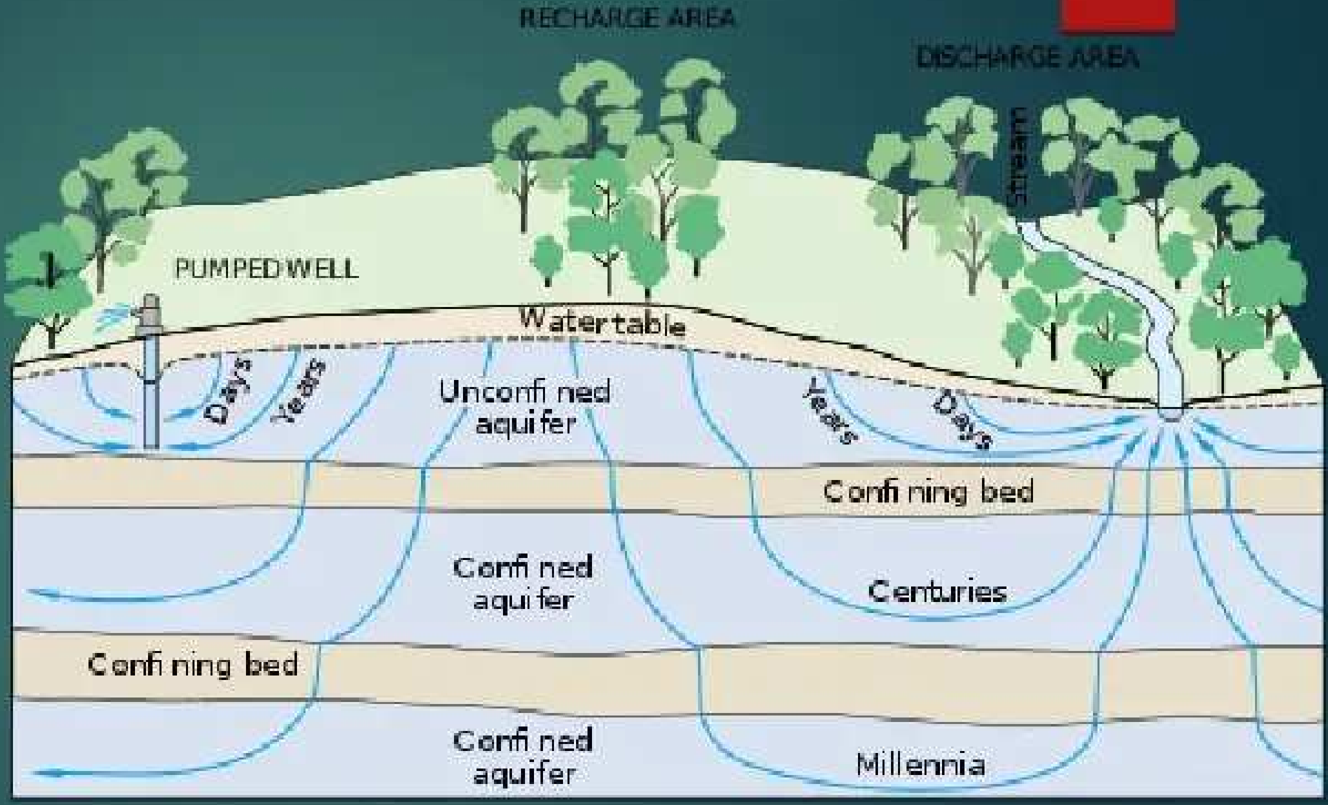
# BASIC SUBSURFACE FLOW

(Steady State Condition)

# GROUNDWATER FLOW

part of streamflow that has infiltrated the ground, has entered the saturated zone, and has been discharged into a stream channel, or springs and seepage water




https://en.wikipedia.org/wiki/Groundwater_flow

# DARCY'S LAW

# Darcy's Law

In 1856, Darcy published a simple equation for the discharge velocity of water through saturated soils, which may be expressed as

$$v = ki \qquad (7.6)$$

where $v$ = *discharge velocity*, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow

$k$ = hydraulic conductivity (otherwise known as the coefficient of permeability)

7.2 Darcy's Law

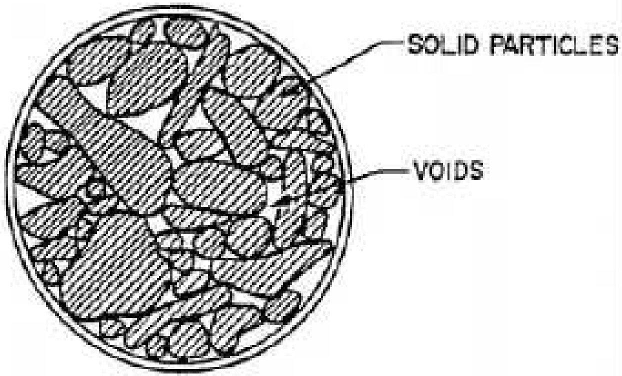
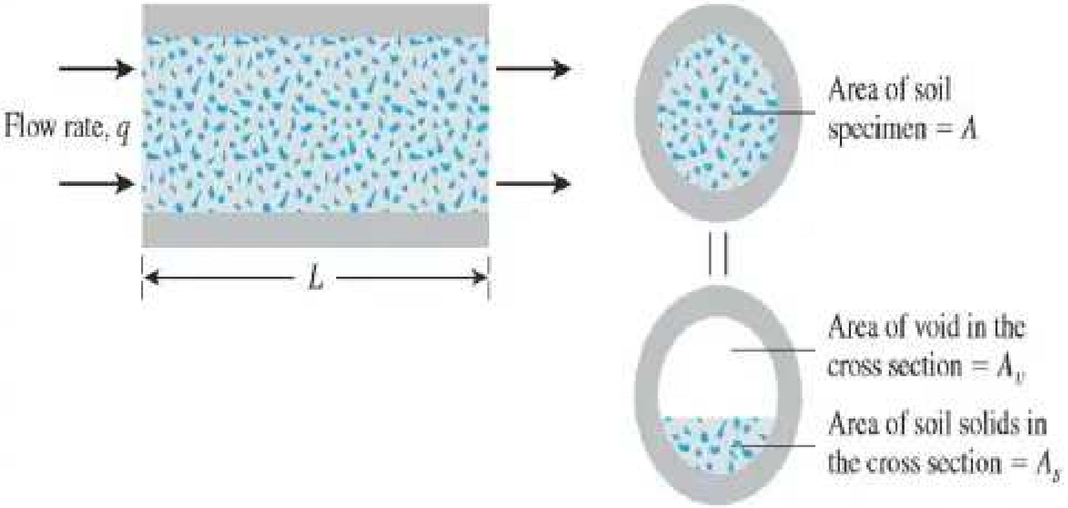



Fig. 4.4 Typical Cross-Section in a Soil Filled Pipe



# Hydraulic Gradient, i

$$i = \frac{\Delta h}{L}$$

where $\Delta h$= head loss

L= length of flow over which the loss of head occur

# NET MASS FLOW PER UNIT TIME:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

where:

$$v_x = k_x \frac{\partial h}{\partial x} \qquad v_y = k_y \frac{\partial h}{\partial y} \qquad v_z = k_z \frac{\partial h}{\partial z}$$

Considering isotropic medium with constant k in all directions:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

# STEADY FLOW IN HOMOGENEOUS ISOTROPIC CONFINED AND UNCONFINED AQUIFER

Steady flow means that the flow rate, piezometric head, and amount of fluid in storage do not change with time.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

REPORTER: ENTORUM, Mia Joy B.

# GROUNDWATER FLOW IN CONFINED AQUIFER

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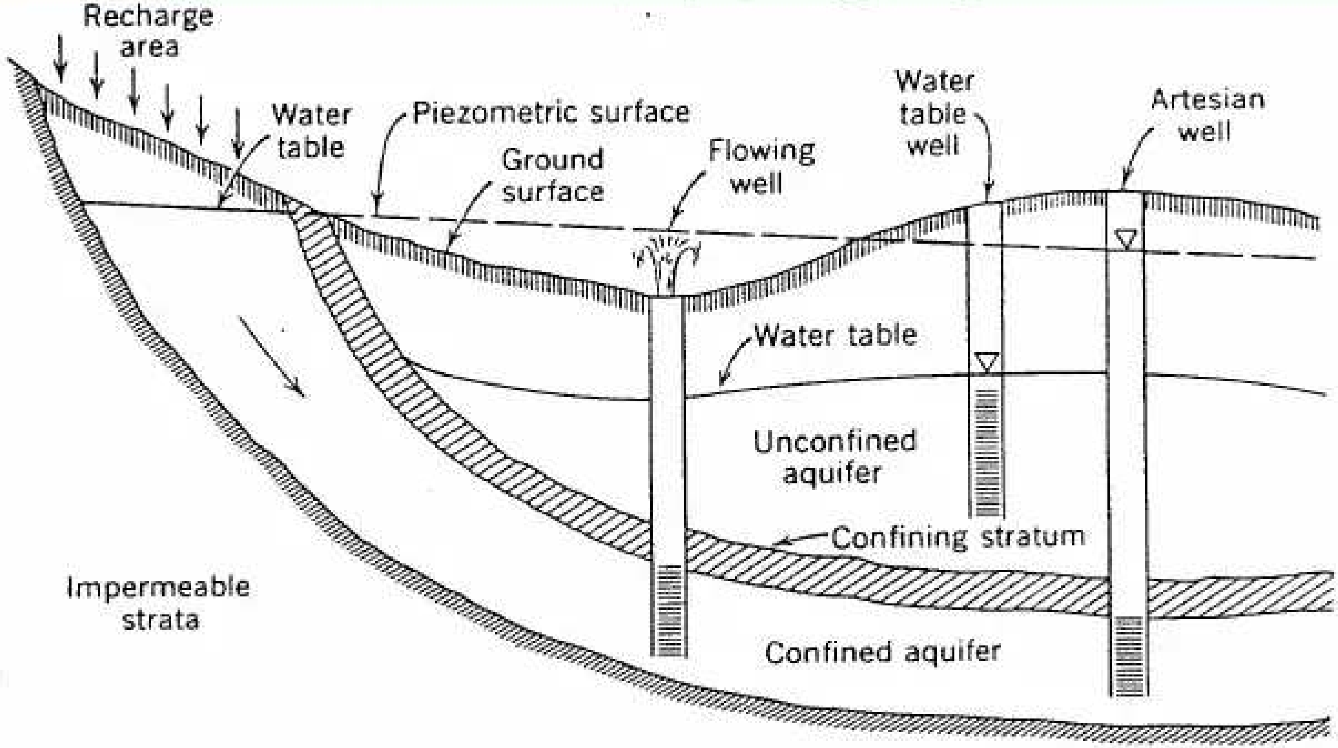
# AQUIFER

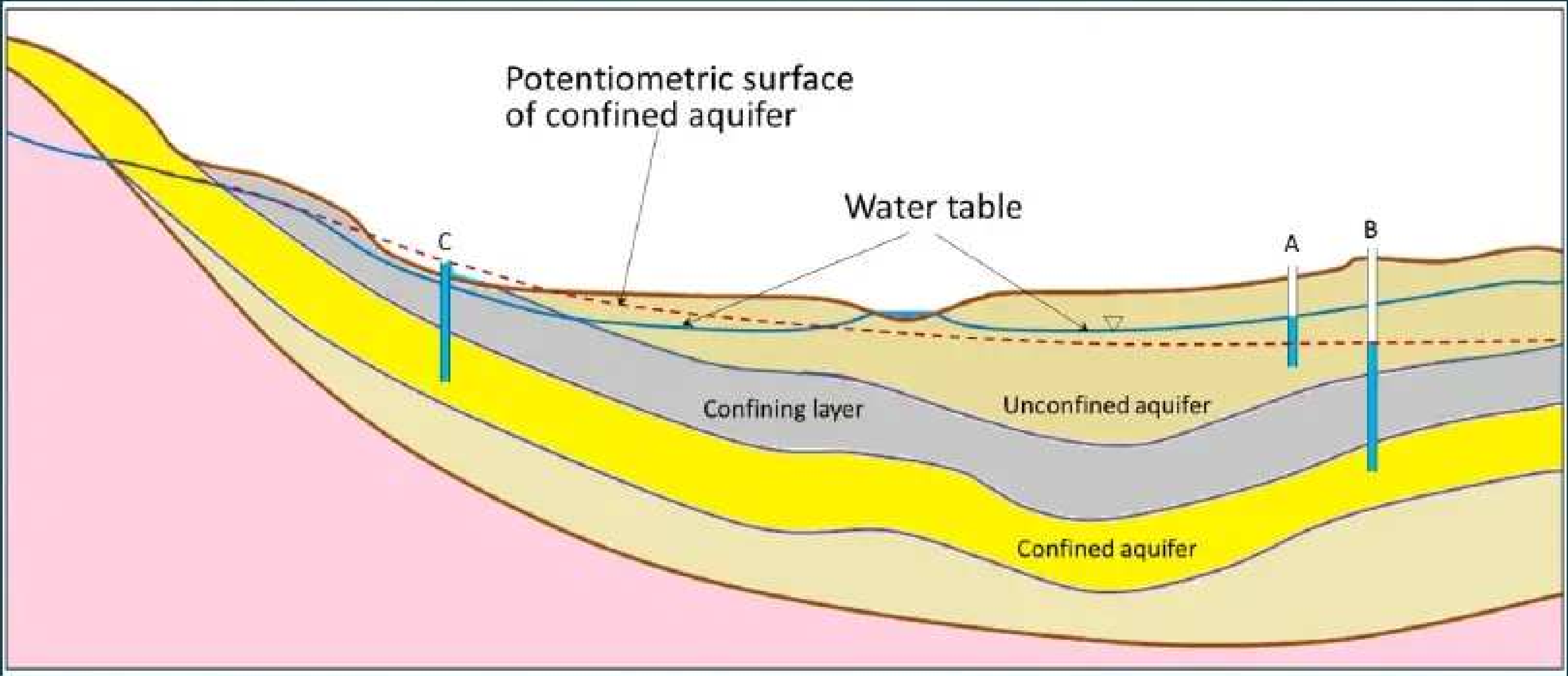
- A geologic formation or stratum containing water in its voids or pores that may be extracted economically and used as a source of water supply.
- An aquifer may be confined or unconfined.

# CONFINED AQUIFER

- One in which groundwater is confined under pressure greater than atmospheric by overlying impermeable strata. It is also known as artesian or pressure aquifer.
- Is an aquifer below the land surface that is saturated with water. Layers of impermeable material are both above and below the aquifer, causing it to be under pressure so that when the aquifer is penetrated by a well, the water will rise above the top of the aquifer.




Schematic cross section illustrating unconfined and confined aquifers.



Potentiometric surface
of confined aquifer
Water table
C
A
B
Confining layer
Unconfined aquifer
Confined aquifer

## 4.7 Equations of Ground-Water Flow**

### 4.7.1 Confined Aquifers

The flow of fluids through porous media is governed by the laws of physics. As such, it can be described by differential equations. Since the flow is a function of several variables, it is usually described by partial differential equations in which the spatial coordinates, $x$, $y$, and $z$, and time, $t$, are the independent variables.

In deriving the equations, the laws of conservation for mass and energy are employed. The *law of mass conservation*, or *continuity principle*, states that there can be no net change in the mass of a fluid contained in a small volume of an aquifer. Any change in mass flowing into the small volume of the aquifer must be balanced by a corresponding change in mass flux out of the volume, or a change in the mass stored in the volume, or both. The *law of conservation of energy* is also known as the *first law of thermodynamics*. It states that within any closed system there is a constant amount of energy, which can be

---

*The terms *discharge velocity* and *Darcian velocity* are synonyms for *specific discharge*. It would be best to avoid these as their use implies that ground water is moving at this velocity.

**The main equation of ground-water flow is derived in this section following a method used by Jacob (1940, 1950) and modified by Domenico (1972).

# NET TOTAL ACCUMULATION OF MASS

$$-\left(\frac{\partial}{\partial x}\rho_w q_x + \frac{\partial}{\partial y}\rho_w q_y + \frac{\partial}{\partial z}\rho_w q_z\right)dxdydz \quad (4.25)$$

# COMPRESSIBILITY

As the pressure in the control volume changes, the fluid density will change, as will the porosity of the aquifer. The compressibility of water, $\beta$, is defined as the rate of change in density with a change in pressure, $P$:

$$\beta dP = \frac{d\rho_w}{\rho_w} \tag{4.27}$$

The aquifer also changes in volume with a change in pressure. We will assume the only change is vertical. The aquifer compressibility, $\alpha$, is given by

$$\alpha dP = \frac{d(dz)}{dz} \tag{4.28}$$

## 4.13 Steady Flow in a Confined Aquifer

If there is the steady movement of ground water in a confined aquifer, there will be a dient or slope to the potentiometric surface of the aquifer. Likewise, we know tha water will be moving in the opposite direction of grad *h*. For flow of this type, Darcy's may be used directly. In Figure 4.16, a portion of a confined aquifer of uniform thickne shown. The potentiometric surface has a linear gradient; that is, its two-directional pr tion is a straight line. There are two observation wells where the hydraulic head ca measured.

# CHANGE IN MASS

$$-\left[\frac{\partial(q_x)}{\partial x}+\frac{\partial(q_y)}{\partial y}+\frac{\partial(q_z)}{\partial z}\right]\rho_w\,dx\,dydz=(\alpha\rho_w g+n\beta\rho_w g)\rho_w\,dx\,dydz\,\frac{\partial h}{\partial t} \qquad \textbf{(4.37)}$$

# FLOW EQUATIONS

**MAIN EQUATION:**

$$K\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right)=(\alpha\rho_w g+n\beta\rho_w g)\frac{\partial h}{\partial t}$$

**TWO DIMENTIONAL FLOW WIT**

**NO VERTICAL COMPONENT:**

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S}{T}\frac{\partial h}{\partial t}$$

**LAPLACE EQUATION:**

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

# FLOW EQUATIONS(CONT.)

GENERAL EQUATION OF FLOW (WITH LEAKAGE):

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{e}{T} = \frac{S}{T}\frac{\partial h}{\partial t}$$

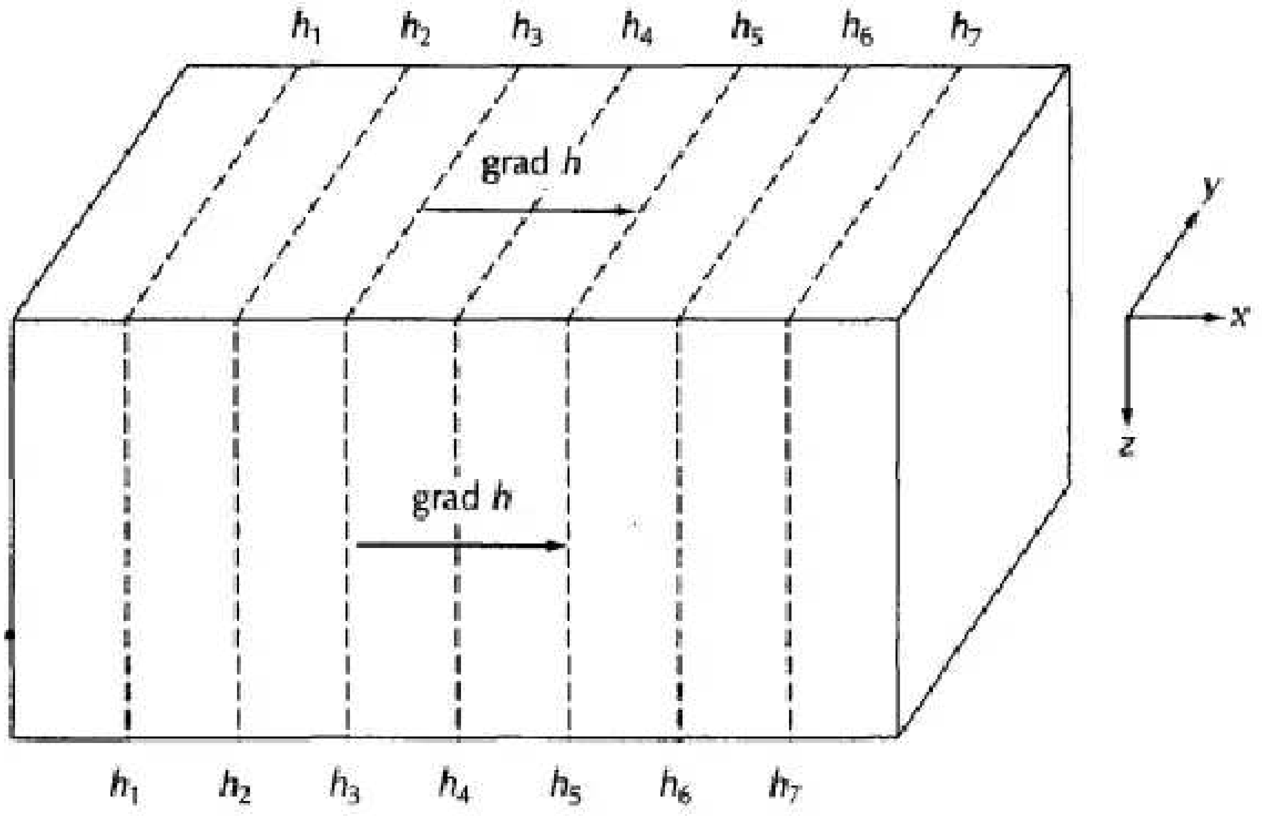
LEAKAGE RATE:

$$e = K'\frac{(h_0 - h)}{b'}$$

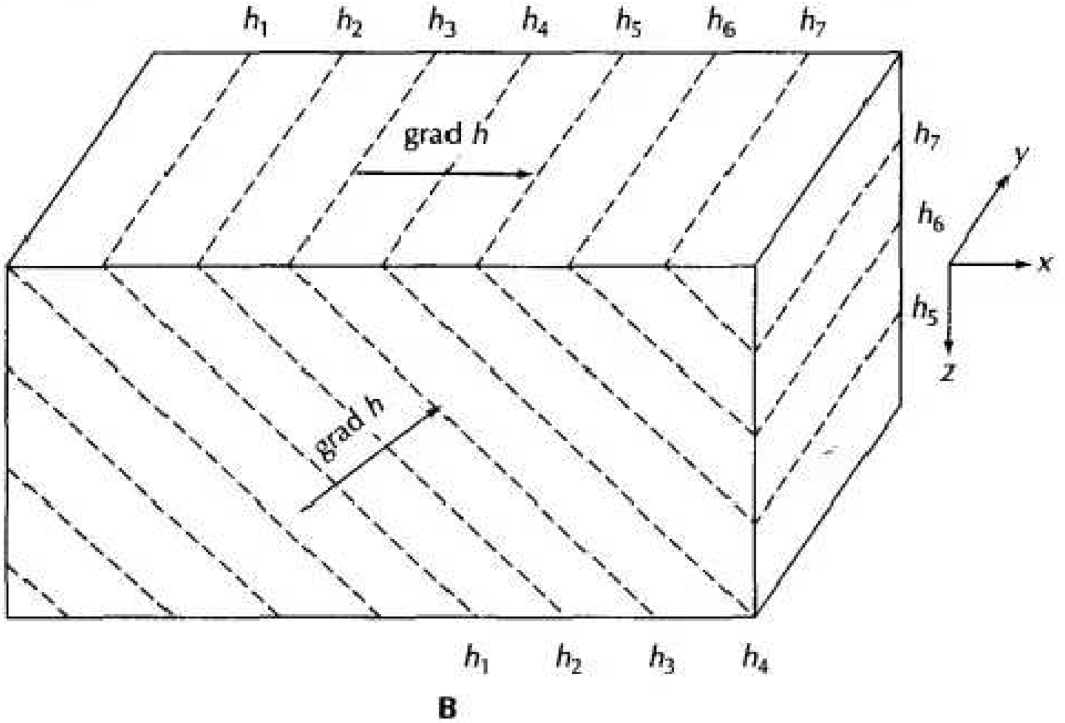
## 4.9 Gradient of Hydraulic Head

The physical quantity that we measure in the field, which represents hydraulic head, is the depth to water in a piezometer. We need to know the elevation of the measuring point, such as the top of the piezometer casing. The depth to water is then subtracted from the measuring point elevation to get the total head of water in the well.

$$\text{grad } h = \frac{dh}{ds}$$




**A**



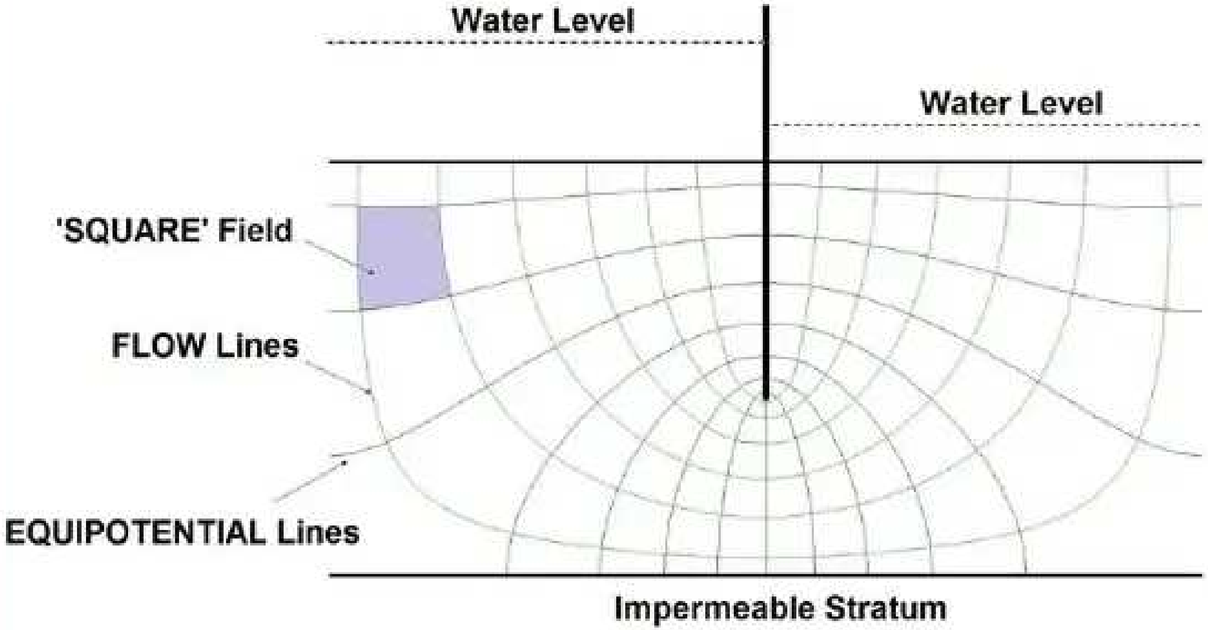

▲ FIGURE 4.8

**A.** Grad *h* in a homogeneous, isotropic aquifer with a uniform horizontal flow field. **B.** Grad *h* in a homogeneous, isotopic aquifer with a uniform flow field with a downward component.

# FLOW LINES AND FLOW NETS

- **FLOW LINES** - show the direction of groundwater **flow**
- **EQUIPOTENTIALS (lines** of constant head) - show the distribution of potential energy.
- **FLOW NETS** – is a graphical representation of two-dimensional steady-state groundwater flow through aquifers.

# FLOW NET
# (GRAPHICAL PROPERTIES)



The method of flow-net construction presented here is based on the following assumptions.

1. The aquifer is homogeneous.
2. The aquifer is fully saturated.
3. The aquifer is isotropic.
4. There is no change in the potential field with time.
5. The soil and water are incompressible.
6. Flow is laminar, and Darcy's law is valid.
7. All boundary conditions are known.

# THREE TYPES OF BOUNDARY

- NO-FLOW BOUNDARY
- CONSTANT-HEAD BOUNDARY
- WATER-TABLE BOUNDARY (unconfined)

$$q' = \frac{Kph}{f}$$

where

$q'$ is the total volume discharge per unit width of aquifer ($L^3/T$; $ft^3/d$ or $m^3/d$)

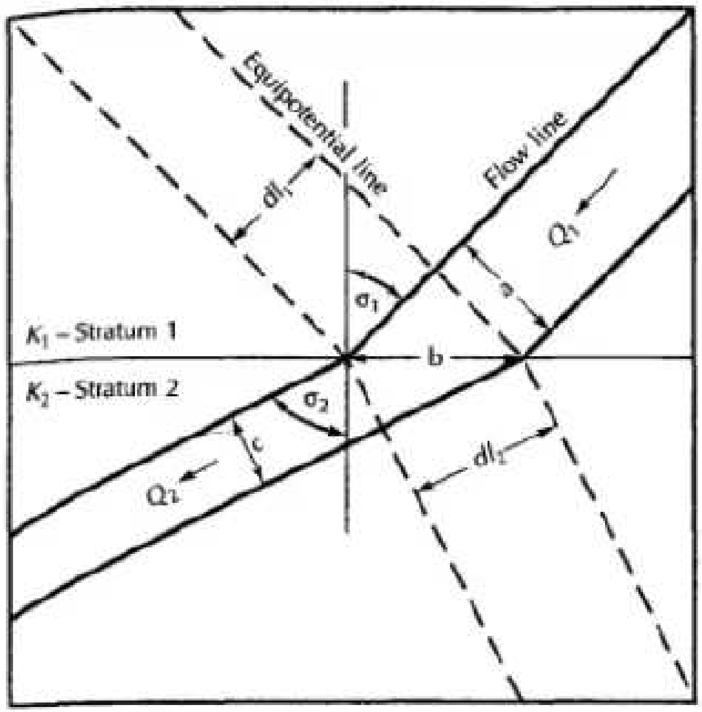
$K$ is the hydraulic conductivity ($L/T$; ft/d or m/d)

$p$ is the number of flowtubes bounded by adjacent pairs of flow lines

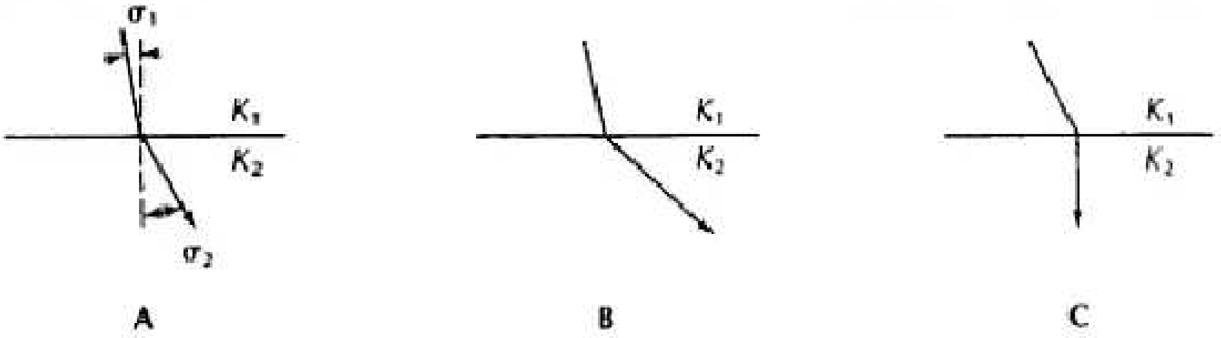
$h$ is the total head loss over the length of the flow lines ($L$; ft or m)

$f$ is the number of squares bounded by any two adjacent flow lines and covering the entire length of flow.

# REFRACTION OF FLOW LINES




◄ FIGURE 4.13
Streamtube crossing a hydraulic conductivity boundary.




▲ FIGURE 4.14

**A.** Refraction of a flowline crossing a conductivity boundary. **B.** Refracted flowline going from a reg of low to high conductivity. **C.** Refracted flowline going from a region of high to low conductivity.

Notice in Figure 4.13 that at the boundary between the two strata are two triangles that have a common leg, $b$. The triangle in stratum 1 is bounded by $a$, $b$, and $dl_1$. The triangle in stratum 2 is bounded by $c$, $b$, and $dl_2$.

The flow through each streamtube is found from Darcy's law:

$$Q_1 = K_1 a \frac{dh_1}{dl_1} \quad \text{and} \quad Q_2 = K_2 c \frac{dh_2}{dl_2} \tag{4.50}$$

From the principle of continuity, $Q_1$ must be equal to $Q_2$; therefore,

$$K_1 a \frac{dh_1}{dl_1} = K_2 c \frac{dh_2}{dl_2} \tag{4.51}$$

Since the head loss between the two equipotential lines is the same in both strata, $dh_1 = dh_2$ and we can divide both sides of Equation 4.51 by $dh_1$:

$$K_1 \frac{a}{dl_1} = K_2 \frac{c}{dl_2} \tag{4.52}$$

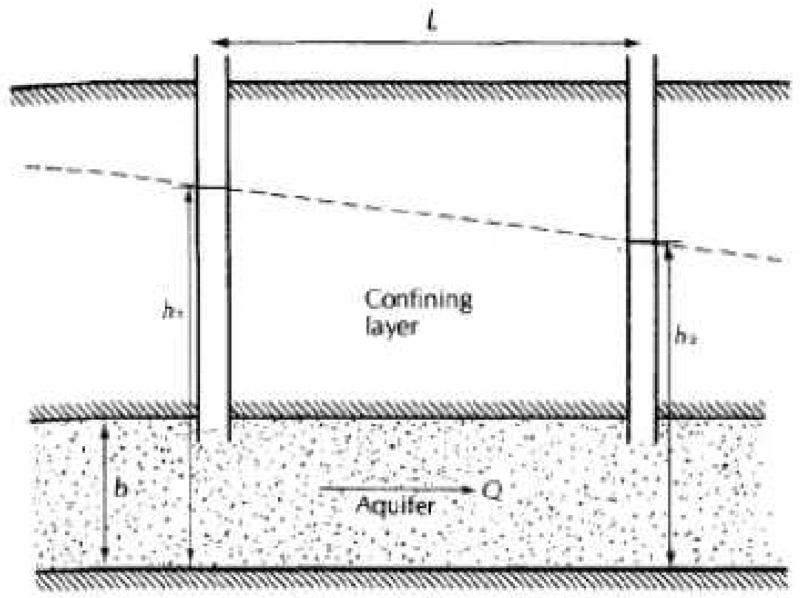
From the geometry of the triangles, $a = b \cos \sigma_1$ and $c = b \cos \sigma_2$. Furthermore, $b/dl_1 = 1/\sin \sigma_1$ and $b/dl_2 = 1/\sin \sigma_2$. Substituting these into Equation 4.52, we obtain

$$K_1 \frac{\cos \sigma_1}{\sin \sigma_1} = K_2 \frac{\cos \sigma_2}{\sin \sigma_2} \tag{4.53}$$

Since $\tan \sigma = (\sin \sigma)/(\cos \sigma)$, Equation 4.53 can be rewritten as the tangent law of refraction.

$$\boxed{\frac{K_1}{K_2} = \frac{\tan \sigma_1}{\tan \sigma_2}} \tag{4.54}$$

# STEADY FLOW IN




◄ FIGURE 4.16
Steady flow through a confined aquifer of uniform thickness.

The quantity of flow per unit width, $q'$, may be determined from Darcy's law:

$$q' = Kb\frac{dh}{dl} \tag{4.55}$$

where

$q'$ is the flow per unit width ($L^2/T$; $ft^2/d$ or $m^2/d$)

$K$ is the hydraulic conductivity ($L/T$; ft/d or m/d)

$b$ is the aquifer thickness ($L$; ft or m)

$\frac{dh}{dl}$ is the slope of potentiometric surface (dimensionless)

One may wish to know the head, $h$ ($L$; ft or m), at some intermediate distance, $x$ ($L$; ft or m), between $h_1$ and $h_2$. This may be found from the equation

$$h = h_1 - \frac{q'}{Kb}x \tag{4.56}$$

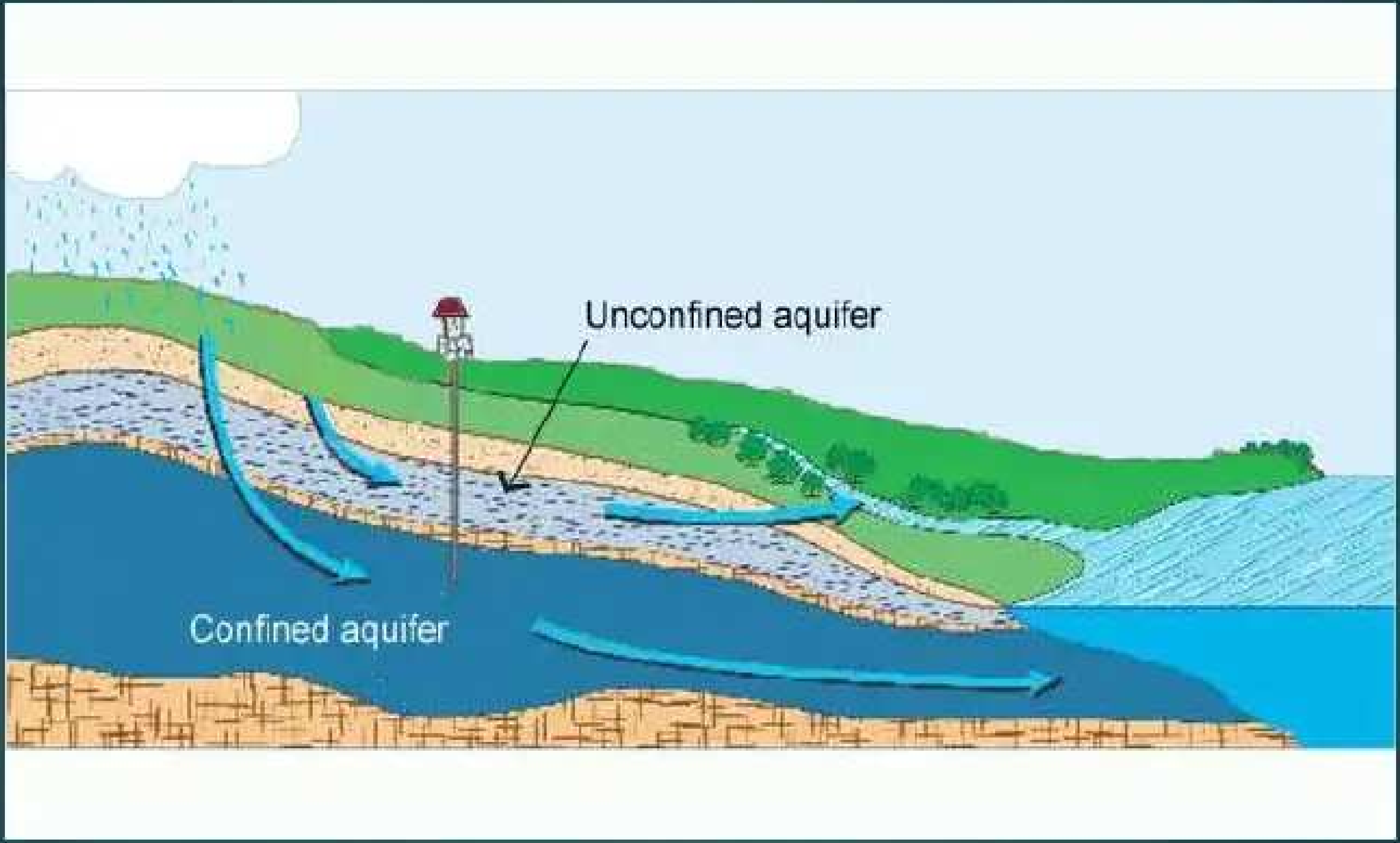
where $x$ is the distance from $h_1$.

# GROUND WATER FLOW IN UNCONFINED ACQUIFER

- Where groundwater is in direct contact with the atmosphere through the open pore spaces of the overlying soil or rock, then the aquifer is said to be unconfined. The upper groundwater surface in an unconfined aquifer is called the water table. The depth to the water table varies according to factors such as the topography, geology, season and tidal effects, and the quantities of water being pumped from the aquifer.

- Unconfined aquifers are usually recharged by rain or stream water infiltrating directly through the overlying soil. Typical examples of unconfined aquifers include many areas of coastal sands and alluvial deposits in river valleys.
- Unconfined aquifers receive recharge directly from rainfall and surface water infiltrating downward. Confined aquifers are connected to unconfined areas where water can flow in.

unconfined areas where water can flow in.



## Unconfined aquifers

In an unconfined aquifer, the saturated flow thickness, h is the same as the hydraulic head at any location, as seen from Figure

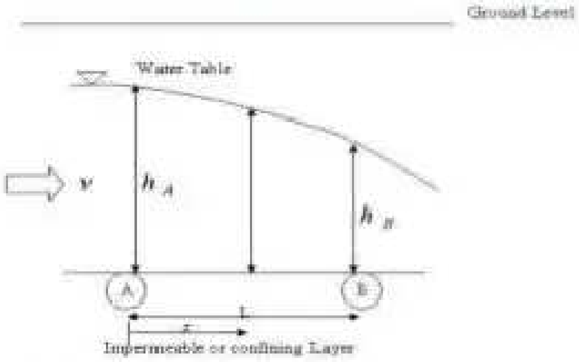



FIGURE 8. Flow through an unconfined aquifer

There are thus some **basic differences between unconfined and confined aquifers** when they are pumped:

· **First**, a confined aquifer is not dewatered during pumping; it remains fully saturated and the pumping creates a drawdown in the piezometric surface;

· **Second**, the water produced by a well in a confined aquifer comes from the expansion of the water in the aquifer due to a reduction of the water pressure, and from the compaction of the aquifer due to increased effective stresses;

· **Third**, the flow towards the well in a confined aquifer is and remains horizontal, provided, of course, that the well and remains horizontal, provided, of course, that the well is a fully penetrating one; there are no vertical flow

## Unconfined aquifer

In case of steady flow in unconfined aquifer, the flow equation becomes

$$\frac{1}{r}\frac{d}{dr}\left(rh\frac{dh}{dr}\right) = 0 \quad \text{or} \quad \frac{d}{dr}\left(rh\frac{dh}{dr}\right) = 0$$

Integrating,

$$\int \frac{d}{dr}\left(rh\frac{dh}{dr}\right) = \int 0 \qquad rh\frac{dh}{dr} = C_1$$

Now, Darcy's law can be expressed as

$$Q = 2\pi r K h \frac{dh}{dr} \rightarrow \frac{Q}{2\pi K} = rh\frac{dh}{dr} = C_1$$

Therefore, the equation can be written as

$$rh\frac{dh}{dr} = \frac{Q}{2\pi T}$$

Now integrating, we have

$$\Rightarrow \int h\,dh = \frac{Q}{2\pi K}\int \frac{dr}{r}$$

$$\Rightarrow \frac{h^2}{2} = \frac{Q}{2\pi K} ln(r) + C_2$$

$$\Rightarrow h^2 = \frac{Q}{\pi K}\ln(r) + C_2$$

Now, Darcy's law can be expressed as

$$Q = 2\pi r K h\frac{dh}{dr} \rightarrow \frac{Q}{2\pi K} = rh\frac{dh}{dr} = C_1$$

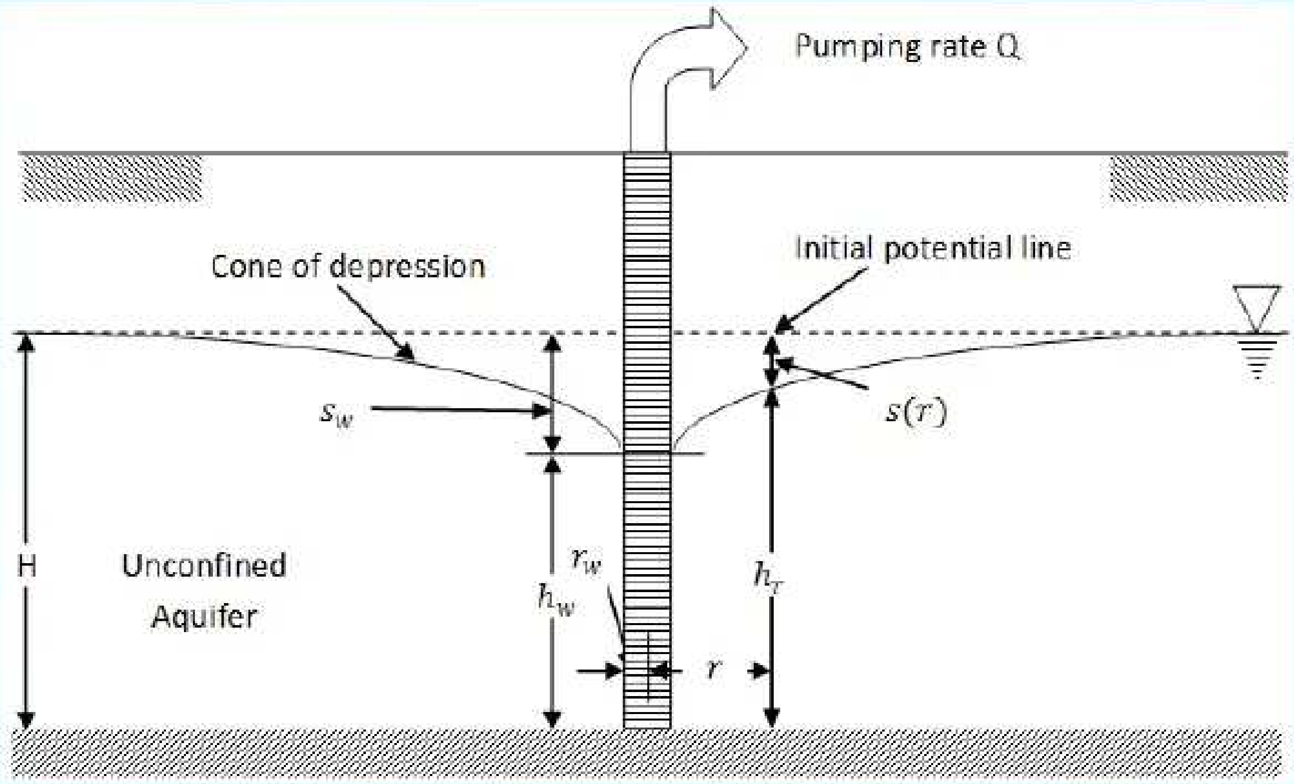



Fig. 1 An unconfined

# aquifer with boundary

Knowing hydraulic head at the well, the equation can be used to calculate steady hydraulic head for any values of r. This equation can also be used for estimation of aquifer conductivity. The equation can be written for calculating aquifer conductivity as,

Flow rate

$$Q = \frac{\pi K\left(h_1^2 - h_2^2\right)}{\ln \frac{r_1}{r_2}}$$

**Permeability coefficient**

$$K = \frac{Q \ln \frac{r_1}{r_2}}{\pi\left(h_1^2 - h_2^2\right)}$$

where:

*Q* = well discharge rate ($m^3/d$)

*K* = hydraulic conductivity of aquifer (m/d)

$r_2$ = radius of the pumping well (m)

$r_1$ = distance from piezometer to the pumping well (m)

$h^2$ = steady-state head in the pumping well (m)

$h_1$ = steady-state head in the piezometer (m)

Pressure head at point 1

$$h_1 = \sqrt{\frac{Q \ln \frac{r_1}{r_2}}{\pi K} + h_2^2}$$

Pressure head at point 2

$$h_2 = \sqrt{h_1^2 - \frac{Q \ln \frac{r_1}{r_2}}{\pi K}}$$

radius at point 1

$$r_1 = r_2 e^{\frac{\pi K\left(h_1^2 - h_2^2\right)}{Q}}$$

radius at point 2

$$r_2 = \frac{r_1}{e^{\frac{\pi K\left(h_1^2 - h_2^2\right)}{Q}}}$$

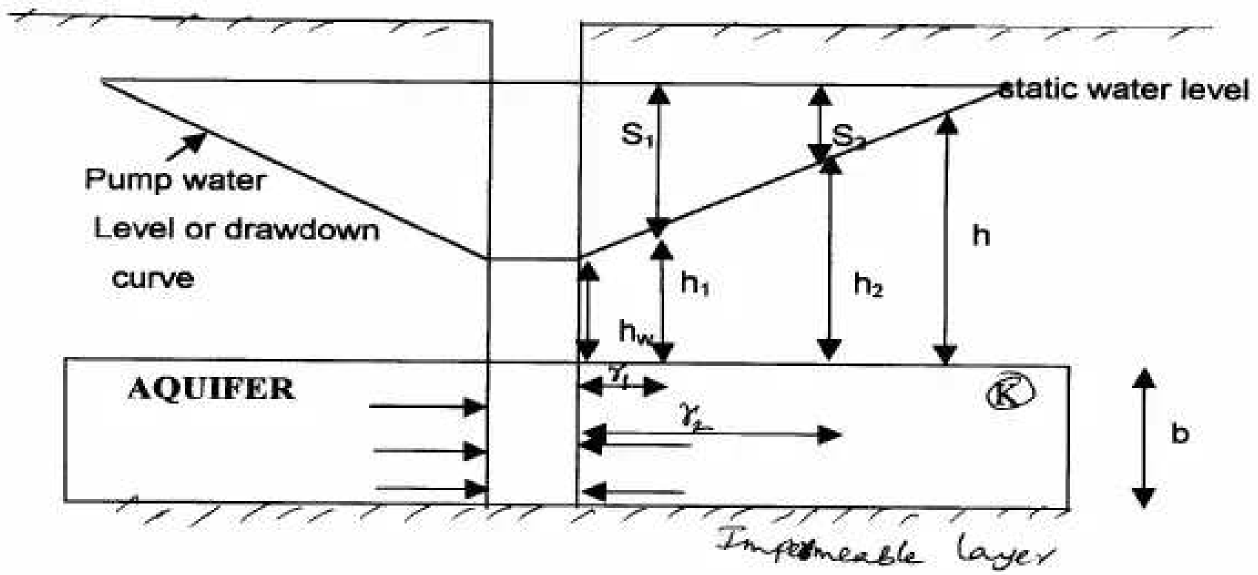
# Steady Radial Flow, Confined

# Steady Radial Flow to a Well

**7.3.2 Steady Radial Flow to a Well**

When a well is pumped, water is removed from the aquifer surrounding the well and the water table or piezometric surface, depending on the type of aquifer is lowered.

**a) Confined Aquifer (Steady State Radial Flow)**



# Steady Radial Flow to a Well- Confined

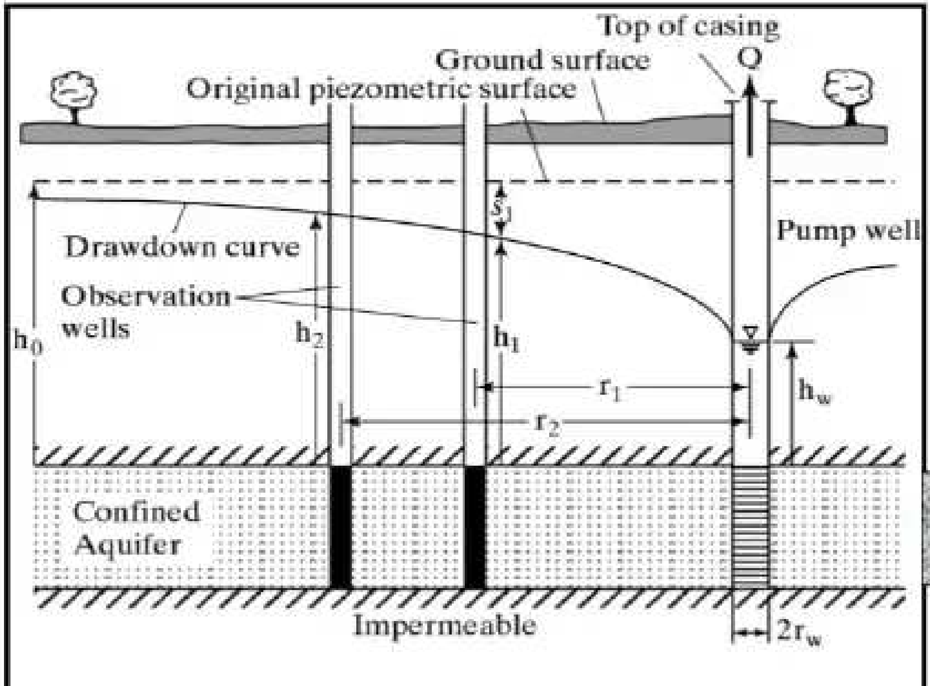
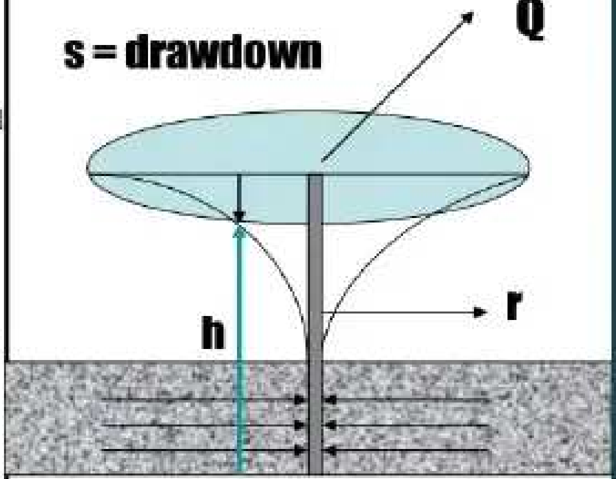



Figure 8.14



Radial flow to a well penetrating an extensive confined aquifer.

# Steady Radial Flow to a Well Confined

- In a confined aquifer, the drawdown curve or cone of depression varies with distance from a pumping well.
- For horizontal flow, Q at any radius r equals, from Darcy's law,

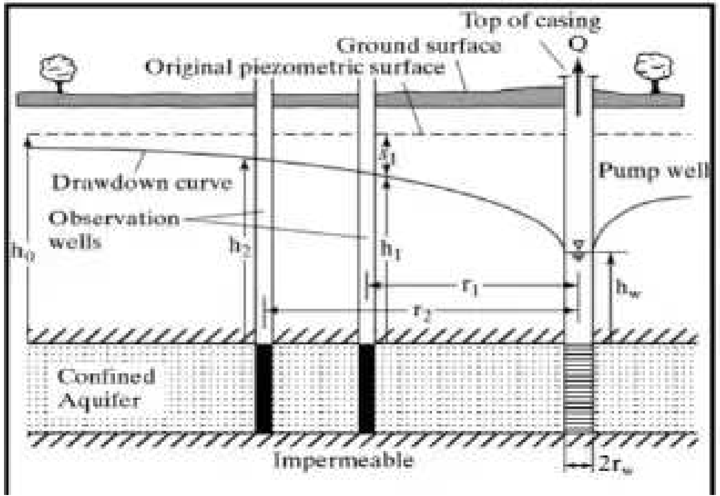
$$Q = 2\pi rbK\, dh/dr$$

for steady radial flow to a well where Q,b,K are const

# Steady Radial Flow to a Well Confined

- Integrating after separation of variables, with $h = h_w$ at $r = r_w$ at the well, yields Thiem Eqn

$$Q = 2\pi Kb[(h-h_w)/(ln(r/r_w))]$$




*Note*, *h* increases indefinitely with increasing *r*, yet the maximum head is $h_0$.

8.14

Radial flow to a well penetrating an extensive confined aquifer.

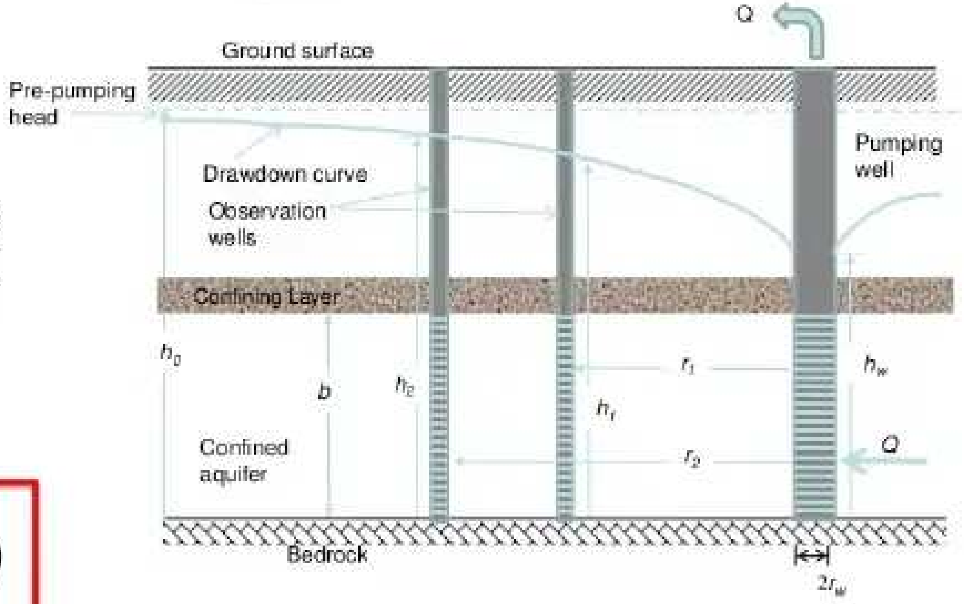
# Steady Radial Flow to a Well Confined

- Near the well, transmissivity, $T$, may be estimated by observing heads $h_1$ and $h_2$ at two adjacent observation wells located at $r_1$ and $r_2$, respectively, from the pumping well

$$T = Kb = Q \ln(r_2 / r_1)$$

$2\pi(h_2 - h_1)$

# Steady Flow to a Well in a Confined Aquifer




$$Q = Aq = (2\pi rb)K\frac{dh}{dr}$$

$$r\frac{dh}{dr} = \frac{Q}{2\pi T}$$

$$h_2 = h_1 + \frac{Q}{2\pi T}\ln\left(\frac{r_2}{r_1}\right)$$

Theim Equation

In terms of head (we can write it in terms of drawdown also)

# RADIAL GROUND WATER FLOW IN UNCONFINED

# AQUIFERS

# Steady Radial Flow in an Unconfined Aquifer

Assumptions:

- The aquifer is unconfined and underlain by a horizontal aquiclude
- The well is pumped at a constant rate
- Equilibrium has been reached; i.e., there is no further change in drawdown with time

# From Darcy’s law:

$$Q = (2\pi r h)K\left(\frac{dh}{dr}\right)$$

$$hdh = \frac{Q}{2\pi K}\left(\frac{dr}{r}\right)$$

$$\int_{h_1}^{h_2} hdh = \frac{Q}{2\pi K}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$h_2^2 - h_1^2 = \frac{Q}{\pi K}\ln\left(\frac{r_2}{r_1}\right)$$

$$K = \frac{Q}{\pi(h_2^2 - h_1^2)}\ln\left(\frac{r_2}{r_1}\right)$$

A well penetrates an unconfined aquifer. The initial water level is ho=25 m. After a long period of pumping at 0.05 m3/s, the drawdowns at 50 and 150 m from the well were observed to be 3 and 1.2 m respectively.

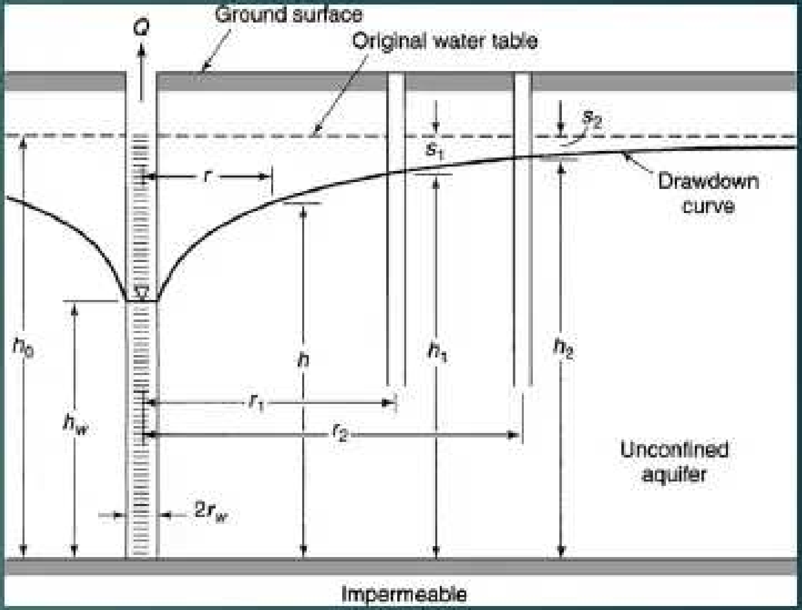
Compute the hydraulic conductivity and radius of influence

What type of deposit is the aquifer

If rw=0.5 m, what is the drawdown at the well

If the water height in the well is hw= 10 m, what are the head losses at the well

What is the drawdown at 500 m.



# RANGE OF VALUES OF HYDRAULIC CONDUCTIVITY

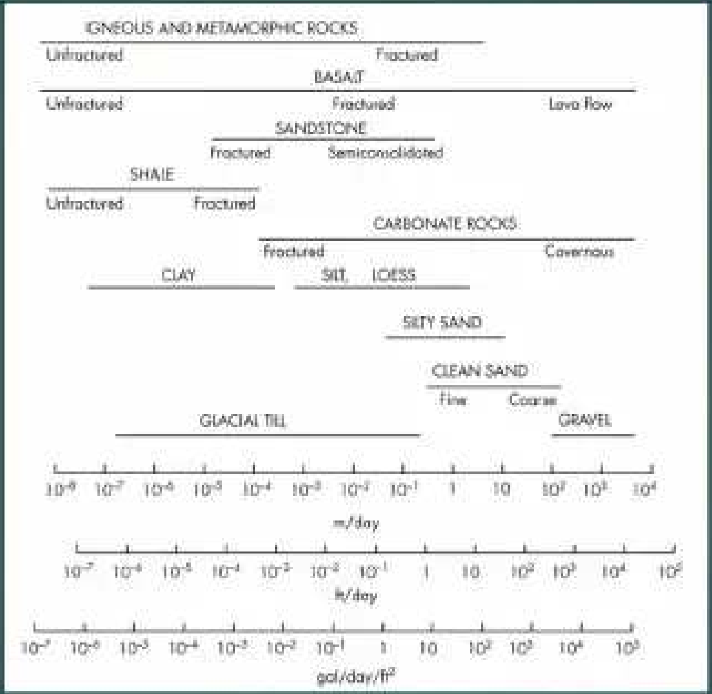


Figure 3.2.2

# TRAVEL TIME OF GROUND WATER IN CONFINED AQUIFER

# FORMULA:

$$T = DT/v_S$$

- WHERE:

T= Time travelled

DT= Distance Travelled

$v_s$=seepage velocity

# FORMULAS:

DARCY'S LAW: $Q = kiA$

where:

k = hydraulic conductivity

i = dh/dL =hydraulic gradient

A = cross-sectional area of the confined acquifer

DARCY VELOCITY: $V_D = Q/A$

SEEPAGE VELOCITY: $V_S = V_D/n$

VOID RATIO: $n = V_v/V_T$

# Example 1:

A confined aquifer has a source of recharge, the piezometric head in the two wells 1000m apart is

55m & 50m respectively, from a common datum. The average thickness of the aquifer is 30m, and the average width of the aquifer is 5km. The hydraulic conductivity for the aquifer is 50m/day, and the porosity is 0.20.

(a) Determine the rate of the flow through the

aquifer

aquifer

(b) Time travelled from the head of the aquifer to a point 4km downstream

GIVEN:

n = 0.2

k = 50m/day

width = 5000m

thickness = 30m

h1 = 55m

h2 = 50m

L = 1000m

SOLUTION:

i= (55-50)/1000 =1/200

A= 30(5000) =150,000

Q= kiA =50(1/200)(150,000) =37,500 cu.m/day

$V_D$= q/a =37,500/150,000 =0.25m/day

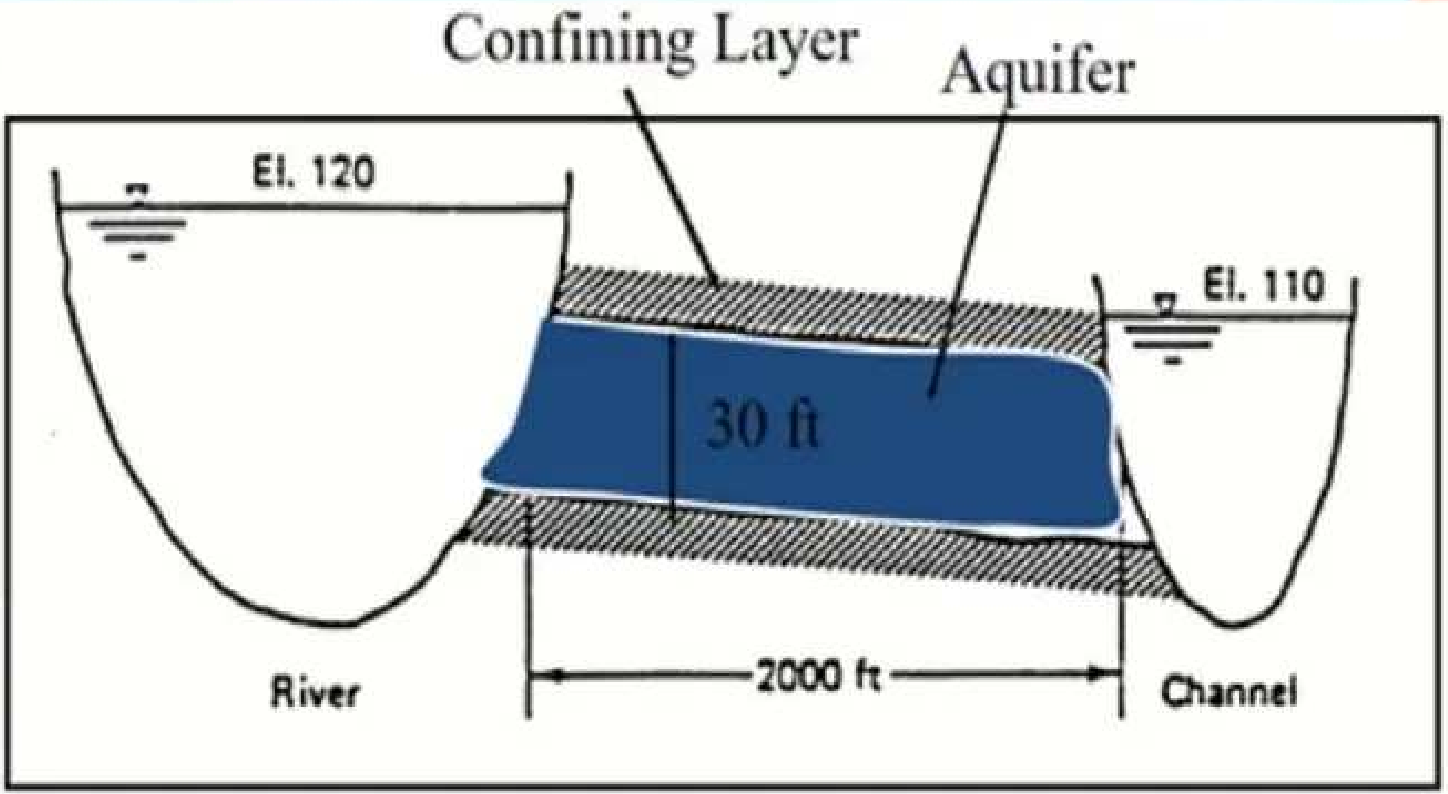
Therefore:

(a) Vs = $V^D$/n = 0.25/0.2 =1.25 m/day   rate of flow

(b) $T = DT/V_s = 400/1.25 = 3{,}200$ days time travelled

# Example 2:

Two channels are 2000ft apart. The water le of two channels are 120ft & 110ft, respectively. A pervious formation averaging 30ft thick with hydraulic conductivity of 0.25ft/hr and porosity of n=0.25. Determine the flow rate of seepage from the river to the channel and the time travelled from the head of the aquifer to 5ft downstream.


Confining Layer
Aquifer
El. 120
El. 110
30 ft
2000 ft
River
Channel

7

# GIVEN:

n= 0.25      k=0.25ft/hr    thickness=30ft

h1=120ft    h2= 110ft       L=2000ft

SOLUTION:

i= (120-110)/2000 =1/200

A= 30x1 =30 sq. ft.

Q= kiA=0.25(1/200)(30)=0.0375 cu. ft/hr

0.0375(cu.ft/hr) x (24hr/1day)=0.9 cu. ft/day

V=Q/A= 0.9/30 =0.03ft/day

Therefore:

(a) Vs = $V^D$/n = 0.03/0.25 =0.12 ft/day   rate of flow

(a) $V_s = V/n = 0.03/0.25 = 0.12$ ft/day rate of flow

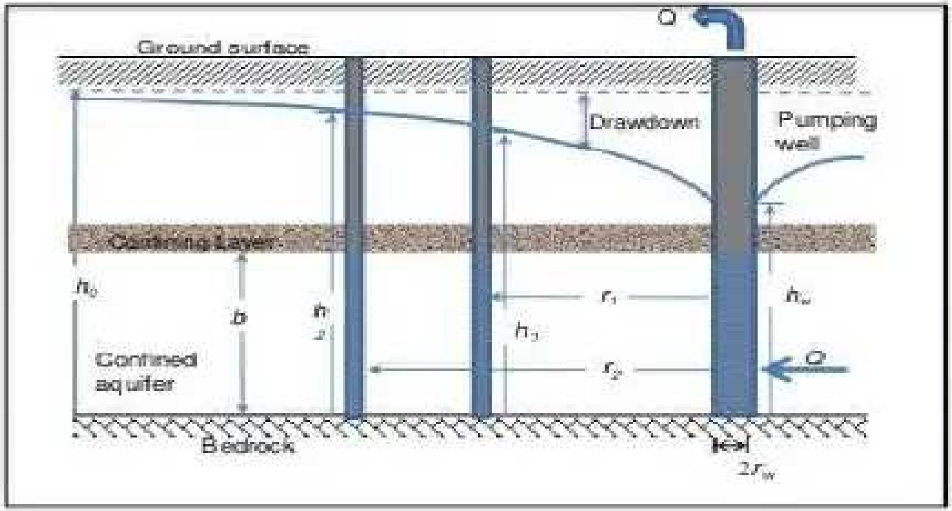
(b) $T = DT/V_s = 5ft/0.12 = 42$ days time travelled

# EXAMPLE PROBLEMS

Steady Flow to a Well in a Confined Aquifer

# Example - Theim Equation

- 1-m diameter well
- $Q = 113\ m^3/hr$
- $b = 30$ m
- $h_0 = 40$ m
- Two observation wells,
  1. $r_1 = 15$ m; $h_1 = 38.2$ m
  2. $r_2 = 50$ m; $h_2 = 39.5$ m
- Find: Head and drawdown in the well



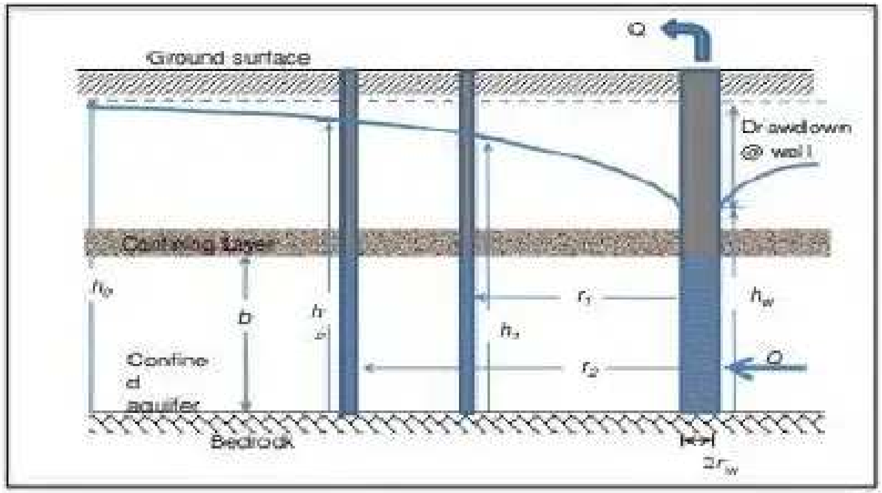

$$s(r) = \frac{Q}{2\ T}\ln\left(\frac{R}{r}\right)$$

$$T = \frac{Q}{2\ (s_1\ \ s_2)}\ln\left(\frac{r_2}{r_1}\right) = \frac{113\,m^3/hr}{2\ (1.8\,m\ \ 0.5\,m)}\ln\left(\frac{50\,m}{15\,m}\right) = 16.66\ m^2/hr$$

Adapted from Todd and Mays, Groundwater Hydr

Steady Flow to a Well in a Confined Aquifer

# Example - Theim Equation



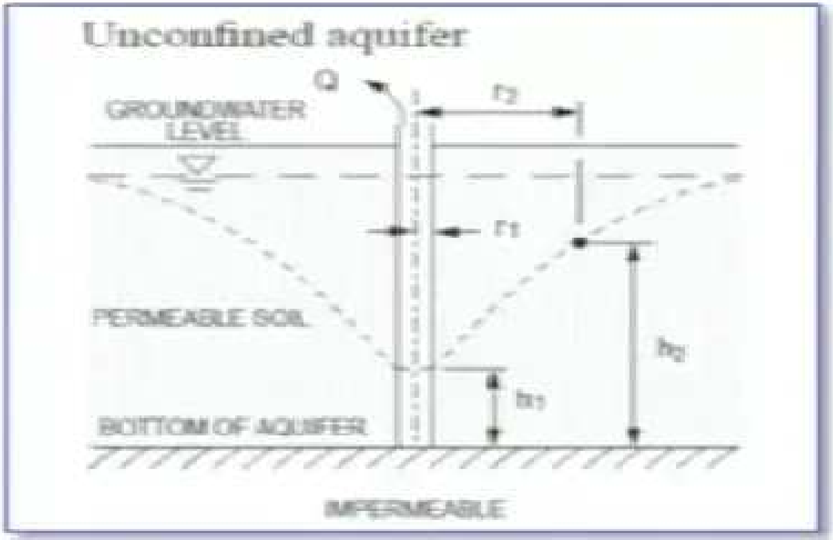

$$h_2 = h_1 + \frac{Q}{2\ \ T}\ln(\frac{r_2}{r_1})$$

$$h_w = h_2 + \frac{Q}{2\ \ T}\ln(\frac{r_w}{r_2}) = 39.5\,m + \frac{113\,m^3/hr}{2\ \ *16.66\,m^2/hr}\ln(\frac{0.5\,m}{50\,m}) = 34.5\,m$$

$s_w = h_0 \quad h_w = 40\,m \quad 34.5\,m = 5.5\,m$ Drawdown at the well

Adapted from Todd and Mays, Groundwater Hydr

An unconfined aquifer is 200 ft deep from its impermeable bottom. Pumping at a rate of 50 gpm is begun from a well with a radius of 1.5 ft. The hydraulic conductivity of the soil is 2.0 ft/day and after a very long time the drawdown is zero at a radial distance of 2000 ft. The water depth in the well is then most nearly:

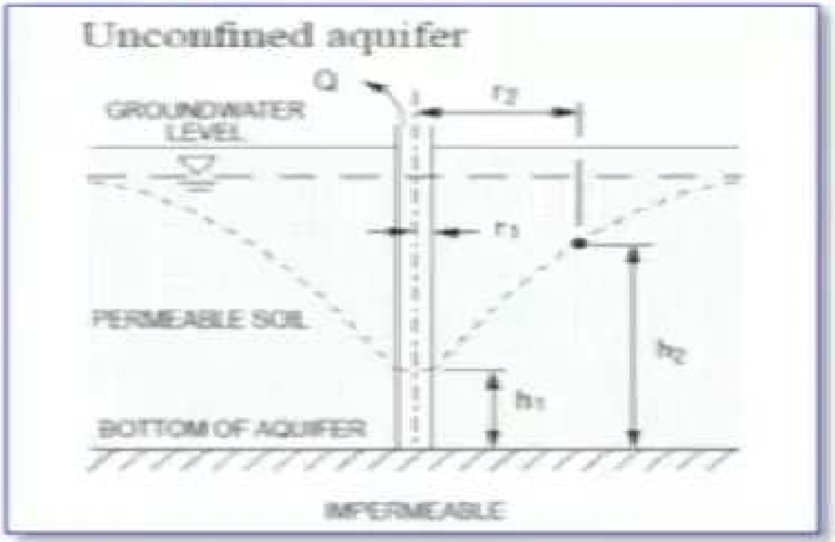



$r_1$=1.5ft Q= 50 gpm

$r_2$=2000ft k= 2ft/day

$h_2$=200ft

An unconfined aquifer is 200 ft deep from its impermeable bottom. Pumping at a rate of 50 gpm is begun from a well with a radius of 1.5 ft. The hydraulic conductivity of the soil is 2.0 ft/day and after a very long time the drawdown is zero at a radial distance of 2000 ft. The water depth in the well is then most nearly:




$r_1$=1.5ft  Q= 50 gpm

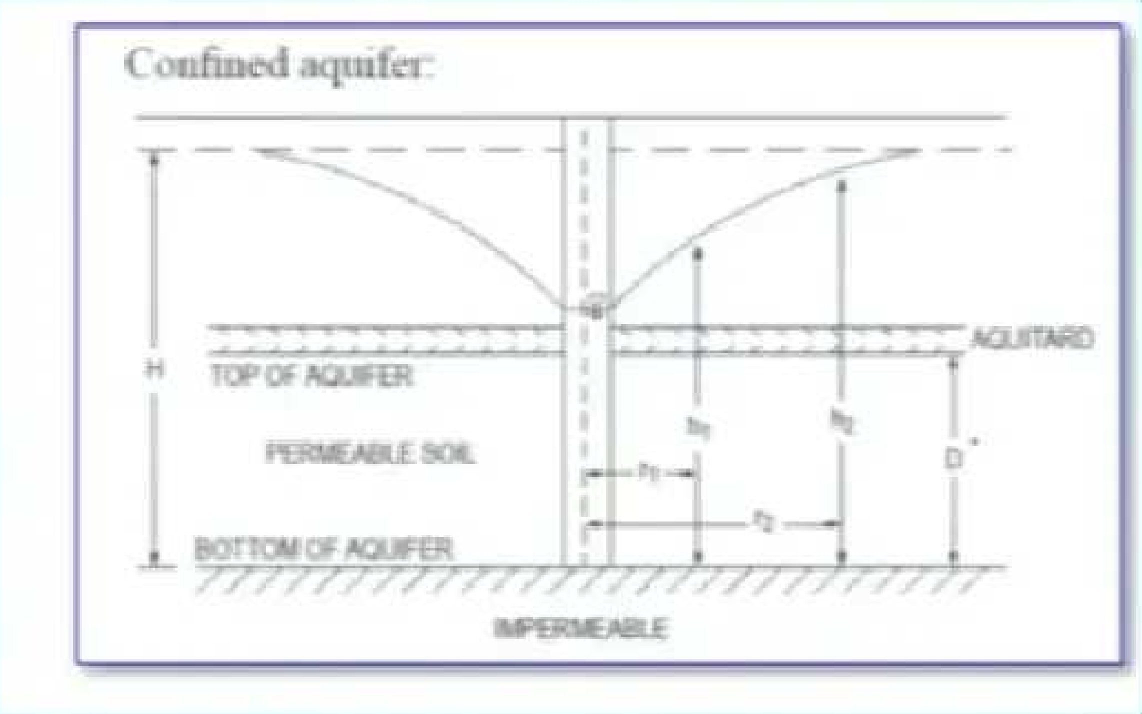
$r_2$=2000ft k= 2ft/day

$h_2$=200ft

$$Q = \left(50\frac{\text{gal}}{\text{min}}\right)\times\left(\frac{60\text{ min}}{1\text{ hr}}\right)\times\left(\frac{24\text{ hr}}{1\text{ day}}\right)\times\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)$$

$$= 9{,}626 \text{ ft}^3/\text{day}$$

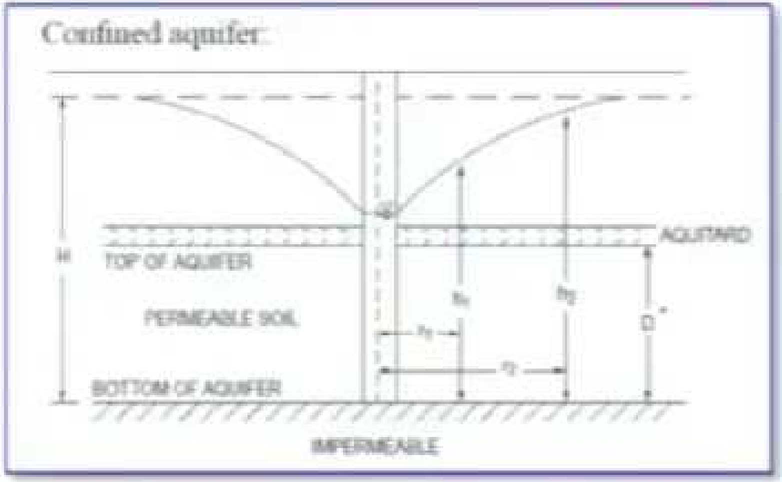
$$Q = \frac{\pi k\left(h_2^2 - h_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)} \quad \Rightarrow h_1 = \sqrt{h_2^2 - \frac{Q}{\pi k}\ln\left(\frac{r_2}{r_1}\right)} \quad = \sqrt{200^2 - \frac{9626}{\pi \times 2.0} \times \ln\left(\frac{2000}{1.5}\right)} \quad = 170 \text{ ft}$$




$$Q = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Transmissivity: $T = kD$ ft²/s or m²/s

A confined aquifer is 200 ft thick. In an experiment to determine its permeability, water is pumped from a well at a rate of 50 gpm. The water table at a radius of 100 ft is observed to be at a depth of 40 ft and at a radius of 200 ft it is 30 ft. The permeability is most nearly:



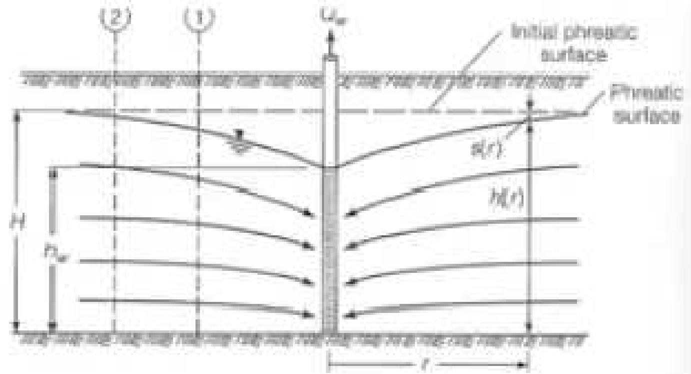

$$Q = 9{,}626\ \text{ft}^3/\text{day}$$

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)} \Rightarrow T = \frac{Q\ln(r_2/r_1)}{2\pi(h_2 - h_1)} \Rightarrow T = \frac{9626 \times \ln(200/100)}{2\pi(40-30)}$$

$$= 106\ \text{ft}^2/\text{day}$$

# Example – Unconfined Aquifer

- A well pumps 0.4 $m^3/s$ from an unconfined aquifer whose saturated thickness is 24 m. If the drawdown 50 m from the well is 1m and the drawdown 100 m from the well is 0.5 m, then calculate the hydraulic conductivity and transmissivity of the aquifer.




$$T = \frac{Q_w}{2\pi(s'_1 - s'_2)} \ln\left(\frac{r_2}{r_1}\right)$$

$$s'_1 = s_1 - \frac{s_1^2}{2H}$$ mod. drawdown

$s_1 = s'_1$ when $s_1^2 << 2H$

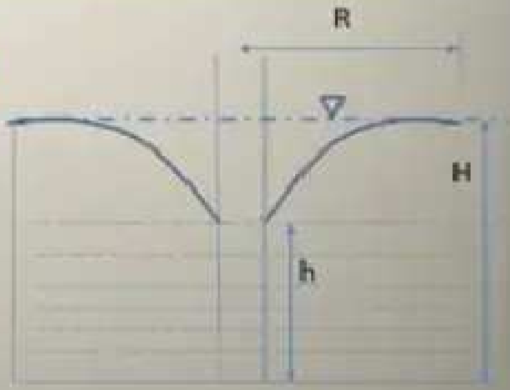
A 30 cm diameter well completely penetrates a confined aquifer of permeability 50 m/day. The length of the strainer is 25 m. Under steady state of pumping the drawdown at the well was found to be 3·5 m and the radius of influence was 300 m. Calculate the discharge.

$$Q = \frac{2\pi k b (H - h)}{\ln\left(\frac{R}{r}\right)}$$

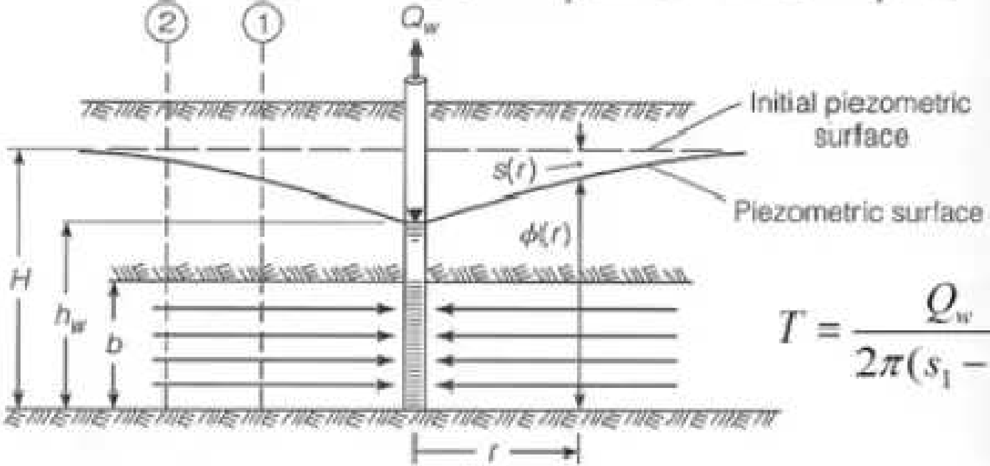
$(H - h) = 3.5$

$$Q = \frac{2\pi \times 50 \times 25 \times 3.5}{\ln\left(\frac{300}{0.15}\right)} \left(\frac{m \times m \times m}{day}\right)$$

$Q =$



# Confined Aquifer Example




$$T = \frac{Q_w}{2\pi(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right)$$

- A confined aquifer has hydraulic conductivity of 20 m/d, thickness of 6.6 m, and initial piezometric surface of 14.53 m above the lower confining layer.
  - What flowrate will cause the piezometric surface to be 13.85

m at a radius of 40 m and 14.31 at a radius of 85 m?

## Gate 2015

Q . An unconfined aquifer covering an area of 50 ha has a hydraulic conductivity of 20 m/day and specific yield of 12%. After a significant rainfall event, the water table rises from 17 m to 14.5 m below the ground level. Assuming no abstraction and outflow of groundwater during the recharge period, the amount of groundwater recharge contributed by the rainfall in $m^3$ is

Solution. Amount = $50 \times 10^4 \times (17 - 14.5) \times 0.12$

$= 150000\ m^3$

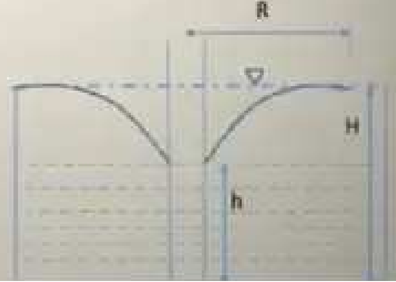
Specific yield is defined as

- the volume of water released from storage by an unconfined aquifer

Q. What discharge can be expected from an unconfined well having diameter as 3 meters. The drawdown in the well is 8 m and the aquifer is saturated to a depth of 15 m. The coefficient of permeability of the aquifer material is 5 m/day and the radius of influence is 150 m.

$$Q = \frac{\pi k\,(H^2 - h^2)}{ln(\frac{R}{r})}$$

$(H - h) = S$
15 − h = 8
h = 7




$$Q = \frac{\pi X 5\,(15^2 - 7^2)}{ln(\frac{150}{1.5})}$$