

PLASTIC ANALYSIS OF BEAMS

LESSON NO. 3

BY:

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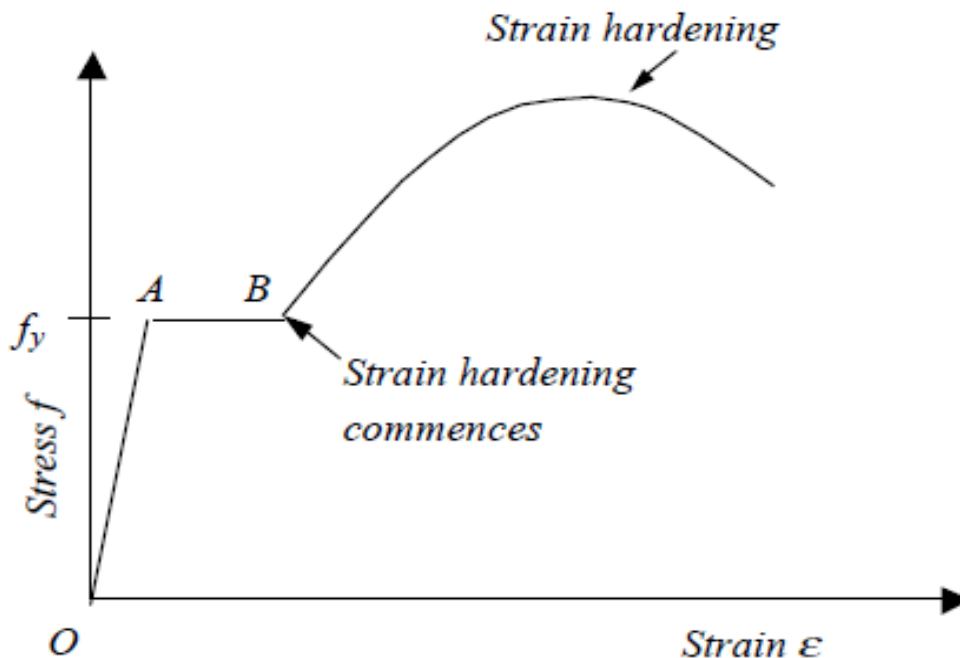
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STEEL DESIGN

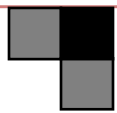
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PLASTIC ANALYSIS

The elastic design method, also termed as allowable stress method (or Working stress method), is a conventional method of design based on the elastic properties of steel. This method of design limits the structural usefulness of the material up to a certain allowable stress, which is well below the elastic limit. The stresses due to working loads do not exceed the specified allowable stresses, which are obtained by applying an adequate factor of safety to the yield stress of steel. The elastic design does not take into account the strength of the material beyond the elastic stress. Therefore the structure designed according to this method will be heavier than that designed by plastic methods, but in many cases, elastic design will also require less stability bracing. The term plastic has occurred due to the fact that the ultimate load is found from the strength of steel in the plastic range. This method is also known as method of load factor design or ultimate load design.



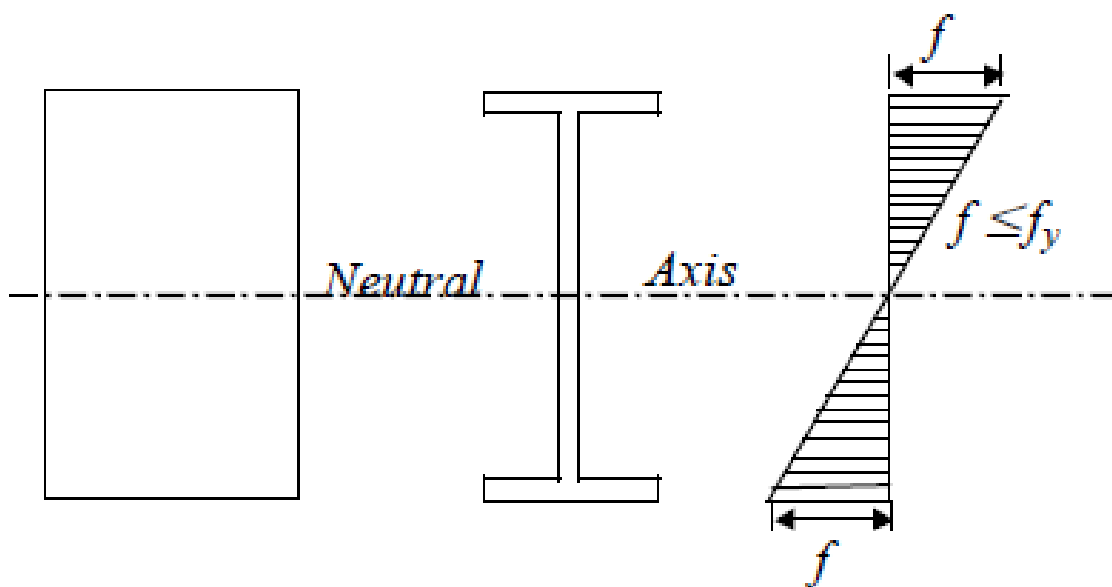
Idealised stress – strain curve for steel in tension



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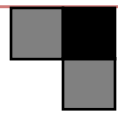
Structural steel is characterized by its capacity to withstand considerable deformation beyond first yield, without fracture. During the process of '**yielding**' the steel deforms under a constant and uniform stress known as '**yield stress**'. This property of steel, known as ductility, is utilised in plastic design methods.

The bending of a symmetrical beam subjected to a gradually increasing moment is considered first. The fibres of the beam across the cross section are stressed in tension or compression according to their position relative to the neutral axis and are strained in accordance with Idealised stress - strain curve for steel in tension.



Elastic stresses in beams

When the beam is subjected to a moment slightly greater than that, which first produces yield in the extreme fibres, it does not fail. Instead the outer fibres yield at constant stress (F_y) while the fibers nearer to the neutral axis sustain increased elastic stresses. Stresses in partially plastic beams the stress distribution for beams subjected to such moments. Such beams are

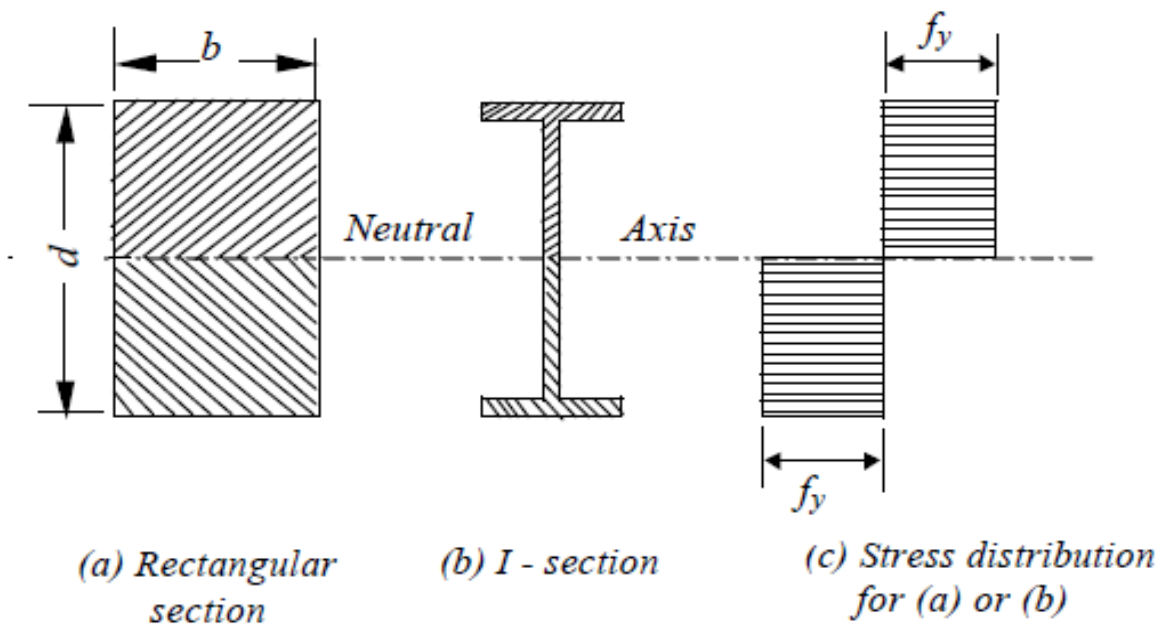


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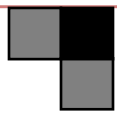
said to be '**partially plastic**' and those portions of their cross-sections, which have reached the yield stress, are described as '**plastic zones**'.

The depths of the plastic zones depend upon the magnitude of the applied moment. As the moment is increased, the plastic zones increase in depth, and, it is assumed that plastic yielding can occur at yield stress (F_y) resulting in two stress blocks, one zone yielding in tension and one in compression. Stresses in fully plastic beams represent the stress distribution in beams stressed to this stage. The plastic zones occupy the whole of the cross section, and are described as being '**fully plastic**'. When the cross section of a member is fully plastic under a bending moment, any attempt to increase this moment will cause the member to act as if hinged at the neutral axis. This is referred to as a plastic hinge. The bending moment producing a plastic hinge is called the full plastic moment and is denoted by ' M_p '.

The neutral axis is an axis in the cross section of a beam or shaft along which there are no longitudinal stresses or strains.



Stresses in fully plastic beams



THE COLLAPSE MECHANISM

A statically determinate beam will collapse if a plastic hinge start to develop. A simple beam, loaded with a concentrated load at its midspan, would become unstable and probably collapse as the concentrated load increases and a plastic hinge is developed at the point where maximum moment occurs. For statically indeterminate beam, it is necessary to develop multiple plastic hinge. The plastic hinge varies from structure but is usually not less than 2.

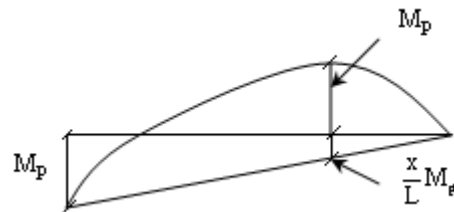
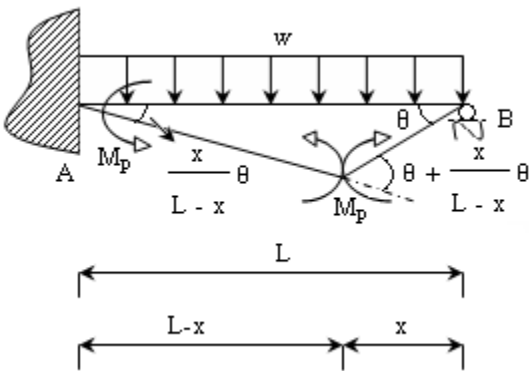
The arrangement of plastic hinges which permit collapse in a structure is called 'mechanism'.

METHODS FOR PLASTIC ANALYSIS OF BEAMS

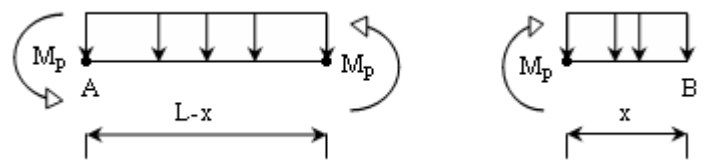
Equilibrium Method or the Upper Bound Theorem

The Equilibrium Method or The Upper Bound Theorem would state that a graph which satisfies the inequality is planar. A load factor computed on the basis of an arbitrarily assumed mechanism will always be greater than, or at best equal to the load factor at rigid plastic collapse.

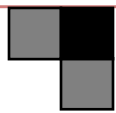
Assume the possible collapse mechanism.



Construct a moment diagram to get the M_p .



Assume the unknown value to be X.



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Solve for the two values of M_p

$$\sum M_A = 0 \qquad \sum M_B = 0$$
$$2M_p = \frac{w(L-x)^2}{2} \qquad M_p = \frac{wx^2}{2}$$

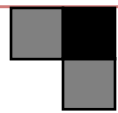
$$x = \frac{-2L \pm \sqrt{(2L)^2 + 4L^2}}{2}$$
$$= \frac{-2 + \sqrt{8}}{2} L$$
$$= (\sqrt{2} - 1)L$$
$$= 0.41421L$$

Solve for the value of X by equating the two M_p .

$$\frac{w(L-x)^2}{4} = \frac{wx^2}{2}$$
$$(L-x)^2 = 2x^2$$
$$L^2 - 2Lx - x^2 = 2x^2$$
$$x^2 + 2Lx - L^2 = 0$$

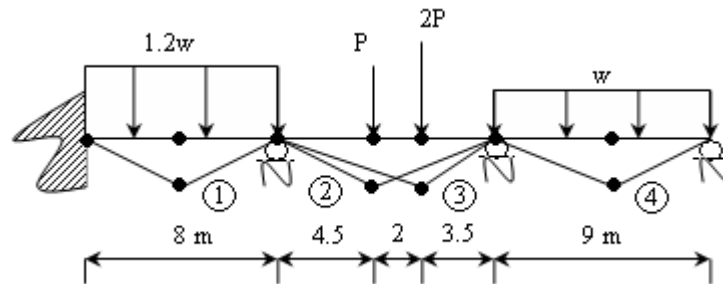
Substitute the value of X to the Equation with the greater equivalent.

$$\therefore M_p = \frac{wL^2}{2} (\sqrt{2} - 1)^2$$



SAMPLE PROBLEMS:

Example No. 1



$$P = 400 \text{ kN}; w = 40 \frac{\text{kN}}{\text{m}}$$

Using A-36 steel, determine the lightest W section that can be used safely for the beam above.

Solution:

Using equilibrium method we can construct four cases to solve for M_p .

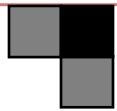
Case 1:

$$M_p = \frac{wL^2}{16} = \frac{1.2w_u L^2}{16}$$

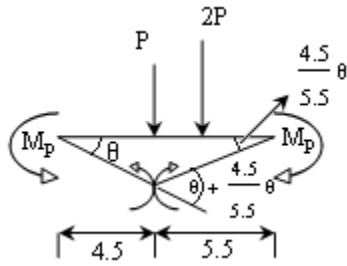
$$= \frac{1.2(40)(8)^2}{16}$$

$$= 192 \text{ kN} \cdot \text{m}$$

Case 2:



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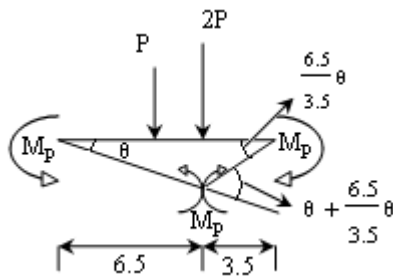


$$EW = IW$$

$$P(4.5\theta) + 2P\left(\frac{4.5}{5.5}\theta\right)(3.5) = M_p\left[\theta + \theta + \frac{4.5}{5.5}\theta + \frac{4.5}{5.5}\theta\right]$$

$$M_p = \frac{400\left[4.5 + \frac{9 \times 3.5}{5.5}\right]}{2\left[1 + \frac{4.5}{5.5}\right]} = 1125.0 \text{ kN} \cdot \text{m}$$

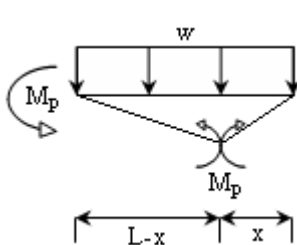
Case 3:



$$P(4.5\theta) + 2P(6.5\theta) = M_p\left[\theta + \theta + \frac{6.5}{3.5}\theta + \frac{6.5}{3.5}\theta\right]$$

$$M_p = \frac{400(4.5 + 13)}{2\left[1 + \frac{6.5}{3.5}\right]} = 1225.0 \text{ kN} \cdot \text{m}$$

Case 4:

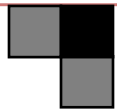


$$M_p = \frac{wL^2}{2}(\sqrt{2} - 1)^2$$

$$= \frac{40(9)^2}{2}(\sqrt{2} - 1)^2$$

$$= 277.95 \text{ kN} \cdot \text{m}$$

Use the M_p with greater value to solve for modulus of plasticity Z_x .



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$$\square M_p = 1225.0 \text{ kN} \cdot \text{m}$$

$$M_p = F_y Z_x$$

$$F_y = (36)(6.9) = 248 \text{ MPa}$$

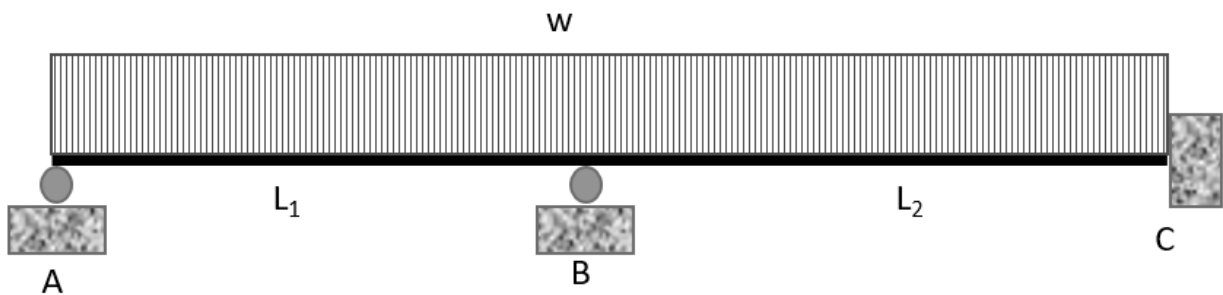
$$Z_x = \frac{1225 \times 10^6}{248} = 4939.5 \times 10^3 \text{ mm}^3$$

$$\text{W30} \times 99; Z_x = 5113 \times 10^3 \text{ mm}^3$$

From Steel Table W shape section, compare the solved Z_x to the given values of Z_x in the Table and use the section with the nearest modulus of plasticity Z_x but still considering the section with the lightest weight W .

Example No. 2 (from Quiz No. 2 - Steel Design 2018)

Determine the maximum safe live load that the beam below can carry using ASD and LRFD.



Given:

Total W_{DL} = 14 kN/m

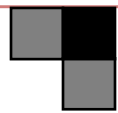
Beam AB = W18x50

Beam BC = W21x73

Steel = A-36

L_1 = 8.3m

L_2 = 11.7m



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Solution:

$$F_y = 6.9(36)$$

$$\hat{=} 248 \text{ MPa}$$

$$Z_{AB} = 1660 \times 10^3 \text{ mm}^3 \text{ (steel table)}$$

$$M_{p,AB} = F_y Z_{AB}$$

$$\hat{=} (248)(1660)(10^{-3})$$

$$\hat{=} 411.68 \text{ kN-m}$$

$$Z_{BC} = 2820 \times 10^3 \text{ mm}^3 \text{ (steel table)}$$

$$M_{p,BC} = F_y Z_{BC}$$

$$\hat{=} (248)(2820)(10^{-3})$$

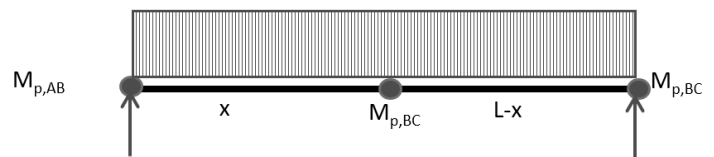
$$\hat{=} 699.36 \text{ kN-m}$$

Case 1, Span AB

$$w = \frac{2M_{p,AB}}{L^2(\sqrt{2}-1)^2}$$

$$\hat{=} \frac{2(411.68)}{8.3^2(\sqrt{2}-1)^2}$$

$$\hat{=} 69.66024 \frac{\text{kN}}{\text{m}}$$



Case 2, Span BC

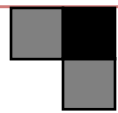
FBD of Left Side:

$$M_{p,AB} + M_{p,BC} = \frac{w x^2}{2} \quad \text{Eq. 1}$$

FBD of Right Side:

$$2M_{p,BC} = \frac{w(L-x)^2}{2} \quad \text{Eq. 2}$$

Equate Eq. 1 and Eq. 2



$$\frac{(M_{p,AB} + M_{p,BC})}{x^2} = \frac{2M_{p,BC}}{(L-x)^2}$$

$$\leq 73.09486 \frac{kN}{m}$$

$$(M_{p,AB} + M_{p,BC})(L^2 - 2Lx + x^2) = 2M_{p,BC}x^2$$

Choose lower "w"

Therefore, w = 69.66024 kN/m

$$(M_{p,BC} - M_{p,AB})x^2 + (M_{p,BC} + M_{p,AB})2Lx - (M_{p,BC} + M_{p,AB})L^2 = 0$$

$$ASD, w = \frac{w}{\Omega}$$

$$(M_{p,BC} - M_{p,AB}) = 699.36 - 411.48$$

$$\leq \frac{69.66024}{1.67}$$

$$\leq 287.68 \text{ kN} - m$$

$$(M_{p,BC} + M_{p,AB})2L = (699.36 + 411.48)(2)(11.7)$$

$$\leq 41.71272 \frac{kN}{m}$$

$$\leq 25998.34 \text{ kN} - m^2$$

$$ASD, w_i = w - w_{DL}$$

$$(M_{p,BC} + M_{p,AB})L^2 = (699.36 + 411.48)(11.7^2)$$

$$\leq 41.71272 - 14$$

$$\leq 152090.3 \text{ kN} - m^3$$

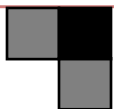
$$\leq 27.71272 \frac{kN}{m}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-25998.34 + \sqrt{25998.34^2 - 4(287.68)(152090.3)}}{2(287.68)}$$

$$\leq 5.513615 \text{ m}$$

$$w = \frac{2(M_{p,AB} + M_{p,BC})}{x^2}$$

$$\leq \frac{2(411.68 + 699.36)}{5.513615^2}$$



$$i 62.69584 \frac{kN}{m}$$

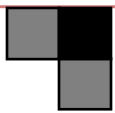
$$LRFD, w_i = w - w_{DL}$$

$$i \frac{(62.69584 - 1.2(14))}{1.6}$$

$$i 28.6849 \frac{kN}{m}$$

$$LRFD, w = \phi w$$

$$i (0.9)(69.66204)$$



SHAPE FACTOR

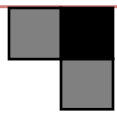
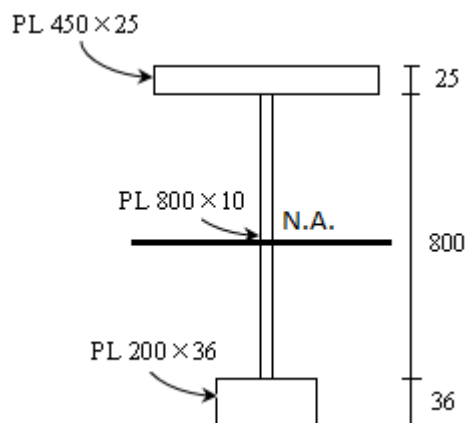
The ratio of the plastic moment of a beam to its yield moment is known as the **shape factor**. Thus

$$\frac{M_P}{M_Y} = \frac{\sigma_Y \cdot S}{\sigma_Y \cdot Z} = \frac{S}{Z}$$

where S is the plastic section modulus and Z is elastic section modulus. It can be seen from the equation that the shape factor is solely a function of the geometry of the beam cross section.

Example No. 1

Solve for the shape factor of the section shown below.



Solution:

Elastic:

Calculate the total area of the section above then Σ Area about N.A. to solve for y_b .

$$A = 11250 + 8000 + 7200 = 26450 \text{ mm}^2$$

$$y_b = \frac{11250(848.5) + 8000(436) + 7200(18)}{26450} = 497.66 \text{ mm}$$

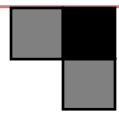
$$y_t = 836 + 25 - 497.66 = 363.34 \text{ mm}$$

Solve for moment of Inertia for modulus of elasticity.

$$\begin{aligned} I &= \frac{1}{12} \left[450(25)^3 + 10(800)^3 + 200(36)^3 \right] + 11250(848.5 - 497.66)^2 \\ &\quad + 8000(497.66 - 436)^2 + 7200(497.66 - 18)^2 \\ &= 3499.7 \times 10^6 \text{ mm}^4 \end{aligned}$$

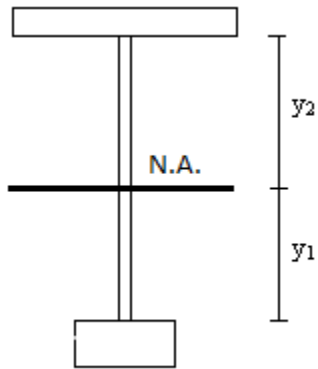
Solve for the section modulus.

$$Z = \frac{3499.7 \times 10^6}{497.66} = 7032.3 \times 10^3 \text{ mm}^3$$



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Plastic:



Find the value of y_1 by equating the area above to area below.

$$450(25) + (800 - y_1)(10) = 10y_1 + 200(36)$$

$$11250 + 8000 - 10y_1 = 10y_1 + 7200$$

$$y_1 = \frac{11250 + 8000 - 7200}{20} = 602.5 \text{ mm}$$

$$y_2 = 800 - 602.5 = 197.5 \text{ mm}$$

Solve for the modulus of plasticity by \int Area about the N.A.

$$S = 11250(12.5 + 197.5) + 197.5(10) \left[\frac{197.5}{2} \right] + 602.5(10) \left[\frac{602.5}{2} \right] + 7200(18 + 602.5)$$

$$= 8840.2 \times 10^3 \text{ mm}^3$$

We may now get the value of the shape factor.

$$f = \frac{S}{Z} = \frac{8840.2}{7032.3} = 1.2571$$