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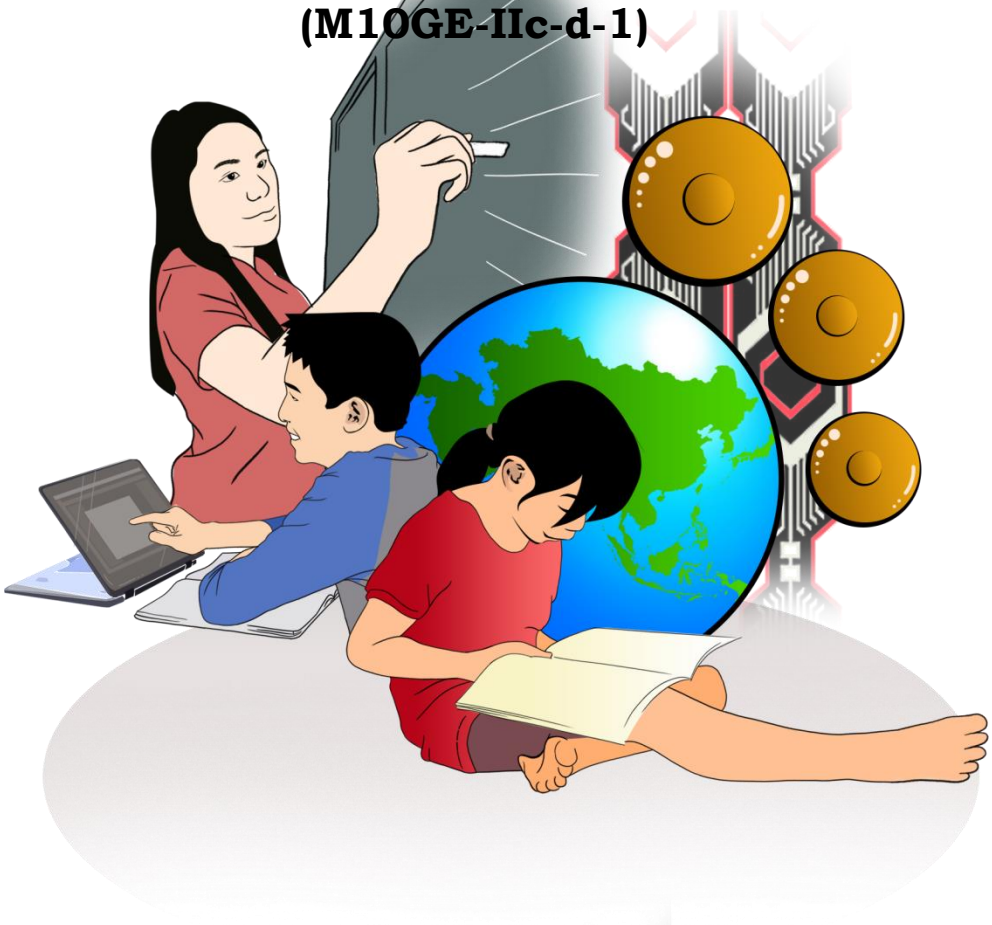
DEPARTMENT OF EDUCATION

Mathematics

Quarter 2 – Module 3:

Proves Theorems Related to Chords, Arcs, Central Angles and Inscribed Angles

(M10GE-IIc-d-1)



SELF-LEARNING MODULE

DEPARTMENT OF EDUCATION - SOCCSKSARGEN

Mathematics – Grade 10
Self-Learning Module (SLM)
Quarter 2 – Module 3: Proving Theorems related to Chords, Arcs, Central Angles and Inscribed Angles
First Edition, 2020

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Introductory Message

For the facilitator:

Welcome to the Mathematics 10 Self-Learning Module (SLM) on Proving theorems related to Chords, Arcs, Central Angles and Inscribed Angles !

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Mathematics 10 Self-Learning Module (SLM) on Proving Theorems related to Chords, Arcs, Central Angles and Inscribed Angles !The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

 ***What I Need to Know***	This will give you an idea of the skills or competencies you are expected to learn in the module.
 ***What I Know***	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
 ***What's In***	This is a brief drill or review to help you link the current lesson with the previous one.
 ***What's New***	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
 ***What is It***	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
 ***What's More***	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
 ***What I Have Learned***	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
 ***What I Can Do***	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.

 ***Assessment***	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
 ***Additional Activities***	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
 ***Answer Key***	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

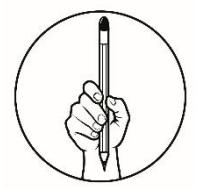
We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the concepts and skills in proving theorems related to chords, arcs, central angles and inscribed angles (M10GE-IIc-d-1). The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

- The module focuses on proving the validity of theorems related to chords, arcs, central angles and inscribed angles.

  After going through this module, you are expected to:
  1. prove theorems involving chords, arcs, central angles and inscribed angles.



What I Know

Let us check your background knowledge about theorems related to chords, arcs, central angles and inscribed angles by answering the questions below. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following states the Perpendicular Bisector Theorem?
   A. In a circle, a radius perpendicular to a chord bisects the chord.
   B. In a circle, a radius that intersects the chord bisects the chord.
   C. In a circle, a radius perpendicular to a chord bisects the arc.
   D. In a circle, a radius that intersects the chord bisects the arc.
2. Which of the following statements is TRUE?
   A. Radii of the different circles are congruent.
   B. All right angles are similar.
   C. The bisector of segment forms two congruent segments.
   D. Corresponding parts of similar triangles are congruent.
3. What property of congruence does line segment CD show if $\overline{CD} \cong \overline{CD}$?
   A. Distributive Property
   B. Symmetric Property
   C. Reflexive Property
   D. Transitive Property
4. What property of equality is used when combining like terms in an equation?
   A. Addition Property
   B. Multiplication Property
   C. Commutative Property
   D. Inverse Property
5. Which of the following statements is TRUE about right angles?
   A. Right angles are closed-figure.
   B. All right angles are similar.
   C. A right angle has an exact measure of $90°$.
   D. Corresponding parts of right angles are congruent.
6. Which of the following states the Congruent Chords Theorem?
   A. In a circle, congruent chords have congruent arcs.
   B. In a circle, congruent chords have congruent sectors.
   C. In a circle, congruent chords have similar arcs .
   D. In a circle, congruent chords have similar sectors.
7. Which of the following statements is TRUE?
   A. Radii of the similar circles are congruent.
   B. Two points determine a line.
   C. In a circle, congruent central angles have different arcs
   D. Corresponding parts of similar triangles are the same.
8. What property of equality is showed in this statement "*If a=b and b=c, then a=c*"?
   A. Distributive Property
   B. Symmetric Property
   C. Reflexive Property
   D. Transitive Property
9. What property of equality is used when using the reciprocal to finally solve for the value of the unknown variable in an equation?
   A. Addition Property
   B. Multiplication Property
   C. Commutative Property
   D. Inverse Property

10. Which of the following statements is TRUE about central angles?
    A. Right angles are central angles
    B. All central angles are congruent.
    C. A central angle has an exact measure of $180°$.
    D. The measure of a central angle is equal to the measure of its intercepted arc.
11. Which of the following states the Inscribed Angle Theorem?
    A. The measure of an inscribed angle is equal the measure of its intercepted arc.
    B. The measure of an inscribed angle is twice the measure of its intercepted arc.
    C. The measure of an inscribed angle is half the measure of its intercepted arc.
    D. The measure of an inscribed angle is one-fourth the measure of its intercepted arc.
12. Which of the following statements is TRUE?
    A. Radii of the same circle are congruent.
    B. The base angles of an isosceles triangle are supplementary.
    C. The measure of a central angle is two times the measure of its intercepted arc.
    D. The measure of an exterior angle of a triangle is equal to the product of the measures of its remote interior angles
13. What is NOT used in proving theorems using the two-column proof?
    A. Definition
    B. Postulate
    C. Property
    D. Example
14. What is a term referred to as "one organize way to show a proof with two columns, the statements and the reasons."
    A. One-Column Proof
    B. Two-Column Proof
    C. Three-Column Proof
    D. Four-Column Proof
15. Which of the following statements is TRUE?
    A. Arc is a portion of the circumference of the circle.
    B. Diameter is a line segment joining any two points on the circle.
    C. Central angle is an angle whose vertex is on a circle and whose sides contain the chords of the circle.
    D. Inscribed angle is an angle whose vertex is the center of the circle and with two radii as its sides.

Lesson 3 Proving Theorems Related to Chords, Arcs, Central Angles and Inscribed Angles

You have derived inductively the relations among chords, arcs, central angles and inscribed angles. Now, it's time for you to investigate deeper by proving theorems related to these concepts.

What's In

In your previous grade level, you have learned about the basic ideas about two-column proofs which are relevant to a better understanding of the validity of theorems specifically about chords, arcs, central angles and inscribed angles. Let's reconsider.

What is the difference?

→ A *postulate* is a statement that is accepted to be true without proof.

For example, *a line contains at least two points*.

→ A *theorem* is a true statement that can be proven.

For example, *if two lines intersect, then they intersect in exactly one point.*

How to prove?

→ There are *different ways* to prove the validity of statements.

One way is by *writing formal arguments of mathematics statements*.

Are you familiar with two-column proof?

→ A *two-column proof* is one common way to organize a *proof in Geometry*.

It always has two columns, the *statements* and the *reasons*.

Questions to Ponder:

→ *What is the basis in writing a theorem?*

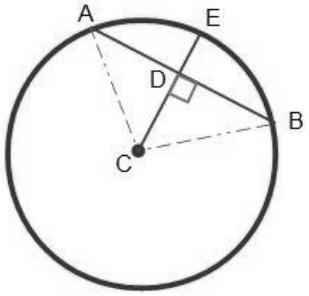
→ *Why do we need to prove statements?*

→ *Is two-column proof also applicable in other branches of Mathematics?*

What's New

This module focuses on the Proving Theorems related to Chords, Arcs, Central Angles and Inscribed Angles. Read the problem below and answer the questions that follow.

Activity: Prove the theorems below using the two-column proof.

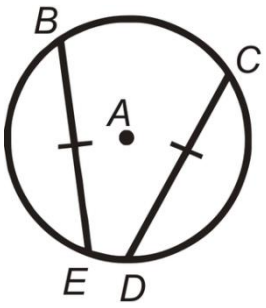


1. If: $\overline{CE} \perp \overline{AB}$ in circle C

Then: $\overline{CE}$ bisects $\overline{AB}$
$\overline{AD} \cong \overline{DB}$

Proof:

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

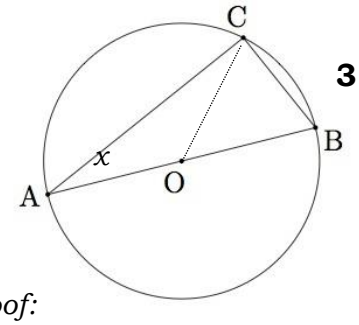


2. If: $\overline{BE} \cong \overline{CD}$

Then: $arc\,AB \cong arc\,CD$

Proof:

Statements	Reasons
1.	1.
2.	2.
3.	3
4.	4
5.	5
6.	6



3. If: $\angle BAC$ is inscribed in circle O and $\overline{BA}$ is a diameter

Then: $m\angle BAC = \frac{1}{2}\, m\, arc\, BC$

Proof:

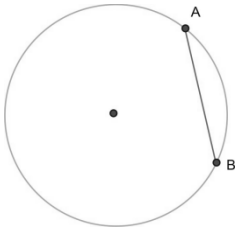
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

What is It

Below are important terminologies, notations and symbols that you must learn and remember in proving theorems related to chords, arcs, central angles and inscribed angles.

Study the concepts below and consider the examples that follow.

Chords and Arcs

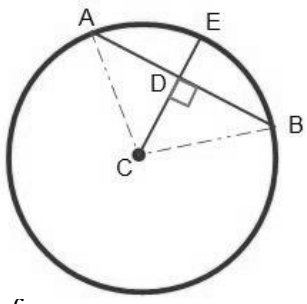


A *chord* is a line segment joining any two points on the circle. In the figure at the right, $\overline{AB}$ is a chord.

On the other hand, An *arc* is a portion of the circumference of the circle. In the figure at the right, $\overset{\frown}{AB}$ is an arc.

One common theorem related to chord is known as Perpendicular Bisector Theorem. It says that *"In a circle, a radius perpendicular to a chord bisects the chord."*

To prove this theorem, we can use the two-column proof.



If: $\overline{CE} \perp \overline{AB}$ in circle C

Then: $\overline{CE}$ bisects $\overline{AB}$

$\overline{AD} \cong \overline{DB}$

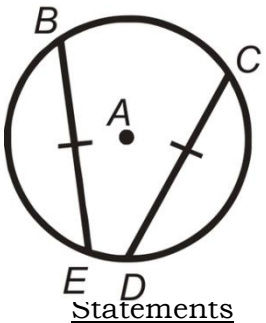
Proof:

Statements	Reasons
1. In circle C, $\overline{CE} \perp \overline{AB}$	1. Given
2. $\overline{CA} \cong \overline{CB}$	2. Radii of the same circle are congruent
3. $\angle CDA, \angle CDB$ are right angles	3. Definition of Perpendicular Lines
4. $\Delta CDA, \Delta CDB$ are right triangles	4. Definition of Right Angles
5. $\angle CDA \cong \angle CDB$	5. Right angles are congruent
6. $\overline{CD} \cong \overline{CD}$	6. Reflexive Property
7. $\Delta CDA \cong \Delta CDB$	7. Hypotenuse Leg (HyL) Theorem
8. $\overline{AD} \cong \overline{DB}$	8. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
9. $\overline{CE}$ bisects $\overline{AB}$	9. Bisector of segment forms two congruent segments

$\therefore$ *A radius perpendicular to a chord bisects the chord in a circle.*

Another common theorem related to arc is known as Congruent Chords Theorem. It says that *"In a circle, or congruent circles, congruent chords have congruent arcs."*

To prove this theorem, we can use again the two-column proof.



If: $\overline{BE} \cong \overline{CD}$

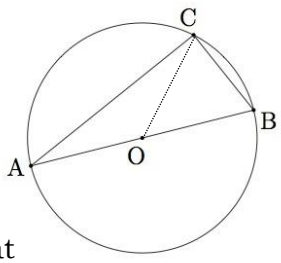
Then: $arc\,AB \cong arc\,CD$

Proof:

Statements	Reasons
10. $\overline{AB} \cong \overline{CD}$	2. Given
11. Draw $\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}$	2. Two points determine a line
12. $\overline{AB} \cong \overline{AC} \cong \overline{AD} \cong \overline{AE}$	3. Radii of the same circle are congruer
13. $\Delta BAE \cong \Delta CAD$	4. Side-Side-Side (SSS) Congruence
14. $\angle BAE \cong \angle CAD$	5. CPCTC
15. $arc\,AB \cong arc\,CD$	6. In a circle, congruent central angles have congruent arcs

$\therefore$ *Congruent chords have congruent arcs in a circle.*

Central Angles and Inscribed Angles

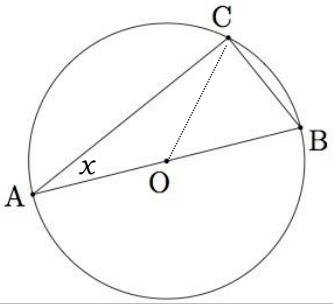


A *central angle* is an angle whose vertex is the center of the circle and with two radii as its sides. In the figure at the right, $\angle COB$ is a central angle.

An *inscribed angle* is an angle whose vertex is on a circle and whose sides contain the chords of the circle. In the figure at the right, $\angle CAB$ is an inscribed angle.

One common theorem related to arc is known as Inscribed Angle Theorem. It says that *"The measure of an inscribed angle is half the measure of its intercepted arc."*

To prove this theorem, we can use again the two-column proof.



If: $\angle BAC$ *is inscribed in circle O* and $\overline{BA}$ is a diameter

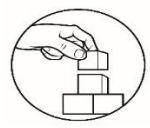
Then: $m\angle BAC = \frac{1}{2}\, m\, arc\, BC$

Proof:

Statements	Reasons
1. $\angle BAC$ is inscribed in circle O and $\overline{BA}$ is a diameter	1. Given
2. $\overline{CO} \cong \overline{AO}$	2. Radii of the same circle are congruer
3. $\angle COA$ is an isosceles triangle	3. Definition of Isosceles Triangle
4. $\angle BAC \cong \angle ACO$	4. The base angles of an isosceles triangle are congruent
5. $m\angle BAC \cong m\angle ACO$	5. The measures of congruent angles a equal
6. $m\angle ACO = x$	6. Transitive Property
7. $m\angle BOC = 2x$	7. The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles
8. $m\angle BOC = m\,arc\,BC$	8. The measure of a central angle is equal to the measure of its intercepted arc.
9. $m\,arc\,BC = 2x$	9. Transitive Property
10. $m\,arc\,BC = 2(m\angle BAC)$	10. Substitution
11. $m\angle BAC = \frac{1}{2} m\,arc\,BC$	11. Multiplication Property of Equality (MPE)

$\therefore$ *The measure of an inscribed angle is half the measure of its intercepted arc*

Concept	Meaning
Arc	It is a portion of the circumference of the circle.
Central Angle	It is an angle whose vertex is the center of the circle and with two radii as its sides.
Chord	It a line segment joining any two points on the circle.
Congruent	It is represented by $\cong$ which means same shape and size.
Inscribed Angle	It is an angle whose vertex is on a circle and whose sides contain the chords of the circle.
Intercepted Arc	It is a section of the circumference of the circle that lies between two lines that intersect it.



What's More

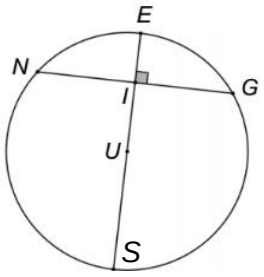
Let us try to answer more challenging set of problems and activities about proving theorems related to chords, arcs, central angles and inscribed angles.

Solve the given problem using two-column proof.

Problem: To prove that in a circle, a diameter bisects a chord and an arc with the same endpoints if and only if it is perpendicular to the chord.

Given: $\overline{ES}$ is a diameter of circle U and perpendicular to chord $\overline{GN}$ at I.

To solve the problem, you need to show these:



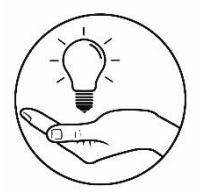
1. $\overline{NI} \cong \overline{GI}$
2. $m\ arcEN \cong m\ arcEG$
3. $m\ arcNS \cong m\ arcGS$

Proof:

Statements	Reasons
1. Circle U with diameter $\overline{ES}$ and chord $\overline{GN}$; $\overline{ES} \perp \overline{GN}$	1. Given
2. _____	2. Definition of Perpendicular Lines
3. $\angle GIU \cong \angle NIU$	3. _____
4. $\overline{UG} \cong \overline{UN}$	4. Radii of the same circle are congruent
5. $\overline{UI} \cong \overline{UI}$	5. _____
6. _____	6. HyL Theorem
7. $\overline{GI} \cong \overline{NI}$	7. _____
8. $\overline{ES}$ bisects $\overline{GN}$	8. _____
16. _____	9. CPCTC
17. $\angle GUI$ and $\angle GUE$ are the same angles; $\angle NUI$ and $\angle NUE$ are the same angles	10. E, I, U are collinear
18. $m\angle GUE = m\angle NUE$	11. _____
19. $m\ arcEG = m\angle GUE$ $m\ arcEN = m\angle NUE$	12. The measure of a central angle is equal to the measure of its intercepted arc.
20. $m\ arcEN = m\ arcEG$	13. Substitution
21. $m\angle GUS = m\angle NUS$	14. Angles that are supplementary to congruent angles are congruent
22. $m\ arcGS = m\angle GUS$ $m\ arcNS = m\angle NUS$	15. The measure of a central angle is equal to the measure of its intercepted arc.
23. $m\ arcNS = m\ arcGS$	16. Substitution
24. $\overline{ES}$ bisects $m\ arcGN$	17. Definition of Arc Bisector

$\therefore$ *A diameter bisects a chord and an arc with the same endpoints if and only if it is perpendicular to the chord.*

Good job! Get ready for another learning battle ahead.



What I Have Learned

Here is an activity that lets you apply what you learned proving theorems related to chords, arcs, central angles and inscribed angles. Just complete the table below by filling in the blanks with correct answer.

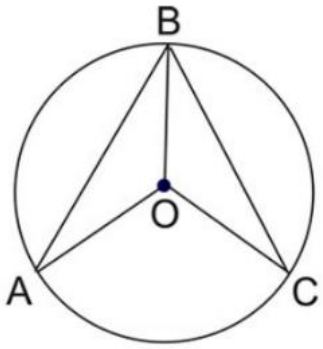
Solve the given problem using two-column proof.

Given:

In circle O, $\overline{AB} \cong \overline{BC}$

Prove:

$\Delta AOB \cong \Delta COB$



Proof:

Statements	Reasons
1. In circle O, $\overline{AB} \cong \overline{BC}$	1. _____
2. $\overline{OA} \cong \overline{OC}$	2. _____
3. $\overline{OB} \cong \overline{OB}$	3. _____
4. $\Delta AOB \cong \Delta COB$	4. _____

Nice work! Now you're up for the final challenge of this module.

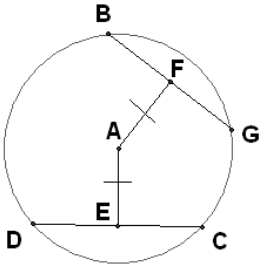
What I Can Do

Here is another activity that lets you apply what you learned about proving theorems related to chords, arcs, central angles and inscribed angles.

Prove the validity of the given theorem using two-column proof.

Theorem:

In a circle, or congruent circles, congruent chords are equidistant from the center.



If: $\overline{BG} \cong \overline{DC}$

Then: $AF = AE$

Proof:

Statements	Reasons
1. _____	1. Given
2. Draw $\overline{AB}, \overline{AG}, \overline{AD}, \overline{AC}$	2. Two points determine a line
3. _____	3. Shortest distance from a point to a line is the perpendicular distance
4. $\angle BFA, \angle GFA, \angle DEA, \angle CEA$ are right angles	4. Definition of Perpendicular Lines
5. _____	5. Radii of the same circle are congruent
6. $\Delta DAC \cong \Delta GAB$	6. SSS Congruence
7. $\angle B \cong \angle C, \angle G \cong \angle D$	7. CPCTC
8. $\Delta DAC, \Delta GAB$ are isosceles triangles	8. _____
9. $\angle B \cong \angle G, \angle D \cong \angle C$	9. The base angles of an isosceles triangle are congruent
10. $\angle B \cong \angle C \cong \angle G \cong \angle D$	10. _____
11. $\angle BFA \cong \angle GFA \cong \angle DEA \cong \angle CEA$	11. All right angles are congruent
12. $\Delta BFA \cong \Delta GFA \cong \Delta DEA \cong \Delta CEA$	12. _____
13. $\overline{AF} \cong \overline{AE}$	1. _____
2. $AF = AE$	14. Congruent segments have congruent length

$\therefore$ *Congruent chords are equidistant from the center of a circle.*

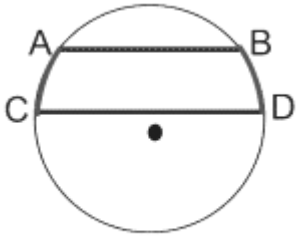
Great work! You did a good job in applying what you have learned!

Assessment

I hope you had a great time going over this module. For you to determine how much you've learned, kindly show the validity of the given theorem using two-column proof.

Theorem:

In a circle, parallel chords intercept congruent arcs.



$\overline{AB} \parallel \overline{CD}$; $\overset{\frown}{AC} \cong \overset{\frown}{BD}$

If: $\overline{AB}$ is parallel to $\overline{CD}$

Then: $arc\,AC \cong arc\,BD$

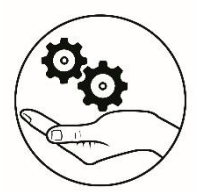
Proof:

Statements	Reasons
1. $\overline{AB}$ is parallel to $\overline{CD}$	1. Given
2. _____	2. _____
3. _____	3. _____
4. _____	4. _____
5. _____	5. _____
6. _____	6. _____
7. _____	7. _____
8. $arc\,AC \cong arc\,BD$	8. Congruent arcs have equal measure

$\therefore$ *Parallel chords intercept congruent arcs in a circle.*

Good Job! You did well on this module! Keep going!

Congratulations on completing the whole module. You are now ready to answer the next module on illustration of secants, tangents, segments and sectors of a circle.



Additional Activities

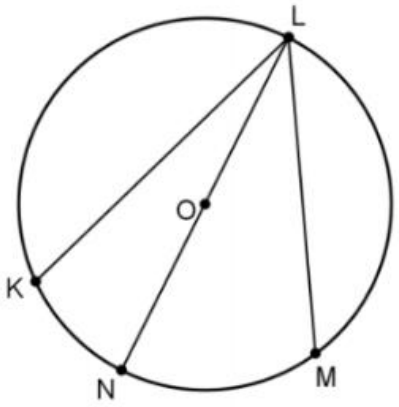
Solve the given problem using two-column proof. Just complete the table below by filling in the blanks with correct answer.

Given:

$\angle KLM$ inscribed in circle O

Prove:

$$m\angle KLM = \frac{1}{2} m\ arcKM$$



Proof:

Statements	Reasons
1. $m\angle KLN = \frac{1}{2} m\ arcKN$ and $m\angle MLN = \frac{1}{2} m\ arcMN$	1. Inscribed Angle Theorem
2. _____	2. _____
3. _____	3. _____
4. _____	4. _____
5. $m\angle KLM = \frac{1}{2} m\ arcKM$	5. Substitution

Answer Key

What I Know

1. A	6. A	11. C
2. C	7. B	12. A
3. C	8. D	13. D
4. A	9. B	14. B
5. C	10. D	15. A

What's More

Statements	Reasons
1. Circle U with diameter $\overline{ES}$ and chord $\overline{GN}$; $\overline{ES} \perp \overline{GN}$	1. Given
2. $\angle GIU$ and $\angle NIU$ are right angles	2. Definition of Perpendicular Lines
3. $\angle GIU \cong \angle NIU$	3. Right angles are congruent
4. $\overline{UG} \cong \overline{UN}$	4. Radii of the same circle are congruent
5. $\overline{UI} \cong \overline{UI}$	5. Reflexive Property
6. $\Delta GIU \cong \Delta NIU$	6. HyL Theorem
7. $\overline{GI} \cong \overline{NI}$	7. CPCTC
8. $\overline{ES}$ bisects $\overline{GN}$	8. Definition of Segment Bisector
9. $\angle GUI \cong \angle NUI$	9. CPCTC
10. $\angle GUI$ and $\angle GUE$ are the same angles; $\angle NUI$ and $\angle NUE$ are the same angles	10. E, I, U are collinear
11. $m\angle GUE = m\angle NUE$	11. Definition of Congruent Angles

What I Can Do

Statements	Reasons
1. $\overline{BG} \cong \overline{DC}$	1. Given
2. Draw $\overline{AB}, \overline{AG}, \overline{AD}, \overline{AC}$	2. Two points determine a line
3. Draw $\overline{AF} \perp \overline{BG}$, $\overline{AE} \perp \overline{DC}$	3. Shortest distance from a point to a line is the perpendicular distance
4. $\angle BFA, \angle GFA, \angle DEA, \angle CEA$ are right angles	4. Definition of Perpendicular Lines
5. $\overline{AB} \cong \overline{AG} \cong \overline{AD} \cong \overline{AC}$	5. Radii of the same circle are congruent
6. $\Delta DAC \cong \Delta GAB$	6. SSS Congruence
7. $\angle B \cong \angle C, \angle G \cong \angle D$	7. CPCTC
8. $\Delta DAC, \Delta GAB$ are isosceles triangles	8. Definition of Isosceles Triangle
9. $\angle B \cong \angle G, \angle D \cong \angle C$	9. The base angles of an isosceles triangle are congruent
10. $\angle B \cong \angle C \cong \angle G \cong \angle D$	10. Substitution
11. $\angle BFA \cong \angle GFA \cong \angle DEA \cong \angle CEA$	11. All right angles are congruent
12. $\Delta BFA \cong \Delta GFA \cong \Delta DEA \cong \Delta CEA$	12. Angle-Angle-Side (AAS) Congruence
13. $\overline{AF} \cong \overline{AE}$	13. CPCTC
14. $AF = AE$	14. Congruent segments have congruent length

What I Have Learned

Statements	Reasons
1. In circle O, $\overline{AB} \cong \overline{BC}$	1. Given
2. $\overline{OA} \cong \overline{OC}$	2. Radii of the same circle are congruent
3. $\overline{OB} \cong \overline{OB}$	4. Reflexive Property
4. $\Delta AOB \cong \Delta COB$	4. SSS Congruence

Additional Activities

Statements	Reasons
1. $m\angle KLN = \frac{1}{2} m\ arcKN$ and $m\angle MLN = \frac{1}{2} m\ arcMN$	1.Inscribed Angle Theorem
2. $m\angle KLN = m\angle MLN = \frac{1}{2} m\ arcKN + \frac{1}{2} m\ arcMN$ or $m\angle KLN = m\angle MLN = \frac{1}{2}(arcKN + m\ arcMN)$	2. Addition Property
3. $m\angle KLN + m\angle MLN = m\angle KLM$	3. Angle Addition Postulate
4. $m\ arcKN + m\ arcMN = m\ arcKM$	4. Arc Addition Postulate
5. $m\angle KLM = \frac{1}{2} m\ arcKM$	5. Substitution

Assessment

Statements	Reasons
1. $\overline{AB}$ is parallel to $\overline{CD}$	1. Given
2. Draw $\overline{BC}$	2. Two points determine a line
3. $\angle ABC \cong \angle BCD$	3. The alternate interior angles are congruent in a transversal
4. $m\angle ABC \cong m\angle BCD$	4. Congruent angles have equal measure
5. $m\angle ABC \cong \frac{1}{2} m\ arcAC$ and $m\angle BCD \cong \frac{1}{2} m\ arcBD$	5. Definition of Inscribed Angle Theorem
5. $\frac{1}{2} m\ arcAC = \frac{1}{2} m\ arcBD$	6. Substitution
6. $m\ arcAC = m\ arcBD$	7. Multiplication Property of Equality (MPE)
7. $arc\,AC \cong arc\,BD$	8. Congruent arcs have equal measure

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EDITOR'S NOTE

This Self-learning Module (SLM) was developed by DepEd SOCCSKSARGEN with the primary objective of preparing for and addressing the new normal. Contents of this module were based on DepEd's Most Essential Learning Competencies (MELC). This is a supplementary material to be used by all learners of Region XII in all public schools beginning SY 2020-2021. The process of LR development was observed in the production of this module. This is version 1.0. We highly encourage feedback, comments, and recommendations.

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