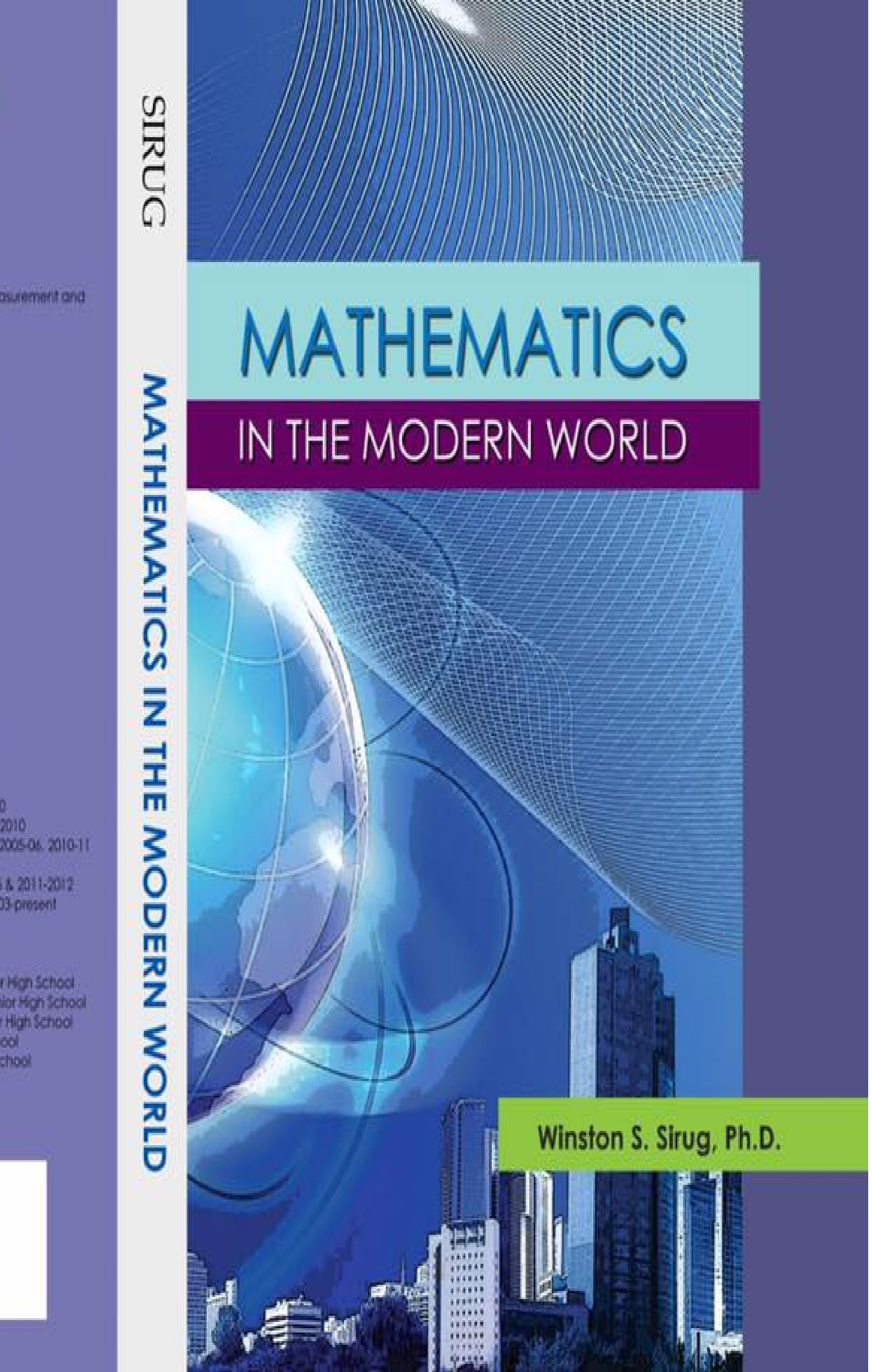


Chapter 3:

Problem Solving and Reasoning

Learning Objectives

- Compare and contrast inductive and deductive reasoning.
- Use different types of reasoning to justify statements and arguments made about mathematics and mathematical concepts.
- Apply the Polya's four-step in problem solving.
- Organize one's methods and procedures for proving and solving problems.
- Perform operations on mathematical expressions correctly.

Learning Objectives

- Articulate the importance of mathematics in one's life.
- Express appreciation for mathematics as a human endeavor.
- Support the use of mathematics in various aspects and endeavors in life.
- Affirm honesty and integrity in the application of mathematics to various human endeavors.

Topic Outline

I. Inductive and Deductive Reasoning

II. Intuition, Proof, and Certainty

III. Polya's Four-Steps in Problem Solving

IV. Problem Solving Strategies

V. Mathematical Problems involving Patterns

IV. Recreational Problems using Mathematics

Inductive Reasoning

Inductive reasoning is drawing a general conclusion from a repeated observation or limited sets of observations of specific examples.

Given data, then we draw conclusion based from the frame these data or simply from specific case to general case.

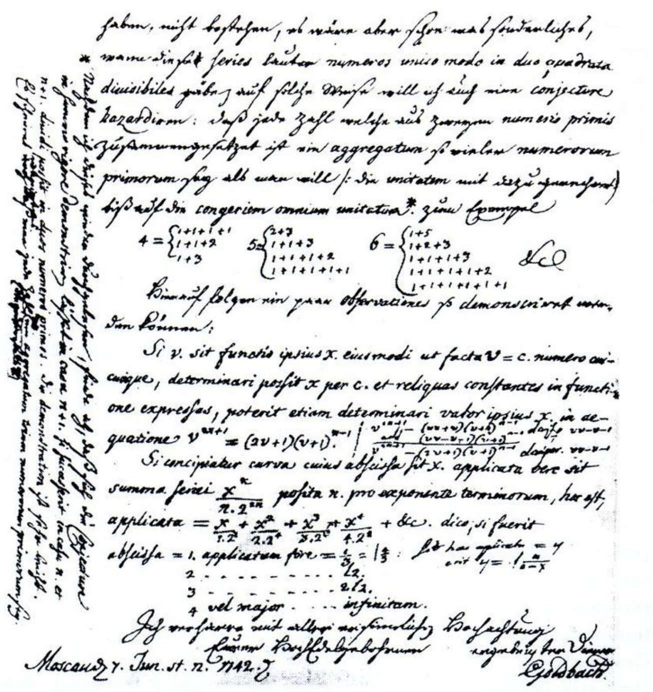
Conjecture is drawing conclusion using inductive reasoning.

The conjecture may be true or false depending on the truthfulness of the argument.

Goldbach Conjecture

Christian Goldbach
(1690-1764)
Russian Mathematician

Leonard Euler
(1707-1783)
Swiss Mathematician



Letter from Goldbach to Euler , 1742

Every even integer greater than 2 can be expressed as the sum of two primes.

Fermat's Last Theorem

Pierre de Fermat
1607-1665) French
Mathematician

Fermat's Last Theorem (or Fermat's conjecture)

No three positive integers *a*, *b*, and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of *n* greater than 2.

Wiles proved the Fermat's Last Theorem on September 19, 1994 and was published in May 1995 in a dedicated volume of the Annals of Mathematics.

Sir Andrew John Wiles
British mathematician

Inductive Reasoning

Counterexample if a statement is a true statement provided that it is true in all cases and it only takes one example to prove the conjecture is false.

Example 1: 1 is an odd number.

11 is an odd number.

21 is an odd number.

Thus, all number ending with 1 are odd numbers.

Inductive Reasoning

Example 2: Essay test is difficult.

Problem solving test is difficult.

Therefore, all tests are difficult

Example 3: Mark is a Science teacher.

Mark is bald.

Therefore, all Science teachers are bald.

Deductive Reasoning

Deductive reasoning is drawing general to specific examples or simply from general case to specific case.

Deductive starts with a general statement (or hypothesis) and examines to reach a specific conclusion.

Example 4: All birds have feathers.

Ducks are birds.

Therefore, ducks have feathers.

Deductive Reasoning

Example 5: Christopher is sick.

If Christopher is sick, he won't be able to go to work.

Therefore, Christopher won't be able to go to work.

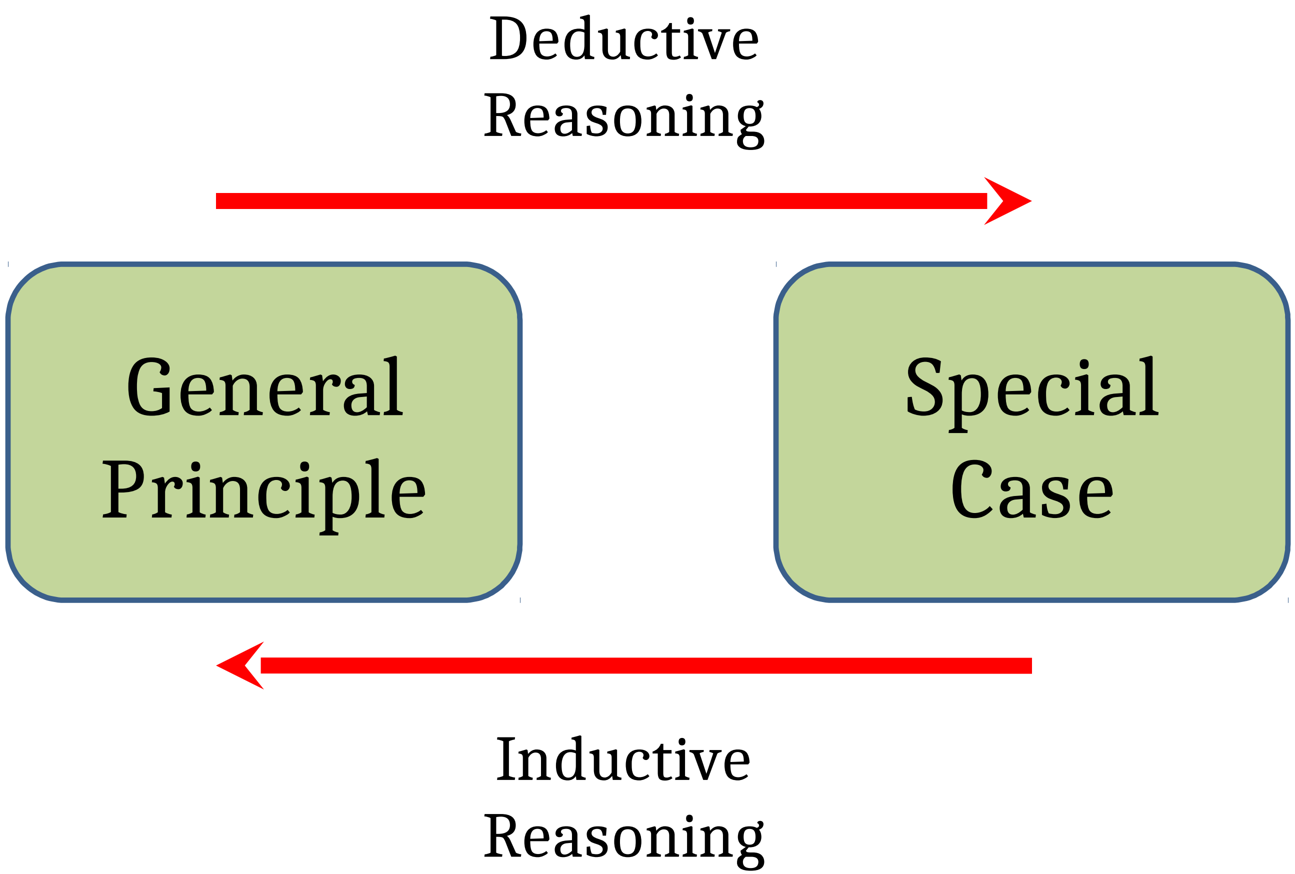
Example 6: All Science teachers are bald.

Mark is a Science teacher.

Therefore, Mark is bald.

Note: Logical reasoning maybe valid but not necessarily true.

Inductive vs Deductive Reasoning



Intuition, Proofs and Certainty

Intuitive can be found in mathematical literature and discovery.

Srinivasa Ramanujan wrote a letter to Godfrey Harold Hardy on infinite sums, products, fractions, and roots.

Ramanujan's formulas prove there is mathematical intuition, though he didn't prove them.

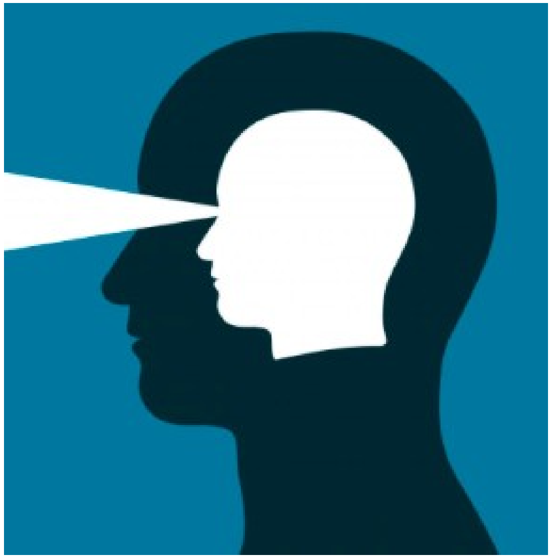
Hardy made a sound judgment without directly proving the formulas of Ramanujan's were correct.

Srinivasa Ramanujan

G. H. Hardy

Intuition

Mathematical intuition. Intuition is a reliable mathematical belief without being formalized and proven directly and serves as an essential part of mathematics.



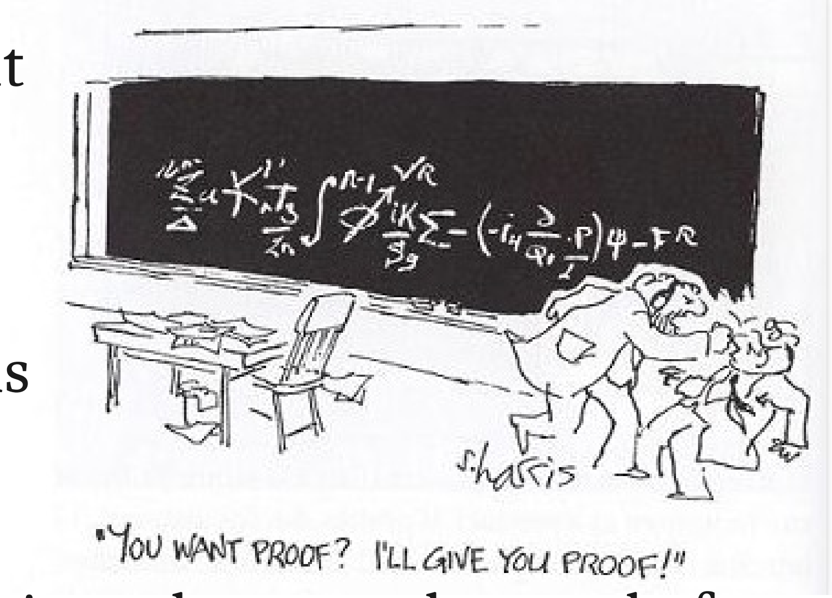
"Intuition" carries a heavy load of mystery and ambiguity and it is not legitimate substitute for a formal proof.

Intuitive is being visual and is absent from the rigorous formal or abstract version.

Proofs

Proof is an inferential argument for a mathematical statement.

Mathematical argument like theorems can only be used if it is already proven.



Axioms may be served as conditions that must be met before the statement applies.

Proofs are examples of exhaustive deductive reasoning and inductive reasoning.

Proofs

A mathematical proof demonstrates that a certain statement is always true in all possible cases.

An unproved proposition that is believed to be true is known as a conjecture.

If one has a conjecture, the only way that it can safely be sure that it is true, is by presenting a valid mathematical proof.

Certainty

Mathematics has a tradition and standard point of view that it provides certainty.

A correct formulated mathematical knowledge is forever beyond error and correction.

Mathematical certainty is an essential defining attribute of mathematics and mathematical knowledge.

Polya's Four-Steps in Problem Solving

George Polya (1887-1985) was a mathematics educator who strongly believed that the skill of problem solving can be taught.

He developed a framework known as Polya's Four-Steps in Problem Solving.

The process addressed the difficulty of students in problem solving.

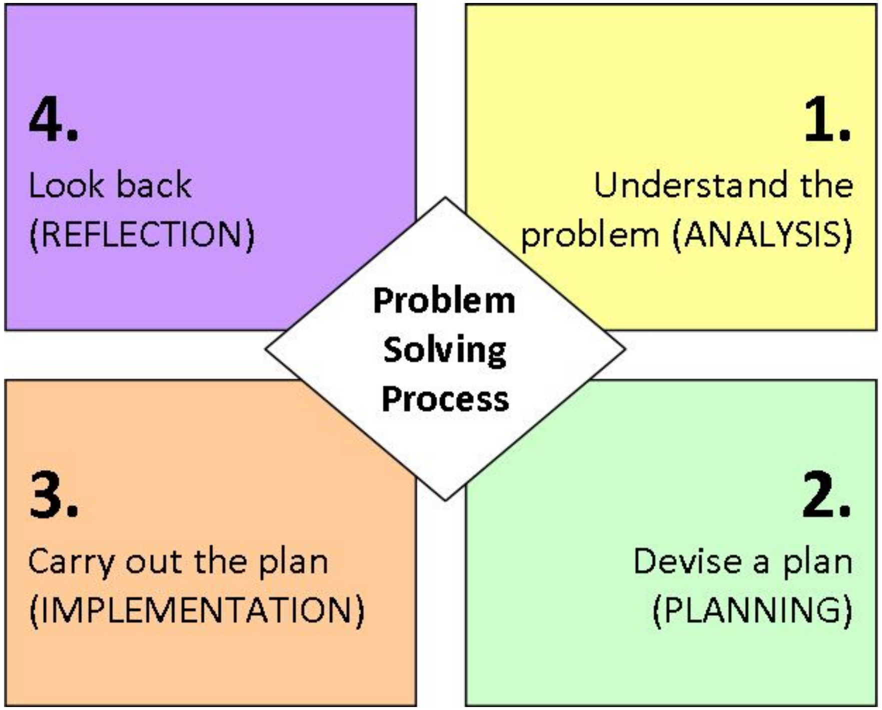
Polya's Four-Steps in Problem Solving

Step 1: Understand the problem.

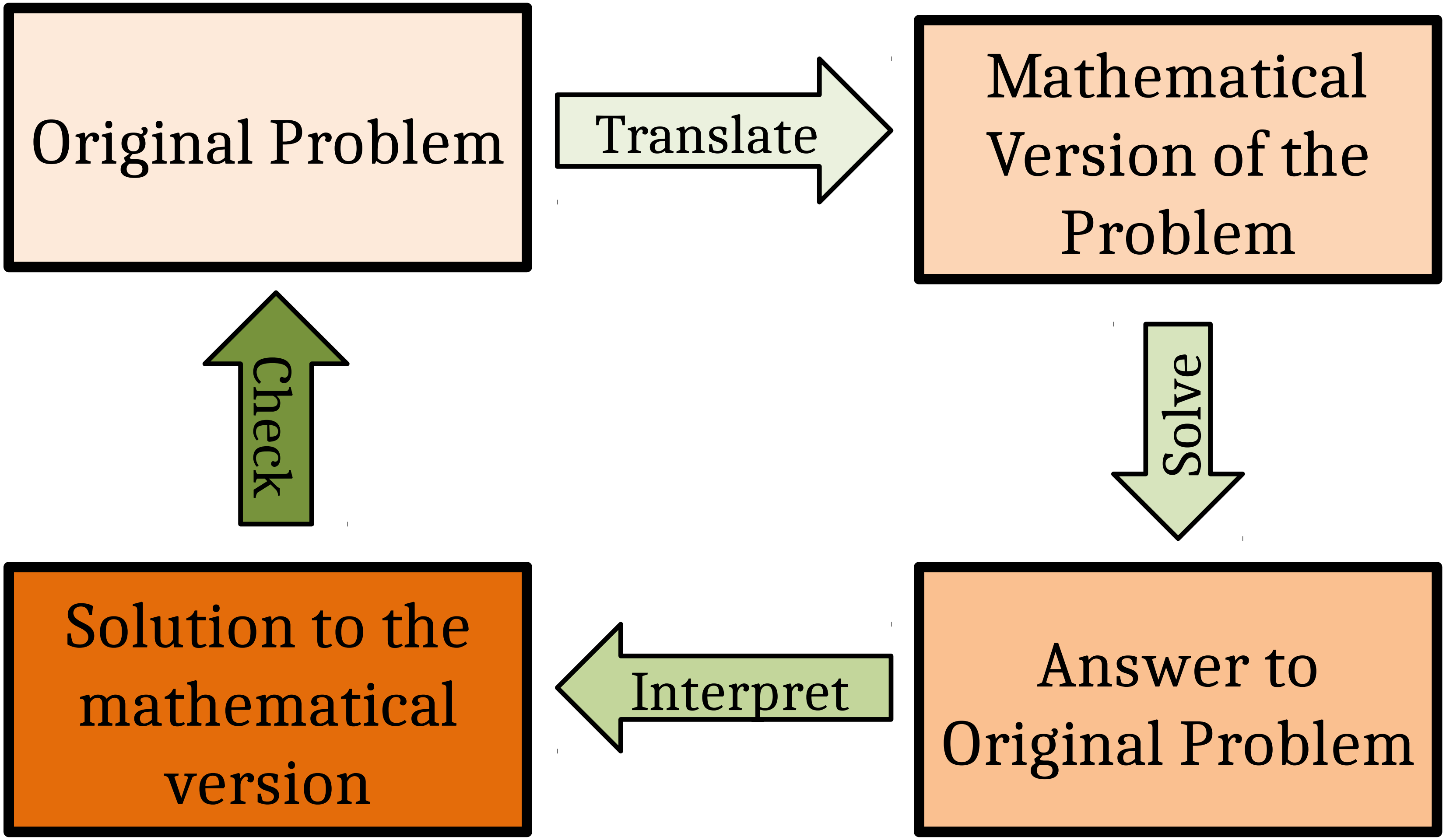
Step 2: Devise a plan.

Step 3: Carry out the plan.

Step 4: Look back.



Process of Problem Solving



Step 1: Understanding the Problem

What is the goal?

What is being asked?

What is the condition?

What sort of a problem is it?

What is known or unknown?

Is there enough information?

Can you draw a figure to illustrate the problem?

Is there a way to restate the problem? In your own words?

Step 2: Devise a Plan

Act it out.

Be systematic.

Work backwards.

Consider special cases.

Eliminate possibilities.

Perform an experiment.

Draw a picture/diagram.

Make a list or table/chart.

Use a variable, such as *x*.

Step 2: Devise a Plan

Look for a formula/formulas.

Write an equation (or model).

Look for a pattern or patterns.

Use direct or indirect reasoning.

Solve a simple version of the problem.

Guess and check your answer (trial and error).

Step 3: Carry Out the Plan

Be patient.

Work carefully.

Modify the plan or try a new plan.

Keep trying until something works.

Implement the strategy and strategies in Step 2.

Try another strategy if the first one isn't working.

Keep a complete and accurate record of your work.

Be determined and don't get discouraged if the plan does not work immediately.

Step 4: Look Back

Look for an easier solution.

Does the answer make sense?

Check the results in the original problem.

Interpret the solution with the facts of the problem.

Recheck any computations involved in the solution.

Can the solution be extended to a more general case?

Ensure that all the conditions related to the problem are met.

Determine if there is another method of finding the solution.

Ensure the consistency of the solution in the context of the problem.

Problem Solving Strategies

Example 1: Suppose the NCAA basketball championships is decided on a best of five series game. In how many diff. way can a team win the championships?

Solution:

Step 1: Understand the Problem.

Many different orders to win the championships.

Team may have won WWW or LLWWW.

Step 2: Devise a Plan.

Make an organized list of all possible orders and ensure that each of the different orders is accounted for only once.

Problem Solving Strategies

Step 3: Carry Out the Plan.

Each entry in the list must contain three Ws and may contain one or two losses.

WWW (Start with three wins)
WWLW (Start with two wins)
WWLLW (Start with two wins)
WLWW (Start with one win)
WLLWW (Start with one win)
WLWLW (Start with one win)
LWWW (Start with one loss)
LWWLW (Start with one loss)
LWLWW (Start with one loss)
LLWWW (Start with two losses)

Problem Solving Strategies

Step 4: Look Back.

Check if the the list contains no duplications.

Includes all possibilities.

Conclude that there are ten (10) different ways in to win the NCAA championships in the best of 5 games.

Example 2: Two times the sum of a number and 3 is equal to thrice the number plus 4. Find the number.

Solution:

Step 1: Understand the Problem.

Read the question carefully several times.

Look for a number, and let *x* be a number.

Step 2: Devise a Plan.

Translate: two times the sum of a number and 3 is equal to thrice the number plus 4. $\Rightarrow 2(x + 3) = 3x + 4$

Problem Solving Strategies

Step 3: Carry Out the Plan.

Solve for the value of x,

$$2(x + 3) = 3x + 4$$
$$2x + 6 = 3x + 4$$
$$3x - 2x = 6 - 4$$
$$x = 2$$

Step 4: Look Back.

If we take two times the sum of 2 and 3, that is the same as thrice the number 2 plus 4 which is 10, so this does check.

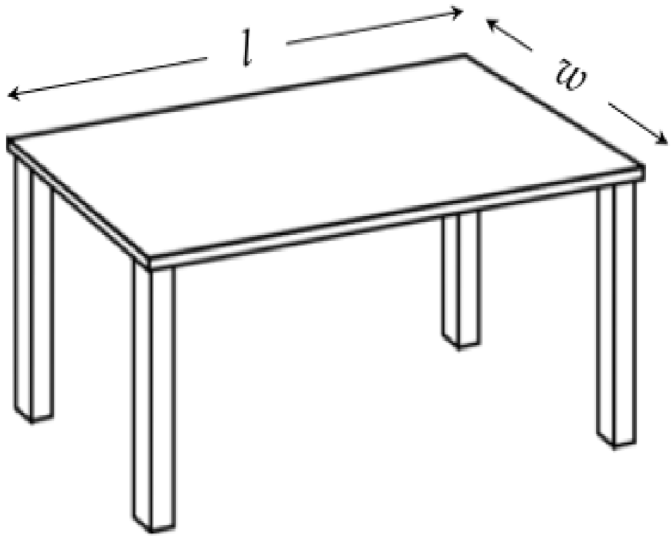
Thus, the number is 2.

Problem Solving Strategies

Example 3: If the length of the top of a rectangle is 15 inches more than its width and the area is 1,350 square inches. Find the dimension of the table.

Solution:

Step 1: Understand the Problem.



Look for the length and width of the table.

Let l be the length

w be the width

The length is 15 inches longer than the width ($l = 15 + w$).

Problem Solving Strategies

Step 2: Devise a Plan.

$$\text{Area} = lw$$
$$1{,}350 = lw = (15 + w)w$$

Step 3: Carry Out the Plan.

$$1{,}350 = (15 + w)w$$
$$1{,}350 = 15w + w^2$$
$$w^2 + 15w = 1{,}350 \longrightarrow w^2 + 15w - 1350 = 0$$
$$(w + 45)(w - 30) = 0$$
$$w + 45 = 0 \qquad w - 30 = 0$$
$$w = -45 \qquad w = 30$$

The width of the rectangle is 30.

The length is $l = 15 + w = 15 + 30 = 45$ inches.

Problem Solving Strategies

Step 4: Look Back.

If the width of a rectangle is 30 inches and the length is 15 inches longer than the width which is 45 inches.

The area of a rectangle is

Area = lw = 30(45) = 1,350 square inches.

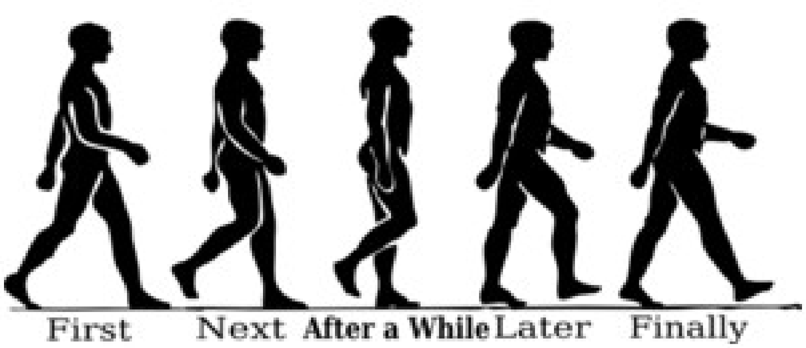
Thus, the width is 30 and the length is 45 inches.

Mathematical Problem involving Patterns

Mathematics is useful to predict.

Number pattern leads directly to the concept of functions in mathematics.

Number pattern is applied to problem-solving whether a pattern is present and used to generalize a solution to a problem.



Pattern can be in the form counting up or down and the missing number is of the form of completing count up or down.

Famous Number Patterns

Fibonacci Sequence – 0, 1, 1, 2, 3, 5, 8, 13, 21, …

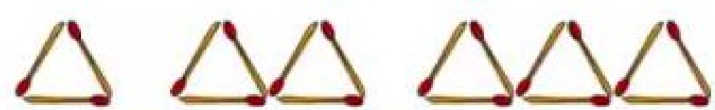
Prime number pattern – 2, 3, 5, 7, 11, 13, 17, 19, …

Imaginary number pattern

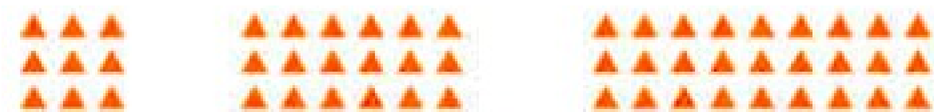
Powers of i

i^n $(n \geq 1)$	Solution
i^1	$\sqrt{-1}$
i^2	-1
i^3	$-i$
i^4	1
i^5	$\sqrt{-1}$
i^6	-1

Geometric number pattern



Growing number pattern



9 18 27

Sequence

Infinite sequence is a function whose domain is the set of positive integers.

Terms of the sequence are $a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n, \ldots$

Finite sequence. If the domain of the function consists of the first n positive integers only, the sequence is a.

Finite and Infinite Sequence

a. 1, 2, 3, 4, 5, 6, 7

b. 2, 4, 6, 8, 10, 12, 14

c. 1, 3, 5, 7, 9, 11, 13, 15

Finite sequences

d. 3, 6, 9, 12, 15, ...

e. 1, 1, 2, 3, 5, 8, 13, ...

f. f. 1, 4, 9, 16, 25, 36, ...

Infinite sequences

General Sequence

A **general sequence** $a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n, \ldots$ having the first term a_1, the second term is a_2, the third term is a_3, and the nth term, also called the **general term** of the sequence, is a_n.

General Sequence

Example 2: Write the first three terms of the sequence whose nth term is given by the formula $a_n = 3n + 1$.

Solution:

$$a_n = 3n + 1$$

$a_1 = 3(1) + 1 = 3 + 1 = 4$ Replace n by 1.

$a_2 = 3(2) + 1 = 6 + 1 = 7$ Replace n by 2.

$a_3 = 3(3) + 1 = 9 + 1 = 10$ Replace n by 3.

Thus, the first three terms of the sequence are 4, 7, and 10.

Difference Table

Difference table shows the differences between successive terms of the sequence..

Differences in rows maybe the first, second, and third differences.

Each number in the first row of the table is the differences between the closest numbers just above it.

If the first differences are not the same, compute the successive differences of the first differences .

Difference Table

Example 3: Construct the difference table to predict the next term of each sequence.

a. 3, 7, 11, 15, 19, ...

b. 1, 4, 9, 17, 28, ...

c. 6, 9, 14, 26, 50, 91, ...

Difference Table

Solution:

a. 3, 7, 11, 15, 19, ...

Sequence	3		7		11		15		19		23
First differences		4		4		4		4		4	

The next term is 23.

Difference Table

Solution:

b. 1, 4, 9, 17, 28, ...

Sequence	1		4		9		17		28		42
First differences		3		5		8		11		14	
Second differences			3		3		3		3		

The next term is 42.

Difference Table

Solution:

c. 6, 9, 14, 26, 50, 91, ...

Sequence	6		9		14		26		50		91		154
First differences		3		5		12		24		41		63	
Second differences			2		7		12		17		22		
Third differences				5		5		5		5			

The next term is 154.

Mathematical Problem involving Patterns

Example 4: Determine the *n*th term formula for the following polygonal numbers in the *n*th figure:

(a) triangular number;

(b) square number;

(c) pentagonal number; and

(d) hexagonal number.

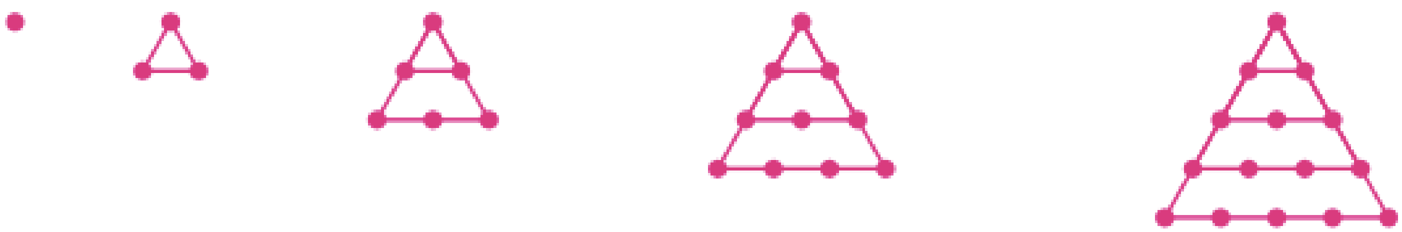
Mathematical Problem involving Patterns

Solution:

A polygonal number is a type of figurative number represented as dots or pebbles arranged in the shape of a regular polygon.

a. Triangular Number

1 3 6 10 15



Mathematical Problem involving Patterns

Solution:

The number sequence is

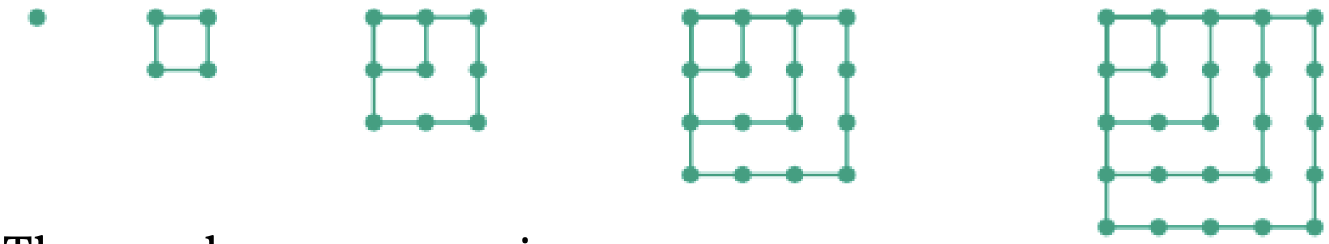
$n = 1$	1
$n = 2$	$1 + 2 = 3$
$n = 3$	$1 + 2 + 3 = 6$
$n = 4$	$1 + 2 + 3 + 4 = 10$
$n = 5$	$1 + 2 + 3 + 4 + 5 = 15$
$n = 6$	$1 + 2 + 3 + 4 + 5 + 6 = 21$

Generalized as $T_n = 1 + 2 + 3 + \ldots + (n - 1) + n = \frac{1}{2}(n^2 + n)$

Expansion: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

b. Square Number

1 4 9 16 25



The number sequence is

$n = 1$ $1^2 = 1$

$n = 2$ $2^2 = 4$

$n = 3$ $3^2 = 9$

$n = 4$ $4^2 = 16$

$n = 5$ $5^2 = 25$

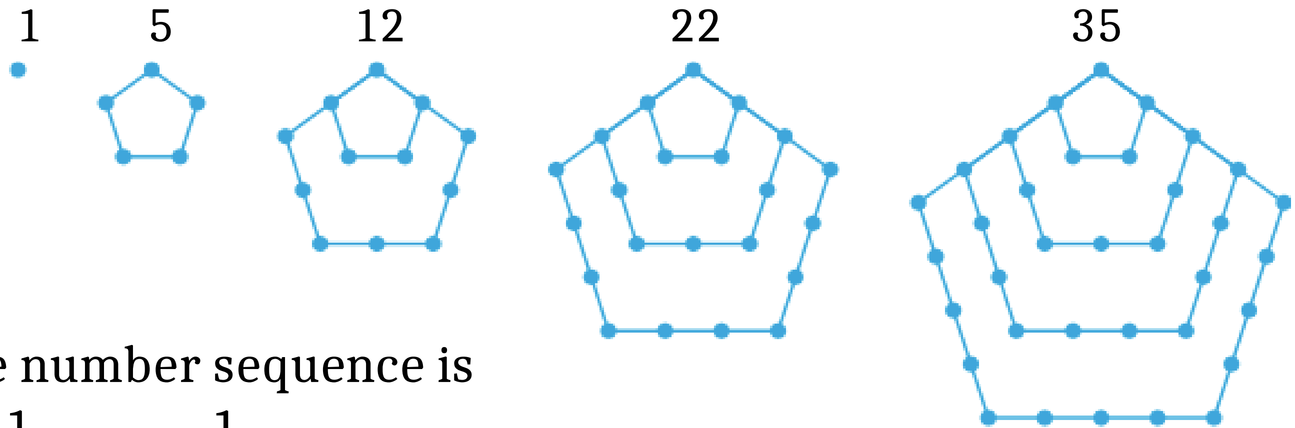
$n = 6$ $6^2 = 36$

Generalized as $S_n = ½(2n^2 – 0n) = n^2$

Expansion: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Mathematical Problem involving Patterns

c. Pentagonal Number



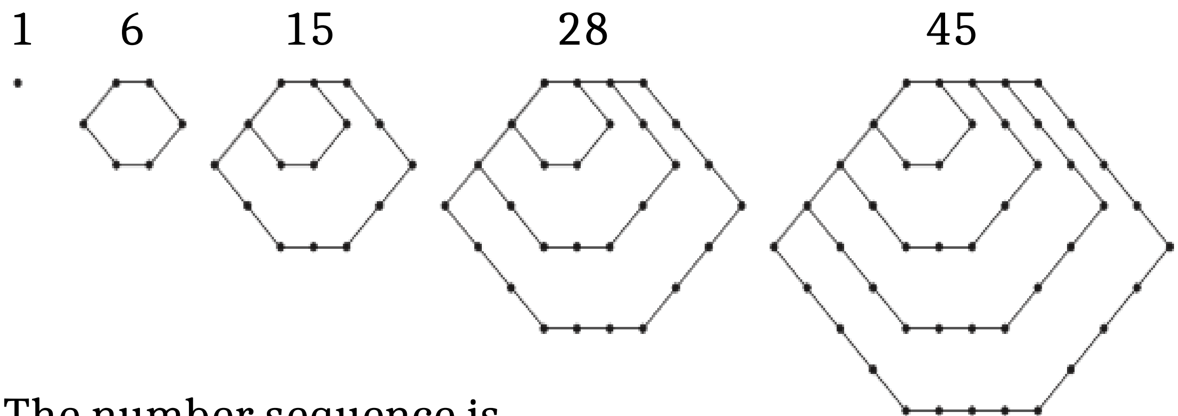
The number sequence is

$n = 1$	1
$n = 2$	$1 + 4 = 5$
$n = 3$	$1 + 4 + 7 = 12$
$n = 4$	$1 + 4 + 7 + 10 = 22$
$n = 5$	$1 + 4 + 7 + 10 + 13 = 35$
$n = 6$	$1 + 4 + 7 + 10 + 13 + 16 = 51$

Generalized as $P_n = \frac{1}{2}(3n^2 - n) = \frac{1}{2}(3n^2 - n)$

Expansion: 1, 5, 12, 22, 35, 51, 70, 90, 117, 145, ...

d. Hexagonal Number



The number sequence is

$n = 1$ 1

$n = 2$ $1 + 5 = 6$

$n = 3$ $1 + 5 + 9 = 15$

$n = 4$ $1 + 5 + 9 + 13 = 28$

$n = 5$ $1 + 5 + 9 + 13 + 17 = 45$

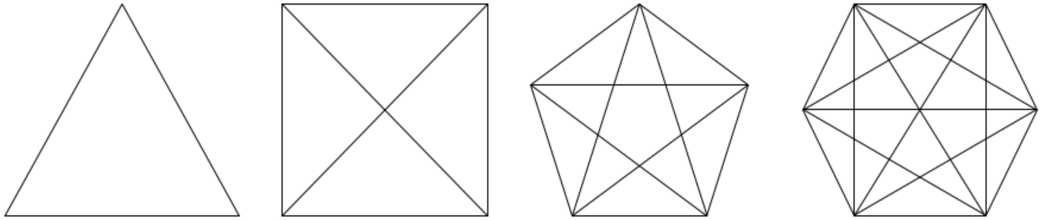
$n = 6$ $1 + 5 + 9 + 13 + 17 + 21 = 66$

Generalized as $H_n = \frac{1}{2}(4n^2 - 2n) = 2n^2 - n$

Expansion: 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, ...

Mathematical Problem involving Patterns

Example 5: A diagonal of a polygon is a line segment that connects vertices (corners) of the polygon. Following are polygons: triangle, quadrilateral, pentagon, and hexagon.



Determine the number of diagonals in a heptagon and an octagon.

Mathematical Problem involving Patterns

Solution:

The sequence of the diagonals

Number of Sides	3	4	5	6	7	8
Number of Diagonals	0	2	5	9		

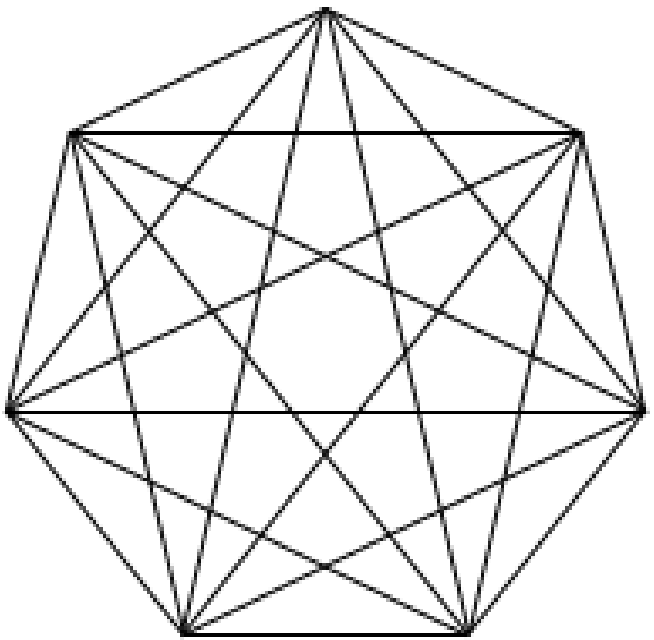
Difference table

Sequence	0		2		5		9		14		20
First differences		2		3		4		5		6	
Second differences			1		1		1		1		

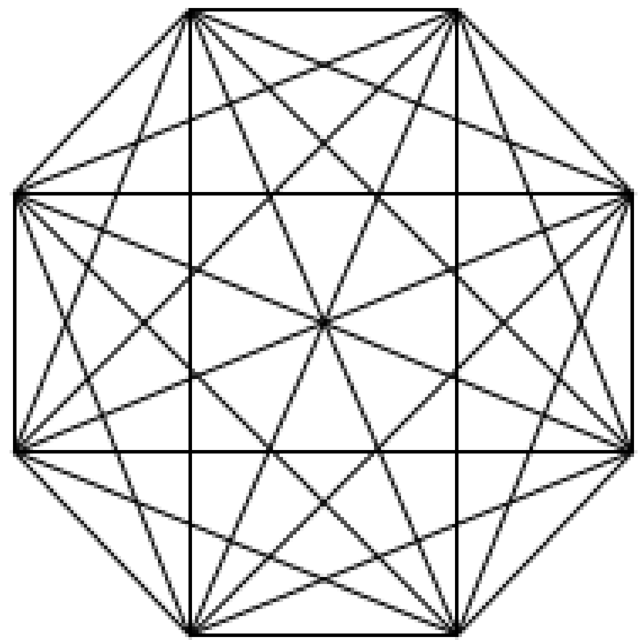
Mathematical Problem involving Patterns

Pentagon has 14 diagonals

Octagon has 20 diagonals



Heptagon



Octagon

Mathematical Problem involving Patterns

Example 6: Given a circle with n points on its perimeter what is the maximum number of regions determined by the chords and the circle rim.

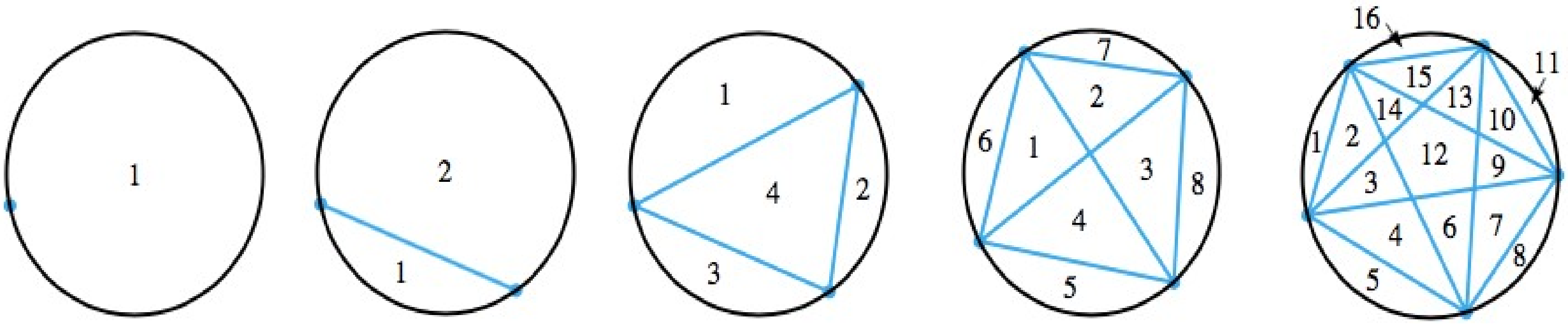
Solution:

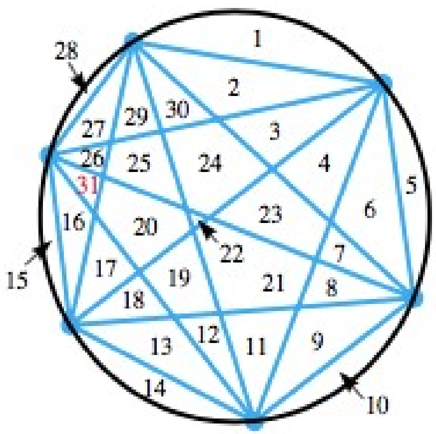
When $n = 0$ and continuing the case to $n = 6$.

Locate a point on a circle and connect it with other point on a circle and count the number of regions it generates.

Mathematical Problem involving Patterns

The number of chords and regions produced with given *n* points on a circle.





Summary Table

No. of Points	1	2	3	4	5	6
No. of Chords	0	1	3	6	10	15
No. of Regions	1	2	4	8	14	31

Mathematical Problem involving Patterns

Seven points on a circle: Maximum number of regions is 57.

Eight points on a circle: Maximum number of regions is 99.

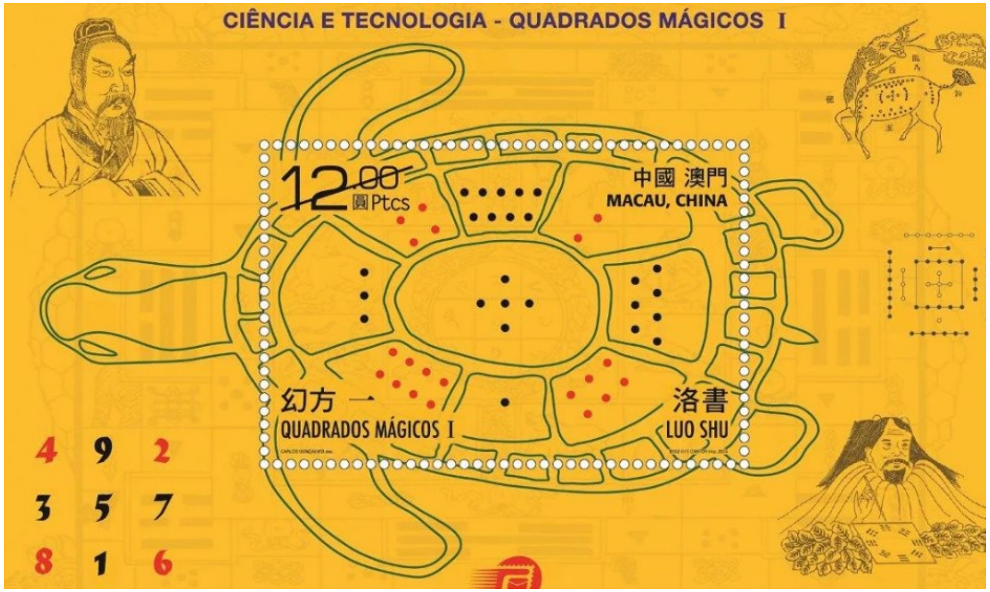
Using the Pascal’s triangle and split the diagram.

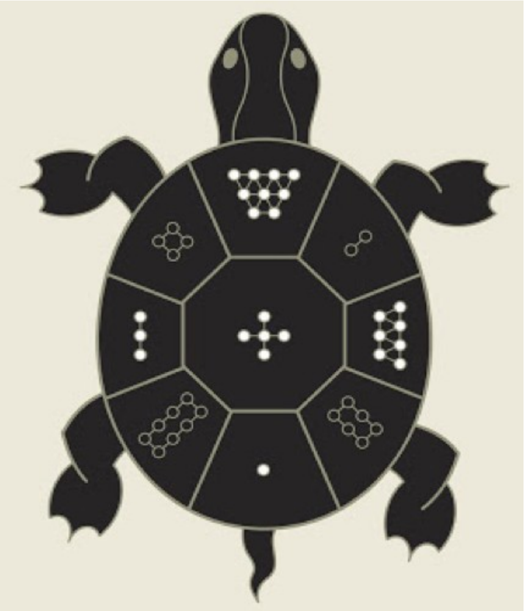
1	= 1
1 1	= 2
1 2 1	= 4
1 3 3 1	= 8
1 4 6 4 1	= 16
1 5 10 10 5 1	= 31
1 6 15 20 15 6 1	= 57
1 7 21 35 35 21 7 1	= 99

Recreational Problems using Mathematics

One of ancient "square" mathematical recreations of all is the magic square.

A Chinese myth, on the time of Emperor Yu, came across a sacred turtle with a strange marking on its shell known as Lo Shu.





Lo Shu

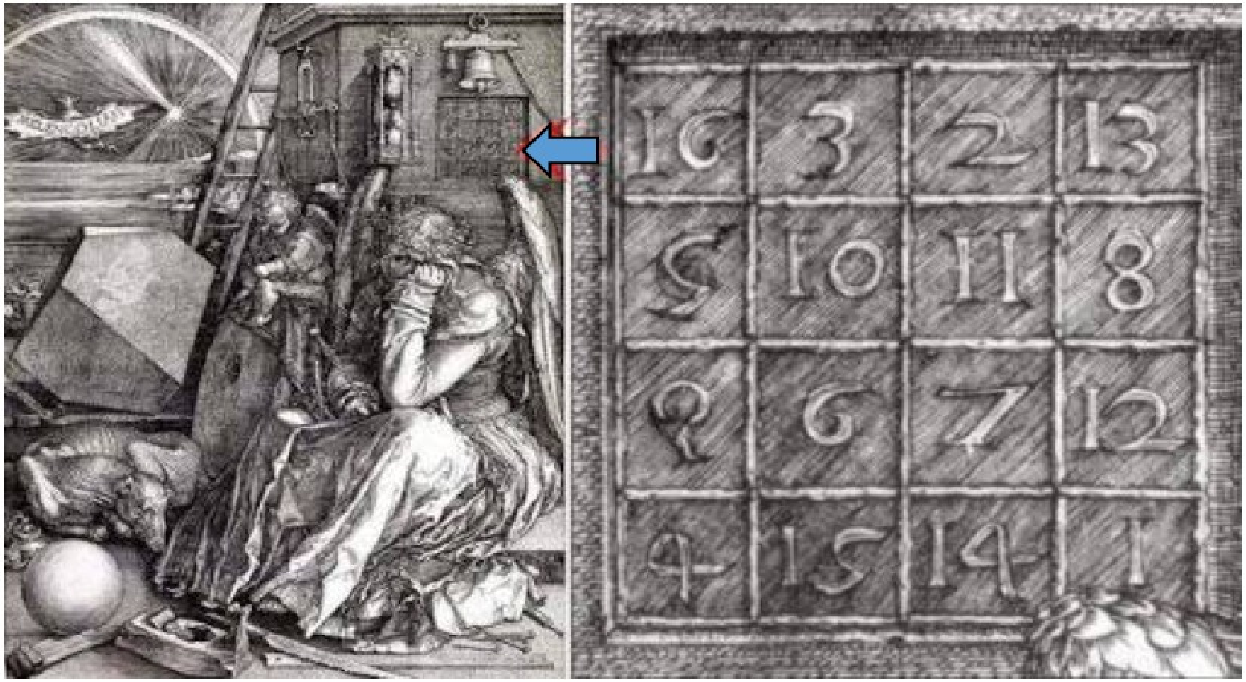
Recreational Problems using Mathematics

The markings are numbers, and they form a square pattern of order 3.

In 1514 the artist Albrecht Durer made an engraving “Melancholia”, containing a square pattern of order 4.

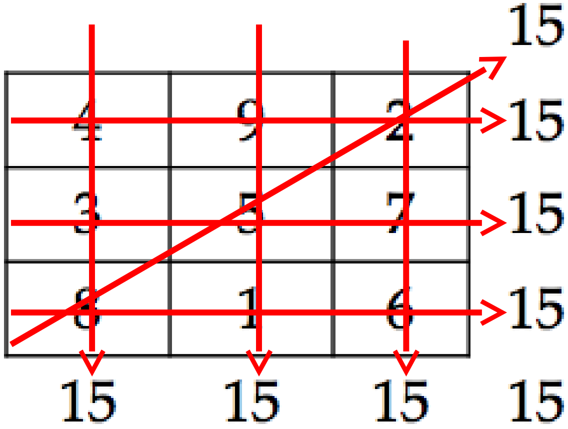
Albrecht Durer

Durer’s Melancholia

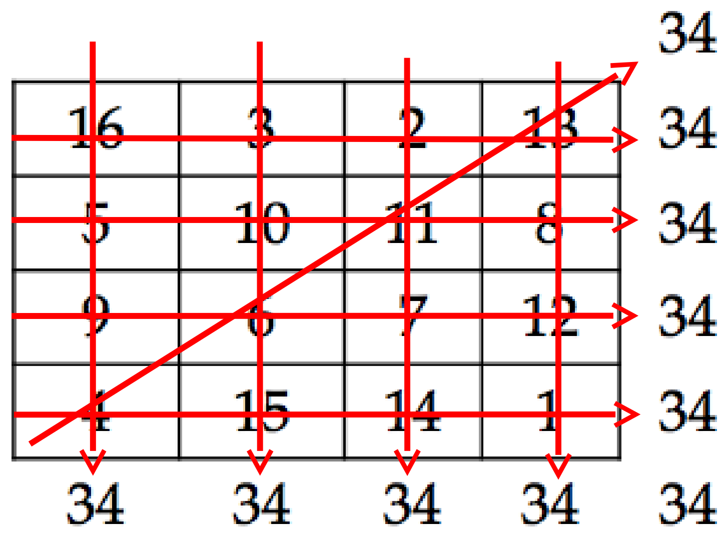


Magic Square

Magic square of order *n* is an arrangement of numbers in a square such that the sum of the *n* numbers in each row, column, and diagonal is the same number.



Magic Square of Order 3



Magic Square of Order 4

Palindrome

A **palindrome** is a number (or word, or phrase) sequence of characters (or symbols) which reads the same backward as forward, for example 131, 12,321, 1,234,321, etc.

Palindromes of squares are as follows:

$1^2 = 1$

$11^2 = 121$

$111^2 = 12{,}321$

$1{,}111^2 = 1{,}234{,}321$

$11{,}111^2 = 123{,}454{,}321$

$111{,}111^2 = 12{,}345{,}654{,}321$

$1{,}111{,}111^2 = 1{,}234{,}567{,}654{,}321$

$11{,}111{,}111^2 = 123{,}456{,}787{,}654{,}321$

:

9-digit Palindromic Primes

Plateau Primes	Smoothly Undulating	5 Consecutive Digits
18888881	323232323	120343021
199999991	727272727	345767543
355555551	919191919	759686957
8 like Digits	**Peak Primes**	**Valley Primes**
111181111	123494321	765404567
111191111	345676543	987101789
777767777	345686543	987646789

Pascal’s Triangle

One of the most famous palindrome triangles is the Pascal’s Triangle named after French mathematician Blaise Pascal (1623-1662).

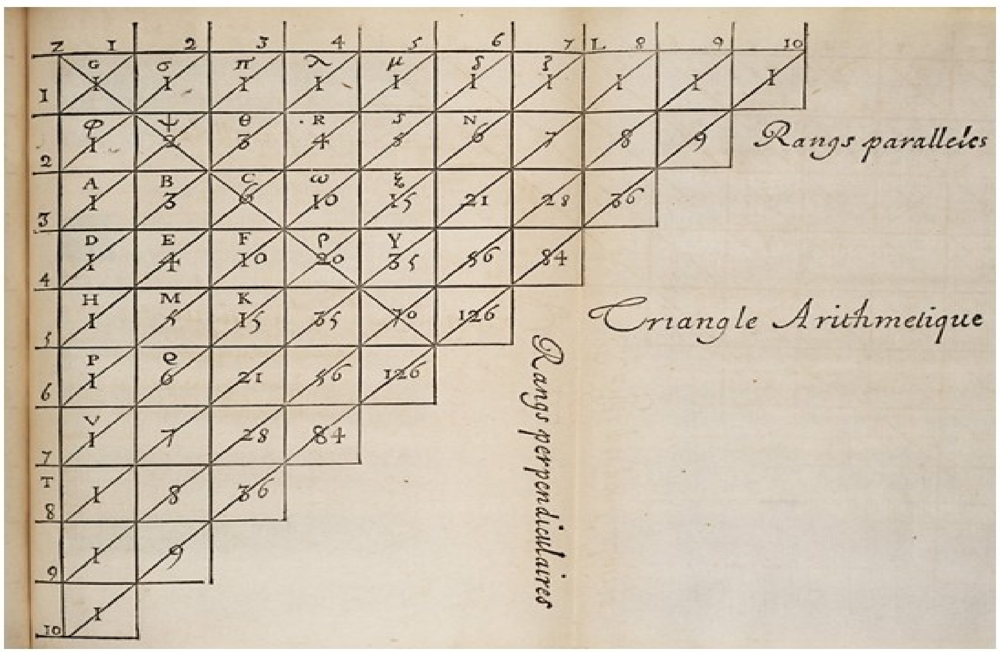
The Pascal’s triangle has intrigued mathematicians for hundreds of years.

Blaise Pascal

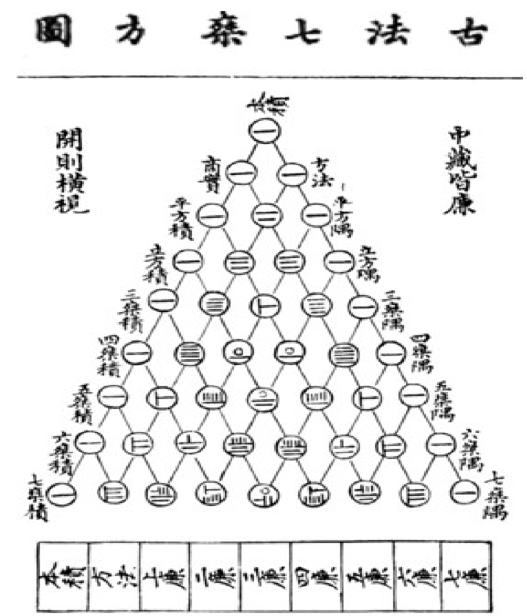
Pascal's Triangle

The Triangle was first published by Zhu Shijie (1260-1320) a Chinese mathematician in 1303 in his "Si Yuan Yu Jian".

It was called Jia Xian Triangle or Yanghui Triangle by the Chinese.

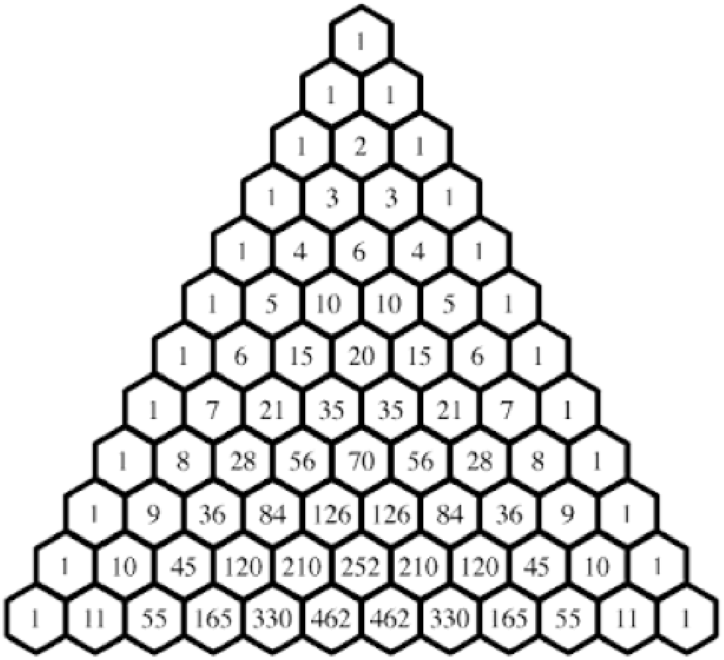


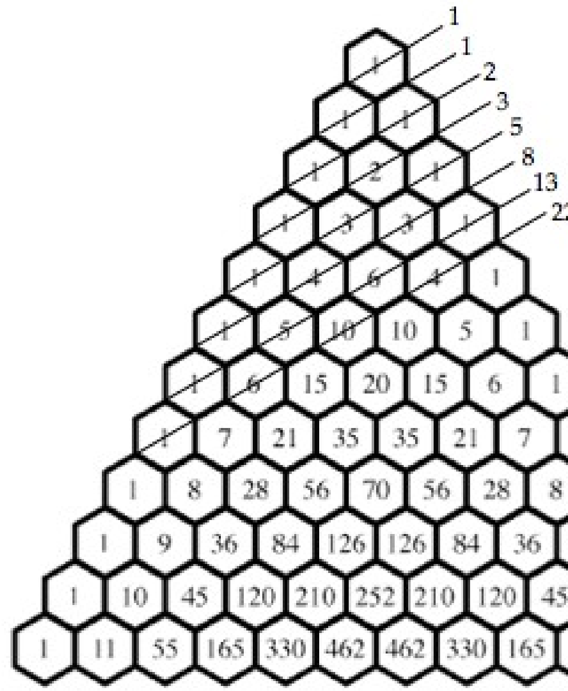
Blaise Pascal's Version



Zhu Shijie' Version

Pascal's and Fibonacci Sequence





Simple Number Triangles

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16
17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36
37 38 39 40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59 60 61 62 63 64
65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81
82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Lazy Caterer's Problem

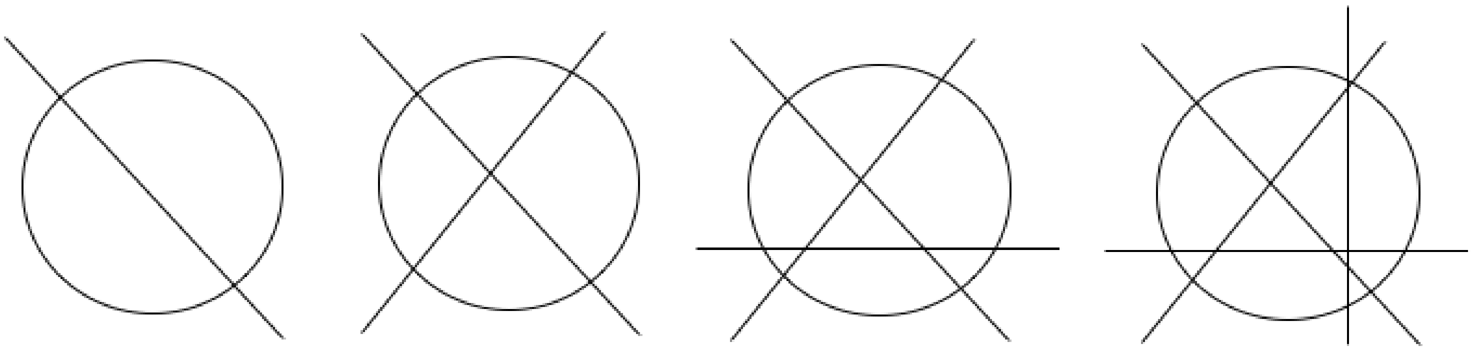
Example 1: The lazy caterer's sequence describes the maximum number of pieces of a pizza that can be made with a given number of straight cuts. One straight cut across a pizza produces 2 pieces. Two straight cuts produce a maximum of 4 pieces. Three straight cuts produce a maximum of 7 pieces.

Four straight cuts produce a maximum of 11 pieces. Determine the number of pieces in which it is the maximum possible pieces to divide the pizza for a given number of straight cuts.

Lazy Caterer's Problem

Solution:

Illustrating the number of cuts and the number of regions it creates.



Cuts	1	2	3	4
Regions	2	4	7	11

To maximize the number of pieces in the next cut, then nth cut must cut each of the $n - 1$ previous one.

Lazy Caterer's Problem

The new cut meets one of the $n - 1$ previous one, a pizza piece is cut in two.

A piece cut in two when the new cut finishes on the opposite side of the pizza.

The total number of pieces of pizza increases by n when we pass from $n - 1$ cuts to n cut, which is exactly what the recurrence reveals.

Lazy Caterer's Problem

The recurrence relation in nth cut creates new regions,

$$f(1) = 2$$
$$f(2) = 4 = 2 + f(1)$$
$$f(3) = 7 = 3 + f(2)$$
$$f(4) = 11 = 4 + f(3)$$
$$\vdots$$
$$f(n) = n + f(n-1)$$

Therefore, $f(n) = n + f(n-1)$.

Lazy Caterer's Problem

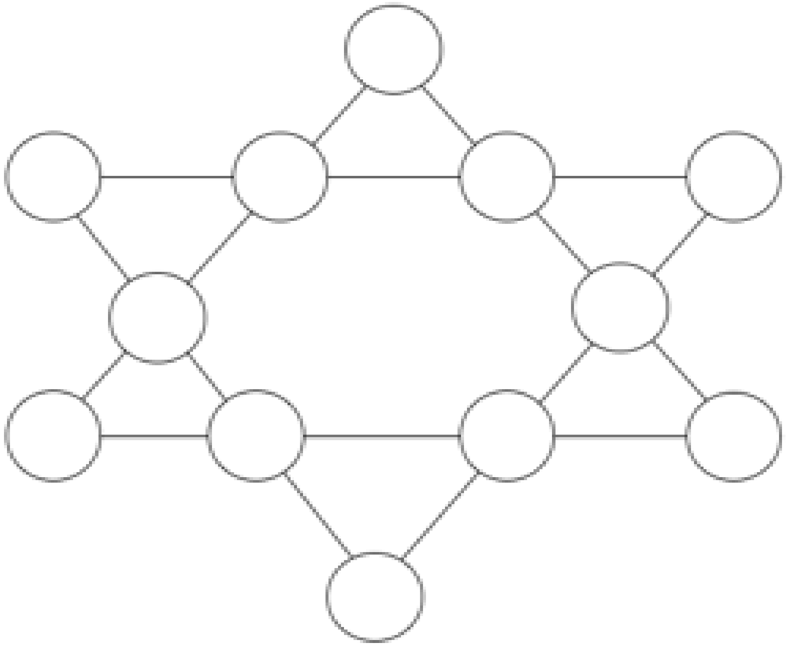
$$
\begin{aligned}
f(n) &= n + (n-1) + (n-2) + \dots + 2 + f(1) \\
&= f(1) + \textstyle\sum_{k=2}^{n} k \\
&= 2 + \tfrac{1}{2}(n+2)(n-1) \\
&= 2 + \tfrac{1}{2}(n^2 + n - 2) \\
&= \tfrac{1}{2}(n^2 + n + 2)
\end{aligned}
$$

Evaluating for

$n = 0, 1, 2, 3, 4, 5, 6, 7, \dots$ gives $1, 2, 4, 7, 11, 16, 22, 29, \dots$

Magic Six Pointed Star

Example 2: Given a magic six pointed star, place the numbers 1 to 12 in the circles, using one of each. Each line of four numbers should have the same total.



Magic Six Pointed Star

Solution:

There are six lines in the magic pointed star.

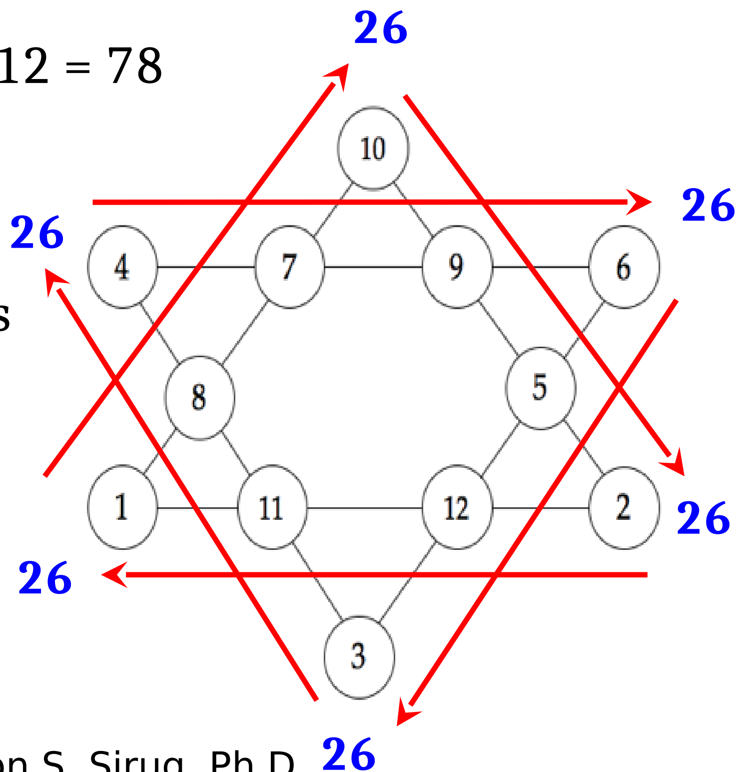
Adding each line, each of the number is considered twice.

Adding the numbers 1 to 12,

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78$$

$$2(78) = 156$$

Divide the sum of the totals 156 by the lines obtaining a value of 26.

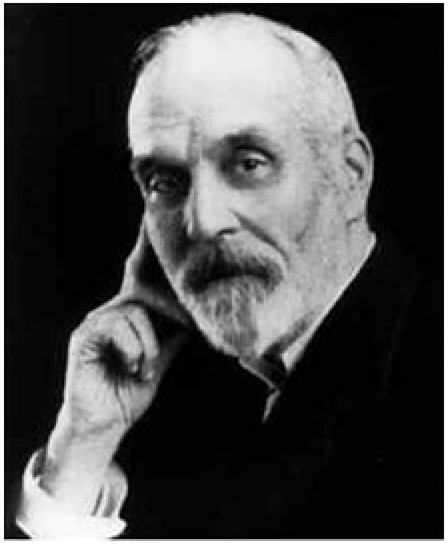


Alphametic

An Alphametic is a type of number puzzle containing sum (or other arithmetic operation) in which digits (0 to 9) are replaced by letters of the alphabet.

One of the most famous alphametic puzzles is the one introduced by Henry Dudeney in 1924.

He was an English author and mathematician who specialized in logic puzzles and mathematical games.



Example 4: Find which digit is equivalent by each of the letters so that the addition is correct.

```
      S E N D
+     M O R E
-------------
    M O N E Y
```

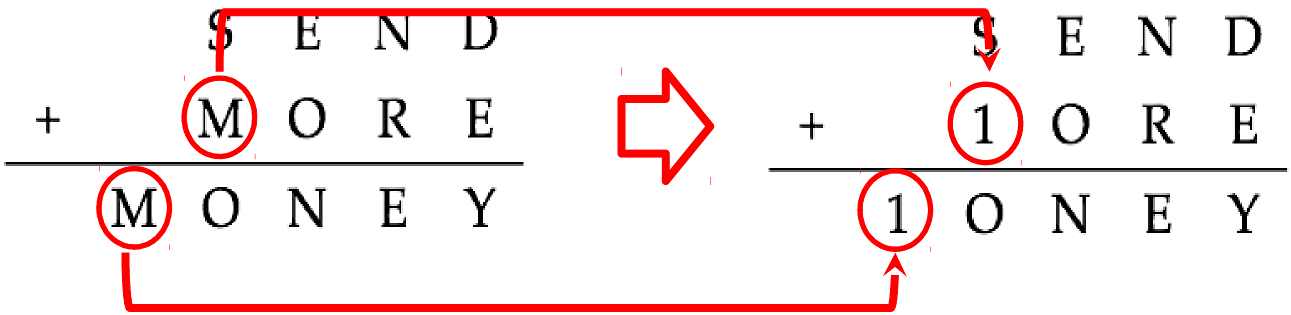
Solution:

The numbers SEND, MORE, and MONEY;

S and M cannot be zero.

Alphametic

The sum of two four-digit numbers cannot exceed 19,998; thus, M is 1.



S + 1 = O.

Recall: S + 1 = 10 + O

(Note S ≠ 1 and O ≠ 1, since M = 1).

Alphametic

Note: There may or may not be a carry into the thousand place.

Thus, O = 0.

No carry into the thousand column, S = 9.

		S	E	N	D
+		1	0	R	E
	1	0	N	E	Y

E + 0 = N.

N = E + 1. (Note: E $\neq$ 9 and N $\neq$ 0).

Alphametic

```
    S E N D            9 E N D
+   1 0 R E    ⇨   +   1 0 R E
-----------        -----------
  1 0 N E Y          1 0 N E Y
```

N + R = E.

Note: N = E + 1, thus 1 + N + R = 9 + N or R = 8

```
    9 E N D
+   1 0 8 E
-----------
  1 0 N E Y
```

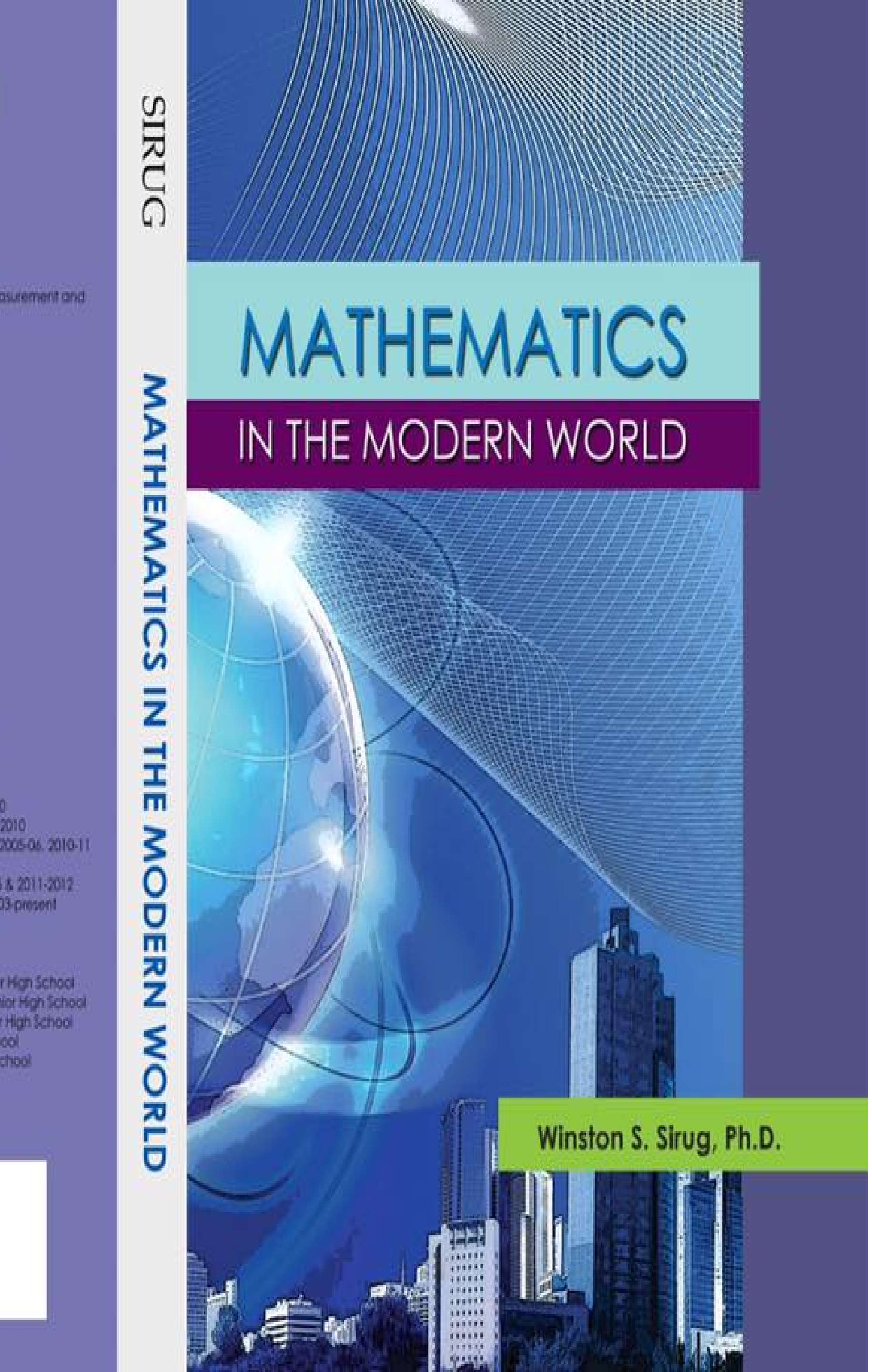
Alphametic

$D + E \geq 12$ (Since $S = 9$, $R = 8$, and $D \neq 1$).

By elimination when $D = 7$, $E = 5$, and $N = 6$, then $Y = 2$.

The sum is

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array} \Rightarrow \begin{array}{rccccc} & & 9 & 5 & 6 & 7 \\ + & & 1 & 0 & 8 & 5 \\ \hline & 1 & 0 & 6 & 5 & 2 \end{array}$$



The definition of a good mathematical problem is the mathematics it generates rather than the problem itself.

–

Andrew Wiles