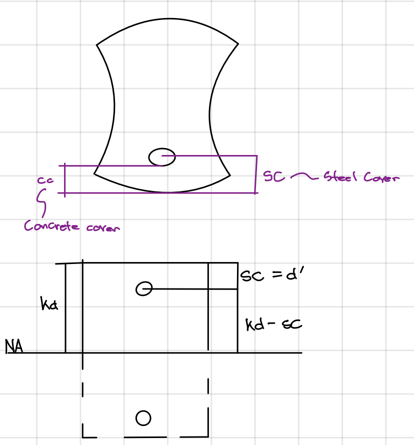
$$E_c = w_c^{1.5} 0.043 \sqrt{f'_c} \quad \text{(in MPa)} \qquad (419.2.2.1.a)$$

b. For normal weight concrete

$$E_c = 4700 \sqrt{f'_c} \quad \text{(in MPa)} \qquad (419.2.2.1.b)$$

Sit 3

8. Transformed Moment of Inertia



$A'_s = \frac{\pi}{4} d_B^2 N$

$= \frac{\pi}{4}(28)^2(2)$

$= 392\pi \text{ mm}$

$A_s = \frac{\pi}{4} d_B^2 N$

$= \frac{\pi}{4}(32)^2(4)$

$= 1\,024 \text{ mm}$

$n = \frac{E_s}{E_c}$

$= \frac{200 \times 10^3 \text{ MPa}}{17\,367.276}$

$n = 11.516$

$n = 12$

$E_c = 0.043 \rho_c^{1.5} \sqrt{f'_c}$

$E_c = 0.043(1950)^{1.5}\sqrt{22}$

$E_c = 17\,367.276 \text{ MPa}$

$\Sigma Ay_{Above\ NA} = \Sigma Ay_{below\ NA}$

$b(kd)\left(\frac{kd}{2}\right) + (2n-1)A'_s(kd-d) = nA_s(d-kd)$

$\frac{1}{2}(kd)\left(\frac{kd}{2}\right) + (2n-1)A'_s(kd-d') = nA_s(d-kd)$

$\frac{1}{2}(400)(kd)^2 + [2(12)-1](392\pi)(kd-70) = 12(1{,}024\pi)(480-kd)$

$kd = 194.009 \text{ mm}$

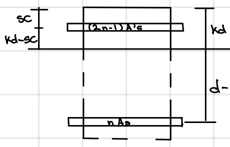
$I_T = \Sigma I_{NA} + Ad^2$

$I_T = \frac{b(kd)^3}{3} + (2n-1)A'_s(kd-d')^2 + nA'_s(d-kd)^2$

$I_T = \frac{400(194.009)^3}{3} + [2(12)-1](392\pi)(194.009-70)^2 + 12(1{,}024\pi)(480-194.009)^2$

$I_T = 4\,566.681 \times 10^6 \text{ mm}^4$

9. Moment Capacity



• Based on Concrete Strength

$$f_c = \frac{M_c(kd)}{I_T}$$

$$0.45f'_c = \frac{M_c(kd)}{I_T}$$

$$0.45(22) = \frac{M_c(194.009)}{4{,}566.681\times10^6}$$

$M_c = 233.031$ KN·m

(Top Steel)

• Based on Steel (A's)

Note: for $f_y = 420$ MPa , $f_s = 170$ MPa

$$\frac{f'_s}{2n} = \frac{M_s(kd-d')}{I_T}$$

$$\frac{170}{2(12)} = \frac{M_s'(194.009-70)}{4{,}566.681\times10^6}$$

$M_{s'} = 260.847$ kN·m

(Bottom Steel)

• Based on Steel Strength (As)

$$\frac{f_s}{n} = \frac{M_s(d-kd)}{I_T}$$

$$\frac{170}{12} = \frac{M_s(480-194.009)}{4{,}566.681\times10^6}$$

$M_s = 226.212$ kN·m

10. Safest uniformly distributed load that the beam can carry if the length of the beam is 8 m.

$$M = \frac{WL^2}{8}$$

$$226.212 = \frac{W(8)^2}{8}$$

$W = 28.276 \frac{kN}{m}$

* Ultimate Strength Design (USD)

Two – Way Bending Members

$$A_{s(min)} = \rho_{min}bd$$

One – Way Bending Members

$$A_{s(min)} = kbh$$

Where:

$$\rho_{min} = \frac{1.4}{f_y} \text{ or } \frac{\sqrt{f'_c}}{4f_y}$$

(whichever is larger)

	2001	2010	2015
f_y	275	280	<420
	400	415	420

Where: (NSCP 2010 Sec 410.6.4)

F_y (MPa)	$k \geq 0.0014$
280	0.002
415	0.0018
> 415	$\frac{0.0018(415)}{f_y}$

Where: (NSCP 2015 Sec 409)

F_y (MPa)	k
< 420	0.002
≥ 420	$\frac{0.0018(420)}{f_y}$
	0.0014

FLEXURAL STRENGTH

NSCP 1992:

$$\beta_1 = 0.85 - 0.008(f'_c - 30)$$

NSCP 2001:

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 30)$$

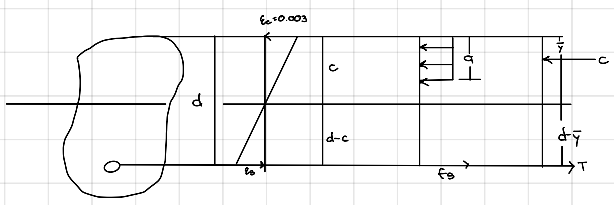
NSCP 2010/2015:

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 28)$$

Use if $f'_c > 28$

Note: $0.65 \leq \beta_1 \leq 0.85$

When $f'_c \leq 28$ MPa, use $\beta_1 = 0.85$
When $f'_c > 56$ MPa, use $\beta_1 = 0.65$



Similar Triangle

$$\frac{\varepsilon_s}{d-c} = \frac{0.003}{c}$$

$$\varepsilon_s = \frac{0.003(d-c)}{c}$$

From Hookes Law

$$\sigma \propto E_s\,\varepsilon_s$$

$$\sigma_s = (200\times10^3)\frac{0.003(d-c)}{c}$$

$$f_s = \frac{600(d-c)}{c}$$

Analysis

1.) $\Sigma F_H = 0$

2.) Check f_s

3. $\Sigma M = 0$

$\Sigma F_H = 0$

$C = T$

$0.85\,\sigma_c A_c = f_s A_s$

Check f_s

$$f_s = \frac{600(d-c)}{c}$$

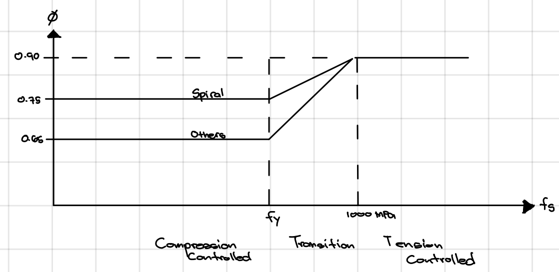
$a = \beta_1 c$

$\Sigma M = 0$

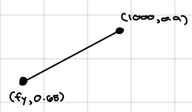
$M_n = (C \text{ or } T)(d-y)$

$M_u = \phi\,(C \text{ or } T)(d-y)$

• Reduction Factor



Others: (Transition)



$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{\phi-0.65}{f_s-f_y} = \frac{0.9-0.65}{1000-f_y}$$

$$\frac{\phi-0.65}{f_s-f_y} = \frac{0.25}{1000-f_y}$$

$$\phi - 0.65 = 0.25\,\frac{f_s-f_y}{1000-f_y}$$

$$\phi = 0.65 + 0.25\,\frac{\varepsilon_s-\varepsilon_y}{0.005-\varepsilon_y}$$

Table 421.2.2

Strength Reduction Factor, ϕ, for Moment, Axial Force, or Combined Moment and Axial Force

Net tensile stain, ε_t	Classification	ϕ Type of transverse reinforcement: Spirals conforming to Sect. 425.7.3		ϕ Type of transverse reinforcement: Other	
$\varepsilon_t \leq \varepsilon_{ty}$	Compression controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < 0.005$	Transition[1]	$0.75 + 0.15\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$	(c)	$0.65 + 0.25\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$	(d)
$\varepsilon_t \geq 0.005$	Tension controlled	0.90	(e)	0.90	(f)

[1] For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

421.2.2.1 For deformed reinforcement, ε_{ty} shall be f_y/E_s . For Grade 280 deformed reinforcement, it shall be permitted to take ε_{ty} equal to **0.002.**

$$\ell_{tr} = \left(\frac{f_{se}}{21}\right)d_b \qquad (421.2.3)$$

421.2.2.2 For all prestressed reinforcement, ε_{ty} shall be taken as 0.002.

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NSCP 1992/2001

$\rho_{max} = 0.75\rho_b$

$A_{s(max)} = 0.75A_{s(b)}$

NSCP 2010/2015

For $A_{s(max)}$ calculation, $\varepsilon_s = 0.005$ ($f_s = 1{,}000$ MPa)

$$f_s = \frac{600(d-c)}{c}$$

$$1{,}000 = \frac{600(d-c_{max})}{c_{max}}$$

$$1{,}000c_{max} = 600d - 600c_{max}$$

$$1{,}600c_{max} = 600d$$

$$c_{max} = \frac{3}{8}d$$

NSCP 2010/2015

For $A_{s(max)}$ calculation, $\varepsilon_s = 0.005$ ($f_s = 1{,}000$ MPa)

$$f_s = \frac{600(d-c)}{c}$$

$$1{,}000 = \frac{600(d-c_{max})}{c_{max}}$$

$$1{,}000c_{max} = 600d - 600c_{max}$$

$$1{,}600c_{max} = 600d$$

$$c_{max} = \frac{3}{8}d$$

$$a_{max} = \beta_1 c_{max}$$

$$\boldsymbol{\sum F_H = 0}$$

$$\leftarrow\; = \;\rightarrow$$

$$C_{max} = T_{max}$$

$$0.85f'_c a_{max} b = f_y A_{s(max)}$$

For Balance Condition, $f_s = f_y$:

$$f_s = \frac{600(d-c)}{c}$$

$$f_y = \frac{600(d-c_b)}{c_b}$$

$$f_y c_b = 600d - 600c_b$$

$$(600 + f_y)c_b = 600d$$

$$c_b = \frac{600d}{600 + f_y}$$

$$a_b = \beta_1 c_b$$

$$\boldsymbol{\sum F_H = 0}$$

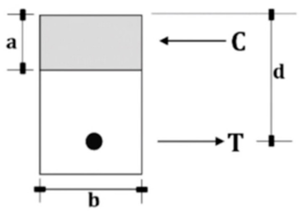
$$\leftarrow\; = \;\rightarrow$$

$$C_b = T_b$$

$$0.85f'_c a_b b = f_y A_{s(b)}$$

Situation 4: A rectangular concrete beam has a width of 350 mm and an effective depth of 620 mm. It is reinforced with five 28-mm diameter bars for tension only. Concrete strength is f'c = 21 MPa and steel yield strength fy= 415 MPa.

11. Depth of the rectangular compression stress block.



$$A_s = \frac{\pi}{4}(28)^2(5)$$

$$A_s = 980\pi \text{ mm}^2$$

$$\boldsymbol{\sum F_H = 0}$$

$$\leftarrow\; = \;\rightarrow$$

$$C = T$$

$$\sigma_c A_c = \sigma_s A_s$$

$$0.85f'_c ab = f_s A_s$$

Assume steel yields

$\boldsymbol{(f_s \geq f_y)}$

$$0.85f'_c ab = f_y A_s$$

$$0.85(21)(a)(350) = 415(980\pi)$$

$$a = 204.512 \text{ mm}$$

Check f_s (Assumption):

$$a = \beta_1 c$$

$$204.512 = 0.85c$$

$$c = 240.602 \text{ mm}$$

$$f_s = \frac{600(d-c)}{c}$$

$$f_s = \frac{600(620 - 240.602)}{240.602}$$

$$f_s = 946.123 \text{ MPa} \quad > \quad f_y$$

$$< \quad 1{,}000 \text{ MPa}$$

Since $f_s > f_y$, steel reinforcement yields, therefore; correct assumption.

Answer: $a = 204.602$ mm

12. Distance of the neutral axis from the extreme compression concrete.

$c = 240.602$ mm

13. Ultimate moment capacity of the beam.

Since $f_y < f_s < 1{,}000$ MPa, therefore; in transition.

$$\emptyset = 0.65 + 0.25\frac{f_s - f_y}{1000 - f_y}$$

$$\emptyset = 0.65 + 0.25\frac{946.123 - 415}{1000 - 415}$$

$$\emptyset = 0.877$$

$$M_u = \emptyset(C \text{ or } T)\left(d - \frac{a}{2}\right)$$

$$M_u = \emptyset(A_s f_y)\left(d - \frac{a}{2}\right)$$

$$M_u = 0.877(980\pi)(415)\left(620 - \frac{204.512}{2}\right)$$

$$M_u = 580.132 \text{ kN.m}$$

Situation 5:

A beam has strengths of f'c = 32 MPa and fy = 415 MPa. Reinforcement used are 4 – 25 mmØ. Dimensions b = 300 mm and d = 450 mm.

15. Depth of compression block.

$$A_s = \frac{\pi}{4}(25)^2(4)$$

$$A_s = 625\pi \text{ mm}^2$$

Solution 1: With assumption

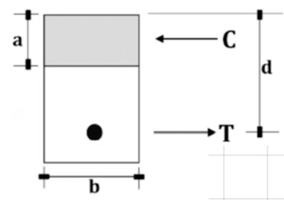
$$\boldsymbol{\sum F_H = 0}$$

$$\leftarrow\; = \;\rightarrow$$

$$C = T$$

$$\sigma_c A_c = \sigma_s A_s$$

$$0.85f'_c ab = f_s A_s$$



Solution 1: With assumption

$$\boldsymbol{\sum F_H = 0}$$

$$\leftarrow\; = \;\rightarrow$$

$$C = T$$

$$\sigma_c A_c = \sigma_s A_s$$

$$0.85f'_c ab = f_s A_s$$

Assume steel yields

$\boldsymbol{(f_s \geq f_y)}$

$$0.85f'_c ab = f_y A_s$$

$$0.85(32)(a)(300) = 415(625\pi)$$

$$a = 99.86 \text{ mm}$$

Check f_s (Assumption):

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 28)$$

$$\beta_1 = 0.85 - \frac{0.05}{7}(32 - 28)$$

$$\beta_1 = 0.8214$$

$$a = \beta_1 c$$

$$99.86 = 0.8214c$$

$$c = 121.57 \text{ mm}$$

$$f_s = \frac{600(d-c)}{c}$$

$$f_s = \frac{600(450 - 121.57)}{121.57}$$

$$f_s = 1{,}620.99 \text{ MPa} \quad > \quad f_y$$

$$> \quad 1{,}000 \text{ MPa}$$

Since $f_s > f_y$, steel reinforcement yields, therefore; correct assumption.

Since $f_s > 1{,}000$ MPa, therefore; tension controlled.

Answer: $a = 99.86$ mm

Solution 2: Without assumption

$$\sum F_H = 0$$
$$\leftarrow = \rightarrow$$
$$C = T$$
$$\sigma_c A_c = \sigma_s A_s$$
$$0.85f'_c ab = f_s A_s$$
$$0.85f'_c(\beta_1 c)b = \frac{600(d-c)}{c}A_s$$
$$0.85(32)(0.8214c)(300) = \frac{600(450-c)}{c}(625\pi)$$
$$6{,}702.86c^2 = 1{,}178{,}097.25(450-c)$$
$$6{,}702.86c^2 + 1{,}178{,}097.25c - 530{,}143{,}760.3 = 0$$

Check f_s:

$$f_s = \frac{600(d-c)}{c}$$
$$f_s = \frac{600(620-206.76)}{206.76}$$
$$f_s = 705.84 \text{ MPa} \quad > \quad f_y$$

Since $f_s > f_y$, therefore; steel reinforcement yields.

Recompute:

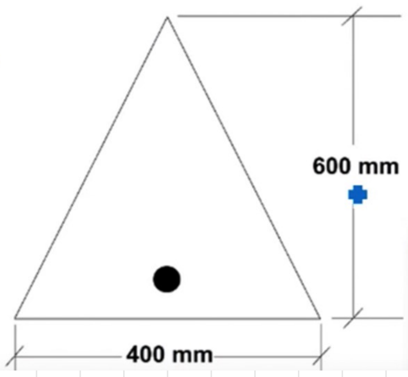
$$0.85f'_c ab = f_s A_s$$
$$0.85f'_c ab = f_y A_s$$
$$0.85(32)(a)(300) = 415(625\pi)$$
$$a = 99.86 \text{ mm}$$

Situation 6: For the beam below;

f'c = 27 MPa
fy = 270 MPa
ML = 20 kN.m
Steel cover = 100 mm
MD = 30 kN.m

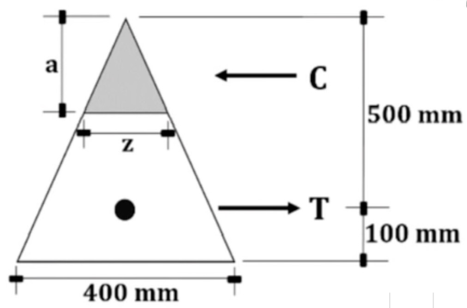
18. Depth of compression block.

$$M_u = 1.2M_{DL} + 1.6M_{LL}$$
$$M_u = 1.2(30) + 1.6(20)$$
$$M_u = 68 \text{ kN.m}$$



Similar Triangles:

$$\frac{z}{a} = \frac{400}{600}$$
$$z = \frac{2}{3}a$$
$$M_u = \emptyset(C \text{ or } T)\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left(\frac{1}{2}za\right)\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left[\frac{1}{2}\left(\frac{2}{3}a\right)a\right]\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left(\frac{a^2}{3}\right)\left(d - \frac{2}{3}a\right)$$



$$M_u = \emptyset 0.85f'_c\left(\frac{a^2}{3}\right)\left(d - \frac{2}{3}a\right)$$

Assume tension controlled (Ø=0.9)

$$M_u = \emptyset 0.85f'_c\left(\frac{a^2}{3}\right)\left(d - \frac{2}{3}a\right)$$
$$68(10^6) = 0.9\left[0.85(27)\left(\frac{a^2}{3}\right)\right]\left(500 - \frac{2}{3}a\right)$$
$$a = 158.22 \text{ mm}$$

Check f_s (Assumption):

$$a = \beta_1 c$$
$$158.22 = 0.85c$$
$$c = 186.14 \text{ mm}$$
$$f_s = \frac{600(d-c)}{c}$$
$$f_s = \frac{600(500-186.14)}{186.14}$$
$$f_s = 1{,}011.65 \text{ MPa}$$
$$> f_y$$
$$> 1{,}000 \text{ MPa}$$

Since $f_s > f_y$, therefore; steel reinforcement yields.

Since $f_s > 1{,}000$ MPa, therefore; tension controlled.

Answer: $a = 158.22$ mm

19. Maximum compressive strength of concrete.

$$c_{max} = \frac{3}{8}d$$
$$c_{max} = \frac{3}{8}(500)$$
$$c_{max} = 187.5 \text{ mm}$$
$$a_{max} = \beta_1 c_{max}$$
$$a_{max} = 0.85(187.5)$$
$$a_{max} = 159.375 \text{ mm}$$

Similar Triangles:

$$\frac{z}{a} = \frac{400}{600}$$
$$z = \frac{2}{3}a$$
$$M_u = \emptyset(C \text{ or } T)\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left(\frac{1}{2}za\right)\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left[\frac{1}{2}\left(\frac{2}{3}a\right)a\right]\left(d - \frac{2}{3}a\right)$$
$$M_u = \emptyset 0.85f'_c\left(\frac{a^2}{3}\right)\left(d - \frac{2}{3}a\right)$$

$$a_{max} = \beta_1 c_{max}$$
$$a_{max} = 0.85(187.5)$$
$$a_{max} = 159.375 \text{ mm}$$
$$C_{max} = 0.85f'_c\left(\frac{{a_{max}}^2}{3}\right)$$
$$C_{max} = 0.85(27)\left[\frac{(159.375)^2}{3}\right]$$
$$C_{max} = 194.313 \text{ kN}$$

20. Required number of 20 mmØ bars.

$\sum F_H = 0$

$\leftarrow = \rightarrow$

$C = T$

$$0.85f'_c\left(\frac{a^2}{3}\right) = f_yA_s$$

$$0.85(27)\left[\frac{(158.22)^2}{3}\right] = 270A_s$$

$A_s = 709.31\ mm^2$

Check $A_{s(max)}$:

$\sum F_H = 0$

$\leftarrow = \rightarrow$

$C_{max} = T_{max}$

$C_{max} = f_yA_{s(max)}$

$194.313(10^3) = 270A_{s(max)}$

$A_{s(max)} = 719.68\ mm^2$

Since $A_s < A_{s(max)}$, therefore; singly reinforced beam.

$\sum F_H = 0$

$\leftarrow = \rightarrow$

$C = T$

$$0.85f'_c\left(\frac{a^2}{3}\right) = f_yA_s$$

$$0.85(27)\left[\frac{(158.22)^2}{3}\right] = 270A_s$$

$A_s = 709.31\ mm^2$

Since $A_s < A_{s(max)}$, therefore; singly reinforced beam.

$$A_s = \frac{\pi}{4}d_B{}^2N$$

$$709.31 = \frac{\pi}{4}(20)^2N$$

$N = 2.26$

$N = 3$ bars