

THE BASICS OF ENGINEERING ECONOMY

Engineering is the profession in which a knowledge of the mathematical and natural science gained by study, experience and practice is applied with judgement to develop ways to utilize, **economically** the materials and forces of nature for the benefit of mankind. In this definition, the economic aspects of engineering are emphasized, as well as the physical aspects. Clearly, it is essential that the economic part of engineering be accomplished well.

In manufacturing, engineering is involved in every detail of a product's production, from the conceptual design to the shipping. In fact, engineering decisions account for the majority of product costs. Engineers must consider the effective use of capital assets such as building and machinery. One of the engineer's primary tasks is to plan for the acquisition of equipment (capital expenditure) that will enable the firm to design and produce products economically.

Engineering economy is the discipline concerned with the economic aspect of engineering. It involves the systematic evaluation with the economic merits of proposed solutions to the engineering problems. To be economically acceptable (i.e., affordable), solutions to engineering problem must demonstrate a positive balance of long term benefits over long term cost.

Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives. The role of engineering economics is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering standpoint.

The General Economic Environment

There are numerous general economic concepts that must be taken into account in engineering studies.

- **Consumer goods and services** are those products or services that are directly used by people to satisfy their wants. Examples are foods, clothing, homes, cars, haircuts and medical services.
- **Producer goods and services** are used to produce consumer goods and services and other producer goods. Examples are machine tools, factory buildings, buses and farm machinery.
- **Price of goods and services** is defined to be the present amount of money or its equivalent which is given in exchange for it.
- **Demand** is a quantity of certain commodity that is bought at a certain price at a given place and time.
- **Supply** is a quantity of a certain commodity that is offered for sale at a certain price at a given place and time.
- **Perfect competition** occurs in a situation in which any given product is supplied by a large number of vendors and there is no restriction in additional suppliers entering the market.
- **Perfect monopoly** exists when a unique product or service is available from a single supplier and that vendor can prevent the entry of all others into the market.
- **Oligopoly** occurs when there are few suppliers and any action taken by anyone of them will definitely affect the course of action of the others.
- **Total Revenue** is the product of the selling price per unit and the number of units sold.
- **Total Cost** is the sum of the fixed costs and the variable costs.
- **Profit/ Loss** is the difference between total revenue and the total costs.

Cost Terminology

Cost considerations and comparisons are fundamental aspects of engineering practice. Before the study of various engineering economic decisions problems, the concept of various costs must be understood. At the level of plant operations, engineers must make decisions involving materials, plant facilities and the in-house capabilities of company level.

- **Fixed costs** are those unaffected by changes in activity level over a feasible range of operations for the capacity or capability available. Examples are insurance and taxes on facilities, general management and administrative salaries, license fees, and interest costs on borrowed capital.
- **Variable costs** are those associated with an operation that vary in total with the quantity of output or other measures of activity level. Examples are the costs of material and labor used in a product or service.
- **Incremental cost** is the additional cost (or revenue) that results from increasing the output of the system by one or more units.
- **Recurring costs** are those that are repetitive and occur when an organization produces similar goods or services on a continuing basis.
- **Nonrecurring costs** are those which are not repetitive even though the total expenditure may become cumulative over a relatively short period of time.
- **Direct costs** are costs that can be reasonably measured and allocated to a specific output or work activity. Examples are labor and material costs.
- **Indirect costs** are those that are difficult to attribute or allocate to a specific output or work activity. Examples are the costs of common tools, general supplies, and equipment maintenance.
- **Overhead cost** consists of plant operating costs that are not direct labor or direct material costs. Examples are electricity, general repairs, property taxes and supervision.
- **Standard costs** are representative costs per unit of output that are established in advance of actual production or service delivery.
- **Cash costs** are that involves payment of cash.
- **Noncash costs** (book costs) are costs that does not involve a cash payment, but rather represent the recovery of past expenditures over a fixed period of time. Example is the depreciation charged.

- **Sunk cost** is one that has occurred in the past and has no relevance to estimates of future costs and revenues related to an alternative course of action.
- **Opportunity cost** is incurred because of the use of limited resources such that the opportunity to use those resources to monetary advantage in an alternative use is foregone.
- **Life-cycle cost** refers to a summation of all the costs, both recurring and nonrecurring, related to product, structure system, or services during its life span.
- **Investment cost** is the capital required for most of the activities in the acquisition phase.
- **Working capital** refers to the funds required for current assets that are needed for the startup and support of operational activities.
- **Operational and Maintenance cost** includes many of the recurring annual expense items associated with the operation phase of the life cycle.
- **Disposal cost** includes those nonrecurring costs of shutting down the operation and the retirement and disposal of assets at the end of the life cycle. These costs will be offset in some instances by receipts from the sale of assets with remaining value.
- **Economic life** coincides with the period of time extending from the date of acquisition to the date of abandonment, demotion in use, or replacement from the primary intended service.
- **Ownership life** is the period between the date of acquisition and the date of disposal by a specific owner.
- **Physical life** is the period between original acquisition and final disposal of an asset over the succession of owner.
- **Useful life** is the time period that an asset is kept in productive service (either primary or backup). It is an estimate of how long an asset is expected to be used in a trade or business to produce income.

Chapter 1

The Economic Environment

Engineering economy is the analysis and the evaluation of the factors that will affect the economic success of engineering projects to the end that a recommendation can be made which will insure the best use of capital.

CONSUMER AND PRODUCER GOODS AND SERVICES

Consumer goods and services are those products or services that are directly used by people to satisfy their wants

Producer goods and services are used to produce consumer goods and services or other producer goods.

NECESSITIES AND LUXURIES

Necessities are those products or services that are required to support human life and activities that will be purchased in somewhat the same quantity even though the price varies considerably.

Luxuries are those products or services that are desired by humans will be purchased if money is available after the required to support human life and activities that will be purchased in somewhat the same quantity even though the price varies considerably.

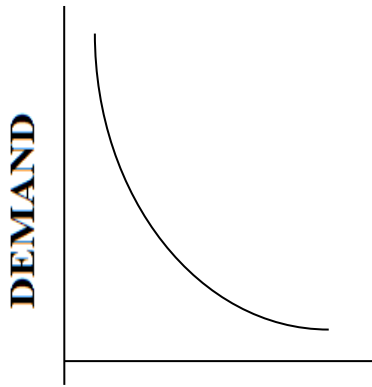
DEMAND

Demand is the quantity of a certain commodity that is bought at a certain price at a given place and time.

Elastic demand occurs when a decrease in selling price result in a greater than proportionate increase in sales.

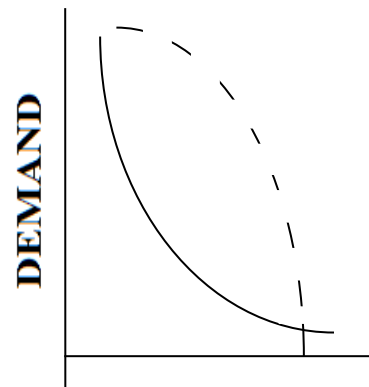
Inelastic demand occurs when a decrease in selling price produces a less than proportionate increase in sales.

Unitary elasticity of demand occurs when the mathematical product of volume and price is constant.



PRICE

FIGURE 1-1. *General Pricedemand forrelationship*



PRICE

FIGURE 1-2. *Price-demand relationship luxuries and necessities*

Competition, Monopoly and Oligopoly

Perfect competition occurs in a situation where a commodity or service is supplied by a number of vendors and there is nothing to prevent additional vendors entering the market.

Monopoly is the opposite of perfect competition. A perfect monopoly exists when a unique product or service available from a single vendor and that vendor can prevent the entry of all others into the market.

Oligopoly exists when there are so few suppliers of a product or service that action by one will almost inevitably result in similar action by the others.

The Law of Supply and Demand

Supply is the quantity of a certain commodity that is offered for sale at ascertain price at a given place and time

The law of supply and demand may be stated as follows:

“Under conditions of perfect competition the price at which a given product will be supplied and purchased is the price that will result in the supply and the demand being equal”

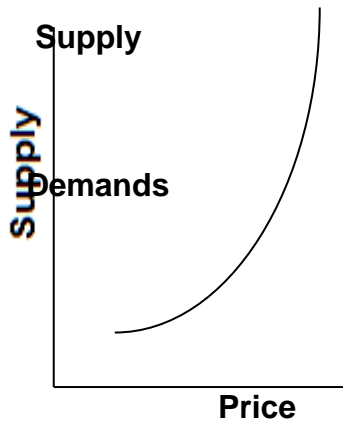


Figure 1-3. *General price-supply relationship*

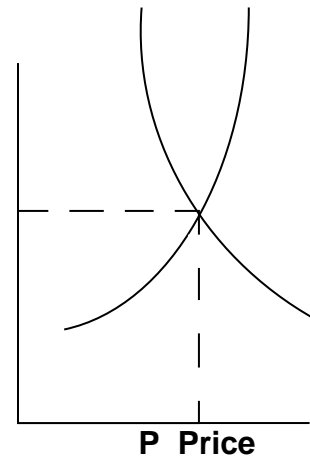
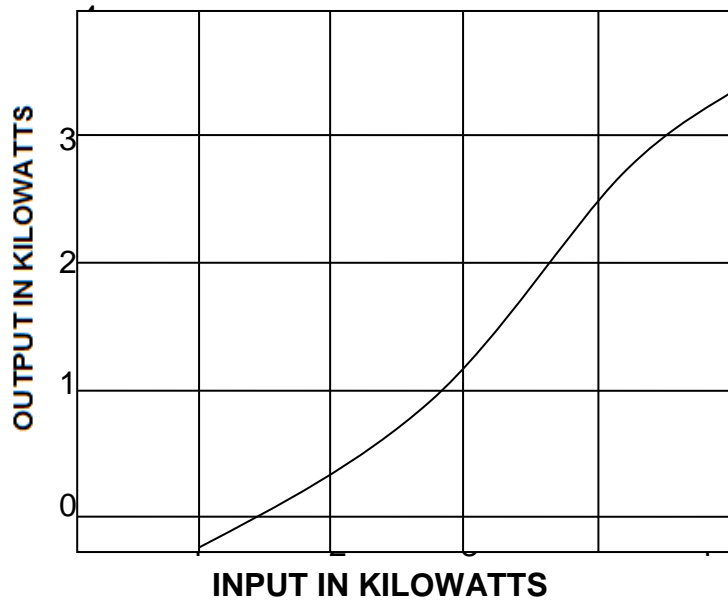


Figure 1-4. *Price-supply demand relationship*

The Law of Diminishing Returns

“When the use of the one of the factors of the production is limited, either in increasing cost or by absolute quantity, a point will be reached beyond which an increase cost or by absolute quantity, a point will be reached beyond which an increase in the variable factors will result in a less than proportionate increase in output.”



5

Figure 1-5. *Performance curve of an electric motor.*

The effect of the law of diminishing returns on the performance of an electric motor is illustrated in Fig. 1-5. For the early increase in input through input of 4.0kw, the actual increase in output is greater than proportional; beyond this point the output is greater than proportional. In case the fixed input factor is the electric motor.

Chapter 2

Interest and Money Time Relationships

Capital

The term **capital** refers to wealth in the form of money or property that can be used to produce more wealth. The majority of engineering economy studies involve commitment of capital for extended periods of time, so effect of time must be considered. It is recognized that a unit of principal today is worth more than a unit of principal one or more years from now because of interest (or profits) it can earn. Therefore, **money has a time value.**

The following reasons why Php100 today is “worth” more than Php100 one year from today:

1. Inflation
2. Risk
3. Cost of money (interest)

Of these, the **cost of money** is the most predictable, and, hence, it is the essential component of economic analysis. Cost of money is represented by money paid for the use of borrowed money, or return of investment.

Cost of money is determined by an **interest rate**. It is established and measured by an interest rate, a percentage that is periodically applied and added to an amount of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender’s property;

when the money is loaned or invested, the interest earned is the lender's gain from providing a good to another. Interest maybe defined as the cost of having money available for use.

Types of Capital

1. **Equity Capital** is that owned by individuals who have invested their money or property in a business project or venture in the hope of receiving a profit.
2. **Borrowed Capital** is obtained from lenders for investment, with a promise to repay the principal and interest on a specific date, whether or not the operations of the business have been profitable or not. In return, the lenders receive interest from the borrowers.

Cash Flow Diagram

Cash flow is the stream of monetary values – costs and benefits – resulting from a project investment. It is difficult to solve a problem is you cannot see it. The easiest way to approach problems in economic analysis is to draw a picture. **Cash flow diagram** is a graphical representation of cash flows drawn in a time scale. It has three elements:

1. A **horizontal line** represents time with progression of time moving from left to right. The period labels can be applied to intervals of time rather than to point on the time scale. A time interval is divided into an appropriate number of equal periods.
2. **Arrows** represent cash flow and are place at the specified period. If distinctions are needed to be made, downward arrows represents cash outflows (expenditures, disbursements) and upward arrows represents cash inflows (income).

3. **Depends on the person's viewpoint.** Unless otherwise indicated, all such cash flows are considered to occur at the end of their respective periods.

The following symbols nomenclatures will be used:

P = Present sum of money
F = Future sum of money
N = Number of interest periods
i = Interest rate per period

Interests the amount of the money paid for the use of borrowed capital or the income produced by money which has been loaned.

Interest from the viewpoint of the lender is the income produced by money which has been borrowed or invested. For the borrower, it is the amount of money paid for the use of borrowed capital.

Simple Interest

When the total interest earned is linearly proportional to the amount of the loan (principal), the number of the interest rate per interest periods for which the principal is committed, and the interest rate per interest period, the interest is said to be simple.

Is the calculated using the principal only, ignoring any interest that had been accrued in preceding periods. In practice, simple interest is paid on short term loans in which the time of the loan is measured in days

$$I = Pni$$

$$F = P + I = P + Pni$$

$$F = P (1+ni)$$

Where: I = interest

P = principal or present worth

n = number of interest periods

i = rate of interest per interest period

F = accumulated amount or future

Types of Simple Interest

1. Ordinary Simple Interest (OSI)

Based on one banker's year which is equivalent to 300 days or 30 days in one month.

is computed on the basis of 12 months of 30 days a year.

$$1 \text{ interest period} = 360 \text{ days}$$

$$\text{OSI} = Pni$$

Example:

Determine the ordinary simple interest on P8,000 for 8 months at 7% per year.

Given: $P = 8,000$

$i = 7\% / \text{yr}$

$n = 8 \text{ monthly}$

Required: OSI

Solution:

$$\text{OSI} = Pni$$

$$\text{OSI} = 8000 \left(\frac{8}{12} \right) (0.07)$$

$$\text{OSI} = P373.33$$

Problem 1

Determine the ordinary simple interest on P700 for 8 months and 25 days if the rate of interest is 15%.

Solution

Number of days = $(8)(30) + 25 = 255 \text{ days}$

$$I = Pni = P700 \times \frac{255}{360} \times 0.15 = P74.38$$

2. Exact Simple Interest (ESI)

Based in the exact number of days which is 365 days in ordinary year or 366 days for leap year.

is based on the exact number of days in year, 365 days for an ordinary year and 366 days for a leap year.

$$1 \text{ interest period} = 365 \text{ or } 366 \text{ days}$$

$$ESI = Pni$$

$$ESI = P \frac{n}{365} (i) \quad \text{for ordinary year}$$

$$ESI = P \frac{n}{366} (i) \quad \text{for leap year}$$

Example:

Determine the exact simple interest on P500 for the period from January 10 to October 28, 1996 at 16% interest.

Solution

Jan 10-31	= 21 (excluding Jan 10)
February	= 29
March	=31
April	=30
May	=31
June	=30
July	=31
August	=31
September	=30
October	= <u>28</u> (including Oct.28)
	292 days

$$\text{Exact simple interest} = P500 \left(\frac{292}{366} \right) (0.16) = P63.83$$

Example

Determine the ordinary exact interest on P15,000 loan at 8% per year made from January 12 to September 24, 2007.

$$\text{Given: } P = P15,000$$

$$i = 8\% \text{ per year}$$

$$n = \text{January 12 to September 24, 2007}$$

Required:

- OSI, ordinary simple interest
- ESI, exact simple interest

Solution:

Solving for the number of days, from Jan 2 to September 24, 2007, there are 256 days.

(*Note: for a specific dates, to count the number of days , exclude the first day, include the last day .)

a. Solving for the ordinary simple interest:

$$\begin{aligned} \text{OSI} &= P\left(\frac{n}{360}\right)i \\ &= P15\,000 (0.08/\text{year}) \left(\frac{256}{360}\right)\text{year} \\ &= P853.33 \end{aligned}$$

b. Solving for the exact simple interest:

$$\begin{aligned} \text{ESI} &= P\left(\frac{n}{365}\right)i \\ &= P15\,000(0.08/\text{year})\left(\frac{256}{365}\right)\text{year} \\ &= P853.34 \end{aligned}$$

Example

What will be the future worth of money after 14months, if a sum of P10,000 is invested today at a simple interest rate of 12% per year?

Solution

$$F = P(1+ni) = P10,000 \left[1 + \frac{14}{12} (0.12) \right] = P 11,400$$

Problem:

Determine the exact simple interest on P50 000 periods for the period June 25, 2005 to September 2, 2006, if the rate of interest is 12% per year?

Problem:

A loan of P2000 is made for a period of 13 months, from January 1 to January 31 the following year, at simple interest rate of 20% . What future amount is due as the end of the loan period?

Solution:

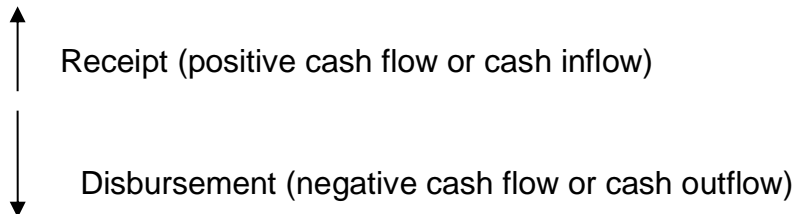
Problem:

If you borrow money from your friend with simple interest of 12%, find the present worth of P20, 000, which is due at the end of nine months.

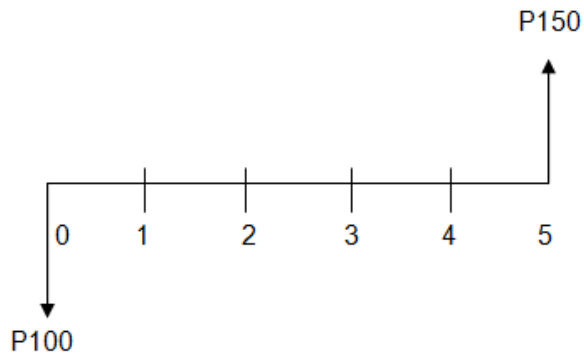
Solution:

Cash-Flow Diagrams

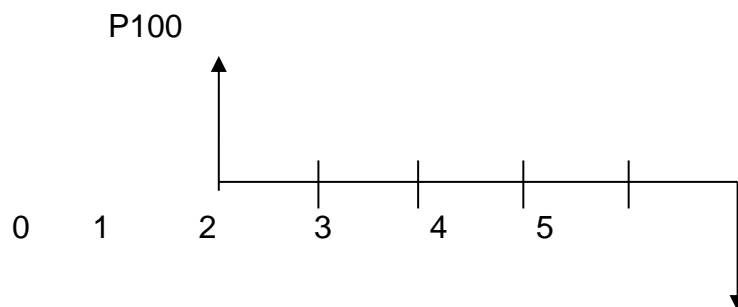
A cash-flow diagram is simply a graphical representation of cash flows drawn on a time scale. Cash-flow diagram for economic analysis problems is analogous to that of free body diagram for mechanics problems.



A loan of 100 at simple interest of 10% will become P150 after 5 years.



Cash flow diagram on the viewpoint of the lender



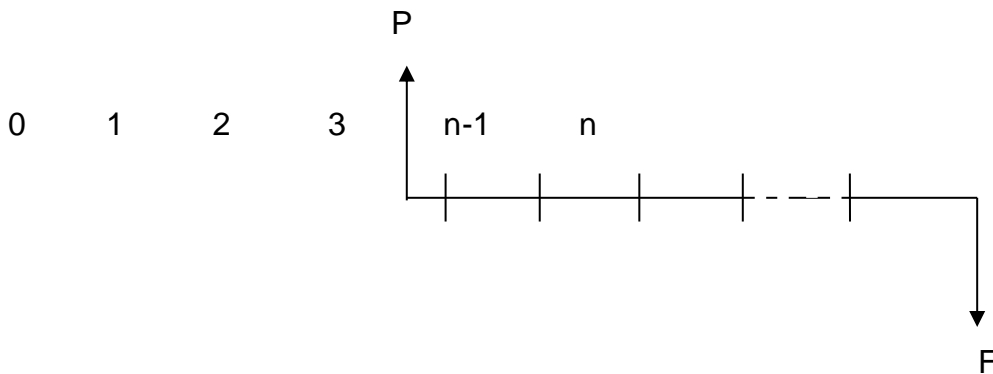
P150

Cash flow diagram on the viewpoint of the borrower

Compound Interest

Whenever the interest charge for any interest period is based on the remaining principal amount plus any accumulated interest charges up to the beginning of that period the interest is said to be compounded.

In calculations of compound interest, the interest for an interest period isn't calculated on the principal plus total amount of interest accumulated in previous periods. Thus compound interest means "interest on top of interest."



Compound Interest (Borrower's Viewpoint)

Discrete Compounding

The formulas are for discrete compounding, which means that the interest is compounded at the end of each finite length period, such as a month or a year.

Discrete Cash Flows

The Formulas also assume discrete (i.e., lump sum) cash flows spaced at the end of equal time intervals on a cash flow diagram.

Derivation of Formula

Interest Period	Principal at Beginning of Period	Interest Earned During Period	Amount at End of Period
1	P	Pi	$P+Pi = P(1+ni)$
2	$P(1+i)$	$P(1+i) i$	$P(1+i) + P(1+i)i$ $= P(1+i)^2$

3	$P(1+i)^2 i$	$P(1+i)^2 i$	$P(1+i)^2 + P(1+i)^2 i$ $=P(1+i)^2$
...
n	$P(1+i)^{n-1} i$	$P(1+i)^{n-1} i$	$P(1+i)^n$

$$F = P (1+i)^n$$

The quantity $(1+i)^n$ is commonly called the “single payment compound amount factor” and is designated by the functional symbol $F/P, i\%, n$. Thus,

$$F = P (F/P, i\%, n)$$

The symbol $F/P, i\%, n$ is read as “F given P at per cent in n interest period.” From Equation (2-3),

$$P = F(1+i)^{-n}$$

The quantity $(1+i)^{-n}$ is called the “single payment present worth factor” and is designated by the functional symbol $P/F, i\%, n$. Thus,

$$P = F (P/F i\% n)$$

The symbol $P/F, i\%, n$ is read as “P given F at i percent in n interest periods.”

The Five Types of Cash Flows

Whenever patterns are identified in cash flow transactions, these patterns can be used to develop concise expressions for computing either the present or future worth of the series. Cash flows can be classified into five categories:

1. **Single Cash Flows:** The simplest case involves the equivalence of a single cash present amount and its future worth. Thus, the single-cash flow formula deal with only two amounts: the single present amount P, and its future worth F.
2. **Equal Uniform Series:** Probably the most familiar category includes transactions arranged as a series of equal cash flows at regular intervals, known as an equal-payment series (or uniform series)
3. **Linear Gradient Series** consists of cash flows that increase or decrease by uniform amount each periods.
4. **Geometric Gradient Series** consist of cash flows that increase or decrease by a fixed percentage.
5. **Irregular Series** consists of cash flows that change with no pattern.

Problem:

How long will it take inventory to triple itself if invested of 12% per unit?

Solution:

Problem:

You deposit \$3,000 in a savings account that 9% simple interest per year. How many years will it take to double your balance? If instead you deposit the \$3,000 in another savings account that earns 8% interest compounded yearly, how many years will it take to double your balance?

Solution:

Interest

Interest from the viewpoint of the lender is the income produced by money which has been borrowed or invested. For the borrower, it is the amount of money paid for the use of borrowed capital.

Is the amount of the money paid for the use of borrowed capital or the income produced by money which has been loaned

Rate of Interest

is defined as the amount earned by one unit of principal during a unit of time

(a) Nominal rate of interest

The nominal rate of interest specifies the rate of interest and a number of interest periods in one year.

Nominal rate of interest specifies the rate of interest and the number of compounding period in one year. It has because customary to wrote interest rates on an annual basics followed by the compounded period if different from one year in length. Even if a financial institution.

$$i = \frac{r}{m}$$

where: i = rate of interest per interest period
 r = nominal interest rate
 m = number of compounding periods per year

$$n = Tm$$

where: n = no. of interest per periods
 T = no. of years
 m = number of compounding periods per year

If the nominal rate o interest is 10%compounded quarterly, then $i = 10\%/4 = 2.5\%$, the rate of interest per interest period.

The following list the different compounding periods and the occurrence in one year

annually	once a year
semi- annually	twice a year
Quarterly	every 3 months
Bi – monthly	every 2 months
Monthly	every month
Semi- monthly	twice a month
Weekly	
Daily	
Continuously	

Example:

The sum of P10, 000 was deposited to a fund earning interest 10% per annum compounded quarterly, what was the principal in the fund at the end of a year?

Given: P = 10,000

r= 10% per year compounded quarterly

n= 3 years (4) = 12 quarters

Required: The amount in the fund at the end of 3 years

Solution: Solving for the rate of interest per period.

$$i = \frac{r}{m} = \frac{10}{12}$$

$$= 2.5 \% \text{ per quarter}$$

(b) Effective rate of interest

Effective rate of interest quotes the exact rate of interest for one year it should be noted that the effective interest rates are always expressed on an annual basis.

Effective rate of interest is the actual or exact rate of interest on the principal during one year. If P1.00 is invested at a nominal rate of 15% compounded quarterly, after one year this will become.

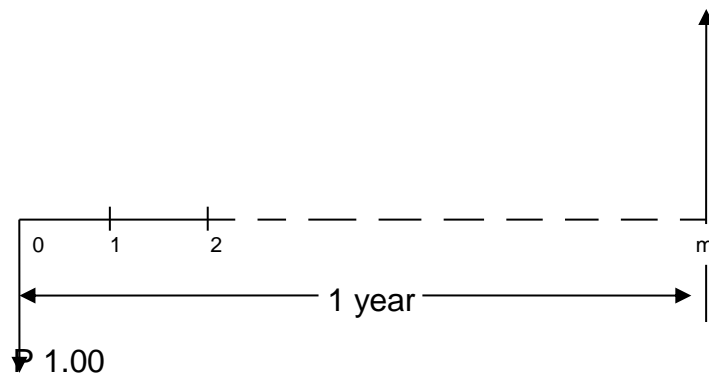
$$P1.00 \left[1 + \left(\frac{0.15}{4} \right) \right]^4 = P1.1586$$

The actual interest earned is 0.1586, therefore, the rate of interest after one year is 15.86%. Hence,

$$\text{Effective rate} = F1 - 1 = (1+i)^m - 1$$

where: $F1$ = the amount P1.00 will be after one year

F_1



Effective Rate of Interest

Example

Consider, one unit of principal for one unit a time invested in a nominal interest of 12% compounded monthly,

$$P = 1.00$$

$N = 12$ months

$$i = 12/12 = 1\% / \text{month}$$

The accumulated amount of P for one year will be

$$\begin{aligned} F &= P(1+i)^n \\ &= 1(1.01)^{12} = P1.1268 \end{aligned}$$

The implication is that, for one unit of principal. The interest earned will be

$$I = 0.1268$$

and in terms of effective or actual annual interest (ERI) the interest can be rewritten as a percentage of the principal amount:

$$i = I / P = 0.1268 / 1 = 12.68\%$$

Therefore, to determine the effective rate of interest for a given nominal rate of interest,

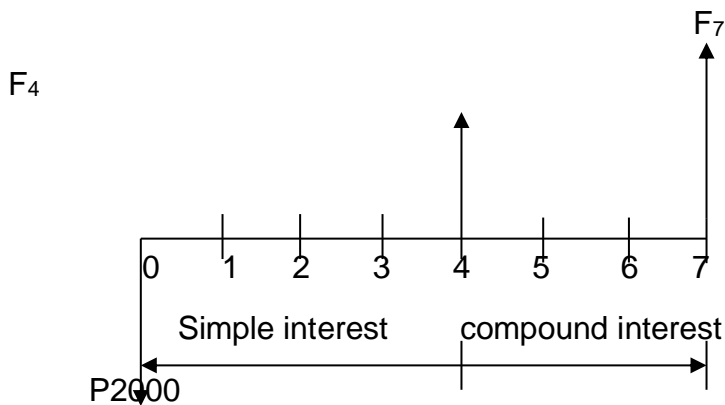
$$ER = (1+i)^m - 1$$

In other words, paying 1% interest per month for 12 months is equivalent to paying 12.68% interest just one time each year.

Example

A P2000 loan was originally made 8% simple interest for 4 years. At the end of this period the loan was extended for 3 years, without the interest being paid, but the new interest rate was made 10% compounded semi annually. How much should the borrower pay at the end of the 7 years?

Solution



$$F_4 = P(1 + ni) = P2000 [1 + (4)(0.08)] = P 2,640$$

$$F_7 = F_4(1 + i)^n = P2,640 (1 + 0.05)^6 = P3,537.86$$

Example

Find the normal rate which if converted quarterly could be used instead of 12% compounded monthly. What is the corresponding effective rate?

Solution:

Let r = the unknown nominal rate

For two or more nominal rates to be equivalent, their corresponding effective rates must be equal.

Nominal rate	Effective rate
r% compounded quarterly	$(1 + \frac{r}{4})^4 - 1$
12% compounded monthly	$(1 + \frac{0.12}{12})^{12} - 1$

$$\left(1 + \frac{r}{4}\right)^4 - 1 = 1(0.01)^{12} - 1$$

$$1 + \frac{r}{4} = (1.01)^3 = 1.0303$$

r=0.1212 or 12.12% compounded quarterly

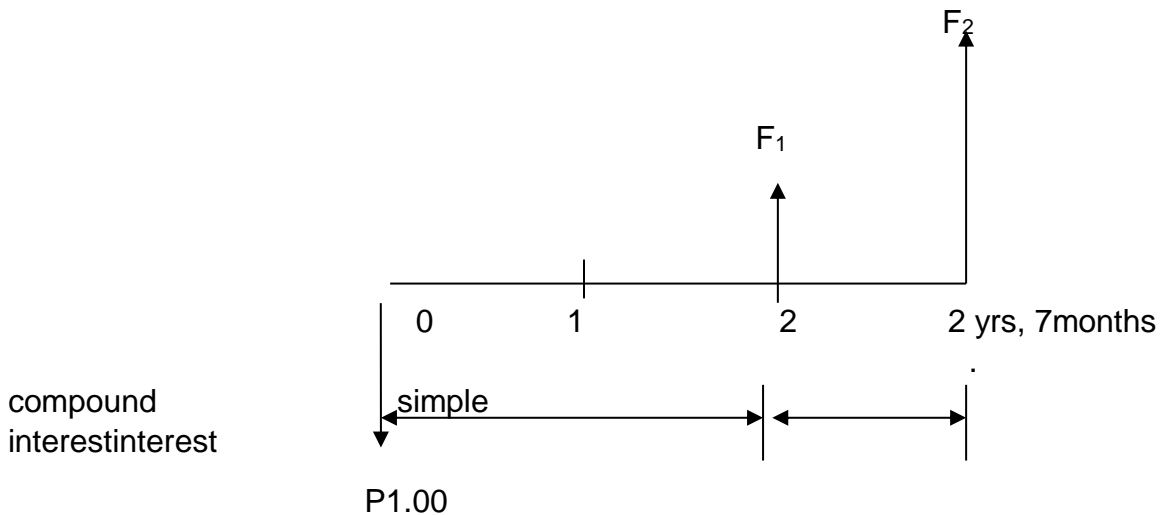
Example

Find the amount at the end of two years and seven months if P100 is invested at 8% compounded quarterly using simple interest for anytime less than a year interest period.

Solution:

For compound interest: $i = \frac{8\%}{4} = 2\%$, $n = (20)(40) = 8$

For simple interest: $i = 8\%$, $n = \frac{7}{12}$



$$F1 = P(1 + i)^n = 1000(1 + 0.02)^8 = P1171.66$$

$$F2 = F1(1 + ni) = P1171.66 [1 + \frac{7}{12}(0.08)] = P 1,226.34$$

Example:

A loan of P10,000 for 3 years at 10% simple interest per year. Determine the interest earned and the accumulated amount.

Given: $P = P10,000$

$n = 3$ years

$i = 10\%$ per year

Required: a. Total Interest, I

b. Accumulated amount, F

Solution:

a. The total interest earned:

$$I = Pni$$

$$I = P10,000 (3 \text{ years}) (0.1/\text{year})$$

$$I = P3,000$$

The accumulated amount at the end of the interest period:

$$F = P + I$$

$$F = P10,000 + 3,000$$

$$F = P13,000$$

Amortization Schedule

- A table showing the mode of payment.

Period	Principal	Interest	Payment
1	10, 000	1, 000	0
2	10, 000	1, 000	0
3	10, 000	1, 000	13, 000
Total		3, 000	

Problem:

Annie buys a television set from a merchant who asks $P12,500$. At the end of 60 days (cash in 60 days). Annie wishes to pay immediately and the merchant offers to compute the cash price in the assumption that money is worth 8% simple interest.

What is the cash price today?

Problem:

What will be the future worth of money after 14 months of a sum of P10, 000 is invested today at a simple interest rate of 12% per year?

Problem:

What is the amount of interest earned on \$2,000 for five years at 10% simple interest per year?

Problem:

Determine the effective rates of interest for the following nominal rates of interest

- a. 9% compounded annually
- b. 9% compounded semi annually
- c. 9% compounded quarterly
- d. 9% compounded monthly
- e. 9% compounded daily

Problem

If you are investing which is better 15% compounded semiannually or 14.5% compounded monthly?

Problems:

What nominal rate of interest compounded quarterly can be used instead of 8% compounded monthly. ($r = 8\%$ compounded monthly but the required interest rate, i , is per quarter.)

Problem:

Supposed that you have just borrowed P75,000 at 12% nominal interest compounded quarterly. What is the total lump-sum, compounded amount to be paid by you at the end of a 10 – year loan period?

Problem:

A sum of P1,000 is invested now and left for 8 years at which time the principal is withdrawn. The interest has accrued is left for another 8 years. If the effective annual interest rate is 5%, what is the amount of the account at the end of the 16th year?

Problem:

A businessman wants to have P500,000 4 years from now. What amount should he invest now if it will earn interest of 6% compounded quarterly for the first two years and 8% compounded semi – annually during the next two years?

Problem:

How long will it take you to become a millionaire if you invest P10, 000 at 20% compounded quarterly?

Problem:

How many years will it take an investment to triple if the interest rate is 8% compounded

- (a). Quarterly?
- (b). Monthly?
- (c). Continuously?

Equation of Value

If cash flows occur on different periods, comparison of such should be made on a same focal date. Equation of values is obtained by setting the sum of one set of obligation on a certain comparison or focal date equal to the sum of another set of obligation on the same date.

An equation of value is obtained by setting the sum of the values on a certain comparison or focal date of one set of obligations equal to the sum of the values on the same date of another set of obligations.

Application of Compound Interest

Deposits ----- withdrawals

Loans ----- repayments

Investment ---- income

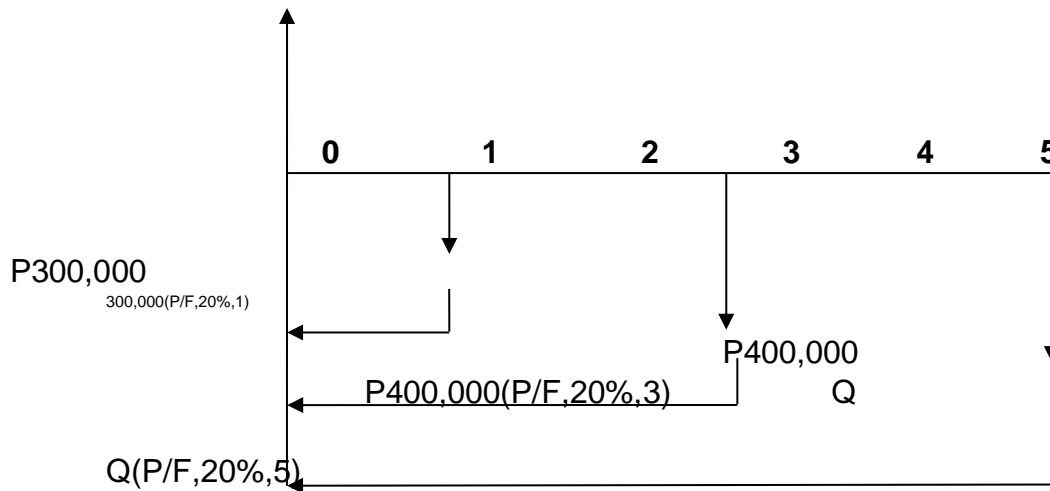
Cash inflow --- cash outflow

Example

A man bought a lot worth P1000,000 if paid in cash. On the installment basis, he paid a down payment of P20,000; P300, at the end of one year, P400, at the end of three years and a final payment at the end of five years. What was the final payment if interest was 20%

Solution

P 800,000



Using today as the focal date, the equation of value is
 $P800,000 = P300,000 (P/F,20\%,1) + P400,000 (P/F,20\%,3) + Q (P/F, 20\%,5)$

$$P800,000 = P300,000 (1.20)^{-1} + P400,000 (1.20)^{-3} + Q (1.20)^{-5}$$

$$P800,000 = P300,000 (0.8333) + P400,000 (0.5787) + Q (0.4019)$$

$$Q = P792,560$$

Problems:

A student plans to deposit P1,500 in a bank now and another P3,000 for the next two years. If he plans to withdraw P5,000 three years after his deposit for the purpose of buying shoes. What will be the amount of money left in the bank one year after his withdrawal? Effective rate is 10%.

Solution:

Problem:

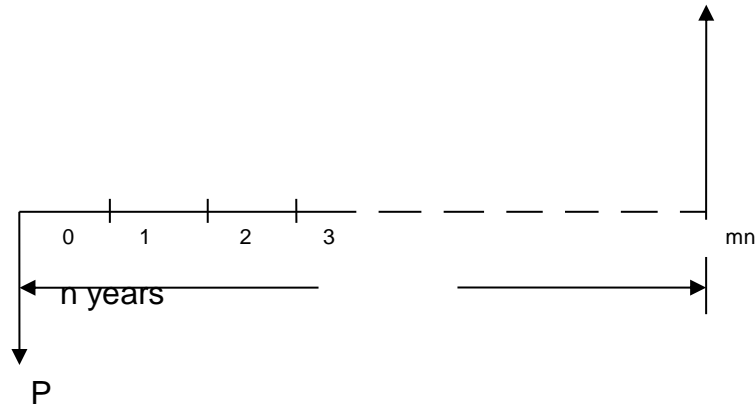
A man purchases a lot worth 1M if paid in cash. On installment basis, he paid P200,000 down payment, P300,000 at the end of the year, P400,000 at the end of the fourth year and the last payment at the end of the 5th year. If money is worth 16%, what is the worth of the last payment?

Continuous Compounding and Discrete Payments

In discrete compounding the interest is compound at the end of each finite- length period, such as a month, a quarter or a year.

In continuous compounding, it is assumed that cash payments occur once per year, but the compounding is continuous throughout the year.

F



Continuous Compounding (Lender's Viewpoint)

r = nominal rate of interest per year

$\frac{r}{m}$ = rate of interest per period

m = number of interest periods per year

n = number of interest periods in n years

$$F = P \left(1 + \frac{r}{m}\right)^{mn}$$

Let $\frac{m}{r} = k$, then $m = rk$, as m increases so must k

$$\left(1 + \frac{r}{m}\right)^{mn} = \left(1 + \frac{1}{k}\right)^{rkn} = \left[\left(1 + \frac{1}{k}\right)^k\right]^m$$

The limit of $\left(1 + \frac{r}{m}\right)^k$ as k approaches infinite is e

$$\left[\left(1 + \frac{1}{k}\right)^k\right]^m = e^m$$

Thus, $F = Pe^m$

$P = Fe^{-m}$

Example

Compare the accumulated amounts after 5 years of P1,000 invested at the rate of 10% per year compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, (e) daily, and (f) continuously.

Solution

Using the formula, $F = P(1+i)^n$

$$(a) F = P 1,000 (1+0.10)^5 = P1,610.51$$

$$(b) F = P 1,000 \left(1 + \frac{0.10}{2}\right)^{10} = P1,628.89$$

$$(c) F = P 1,000 \left(1 + \frac{0.10}{4}\right)^{20} = P1,638.62$$

$$(d) F = P 1,000 \left(1 + \frac{0.10}{12}\right)^{60} = P1,645.31$$

$$(e) F = P 1,000 \left(1 + \frac{0.10}{365}\right)^{1825} = P1,628.89$$

$$(f) F = P e^{0.10 \times 5} = P 1,648.72$$

Discount

Discount is defined as interest deducted in advance. In negotiable paper, it is the difference between the present worth of the paper and its value sometime in the future.

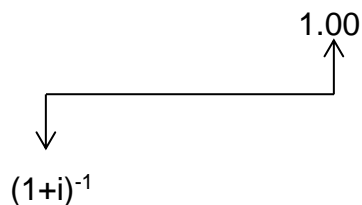
Discount on a negotiable paper is the difference between the present worth (the amount received for the paper in cash) and the worth of the paper at some time in the future (the face value of the paper or principal). Discount is interest paid in advance.

$$\text{Discount} = \text{Future Worth} - \text{Present Worth}$$

The rate of discount is the discount on one unit of principal for one unit of time.

Rate of Discount

Rate of discount is defined as the discount of one unit of principal for one unit of time.



In the cash flow diagram,

$$P = (1+i)^{-1}$$

And $F = 1.00$

Using the formula of discount,

$$d = F - P$$

$$d = 1 - (1+i)^{-1}$$

$$d = 1 - \frac{1}{1+i}$$

Simplifying the equation will give us;

$$i = \frac{d}{1-d}$$

and,

$$d = \frac{i}{1+i}$$

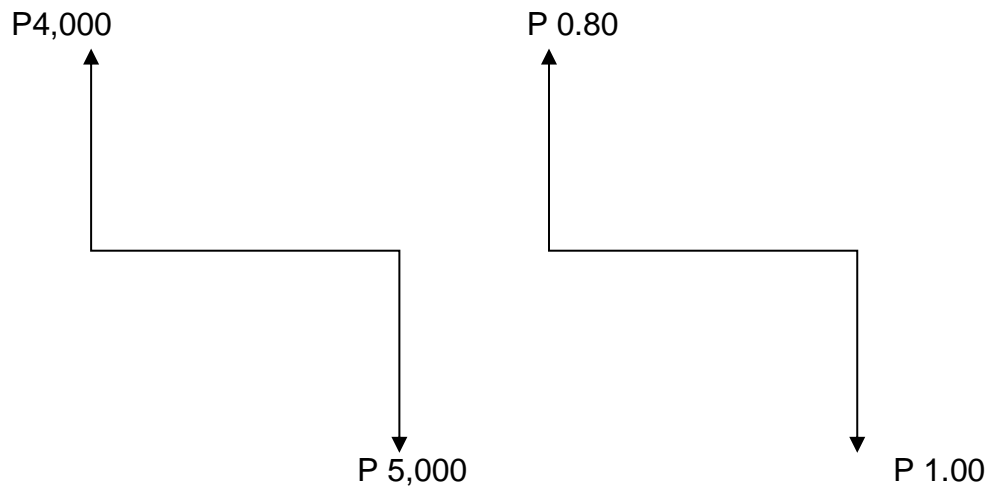
where d = rate of discount for the given period

i = rate of interest for the same period

Example

A man borrowed P5,000 from a bank and agreed to pay the loan at the end of 9 months. The bank discounted the loan and gave him P4,000 in cash. (a) What was the rate of discount? (b) What was the rate of interest? (c) What was the rate of interest for one year?

Solution



$$(a) d = \frac{\text{discount}}{\text{principal}} = \frac{P1,000}{P 5,000} = 0.20 \text{ or } 20\%$$

Another solution, using equation (2-12)

$$d = 1 - 0.80 = 0.20 \text{ or } 20\%$$

$$(b) i = \frac{d}{1-d} = \frac{0.20}{1-0.20} = 0.25 \text{ or } 25\%$$

Another solution,

$$i = \frac{\text{interest}}{\text{present worth}} = \frac{P1,000}{(P 4,000)} = 0.25 \text{ or } 25\%$$

$$(c) i = \frac{I}{Pn} = \frac{P1,000}{(P 4,000)\left(\frac{9}{12}\right)} = 0.3333 \text{ or } 33.33\%$$

Problem:

Mr. Cadevida was granted a loan of P20,000.00 by his employer CPM Industrial Fabricator & Construction Corporation with an interest of 6% for 180 days on the principal collected in advance. The corporation would accept a promissory note for P20,000 non-interests for 180 days. If discounted at once, find the proceeds on the note.

Problem:

A man applied for a loan with 16% interest. The interest will be automatically deducted from the loan at the time the money is released, and after one year will have to pay the same amount as stated in the loan. Determine the interest actually charged to him.

Problem:

How much interest is deducted in advance from a loan of P18,000.00 for one year and 6 months if the discount rate is 9%. How much is the proceeds after deducting the interest?

Problem:

Mr. de la Cruz borrowed money from a bank. He received from the bank P1, 342 and promised to repay P1, 500 at the end of 9 months. Determine the corresponding discount rate.

Problem:

John is deciding between a loan charged at 5% bank discount, and one charged at 6% simple interest. Either would require John to repay P1000 on August 3, 2010. It is now August 3, 2010, which loan should John get?

Problem:

How much should the engineer borrow from a bank that charges 10% simple discount for 6 months if he received an amount of P14, 500?

Solution:

Inflation

Inflation is the increase in the prices for goods and services from one year to another, thus decreasing the purchasing power of money.

$$FC = PC (1 + f)^n$$

where: PC = present cost of a commodity

FC = future cost of the same commodity

f = annual inflation

n = number of years

Example

An item presently costs P1000. If Inflation is at the rate of 8% per year. What will be the cost of the item in two years?

Solution

$$FC = PC (1 + f)^n = P1000 (1 + 0.08)^2 = P116.40$$

In an inflationary economy, the buying power of money decrease as cost increase. Thus,

$$F = \frac{P}{(1+f)^n}$$

where F is the future worth, measured in today's pesos, of a present amount P.

Example

An economy is experiencing inflation at the annual rate of 8%. If this continuous, what will P1000 be worth two years from now, in terms of today's peso?

Solution

$$F = \frac{P}{(1+f)^n} = \frac{P1000}{(1+0.08)^n} = P857.34$$

If interest is being compounded at the same time that inflation is occurring. The future worth will be

$$F = \frac{P(1+i)^n}{(1+f)^n} = P \left(\frac{(1+i)}{(1+f)} \right)^n$$

Example

A man invested P10,000 at an interest rate of 10% compounded annually. What will be the final amount of his investment, in terms of today's pesos, after five years, if inflation remains the same at the rate of 8% per year?

Solution

$$F = P \left(\frac{(1+i)}{(1+f)} \right)^n = P10,000 \left(\frac{(1+0.10)}{(1+0.08)} \right)^5 = P10,960.86$$

ANNUITY

An **annuity** is a series of equal payments made at equal intervals of time. Financial activities like installment payments, monthly rentals, life-insurance premium, monthly retirement benefits, are familiar examples of annuity.

Annuity can be certain or uncertain. In annuity certain, the specific amount of payments are set to begin and end at a specific length of time. A good example of annuity certain is the monthly payments of a car loan where the amount and number of payments are known. In annuity uncertain, the annuitant may be paid according to certain event. Example of annuity uncertain is life and accident insurance. In this example, the start of payment is not known and the amount of payment is dependent to which event.

Annuity certain can be classified into two, **simple annuity** and **general annuity**. In **simple annuity**, the payment period is the same as the interest period, which means that if the payment is made monthly the conversion of money also occurs monthly. In **general annuity**, the payment period is not the same as the interest period. There are many situations where the payment for example is made quarterly but the money compounds in another period, say monthly. To deal with general annuity, we can convert it to simple annuity by making the payment period the same as the compounding periods by the concept of effective rates

Types of Annuities

In engineering economy, annuities are classified into four categories. These are (1) ordinary annuity, (2) annuity due, (3) deferred annuity, and (4) perpetuity.

An annuity is a series of equal payments occurring at equal periods of time.

Symbol and Their Meaning

P = Value or sum of money at present

F = Value or sum of money at some future time

A = A series of periodic, equal amounts of money

n = Number of interest periods

i = Interest rate per interest period

Ordinary Annuity

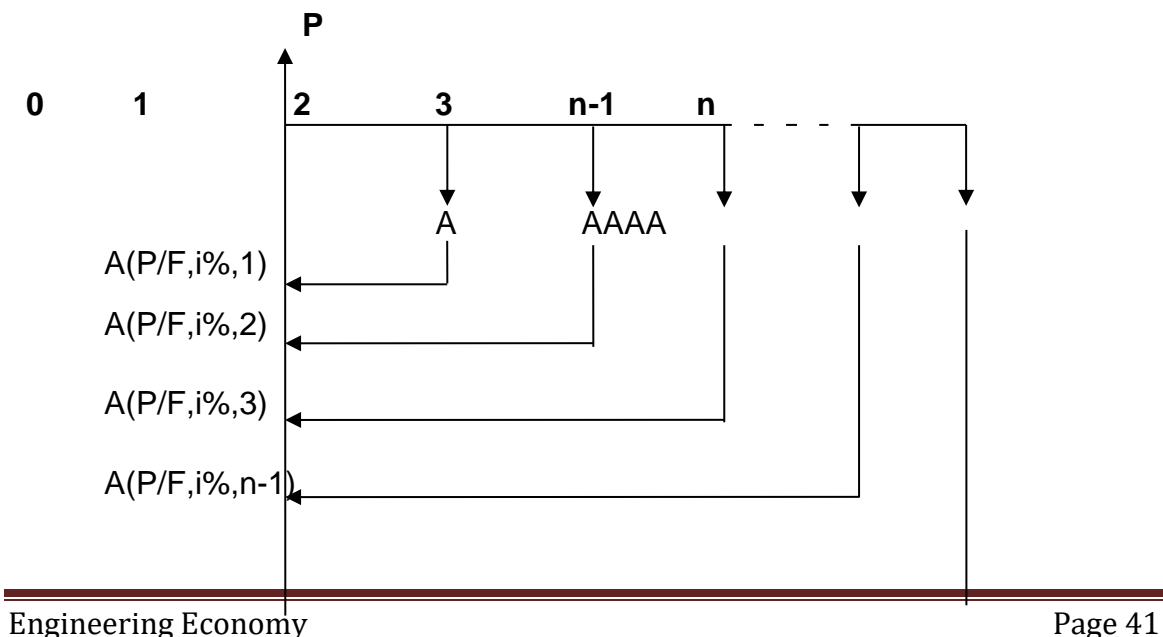
An ordinary annuity is a series of uniform cash flows where the first amount of the series occurs at the end of the first period and every succeeding cash flow occurs at the end of each period.

An ordinary annuity is one where the payments are made at the end of each period.

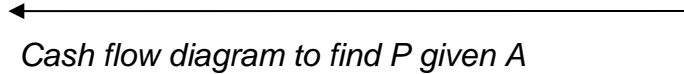
Characteristics of ordinary annuity:

- a.) P (present equivalent value)
 - Occurs one interest period before the first A (uniform amount)
- b.) F (future equivalent value)
 - Occurs at the same time as the last A and n intervals after P
- c.) A (annual equivalent value)
 - Occurs at the end of each period

Finding P when A is given



$A(P/F, i\%, n)$



$$P = A (P/F, i\%, 1) + A (P/F, i\%, 2) + (P/F, i\%, 3) + \dots + A (P/F, i\%, n-1) + A (P/F, 1\%, n)$$

$$P = A (1+i)^{-1} + A (1+i)^{-2} + \dots + A (1+i)^{-(n-1)} + A (1+i)^{-n}$$

Multiplying this equation by $(1+i)$ result in

$$P + Pi = A + A (1+i)^{-1} + A (1+i)^{-2} + \dots + A (1+i)^{-n+2} + A (1+i)^{-n+1}$$

Subtracting the first equation from the second gives

$$P + Pi = A + \cancel{A(1+i)^{-1}} + \cancel{A(1+i)^{-2}} + \dots + \cancel{A(1+i)^{-n+1}} + A(1+i)^{-n+1}$$

$$-P = -\cancel{A(1+i)^{-1}} - \cancel{A(1+i)^{-2}} + \dots + \cancel{A(1+i)^{-n+1}} - A(1+i)^{-n}$$

$$Pi = A - A(1+i)^{-n}$$

Solving For P gives

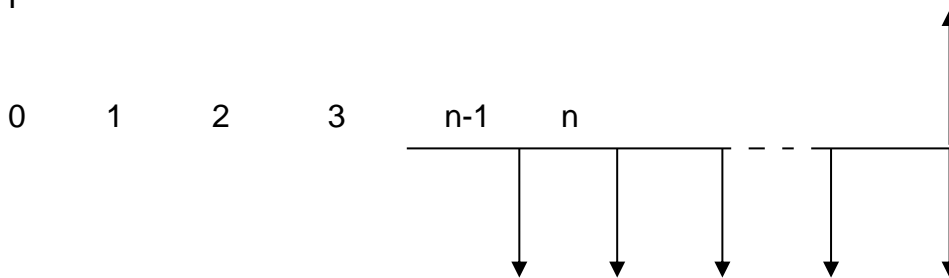
$$P = A \left[\frac{1 - (1+i)^{-n}}{i} \right] = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

The quantity in brackets is called the “ uniform series present worth factor” and is designated by the functional symbol $P/A, i\%, n$, read as “P given A at I percent in the interest periods. “ Hence Equation can be expressed as

$$P = A (P/A, i\%, n)$$

Finding F when A is Given

F



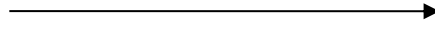
A AAAA

$A(F/P, i\%, 1)$

$A(F/P, i\%, n-3)$

$A(F/P, i\%, n-2)$

$A(F/P, i\%, n-1)$



Cash flow diagram to find F given A

$$F = A + A (F/A, i\%, 1) + \dots + A (F/P, i\%, n-3) + A (F/P, 1\%, n-2) + A(F/P, i\%, n-1)$$

$$F = A + A (1+i) + \dots + A (1+i)^{-(n-3)} + A (1+i)^{-n-2} + A (1+i)^{-n-1}$$

Multiplying this equation by (1+i) results in

$$F + Fi = A + A (1+i) + A (1+i)^2 + \dots + A (1+i)^{n-2} + A (1+i)^{n-1} + A (1+i)^n$$

Subtracting the First equation from the second gives

$$F + Fi = \cancel{A(1+i)} + \dots + \cancel{A(1+i)^{n-2}} + \cancel{A(1+i)^{n-1}} + A (1+i)^n$$

$$\begin{array}{r} - F = \cancel{-A(1+i)} + \dots + A (1+i)^{n-2} + \cancel{A(1+i)^{n-1}} \\ \hline Fi = -A \qquad \qquad \qquad + A (1+i)^n \end{array}$$

Solving for F gives

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

The quantity in brackets is called “Uniform series compound amount factor” and is designated by the functional symbol $F/A, i\%, n$, read as “F given at I percent in n interest period.” Equation can now be written as

$$F = A (F/A, i\%, n)$$

Finding A when P is Given

Taking Equation and solving for A, we have

$$A = \left[\frac{i}{1 - (1+i)^{-n}} \right]$$

The quantity in brackets is called the “capital recovery factor.” It will be denoted by the functional symbol $A/P, i\%, n$ which is read as “A given at I percent in an interest periods.” Hence

$$A = P (A/P, i\%, n)$$

Finding A When F is Given

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

The quantity in brackets is called the “sinking fund factor”. It will be denoted by the functional symbol $A/F, i\%, n$ which is read as “A given F at I per n interest periods.” Hence

$$A = F (A/F, i\%, n)$$

Relation $A/P, i\%, n$ and $A/F, i\%, n$

$$\frac{i}{(1+i)^n - 1} + i = \frac{i + (1+i)^n - i}{i(1+i)^n - 1} \times \frac{(1+i)^{-n}}{(1+i)^{-n}}$$

$$\frac{i}{(1+i)^n - 1} + i = \frac{i}{1 - (1+i)^{-n}}$$

$$A/F, i\%, n+1 = A/P, i\%, n$$

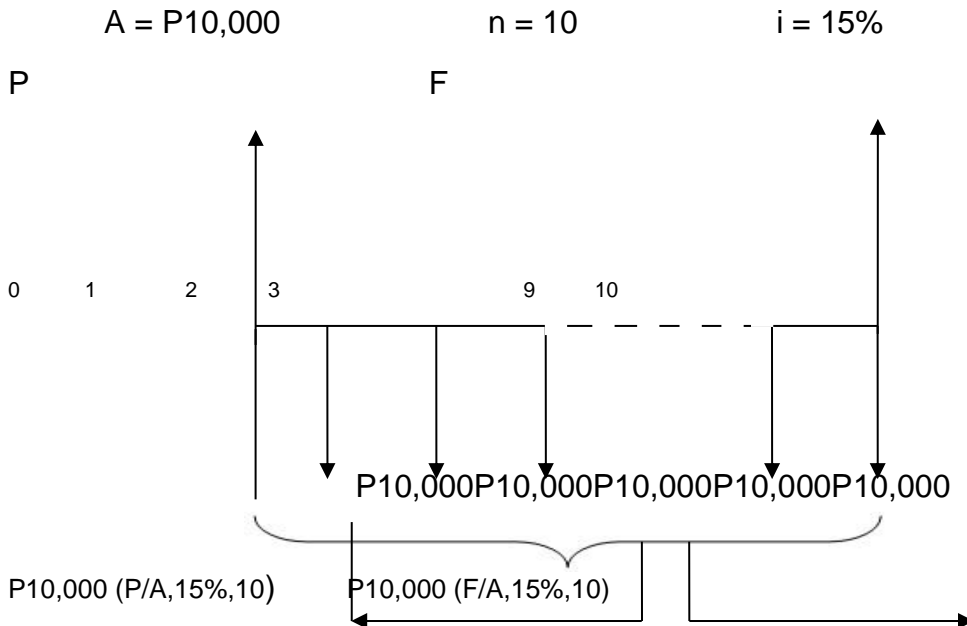
Thus,

Sinking fund factor + i = capital recovery factor

Example

What are the present worth and the accumulated amount of a 10-year annuity paying P10,000 at the end of each year, with interest at 15% compounded annually?

Solution



$$P = A(P/A, i\%, n) = P10,000 (P/A, 15\%, 10)$$

$$= P10,000 \left[\frac{1-(1.15)^{-10}}{.15} \right] = P50,188$$

$$F = A(F/A, i\%, n) = P10,000 (F/P, 15\%, 10)$$

$$= P10,000 \left[\frac{(1.15)^{10}-1}{.15} \right] = P203,037$$

Example

What is the present worth of P500 deposited at the end of every three months for 6 years if the interest rate is 12% compounded semiannually?

Solution

Solving for the interest rate per quarter,

$$(1+i)^4 - 1 = \left(1 + \frac{0.12}{2}\right)^2 - 1$$

$$1 + i = (1.06)^{.5}$$

$$i = 0.0296 \text{ or } 2.96\% \text{ per quarter}$$

$$P = A (P/A, 2.96\%, 24)$$

$$= P500 \frac{1-(1+0.0296)^{-24}}{0.0296}$$

$$= P500 (17.0087)$$

$$= P 8,504$$

Example

A businessman need P50,000 for his operation. One financial institution is willing to lend him the money for one year at 12.5% interest per annum (discounted). Another lender is charging 14% with the principal and interest payable at the end of one year. A third financier is willing to lend him P50,000 payable in 12 equal monthly installments of P4,600. Which offer best for him?

Solution

Compare the effective rate each offer and select the one with the lowest effective rate.

First offer:

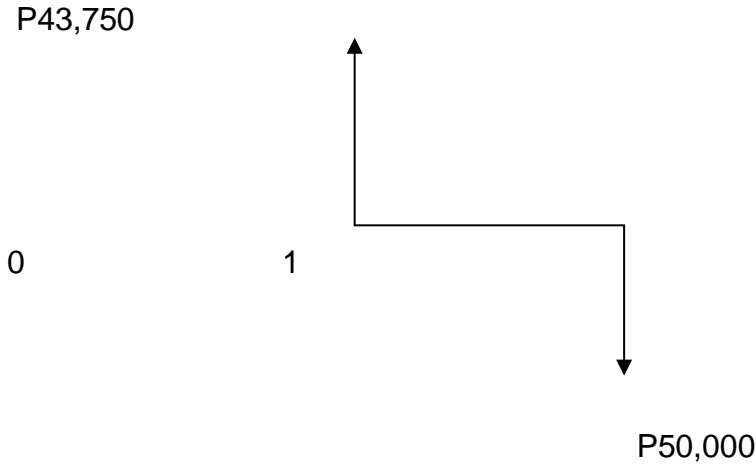
Rate of discount, $d = 12.5\%$

$$\text{Rate of interest, } i = \frac{d}{1-d} = \frac{0.125}{1-0.125} = 14.29\% \text{ per year}$$

Effective rate = 14.29%

Another solution

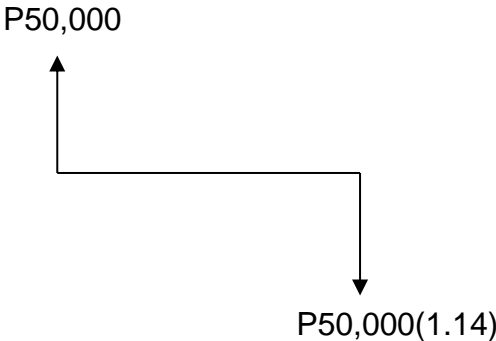
$$\text{Amount received} = P50,000 (0.875) = P43,750$$



$$\text{Rate of interest} = \frac{P50,000 - P43,750}{P43,750} = 14.29\% \text{ per year}$$

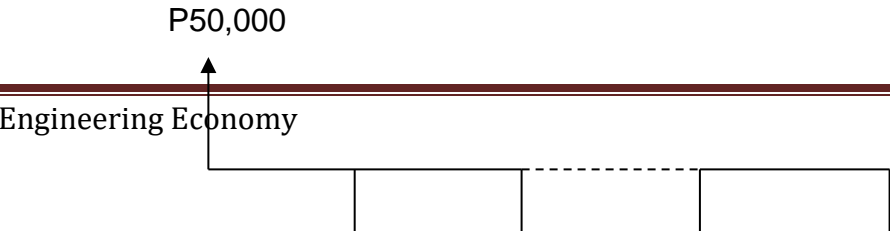
Effective rate = 14.29%

Second Offer:



Effective Rate = 14%

Third Offer:



0 1 2 11 12

P4,600 P4,600 P4,600 P4,600

$$P/A, i\%, 12 = \frac{P}{A} = \frac{P50,000}{P4,600}$$

$$\frac{1-(1+i)^{-12}}{i} = 10.6896$$

Try $i = 1\%$,

$$\frac{1-(1+0.01)^{-12}}{0.01} = 11.2551$$

Try $i = 2\%$

$$\frac{1-(1+0.02)^{-12}}{0.02} = 10.5753$$

	X0.3855									
i%	10.8696	<table border="1"> <tr> <td>1%</td> <td>11.2551</td> </tr> <tr> <td>0.6798</td> <td></td> </tr> <tr> <td>2%</td> <td>10.5753</td> </tr> </table>	1%	11.2551	0.6798		2%	10.5753		
1%	11.2551									
0.6798										
2%	10.5753									

$$\frac{x}{1\%} = \frac{0.3855}{0.6798}$$

$$x = 0.57\%$$

$i = 1\% + 0.57\% = 1.57\%$ per month

Effective rate = $(1+0.0157)^{12} - 1 = 20.26\%$

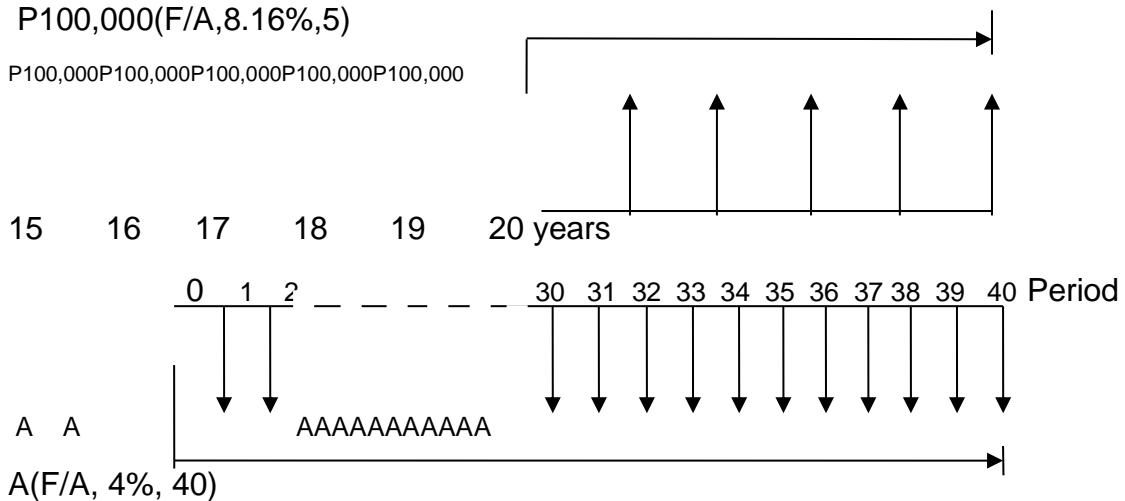
The second is the best offer.

Example

A chemical engineer wishes to set up a special fund by making uniform semiannual end-of-period deposits for 20 years. The fund is to provide P100,000

at the end of each of the last five years of the 20-year period. If interest is 8% compounded semiannually, what is the required semiannual deposit to be made?

Solution



For the deposits, $i = \frac{8}{2} = 4\%$

For the withdrawals, $i = (1+0.04)^2 - 1 = 0.0816$ or 8.16%

Using 20 years from today as the focal date, the equation of value is

$$A (F/A, 4\%, 40) = P100,000 (F/A, 8.16\%, 5)$$

$$A \left[\frac{(1+.04)^{40} - 1}{.04} \right] = P100,000 \left[\frac{(1+.0816)^5 - 1}{.0816} \right]$$

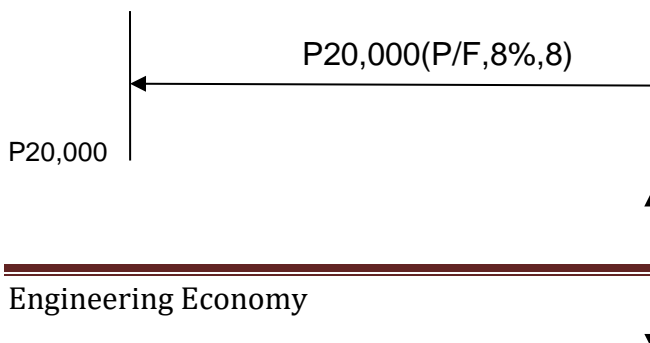
$$A (95.0255) = P100,000 (5.8853)$$

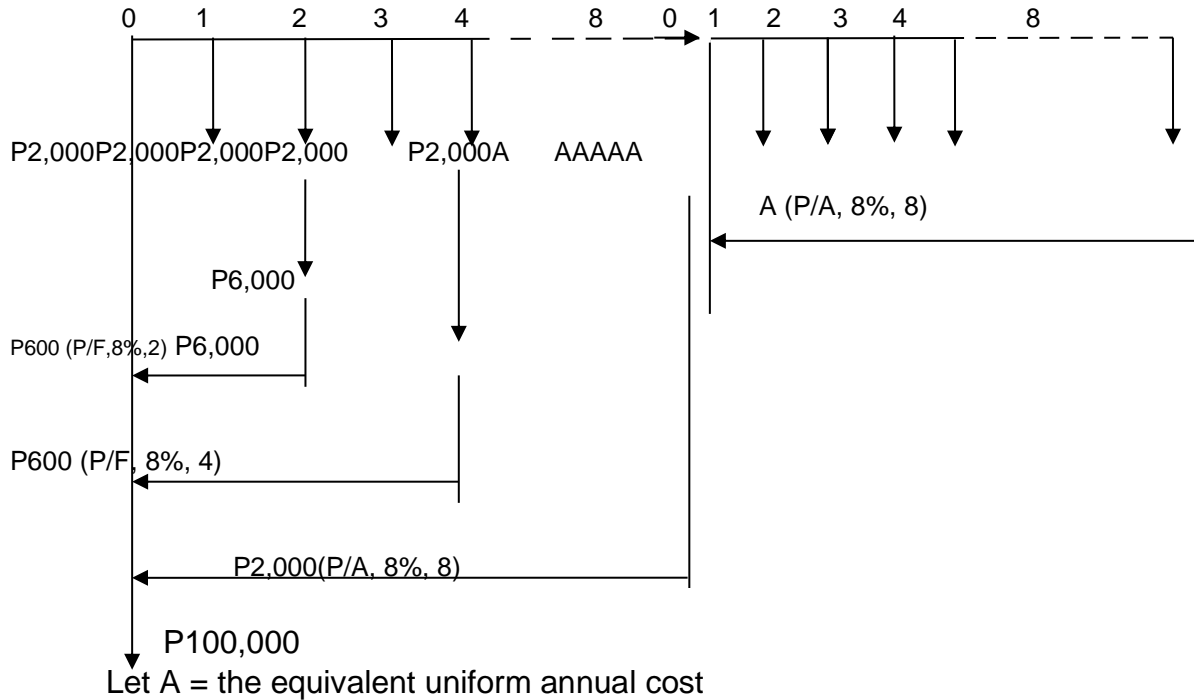
$$A = P6,193.39$$

Example

Using a compound interest of 8%, find the equivalent uniform annual cost for a proposed machine that has a first cost of P100,000 an estimated salvage value of P20,000 an estimated economic life of 8 years. Annual maintenance will amount to P2,000 a year and periodic overhaul costing P6,000 each will occur at the end of the second and fourth year.

Solution





Using today as the focal date the equation of value is

$$A (P/A, 8\%, 8) = P100,000 + P2,000 (P/A, 8\%, 8) + P6,000 (P/F, 8\%, 2) + P6,000 (P/F, 8\%, 4) - P20,000 (P/F, 8\%, 8)$$

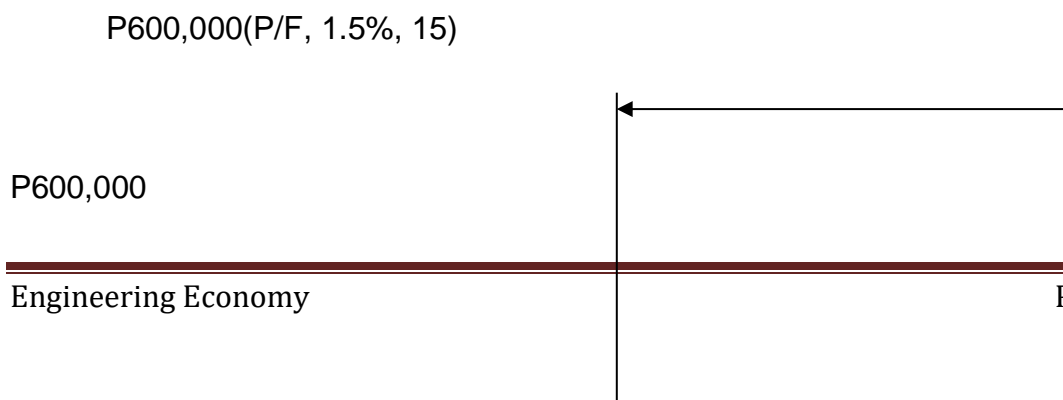
$$A (5.7466) = P100,000 + P2,000 (5.7466) + P6,000 (0.8573) + P6,000 (0.7350) - P20,000 (0.5403)$$

$$A = P19,183$$

Example

A man purchased a house for P425,000. In the first month that he owned the house, he spent P75,000 on repairs and remodeling. Immediately after the house was remodeled, he was offered P545,000 to sell the house. After some consideration, he decided to keep the house and have it rented for P4,500 per month starting two months after the purchase. He collected rent for 15 months and then sold the house for P600,000. If the interest rate was 1.5% per month, how much extra money did he make or lose by not selling the house immediately after it was remodeled?

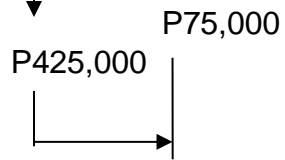
Solution



P4,500(P/A, 1.5%, 15)

P545,000 P4,500 P4,500 P4,500 P4,500

1



P425,000(F/P, 1.5%, 1)

The only focal date that can be used is one month after purchase.

Profit if the house was sold immediately after remodeling
(1 month after purchase) = P545,000 – P75,000 – P425,000 (F/P, 1.5%, 1) =
P38,625

Profit if the house was sold later (16 months after purchase)

= P600,000 (P/F, 1.5%, 15) + P4,500 (P/A, 1.5%, 15) – P75,000 –
P425,000 (F/P, 1.5%, 1)
= P600,000 (0.7999) + P4,500 (13.3432) – P75,000 – P425,000 (1.015)
= P33,610

Loss by not selling immediately after remodeling = P38,625 – P33,610 = P5,105

Another solution:

Loss by not selling immediately after remodeling

= P545,000 – P4,500 (P/A, 1.5%, 15) – P600,000 (P/F, 1.5%, 15)
= P545,000 – P4,500 (13.3432) – P600,000 (0.7999)
= P5,105

Example:

Determine the present equivalent value of P5,000 paid every 3 months for period of seven years and the rate of interest is 12% compounded quarterly.

Required: The present value, P

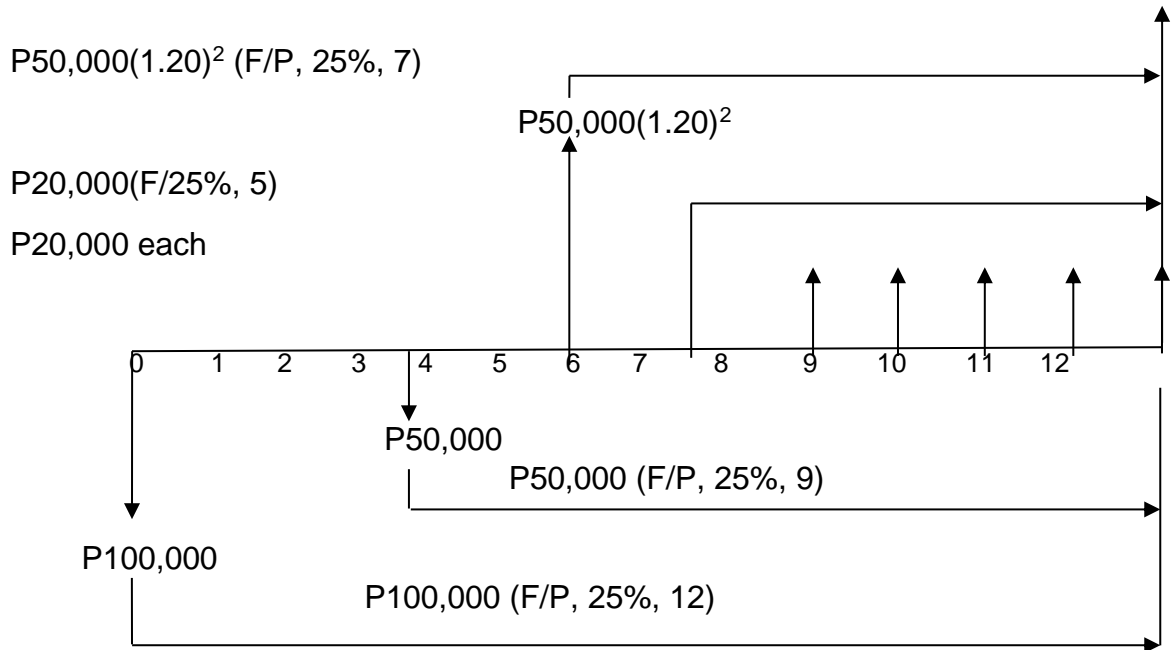
$$\begin{aligned}\text{Solution: } P &= A \left[\frac{1 - (1+i)^{-n}}{i} \right] \\ &= P5,000 \left[\frac{1 - 1 + 0.03^{-28}}{0.03} \right] \\ &= P93,820.54\end{aligned}$$

Example

Today, you invest P100,000 into a fund that pays 25% interest compounded annually. Three years later, you borrow P50,000 from a bank at 20% annual interest and invest in the fund. Two years later, you withdraw

enough money from the fund to repay the bank loan and all interest due on it. Three years from this withdrawal you start taking P20,000 per year out of the fund. How much was withdrawn?

Solution



Let Q = the amount withdrawn after 12 years

Using 12 years from today as the focal date, the equation of value is

$$Q + P20,000 (F/A, 25\%, 5) + P50,000 (1.20)^2 (F/P, 25\%, 7) = P100,000 (F/P, 25\%, 12) + P50,000 (F/P, 25\%, 9)$$

$$Q + P20,000 \left(\frac{(1+.25)^5 - 1}{.25} \right) + P50,000 (1.20)^2 (1.25)^7 = P100,000 (1.25)^{12} + P50,000 (1.25)^9$$

$$Q + P20,000 (8.2070) + P50,000 (1.20)^2 (4.7684) = P100,000(14.5519) + P50,000 (7.4506)$$

$$Q = P1,320,255$$

Problem:

If P25, 000 is deposited now into a savings account that earns 6% per year, what uniform annual amount could be withdrawn at the end of each year for ten years so that nothing would be left in the account after the 10th withdrawal?

Problem:

You can buy a machine for P100, 000 that will produce a net income, after operating expenses, of P10, 000 per year. If you plan to keep the machine for four years, what must be the market resale value be at the end of four years to justify the investment? You must make a 15% annual return on your investment.

Problem:

Sixty monthly deposits are made into an account paying 6% nominal interest compounded monthly. If the subject of these is deposits is to accumulate P100,000 by the end of the fifth year, what is the amount of each deposits?

Problem:

What is the principal remaining after 20 monthly payments have been made on a P20,000 five-year loan? The annual rate is 12% nominal compounded monthly.

Solution:

Problem:

How much should be deposited each year to 12 years if you wish to withdraw P309 each per years for five years beginning at the end of the 15th year? Let the interest rate be 8%per year.

Solution:

Problem:

An individual is borrowing P100, 000 at 8% interest compounded annually. The loan is to be repaid in equal payments over 30 years. However, just after eight payment is made, lender allows the borrower to triple the payment. The borrower agrees to this increase payment. If the lender is still charging 10% per year, compounded annually, on the unpaid balance of the loan, what is the balance still owed just after the 12th payment is made?

Problem:

A woman arranges to repay a P 102,000 bank loan into equal payment at a 10% effective annual interest rate. Immediately after her third payment, she borrows another P5, 000 also at 10% per year. When she borrows the P5, 000 she talks the banker into letting her to repay the remaining debt of the first loan and the entire amount of second loan in 12 equal payments. The first of these 12 payments would be made one year after she receives the P5, 000. Compute the amount of each of the 12 payments?

Problem:

A loan of P10, 000 is to be repaid over a period of eight years. During the first four years, exactly half of the loan principal is to be repaid (along with the accumulated compound interest) by uniform series of payments of A_1 per year. The other half of the loan principal is to be repaid over four years with accumulated interest by a uniform series of payments of A_2 per year. If $i = 9\%$ per year, what are the value of A_1 and A_2 ?

PROBLEMS

1. What is the annual rate of interest if P265 is earned in four months on an investment of P15, 000?
2. A loan of P2, 000 is made for a period of 13 months, from January 1 to January 31 the following year, at a simple interest of 20%. What is the future amount is due at the end of the loan period?
3. If you borrow money from your friend with simple interest of 12%, find the present worth of P20, 000, which is due at the end of nine months.
4. Determine the exact simple interest on P5, 000 for the period from Jan.15 to Nov.28, 1992, if the rate of interest is 22%.

5. A man wishes his son to receive P200, 000 ten years from now. What amount should he invest if it will earn interest of 10% compounded annually during the first 5 years and 12% compounded quarterly during the next 5 years?

6. By the condition of a will the sum of P25, 000 is left to be held in trust by her guardian until it amounts to P45, 000. When will the girl receive the money if the fund is invested at 8% compounded quarterly?

7. At a certain interest rate compounded semiannually P5, 000 will amount to P20, 000 after 10 years. What is the amount at the end of 15 years?

8. A man borrows money from a bank which uses a simple discount rate of 14%. He signs a promissory note promising to pay a P500 pre-month at the end of 4th, 6th and 7th months, respectively. Determine the amount he received from the bank?

9. An amount of P3, 000 is invested for 9 years at 6% compounded quarterly. Determine the amount of interest from the earned by the investment.

10. If P10, 000 is invested at 12%, interest compounded annually, determine how many years it will take to double?

11. Which is better for an investor, to invest at $5\frac{1}{2}\%$ compounded semi-annually as 5% compounded monthly?

12. Mr. W borrowed P2, 000 from Mr. Y on June 1, 1928 and P500 one June 1, 1930. Mr. W paid P500 on June 1, 1931, P400 one June 1932 and P700 on June 1, 1933. If money is worth 5% compounded annually, what additional payment should Mr. W pay on June 1, 1936 to discharge all remaining liability?

13. How long will it take P500 to be four times in value if invested at the rate of 7% compounded semi-annually? What is the effective rate of interest?

14. The sum of P26, 000 was deposited in a fund earning interest at 8% interest per annum compounded quarterly. What was the amount in the fund at the end of 3 years?

15. The Philippines Society of Mechanical Engineer is planning to construct its own building. Two proposals are being considered:

- a. The construction of the building now to cost P400, 000.
- b. The construction of a smaller building now to cost P300, 000 and at the end of 5 years, an extension to be added to cost P200, 000, which proposal is more economical, if interest is 20% depreciation is disregarded and by how much?

16 If the sum of P15, 000 is deposited in an account earning 4% per annum compounded quarterly, what will be the amount of the deposited at the end of 5 years?

17. Jones Corporation borrowed P9, 000 from Brown Corporation on Jan. 1, 1978 and P12, 000 on Jan. 1, 1980. Jones Corporation made a partial payment of P7, 000 on Jan. 1, 1981. It was agreed that the balance of the loan would be amortized by two payments one of Jan. 1, 1982 and the other on Jan. 1, 1983, the second being 50% larger than the first. If the interest rate is 12%. What is the amount of each payment?

18. A woman borrowed P3,000 to be paid after $1\frac{1}{2}$ years with interest at 12% compounded semi-annually and P5,000 to be paid after 3 years at 12% compounded monthly. What single payment must she pay after $3\frac{1}{2}$ at an interest rate of 16% compounded quarterly to settle the two obligations?

19. Mr. J. de la Cruz borrowed money from a bank. He received from the bank P1,342 and promised to repay P1,500 at the end of 9 months. Determine the simple interest rate and the corresponding discount rate or often referred to as the “Banker’s discount”.

20. A man deposits P50,000 in a bank account at 6% compounded monthly for 5 years. If the inflation rate of 6.5% per year continues for this period, will this effectively protect the purchasing power of the original principal?

21. What is the future worth of P600 deposited at the end of every month for 4 years if the interest rate is 12% compounded quarterly?

22. A young woman, 22 years old, has just graduated from college. She accepts a good job and desires to establish her own retirement fund. At the end of each year thereafter she plans to deposit P2,000 in a fund at 15% annual interest. How old will she be when the fund has an accumulated value of P1,000,000?

23. Mr. Reyes borrows P600,000 at 12% compounded annually, agreeing to repay the loan in 15 equal annual payments. How much of the original principal is still unpaid after he has made the 8th payment?

24. A man who won P300,000 in a lottery decided to place 50% of his winnings in a trust fund for the college education of his son. If the money will earn 14% a year compounded quarterly, how much will the man have at the end of 10 years, when his son will be starting his college education?

25. A man borrowed P25,000 from a bank which he signed a promissory note to repay the loan at the end of one year. He only received the amount of P21,915 after the bank collected the advance interest and an additional amount of 85.00 for legal fees. What was the rate of interest that the bank charged?

26. A man wishes to have P35,000 when he retires 15 years from now. If he can expect to receive 4% annual interest, how much must he set aside beginning at the end of each of the 15 years?

27. An investor deposits P1,000 per year in a bank which offers an interest of 18% per annum for time deposits of over 5 years. Compute how much the investor can collect at the end of 13 years, assuming that he never withdraws any amount before the 13th year.

28. A man wishes to prepare the future of his ten-year old son. Determine the monthly savings that he should make with interest at 5.41% per annum to amount to P120,000 at the time his son will be 18 years old.

29. A building and loan association requires that loans be repaid by uniform monthly payments which include monthly interest calculated on the basis of nominal interest of 5.4% per annum. If P5,000 is borrowed to be repaid in 10 years, what must the monthly payment be?

30. You want to start saving for your ten-year old son's college education. If you are guaranteed 6% interest compounded quarterly, how much would you have to save each month to amount to 12,000 by the time he is 18 years old?

31. Find the annual payments to extinguish a debt of P10,000 payable in 5 years at 12% compounded annually?

32. A man owes P10,000 with interest at 6% payable semi-annually. What equal payments at the beginning of each 6 months for 8 years will discharge his debt?

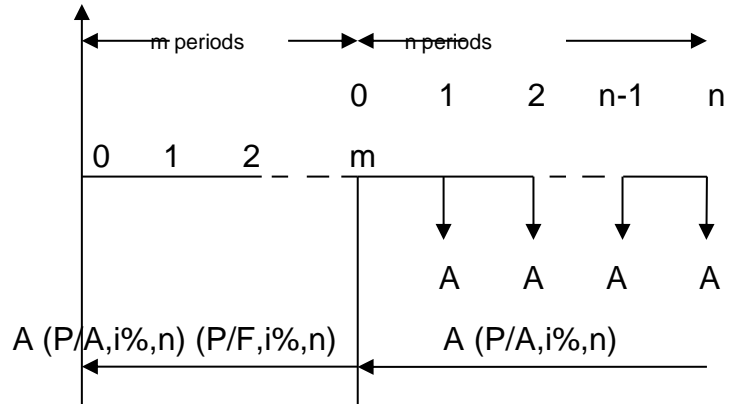
33. The P100,000 cost of an equipment was made available through a loan which earns 12% per annum. If the loan will be paid in 10 equal annual payments, how much is the annual installment?

34. An individual is borrowing P100, 000 at 8% interest compounded annually. The loan is to be repaid in equal payments over 30 years. However, just after eight payment is made, lender allows the borrower to triple the payment. The borrower agrees to this increase payment. If the lender is still charging 10% per year, compounded annually, on the unpaid balance of the loan, what is the balance still owed just after the 12th payment is made

Deferred Annuity

A deferred annuity is one where the first payment is made several periods after the beginning of the annuity.

An ordinary annuity where the first cash flow of the series is not at the end of the lot period or it is deferred for some time



Cash flow diagram given A to find P

$$P = A (P/A, i\%, n) (P/F, i\%, m)$$

$$P = A \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)^{-m}$$

Where :

m = deferred period

n = number of annuities

*Note that the counting of m is up to the period before the first A.

Example

On the day his grandson was born, a man deposited to a trust company a sufficient amount of money so that the boy could receive five annual payments of P80,000 each for his college tuition fees, starting with his 18th birthday. Interest at the rate 12% per annum was to be paid on all amounts on deposit. There was also a provision that the grandson could select to withdraw no annual payments and received a single lump amount on his 25th birthday. The grandson chose this option.

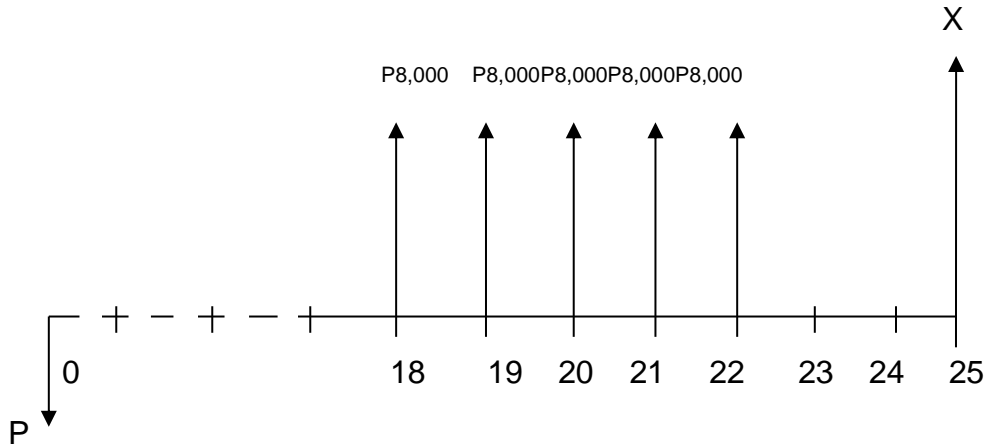
(a) How much did the boy received as the single payments?

(b) How much did the grandfather deposit?

Solution

Let P=the amount deposited

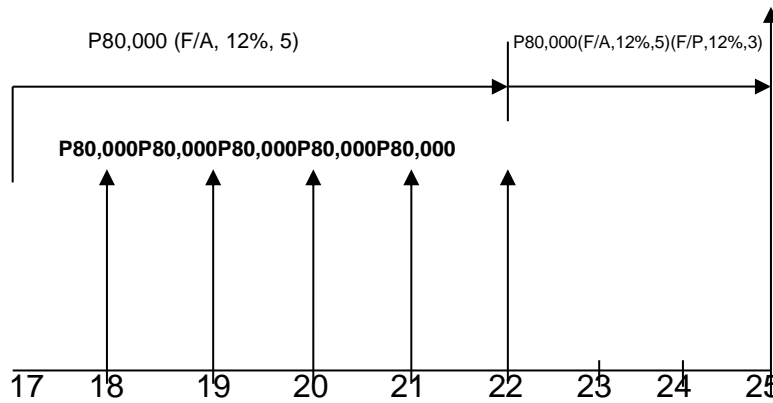
X=the amount withdrawn



The P80,000 supposed withdrawals are represented by broken lines, since they did not actually occur. Three separated cash flows diagrams can be drawn.

(a)

X



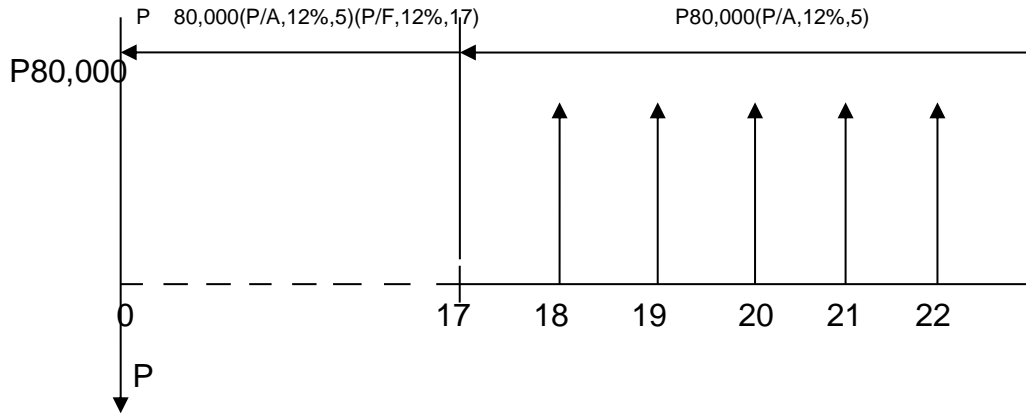
X and the P80,000 supposed withdrawals are equivalent.

Using 25 years of age as the focal date, the equation of value is

$$\begin{aligned}
 X &= P80,000 (F/A, 12\%, 5)(F/P, 12\%, 3) \\
 &= P80,000 \left[\frac{(1+.12)^5 - 1}{.12} \right] (1+.12)^3 \\
 &= P714,023.4509
 \end{aligned}$$

The other good focal dates are 17 and 22 years from today.

(b)

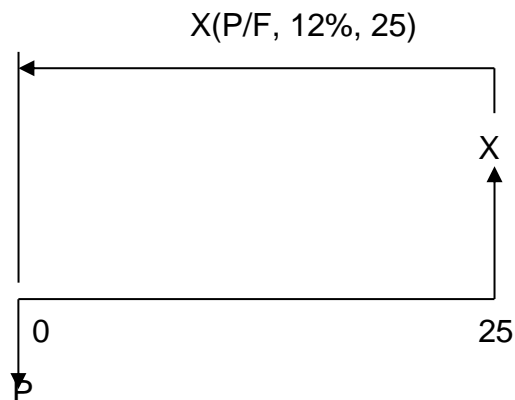


P and P80,000 supposed withdrawals are equivalent.

Using today as the focal date, the equation of value is

$$\begin{aligned}
 P &= P80,000 (P/A, 12\%, 5) (P/F, 12\%, 17) \\
 &= P80,000 \left[\frac{1 - (1 + .12)^{-5}}{.12} \right] (1 + .12)^{-17} \\
 &= P42,001.22
 \end{aligned}$$

Another solution:



This was what actually happened, P was deposited today and X was withdrawn 25 years later.

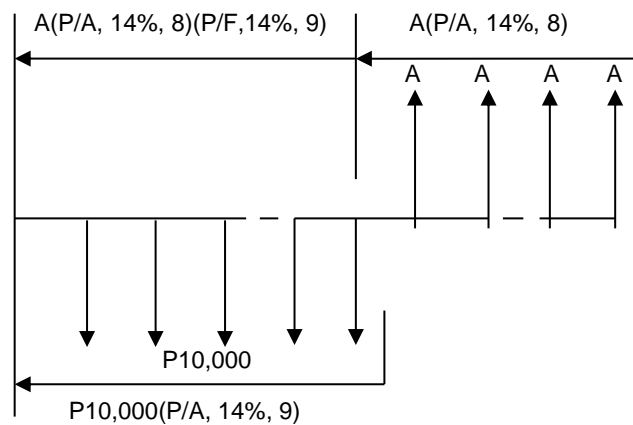
Using today as the focal date, the equation of value is

$$\begin{aligned} P &= X (P/F, 12\%, 25) \\ &= P714,023.4509 (1+.12)^{-25} \\ &= P42,001.22 \end{aligned}$$

Example

If P10, 000 is deposited each years for 9 years, how much annuity can a person get annually from the bank every year for 8 years starting 1 year after the 9th deposit is made. Cost of money is 14%.

Solution:



Using today as the focal date, the equation of value is

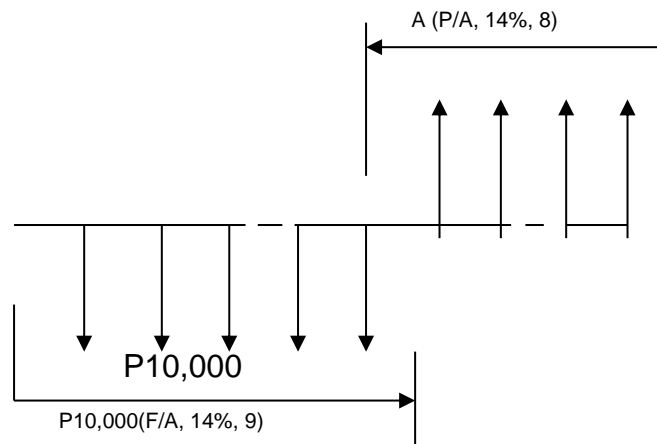
$$A (P/ A, 14\%, 8) (P/F, 14\%, 9) = P10,000 (P/ A, 14\%, 9)$$

$$A \left[\frac{1-(1+.14)^{-8}}{.14} \right] (1+.14)^{-9} = P10,000 \left[\frac{1-(1+.14)^{-9}}{.14} \right]$$

$$A (4.63886) (0.30751) = P10,000 (4.94637)$$

$$A = P34,675$$

Another solution



Using 9 years from today as the focal date, the equation of value is

$$A (P/A, 14\%, 8) = P10,000(F/A, 14\%, 9)$$

$$A \left[\frac{1 - (1 + .14)^{-8}}{.14} \right] = P10,000 \left[\frac{(1 + .14)^9 - 1}{.14} \right]$$

$$\begin{aligned} A (4.63886) &= P10,000(16.08535) \\ A &= P34,675 \end{aligned}$$

Example

A debt of P40,000, whose interest rate is 15% compounded semiannually, is to be discharged by a series of 10 semiannual payments, the first payment to be made 6 months after consumption of the loan. The first 6 payments will be P6,000 each, while the remaining 4 payments will be equal and such amount that the final payment will liquidate the debt. What is the amount of the last 4 payments?

Solution

Problem

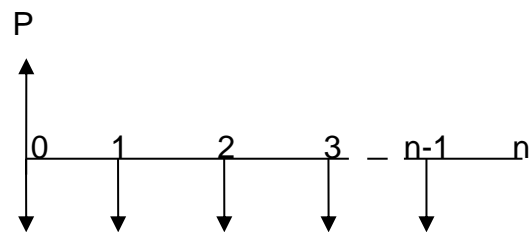
A man invested P50,000 now for the college education of his 2-year old son. If the fund earns 14% effective, how much will the son get each year starting from his 17th to the 22th birthday?

Annuity Due

An annuity due is one where the payments are made at the beginning of each period

An annuity due is series of uniform cash flows that occur at the beginning of each period.

Finding P When A is Given



Cash flow diagram given A to find P

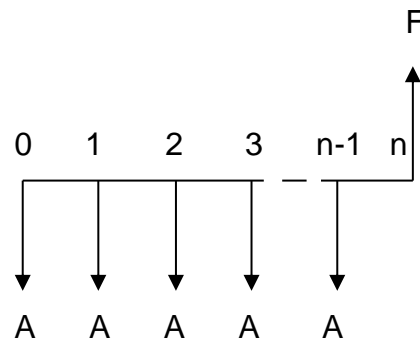
$$P = A + A (P/A, i\%, n-1)$$

$$P = A (1 + P/A, i\%, n-1)$$

$$P = A \left[\frac{1 - (1+i)^{-(n-1)}}{i} \right] + A$$

$$P = A \left[\frac{1 - (1+i)^{-(n-1)}}{i} + 1 \right]$$

Finding F When A is Given



Cash flow diagram given A to find F

$$F = A (F/A, i\%, n+1) - A$$

$$F = A [(F/A, i\%, n+1) - 1]$$

$$F = A \left[\frac{1 - (1+i)^{-(n+1)}}{i} \right] - A$$

$$F = A \left[\frac{(1+i)^{n+1} - 1}{i} - 1 \right]$$

Example

A man bought an equipment costing P60,000 payable in 12 quarterly payments, each installment payable at the beginning of each period. The rate of interest is 24% compounded quarterly. What is the amount of each payment?

Solution

$$P = \text{P}60,000$$

$$n = 12$$

$$i = \frac{24\%}{3} = 8\%$$

$$P = A (1 + P/A, 2\%, n-1)$$

$$\text{P}60,000 = A (1 + P/A, 8\%, 11)$$

$$\text{P}60,000 = A \left[1 + \frac{1 - (1.08)^{-11}}{.08} \right]$$

$$\text{P}60,000 = A (8.1390)$$

$$A = \text{P}7,371.91$$

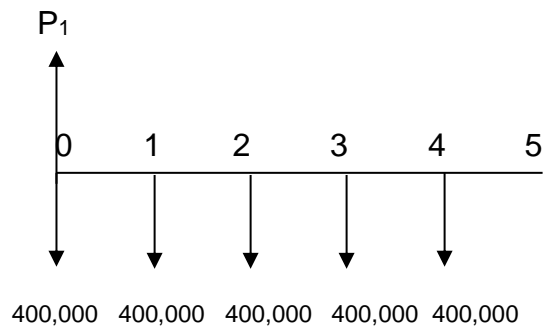
Example

A certain property is being sold and the owner received two bids. The first bidder offered to pay P400,000 each year for 5 years, each payment is to be made at the beginning of each year. The second bidder offered to pay P240,000 first year, P360,000 the second year and P540,000 each year for the next years, all payments will be made at the beginning of each year.

If money is worth 20% compounded annually, which did should the owner of the property accept?

Solution

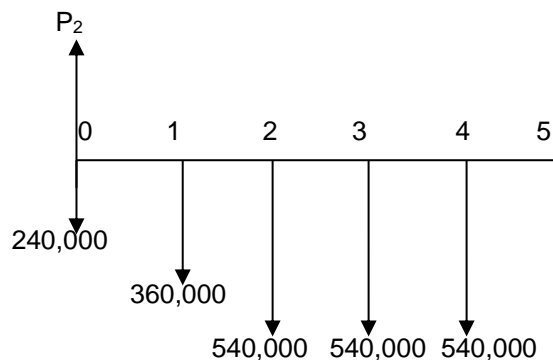
First bid:



Let P_1 = present worth of the first bid

$$\begin{aligned} P_1 &= A (1 + P/A, 20\%, 4) \\ &= P400,000(1 + P/A, 20\%, 4) \\ &= P400,000 \left[1 + \frac{1 - (1 + .20)^{-4}}{.20} \right] \\ &= P 400,000 (3.5887) \\ &= P1,435,480 \end{aligned}$$

Second bid:



Let P_2 = present worth for the second bid

$$\begin{aligned}
P_2 &= P240,000 + P360,000(P/F, 20\%, 1) + P540,000(P/A, 20\%, 3)(P/F, 20\%, 1) \\
&= P240,000 + P360,000(1+.20)^{-1} + P540,000 \left[\frac{1-(1+.20)^{-3}}{.20} \right] (1+.20)^{-1} \\
&= P240,000 + 360,000(0.8333) + P540,000(2.1065)(0.8333) \\
&= P1,487,875
\end{aligned}$$

The owner of the property should accept the second bid.

Problem:

Suppose that P 400 is deposited each year into a bank account that pays 8% interest annually. If 12 payments are made into the account, how much would be accumulated in his fund by the end of the 12th year? The first payment occurs at time zero (now).

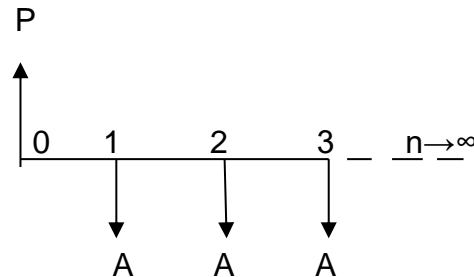
Problem:

A farmer bought a tractor costing P 25,000 payable in to semi-annual payments, each installment payable at the beginning of each period. If the rate of interest is 12% compounded semiannually, determine the amount of each installment

Perpetuity

A perpetuity is an annuity in which the payments continue indefinitely.

Perpetuity is a series of uniform cash flows where they extend for a long time or forever.



Cash flow diagram to find P given A

From the formula of present worth of an ordinary annuity;

$$P = A \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

The expression $(1 + i)^{-\infty}$ approaches 0 when n approaches infinity. Thus, the present worth of perpetuity becomes;

$$P = \frac{A}{i}$$

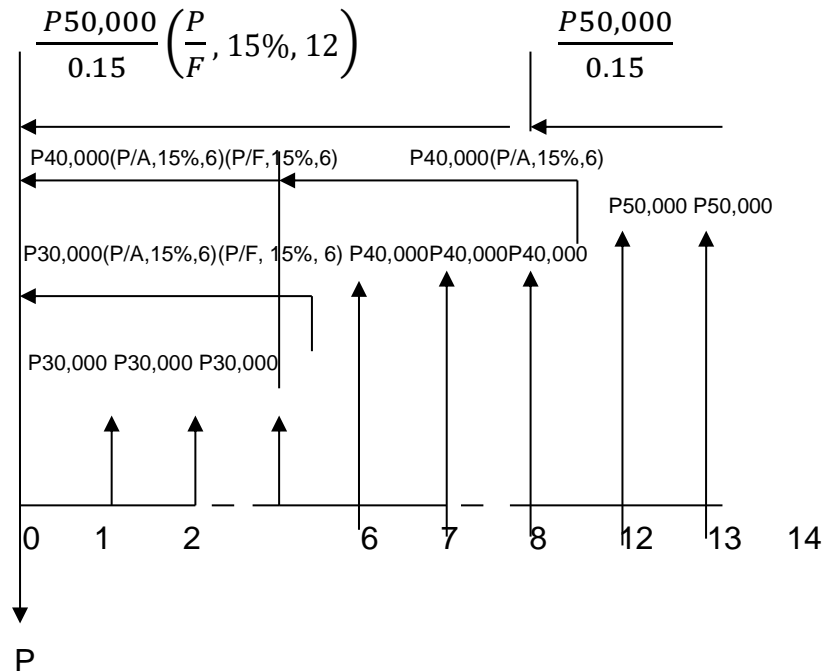
Solving for A:

$$A = Pi \text{ (the formula for periodic interest)}$$

Example

What amount of money invested today at 15% interest can provide the following scholarship: P30,000 at the end of each year for 6 years; P40,000 for the next years and P50,000 thereafter?

Solution



Using today as the focal date, the equation of value is

$$\begin{aligned}
 P &= P30,000 (P/A, 15\%, 6) + P40,000 (P/A, 15\%, 6) (P/F, 15\%, 6) + \\
 &\quad \frac{P50,000}{.15} (P/F, 15\%, 12) \\
 = & P30,000 \left[\frac{1-(1+.15)^{-6}}{.15} \right] + P40,000 \left[\frac{1-(1+.15)^{-6}}{.15} \right] (1+.15)^{-6} + \\
 &\quad \frac{P50,000}{.15} (1+.15)^{-12} \\
 = & P30,000 (3.7845) + P40,000 (3.7845) (0.4323) + \frac{P50,000}{.15} (0.1869) \\
 = & P241,477
 \end{aligned}$$

Problem:

What amount should be invested today at 5% per year to provide for the following scholarship: P30,000 every year for the first four years, P40,000 every year for the next four years and P 50,000 thereafter?

Capitalized Cost

One of the most important applications of perpetuity is in capitalized cost. The capitalized cost of any property is the sum of the first cost and the present worth of all costs of replacement, operation and maintenance for a long time or forever.

Capitalized cost is an application for perpetuity. It is one method used in comparing alternatives. It is defined as the sum of the first cost (FC) and the present worth of all perpetual maintenance and replacement cost.

Case 1. No replacement, only maintenance and or operation every period.

Capitalized cost = First cost + Present worth of Perpetual operation and/or maintenance.

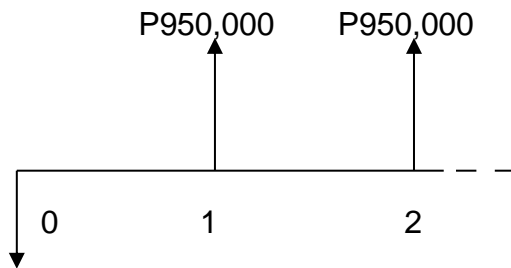
Where: FC = first cost (initial investment)

$P_{O\&M}$ = Present worth of all perpetual operation and maintenance cost

Example

Determine the capitalized cost of a structure that requires an initial investment of P10,000,000 and an annual maintenance of P950,000. $i = 15\%$

Solution:



$$P = \frac{A}{i} = \frac{P950,000}{0.15} = P6,333,334.00$$

$$\begin{aligned} \text{Capitalized cost} &= \text{First cost} + P \\ &= P10,000,000 + P6,333,334 \\ &= P16,333,334.00 \end{aligned}$$

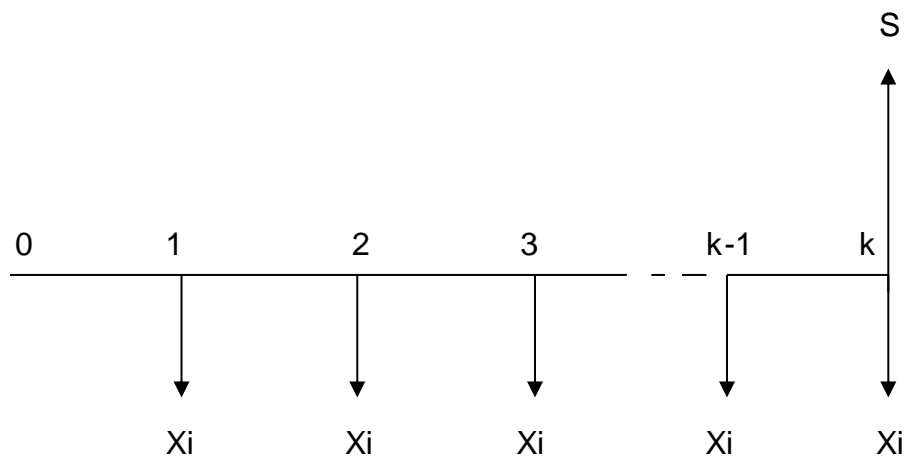
Case 2.Replacement only, no maintenance and/ or operation.

Capitalized cost = First cost + Present worth of perpetual replacement

Let S = amount needed to replace a property every k period

X = amount of principal invested at rate i% the interest on which will amount to S every k periods

Xi = interest on X every period, the periodic deposit towards accumulation of S



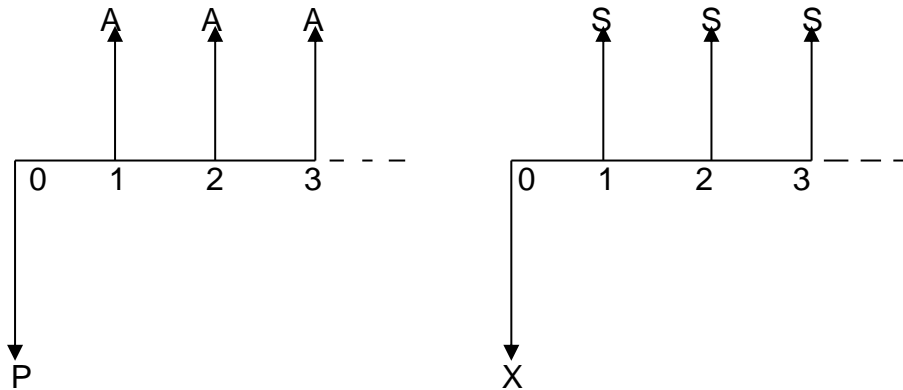
Cash flow diagram to find X given S

$$S = Xi (F/A, i\%, k)$$

$$X = \frac{S}{i} \left[\frac{1}{\frac{F}{A}, i\%, k} \right] = \frac{S}{i} \left[\frac{i}{(1+i)^k - 1} \right]$$

$$X = \frac{S}{(1+i)^k - 1}$$

Difference between P and X in a perpetuity



$$P = \frac{A}{i} X = \frac{S}{(1+i)^k - 1}$$

P is the amount invested now at $i\%$ per period whose interest at the end of every period forever is A while X is the amount invested now at $i\%$ per period whose interest at the end of every k periods forever is S. If $k = 1$, then $X = F$.

Replacement cost

Let: X = present worth of perpetual replacement
 S = amount of replacement
 K = replacement period or the life of the property

Salvage Value (SV) is defined as the worth of the property at the end of useful life

Scrap value (SV) is defined as the worth of the property if disposed of as a junk.

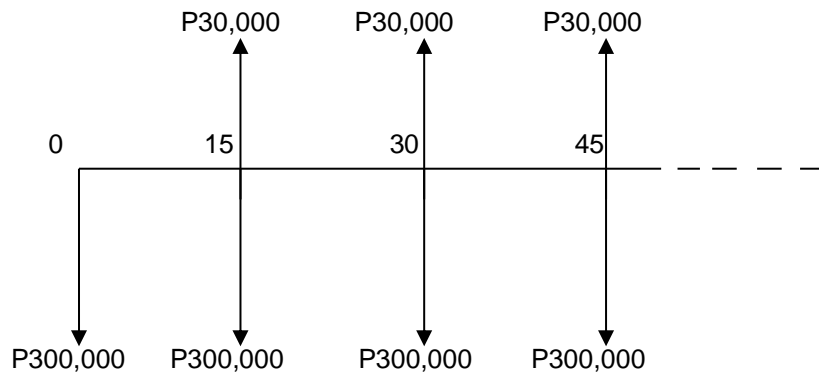
Then the amount needed for replacement will be:

$$S = FC - SV$$

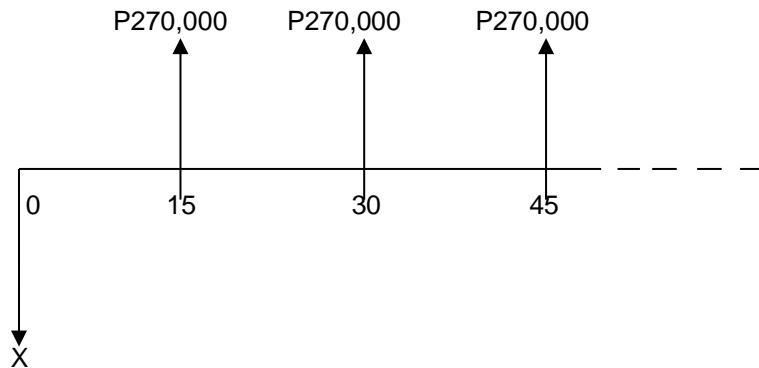
Example

A new engine was installed by a textile plant at a cost of P300,000 and projected to have a useful life of 15 years. At the end of its useful life, it is estimated to have a salvage value of P30,000. Determine its capitalized cost if interest is 18% compounded annually.

Solution:



Cash flow diagram for the engine



$$X = \frac{S}{(1+i)^k - 1} = \frac{P270,000}{(1+0.18)^{15} - 1} = P24,604.17$$

$$\begin{aligned} \text{Capitalized cost} &= \text{First cost} + X \\ &= P300,000 + P24,604.17 \\ &= P324,604.17 \end{aligned}$$

Example

A manufacturing plant installed a new boiler at a cost of P150,000 and estimated to have a useful life of 10 years. It is estimated to have a scrap value at the end of the useful life of P5,000. If the interest is 10% compounded annually, determine the capitalized cost.

Given:

First Cost (FC) = P150,000

$i = 10\%$

$n = 10$ years

Salvage value = P5,000

Solution

Cost + Replacement

$$S = P150,000 - P5,000 = P145,000$$

$$CC = FC + \frac{S}{(1+i)^k - 1}$$

$$= 150,000 + \frac{145,000}{(1.12)^{10} - 1}$$

$$= P218,855.87$$

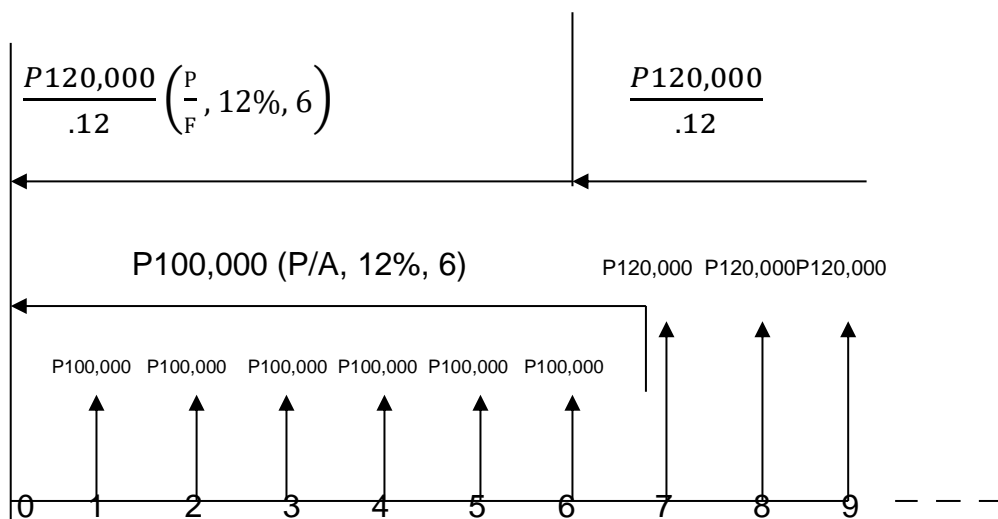
Case 3. Replacement, maintenance and/or operation every period

Capitalized cost = First cost + Present worth of cost of perpetual operation and/ or maintenance + Present worth of cost of perpetual replacement.

Example

Determine the capitalized cost of a research laboratory which requires P5,000,000 for original construction; P100,000 at the end of every year for the first 6 years and then P120,000 each year thereafter for operating expenses, and P500,000 every 5 years for replacement of equipment with interest at 12% per annum?

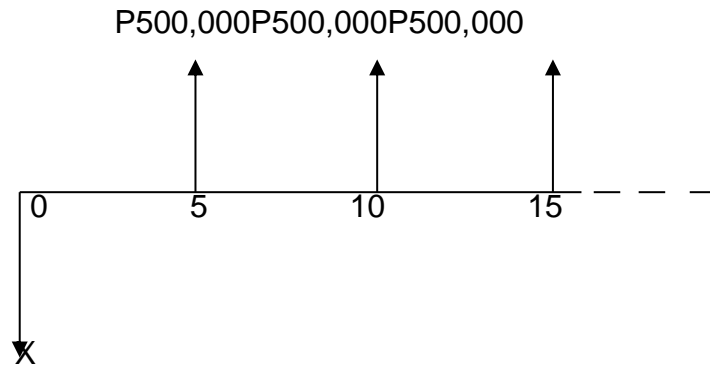
Solution



Let P = the present worth of cost of perpetual operation

$$\begin{aligned}
 P &= P100,000 (P/A, 12\%, 6) + \frac{P120,000}{.12} (P/F, 12\%, 6) \\
 &= P100,000 \left[\frac{1-(1+.12)^{-6}}{.12} \right] + \frac{P120,000}{.12} (1.12)^{-6} \\
 &= P100,000 (4.1114) + \frac{P120,000}{.12} (0.5066) \\
 &= P917,740
 \end{aligned}$$

Replacement:



Let X = the present worth of cost of perpetual replacement

$$X = \frac{S}{(1+i)^k - 1} = \frac{P500,000}{(1+.12)^5 - 1} = P655,873.88$$

$$\begin{aligned}
 \text{Capitalized cost} &= \text{First cost} + Q + X \\
 &= P5,000,000 + P917,740 + P655,873.88 \\
 &= P6,573,613.88
 \end{aligned}$$

Problem:

A rich man left 5,000,000 to establish a perpetual charitable foundation. The foundation trustees decided to spend 1,100,000 to provide for facilities immediately and to provide 100,000 of capital replacement at the end of each 5 year period. If the invested fund earns 12% per annum, what would be the year end amount available in perpetuity from the endorsement for the charitable purposes?

Amortization

Amortization is any method of repaying a debt, the principal and interest included, usually by a series of equal payments at equal intervals of time.

Amortization is any mode of paying debt, the principal and interest included, usually by a series of uniform amount every period.

Amortization Schedule

-is a table showing the payments throughout the total interest period.

Example

A debt of P5,000 with interest at 12% compounded semiannually is to be amortized by equal semiannual payments over the next 3 years, the first due in 6 months. Find the semiannual payment and construct an amortization schedule.

Solution

$$A = \frac{P}{\frac{P}{A}, 6\%, 6} = \frac{P5,000}{\left[\frac{1 - (1.06)^{-6}}{.06} \right]} = P1,016.81$$

--	--	--	--	--	--

Example:

A loan of P100,000 must be repaid by a uniform amount every year for 10 years at 10% interest per year. Determine the amount of periodic payments and construct the amortization schedule.

GRADIENT SERIES

Gradient series is the series of cash flow where the amounts change every period.

Uniform Arithmetic Gradient

In certain cases, economic analysis problems involve receipts or disbursement that increase or decrease by a uniform amount each period. For example, maintenance and repair expenses on specific equipment or property may increase by a relatively constant amount each period. This is known as a uniform arithmetic gradient.

Suppose that the maintenance expense on a certain machine is P1,000 at the end of the first year and increasing at a constant rate of P500 each year for the next four years.

Some problems involve cash flows that are projected to increase or decrease by a uniform amount each period, thus constituting an arithmetic sequence of cash flows.

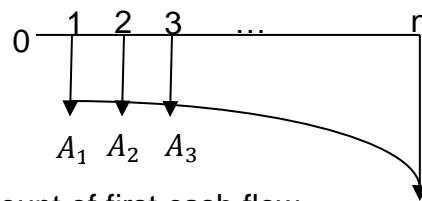
Geometric Gradient Series

Deposits or payment where there is an increase or decrease by a fixed percentage.

Some problems involve projected cash flow patterns that are changing at an average rate f each period

Let: f = fixed percentage (increase or decrease)

A_1 = the amount of the first cash flow of the series



A_1 = the amount of first cash flow

$$A_2 = A_1 + A_1f = A_1(1+f)$$

$$A_3 = A_1 (1+f)^2$$

$$A_4 = A_1 (1+f)^{n-1}$$

Getting the present value of each cash flow:

$$P = A_1 (1+i)^{-1} + A_1 (1+f)(1+i)^{-2} + A_1 (1+f)^2 (1+i)^{-3} \dots + A_1 (1+f)^{n-1} (1+i)^{-n}$$

Multiply the equivalent by $(1+i)$:

$$P = A \left[\left(\frac{1+f}{1+i} \right)^1 + \left(\frac{1+f}{1+i} \right)^2 + \left(\frac{1+f}{1+i} \right)^3 + \dots + \left(\frac{1+f}{1+i} \right)^n \right]$$

CONVENIENCE RATE (icr)

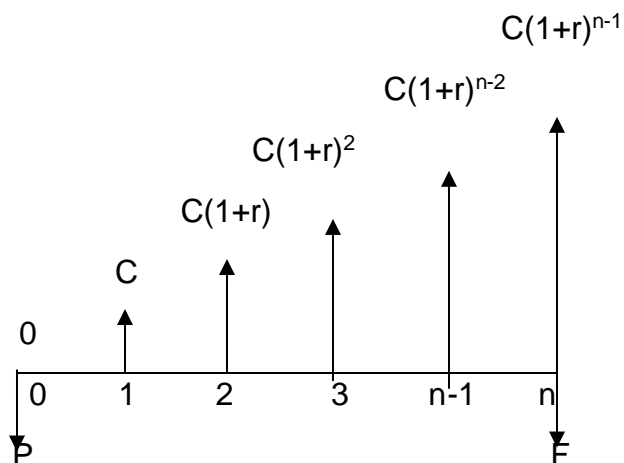
$$icr = \frac{1+i}{1+f} - 1$$

$$\frac{1+f}{1+i} icr - 1$$

$$P = A_1 (1+icr)^{-1} [(1+icr)^{-1} + (1+icr)^{-2} + \dots + (1+icr)^{-n}]$$

Simplifying:

$$P = \frac{A_1}{(1+i)} \left[\frac{1+(1+i)^n}{i} \right]$$



$$P = \frac{C}{1+i} \left[\frac{1-x^n}{1-x} \right]$$

$$\text{Where: } x = \frac{1+r}{1+i}$$

When: $x = 1.0$ that is $i = r$

$$P = \frac{Cn}{1+i}$$

$$F = C (1+i)^{n-1} \left[\frac{1-x^n}{1-x} \right]$$

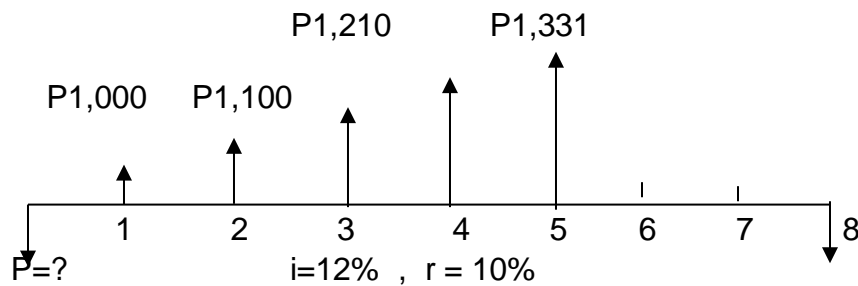
$$\text{where: } x = \frac{1+r}{1+i}$$

when: $x = 1.0$ that is $i = r$

$$F = Cn (1+i)^{n-1}$$

Example

P1464.1



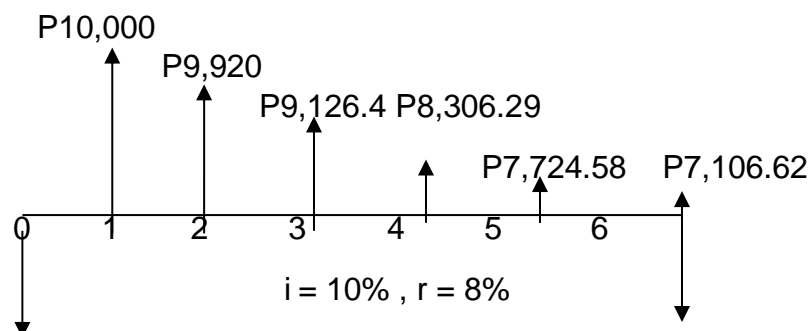
Solution

$$x = \frac{1+r}{1+i} = \frac{1+.10}{1+.12} = \frac{55}{56}$$

$$P = \frac{P1,000}{1+.12} \left[\frac{(1-\frac{55}{56})^5}{(1-\frac{55}{56})} \right] = P4,307.67$$

$$F = P1,000 (1+.12)^{5-1} \left[\frac{(1-\frac{55}{56})^5}{(1-\frac{55}{56})} \right] (1.12)^3 = P10,665.63$$

Example



$$P = ?$$

$$F = ?$$

Solution

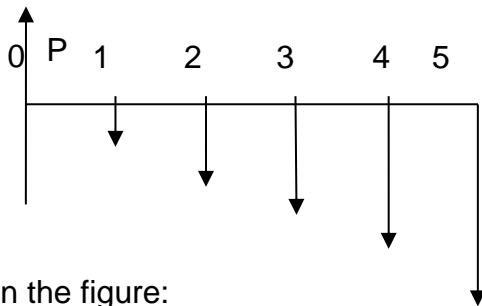
$$x = \frac{1-r}{1-i} = \frac{1-.08}{1-.10} = \frac{46}{45}$$

$$P = \frac{P10,000}{1+.10} \left[\frac{\left(1 - \frac{46}{45}\right)^6}{\left(1 - \frac{46}{45}\right)} \right] = P36,540.47$$

$$F = P10,000 (1+.10)^{6-1} \left[\frac{\left(1 - \frac{46}{45}\right)^6}{\left(1 - \frac{46}{45}\right)} \right] = P64,733.67$$

Example:

Determine the present worth of the following series.



In the figure:

$$A_1 = 100$$

$$F = 10\%$$

$$I = 12\%$$

$$N = 5$$

Solution:

$$P = \frac{A_1}{(1+f)} \left[\frac{1 - (1+i_{CR})^{-n}}{i_{CR}} \right]$$

Solving for the convenience rate:

$$I_{CR} = \frac{1+i}{1+f} - 1$$

$$I_{CR} = \frac{1.12}{1.10} - 1$$

$$I_{CR} = 0.0182$$

Therefore:

$$P = \frac{100}{1.10} \left[\frac{1 - (1.0182)^{-5}}{0.0182} \right]$$

$$P = 430.74$$

Problem:

An EOY Geometric gradient series for 10 years whose initial values at EOY three is P5, 000 and $f=6.04\%$ per year thereafter. Find the equivalent uniform gradient amount G over the same period (beginning in year 1 and ending in year 12) if the initial value of the series at EOY one is P4,000. Answer the following questions in determining the value of the gradient amount G . The interest rate is 8% nominal, compounded semi-annually,

- a. What is the i ?
- b. What is P_T for the geometric gradient
- c. The is P_T of the uniform (arithmetic) gradient?
- d. What is the value of G ?

Problem:

Ronald makes year-end deposit of 500, the first year increasing the year's deposits by 10% until the end of the 10th year. Ellen makes equal year-end deposits of 700 each year for 10 years. If interest on both funds is 12% compounded annually, who will be able to save more at the end of ten years?

Problems

1. M purchased a small lot in a subdivision, paying P200, 000 down and promising to pay P15, 000 every 3 months for the next 10 years. The seller figured interest at 12% compounded quarterly.

a).What was the cash price of the lot?

b.) If M missed the first 12 payments, what must he pay at the time the 13th is due to bring him up to date?

c.) After making 8 payments, M wished to discharge his remaining indebtedness by a single payment at the time when the 9th regular payment was due, what must he pay in addition to the regular payment then due?

d.)If M missed the first 10 payments, what must he pay when the 11th payment is due to discharge his entire indebtedness?

2. A man approaches the ABC Loan Agency for P100, 000 to be paid in 24 monthly installments. The agency advertises an interest rate of 1.5% per month. They proceed to calculate the amount of his monthly payment in the following manner

Amount requested	P100,000
Credit investigation	500
Credit risk insurance	1,000
Total	P101,500

Interest: $(P101,500) (24) (0.015) = P36,540$

Total owed: $P101,500 + P36,540 = P138,040$

Payment = $P138,040/24 = P5,751.67$

What is the effective rate of interest of the loan?

3. A new office building was constructed 5 years ago by a consulting engineering firm. At that time the firm obtained the bank loan for P 10,000,000 with a 20% annual interest rate, compounded quarterly. The terms of the loan called for equal quarterly payments for a 10-year period with the right of prepayment any time without penalty.

Due to internal changes in the firm, it is now proposed to refinance the loan through an insurance company. The new loan is planned for a 20- year term with an interest rate of 24% per annum, compounded quarterly. The insurance company has a onetime service charge 5% of the balance. This new loan also calls for equal quarterly payments.

a.) What is the balance due on the original mortgage (principal) if all payments have been made through a full five years?

b.) What will be the difference between the equal quarterly payments in the existing arrangement and the revised proposal?

4. An asphalt road requires no upkeep until the end of 2 years when P60, 000 will be needed for repairs. After this P90, 000 will be needed for repairs at the end of each year for the next 5 years, then P120, 000 at the end of each year for the next 5 years.

If money is worth 14% compounded annually, what was the equivalent uniform annual cost for the 12-year period?

5. A man wishes to provide a fund for his retirement such that from his 60th to 70th birthdays he will be able to withdraw equal sums of P18, 000 for his yearly expenses. He invests equal amount for his 41st to 59th birthdays in a fund earning 10% compounded annually. How much should each of these amounts be?

6. Juan Reyes inherited P280,000 on his 30th birthday. This he placed in his own person fund which earns interest 12% compounded annually. He withdraws P450,000 on his 35th birthday to make the initial payment on a house. He drew his annual uniform payment for the person fund on his 65th birthday and planned to make 20 such withdraws so as to just exhaust the fund. He died at the age of 71, however, just before withdrawing the amount due on that date. How much is left in the fund to be passed on the heirs at the time of Juan's death?

7. Determine the present worth and the accumulated amount of an annuity consisting of 6 payments of P120, 000 each, the payment are made at the beginning of each year. Money is worth 15% compounded annually.

8. Calculate the capitalized cost of a project that has an initial cost of P3, 000,000 and an additional cost of P100, 000 at the end of every 10 yrs. The annual operating costs will be P100, 000 at the end of every year for the first 4 years and P160, 000 thereafter. In addition, there is expected to be recurring major rework cost of P300, 000 every 13 yrs. Assume $i = 15\%$.

9. The will of a wealthy philanthropist left P5, 000,000 to establish a perpetual charitable foundation. The foundation trustees decided to spend P1, 200,000 to provide facilities immediately and to provide P100, 000 of capital replacement at the end of each 5 year period. If the invested funds earned 12% per annum, what would be the year end amount available in perpetuity from the endowment for charitable purposes?

10. The surface area of a certain plant requires painting is 8,000 sq. ft. Two kinds of paint are available whose brands are A and B. Paint A cost P 1.40 per sq. ft. but needs renewal at the end of 4 yrs., while paint B cost P 1.80 per sq. ft. If money is worth 12% effective, how often should paint B be renewed so that it will be economical as point A?

11. A contract has been signed to lease a building at P20,000 per year with an annual increase of P1,500 for 8 years. Payments are to be made at the end of each year, starting one year from now. The prevailing interest rate is 7%. What lump sum paid today would be equivalent to the 8-year lease-payment plan?

12. A mechanical engineer designed a mechanical device costing P20,000. Maintenance will cost P1,000 each year. The device will generate revenues of P5,000 each year for 5 years, after which the salvage value is expected to be P7,000. Draw the cash flow diagram.

13. A father wishes to provide P4,000 for his son on his 21th birthday. How much should he deposit every 6 months in a savings bank which pays 3% compounded semi annually, if the first deposit is made when the sons is $3\frac{1}{2}$ years old?

14. If money is worth 12% compounded quarterly and $n = 5$, compute the present value of the following.

- a.) P1.00 received monthly in year n
- b.) P1.00 received monthly for n years

15. A Filipino contractor in the Middle east plans to purchase a new office Building costing P1,000,000. He agrees to pay P150,000 annually for 20 years. At the end of this time he estimate that he can resell the building for at least P600,000. Instead of buying the building he can lease it for P140,000 a year. All payments are to be made at the beginning of each year and the rate of interest is 10%. Should the contractor purchase or lease the building?

16. XYZ Inc. plans to construct an additional building at the end of 10 years at an estimated cost of P5,000,000. To accumulate this amount it will deposit equal year end amounts in a fund earning 13%. However at the end of the 5th year it decrease to have a longer building estimated to cost P8,000,000. What should be the annual deposit for the last 5 years?

17. A computer costs P105,000. It can be leased for an annual rent of P30,000 payable at the beginning of each year throughout its economic life of 5 years. No salvage value is expected at the end of 5 years. Cost of capital is 10% and the applicable interest rate is 40%. Should the computer be purchased or leased?

Solution:

18. A refrigerator equipment fabricator sold an ice drop making machine with condition that in case the machine will not be able to produce the guaranteed capacity, he will deduct from the agreed price of the machine the loss of revenue that the buyer will incur during the economic life of the machine which is 5 years, plus 12% cost of money. One month after the machine was delivered the buyer complained to the fabricator that he was not getting the guaranteed production. After a series of test it was determined that the buyer would be costing P4,000 worth of unrealized sales per year. Assuming no increase in operation cost if guaranteed production is attained, how much must the buyer deduct from the agreed price of the machine?

19. A company manufacturing acids, upon inspection of the plant, found out that it is badly corroded from the acid fumes and would need to be replaced, to try to get some more life out of the roofing, the company consulted a roofing coating contractor who presented the company with two options. The first option is a coating that will cost P20,000 which would extend the life of the roofing for 3 years from the date of application, and the second option will cost P30,000 and which would extend the life of the roofing for 5 years from the date of application. At what rate of return are the investment equal?

20. A fully secured loan of P30,000 was to be amortized by 10 equal payment semi-annual, the first payment to be made 6 months after the loan is approved. After the 6th payment was made the debtor was in a position to settle the entire balance remaining by a single payment on the date. If the interest on the loan is 12% compounded semi-annually, what be the amount of single payment?

21. A machine has just been purchased for P1.2 million. It is anticipated that the machine will be held 5 years, that it will have a salvage value of P80,000 based on current prices, and that the annual rate of inflation during the next 5 years will be 7%. The machine will be replaced by a duplicate and the firm will accumulate the necessary capital by making equal year end deposits in a reserve fund that earns 6% per annum. Determine the amount of the annual deposit.

22. A man agreed to pay the loan he is borrowing from a bank in six equal year end payments of P71,477.70. Interest is 18% per annum, compounded annually, and is included in the yearly amount he will be paying the bank. How much money is the man borrowing from the bank?

23. A man wishes to purchase a set of appliances with a total cost of P20,000. He made a down payment of P5,000 and the balance payable in 24 equal monthly installments. If financing charge is 12% each year computed on the total balance to be paid by installment and interest is 12%,

a.) how much would each installment payment be?

b.) what will be the actual cost of money?

24. A new company developed a program in which employees will be allowed to purchase shares of stock of the company at the end of its fifth year of operation, when the company is thought to have gained stability, at par value of P100 per share. Believing in the good potential of the company, an employee decided to save in a bank the amount of P8,000 at the end of every year which will earn 9% interest, compounded annually. How many shares will he be able to purchase at the end of the fifth year?

25. If P10,000 is deposited each year for 9 years, how much annuity can a person get annually from a bank every year for 8 years starting one year after the ninth deposit is made? Money is worth 14%.

26. A company has approved a car plan for its six senior officers in which the company will shoulder 25% of the cost and the difference payable by each officer to a financing company in 48 equal end-of-the-month installments at an interest rate of 1.5% per month. If the cost of each car is P350,000 determine the following:

27. A machinery supplier is offering a certain machine on a 10% down payment and the balance payable in equal year end payments without interest for 2 years. Under this arrangement the price is pegged at P250,000. However, for cash purchase the machine would only cost P195,000. What is the equivalent interest rate that is charged on the two year payment plan if interest is compounded quarterly?

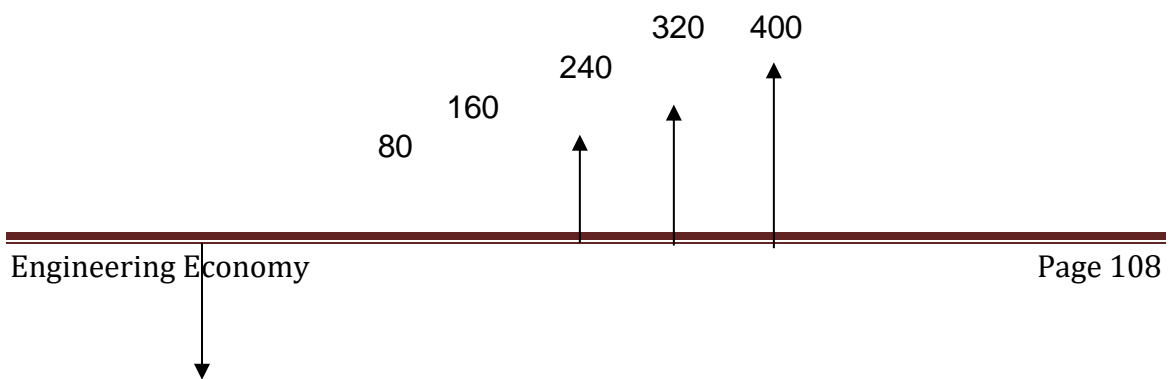
28. Determine the amount needed now to purchase a machine for P100,000, provide an annual fund of P15,000 for operation and maintenance, and to replace it at the same cost at the end of every ten-year period. Money is worth $7\frac{1}{2}\%$ compounded annually.

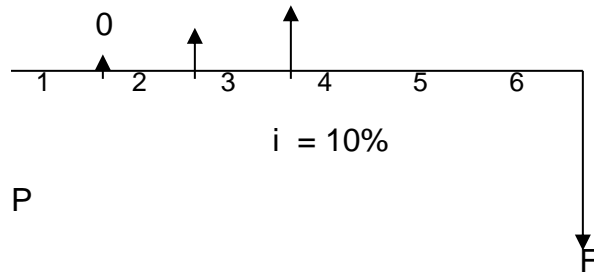
29. A new storage tank can be purchased and installed for P50,000 and is expected to last for 10 years. However, an existing tank of equivalent capacity may be continued to be used provided it is repaired. If this tank were repaired its useful life is estimated to be done again. Assume that future costs will remain the same and that the tanks will have no scrap value. Money is worth 10%

compounded annually. How much can be spent for repairing the existing tank if the capitalized cost of the two tanks are the same?

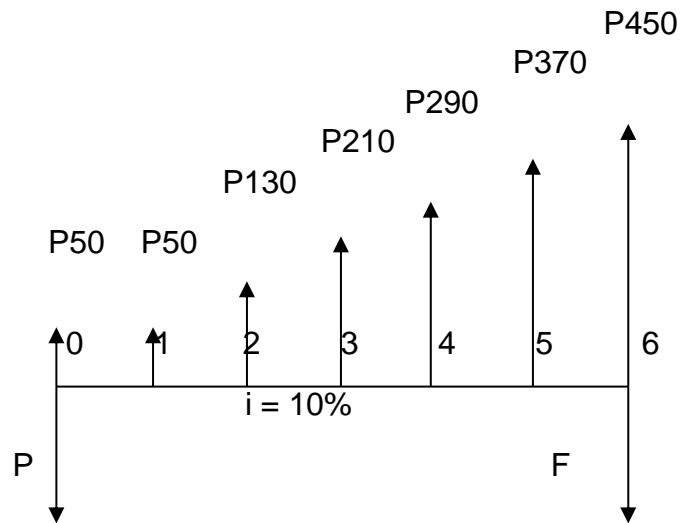
30. A chemical reactor , which will contain corrosive liquids, has been designed. The reactor may be made of mild steel or stainless steel. The initial cost, including installation, of a mild steel reactor is expected to be P72,000 and would last 5 years. Stainless steel is more resistant to be corrosive action of the liquids, but is more expensive and will cost P120,000. Assuming that both types of reactors will have no salvage or scrap value at the end of their useful lives and could be replaced at the same costs as before, and if money is worth 12% compounded annually, what should the useful life of the stainless reactor be so that the capitalized costs are equal?

31. Compute the values of P and F

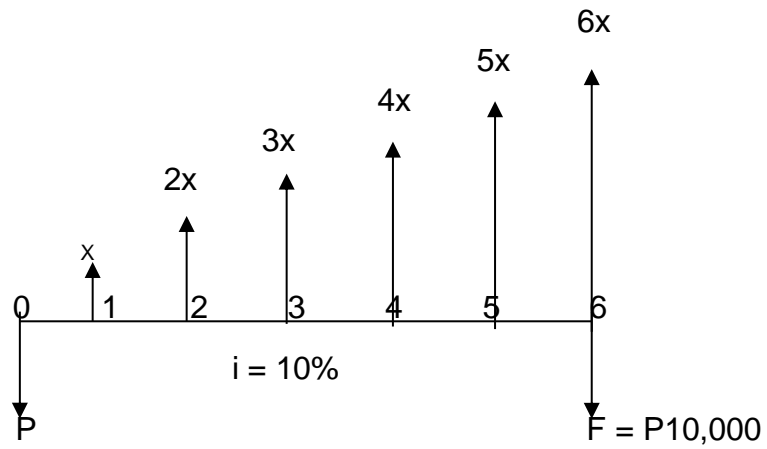




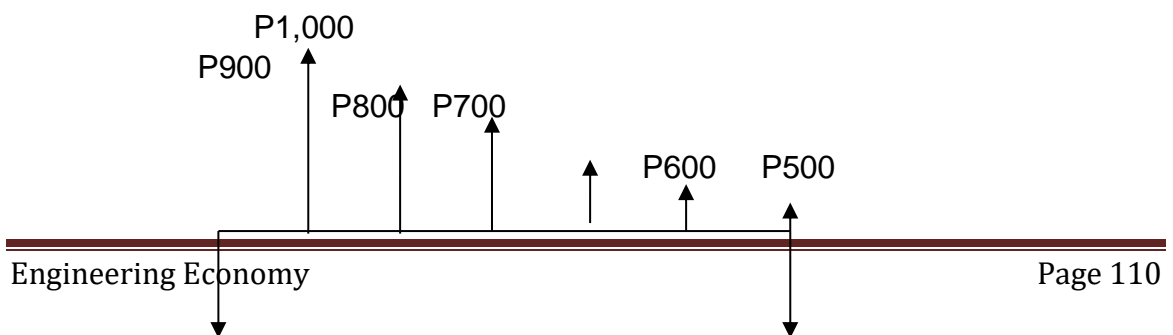
32. Compute the values of P and F



32. Compute the values of X and P



33. For the gradient series, compute the values of P and F, and the value of A for the equivalent uniform series.



35. Suppose an engineer receives an initial annual salary of P24,000 increasing at the rate of P1,200 a year. If money is worth 12%, determine his equivalent annual salary for a period of 10 years.

36. The year end operating and maintenance costs of a certain machine are estimated to be P4,800 the first year and to increase by P800.

37. An inventor was offered the following alternatives as royalties for his invention:

a.) A single lump sum payment of P350,000 for the right to manufacture the invention for the next 5 years

b.) An initial payment of P100,000 and year end payments as follows:

End of Year	Royalty Payments
1	P 0
2	50,000
3	100,000
4	150,000
5	200,000

- 38.** For the following arithmetic gradient series, with interest at 10% find:
- a.) the present worth
 - b.) the value at the end of year 6
 - c.) the value at the end of year 10, assuming no further deposits after the sixth year
 - d.) the equivalent uniform series for 6 years.

End of year	Deposits
1	P 0
2	800
3	1,600
4	2,400
5	3,200
6	4,000

39. A machine in a factory has the following end-of-year maintenance costs

Year	Maintenance Costs
1	P 720
2	800
3	880
4	960
5	1,040
6	1,120
7	1,200

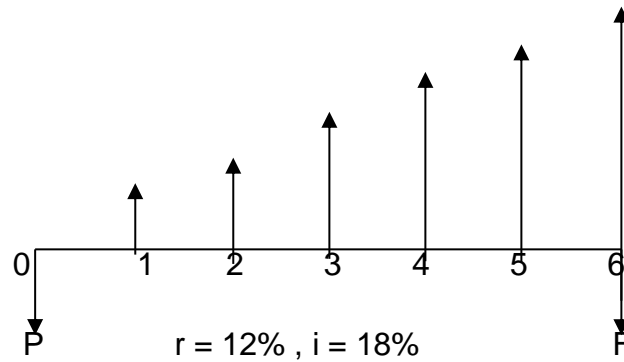
If Interest is 14.8% compounded annually, determine the present worth of this series.

Solution

40. A company owes P 80,000, which includes the interest, to be paid one year from now. To provide for the repayment of this debt, the company deposits P2,000 at the beginning of the first month, P4,000 at the beginning of the second month, increasing the deposits in each succeeding month by P2,000 until the beginning of the eighth month. These amounts earn interest at the rate of 9% compounded monthly. Will the total amount in the fund at the end of 12 months be sufficient to repay the debt? If not, how more is needed?

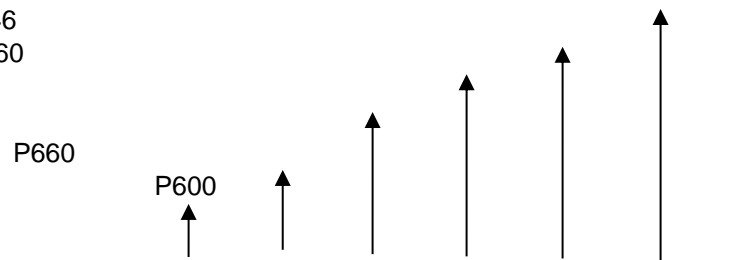
41. For the cash flow, find P and F if $i = 18\%$ per year

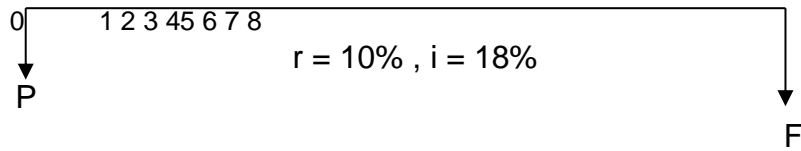
P881.17
 P786.76
 P702.46
 P627.20
 P560
 P500



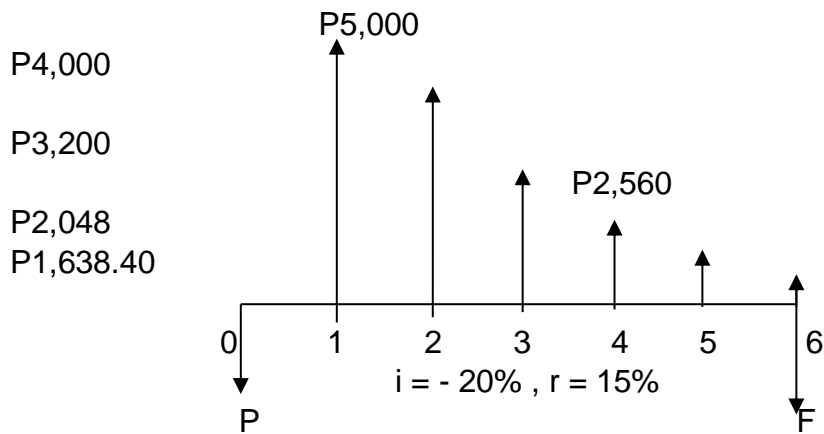
42. For the cash flow , find P and F if $i=18\%$ per year.

P966.31
 P878.46
 P798.60
 P726

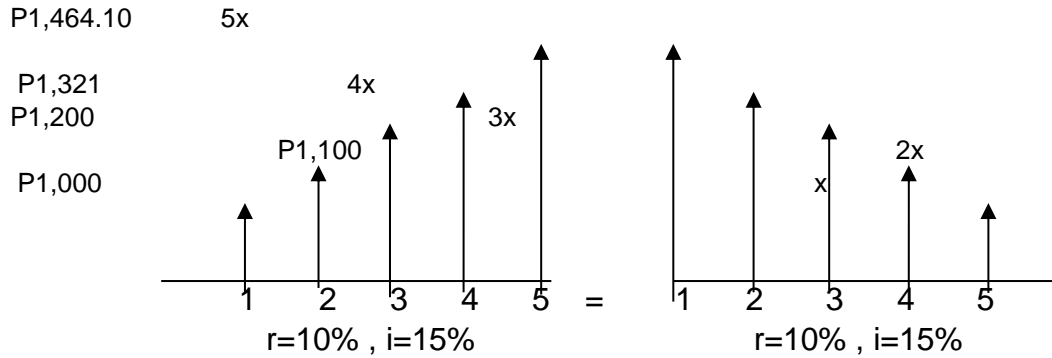




43. For the cash flow, find P and if $i = 15\%$.



44. Find the value of x in the arithmetic gradient, so that the two cash flows will be equivalent if $i = 15\%$ per year.



45. A man makes a series of ten annual deposits starting at P2,000 at the end of the first year and increasing the amount deposited by 10% every year thereafter. Find the total amount on deposit in (a) 8% (b) 10% (c) 12%.

46. A parcel of land in a downtown area, suitable for a making lot, can be leased for a period of 10 years. Initial development costs for clearing, paving and constructing a small office shed on the lot is estimated to be P150,000. If the net annual revenue for the first year is P22,000 and increases by 10% per year thereafter until the tenth year, and the improvement revert to the owner of the land at the end of 10 years, what is the rate of return on the investment?

47. A man invested P50, 000 now for the college education of his 2-year old son. If the fund earns 14% effective, how much will the son get each year starting from his 17th to the 22th birthday?

48. Find the uniform annual amount that is equivalent to a uniform gradient series in which the first year's payment is P500, the second year's payment is P600, third year's payment is P700 and so on and there is a total of 20 payments. The annual interest is 8%.

49. You are purchasing a P900,000 used automobile, which is to be paid in 36 monthly installment of P28,872.00. What nominal interest rate are you paying on this financing arrangement?

50. What is the future worth of the following series of payments?

(a) \$3,000 at the end of each six-month period for 10 years at 6% compounded semiannually.

(b) \$4,000 at the end of each quarter for six years at 8% compounded quarterly.

(c) \$7,000 at the end of each month for 13 years at 9% compounded monthly.

51. What equal series of payments must be paid into a sinking fund in order to accumulate the following amounts?

(a) \$12,000 in 10 years at 6% compounded semi annually when payments are semi annually

(b) \$7,000 in 15 years at 9% compounded quarterly when payments are quarterly

(c) \$34,000 in 5 years at 7.55% compounded monthly when payments are month.

52. What is the present worth of the following series of payments?

(a) \$500 at the end of each six-month period for 10 years at 8% compounded semiannually

(b) \$2,000 at the end of each quarter for five years at 8% compounded quarterly

(c) \$3,000 at the end of each month for eight years at 9% compounded monthly

53. Find the effective interest rate per payments period for an interest rate at 9% compounded monthly if the payment period is

(a) Monthly

(b) Quarterly

(c) Semiannual

(d) Annually

54.What will be the amount accumulated by each of the following present investments?

(a) \$4,638 in 10 years at 6% compounded semiannually

(b) \$6,500 in 15 years at 8% compounded quarterly

(c) \$28,300 in seven years at 9% compounded monthly

55. A man is planning to retire in 25 years. He wishes to deposit a regular amount every three months until he retires so that, beginning one year following his retirement, he will receive annual payments of \$32,000 for the next 10 years. How much he deposit if the interest rate is 8% compounded quarterly?

56. A building is priced at \$125,000. If the buyer makes a down payment of \$25,000 and a payment of \$1,000 every month thereafter, how many months will it take for the buyer to completely pay for the building? Interest is charged at a rate of 9% compounded monthly

57. A couple is planning to finance its three-year old son's college education. The couple can deposit money at 6% compounded quarterly. What quarterly deposit must be made from the son's 3rd birthday to his last birthday in order to provide \$50,000 on each birthday from the 18th to the 21st? (Note that the last deposit is made on the date of the first withdrawal)

58.Sam Santos is planning to retire in 15 years. He can deposit money at 8% compounded quarterly. Which deposit must he make at the end of each quarter until he retires so that he can make a withdrawal of \$25,000 semiannually over the five years after his retirement? Assume that his first withdrawal occurs at the end of six months after his retirement.

59.Emily Lacy received \$500,000 from an insurance company after her husband's death. She wants to deposit this amount in a savings account that earns interest at a rate of 6% compounded monthly. Then she would like to make 60 equal withdrawals over five years such that, when she makes the last withdrawal the savings account will have a balance of zero. How much should she withdraw each month?

60.Bob Pearson borrowed \$20,000 from a bank at an interest rate of 12% compounded month. This loan is to be repaid in 36 equal monthly installments over three years. Immediately after his 20th payment, Bob desires to pay the remainder of the load in a single payment. Compute the total amount he must pay at the time.

61. You are buying a home for \$190,000. If you make a down payment of \$40,000 and take out a mortgage on the rest at 8.5% compounded monthly, what will be your monthly payment if the mortgage is to be paid off in 15 years?

Annuity with Continuous Compounding

62. What is the present equivalent of the following continuous fund flow situation?

- a) P1,000,000 per year for 4 years at 10% compounded continuously
- b) P500 per quarter for 6.75 years at 20% compounded continuously

63. A series of equal quarterly payments of \$5.0(1 for 12 years is equivalent to what present amount at an interest rate of 9% compounds

- (a) Quarterly?
- (b) Monthly?
- (c) Continuously?

64. What is the future worth of an equal-payment series of \$5,000 per year for five years if the interest rate is 8% compounded continuously?

65. Suppose that \$1,000 is placed in the bank account at the end of each quarter over the 20 years. What is the account's future worth at the end of 20 years when interest rate is 8 compounded.

- (a) Quarterly?
- (b) Monthly?
- (c) Continuously?

66. If the interest rate is 7.5% compounded continuously, what is the required quarterly payment to repay a loan of \$10,000 in four years?

67.What is the required payment to repay a loan of \$20,000 in five years if the interest rate is 8% compounded continuously?

68.A series of equal quarterly payments of \$1,000 extends over a period of five years. What is the present worth of this quarterly-payment series at 9.75% interest compounded continuously

69.A series of equal quarterly payments of \$2,000 for 15 years is equivalent to what future lump sum amount at the end of 10 years at an interest rate of 8% compounded continuously?

Chapter 3

Depreciation

Definitions

Depreciation is the decrease in the value of physical property with the passage of time.

More specifically, depreciation is an accounting concept that establishes an annual deduction against before tax income such that the effect of time and use on the asset's value can be reflected in a firm financial statement.

Value, in a commercial sense, is the present worth of all future profits that are to be received through ownership of a particular property.

The market value of a property is the amount which a willing buyer will pay to a willing seller for the property where each has equal advantage and is under no compulsion to buy or sell.

The utility or use value of a property is what the property is worth to the owner as an operating unit.

Fair value is the value which is usually determined by a disinterested third party in order to establish a price that is fair to both seller and buyer.

Book value, sometimes called depreciated book value, is the worth of a property as shown on the accounting records of an enterprise.

Salvage, or resale, value is the price that can be obtained from the sale of the property after it has been used.

Scrap value is the amount the property would sell for, if disposed off as junk.

Purposes of Depreciation

1. To provide for the recovery of capital which has been invested in physical property.
2. To enable the cost of depreciation to be changed to the cost of producing products or services that results from the use of the property.

Types of Depreciation

1. Normal depreciation
 - a.)Physical
 - b.)Functional
2. Depreciation due to changes in price levels
3. Depletion

Physical depreciation is due to the lessening of the physical ability of a property to produce results. Its common causes are wear and deterioration. Functional depreciation is due to the lessening of the demand for the function which the property was designed to render. Its common causes are inadequacy, changes in styles, population-center shifts, saturation of markets or more efficient machines are produced (obsolescence).

Depreciation due to changes in price levels is most impossible to predict and therefore is not considered in economy studies.

Depletion refers to the decrease in the value of a property due to the gradual extraction of its contents.

Purpose of Depreciation

1. To enable the cost of depreciation to be included as a cost in the production of goods and services.
2. Annual cost of depreciation are being put in a fund called depreciation reserve for replacement of the property
3. To recover the capital invested in the property.
4. Provide as an additional capital termed as *depreciation reserve*

Properties Depreciable Assets:

1. It must have a determinable life and the life must be greater than 1 year.
2. It must be something used in business or held to produce income.
3. It must be something that gets used up, wears out decays, become obsolete, or loses its value due to natural causes.
4. It must not be an inventory or stock in trade or investment

Definition of terms:

1. **Initial Investment (cost)** – the cost of acquiring an asset (purchase price plus any asset taxes), including transportation expenses and other normal costs of making the asset serviceable for its intended use.
2. **Book Value** – worth of property or an asset as shown on the accounting records of the company. It is the original cost of the property less all allowable depreciation deductions.
3. **Market Value** – the amount that will be paid by a willing buyer to a willing seller for a property where each has equal advantage and is under no compulsion to buy or sell.
4. **Useful Life** – the expected (estimated) period that a property will be used in trade or business to produce income. It is not how long the property will last, but how long the owner expects to productively use it.

Physical and Economical Life

Physical life of a property is the length of time during which it is capable of performing the function for which it was designed and manufactured.

Economic life is the length of time during which the property may be operated at a profit.

Requirements of a Depreciation Method

1. It should be simple.
2. It should be recover capital.
3. The book value will be reasonably close to the market value at any time.
4. The method should be accepted by the Bureau of Internal Revenue.

Depreciation Methods

We shall use the following symbols for different depreciation methods.

L	=	useful life of the property in years
C _o	=	the original cost
C _L	=	the value at the end of the life, the scrap value (including gain or loss due to removal)
d	=	the annual cost of depreciation
C _n	=	the book value at the end of n years
D _n	=	depreciation up to age n years

1. The Straight Line Method

The method assumes that the loss in value is directly proportional to the age of the property.

is the simplest depreciation method. It assumes that a constant amount is depreciated each year over the depreciable (useful) life of the asset

$$d = \frac{C_o - C_L}{L}$$
$$D = \frac{n(C_o - C_L)}{L}$$
$$C_n = C_o - D_n$$

Example

An electronic balance costs P90,000 and has an estimated salvage value of P8,000 at the end of 10 years life time. What would be the book value after three years, using the straight line method in solving for the depreciation?

Solution:

$$C_0 = P90,000 \quad C_L = P8,000 \quad L = 10 \quad n = 3$$

$$d = \frac{C_0 - C_L}{L} = \frac{P90,000 - P8,000}{10} = P8,200$$

$$D_3 = (n)(d) = (3)(8,200) = 24,600$$

$$C_3 = C_0 - D_3 = P90,000 - P24,600 = P65,400$$

Problem

An elective saw for cutting small pieces of lumber in a furniture manufacturing plant has an initial investment of ₱ 4,000 and a 10 year depreciable life. The estimated SV of the saw is zero at the end of 10 years. Determine the annual depreciation amounts using SL method. Tabulate the annual depreciation amounts and the book value of the saw at the end of each year.

Given:

$$C_0 = ₱ 4,000$$

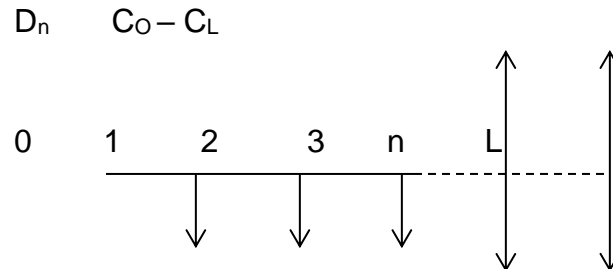
$$SV = 0$$

$$L = 10 \text{ years}$$

- Required:
- annual depreciation charge, d
 - Book value, C_n , for year 1-10

The Sinking Fund Formula

This method assumes that a sinking fund is established in which funds will accumulate for replacement. The total depreciation that has taken place up to any given time is assumed to be equal to the accumulated amount in the sinking fund at that time.



$$d = \frac{C_0 - C_L}{F[A, i\%, L]} \text{ or } d = \frac{C_0 - C_L}{\frac{(1-i)^L - 1}{i}}$$

$$D_n = d(F/A, i\%, n) \text{ or } D_n = d \frac{(1-i)^n - 1}{i}$$

$$C_n = C_0 -$$

Example

A broadcasting corporation purchased an equipment for P53,000 and paid P1,500 for freight and delivery charges to the job site. The equipment has a normal life of 10 years with a trade-in value of P5,000 against the purchase of a new equipment at the end of the life.

- Determine the annual depreciation cost by the straight line method.
- Determine the annual depreciation cost by the sinking fund method. Assume interest at $6\frac{1}{2}\%$ compounded annually.

Solution:

$$C_0 = P53,000 + P1,500 = P54,500$$

$$C_L = P5,000$$

$$\text{a) } d = \frac{C_0 - C_L}{L} = \frac{P54,500 - P5,000}{10} = P4,950$$

$$\text{b) } d = \frac{C_0 - C_L}{F[A, i\%, L]} = \frac{P54,500 - P5,000}{\frac{(1+0.065)^{10} - 1}{0.065}} = \frac{P49,500}{13.49} = P3,668.18$$

Example

A firm bought an equipment for P56,000. Other expenses including installation amounted to P4,000. The equipment is expected to have a life of 16 years with a salvage value of 10% of the original cost. Determine the book

value at the end of 12 years by (a) the straight line method and (b) sinking fund method at 12% interest.

Solution:

$$C_0 = P65,000 + P4,000 = P60,000$$

$$C_L = P60,000(0.10) = P6,000$$

$$L = 16 \quad n=12 \quad i=12\%$$

a) Straight line method

$$d = \frac{C_0 - C_L}{L} = \frac{P60,000 - P6,000}{16} = P3,375$$

$$D_{12} = (d)(n) = P3,375(12) = P40,500$$

$$C_{12} = C_0 - D_{12} = P60,000 - P40,500 = P19,500$$

b) Sinking fund method

$$d = \frac{C_0 - C_L}{F[A, i\%, L]} = \frac{P60,000 - P6,000}{\frac{(1+0.12)^{16} - 1}{0.12}} = \frac{P54,000}{42.7533} = P1,263.06$$

$$D_{12} = d \frac{(1+0.12)^{12} - 1}{0.12} = P1,263.06(14.1331) = P30,481.60$$

$$C_{12} = C_0 - D_{12} = P60,000 - P30,481.60 = P29,518.40$$

Problem

Solve the above problem using sinking fund method at $i = 5\%$

Given:

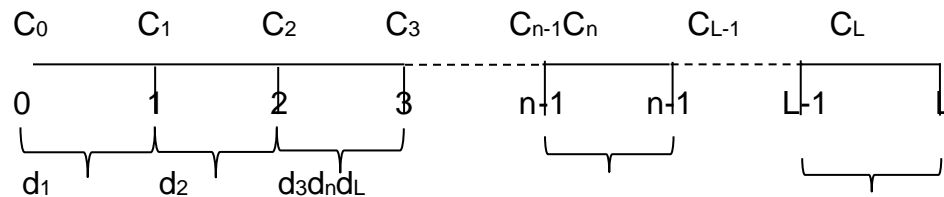
$$C_0 = P4,000$$

$$SV = 0$$

$$L = 10 \text{ years}$$

Declining Balance Method

In this method, sometimes called the constant percentage method or the Matheson Formula, it is assumed that the annual cost of depreciation is a fixed percentage of the salvage value at the beginning of the year. The ratio of the depreciation in any year to the book value at the beginning of that year is constant throughout the life of the property and is designed by k , the rate of depreciation



d_n = depreciation during the n^{th} year

Year	Book Value at beginning of year	Depreciation during the year	Book Value at the end of the year
1	C_0	$d_1 = kC_0$	$C_1 = C_0 - d_1 = C_0(1-k)$
2	$C_0(1-k)$	$d_2 = kC_1$	$C_2 = C_1 - d_2 = C_0(1-k)^2$
3	$C_0(1-k)^2$	$d_3 = kC_2$	$C_3 = C_2 - d_3 = C_0(1-k)^3$
n	$C_0(1-k)^{n-1}$	$d_n = kC_{n-1}$	$C_n = C_{n-1} - d_n = C_0(1-k)^n$
L	$C_0(1-k)^{L-1}$	$d_{L-1} = kC_L$	$C_L = C_{L-1} - d_{L-1} = C_0(1-k)^L$

$$d_n = C_0(1-k)^{n-1}k \quad (3 - 7)$$

$$C_n = C_0(1-k)^n = C_0 \left[\frac{C_L}{C_0} \right]^{\frac{n}{L}} \quad (3 - 8)$$

$$C_L = C_0 (1-k)^L \quad (3 - 9)$$

$$k = 1 - \sqrt[n]{\frac{C_n}{C_0}} = 1 - \sqrt[L]{\frac{C_L}{C_0}} \quad (3 - 10)$$

This method does not apply, if the salvage value is zero, because k will be equal to one and d_1 will be equal to C_0 .

Example

A certain type of machine loses 10% of its value each year. The machine costs P2,000 originally. Make out a schedule showing the yearly depreciation, the total depreciation and the book value at the end of each year for 5 years.

Solution:

Year	Book value at beginning of year	Depreciation during the year 10%	Total depreciation at end of year	Book value at end of year
1	P2,000.00	P200.00	P200.00	P1,800.00
2	P1,800.00	P180.00	P380.00	P1,620.00
3	P1,620.00	P162.00	P542.00	P1,458.00
4	P1,458.00	P145.80	P687.80	P1,312.20
5	P1,312.20	P131.22	P819.02	P1,180.98

Double Declining Balance(DDB) Method

This method is very similar to the declining balance method except that the rate of depreciation k is replaced by $2/L$.

$$d_n = C_0 \left(1 - \frac{2}{L}\right)^{n-1} \frac{2}{L} \quad (3 - 11)$$

$$C_n = C_0 \left(1 - \frac{2}{L}\right)^n \quad (3 - 12)$$

$$C_L = C_0 \left(1 - \frac{2}{L}\right)^L \quad (3 - 13)$$

When the DDB method is used, the salvage value should not be subtracted from the first cost when calculating the depreciation charge.

Example

Determine the rate of depreciation, the total rate of depreciation up to the end of the n th year and the book value at the end of 8 years for an asset that costs P15,000 new and has an estimated scrap value of P2,000 at the end of 10 years by (a) the declining balance method and (b) the double declining balance method.

Solution:

$$C_0 = \text{P15,000} \quad C_L = \text{P2,000} \quad L = 10 \quad n = 8$$

(a) Declining balance method

$$k = 1 - \sqrt[L]{\frac{C_L}{C_0}} = 1 - \sqrt[10]{\frac{P2,000}{P15,000}} = 0.1825 \text{ or } 18.25\%$$

$$C_8 = C_0 (1-k)^8 = P15,000(1 - .0825)^8 = P2,992$$

$$D_8 = C_0 - C_8 = P15,000 - P2,992 = P12,008$$

(b) Double declining balance method

$$\text{Rate of depreciation} = \frac{2}{L} = \frac{2}{20} = 0.20 \text{ or } 20\%$$

$$C_8 = C_0 \left(1 - \frac{2}{L}\right)^8 = P15,000 \left(1 - \frac{2}{10}\right)^8 = P2,517$$

$$D_8 = C_0 - C_8 = P15,000 - P2,517 = P12,483$$

Example

A plant bought a calciner for P220,000 and used it for 10 years, the life span of the equipment. What is the book value of the calciner after 5 years of use? Assume a scrap value of P20,000 for straight line method; P22,000 for textbook declining balance method and P20,000 for the double declining balance method.

Solution:

$$C_0 = P220,000 \quad C_L = P20,000 \quad L = 10 \quad n = 5$$

(a) Straight line method

$$D_5 = \frac{n(C_0 - C_L)}{L} = \frac{5(P220,000 - P20,000)}{10} = P100,000$$

$$C_5 = C_0 - D_5 = P220,000 - P100,000 = P120,000$$

(b) Declining balance method $C_L = P22,000$

$$C_5 = C_0 \left[\frac{C_L}{C_0}\right]^{\frac{n}{L}} = P220,000 \left[\frac{P22,000}{P220,000}\right]^{\frac{5}{10}} = P69,570$$

(c) Double declining balance method $C_L = P22,000$

$$C_5 = C_0 \left(1 - \frac{2}{L}\right)^5 = P220,000 \left(1 - \frac{2}{10}\right)^5 = P72,090$$

The Sum-of-the-Years'-Digits (SYD) Method

Let d_n = depreciation charge during the n^{th} year

d_n = (depreciation factor)(total depreciation)

$$d_n = \frac{\text{reverse digit}}{\text{sum of digits}} (C_0 - C_L)(3 - 14)$$

For example, for a property whose life is 5 years.

Year	Year in Reverse Order	Depreciation Factor	Depreciation during the year
1	5	5/15	(5/15)(C ₀ - C _L)
2	4	4/15	(4/15)(C ₀ - C _L)
3	3	3/15	(3/15)(C ₀ - C _L)
4	2	2/15	(2/15)(C ₀ - C _L)
5	1	1/15	(1/15)(C ₀ - C _L)

Σ of digits=15

Example

A structure costs P12,000 new. It is estimated to have a life of 5 years with a salvage value at the end of life of P1,000. Determine the book value at the end of each year of life.

Solution:

$$C_0 - C_L = P12,000 - P1,000 = P11,000$$

Year	Year in reverse order	Depreciation during the year	Depreciation during the year
1	5	(5/15)(P11,000)= P3,667	P8,333
2	4	(4/15)(P11,000)= P2,933	P5,400
3	3	(3/15)(P11,000)= P2,200	P3,200
4	2	(2/15)(P11,000)= P1,467	P1,733
5	1	(1/15)(P11,000)= P733	P1,000

Σ of digits = 15

Problem

A consortium of international telecommunication companies contracted for the purchase and installation of a fiber optic cable linking two major cities at a total cost of US \$960million. This amount includes freight and installation charges estimated at 10% of the above contact price. If the cable shall be depreciated over a period of 15years with zero salvage value:

- (A) Given the sinking fund deposit factor of 0.0430 at 6% interest where n= 15, what is the annual depreciation charge?
- (B) What is the depreciation charge during the 8th year using the-sum-of-the-years-digits method?

The Service-Output Method

This method assumes that the total depreciation that has taken place is directly proportional to the quantity of output of the property up to that time. This method has the advantage of making the unit cost of depreciation constant and giving low depreciation expense during periods of low production.

Let T = total units of output up to the end of life

Q_n = total number of units of product during the n^{th} year

$$\text{Depreciation per unit of output} = \frac{C_0 - C_L}{T}$$

$$d_n = \frac{C_0 - C_L}{T} (Q_n) \quad (3 - 15)$$

Example

A television Company purchased machinery for P100,000 on July 1, 1979. It is estimated that it will have a useful life of 10 years; scrap value of P4,000, production of 400,000 hours and working hours of 120,000.

The company uses the machinery for 14,000 hours in 1979 and 18,000 hours in 1980. The machinery produces 36,000 units in 1979 and 44,000 units in 1980. Compute the depreciation for 1980 using each method given below:

- (1) Straight line
- (2) Working hours
- (3) Output method

Solution:

$$C_0 = P100,000 \quad C_L = P4,000 \quad L = 10 \text{ years}$$

$$T = 400,000 \text{ units} \quad H = 120,000 \text{ hours}$$

$$(1) d_{80} = \frac{C_0 - C_L}{L} = \frac{P100,000 - P4,000}{10} = P9,600$$

$$(2) d_{80} = \frac{C_0 - C_L}{H} (H_{80}) = \frac{(P100,000 - P4,000)}{120,000} (18,000) = P14,400$$

$$(3) d_{80} = \frac{C_0 - C_L}{T} (Q_{80}) = \frac{(P100,000 - P4,000)}{400,000} (44,000) = P10,560$$

What Depreciation Method Should Be Used?

It is worth mentioning what the National International Revenue Code says about depreciation specifically Section 29(f).

- (1) General rule. "There shall be allowed as a depreciation deduction a reasonable allowance for the exhaustion, wear and tear (including reasonable allowance for obsolescence) of property used in the trade or business.
- (2) Use of certain methods and rates. The term "reasonable allowance" as used in the preceding paragraph shall include (but not limited to) an allowance computed in accordance with regulations prescribed by the Secretary of Finance, under any of the following methods:
 - (A) The straight line method.
 - (B) Declining balance method, using a rate not exceeding twice the rate which would have been used had the annual allowance been computed under the method described in the paragraph (f)(1).
 - (C) The sum-of-the-years-digits method, and
 - (D) Any other method which may be prescribed by the Secretary of Finance upon recommendation of the Commissioner of Internal Revenue.

Notes: Taxpayer may enter into an "agreement in writing specifically dealing with the useful life and rate of depreciation of any property, the rate so agreed upon shall be binding."

Example

A machine costs P7,000, last 8 years and has a salvage value at the end of life of P350. Determine the depreciation charge during the 4th year and the book value at the end of 4 years by the (a) straight line method, (b) declining balance method, (c) SYD method, and (d) sinking fund method with interest at 12%.

Solution:

$$C_0 = P7,000$$

$$C_L = P350$$

$$L = 8 \quad n = 4$$

(a) Straight line method

$$d = \frac{C_0 - C_L}{L} = \frac{P7,000 - P350}{8} = P831$$

$$C_4 = C_0 - D_4 = P7,000 - P831(4) = P3,676$$

(b) Declining balance method

$$k = 1 - \sqrt[L]{\frac{C_L}{C_0}} = 1 - \sqrt[8]{\frac{P350}{P7,000}} = 0.3123$$

$$d_4 = C_0(1 - k)^{4-1}k = P7,000(1 - 0.3123)^3(0.3123) = P711$$

$$C_4 = C_0(1 - k)^4 = P7,000(1 - 0.3123)^4 = P1,566$$

(c) SYD method\

1	8	<i>Sum of digits</i> = $\frac{8(8-1)}{2} = P36$
2	7	$d_4 = (5/36)(P7,000 - P350) = P924$
3	6	$D_4 = (26/36)(P7,000 - P350) = P4,803$
4	<u>5</u>	
5	26	
6		$C_4 = C_0 - D_4 = P7,000 - P4,803 = P2,197$
7		
8	<u>8</u>	
	36	

(d) Sinking fund method

$$d = \frac{C_0 - C_L}{\frac{(1 - .12)^8 - 1}{.12}} = \frac{P7,000 - P350}{12.2997} = P541$$

$$D_4 = d \frac{(1 - .12)^4 - 1}{.12} = P541(4.7793) = P2,586$$

$$C_4 = C_0 - D_4 = P7,000 - P2,586 = P4,414$$

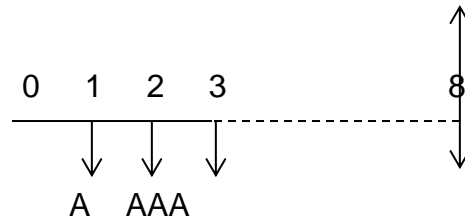
Example

A machine costs P20,000 and has a salvage value of P2,000 after a useful life of 8 years. Money is worth 12%. If average inflation is 8% per year during this period, what is the annual cost of depreciation to replace the machine after 8 years? What is the annual cost of depreciation if inflation is not considered?

Solution:

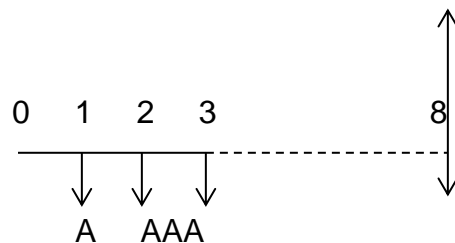
With inflation:

$$\text{Future cost} = PC(1 + i)^n = P20,000(1 + 0.08)^8 = P37,018$$



$$A = \frac{F}{\frac{(1 - .12)^8}{.12}} = \frac{F}{12.2997}$$

P18,000



$$= \frac{P18,000}{\frac{(1 - .12)^8}{.12}} = \frac{P18,000}{12.2997} = P1,463$$

Problem:

An asset for drilling was purchased and placed in service by a petroleum production company. Its initial investment is ₱ 60,000 and it has an estimated SV of ₱12,000 at the end of an estimated useful life of 14 years. Compute the depreciation amount in the third year and the BV at the end of the 5th year of life by each of these methods:

- a. Straight Line Method
- b. Sinking Fund Method at 5 % per year
- c. Declining Balance Method
- d. Double Declining Balance Method
- e. Sum of the Year's Digit Method

Solution

Problem:

A bowling alley cost ₱ 500,000 and has a useful life of 10 years. Its estimated SV at the end of year 10 is ₱20,000.

- a. Determine the depreciation for years 1-10 using each method discussed.
 - **Straight Line Method Solution:**
- b. Compute the present worth of the depreciation for each of the methods using 10% per year.
- c. If a large present worth in part b is desirable, what do you conclude regarding which method is preferred?

Problem:

The large profitable corporation has purchased a jet plane for use by its executives. The cost of the plane is ₱ 76,000,000. It has a useful life of 5 years. The estimated resale value at the end of five years is ₱6,000,000. Using sum of the years digit method of depreciation, what is the book value of the jet plane at end of 3 years?

Problem:

A certain equipment costs ₱7,000 has an economic life of n years and a salvage value of ₱350 at the end of n years. If the book value at the end of 4 years is equal to ₱2,197.22, compute the economic life of the equipment using sum of the years digit method.

Valuation

Valuation or appraisal is the process of determining the value of certain property for specific reasons. The person engaged in the task of valuation is called an appraiser.

Intangible Values

In the determination of the value of industrial property or equipment, four intangible items are often encountered.

Goodwill is that element of value which a business has earned through the favorable consideration and patronage of its customers arising from its well-known and well-conducted policies and operation.

Franchise is an intangible item of value arising from the exclusive right of a company to provide a specific product or service in a stated region of the company.

Going value is an intangible value which an actually operating concern has due to its operation.

Organization cost is the amount of money spent in organizing a business and arranging for its financing and building.

DEPLETION

Depletion is the decrease in the value of a property due to gradual extraction of its contents. The term is commonly used in connection with mining properties, oils and gas wells, timberlands and so on. The purpose of determining the depletion per year is to recover the capital invested in the property. Owners of such properties received two types of income:

- a.) the profit that has been earned
- b.) a portion of the owners capital that is being returned, marked as depletion

These are two ways to compute depletion allowances:

1. The Cost Method

In this method, depletion per units is determined.

$$\text{Depletion per unit} = \frac{\text{Cost Basis}}{\text{estimated no of units to be mined}}$$

The depletion allowance for a given year is calculated as:

$$D = \text{Depletion per unit} \times \text{No of units sold}$$

2. Percentage Method

Depletion allowances on mines and other natural deposits, maybe computed as percentage of gross income for that year, provided that the amount charged for depletion does not exceed 50% of the net income (100% of oils and gas property) before deduction of the depletion allowance.

Some example of depletion allowance	Max Percentage
Sulfur and uranium, domestically mined	
Lead, zinc, nickel and asbestos	22%
Gold, silver, copper, iron ore	15%
Oils and gas wells	15%
Coal, lignite and sodium chloride	10%
Clay, gravel, sand and stone	5%

Therefore,

$$D = \% \text{ gross income}$$

Or
$$D = 50\% \text{ net income}$$

Where: Gross Income = Price/unit x no. of units sold

$$\text{Net Income} = \text{Gross Income} - \text{expenses}$$

Problems

1. A machine shop purchased 10 years ago a milling machine for P60,000. A straight-line depreciation reserve had been provided based on a 20-year life of the machine. The owner of the machine shop desires to replace the old milling machine with a modern unit having many advantages costing P100,000. It can sell the old unit for P20, 000. How much new capital will be required for the purchase?

2. A tax and duty free importation of a 30-horsepower sand mill for paint manufacturing costs P360,000, CIF Manila. Bank charges, arrester and brokerage cost P5,000. Foundation and installation costs were P25,000. Other incidental expenses amount to P20,000. Salvage value of the mill is estimated to be P60,000 after 20 years. Find the appraisal value of the mill using straight-line depreciation at the end of

- a.) 10 years,
- b.) 15 years

3. On January 1, 1978, the purchasing engineer of a Cement Co. purchased a new machine at a cost of 140,000. Depreciation has been computed by the straight-line method based on an estimated useful life of five years and residual scrap value of 12,800. On January 2, 1981, extraordinary repairs (which were almost equivalent to a rebuilding of machinery) were performed at a cost of 30,400. Because of the thorough going nature of these repairs, the normal life of the machinery was extended materially. The revised estimate of useful life was four years from January 1, 1981.

Determine the annual provision for depreciation for the years 1978 to 1980 and the adjusted provision for depreciation on December 31, 1981. Assume payment in cash for the machine and extraordinary repairs.

4. Power to a remote transmitting station is provided by a Diesel-generator unit. The original cost of the unit is P65,000. It costs P2,000 to ship the unit to the job site. An additional cost of P3,000 was incurred for installation.

(a) Determine the annual depreciation cost by the sinking fund method, if the unit has an expected life of 10 years. The salvage value of the unit at the end of its life was estimated at P5000.

(b) Determine the annual depreciation cost by the sinking fund method. Assume that the annual charge for depreciation was deposited in a fund drawing compound interest at the rate of 5%.

5. An industrial plant bought a generator set for 90,000. Other expenses including installation amounted to 10,000. The generator set is to have a life of 17 years with a salvage value at the end of life of 5,000. Determine the depreciation charge during the 13th year and the book value at the end of 13 years by the (a) declining balance method, (b) double declining balance method, (c) sinking fund method at 12% and (d) SYD method.

- 6.** A telephone company purchased a microwave radio equipment for P6,000,000.00. Freight and installation charges amounted to 3% of the purchased price. If the equipment shall be depreciated over a period of 8 years with a salvage value of 5%, determine the following:
- a.) Annual depreciation charge using the straight line method.
 - b.) Depreciation charge during the 5th year using the sum-of-the year's digits method.

Problems

(Depletion)

1. A gold mine that is expected to produce 30,000 ounces of gold is purchased for P2,400,000. The gold can be sold for P450 per ounce; however, it cost P265

per ounce for mining and processing costs. If 3,500 ounces are produced this year, what will be the depletion allowance for

- (a) Unit depletion
- (b) Percentage depletion

2. A marble quarry is estimated to contain 900,000 tons of stone, and a company just purchased this quarry for P1,800,000. If 100,000 tons of marble can be sold each year and the average selling price per ton is P8.60, calculate the first year's depletion allowance for

- (a) Cost depletion method
- (b) Percentage depletion at 5% per year. The company's net income before deduction of a depletion allowance is P350,000.

Chapter 4

Capital Financing

Capital Financing

Types of business organization:

1. Individual Ownership or Sole Proprietorship
2. Partnership
3. Corporation

Sole Proprietorship

The sole proprietorship or individual ownership is the simplest form of business organization, wherein the business is owned entirely by one person who is responsible for the operation. All profits that are obtained from the business are his alone, but he must also bear all losses should they be incurred.

Equity and Borrowed Capital

Equity capital or ownership funds those supplied and used by the owners of an enterprise in the expectation that a profit will be earned.

Borrowed funds or capital are those supplied by others on which a fixed rate of interest must be paid and the debt must be repaid at a specified time.

Types of Business Organization

- A. Individual Ownership. The individual ownership or sole proprietorship is the simplest form of business organization, wherein a person uses his or her own capital to establish a business and is the sole owner.

Advantages of the individual ownership.

1. It is easy to organize
2. The owner has full control of the enterprise.
3. The owner is entitled to whatever benefits and profits that accrue from the business.
4. It is easy to dissolve.

Disadvantages of the Individual Ownership

1. The amount of equity capital which can be accumulated limited.
2. The organization ceases upon the death of the owner.
3. It is difficult to obtain borrowed capital, owing to the uncertainty of the life of the organization.
4. The liability of the owner of debt unlimited.

The Partnership. A partnership is an association of two or more persons for the purpose of engaging in a business for profit.

Advantages of the partnership

1. More capital may be obtained by the partners pooling their resources together.
2. It is bound by few legal requirements as to its account, procedures, tax form and other items of operation.
3. Dissolution of the partnership may take place at any time by mere agreement of the partners.
4. It provides an easy method whereby two or more persons of differing talents may enter into business, each carrying those burdens that he can best handle.

Disadvantages of the Partnership

1. The amount of capital that can be accumulated is definitely limited.
2. The life of the partnership is determined by the life of the individual partners. When any partner dies, the partnership automatically ends.
3. There may be serious disagreement among the individual partners.
4. Each partner is liable for the debts of the partnership.

The Corporation. A corporation is a distinct legal entity, separate from the individuals who own it, and which can engage in almost any type of business transaction in which a real person could occupy himself or herself.

Advantages of the Corporation

1. It enjoys perpetual life without regard to any change in the person of its owner, the stockholders.
2. The stockholders of the corporation are not liable for the debts of the corporation.
3. It is relatively easier to obtain large amounts of money for expansion, due to its perpetual life.
4. The ownership in the corporation is readily transferred.
5. Authority is easily delegated by the hiring of managers.

Disadvantages of the Corporation

1. The activation of corporation are limited to those stated in its charter.
2. It is relatively complicated in formation and administration.
3. There is a greater degree of governmental control as compared to other types of business organizations.

Capitalization of A Corporation

The capital of a corporation is acquired through the sales of stock. There are two principal types of capital stock: common stock and preferred stock.

Common Stock

Common stock represents ordinary ownership without special guarantees of return. Common stockholders have certain legal rights, among which are the following:

1. Vote at stockholders' meetings.
2. Elect directors and delegates to them power to conduct the affair of the business.
3. Sell or dissolve the corporation.
4. Make and amend by the laws of the corporation.
5. Subject to government approval, amend, or change the character or capital structure.
6. Participate in the profits.
7. Inspect the books of the corporation.

Preferred Stock

Preferred stockholders are guaranteed a definite dividend on their stocks. In case the corporation is dissolved, the assets must be used to satisfy the claims of the preferred stockholders before those of the holders of the common stock. Preferred stockholders usually have the right to vote in meetings, but not always.

Financing With Bonds

A bond is a certificate of indebtedness of a corporation usually for a period not less than ten years and guaranteed by a mortgage on certain assets of the corporation or its subsidiaries. Bonds are issued when there is need for more capital such as for expansion of the plant or the services rendered by the corporation.

The face or par value of a bond is the amount stated on the bond. When the face value has been repaid, the bond is said to have been retired or redeemed. The bond rate is the interest rate quoted on the bond.

Classification of Bonds

1. Registered bonds. The name of the owner of this bond is recorded on the record books of the corporation and interest payments are sent to the owner periodically without any action on his part.
2. Coupon bonds. Coupon bonds have coupons attached to the bond for each interest payment that will be collected during the life of the bond. The owner of the bond can collect the interest due by surrendering the coupon to the offices of the corporation or at specified banks.

Methods of Bond Retirement

1. The corporation may issue another set of bonds equal to the amount of bonds due for redemption.

2. The corporation may set upon sinking fund into which periodic deposits of equal amount are made. The accumulated amount in the sinking fund is equal to the amount needed to retire the bonds at the time they are due.

A = periodic deposit to the sinking fund

F = accumulated amount, the amount needed to retire the fund

i = rate of interest in the sinking fund

r = bond rate per period

I = interest on the bonds per period

A+I = total periodic expense

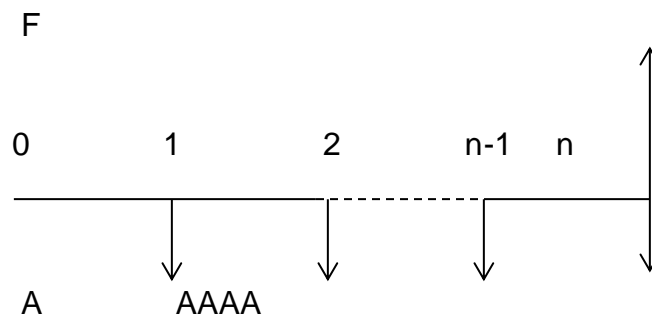


Figure 4-1. Bond retirement by sinking fund

$$A = \frac{F}{\frac{(1-i)^n - 1}{i}} = F \frac{i}{(1-i)^n - 1} \quad (4-1)$$

$$I = Fr \quad (4-2)$$

Example

A bond issue of P200,000 in 10-year bonds, in P1,000 units, paying 16% nominal interest in semiannual payments must be retired by use of sinking fund that earns 12% compounded semiannually. What is the total semiannual expense?

Solution:

$$F = P200,000 \quad r = \frac{16\%}{2} = 8\% \quad i = \frac{12\%}{2} = 6\% \quad n = (10)(2) = 20$$

$$A = \frac{F}{\frac{(1-i)^n - 1}{i}} = \frac{P200,000}{\frac{(1-0.06)^{20} - 1}{0.06}} = P5,437$$

$$I = Fr = (P200,000)(0.08) = P16,000$$

$$\text{Total semiannual expense} = P5,437 + P16,000 = P21,437$$

Bond value

The value of a bond is the present worth of all future amounts that are expected to be received through ownership of this bond.

Let F = face, or par, value

C = redemption or disposal price (often equal to F)

r = bond rate per period

n = number of periods before redemption

i = investment rate or yield per period

P = value of the bond n periods before redemption

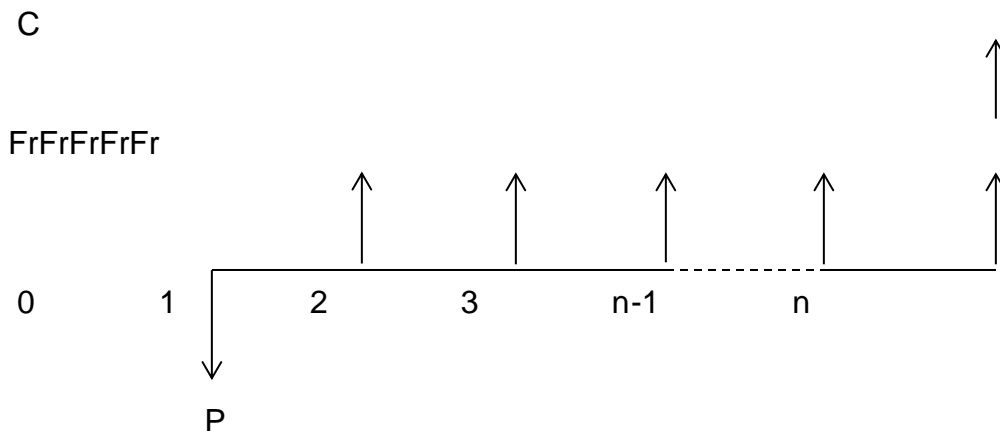


Figure 4-2. Cash flow for a bond investment

$$P = Fr \left(\frac{P}{F}, i\%, n \right) + C \left(\frac{P}{F}, i\%, n \right) \quad (4-3)$$

$$P = Fr \left[\frac{1 - (1+i)^{-n}}{i} \right] + C(1+i)^{-n} \quad (4-$$

4)

Example

A man wants to make 14% nominal interest compounded semiannually on a bond investment. How much should the man be willing to pay for a 12% P10,000-bond that will mature in 10 years and pays interest semiannually?

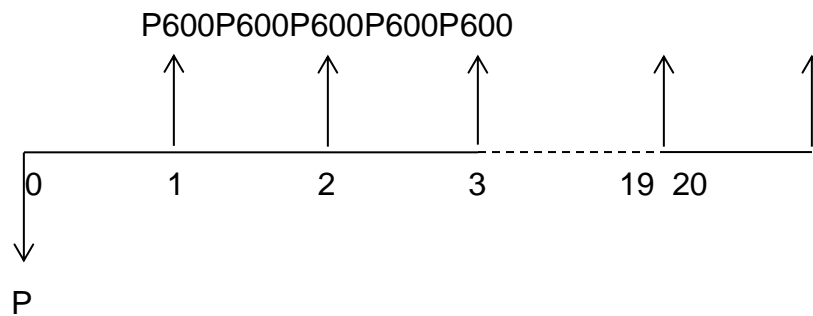
Solution:

$$F = P10,000 \quad r = \frac{12\%}{2} = 6\% \quad i = \frac{14\%}{2} = 7\%$$

$$I = Fr = (P10,000)(0.06) = P600$$

P10,000





$$P = P600 \left[\frac{1 - (1 + .07)^{-20}}{.07} \right] + P10,000(1 + .07)^{-20}$$

$$P = 600(10.5940) + P10,000(0.2584)$$

$$P = P8,950$$

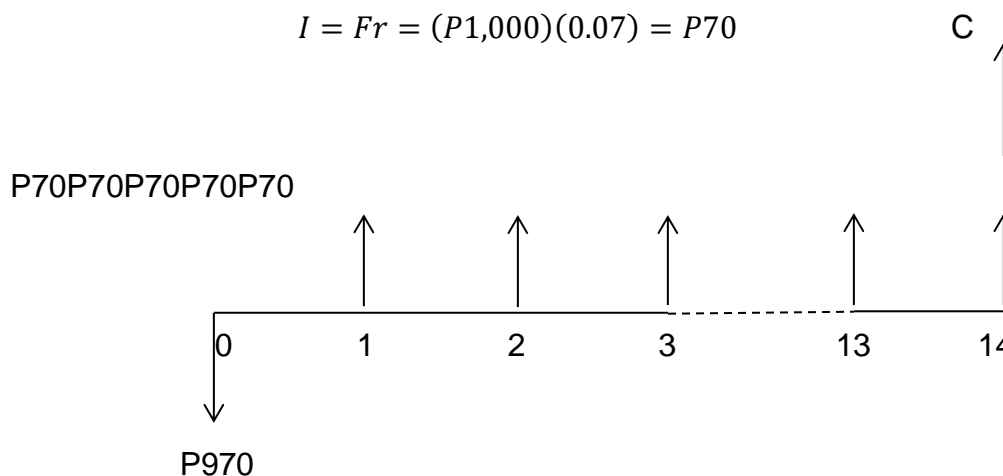
Example

Mr. Romualdo bought a bond having a face value of P1,000 for P970. The bond rate was 14% nominal and interest payments were made to him semiannually for a total of 7 years. At the end of the seventh year, he sold the bond to a friend at a price that resulted to a yield of 16% nominal on his investment. What was the selling price?

Solution:

$$F = P1,000i = \frac{16\%}{2} = 8\%r = \frac{14\%}{2} = 7\%$$

$$I = Fr = (P1,000)(0.07) = P70$$



$$P970 = P70 \left[\frac{1 - (1 + .08)^{-14}}{.08} \right] + C(1 + .08)^{-14}$$

$$P970 = P70(10.5940) + C(0.2584)$$
$$P = P1,153.90$$

Problem

A P1,000-bond which will mature in 10 years and with a bond rate of 8% payable annually is to be redeemed at par the end of this period . If it is sold at P1,030, determine the yield at this price.

Problem

A mine will have a net annual income of P400,000 annually for 15 years. Determine its value if the annual dividend rate is to be 12%, payable annually and the sinking fund is to accumulate 7% annually.

PROBLEMS**Capital Financing**

1. A Corporation sold an issue of 20-year bonds, having a total face value of 10,000,000 for 9,500,000. The bonds bear interest at 16%, payable semiannually. The company wishes to establish a sinking fund for retiring the bond issue and will make semiannual deposit that will earn 12%, compounded semiannually. Compute the annual cost for interest and redemption of these bonds.

2. A company has issued 10-year bonds, with face value of 1,000,000 in 1,000 units. Interest at 16% is paid quarterly. If an investor desires to earn 20% nominal interest on 100,000 worth of these bonds, what would the selling price have to be?

3. A 1,500-bond which will mature in 10 years and with a bond rate of 15% payable annually is to be redeemed at par at the end of this period. If it is sold now for 1,390, determine the yield at this price.

Chapter 5

Basic Methods for Making Economy Studies

BASIC METHODS OR PATTERNS FOR MAKING ECONOMY STUDIES

Engineering economy studies are made for the purpose of determining whether capital should be invested in a project or whether it should be used differently than it presently is being used. They should consider the return that a given project will or should produce.

The basic question is; whether a proposed capital investment and its associated expenditures can be recovered by revenue overtime in addition to return on the capital that is sufficiently attractive in view of the risk involved and the potential alternative uses.

THE RATE OF RETURN (ROR) METHOD

The rate of return on the capital invested is given by the formula,

$$\text{Rate of return} = \frac{\text{net annual profit}}{\text{capita invested}}$$

Minimum Attractive Rate of Return

The MARR, sometimes called the hurdle rate, should be chosen to maximize the economic well-being of an organization, subject to the following considerations:

1. The amount of money available for investment, and the source and cost of these funds (equity or borrowed capita).
2. The number of good projects available for investment and their purpose.
3. The amount of perceived risk associated with investment opportunities available to the firm and the estimated cost of administering projects over short planning horizons versus long planning horizons.
4. The type of organization (government, public utility, or competitive industry).

Methods in Investment of Capital

Let I = initial Investment
 R = annual revenue or income
 E = annual expenses or costs
 Sv = salvage value
 n = useful life

Rate of return is a measure of the effectiveness of an investment of capital. It is a financial efficiency. When this method is used, it is necessary to decide whether the computed rate of return is sufficient to justify the investment. The advantage of this method is that it is easily understood by management and investors. The applications of the rate of return method are controlled by the following conditions. A single investment of capital at the beginning of the first year of the project life and identical revenue and cost data for each year. The capital invested is the total amount of capital investment required to finance the project, whether equity or borrowed.

Example

An investment of P10,000 can be made in project that will produce a uniform annual revenue of P5310 for five years and then have a salvage value of P2000. Annual expenses will P3000 each year. The company is willing to accept any project that will earn 10% per year or more on all investment capital. Show whether this is desirable investment.

Given: I = investment 10,000
 R = revenue/income 5130
 n = 5 years
 Sv = 2000
 E = 3000/year
 MARR = 10%/year
 Required: Evaluation of the investment of capital

Solution:

To evaluate the problem by rate of return method, determine the total expenses including the annual cost of depreciation (sinking fund method):

$$d = (C_o - C_L) \left[\frac{i}{(1 + i)^n - 1} \right]$$

$$d = (P5000 - 2000) \left[\frac{0.1}{(1.1)^5 - 1} \right]$$

d = 1310

Determine the annual profit:

$$\text{Profit} = \text{Income} - \text{Expenses}$$

$$\text{Profit} = 5310 - 4310 = \text{P}1,000$$

$$\text{RoR} = \frac{1000}{10000} \times 100 = 10\%$$

Since the computed rate of return is equal to MARR, then the investment of capital is desirable.

THE ANNUAL WORTH (AW) METHOD

In this method, interest on the original investment (sometimes called minimum required profit) is included as a cost. If the excess of annual cash inflows over cash outflows is not less than zero the proposed investment is justified – is valid. This method is covered by

In this method, the minimum required profit (MRP) is included as a cost or expenses. This computed as:

$$MRP = \text{InitialInvest} \times MARR$$

Then excess is computed as:

$$\text{Excess} = \text{Income} - \text{Expenses}$$

If the computed excess is

Excess ≥ 0 , the investment of capital is justified

Excess ≤ 0 , the investment of capital is not justified

EXAMPLE

Solve the same problem using annual cost method

To evaluate the problem using annual cost method, determine the minimum required profit and include this in the total expenses:

$$\begin{aligned} \text{MRP} &= I \times MARR \\ &= 10000 (0.1) = 1000 \end{aligned}$$

$$\begin{aligned} \text{Total Expenses} &= E + \text{annual depreciation} + \text{MRP} \\ &= \text{P}3000 + \text{P}1310 + \text{P}1000 \\ &= \text{P}5310 \end{aligned}$$

Compute the excess as:

$$\begin{aligned} \text{Excess} &= \text{Income} - \text{Expenses} \\ &= \text{P}5310 - \text{P}5310 \\ &= 0 \end{aligned}$$

Since the computed excess is equal to zero, the investment of capital is justified

THE FUTURE WORTH (FW) METHOD

The future worth method for economy studied is exactly comparable to the present worth method except that all cash inflows and outflows are compounded forwards to a reference point in time called the future. If the future worth of the net cash flows is equal to, or greater than, zero, the project is justified economically.

THE PAYBACK (PAYOUT) PERIOD METHOD

The payback period is commonly defined as the length of time required to recover the first cost of an investment from the net cash flow produced by that investment for an interest rate of zero.

Payout period (years) = investment – salvage value over net annual cash flow

Example

An investment of P270, 000 can be made in a project that will produce a uniform annual revenue of P185, 400 for 5 years and then have a salvage value of 10% of the investment. Out-of-pocket costs for operation and maintenance will be P81, 000 per year. Taxes and insurance will be 4% of the first cost per year. The company expects capital to earn not less than 25% before income taxes. Is this a desirable investment? What is the payback period of the investment?

Solution

By the rate of return method

Annual revenue P185, 000

Annual cost:

$$\text{Depreciation} = \frac{P270,000 - P27,000}{FA, 25\%, 5} = \frac{P243,000}{8.2070} = P29, 609$$

Operations and maintenance =81, 000

Taxes and insurance = P270, 000(0.04) =10, 800

Total annual cost P121, 409

Net annual profit P63, 991

$$\text{Rate of return} = \frac{P63,991}{P270,000} \times 100 = 23.70\%$$

Since the rate of return is less than 25%, the investment is not justified.

By the annual worth method

Annual revenue P185, 000

Annual cost:

$$\text{Depreciation} = \frac{P270,000 - P27,000}{F A, 25\%, 5} = P29, 609$$

Operations and maintenance = 81, 000

Taxes and insurance = 10, 800

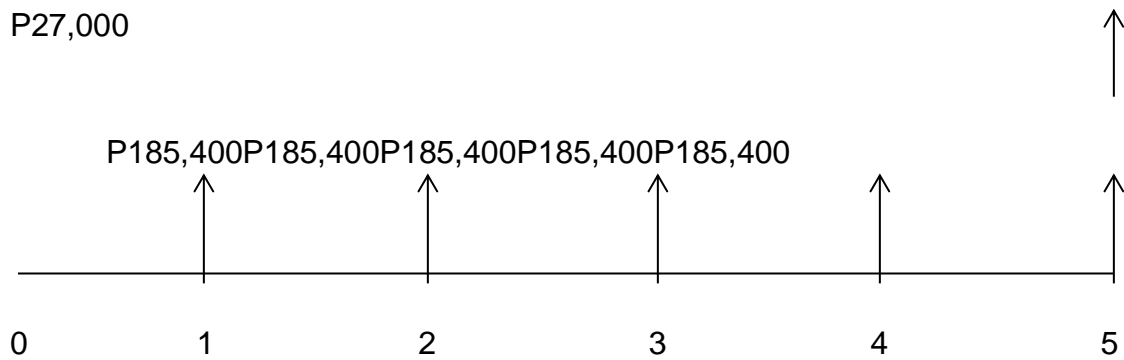
Interest on capital = 67, 500

Total annual cost P188, 909

Excess -P 3, 509

Since the excess of annual cash inflows over cash outflows is less than zero (-P3, 509), the investment is not justified.

By the present worth method

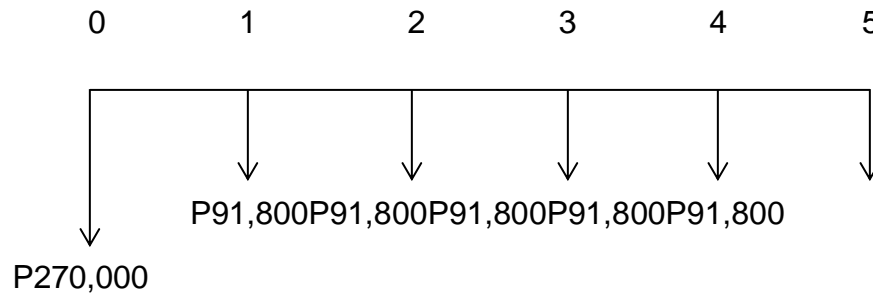


$$\text{PW of cash inflows} = P185, 400 (P/A, 25\%, 5) + P27, 000 (P/F, 25\%, 5)$$

$$= P185, 400 (2.6893) + P27, 000 (0.3277)$$

$$= P506, 370$$

$$\begin{aligned}\text{Annual cost (excluding depreciation)} &= P81,000 + P270,000 (0.04) \\ &= 91,800\end{aligned}$$



$$\text{PW of cash outflows} = P270,000 + P91,800(P/A\ 25\%,\ 5) = P516,880$$

Since the PW of the net cash flows is less than zero (-P10,510), the investment is not justified.

By the future worth method

Referring to the cash flow diagrams in the solution by the PW method.

$$\begin{aligned}\text{FW of cash inflows} &= P27,000 + P185,400 (F/A, 25\%, 5) \\ &= P27,000 + P185,400 (8.2070) \\ &= P1,548,580\end{aligned}$$

$$\begin{aligned}\text{FW of cash outflows} &= P91,800 (F/A, 25\%, 5) + P270,000 (F/P, 25\%, 5) \\ &= P91,800 (8.2070) + P270,000 (3.0518) \\ &= P1,577,390\end{aligned}$$

Since the FW of the net cash flows is less than zero (-P28,810), the investment is not justified.

By the payback period

$$\text{Total annual cost} = P81,000 + P270,000 (0.04) = P91,800$$

$$\text{Net annual cash flows} = P185,400 - P91,800 = P93,600$$

$$\text{Payback period} = \frac{\text{investment} - \text{salvage value}}{\text{net annual cash flows}} = \frac{P270,000 - P27,000}{P93,600}$$

= 2.6 years

In computing the total annual cost, depreciation was not included because the method does not consider the time value of money or interest. The use of payback period for making investments decisions should be avoided as it may produce misleading results.

Example

a businessman is considering building a 25-unit apartment in a place near a progressive commercial center. He felt that because of the location of the apartment it will be occupied 90% at all time. He desires a rate of return of 20%. Other pertinent data are the following:

Land investment	P 5, 000, 000
Building investment	P 7, 000, 000
Study period	20 years
Cost of land after 20 years	20, 000, 000
Cost of building after 20 years	2, 000, 000
Rent per unit per month	6, 000
Upkeep per unit per year	500
Property taxes	1%
Insurance	0.50%

Is this a good investment?

Solution

Annual income:

$$\text{Rental} = (P6, 000) (12) (25) (0.90) = P1 620,000$$

$$\text{Land} = \frac{P20,000,000 - P5,000,000}{F A, 20\%, 20} = \frac{P15,000,000}{186.688}$$
$$= 80, 350$$

Total annual income P1, 700, 350

Annual cost:

$$\text{Depreciation} = \frac{P7,000,000 - P2,000,000}{F A, 20\%, 20} = P 26, 780$$

$$\text{Upkeep} = P500 (25) = P12, 500$$

$$\text{Taxes} = P12, 000, 000 (0.01) = P120, 000$$

$$\text{Insurance} = P7, 000, 000 (0.005) = P35, 000$$

$$\text{Total annual cost} = P194, 280$$

$$\text{Net annual profit} = P 1, 506, 070$$

$$\text{Rate of return} = \frac{P 1,506,070}{P 12,000,000} \times 100 = 12.55\% < 20\%$$

The businessman should not invest.

Another solution

$$\text{Investment} = P 5, 000, 000 + P7, 000, 000 = P12, 000, 000$$

$$\begin{aligned} \text{Amount of investment after 20 years} &= P20, 000, 000 + 2, 000, 000 \\ &= P22, 000, 000 \end{aligned}$$

$$\text{Annual income} = (P6, 000) (12) (25) (0.90) = P1, 620, 000$$

Annual cost:

$$\text{Depreciation} = \frac{P12,000,000 - P22,000,000}{F A, 20\%, 20} = P-53, 570$$

$$\text{Upkeep} = P500 (25) = 12, 500$$

$$\text{Taxes} = P12, 000, 000 (0.01) = 120, 000$$

$$\text{Insurance} = P7, 000, 000 (0.005) = 35, 000$$

$$\text{Rate of return} = \frac{P1,506,070}{P12,000,000} \times 100 = 12.55\% < 20\%$$

The businessman should not invest.

(The negative sign for depreciation means that the value of the investment has increased after 20 years.)

Example

A man is considering investing P500, 000 to open a semi-automatic auto-washing business in a city of 400, 000 population. The equipment can wash, on the average, 12 cars per hour, using two men to operate it and to do small

amount of hand work. The man plans to hire two men, in addition to himself, and operate the station on an 8-hour basis, 6 days per week, 50 weeks per year. He will pay his employees P25. 00 per hour. He expects to charge P25. 00 for a car wash. Out-of-pocket miscellaneous cost would be P8, 500 per month.

He would pay his employees for 2 week for vacations each year. Because of the length of his lease, he must write off his investment within 5 years. His capital now is earning 15%, and he is employed at a steady job that pays P25, 000 per month. He desires a rate of return of at least 20% on his investment.

Would you recommend the investment?

Solution

By the rate of return method

$$\text{Annual revenue} = (12) (25) (8) (6) (50) = \text{P}720,000$$

Annual cost:

$$\text{Depreciation} = \frac{P500,000}{F A, 15\%, 5} = \frac{P 500,000}{6.7424} = 74,160$$

$$\text{Labor} = (2) (48) (50) (\text{P}25.00) = 120,000$$

$$\text{Vacation pay} = (2) (2) (48) (25.00) = 4,800$$

$$\text{Miscellaneous} = \text{P}8,500 (12) = 102,000$$

$$\text{Owner's salary} = \text{P}25,000 (12) = 300,000$$

$$\text{Total annual cost} = \text{P}600,960$$

$$\text{Net annual profit} = \text{P}119,040$$

$$\text{Rate of return} = \frac{P119,040}{P500,000} \times 100 = 23.81\% > 20\%$$

The man should invest.

By the annual worth method

$$\text{Annual revenue} = (12) (\text{P}25) (8) (6) (50) = \text{P}720,000$$

Annual cost:

$$\text{Miscellaneous} = \text{P}8,500 (12) = 102,000$$

$$\text{Depreciation} = \frac{P 500,000}{F / A, 15\%, 5} = 74,160$$

Labor = (2) (48) (50) (P25)	= 120, 000
Vacation pay = (2) (2) (48) (P25. 00)	= 4, 800
Owner's salary = P25, 000 (12)	= 300, 000
Interest on capital = P500, 000 (0.20)	= 100, 000
Total annual cost	P700, 960
	Excess P19, 040

Since the excess of annual revenue over annual cost is greater than zero, the investment is justified. The man should invest.

Example

A firm is considering purchasing equipment that will reduce cost by P 400, 000. The equipment costs P300, 000 and has a salvage value of P50, 000 and a life of 7 years. The annual maintenance cost is P6, 000. While not in use by the firm, the equipment can be rented to others to generate an income of P10, 000 per year. If money can be invested for an 8 per cent return, is the firm justified in buying the equipment?

Solution

Annual savings

Reduction in annual cost	=P40, 000
Rental	= 10, 000

Annual cost:

$$\text{Depreciation} = \frac{P300,000 - P50,000}{F/A, *%, 7} = \frac{P250,000}{8.9228} = P28, 018$$

Maintenance	= 6, 000
-------------	----------

Total annual cost	P34, 018
-------------------	----------

Annual net savings	P15, 982
--------------------	----------

$$\text{Rate of return} = \frac{\text{annual net savings}}{\text{capital}} = \frac{P15,982}{P300,000} \times 100$$

$$= 5.33\% < 8\%$$

The equipment should not be purchased.

Example

The MGC Company has a contract with a hauler to transport its naphtha requirements of 3, 600, 000 liter per year from a refinery in Batangas to its site in Paco at a cost of P1. 05 per liter. It is proposed that the company buys a tanker with a capacity of 18, 000 liters to service its requirements at a first cost of P 8, 000, 000 life is 6 years and a salvage value of P 800, 000. Other expenses are as follows:

- (a) Diesel fuel at P7, 95 per liter and the tanker consumes 120 liter per round trip from Paco to Batangas and back.
 - (b) Lubricating oil servicing is P3, 200 per month.
 - (c) Labor including overtime and fringe benefits for one driver and one helper is P21, 000 per month.
 - (d) Annual taxes and insurance. 5% of first cost.
 - (e) General maintenance per year is P40, 000
 - (f) Tires cost P 32, 000 per set and will be renewed every 150 round trips.
- What should the MGC Company do if a 5% interest rate on investment is included in the analysis?

Solution

Hauling

$$\text{Annual cost} = (P1. 05) (3, 600, 000) = P3, 780, 000$$

Buying Tanker

Annual cost:

$$\text{Depreciation} = \frac{P8,000,000 - P800,000}{F/A, 15\%, 6} = \frac{P7,200,000}{8.753} = P822,575$$

575

$$\text{Fuel} = \frac{3,500,000}{18,000} (120) (P7. 95) = 190, 800$$

$$\text{Oil} = (P3, 200) (12) = 38, 400$$

$$\text{Labor} = (P21, 000) (12) = 252, 000$$

$$\text{Taxes \& insurance} = (P8, 000, 000) (0.05) = 400, 000$$

$$\text{Maintenance} = 40, 000$$

$$\begin{aligned} \text{Tires} &= \frac{3,600,000}{(18,000)(150)} (P32,000) &&= 42,667 \\ \text{Total annual cost} &&&= P1,786,442 \\ \text{Annual savings} &= P3,780,000 - P1,786,442 &&= P1,993,550 \\ \text{Rate of return} &= \frac{P1,993,558}{P8,000,000} \times 100 = 24.94\% > 25\% \end{aligned}$$

The company should buy the tanker.

Example

A machine that is not equipped with a brake “coast” 30 seconds after the power is turned off upon completion of each piece, thus preventing removal of the work from the machine. The time per piece, exclusive of this stopping time in 2 minutes. The machine is used to produce 40,000 pieces per year. The operator receives P35.00 per hour and the machine overhead rate is P20.00 per hour.

How much could the company afford to pay for a brake that would reduce the stopping time to 3 seconds, if it would have a life of 5 years? Assume zero salvage value, capital worth 18% and that repairs and maintenance would total not over P300 per month.

Solution

$$\text{Annual savings} = \frac{27}{3,600} (P35 + P20) (40,000) = 16,500$$

Let C = the amount that can be invested for the brake

Annual cost:

$$\text{Depreciation} = \frac{C}{F A, 10\%, 5} = \frac{C}{6.1051} = P 0.1638C$$

$$\text{Repairs and maintenance} = (P300) (12) = 3,600$$

$$\text{Interest on capital} = P 0.1800C$$

$$\text{Total annual cost} = P 0.3438C + P3,600$$

Equating annual savings to annual cost

$$P0.3438C + P3,600 = P16,500$$

$$C = 37,522$$

In any problem where the unknown quantity is the investment or capital, the best method to use is the annual worth method.

Example

A newly-built business property, containing space for a store and two offices, can be purchased for P1, 200, 000. A prospective buyer estimates that during the next 10 years he can obtain annual rentals of at least P458, 460 from the property and that the annual out-of-pocket disbursements will not exceed P60, 000. He believes that he should be able to dispose of the property at the end of 10 years at not less than P700, 000. Annual taxes and insurance will total 2.5% of the first cost.

- (a) Assume he has sufficient equity capital to purchase the property, and that the average return he is obtaining from his capital is 20%. Would you recommend the investment?
- (b) What recommendation would you make if he had to borrow 25% of the required capital, on the basis of a 10-year amortization with interest of 18%?
- (c) If the entire capital can be obtained by floating bonds at 15% that will mature in 10 years, what would you recommend? Sinking fund interest is 15%

Solution

(a) Annual revenue	P458, 460
Annual cost:	
Depreciation = $\frac{P1,200,000 - P700,000}{F A, 20\%, 10}$	= P19, 260
Disbursements	= 60, 000
Taxes & insurance = P1, 200, 000(0.025)	= 30, 000
Total annual cost	P109, 260
Net annual profit	P349, 200
Rate of return = $\frac{P349,000}{P1,200,000} \times 100 =$	29.1 >20%

The investment is justified.

(b) Annual revenue	P458, 460
Annual cost:	

$$\text{Depreciation} = \frac{P900,000 - P700,000}{F/A, 20\%, 10} = P7,700$$

$$\text{Amortization} = \frac{P1,200,000 (0.25)}{P/A, 18\%, 10} = 66,760$$

$$\text{Disbursements} = 60,000$$

$$\text{Taxes \& insurance} = P1,200,000 (0.025) = 30,000$$

$$\text{Total annual cost} = P164,460$$

$$\text{Net annual profit} = P294,000$$

$$\text{Rate of return} = \frac{P294,000}{P1,200,000} \times 100 = 24.50\% > 20\%$$

The investment is justified.

$$(c) \text{ Annual revenue} = P458,460$$

Annual cost:

$$\text{Sinking fund deposit} = \frac{P1,200,000 - P700,000}{F/A, 15\%, 10} = P24,630$$

$$\text{Bond interest} = P1,200,000 (0.15) = 180,000$$

$$\text{Disbursements} = 60,000$$

$$\text{Taxes and insurance} = P1,200,000 (0.025) = 30,000$$

$$\text{Total annual cost} = P294,630$$

$$\text{Net annual profit} = P163,830$$

$$\text{Rate of return} = \frac{P163,830}{P1,200,000} \times 100 = 13.65\% < 20\%$$

The investment is not justified.

Problem

proposed project will require the immediate investment of P50,000 and is estimated to have year-end revenues and costs as follows:

Year	revenue	costs
1	P75,000	P60,000
2	90,000	77,500
3	100,000	75,000
4	95,000	80,000
5	60,000	47,500

An additional investment P20, 000 will be required at the end of the second year. The project would terminate at the end of the 5th year, and the assets are estimated to have of P25, 000 at that time.

Is this a good investment?

Solution

Solve for the internal rate of return (IRR) and then decide whether the investment justified. The internal rate of return is that rate of return that will exactly reduce the worth of the investment to zero at the end of the life of the investment. Thus, present worth of cash inflows minus the present worth of cash outflows must equal to zero.

Problem

.A company is considering constructing a plant to manufacture a proposed new product. The land costs P300,000, the building costs P600,000, the equipment costs P250,000 and P100,000 additional working capital is required. It is expected that the product will result in sales of P750,000 per year for 10 years, at which time the land can be sold for P400,000, the building for P350,000 and the equipment for P50,000. All of the working capital would be recovered at the end of year 10. The annual expenses for labor, materials, and all other items are estimated to total P475,000. If the company requires a MARR of 15% per year on projects of comparable risk, determine if it should invest in the new product line. Evaluate using all methods.

a.) ROR

b.) Annual Worth Method

c.) Present Worth Method

d.) Future Worth Method

e.) Payback Period

Problem

Your firm is considering the purchase of an old office building with an estimated remaining service life of 25 years. The tenants have recently signed long-term leases, which leads you believe that the current rental income of \$150,000 per year will remain constant for the first five years. Then the rental income will increase by too for every five-year interval over the remaining asset life. For example, the annual rental income would be \$165,000 for years six through to, \$181,500 for years n through 15. \$199, 650 for year's i6 through 20. And \$21 9.615 for years 21 through 25. You estimate that operating expenses including income taxes. will be \$4.5,000 for the first year and that they will increase by \$3,000 each year thereafter. You estimate that razing the building and selling the lot on which it stands will realize a net amount of \$50,000 at the end of the 25 year period. If you had the opportunity to invest your money elsewhere and thereby earn interest at the rate of 12% per annum, what would be the maximum amount you would be willing to pay for the building and lot at the present time?

Problem

.A large food-processing corporation is considering using laser technology to speed up and eliminate waste in the potato-peeling process. To implement the system, the company anticipates needing P3 million to purchase the industrial strength lasers. The system will have \$1,200,000 per year in labor and materials. However, it will incur an additional operating and maintenance cost of P250, 000 per year. Annual income taxes will also increase by P150, 000. The system is expected to have a 10-year service life and a salvage value of about P200, 000. If the company's MARR is 15%, justify the economics of the project, based on the PW method.

Problem

.You are face with making a decision on a large capital investment proposal. The capital investment amount is P640, 000. Estimated annual revenue at the end of the year in the eight year study period is P180, 000. The estimated annual year-end expenses are P420, 000 starting in year one. These expenses begin decreasing by P4, 000 per year at the end of the year four and continue decreasing through the end of the eight. Assuming a P20,000 market value at the

end of the year eight and a MARR of 12% per year, answer the following questions:

- a. What is the PW of this proposal?
- b. What is the IRR of this proposal?
- c. What is the simple payback period for this proposal?
- d. What is your conclusion about the acceptability of this proposal?

Problem

.Uncle Wilbur's trout ranch is now for sale for P300, 000. Annual property taxes, maintenance, supplies, and so on are estimated to continue to be P30, 000 per year. Revenues from the ranch are expected to be P100, 000 next year and then

to be decline by P400 per year thereafter through the 10th year. If your bought the ranch, you would plan to keep it for only five years and at that time to sell it for the value of the land, which is P150,000. If your desired MARR is 12% per year should you become a trout rancher? Use the PW method.

PROBLEMS

Basic method for making economy studies

1. A young mechanical engineer is considering establishing his own small company. An investment of P400,000 will be required which will be recovered in 15 years.

It is estimated that sales will be P800,000 per year and that operating expenses will be as follows.

Materials	P160,000 per year
Labor	P280,000 per year
Overhead	P40,000 +10% of sales per year
Selling expense	P60,000

The man will give u his regular job paying P216,000 per year and devote full time to the operation of the business; this will result in decreasing labor cost by P40,000 per year, material cost by P28,000 per year and overhead cost by P32,000 per year. If the man expects to earn at least 20% of his capital, should he invest?

2. The ABC company is considering constructing a plant to manufacture a proposed new product. The land costs P15,000,000, the building costs P30,000,000, the equipment costs P12,500,000, and P5,000,000 working capital is required. At the end of 12 years, the land can be sold for P25,000,000, the building for P12,000,000, the equipment for P250,000 and all of the working capital recovered. The annual disbursements for labor, materials, and all other expenses are estimated to cost P23,750,000. If the company requires a minimum return of 25%, what should be the minimum annual sales for 12 years to justify the investment?

3. A man formerly employed as chief mechanic of an automobile repair shop has saved P1,000,000.00 which are now invested in certain securities giving him an annual dividend of 15%. He now plans to invest this amount in his own repair shop. In his present job, he is earning P25,000.00 a month, but he has to resign to run his own business. He will need the services of the following: 2 mechanics each earning P400.00 a day and 8 helpers each are earning P200.00 a day. These men will work on the average 300 days per year. His other expenses are the following:

Rental	P30,000.00 a month
Miscellaneous	P25,000.00 a month

Sales tax	3% of gross income
Insurance	2%

The length of his lease is 5 years. If the average charge for each car repaired by his shop is P1,000.00. Determine the number of cars he must service in one year so that he will obtain a profit of at least 20% on his investment?

4. A firm is charged P150 per ton for hauling its raw materials by a trucking company. Forty tons per day are hauled for 300 days a year. It is desired to install a railway system which would bring down the cost of hauling to P6.60 per ton. Maintenance cost of this is P12,000 per month. Tax is 1%. Average rate of earning is 20%.

a. If the company has the cash necessary for the installation, would you recommend the change?

b. If the company has to float P5,000,000 worth of noncallable bonds at 15% that will mature in 10 years to have the capital for the project, would you recommend the change?

5. A food processing plant consumed 600,000 kW of electric energy annually and pays an average of P2.00 per kWh. A study is being made to generate its own power to supply the plant the energy required, and that the power plant installed would cost P2,000,000. Annual operation and maintenance, P800,000. Other expenses P100,000 per year. Life of power plant is 15 years; salvage value at the end of life is P200,000; annual taxes and insurances, 6% of first cost; and rate of interest is 15%. Using the sinking fund method for depreciation, determine if the power plant is justifiable.

6. A fixed capital investment of P10, 000,000.00 is required for a proposed manufacturing plant and an estimated working capital of P2,000,000.00. Annual depreciation is estimated to be 10% of the fixed capital investment. Determine the rate of return on the total investment and the payout period if the annual profit is P2,500,000.00.

7. A small business purchased now for P50, 000 will lose P9, 600 each year for first 4 years. An additional investment of P30,000 in the business will required at the end of the fourth year. After 15 years the business can sold for P70, 000. What should be the profit each year from the fifth through the fifteenth year to obtain a rate return of 25%?

Chapter 6

Comparing Alternatives

In chapter 5, the different methods for investment of capital were discussed. In that chapter, the problem that was resolved was whether the investment of capital in a certain enterprise was justified or not. However, most engineering and business project can be accomplished by more than one method or alternative. This chapter will deal with this type of problems.

The fundamental principle on which alternative should be used is stated as follows.

The alternative that requires the minimum investment of capital and will produce satisfactory functional result will always be used unless there are

definite reasons why an alternative requiring a larger investment should be adopted

When the selection of the one of the alternatives excludes the choice of any of the others, the alternatives are called **mutually exclusive alternatives**. The basic principle in selecting among alternatives that requires the minimum investment of the capital and produces satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a large investment can be justified with respect to its incremental benefits.

Two types of projects and alternatives

- a. **Investment alternatives** are those with initial capital investments that produce positive cash flows from increased revenues, saving through reduced costs, or both.
- b. **Cost alternatives** are those with all negative cash flows, except for a possible positive cash flow element from disposal of assets at the end of the project's useful life.

Rule 1: when revenues and other economic benefits are present and vary among the alternatives, choose the alternative that maximizes overall profitability. That is, select the alternative that has the greatest positive equivalent worth at $i = \text{MARR}$ and satisfies all project requirements.

METHODS OR PATTERNS IN COMPARING ALTERNATIVES

There are several methods for comparing alternatives, but only six patterns will be discussed

1. THE RATE OF RETURN ON ADDITIONAL INVESTMENT METHOD

This method assumes that unlimited capital is available; therefore an alternative requiring a bigger investment may be adopted provided the rate of return on the additional investment justifies the bigger outlay of capital by savings or benefits. In this method the rate of return on additional investment is calculated as:

$$\text{Rate of return on additional investment} = \frac{\text{savingsofbenefits}}{\text{additionalcapital}} \times 100\%$$

- | | |
|-----------------|---|
| ROR \geq MARR | (Choose the alternative with larger capital) |
| ROR \leq MARR | (Choose the alternative with lesser capital investment) |

If the rate of return on additional investment is satisfactory, then, the alternative requiring a bigger investment is more economical and should be chosen.

Example

A piece of production equipment is to be replaced immediately because it no longer meets quality requirements for the end product. The two best alternatives are a used piece of equipment (E1) and a new automated model (E2). The economic estimates for each are shown below

	E ₁	E ₂
Capital Investment	14,000	65,000
Annual Expenses	14,000	9,000
Useful life (years)	5	20
Salvage Value	8,000	13,000

The MARR is 15% per year. Which alternative is preferred?

Solution: To compare the two alternatives, determine first the total annual cost for each alternative, including annual depreciation.

	E ₁	E ₂
Expenses	14,000	9,000
Depreciation	$\frac{889.89}{14,889.89}$	$\frac{507.6}{9507.6}$

Consider B

$$\text{ROR} = \frac{14,889.89 - 9,507.6}{65,000 - 14,000} \times 100\%$$

$$= 10.55\%$$

Since ROR < MARR Choose E₁

2. Annual Cost of Method

In this method, the annual cost for each alternative is determine and the alternative with the least AC should be chosen.

To apply this method, the annual cost of alternatives including interest on investment is determined. The alternative with the least annual cost is chosen. This pattern, like the rate of return on additional investment pattern, applies only to alternatives which has a uniform cost data for each year and a single investment of capital at the beginning of the first year of the project life.

Example

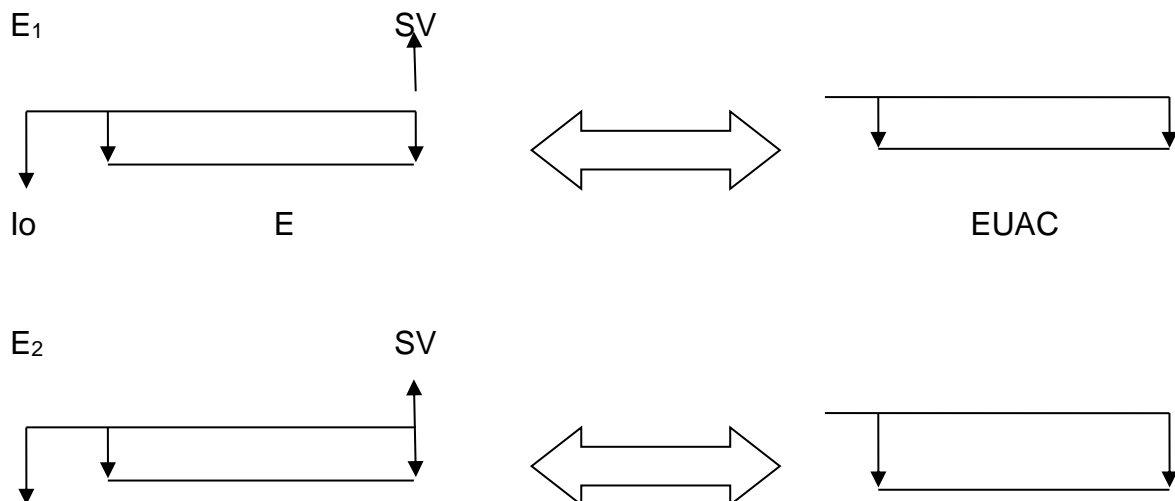
	E ₁	E ₂
	14,000	9,000
Expenses		
Depreciation	889.89	507.6
MRP	<u>15%(14,000)</u>	<u>45%(65,000)</u>
	16,889.89	19,257.6

Since AC of E₁, is less than AC of E₂, Choose E₁

3. THE EQUIVALENT UNIFORM ANNUAL COST (EUAC) METHOD

The most straightforward technique for comparing mutually exclusive alternatives is to determine the equivalent worth of each alternative over the same study period based on the total investment at $i = MARR$. The for investment alternative, the one with the greatest positive equivalent worth is selected.

In this method, all cash flows (irregular or uniform) must be converted to equivalent uniform annual cost, that is, a year-end amount which is the same each year. The alternative with the least equivalent annual cost is preferred. When the euac method is used, the equivalent uniform annual cost of alternatives must be calculated for one life cycle only. This method is flexible and can be used for any type of alternative selection problems. The method is modification of the annual cost pattern.



lo

EUAC

Solve the above problem using EUAC method:

Annual Worth for A

$$AW = 18,000 \left[\frac{.15}{1-(1.15)^{-5}} \right] + 14,000 - 9,000 \left[\frac{.15}{(1.15)^5 - 1} \right]$$

$$AW = P16,985.89$$

Annual Worth for B

$$AW = 65,000 \left[\frac{.15}{1-(1.15)^{-20}} \right] + 5,000 - 13,000 \left[\frac{.15}{(1.15)^{20} - 1} \right]$$

$$AW = P17,257.59$$

Choose A as alternative.

4.THE PRESENT WORTH COST (PWC) METHOD

In comparing alternatives by this method, determine the present worth of the net cash outflows for each alternative for the same period of time. The alternative with least present worth of cost is selected.

In this method, the present worth of all cash flows each alternative is computed for the same study period.

Example

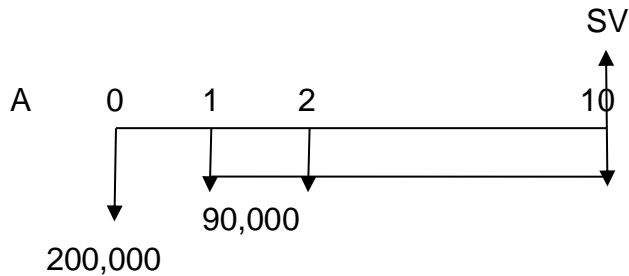
Economic estimates for two alternatives are follows:

	A	B
FC	200,000	300,000
Annual Op. Cost	32,000	24,000
Labor Cost	50,000	32,000
Insurance	3%	3%
Payroll Tax	4%	4%
Life	10	10

If the MARR = 15%, compare the two alternatives.

For alternatives with the same lives, the study period will lives of the alternatives. Using PWC method, determine the PWC of cost for each alternative.

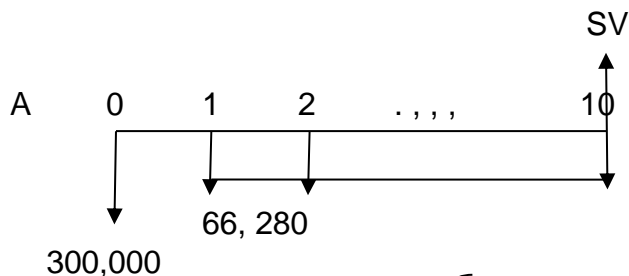
For Alternative A:



$$P_A = 200,000 + 90,000 \left[\frac{1 - (1.05)^{-10}}{0.05} \right]$$

$$P_A = 651,689.18$$

For Alternative B:



$$P_B = 300,000 + 66,280 \left[\frac{1 - (1+0.05)^{-10}}{0.05} \right]$$

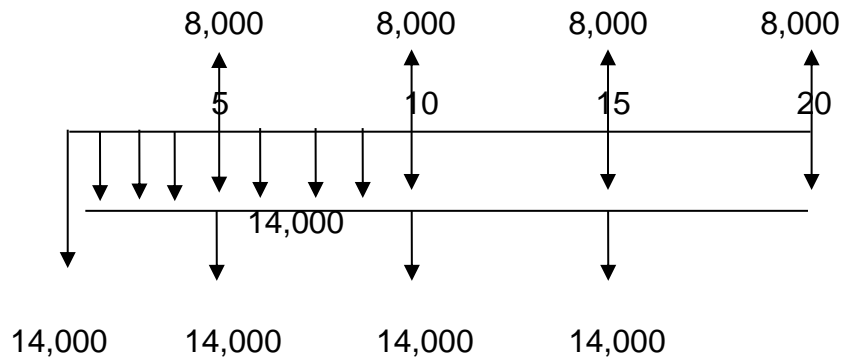
$$P_B = 632,543.98$$

Since PWC of Alternative B less that PW of Alternative A, therefore Choose B

Example: Solving problem 1 using present worth method:

Since the alternatives' lives are different, the study period will the common multiple of their lives. In this case, 5 and 20, study period will be 20 years.

For alternative E₁, there will be 4 Life cycle for 20 year-study period.



Solving for the PW of all cash flows:

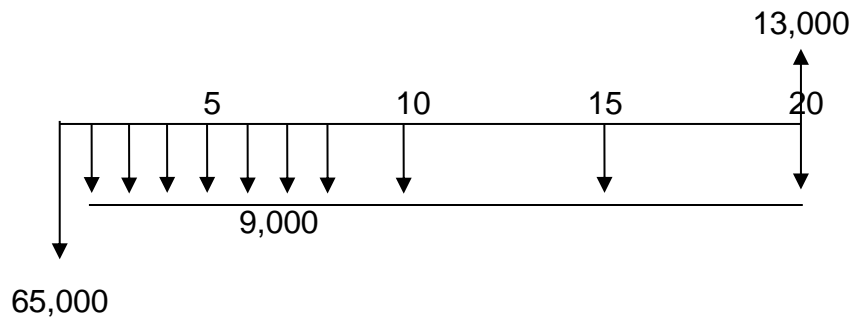
$$PW = 14,000 + 14,000 \left[\frac{1 - (1.15)^{-5}}{.15} \right] - 8,000 (1.15)^{-5}$$

$$= \mathbf{P56952.75749}$$

$$PW = 56952.75749 [1 + (1.15)^{-5} + (1.15)^{-10} + (1.15)^{-15}]$$

$$\mathbf{PW = P106,345.374}$$

For alternative E₂, there will be one life cycle only.



Solving the PW of all cash flows:

$$PW = 65,000 + 9,000 \left[\frac{1 - (1.15)^{-20}}{.15} \right] - 13,000 (1.15)^{-20}$$

$$\mathbf{PW = P120,359.6796}$$

5. THE CAPITALIZED METHOD

The capitalized cost method is a variation of the present worth cost pattern. This method is used for alternatives having long lives. To use the method, determine the capitalized cost of all the alternatives and choose that one with the least capitalized cost.

6. PAYBACK (PAYOUT) PERIOD METHOD

To use this method, the payback period of each alternative is computed. The alternative with the shortest payback period is adopted. This method is seldom used.

Example

A company is considering two types of equipment for its manufacturing plant. Pertinent data are as follows:

	Type a	Type b
First cost	P200,000	P300,000
Annual operating cost	32,000	24,000
Annual labor cost	50,000	32,000
Insurance and property tax	3%	3%
Payroll taxes	4%	4%
Estimated life	10	10

If the minimum required rate of return is 15%, which equipment should be selected?

SOLUTION

By the **rate of return** on additional investment method

TYPE A

Annual cost:

Depreciation = $\frac{P200,000}{F/A, 15\%, 10} = \frac{P200,000}{20.30337}$	= P 9,850
Operation	= 32,000
Labor	= 50,000
Payroll taxes = (50,000) (0.04)	= 2,000
Taxes & insurance = (P200,000) (0.03)	= <u>6,000</u>
Total annual cost	P99,850

TYPE B

Annual costs:

Depreciation = $\frac{300,000}{F/A, 15\%, 10}$	=	$\frac{300,000}{20.3037}$	= P 14,776
Operation			= 24,000
Labor			= 32,000
Payroll taxes = (P32,000)(0.04)			= 1,280
Taxes & insurances = (P300,000) (0.03)			= <u>9,000</u>
		Total annual cost	= P 81,056

$$\text{Annual savings} = P99,850 - P81,056 = P18,794$$

$$\text{Additional investment} = P300,000 - P200,000 = P100,000$$

$$\text{Rate of return on additional investment} = \frac{P18,794}{P100,000} \times 100 = 18.79 > 15\%$$

Type b should be selected.

By annual cost method

Type A

Annual cost:

Depreciation = $\frac{P200,000}{(F/A, 15\%, 10)}$	=	$\frac{P200,000}{20.3037}$	= P 9,850
Operation			= 32,000
Labor			= 50,000
Payroll taxes = (P50,000)(0.04)			= 2,000
Taxes & insurances = (P200,000) (0.03)			= 6,000
Interest on capital = (P200,000) (0.15)			= <u>30,000</u>
		Total annual cost	= P126,000

Type B

Annual cost:

Depreciation = $\frac{P300,000}{F/A, 15\%, 10}$	=	$\frac{P300,000}{20.3037}$	= P 14,776
Operation			= 24,000
Labor			= 32,000
Payroll taxes = (P32,000)(0.04)			= 1,280
Taxes & insurances = (P300,000) (0.03)			= 9,000
Interest on capital = (P300,000) (0.15)			= <u>45,000</u>

Total annual cost

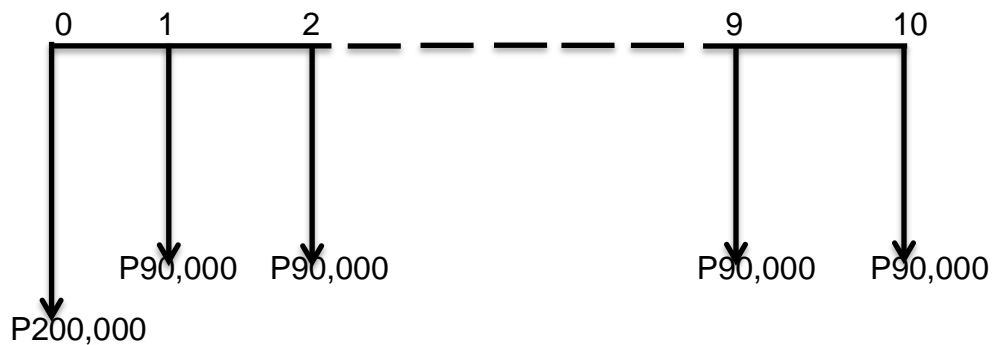
P126,056

Since $AC < AC$, TYPE B SHOULD BE SELECTED

By the present worth cost method

Type A

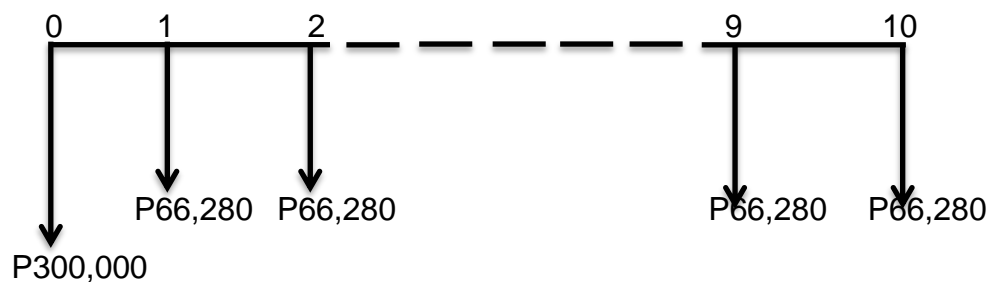
$$\text{Annual cost (excluding depreciation)} = P32,000 + P50,000 + (P50,000)(0.04) = (P200,000)(0.03) = P90,000$$



$$\begin{aligned} PWCA &= P200,000 + P90,000 (P/A, 15\%, 10) \\ &= P200,000 + P90,000 (5.0188) = P651,692 \end{aligned}$$

Type B

$$\text{Annual cost (excluding depreciation)} = P24,000 + P50,000 + P32,000 + (P32,000)(0.04) = (P300,000)(0.03) = P66,280$$

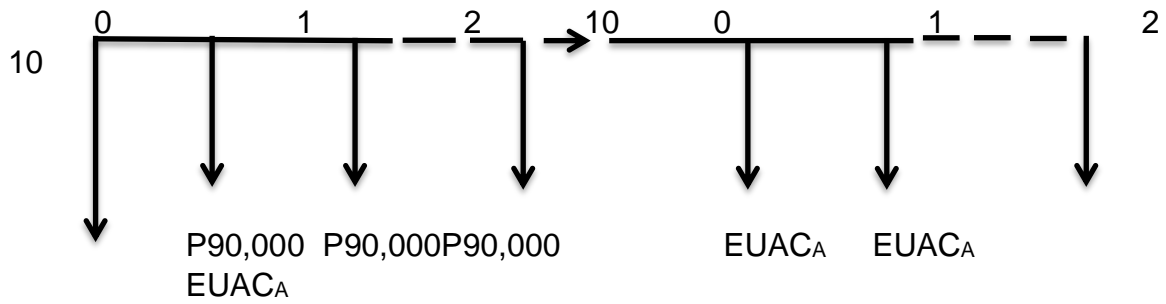


$$\begin{aligned} PWCB_B &= P300,000 + P66,280 (P/A, 15\%, 10) \\ &= P300,000 + P66,280 (5.0188) = P632,646 \end{aligned}$$

Since $PWC_B < PWC_A$ for the same period of time, type B should be selected.

By equivalent uniform annual cost method

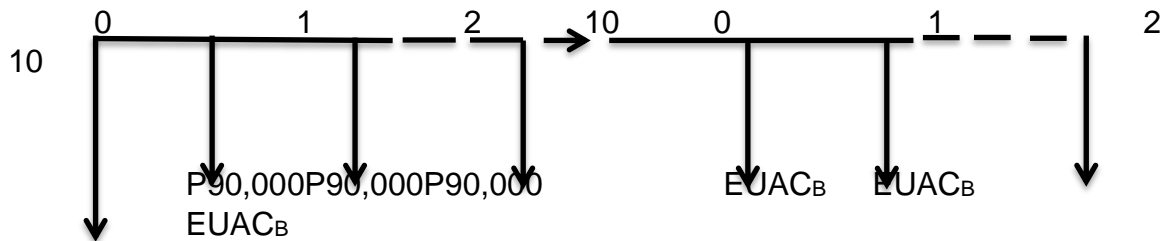
Type A



P200,000

$$\begin{aligned}
 \text{UEAC} &= P200,000 (A/P, 15\%, 10) + P90,000 \\
 &= (P200,000) (0.1993) + P90,000 = P129,860
 \end{aligned}$$

Type B



P200,000

$$\begin{aligned}
 \text{EUAC}_B &= P300,000(A/P, 15\%, 10) + P66,280 \\
 &= (P300,000) (0.19993) + P66,280 = P126,070
 \end{aligned}$$

Since $\text{EUAC}_B < \text{EUAC}_A$, type B is more economical.

Example

Choose from the two machines which is more economical

Machine A	Machine B
First cost	P8,000
Salvage value	0
Annual operation	3,000
Annual maintenance	1,200
Taxes and insurance	3%
Life years	10
	P14,000
	2,000
	2,400
	1,000
	3%
	15

Money is worth at least 16%

Solution

By **rate of return** on additional investment

Machine A

Annual cost:

$$\text{Depreciation} = \frac{P8,000}{\frac{F}{A}, 16\%, 10} = \frac{P8,000}{21.3215} = P 375$$

$$\text{Operation} = 3,000$$

$$\text{Maintenance} = 1,200$$

$$\text{Taxes \& insurances} = (P8,000) (0.03) = \underline{240}$$

$$\text{Total annual cost} = P4,815$$

Machine B

Annual cost

$$\text{Depreciation} = \frac{P12,000}{\frac{F}{A}, 16\%, 15} = \frac{P12,000}{51.6595} = P 232$$

$$\text{Operation} = 2,400$$

$$\text{Maintenance} = 1,000$$

$$\text{Taxes \& insurances} = (P8,000) (0.03) = \underline{420}$$

$$\text{Total annual cost} = P4,052$$

$$\text{Annual savings} = P4,815 - P4,052 = P763$$

$$\text{Additional investment} = P14,000 - P8,000 = P6,000$$

$$\text{Rate of return on additional investment} = \frac{P763}{6,000} \times 100$$

$$= 12.72\% < 16\%$$

Machine A is more economical.

By annual cost methods

Machine A

Annual cost:

$$\text{Depreciation} = \frac{P8,000}{\frac{F}{A}, 16\%, 10} = \frac{P8,000}{21.3215} = P 375$$

$$\text{Operation} = 3,000$$

$$\text{Maintenance} = 1,200$$

$$\begin{array}{rcl}
 \text{Taxes \& insurances} & = (\text{P}8,000) (0.03) & = 240 \\
 \text{Interest on capital} & = (\text{P},000) (0.16) & = 1,280 \\
 \text{Total annual cost} & & \underline{\text{P}6,095}
 \end{array}$$

Machine B

Annual cost:

$$\begin{array}{rcl}
 \text{Depreciation} & = \frac{\text{P}12,000}{\frac{F}{A}, 16\%, 15} = \frac{\text{P}8,000}{51.695} & = \text{P } 232 \\
 \text{Operation} & & = 2,400 \\
 \text{Labor} & & = 1,000 \\
 \text{Taxes \& insurances} & = (\text{P}14,000) (0.03) & = 9,000 \\
 \text{Interest on capital} & = (\text{P}4,000) (0.16) & = \underline{2,240} \\
 \text{Total annual cost} & & \text{P}6,292
 \end{array}$$

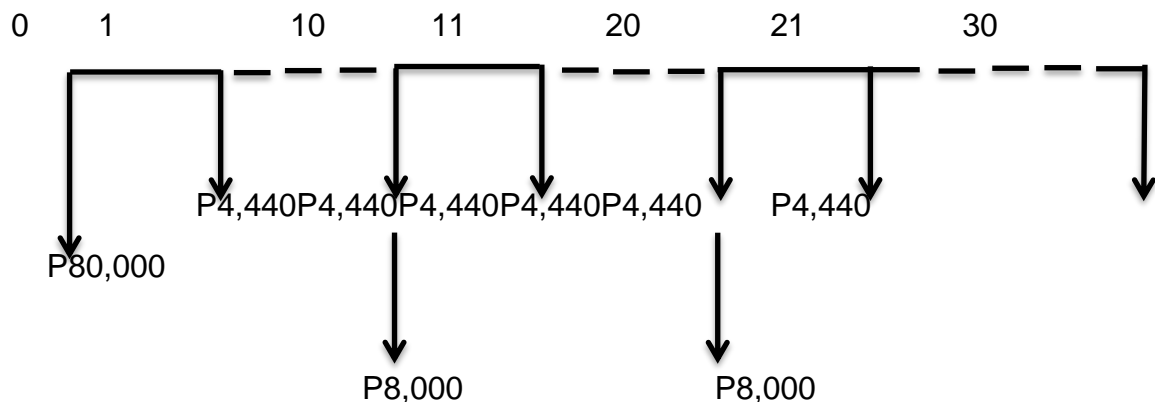
Since $AC_A < AC_B$, Machine A is more economical.

By present worth cost method

Use 30-year study period, which is the least common multiple of 10 and 15.

Machine A

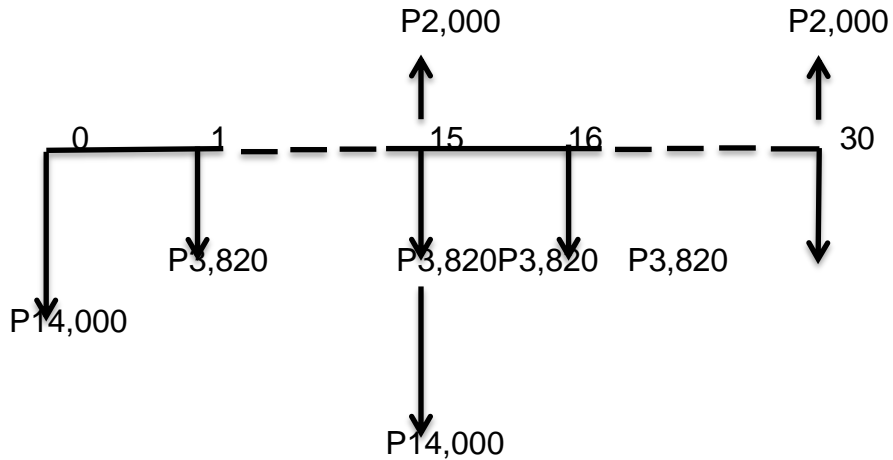
$$\text{Annual cost} = \text{P}3,000 = \text{P}1,200 + (\text{P}8,000) (0.03) = \text{P}4,440$$



$$\text{PWC}_A = \text{P}8,000 + \text{P}4,440 (P/A, 16\%, 30) + \text{P}8,000 (P/F, 16\%, 10) + \text{P}8,000 (P/F, 16\%, 20) = \text{P}37,652$$

Machine B

$$\text{Annual cost} = \text{P}2,400 + \text{P}1,000 + (\text{P}14,000) (0.03) = \text{P}3,820$$

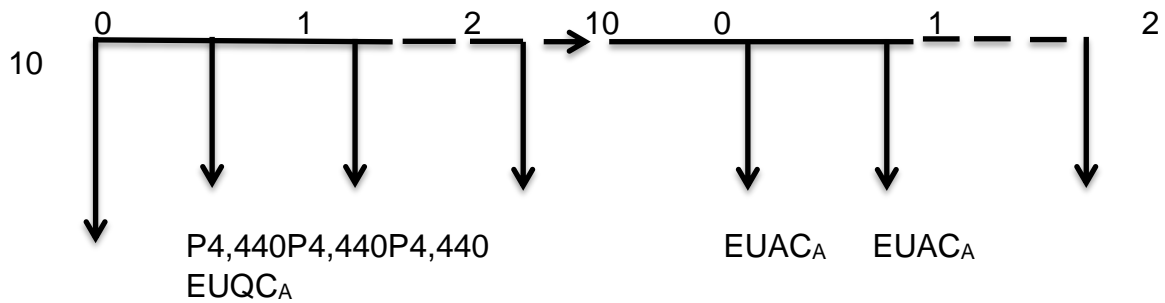


$$PWC_B = P14,000 + P3,820 (P/A, 16\%, 30) + 12,000 (P/F, 16\%, 15) - 2,000 (P/F, 16\%, 30) = P38,869$$

Machine A should be chosen, since $PWC_A < PWC_B$

By the equivalent uniform annual cost method

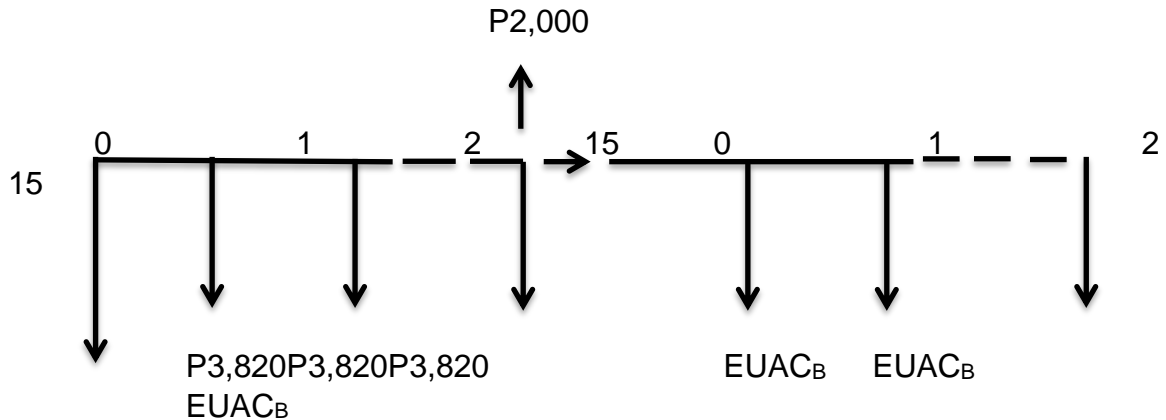
Machine A



P8,000

$$EUAC_A = P8,000 (A/P, 16\%, 10) = P4,440 = P6,095$$

Machine B



P14,000

$$EUAC_B = P14,000 (A/P, 16\%, 15) + P3,820 - 2,000 (A/F, 16\%, 15) = P6,292$$

Machine A should be chosen, since $EUAC_A < EUAC_B$

Example

A company is going to buy a new machine for manufacturing its product. Four different machines are available. Cost, operating and other expenses are as follows:

	A	B	C	D
First cost	P24,000	P30,000	P49,600	P52,000
Power per year	1,300	1,360	2,400	2,020
Labor per year	11,600	9,320	4,200	2,000
Maintenance per year	2,800	1,900	1,300	700
Taxes and insurance	3%	3%	3%	3%
Life, years	5	5	5	5

Money is worth 17% before taxes to the company. Which machine to be chosen?

Solution

By the annual cost method

Depreciation:

$$\text{Machine A} = \frac{P24,000}{\frac{F}{A}, 17\%, 5} = \frac{P24,000}{7.0144} = P3,422$$

$$\text{Machine B} = \frac{P30,000}{7.0144} = P4,277$$

$$\text{Machine C} = \frac{P49,600}{7.0144} = P7,071$$

$$\text{Machine D} = \frac{P52,000}{7.0144} = P7,413$$

Comparative total annual cost for four machines

	A	B	C	D
Power	P 1,300	P1,360	P2,400	P2,020
Labor	11,600	9,320	4,200	2,000
Maintenance	2,800	1,900	1,300	700
Taxes and insurance (3%)	720	900	1,488	1,560
Depreciation	3,422	4,277	7,071	7,413
Interest on capital (17%)	<u>4,080</u>	<u>5,100</u>	<u>8,432</u>	<u>8,840</u>
Total annual cost	P23,922	P22,857	P24,891	P22,533

The economic criterion is to choose that alternative with the minimum annual cost, which is machine D. however, it should be noted that machine B is very close, showing a total annual cost of only P22,857 - P22,533 = P324 more than machine D.

By the rate of return on additional investment method

	A	B	C	D
Power	P 1,300	P1,360	P2,400	P2,020
Labor	1,600	9,320	4,200	2,000
Maintenance	2,800	1,900	1,300	700
Taxes and insurance (3%)	720	900	1,488	1,560
Depreciation	<u>3,422</u>	<u>4,277</u>	<u>7,071</u>	<u>7,413</u>
Total annual cost	P19,842	P17,757	P16,459	P13,693

Comparing Machine A with Machine B

$$\text{ROR on additional investment on B} = \left(\frac{P19,48 - P17,757}{P30,000 - P24,000} \right) (100)$$

Machine B is more economical than machine A

Comparing machine b with machine c

$$\text{ROR on additional investment on c} = \left(\frac{P17,757 - 16,459}{P49,600 - P30,000} \right) (100)$$

$$= 6.62\% < 17\%$$

Machine B is more economical than Machine C

Comparing Machine B with Machine D

$$\text{ROR on additional investment D} = \left(\frac{P17,757 - P3,693}{P52,000 - P30,000} \right) (100)$$

Machine D is more economical than Machine B, choose machine D.

Example

An untreated electric wooden pole that will last 10 years under certain condition costs P20,000. If treated pole will last for 20 years, what is the maximum justifiable amount that can be paid for the treated pole, if the maximum return on investment is 20%. Consider annual taxes and insurance amount to be 1% of first cost.

Solution

Let c = maximum amount that can be invested on the treated pole

Untreated pole

Annual cost:

$$\text{Depreciation} = \frac{P20,000}{\frac{F}{A}, 20\%, 10} = \frac{P20,000}{25.90587} = P770$$

$$\text{Taxes and insurances} = (20,000) (0.01) = 200$$

$$\text{Interest on capital} = (20,000) (0.20) = \underline{4,000}$$

$$\text{Total annual cost} \qquad \qquad \qquad P4,970$$

Treated pole

Annual cost:

$$\text{Depreciation} = \frac{C}{\frac{F}{A}, 20\%, 20} = \frac{C}{186.688} = 0.005356C$$

$$\text{Taxes and insurances} = (C) (0.01) = 0.01C$$

$$\text{Interest on capital} = (C) (0.20) = \underline{0.20C}$$

Equating annual costs,

$$P0.215356C = P4,970$$

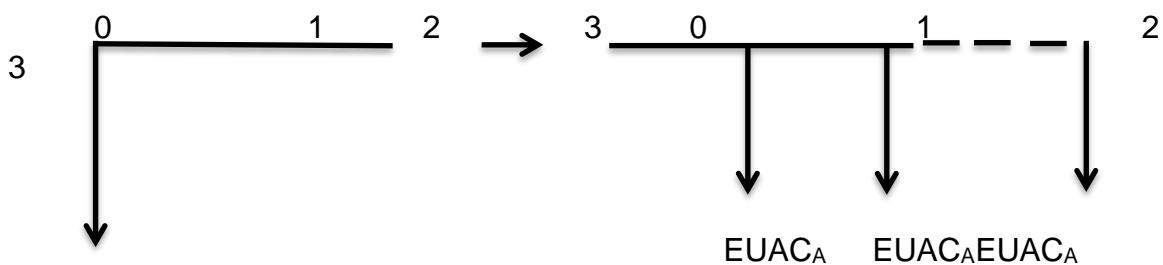
$$C = P23,078$$

Example

A company manufacturing acids, upon inspection of the roofing of the plant, found out that it is badly corroded from the acids fumes and would need to be replaced. To try to get some more life out of the roofing, The company consulted a roofing coating constructor who presented the company with two options. The first option is a coating that will cost P200,000 which would extend the life of the roofing for 3 years from date of application, and the second option will cost P30,000 and which would extend the life of the roofing for 5 years from the date of application. at what rate of return are the two investment equal?

SOLUTION

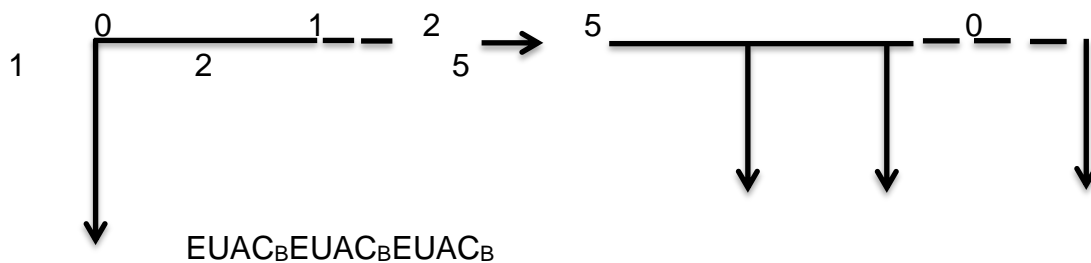
First option



P20,000

$$EUAC_A = P20,000 (A/P, i\%, 3)$$

Second option



P30,000

$$EUAC_B = P30,000 (A/P, i\%, 5)$$

For the two alternatives to be economical, $EUAC_A = EUAC_B$

$$P20,000(A/P, i\%3) = P30,000(A/P, i\%, 5)$$

$$\frac{1-(1+i\%)^{-3}}{1-(1+i\%)^{-5}} = 0.6667$$

$$1-(1+i\%)^{-5}$$

Try $i = 12\%$

$$\frac{1-(1+1.12)^{-3}}{1-(1+1.12)^{-5}} = 0.6667$$

$$1-(1+1.12)^{-5}$$

$$0.6662 = 0.6667$$

Rate of return = 12%

Example

The engineer of a medium scale industry was instructed to prepare at least two plans which is to be considered by management for the improvement of their operations. plan "a" calls for an initial investment of P200,000 now with a prospective salvage value of 20% of the first cost 20 years hence. the operation and maintenance disbursement are estimated to be P15,000 a year and taxes will be 2% of first cost.

Plan b calls for an immediate investment of P140,000 and a second investment of 160,000 eight years later. the operation and maintenance disbursements will be P9,000 a year for initial installation and P8,000 a year for the second installation. At the end of 20 years the salvage value shall be 20% of the investments. Taxes will be 2% of the first cost.

If money is worth 12%, which plan would you recommend?

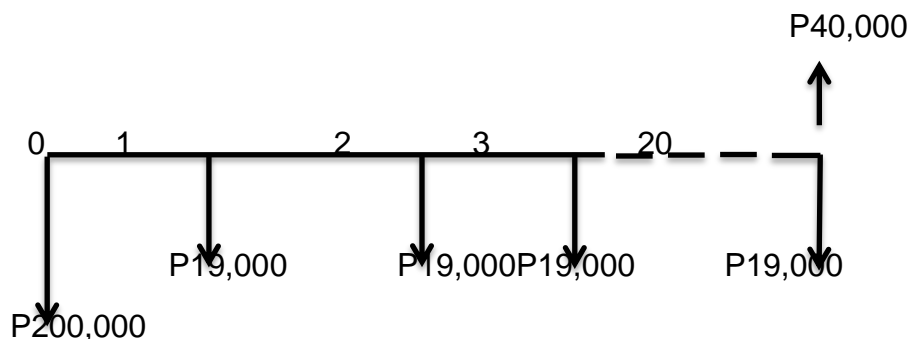
Solution

By present worth cost method

Plan A

$$\text{Annual cost} = P15,000 + (P200,000)(0.02) = P19,000$$

$$\text{Salvage value} = (200,000)(0.20) = P40,000$$



$$PWC_A = P200,000 + (19,000)(P/A, 12\%, 20) - (P40,000)(P/F, 12\%, 20)$$

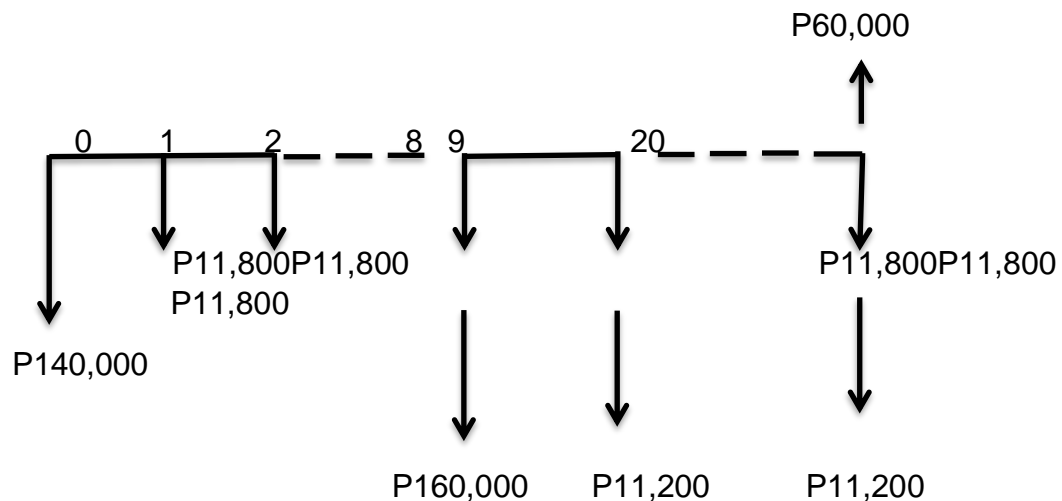
$$= 337,771$$

Plan B

$$\text{Annual cost} = P9,000 + (P140,000)(0.02) = P11,800$$

$$\text{Additional annual cost after 8 years} = P8,000 + (P160,000)(0.02) = P11,200$$

$$\text{Salvage value} = (140,000 - 160,000)(0.20) = P60,000$$



$$PWC_B = P140,000 + P11,800(P/A, 12\%, 20) + P160,000(P/F, 12\%, 8) + P11,200(P/A, 12\%, 12)(P/F, 12\%, 8) - P60,000(P/F, 12\%, 20) = 314,564$$

Plan B is more economical

By the equivalent uniform annual cost method

$$EUAC_A = P200,000(A/P, 12\%, 20) + P19,000 - P40,000(A/F, 12\%, 20) = P45,224$$

$$EUAC_B = P140,000(A/P, 12\%, 20) + 11,800 + P160,000(P/F, 12\%, 8)(A/P, 12\%, 20) + P11,200(F/A, 12\%, 12)(A/F, 12\%, 20) - P60,000(A/F, 12\%, 20) = 34,019$$

Plan B is more economical

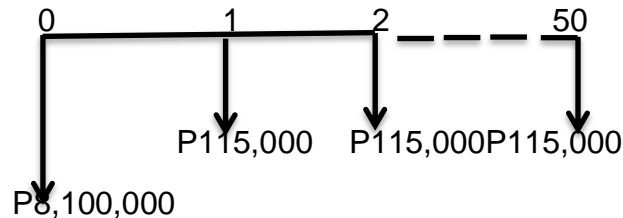
Example

The national government is planning to construct a bridge. At present a 4-lane bridge will be built, but it is expected that at a later date a second 4-lane bridge will be added.

If an 8-lane bridge is built now, it will cost P8,100,000. A single 4-lane bridge can be built now for P5,500,000. It is estimated that a second 4-lane bridge at a later date will cost P8,000,000. Annual upkeep on each 4-lane bridge would be P91,500 and for the 8-lane bridge would be P115,000. Money to build the bridge will cost 12%. Assume a 50-year functional life for the project and determine the earliest time at which the additional 4 lanes would be required to make it equally economical to build the 8-lane bridge immediately.

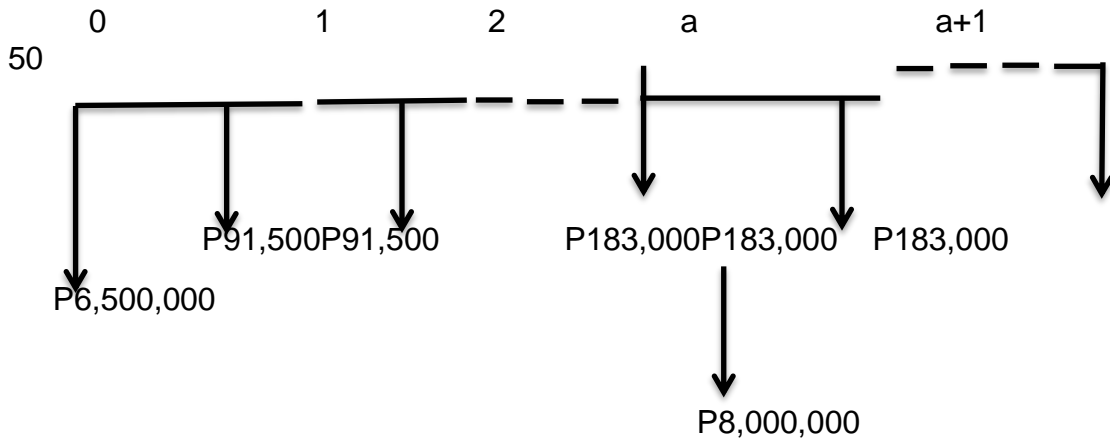
Solution

8-lane Bridge



$$PWC_B = P8,100,000 + P115,000(P/A, 12\%, 50) = P9,055,000$$

4-lane Bridge



$$PWC_A = P5,500,000 + P91,500(P/A, 12\%, n) + P8,000,000(P/F, 12\%, N) + P183,000[P/A, 12\%, (50-n)](P/F, 12\%, n)$$

When $n = 10$

$$= P5,500,000 + P91,500(P/A,12\%,10) + P8,000,000(P/F,12\%,10) + P183,000(P/A,12\%,40)(P/F,12\%,10)$$

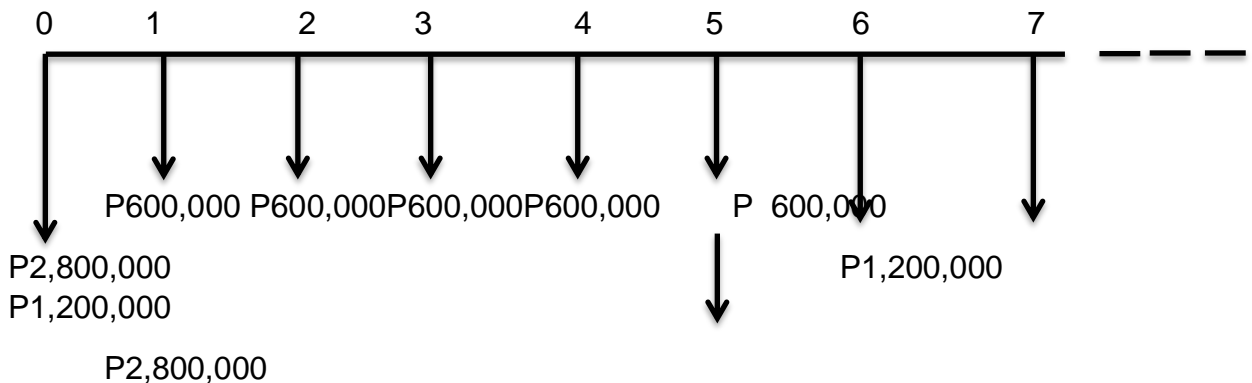
$$= P9,079,000$$

Example

A plant to provide the company's present needs can be constructed for P2,800,000 with annual operating disbursements of P600,000. It is expected that at the end of 5 years the production requirements could be doubled, which will necessitate the addition of an extension costing P2,400,000. The disbursement after 5 years will likewise double. A plan to provide the entire expected capacity can be constructed for P4,000,000 and its operating disbursement will be P640,000 when operating on half capacity (for the first 5 years) and P900,000 on full capacity. The plants are predicted to have indeterminately long life. The required rate of return is 20%. What would you recommend?

Solution

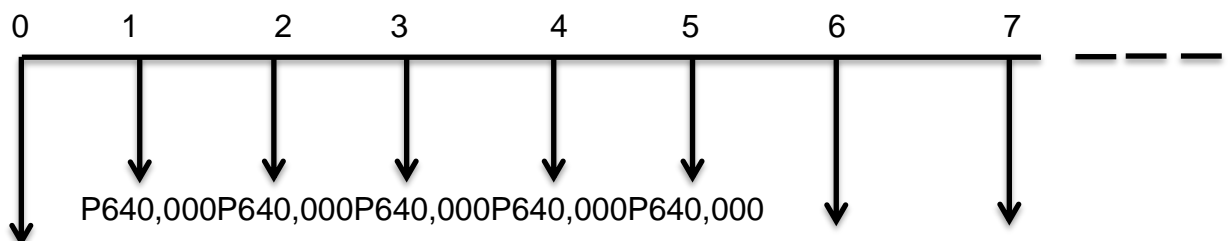
DEFERRED EXPANSION



$$\text{Capitalized cost} = P2,800,000 + P600,000(P/A,20\%,5) + P2,400,000(P/F,20\%,5)$$

$$+ \frac{P1,200,000}{0.20} (P/F,20\%,5) = P7,970,152$$

Full-size plant



P2,800,000

P900,000 P900,000

$$\text{Capitalized cost} = P4,000,000 + P640,000(P/A, 20\%, 5) + \frac{P900,000}{0.20} (P/F, 20\%, 5) = P7,722,444$$

The Full-size plant should be constructed.

Problem

Giving the following information with 15% interest rate compounded annually, solve using the comparing alternatives:

- (a) Rate of Return in Additional Investment
- (b) Annual Cost Method
- (c) Present Worth Method
- (d) Equivalent Annual Worth Method

Technology	A	B
Initial Cost	400 000	500 000
Annual Operational Cost	64 000	55 000
Service Life	25 years	25 years
Salvage Value	120 000	150 000

Problem

Three mutually exclusive design alternatives are being considered. The estimated cash flows for each alternative are given next. The MARR is 20% per year: At the conclusion of the useful life, the investment will be sold.

	A	B	C
Investment cost	P 28,000	P55,000	P40,000
Annual expenses	P15,000	P13,000	P22,000
Annual revenues	P23,000	P29,000	P32,000
Market value	P 6,000	P8,000	P10,000
Useful life	10 years	10 years	10years
IRR	26.4%	24.7%	22.4%

A decision – maker can select one of these alternatives or decide to select none of them. Make a recommendation, using PW method

Problems

1. An oil company is being offered a special coating for the gasoline underground tank installation in its service stations which will increase the life of tank from usual 10 years to 15 years. The cost of special costing will increase the cost of the tanks is P240,000. If salvage value for both is zero, and interest rate is 26%, would you recommend the use of the special coating?

2. An electric cooperative is considering the use of concrete electric pole in the expansion of its power distribution lines. a concrete pole cost P180,000 each and will last 20 years. The company is presently using creosoted wooden poles which cost P12,000 per pole and will last 10 years. if money is worth 12 per cent of first cost and zero salvage value in both cases.

3. It is proposed to place a cable on existing pole line along the shore of a lake to connect two points on opposite sides.

	Land Route	Submarine Route
Length, miles	10	5

First cost of cable per mile	P40, 000	P68, 000
Annual maintenance per mile	P950	P3, 500
Interest on investment	18%	18%
Taxes	3%	3%
Net Salvage value per mile	P12, 000	P22, 000
Life, years	15	15

Which is more economical?

4. In a cold storage plant, it is desired to determine whether to use insulation two inches thick or three inches in insulating the walls of the cold storage warehouse. Heat Absorbed through the walls without insulation would cost P96.00 per year per square meter and will cut out 89% of the loss. a three - inch insulation will cut out 92% of the loss and will cost 65% per square meter. Using a life of 15 years for the insulation with no salvage value and a minimum attractive return of 8%, what thickness of insulation should be used?

Solution

	TWO INCHES	THREE INCHES
Life	15	25
First Cost	30.4	65
E.R	8% =2.432	8%=5.2
Loss	11% of 96.00 = 10.56	8% of 96 = 7.58

Depreciation	$\frac{30 - 40}{\frac{(1.08)^{15} - 1}{0.08}} = 1.12$	$\frac{65}{\frac{(1.08)^{15} - 1}{0.08}} = 2.39$
Net Annual Savings	10.6	10.07

5. In building their plant, the officers of the international leather company had the choice between alternatives:

One alternative is to build in metro manila where the plant would cost P2,000,000. Labor would cost annually P120,000 and annual overhead P40,000. Taxes and insurance would total 5% of the first cost of the plant.

The second alternative would be to build in bulcan a plant costing P2,250,000. Taxes and insurances would be 3% of the first cost. the cost of raw materials would be the same in either plant. if capital must be recovered within 10 years and money is worth at least 20%. which site should the officers of the company choose?

6. A utility company is considering the following plans to provide a certain service required by present demand and the prospective growth of demand for coming 18 years.

Plan R requires an immediate investment of P500,000 in property that has an estimated life of 18 years and with 20% terminal salvage value. Annual disbursements for operation and maintenance will be P50,000. Annual property taxes will be 2% of first costs.

Plan S requires an immediate investment of P300,000 in property that has an estimated life of 18 years with 20% terminal salvage value. Annual disbursements for its operation and maintenance during the first 6 years will be P40,000. After 6 years, an additional investment of P400,000 will be required in property having an estimated life of 12 years with 40% terminal salvage value. After this additional property is installed, annual disbursement for operation and maintenance of the combined property will be P60,000. Annual property taxes will be 2% of the first cost of property in service at any time. Money is worth 12%. What would you recommend?

Chapter 7

Fixed, Increment, and Sunk Costs

Type of Costs

Fixed costs are those costs which remain constant, whether or not a given change on operations or policy is adopted.

Variable costs are those costs which vary with output or any change in the activities of an enterprise.

Increment costs are those that arise as a result of a change in operations or policy.

Marginal cost is the additional cost of producing one more unit of a product.

Sunk cost represents money which has been spent or capital which has been invested and which cannot be recovered due to certain reasons.

Example

A machine costing P30,000 is expected to produce 10,000 units of a certain product during its entire life before being replaced. At the end of life, it will have a scrap value of P2,000. The cost of housing the machine will be P900 a year. The power consumption per unit is P0.85 and the maintenance per unit will be P0.55. Labor will cost P1.10 per unit. If depreciation is by sinking fund method at 15%, determine the cost per unit produced if the annual production is (a)10,000 units and (b)2,500 units.

Solution

$$\text{Fixed cost per unit} = P0.85 + P0.55 + P1.10 = P2.50$$

(a) Annual production 10,000 units

The machine will last for 1 year

$$\begin{array}{rcl} \text{Depreciation} & = & P30,000 - P2,000 = P28,000 \\ \text{Housing} & & = \underline{900} \\ \text{Total} & & P28,900 \end{array}$$

$$\text{Total cost per unit} = P2.50 + \frac{P28,900}{10,000} = P5.39$$

(b) Annual production 2,500 units

The machine will last for 4 years

Annual costs:

$$\begin{array}{rcl} \text{Depreciation} & = & \frac{P30,000 - P2,000}{F/A, 15\%, 4} = P5,607 \\ \text{Housing} & & = \underline{900} \\ \text{Total} & & P6,507 \end{array}$$

$$\text{Total cost per unit} = P2.50 + \frac{P6,507}{2,500} = P5.10$$

Example

A company having a capacity of 1,600 units per year currently operating at a sales level of only 1,200 units with a selling price of P720 per unit. The fixed costs of the plant are P365,000 per year, and the variable costs are P416 per

unit. It has been estimated that a reduction of P50 per unit in the selling price would increase sales by 300 units per year.

(a) Would this be a good program to follow?

(b) An alternative being considered is to engage in a modernization plan that would increase the fixed costs by P58,000 per year, but that would reduce the variable costs by P56 per unit. Would this be a better procedure than the price-reduction program?

(c) Can you suggest any other program that might be superior to the foregoing?

Solution

(a) Present Revenue = P720(1,200) = P864,000

Present costs:

Fixed	= P365,000	
Variable = P416(1,200)	= <u>624,000</u>	
Total		<u>P 864,200</u>
Loss	(-) P 200	
New Revenue = P670(1,500)	= P1,005,000	

Costs:

Fixed	= 365,000	
Variable = (P416)(1,500)	= <u>624,000</u>	
Total		<u>P989,000</u>
Profit		P 16,000

Reducing the price would be profitable program.

(b) Revenue

Costs:

Fixed	= P423,000	
Variable = (P360)(1,200)	= <u>432,000</u>	
Total		= <u>P 855,000</u>
Profit		P 9,000

Modernization would be profitable, but it would not be as good a procedure as the price reduction.

(c) Both programs should be combined.

Example

The ABC Company has two plants in the same city, each having a capacity of ten units of the same product per month. Fixed costs of plant I are P50,000 per month and its variable costs are $P1000X - P10X^2$. Fixed costs of plant II are P50,000 per month and the variable costs are $P1,200Y - P5Y^2$. X and Y are the number of units produced. At present the sales have been established at 14 units per month with each plant producing 7 units. Should the production volume at the plant be changed, and to what extent?

Solution

Fixed costs at both plants will be unaffected and are therefore neglected.

Plant I Variable cost = $P1000X - P10X^2$

X	Variable costs
4	P3,840
5	4,750
6	5,640
7	6,510
8	7,360
9	8,190
10	9,000

Plant II

Variable costs = $P1,200Y - P5Y^2$

Y	Variable costs
10	P7,000
9	7,155
8	7,040
7	6,685
6	6,120
5	5,375
4	4,480

Units per month		Variable costs	Total variable	
Plant I	Plant II	Plant I	Plant II	Costs
4	10	P3,840	P7,000	P10,840
5	9	4,750	7,155	11,905
6	8	5,640	7,040	12,680
7	7	6,510	6,685	13,195

8	6	7,360	6,120	13,480
9	5	8,190	5,375	13,565
10	4	9,000	4,480	13,480

Operate Plant II at full capacity

The manufacturer should produce the new line of product.

Example

A manufacturing firm has a waste product that is now being dumped into a river. The waste material could be processed to produce fertilizer. This would require the investment of P320,000 in equipment and P53,000 per month for other materials. Overhead expenses will cost P55,000 per year. Taxes and insurance amount to 3% of the first cost of the equipment per year. Payroll taxes and other benefits amount to 4%. The life of the equipment is 5 years with no salvage value at the end of its life. Money is worth 20% to the firm.

If 3,600 kg of fertilizer are produced per month, which can be sold for P22.00 per kg, should the firm make the investment?

$$\text{Increment revenue} = 3,600(12)(P22.00) = P950,400$$

Increment costs:

$$\begin{aligned} \text{Depreciation} &= \frac{P320,000}{F/A, 20\%, 5} = \frac{P320,000}{7.4416} = P43,002 \\ \text{Materials} &= P53,000(12) = 636,000 \\ \text{Labor} &= P14,000(12) = 168,000 \\ \text{Overhead} &= 56,000 \\ \text{Payroll taxes and benefits} &= P168,000(0.12) = 6,720 \\ \text{Taxes and insurance} &= P320,000(0.03) = 9,600 \\ \text{Interest on capital} &= (P320,000)(0.20) = \underline{64,000} \\ \text{Total annual cost} &= P983,322 \end{aligned}$$

The firm should not invest.

Example

An old machine was purchased 3 years ago at a cost of P50,000. It was estimated to have a useful life of years, with a salvage value of P5,000. It is now going to be replaced by a new old machine costing P70,000 and P30,000 trade-in will be allowed for the old machine. Determine the sunk cost if depreciation has been computed by the (a) straight-line method, (b) sinking fund method at 16% and, (c) SYD method.

Solution

$$C_o = P50,000 \quad C_L = P5,000 \quad n = 3$$

(a) straight-line method

$$D_3 = \frac{(P50,000 - P5,000)(3)}{8} = P16,875$$

$$c_3 = C_o - D_3 = P50,000 - P16,875 = P33,125$$

Sunk cost = book value – resale value

$$= P33,125 - P30,000$$

$$= P3,125$$

(b) Sinking fund method

$$d = \frac{P50,000 - P5,000}{F/A, 16\%, 8} = \frac{P45,000}{13.2401} = P3,160$$

$$D_3 = d(F/A, 16\%, 3) = P3,160(3.5056) = P11,078$$

$$C_3 = C_o - D_3 = P50,000 - P11,078 = P38,922$$

Sunk cost = book value – resale value

$$= P38,922 - P30,000$$

$$= P8,922$$

(c) SYD method

$$\text{Sum-of-digits} = 1+2+3+\dots+7+8 = 8/2(1+6) = 36$$

$$D_3 = \frac{(8 + 7 + 6)}{36} (P50,000 - P5,000) = P26,250$$

$$C_3 = C_o - D_3 = P50,000 - P26,250 = P23,750$$

$$\text{Sunk cost} = P23,750 - P30,000 = -P6,250$$

The negative sign means that the amount is not a loss but a gain.

Problems

1. The XYZ Company has two plants producing “K Specials”. It has the following expected data for the next month’s operations. Variable incremental costs vary linearly from zero production to maximum-capacity production

	Plant A	Plant B
Maximum capacity, Units	1,000	800
Total fixed cost	P750,000	P480,000
Variable (Incremental) Costs Maximum capacity	P900,000	P800,000

(a) Performance has not been good, so the company expects to received domestic orders for only 1,200 units next month at a price of 1,400 per

unit. How should the production be distributed between the plants for optimum economic operation?

Expected orders = 1,200 units

Plant A

$$1000x = 900,000$$

$$x = 900 \text{ per unit}$$

Plant B

$$800y = 800,000$$

$$Y = 1000 \text{ per unit}$$

Unit per month		Variable Costs		Total Variable cost
Plant A	Plant B	Plant A	Plant B	
400	800	360,000	800,000	1,160,000
500	700	450,000	700,000	1,150,000
600	600	540,000	600,000	1,140,000
700	500	630,000	500,000	1,130,000
800	400	720,000	400,000	1,120,000
900	300	810,000	300,000	1,110,000
1000	200	900,000	200,000	1,100,000

Therefore Plant A should produced 1,000 units and 200 units for Plant B.

(b) If a friendly foreign offers to by 350 additional units at P1,100 per unit, should the company accept the offer? Show the incremental gain or loss.

Ans.(a) Produce 1000 units at Plant A and 200 units at Plant B, (b) Incremental gain is P35,000, the company should accept the offer.

Increment Revenue for 350 units:

$$(350)(1,100) = 385,000$$

Increment cost:

$$\text{Plant B } (350)(1000) = \underline{350,000}$$

Therefore, with gain 35,000, the company should accept the offer.

(2) A company has a new Plant A and an old Plant B in the same metropolitan area, each with a capacity of 12 units of product per month. Fixed expense at A is P40,000 per month and at B is P20,000 per month. Variable expense per month at A is $P1,000 - N^2$ where N = the number of units produced. At B it is $P2,000 \times M^2$, where M = the number of units produced. At present the sales have been established at 14 units per month with each plant producing 7 units. Should the interplant load be distributed? Why? How? Ans. 9 units and 5 units

Note: Plant A should produce 9 units per month and Plant B should produce 5 units because it's more profitable when you have the lowest total Variable Cost.

Unit per Month		Variable Costs		Total Variable cost
Plant A	Plant B	Plant A	Plant B	
3	11	9,000	242,000	251,000
4	10	16,000	200,000	216,000
5	9	25,000	162,000	187,000
6	8	36,000	128,000	164,000
7	7	49,000	98,000	147,000
8	6	64,000	72,000	136,000
9	5	81,000	50,000	131,000
10	4	100,000	32,000	132,000
11	3	121,000	18,000	139,000

(3) An asset is purchased six years ago at a cost of P70,000. It was estimated to have a useful life of ten years with a salvage value of P300 at the end of the time. It is now of no future use and can be sold for only P800. Determine the sunk cost if depreciation has been computed by

(a) The straight-line method

(b) The sum-of-the-year's digits method

a) Straight-line method

$$D_6 = \frac{(7,000 - 3,000)(6)}{10} = 4,020$$

$$C_6 = C_o - D_6 = 7,000 - 4,000 = 2,980$$

$$\begin{aligned} \text{Sunk cost} &= \text{book value} - \text{resale value} \\ &= 2,980 - 800 \\ &= \mathbf{2,180.00} \end{aligned}$$

b) Sum-of-the-year's digits method

Sum of digits = $1+2+3+\dots+9+10 = 55$

$$D_6 = \frac{(10 + 9 + \dots + 5 + 6)}{55} (7,000 - 3,000) = 5,481.2$$

$$C_6 = 7,000 - 5,481.82 = 1,518.18$$

$$\text{Sunk cost} = 1,518.18 - 800 = \mathbf{718.18}$$

Chapter 8

Replacement Studies

A decision often encountered in business firms and government organization as well as by individuals is whether an existing asset should be retired from use continued in service replaced with new asset. Engineering economy studies of replacement situations are performed using the same methods involving two or more alternatives. However, the decision is whether to replace an existing (old) asset called the defender with a new asset called the challenger.

Reasons for Replacement Analysis

1. Physical Impairment (Deterioration)

These are the changes that occur in the physical condition of the asset. Normally, continued use (aging) results in the less efficient operation of the asset. Routine maintenance and breakdown repair costs increase, energy use may increase, more operator time may be required and so forth.

The existing asset is completely or partially worn out and will no longer function satisfactorily without extensive repairs.

2. Altered Requirement

Capital assets are used to produce goods and services that satisfy human wants. When the demand for a good or service either increases or decreases or the design of a good service, or changes, the related assets may have the economics of the use affected.

The existing asset does not have sufficient capacity to meet the present demands that are placed on it.

3. Technology

The impact of changes in technology varies among different types of assets. In general the costs per unit of production as well as quality and other factors are favorably impacted by changes in technology.

This may be caused either by a lessening in the demand for the service rendered by the asset or the availability of more efficient assets which will operate with lower out-of-pocket costs.

4. Financing

Financing factors involve economic opportunity changes extended to the physical operation or use of assets and may involve income consideration.

It is possible to rent an identical or comparable asset or property, thus freeing capital for other and more profitable use.

SUNK COST DUE TO UNAMORTIZED VALUE

Unamortized value of an equipment or property is the difference between its book value and its resale value when replaced. Unamortized value should be considered as a sunk cost or loss.

BASIC PATTERNS FOR REPLACEMENT STUDIES

Replacement economy studies may be made by any of the basic procedures or patterns which have been discussed previously. However, in most cases either the rate of return method or the annual cost method is used.

Example

An existing factory must be enlarged or replaced to accommodate new production machinery. The structure was built at a cost of P2.6 million. Its present book value, based on straight line depreciation is P700,000 but it has been appraised at 800,000. If the structure is altered, the cost will be P1.6 million and its service life will be extended 8 years with a salvage value of P600,000. A new factory could be purchased or built for P5.0 million. It would have a life of 20 years and a salvage value of P700,000. Annual maintenance of the new building would be 160,000 compared with P100,000 in the enlarged structure. However, the improved layout in the new building would reduce annual production cost by P240,000. All other expenses for the new structure are estimated as being equal. Using an investment rate of 8 per cent, determine which is the more attractive investment for this firm.

SOLUTION

Enlarged building

Annual costs:

$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}800,000 + \text{P}1,600,000 - \text{P}600,000}{F/A, 8\%, 8} \\ &= \frac{\text{P}1,800,000}{10.6366} &&= \text{P}169,227 \\ \text{Maintenance} &&&= 100,000 \\ \text{Production (excess)} &&&= \underline{\text{P}240,000} \\ \text{Total annual cost} &&&= \text{P}509,227 \end{aligned}$$

New building

Annual costs:

$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}500,000 - \text{P}700,000}{F/A, 8\%, 20} \\ &= \frac{\text{P}4,300,000}{45.7620} &&= \text{P}93,964 \\ \text{Maintenance} &&&= \underline{\text{P}160,000} \\ \text{Total annual cost} &&&= \text{P}253,964 \end{aligned}$$

$$\text{Annual savings} = \text{P}509,227 - \text{P}253,964 = \text{P}255,263$$

$$\text{Additional investment} = \text{P}5,000,000 - \text{P}800,000 - \text{P}1,600,000 =$$

P2,600,000

$$\begin{aligned}\text{Rate of return on additional investment} &= \frac{P255,263}{P2,600,000} \times 100 \\ &= 9.82\%\end{aligned}$$

Construct a new building.

Example

A decision must be made whether to replace a certain engine with a new one, or to rebore the cylinder of the old engine and thoroughly recondition it. The original cost of the old engine 10 years ago was P700,000; to rebore and recondition it will cost P28,000, but would extend its useful life for 5 years. A new engine will have a first cost of P62,000 and will have an estimated life of 10 years. It is expected that the annual cost of fuel and lubricant with the reconditioned engine will be about P20,000 and that this cost will be 15% less with the new engine. It is also believed that repairs will be 2500 a year less with the new engine than with the reconditioned one. Assume that neither engine has any net realizable value when retired. If money is worth 16%, what would you recommend?

Solution

Reconditioned engine

Annual cost:

$$\begin{aligned}\text{Depreciation} &= \frac{P28,000}{\frac{F}{A}, 16\%, 5} = \frac{P28,000}{6.877} = P 4,071 \\ \text{Fuel and lubricants} &= 20,000 \\ \text{Repairs excess} &= 2,500 \\ \text{Interest on capital} &= P28,000 (0.16) = \underline{4,480} \\ \text{Total annual cost} &= 31,051\end{aligned}$$

New engine

Annual cost:

$$\begin{aligned}\text{Depreciation} &= \frac{P28,000}{\frac{F}{A}, 16\%, 10} = \frac{P28,000}{21.32} = P 2,908 \\ \text{Fuel and lubricants} &= P20,000 (0.85) = 20,000 \\ \text{Interest on capital} &= P62,000 (0.16) = \underline{4,480} \\ \text{Total annual cost} &= 29,828\end{aligned}$$

The old engine should be replaced

Example

Four years ago an ore-crushing unit was installed at a mine which cost P81,000. Annual operating cost for this unit are P3,540. This unit was estimated to have a life of 10 years. The quantity of ore to be handled is to be doubled and is expected to continue at this higher rate for at least 10 years. A unit that will handle the same quantity of ore and have same operating cost as the one now in service can be installed for P75,000. This unit will have a useful life of 6 years.

A unit with double the capacity of the one now in use can be installed for P112,000. Its life is estimated at 6 years and its annual operating cost are estimated at P4,950. The present realizable value of the unit now in use is P26,000. All units under consideration will have an estimated salvage value at retirement age of 12% of the original cost. Interest rate is 20%. Annual taxes and insurance are 2.5% of the original cost.

What would you recommend?

Solution

Augmentation

Annual cost:

Old unit

$$\begin{aligned} \text{Depreciation} &= \frac{P26,000 - (81,000)(0.12)}{\frac{F}{A}, 20\%, 6} \\ &= \frac{P16,280}{9.9299} &&= P1,639 \\ \text{Operation} &&&= 3,540 \\ \text{Taxes and insurance (P81,000) (0.025)} &&&= 2,025 \end{aligned}$$

New small unit

$$\begin{aligned} \text{Depreciation} &= \frac{P75,000 - (75,000)(0.12)}{\frac{F}{A}, 20\%, 6} \\ &= \frac{P66,000}{9.9299} &&= 6,647 \\ \text{Operation} &&&= 3,540 \\ \text{Taxes and insurance (P75,000) (0.025)} &&&= \underline{1,875} \\ \text{Total annual cost} &&&= 19,266 \end{aligned}$$

Replacement

Annual cost:

New big unit

$$\begin{aligned} \text{Depreciation} &= \frac{P112,000 - (112,000)(0.12)}{\frac{F}{A}, 20\%, 6} \\ &= \frac{P98,560}{9.9299} = P9,926 \end{aligned}$$

Operation	= 4,950
Taxes and insurance (P112,000) (0.025)	= <u>2,800</u>
Total annual cost	= 17,676

Annual savings = P19,266 – P17,676 = P1,590

Additional investment = P112,000 – P75,000 – P26,000 = P11,000

Rate of return on additional investment = $\frac{P1,590}{11,000} \times 100 = 14.5\%$

Buy a new small unit to augment the old unit.

Problem

A car be purchased for P600,000 when new. There follows a schedule of annual operating expenses for each year and trade in values at the end of each year. Assume that these amounts would be repeated for future replacements, and that the car will not be kept more than 3 years. if interest on invested capital is 15% before taxes. Determine at which year's end the car should be replaced so that cost will be minimized.

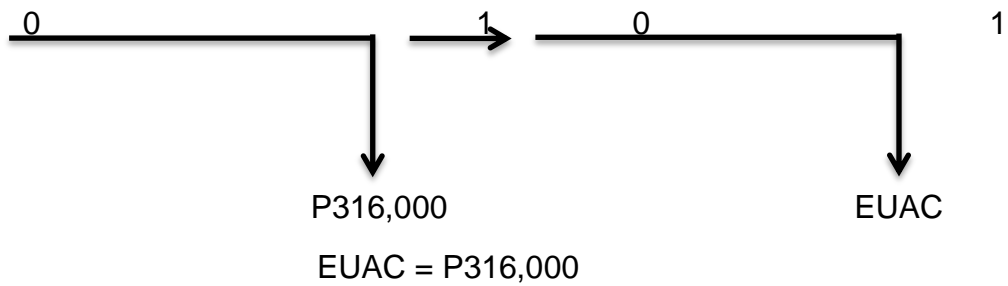
	year 1	year 2	year 3
Operating expenses for year	P34.00	P38.000	P41.000
Trade in value at the end of year	P408.000	P336.000	P240,000

SOLUTION

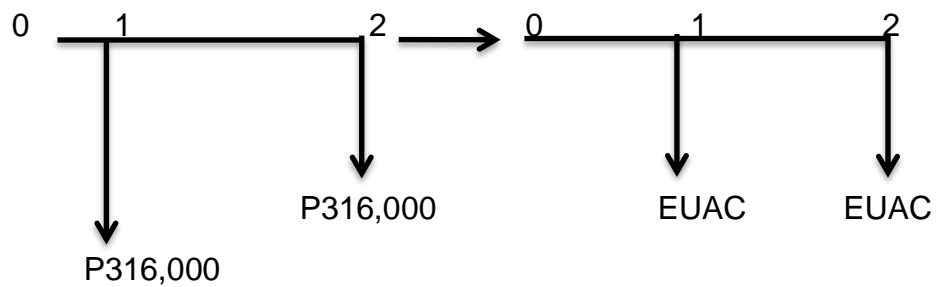
Cost of keeping each year

	1	2	3
Operation	P34,000	38,000	41,000
Depreciation	192,000	144,000	96,000
Interest on capital (15%)	<u>P90,000</u>	<u>P61,200</u>	<u>P50,400</u>
Total	P316,000	P243,200	P187,400

1 year:



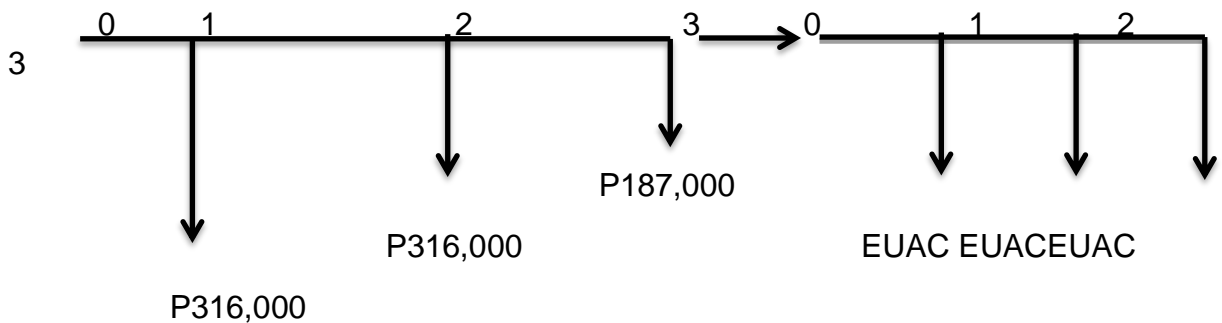
2 years:



$$EUAC = [316,000 + P 243,000 (P/F, 15\%, 2)] (A/P, 15\%, 2)$$

$$= P307,385$$

3 years



$$\begin{aligned} \text{EUAC} &= [P316,000 + P243,000 (P/F, 15\%, 2) + P187,400 (P/F, 15\%, 3)] \\ &\quad (A/P, 15\%, 3) \\ &= P272,851 \end{aligned}$$

Thus it is cheaper to keep the car three years.

Problems

1. A recapping plant is planning to acquire a new diesel generating set to replace its present unit which they run during brownouts. The new set would cost P135,000 with a five (5) year-life, and no estimated salvage value. Variable cost would be P150,000 a year. The present generating set has a book value of 75,000 and a remaining life of 5 year. Its disposal value is now P7,500, but it would be zero after 5

years. Variable operating cost would be P187,500 a year. Money is worth 10%.

Which is profitable, to buy the new generator set or retain the present set? Support your answer by showing your computation.

2. A company that sells computers has proposed to a small public utility company that it purchase a small electronic computer for P1,000,000 to replace ten calculating machines and their operators. An annual service maintenance contract for the computer will be provided at a cost of P100,000 per year. One operator will be required at a salary of P140,000 per year. The estimated economical life of the computer is 10 years. The calculating machine cost P7,000 each when new, 5 years ago and presently can be sold for P2,000 each. They have an estimated life of 8 years and an expected ultimate trade-in value of P1,000 each. Each calculating machine has cost P500 each. Taxes and insurance on all equipment is 2% of the first cost per year.

If capital cost the company about 25%, would you recommend the computer installation?

3. It is desired to determine the present economic value of an old machine by considering of how it compares with the best modern machine that could replace it. The old machine is expected to require out of pocket cost of P85,000 each year for 4 years and then scrapped for 5,000 residual value. The new machine requires an investment of P40,000 and would have out of pocket cost of P79,000 a year for 8 years and then zero salvage value. Invested capital should earn a minimum return of 15% before taxes. Determine the present value of the old machine.

Chapter 9

Break-even Analysis

In engineering economy, many situations are encountered where the cost of two or more alternatives maybe affected by a common variable. Break-even

point is the value of the variable for which the costs for the alternatives will be equal.

An economic breakeven point for an operation occurs when total revenue equals total cost.

$$C_1 = f_1(x) \qquad C_2 = f_2(x)$$

Where:

C_1 = certain specified total cost applicable to Alternative 1

C_2 = certain specified total cost applicable to Alternative 2

X = a common independent variable affecting alternative 1 and Alternative 2

The break-even point is where C_1 and C_2 are equal,

$$f_1(x) = f_2(x)$$

Which maybe solve for x, the break-even point.

Total revenue, TR that will result from a business venture during a given period is the product of the selling price per unit, p, and the numbers of units sold, D. Thus,

$$\text{TR} = \text{price/unit} \times \text{no of units sold}$$

Total cost consists of fixed cost and variable cost. Fixed costs remain constant over a wide range of activities as long as the business does not permanently discontinue operations, but variable costs vary in total with the volume of output, Thus at any output, total cost is

$$C_T = C_F + C_v$$

Where C_T = total cost

C_F = Fixed Cost

C_v = variable cost

= variable cost per unit X no of units sold

Hence to break even,

$$\text{TR} = C_T$$

Let N = no of unit sold

Price / unit x N = CF+ Variable cost/ unit x N

Therefore, N is the break – even point.

BREAK-EVEN CHART

Break-even chart is a graphical representation of break-even analysis. The break-even point is the quantity of production at which the income is equal to total cost. It is the intersection of the income line and the total cost line on the break-even chart.

When two alternatives are to be compared, the break-even point is the intersection of the total cost line for each alternative on the break-even chart.

Example

Two alternatives are being considered for the production of a particular part for which there is a long term demand. Machine A cost P50,000 and is expected to last 3 years and have a P10,000 salvage value. Machine B cost P75,000 and is expected to last 6 years and have a zero salvage value. Machine A can produce a part in 18 seconds; Machine B requires only 12 seconds per part. The out-of-pocket hourly cost of operation is P38 for A and P30 for B. monthly maintenance costs are P200 for A and P220 for B.

If interest on invested capital is 25%, determine the number of parts per year at which the machines are equally economical. If the expected number of parts per year is greater than this break-even quantity, which machine would be favored?

Algebraic Solution

Let N = number of parts per year for equal costs

By the Annual cost method

Machine A

Annual costs:

$$\text{Depreciation} = \frac{P50,000 - P10,000}{F/A, 25\%, 3} = \frac{P40,000}{3.8125} = P10,492$$

$$\text{Maintenance} = (P200)(12) = P 2,400$$

$$\text{Operation} = (38)\left(-\frac{18}{3,600}\right)(N) = 0.19N$$

$$\text{Interest on capital} = (P50,000)(0.25) = \underline{12,500}$$

$$\text{Total Annual cost} \quad P25,392 \quad + \quad P0.19N$$

Machine B

Annual costs:

$$\text{Depreciation} = \frac{P75,000}{F/A, 25\%, 6} = \frac{P75,000}{11.2588} = P6,661$$

$$\text{Maintenance} = (P220)(12) = P2,640$$

$$\text{Operation} = (P30)\left(\frac{12}{3,600}\right)(N) = 0.10N$$

$$\text{Interest on capital} = P75,000(0.25) = \underline{18,750}$$

$$\text{Total Annual cost} = P 28,051 + 0.10N$$

Equating total annual cost,

$$P25,392 + 0.19N = P28,051 + 0.10N$$

$$N = 29,544$$

Machine B will be more economical for number of parts more than 29,544

GRAPHICAL SOLUTION

Machine A

$$\text{Fixed costs} = P25,392$$

$$\text{Variable costs} = P0.19N$$

When $N = 50,000$ parts

$$\text{Variable costs} = (0.19)(50,000) = P9,500$$

$$\text{Total cost} = P25,392 + P 9,500 = P34,892$$

Machine B

$$\text{Fixed costs} = P28,051$$

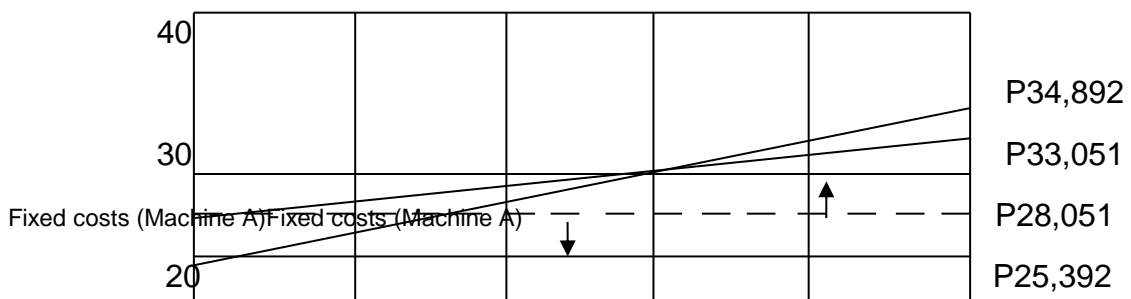
$$\text{Variable costs} = P0.10N$$

When $N = 50,000$ parts

$$\text{Variable costs} = (0.10N)(50,000) = P5,000$$

$$\text{Total cost} = P28,051 + P5,000 = P33,051$$

Break-even Chart



10

0 10 20 30 40 50
No. of parts x 1000

Break-even point = 29,500 parts (approximately)

Example

Two electric motors are being considered to power an industrial hoist. Each is capable of providing 100 hp. Pertinent data for each motor are as follows:

	Motor A	Motor B
Investment	P25,000	P32,000
Electrical efficiency	84%	88%
Maintenance per year	400	600
Life, years	10	10

Money is worth 20%. If the expected usage of the hoist is 700 hours per year, what would the cost of electrical power have to be before Motor A is favored over Motor B?

Solution

Let x = cost of electrical power for both motors to be equally economical

Motor A

Annual costs:

$$\text{Depreciation} = \frac{P25,000}{F/A, 20\%, 10} = \frac{P25,000}{25.9587} = P963$$

$$\text{Power} = \frac{(100)(0.746)(700)(x)}{0.84} = 62,167x$$

$$\text{Maintenance} = 400$$

$$\text{Interest on capital} = P25,000(0.20) = 5,000$$

$$\text{Total Annual cost} = P6,363 + 62,167x$$

Motor B

Annual Costs:

$$\text{Depreciation} = \frac{P32,000}{F/A, 20\%, 10} = \frac{P32,000}{25.9587} = P1,232$$

$$\text{Power} = \frac{(100)(0.746)(7000(x))}{0.88} = P59,341x$$

$$\text{Maintenance} = 600$$

$$\text{Interest on capital} = P32,000 (0.20) = \underline{6,400}$$

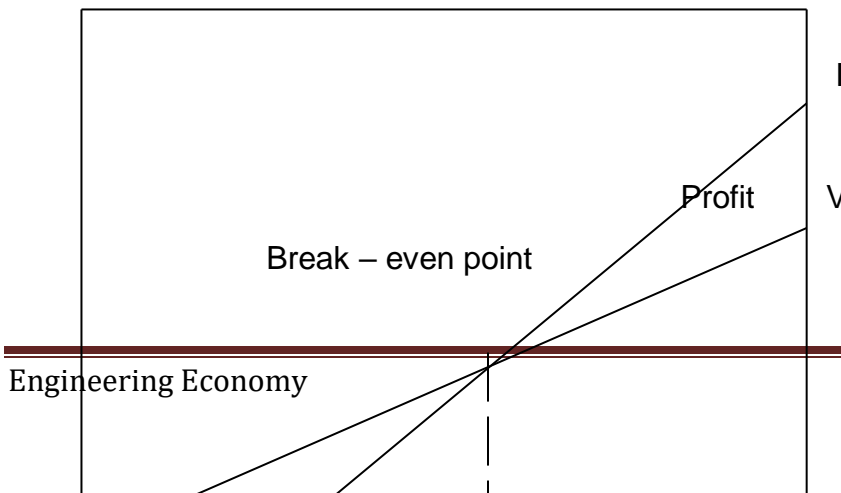
$$\text{Total Annual cost} = P8,232 + 59341x$$

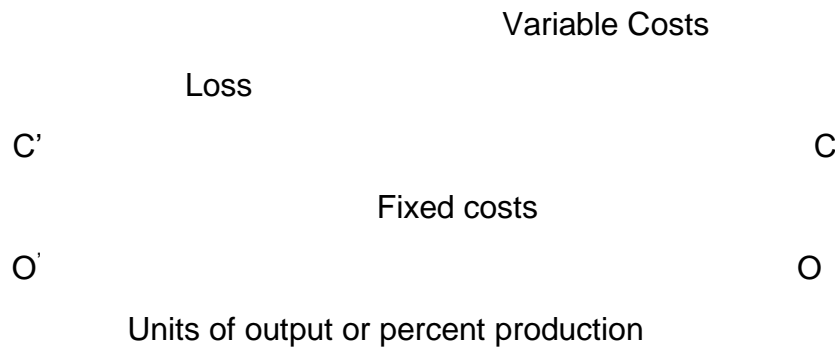
$$P6,363 + P62,167 x = P8,232 + P59,341 x$$

$$x = P0.6614 \text{ kwh}$$

Motor A will be more economical for electrical power cost less than P0.6614 per kwh.

BREAK-EVEN CHART FOR BUSINESS ENTERPRISE





Line C'C represents the fixed costs of production. Area C'COO'C' is the fixed costs region.

Line C'V shows the variation in total variable costs since its starting point is at C'. Area C'VCC' is the variable costs region and area C'VOO'C' is the total cost region.

Line OI' represents the gross income from sales. Area O'I'OO' is the income region. Area BIVB is the profit region and area BO'C'B' is the loss region.

Example

The cost of producing small transistor radio set consists of P23.00 for labor and P37.00 for materials. The fixed charges in operating the plant are P100,000 per month. The variable cost is P1.00 per set . The radio set can be sold for P75.00 each. Determine how many sets must be produced per month to break-even.

Algebraic Solution

Let x = the number of sets to be produced per month to break-even

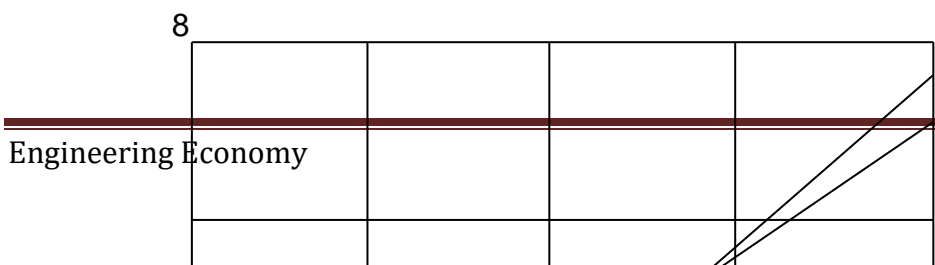
Total income = P75(x)

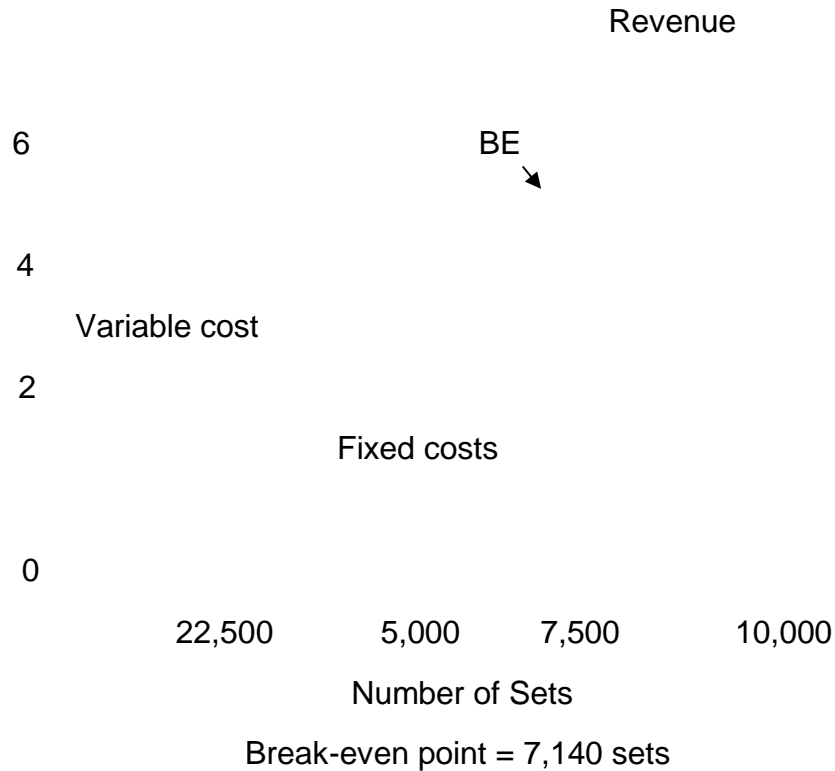
Total cost = P100,000 + (P23 + P37 + P1) x
 = P100,000 + P61x

To break-even,

P75x = P100,000 + P61x
 = 7,143 sets

Graphical Solution





Example

A company has a production capacity of 500 units per month and its fixed costs are P250,000 a month. The variable cost per unit are P1,150 and each unit can be sold for P2,000. Economy measures are instituted to reduce the fixed costs by 10 per cent and the variable cost by 20 per cent. Determine the old and the new break-even points. What are the old and the new profit at 100 per cent capacity?

Algebraic solution

Income = P2,000 per unit

Old fixed costs = P250,000 per month

Old variable costs = P1,150 per unit

Let x = old break-even point

Income = $P2,000x$

Total cost = $P250,000 + P1,150x$

To break-even,

$$P2,000x = P250,000 + P1,150x$$

$$x = 294 \text{ units per month}$$

$$\text{Old profit} = (P2,000)(500) - P250,000 - (P1,150)(500)$$

= P175,000 per month
New Fixed costs = (P250,000)(0.90) = P225,000
New variable costs = (P1,150)(0.80) = P920 per unit

Let y = the new break-even point

Income = P2,000 y

Total cost = P225,000 + P920 y

To break-even,

P2,000 y = P250,000 + P920 y

y = 208 units per month

New profit = (P2,000)(500) – P225,000 – (P920)(500)

= P315,000 per month

Old break-even point = 294 units per month

Old profit = P175,000 per month

New break-even point = 208 units per month

New profit = P315,000 per month

Example

A local factory assembling calculators produces 400 units per month and sells them at P1,800 each. Dividends are 8% on 8000 shares with par value of P250 each. The fixed operating cost per month is P25,000. Other costs are P1,000 per unit. Determine the break-even point. If only 200 units were produced per month determine the profit or loss.

Solution

Income = P1,800 per unit

Fixed costs = P25,000 per month

Variable costs = P1,000 per unit

Dividend = 8%

Let x = number of calculators per month to break-even

Income = P1,800 x

Total cost = P25,000 + P1000 x

To break-even,

$$\begin{aligned} \text{Income} &= \text{Total cost} \\ P1,800x &= P25,000 + P1,000x \\ x &= 31.25 \text{ say } 32 \text{ units} \end{aligned}$$

$$\text{Dividend} = \frac{(P250)(0.08)(8,000)}{12} = P13,333 \text{ per month}$$

$$\text{Income} = \text{Total cost} + \text{Dividend} + \text{Profit/loss}$$

For 200 units,

$$(P1,800)(200) = P25,000 + (P1,000)(200) + P13,333 + \text{Profit/loss}$$

$$\text{Profit} = P121,667 \text{ per month}$$

Example

A company manufacturing calculators has a capacity of 200 units a month. The variable costs are P1,000 per unit. The average selling price of the calculators is P2,500. Fixed cost of the company amount to P150,000 per month, which include all taxes. The company pays an annual dividend of P12 per share on each of the 30,000 shares of common stocks.

- (a) Determine the number of calculators that must be sold each month to break-even and the sales volume corresponding to the unhealthy point.
- (b) What is the profit or loss if 150 units were produced and sold a month/

(*Unhealthy point* is the sales volume at which the business will be able to pay exactly the desired rate of dividend)

Algebraic Solution

Income = P2,500 per unit
 Fixed costs = P150,000 per month
 Variable costs = P1,000 per unit
 Dividend = P12 per share per year
 (a) Let x = the break-even point

$$\begin{aligned} \text{Income} &= P2,500x \\ \text{Total cost} &= P150,000 + P1000x \\ \text{To break-even,} \end{aligned}$$

$$\begin{aligned} \text{Income} &= \text{Total cost} \\ P2,500x &= P150,000 + P1,000x \\ x &= 100 \text{ units per month} \end{aligned}$$

$$\text{Dividend} = \frac{(P12)(30,000)}{12} = P30,000 \text{ per month}$$

Let y = the unhealthy point

At the unhealthy point,

$$\text{Income} = \text{Total cost} + \text{Dividend}$$

$$P2,500y = P150,000 + 1,000y + P30,000$$

$$Y = 120 \text{ units per month}$$

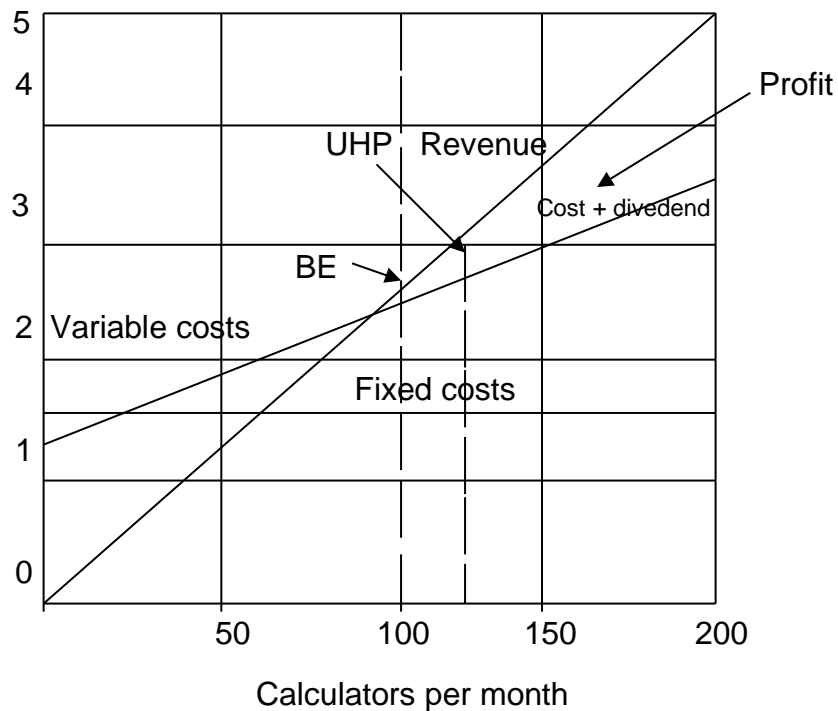
(b) Income = Total costs + Dividend + Profit/loss

For 150 units,

$$(P2,500)(150) = P150,000 + (P1000)(150) + P30,000 + \text{Profit/loss}$$

$$\text{Profit} = P45,000 \text{ per month}$$

Graphical Solution



(a) Break-even point = 100 units per month

Unhealthy point = 120 units per month

(b) Profit = P45,000 per month

Problem

A plant has a capacity of 4,100 hydraulic pumps per month. The fixed cost is P504, 000 per month. The variable cost is P166 per pump, and the sales price is P328 per pump (assume that sales equal output volume). What is the break – even point in number of pumps per month? What is percentage reduction will occur in the break – even point if fixed costs are reduced by 18% and unit variable cost by 6%?

Problem

A stereo manufacturer can produce 1 unit at labor cost of P1, 700, material cost of P2,400 per unit, variable cost of P600 per unit and fixed cost of P66, 000 per year. If each unit are sold at P8, 000. How many units must be produced and sold per annum to break – even?

Problem

An item which can be sold for P60 per unit is produced with the following cost, material cost, P14 per unit; labor cost of P12 per unit; fixed cost P12,000; variable cost of P8 per unit. What is the break – even sale in volume if 2 out of every 12 units produced is defective and is rejected with only full recovery on material.

Problem

A factory with capacity of 700,000 units per year operates at 62% capacity. The annual income is P430, 000 annual fixed cost are P190, 000 and the variable costs are P0.348 per unit. What is the break – even point in units?

Problem

A machine part to be machined may be made either from an alloy of aluminum or steel. There is an order for 8,000 units. Steel costs P3.80 per kg, while aluminum costs P8.70 per kg. If steel is used, the steel per unit weighs no grams; for aluminum, 30 grams. When steel is used, 50 units can be produced per hour, for aluminum, 80 units per hour with the aid of a tool costing P640, which will be useless after 8,000 units are finished. The cost of the machine and operation is P10.80 per hour. If all other cost are identical, determine which material will be more economical.

Problem

A contractor has a job which should be completed in 100 day. At present, he has 80 men on the job and it is estimated that they will finish the work in 130 days. Of the 80 men. 50 are each paid P120 a day; 25 at P180 a day; and 5 at P250 a day. For each day beyond the original 100 days, the contractor has to pay P500 liquidated damages/

- a. How many more man should the contractor add so that he can complete the work on time?
- b. If of the additional men, 2 are paid P180 a day, and the rest at P120 a day would the contractor save money by employing more men and not paying the fine?

Problem

One method for developing a mine containing an estimated 100,000 tons of ore will result in the recovery of 62% of the available ore deposit and will cost P23 per ton of material removed. A second method of developments will recover only 50% of the ore deposit, but will cost only P15 per ton of material removed. Subsequent processing of the removed ore recovers 300 pound of material form each ton of processed ore and costs P40 per ton of ore processed. The recovered metal can be sold for P0.80 per pound. Which method for developing the mine should be used if your objective is to maximize total profit from the mine?

Problem

Ocean water contains 0.9 ounce of gold per ton. Method A costs P220 per ton of water processed and will recover 80% of the metal. Method B costs P160 per ton of water processed and will recover 65% of the metal. The two methods require the same investment and are capable of producing the same amount of gold each day. If the extracted gold can be sold for P350 per ounce, which method of extraction should be used? Assume that the ocean water is unlimited. Work this problem on the basis of profit per ounce of gold extracted.

Problem

Two methods are being used for developing a mine. Method A contains an estimated of 70,000 tons of ore resulting in the recovery of 55% of the available ore deposit and will cost P21 per ton of material removed. Method B will recover only 45% of the ore deposit, but will cost only P16 per ton of material removed. Which method for developing the mine used be used if the objective is to maximize total profit from the mine? Note that the subsequent processing of the removed ore recovery 280 pound of metal form each ton of processed ore and costs P35 per ton of ore processed. The recovered metal can be sold for P0.80 per pound.

Problem

The one of a gold mine in Diwalwal, contains on the average, 1.2 ounce of gold per tons of ore. The method of processing cost P2000 per ton and recovers 90% of the gold.

The ore of a gold mine in Mountain Province contains on the average, 1 ounce of gold per ton of ore and processing costs P1800 per ton and recover 85% of the gold. God is sold at P 3,000 per ounce. Which of the two gold mines is better processing method?

Problem

An equipment installation job in the completion stage can be completed 40 days of 8 hour day work, with 40 men working. With the contract expiring the mechanical engineering contractor decided to add 10 men on the job, overtime not being permitted. If the liquidated damage is P 2000 per day of delay, and the men paid P 80 per day will the engineering be able to complete the job on time? How much would he save money with addition of workers?

Problems

1. A Company is considering two alternatives with regards to an equipment which it needs. The alternatives are as follows:

Alternative A; Purchase

Cost of equipment	P700,000
Salvage value	100,000
Daily operating cost	500
Economic life, years	10

Alternative B: Rental at P1,500 per day

At 18% interest, how many days per year must the equipment be in use if Alternative A is to be chosen. Ans. 152 days

2. Data for two 30 hp motors are as follows:

	Alpha Motor	Beta motor
Original cost	P37,500	P48,000
Annual maintenance	1,500	750
Life, years	10	10
Efficiency	87%	92%
Taxes and insurance	3%	3%

Power cost is P2.00 per kwh. If money is worth 20%, how many hours per year would the motors have to be operated at full load for them to be equally economical? If the expected number of hours of operation per year exceeds the break-even point, which motor is more economical? Ans. 444 hours, Beta Motor

3. A small shop on Bulacan fabricates portable threshers for palay producers in the locality. The shop can produce each thresher at a labor cost of P1,500. The cost of material for each unit is P2,500. The variable costs amount to P650 per unit, while fixed charges incurred per annum totals P69,000. If the portable threshers are sold at P7,800 per unit, how many units must be produced and sold per annum to break-even?

4. Compute for the number of blocks that an ice plant must be able to sale per month to break-even based on the following data:

Cost of electricity per block	P20	
Tax to be paid per month	2	
Real estate tax		3,500/month
Salaries and wages	25,000/month	
Others		12,000/month
Selling price of ice		55/block

5. A local company assembling stereo radio cassette produces 500 units per month at a cost of P800 per unit. Each stereo radio cassette sells for P1,200. If the firm makes a profit of 10% on its 10,000 shares with par value of P200 per share, and the fixed costs are P20,000,000 per month.
- (a) What is the break-even point?

(b) How much is the loss or profit if only 100 units are produced in a given month?

6. A plant produces 300 units of an equipment a month of P3,600 each. A unit sells for P4.00 . The company has 10,000 shares of stock at P200 par value whose annual dividend is 20%. The fixed cost of production is P120,000 a month.
- (a) What is the break-even point?

(b) What is the “unhealthy point”?

(c) What is the profit or loss if production is 60% of capacity?

Chapter 10

Benefit/Cost Ratio

The method of selecting of alternatives that is most commonly used by government agencies for analyzing the desirability of public projects is the benefit/cost ratio (B/C ratio). The B/C method of analysis is based on the ratio of the benefits to cost associated with a particular project.

$$B/C = \frac{BENEFITS - DISBENEFITS}{COST}$$

Benefits are advantages, expressed in terms of pesos which happen to the owner. On the other hand, when the project under consideration involves disadvantage to the owner, these are known as disbenefits. The costs are anticipated expenditure for construction, operation, maintenance, etc. AB/C ratio greater than or equal to 1.0 indicates that the project under consideration is economically advantageous.

Public projects are those authorized financed, and operated by federal, state or local government agencies. The benefit-cost ratio method is used for the evaluating of such projects. It is also known as the Savings-investment Ratio (SIR). It involves the calculation of a ratio of benefits to costs. A project is acceptable when the B-C ratio, is greater than or equal to 1.0.

Definition of Terms:

Project Benefits (B) are defined as the favorable consequence to the public.

Project Cost (C) represents the monetary disbursement required of the government.

Project Disbenefits (D) represents the negative consequences of the project to the public.

Whether evaluating a project in the private sector or in the public sector time value of money must be considered to account for the timing of cash flows occurring after the inception of the project.

Benefit-Cost ratio with present worth

$$B/C = \frac{PW(\text{benefits of the proposed project})}{PW(\text{total cost of the proposed project})}$$

$$B-C = \frac{PW(B)}{I - PW(SV) + PW(O\&M)}$$

Where: PW (.) = present worth of (.)
 B = benefits of the proposed project
 I = initial investment in the proposed project
 SV = salvage value
 O&M = operating and maintenance cost of the proposed project

Benefit-Cost ratio with Annual Worth

$$B-C = \frac{AW(\text{benefits of the proposed project})}{AW(\text{total cost of the proposed project})}$$

$$B-C = \frac{AW(B)}{CR+AW(O\&M)}$$

Where: AW (.) = annual worth of (.)
 B = benefits of the proposed project
 CR = capital recovery amount
 = $I(A/P, i\%, n) - SV(A/F, i\%, n)$
 O&M = operating and maintenance costs of the proposed project

Comparing Alternatives by B/C Ratio Method

When comparing mutually exclusive alternatives with the B-C ratio method the following steps should be considered.

1. The alternatives are ranked first cost (PW, AW, or FW). The do-nothing alternative is selected as a base alternative.
2. The B-C ratio is then calculated for the alternative having the lowest equivalent cost. It is the greater than or equal to 1.0, then it will be the new alternative; otherwise, do nothing remains as the base alternative.
3. 3. The next least equivalent cost alternative is then selected and determined the increment B–C ratio ($\Delta B/\Delta C$) between this alternative and baseline.
4. If ($\Delta B/\Delta C$) \geq 1.0, the nest higher equivalent becomes the new baseline; otherwise the last baseline alternative is maintained.
5. The increment B-C ratios are determined for each successively higher equivalent cost alternative until the last alternative has been compared.

Example

A nonprofit educational research organization, is contemplating an investment of P1,500,000 in grants to develop new ways to teach people the rudiments of profession. The grants would extend over a ten-year period and would achieve an estimated savings of P500,000 per year in professors salaries, student tuition, and other expenses. The program would be an addition to ongoing and planned activities, thus an estimated P100,000 a year would have to be released from other program to support the education research a rate of return of 15% is expected is this a good program?

SOLUTION

BENEFIT = P500,000 per year

DISBENEFIT P100,000 per

COST = P1,500,000 (A/P, 5%, 10) = P298,950 per year

$$B/C = \frac{P500,000 - P100,000}{P298,950} = \mathbf{1.34}$$

The project is justified, since $B/C > 1.00$.

Example

The national government intends to build a dam and hydroelectric project in the Cagayan valley at a total cost of 455,500,000. The project will be financed by soft foreign loan with a rate of interest of 5% per year. The annual cost for operation, maintenance, distribution facilities and others would total P15,100,000. Annual revenues and benefits are estimated to be P56,500,000. If the structures are expected to last for 50 years with no salvage value, determine the B/C ratio of the project.

SOLUTION

By the equivalent uniform annual cost method

$$\begin{aligned} \text{Annual benefit} &= P56,500,000 \\ &= P455,500,000(A/P, 5\%, 50) + P15,100,000 \\ &= P40,061,400 \end{aligned}$$

$$B/C = \frac{P56,500,000}{P40,061,400} = \mathbf{1.410}$$

This is a good project, since $B/C > 1.0$.

By the present worth cost method

$$PW_{\text{BENEFIT}} = P56,500,000 (P/A, \%, 50) = P1,031 \times 10^6$$

$$\begin{aligned} PW_{\text{COST}} &= P455. \times 10^6 + P15.1 \times 10^6 (P/A, 5\%, 50) \\ &= P731.164 \times 10^6 \end{aligned}$$

$$B/C = \frac{P1,031 \times 10^6}{P731.164 \times 10^6} = \mathbf{1.41}$$

This is a good project, since $B/C > 1.0$

Alternative comparison by Benefit/cost analysis

In computing the benefit cost ratio by eq(11-1) for a given alternative the benefits and cost used in the calculation represent the differences between the alternatives.

Example

The routes are under consideration for a new highway route A would be located about five miles from central business district and would require longer travel distances by local commuter traffic. Route B would pass directly through the downtown area and although its construction would be higher. It would reduce the travel time and distance for local commuters.

The costs for two routes are as follows.

	ROUTE A	ROUTE B
INITIAL COST	P200,000,000	P250,000,000
Maintenance/ year	P700,000	P1,100,000
Road upkeep cost/year	10,000,000	4,000,000

If the roads are assumed to last 30 years w/ no salvage value, which route should be accepted on the basis of a benefit/cost analysis using an interest rate of 15%.

solution

$$EUAC = P200,000,000 (A/P, 15\%, 30) + P700,000 = 31.56 \times 10^4$$

$$EUAC = P250,000,000 (A/P, 15\%, 30) + P1,100,000 = 39.175 \times 10^4$$

$$\text{Incremental annual benefit} = 10 \times 10^4 - 4 \times 10^4 = 6 \times 10^4$$

$$\text{Incremental annual cost} = EUAC - EUAC$$

$$= 8.015 \times 10^4$$

$$B/C = \frac{6 \times 10^4}{8.015 \times 10^4}$$

Route A should be selected for construction.

Example

Four alternatives for providing electric supply to a small town have been identified with the ff. annual benefits and cost.

Alternative	Annual benefits	Annual cost
A	1,528,000	780,000
B	1,398,000	664,000

C	960,000	742,000
D	810,000	420,000

Solution

Comparing alternative A with alternative B

$$B/C \text{ A over B} = \frac{1,528,000 - 1,398,000}{780,000 - 664,000} = 1.12$$

Comparing alternative A with alternative C

$$B/C \text{ A over C} = \frac{1,528,000 - 960,000}{780,000 - 742,000} = 14.95$$

Comparing alternative A with alternative D

$$B/C \text{ A over D} = \frac{1,528,000 - 810,000}{780,000 - 420,000} = 1.99$$

Select alternative A.

Problem

To increase accessibility to some beautiful scenery along the coastal line of Batangas, a new highway is being proposed for construction. The initial cost is expected to be P9,600,000, with annual costs P36,000. Every three years minor improvement costing P20,000 are expected to be made. It is estimated that income from tourist from foreign countries will be P1,200,000 annually. Using a planning horizon of 30 years and interest rate of 10%, determine if the highway should be constructed using B-C ratio method.

Problem

To avoid double handling of cargo and to enable larger ships to dock at the copiers of a coastal city, it becomes necessary to dredge the harbor to a sufficient depth dredging is estimated to cost P4, 000, 000, but it will decrease shipping and cargo handling costs by P1.50 per ton. The port is presently processing 800,000 tons of cargo per year but, this expected to increase by 60,000tons each year for the next 20 years. Maintenance costs of the harbor are currently P500, 000 a year, increasing P70, 000 each year. If the harbor is dredged, maintenance will cost P600, 000 per year and increase by P50, 000 per year each year. Funding for the project is available at 5% interest. Determine the feasibility of the project using B-C ratio method and assuming a time horizon of 20 years.:

Problem

A nonprofit government corporation is considering two alternatives for generating power.

Alternative A. Build a coal-powered generating facility at a cost of \$20,000,000. Annual power sales are expected to be \$1,000,000 per year. Annual O&M costs are \$200,000 per year. A benefit of this alternative is that it is expected to attract new industry, worth \$500,000 per year, to the region.

Alternative B. Build a hydroelectric generating facility. The capital investment, power sales, and operating costs are \$30,000,000,

\$800,000, and \$100,000 per year, respectively. Annual benefits of this alternative are as follows.

Flood- control saving	\$600,000
Irrigation	\$200,000
Recreation	\$100,000
Ability to attract new industry	\$400,000

The useful life of both alternatives is 50 years. Using an interest rate of 5%. Determine which alternative should be selected.

Solution

Problems

1.The DPWH is considering the construction of a new highway through a scenic rural area. The road is expected to cost 50 million with annual upkeep estimated at 400,000. The improved accessibility is expected to result in additional income from tourist of 7 million/ year. The road is expected to have a useful life of 25 years. If the rate of interest is 15% should the road be constructed?

2. Det. the B/C ratio for the ff. project.

First cost	100,000
Project life years	5
Salvage value	10,000
Annual benefits	60,000
Annual O&M	22,000
Interest rate	15

3. Data for two alternatives are as follows:

Alternative	A	B
Investment	35,000	50,000
Annual benefit	20,000	25,000

Annual O&M	6,450	13,030
Estimated life years	4	8
Net salvage value	3,500	0

Using an interest rate of 20% which alternative should be chosen?

Solution:

4. There are 5 alternatives for improvement of a road. Determine which alternative should be chosen if the highway department is willing to invest money as long there is B/C ratio at least 1.0

Alternatives	Annual Benefits	Annual Cost
E	P4,200,000	P3,400,000
D	P3,300,000	P2,700,000
C	P2,800,000	P2,100,000
B	P1,300,000	P1,400,000
A	P900,000	P1,000,000

ACCOUNTING PRINCIPLES

Accounting is the process of recording all the transactions of the company which affect any investment of capital so that at any given time results of the investment may be known.

Bookkeeping is the systematic recording of all business transactions in financial.

After a decision to invest capital in a project has been made and the capital has been invested, those who supply and manage the capital want to know the financial results. Financial accounting and cost accounting are the procedures that provide these necessary services in a business organization.

All accounting is based on the fundamental accounting equation, which is

$$\text{Assets} = \text{liabilities} + \text{owner's equity}$$

Assets are these things of monetary value that the firm possesses

Liabilities are those things of monetary value that the firm owes

Owner's Equity is the worth of what the firm owes to its stockholders

Balance Sheet

This fundamental accounting equation defines the format of the balance sheet

Which is the two most common accounting equation defines the format of the balance sheet position of the firm at any given point in time

Typical Form of Balance Sheet

BALANCE SHEET

Name of Company

Date

ASSETS:

Current Assets

Cash on hand and in banks.....
 Notes and Accounts Receivable.....
 Notes receivable customers.....
 Accounts receivable customers.....
 Other receivable (give details).....
 Less: reserve for bad debts.....
 Inventories (goods and materials on hand).....
 Total Current Assets.....

Fixed Assets

Land
 Building or Manufacturing plant -----
 Less Reserve for depreciation -----
 Properties other than buildings -----
 Less Reserve for depreciation -----

	Total Fixed Assets -----	
Prepaid Expenses		
	These includes amount paid in advance -----	
	For insurance, rental, interest, supplies, etc -----	_____
	Total Prepaid Expenses -----	
Other Assets		
	Sinking funds -----	
	Investments in securities such as bonds, etc -----	
	Goodwill -----	
	Patents, franchises, licenses -----	
	Other intangibles -----	
	Miscellaneous (give details) -----	_____
	Total of Other Assets -----	
	TOTAL ASSETS -----	_____

LIABILITIES:

Current Liabilities		
	Notes Payable -----	
	Accounts Payables -----	
	Accrued Expenses (taxes, wages, interest, etc) -----	
	Declared and unpaid dividends -----	
	Other current liabilities (give details) -----	_____
	Total Current Liabilities -----	
Fixed Liabilities		
	Mortgage payable -----	
	Indebtedness in the form of bonds -----	
	Reserve for expansion -----	
	Other long term liabilities (give details) -----	_____
	Total Fixed Liabilities -----	
Prepaid Income		
	Advance payment on orders from the company -----	
	Other income paid in advance to the company -----	_____
	Total Prepaid Income -----	
	TOTAL LIABILITIES -----	

OWNERSHIP

	Preferred Shares	
	Common Shares	
	Undivided Surplus (retained earnings).....	
	TOTAL OWNERSHIP	
	TOTAL : LIABILITIES - OWNERSHIP	

INCOME STATEMENT

Another Important accounting relationship is

Revenue – Expenses = Profit (or loss)

This relationship defines the format of the income statement (also known as a profit and loss statement), which summarizes the revenue and expense result of operation over a period of time.

Example of Form of Income Statement

Income Statement
Company
For the year ending December 31. _____

Income Form Sales

Gross Income -----
Less : Return and allowances
Net Sales -----

Cost of Good Sold

Merchandise inventory at beginning of the year
Purchases and freight
Less : Purchases return and allowances
Cost of Merchandise available for sale
Merchandise inventory at year end
Cost of goods sold
Gross Profit

Operating Expenses:

Selling expenses :
Sale salaries and commissions
Advertising
Other selling expenses (depreciation of sales equipment
Bad debts, delivery expense , insurance on sales
Equipment, etc.)

General and Administrative Expenses:

Office Salaries
Other expenses (rent, telephone, light, taxes, insurance
Depreciation, office supplies, etc)
Total Operating Expenses -----

Net Operating Profit

Non-Operating income and Expenses

Non-operating income (interest, rent, etc)
Non-operating expenses (interest, sales, discount, etc)
Net Profit -----