HYPOTHESIS TESTING FOR THE POPULATION MEAN

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In this section, you will test the null hypothesis

$$H_0 : \mu = \mu_0$$

against the alternative hypothesis

$$H_1 : \mu \neq \mu_0, \mu > \mu_0, \text{ or } \mu < \mu_0,$$

where μ is the population mean under consideration and μ_0 is the hypothesized value of μ.

σ

In dealing with hypothesis tests for the population mean, three cases may arise:

1. The population standard deviation is known.
2. The population standard deviation is not known.
3. The population standard deviation is not known but the sample size exceeds 30.

Table 8.1 summarizes the formulas to be used in computing for the test statistics for each case.

Table 8.1 Summary of Hypothesis Tests for the Population Mean

Case 1: σ is known $\mu = \mu_0$	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$z = \dfrac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$z < z_\alpha$ $z > z_\alpha$ $\lvert z \rvert < z_{\alpha/2}$
Case 2: σ is not known $n \leq 30$ $\mu = \mu_0$	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$t = \dfrac{\overline{x} - \mu_0}{s_x / \sqrt{n}}$	$t < {}_{-n-1}t_a$ $t > {}_{-n-1}t_a$ $\lvert t \rvert > {}_{-n-1}t_a$
Case 3: σ is not known but $n > 30$ $\mu = \mu_0$	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$z = \dfrac{\overline{x} - \mu_0}{s_x / \sqrt{n}}$	$z < z_\alpha$ $z > z_\alpha$ $\lvert z \rvert < z_{\alpha/2}$

In most situations, the population standard deviation is unknown and is estimated from the sample. Consider the problem below.

An admission officer in a university claims that the overall mean score of all applicants who are admitted in the university is less than the mean score of applicants who are admitted in an engineering course. In a sample of 30 applicants' entrance scores, the mean score and the standard deviation are 100 and 13, respectively. Test the admission officer's claim at 0.05 level of significance that the mean score of applicants who are admitted in an engineering course is 109.

To answer the problem, state first the null and alternative hypotheses. Let μ be the mean score of all applicants who are admitted in the university.

$$H_0 : \mu = 109$$

$$H_1 : \mu < 109$$

Then compute for the test statistic. Since the population standard deviation is unknown, use the formula for case 2. The test statistic is computed as follows:

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

$$= \frac{100-109}{13/\sqrt{30}}$$

$$= -3.79$$

Determine the critical value. Since n=30 and α=0.05, the critical value is ${}_{29}t_{0.05}$ = 1.699 from table C of appendix A. Decide whether to reject the null hypothesis or not. Since -3.79 < -1.699, the test statistic is in the rejection region, as shown fig. 8.2. Thus, H_0 is rejected. This means that the admission officer has sufficient evidence to claim that the overall mean score of all applicants who are admitted in the university is less than the mean score of applicants who are admitted in an engineering course.

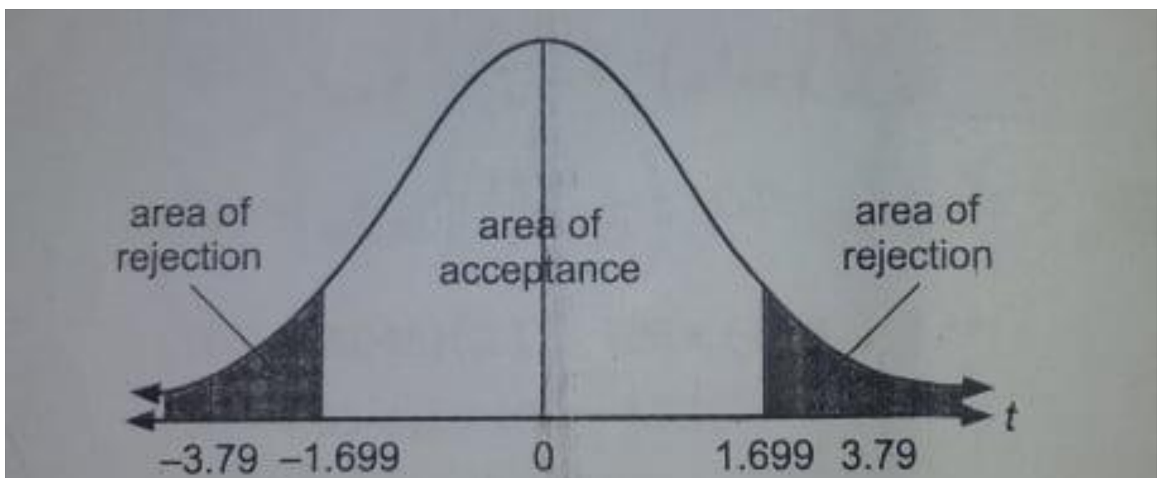

Figure 8.2

One way to verify the result of the hypothesis test is by computing for the **confidence interval.** Confidence interval is the range of values where the actual value of the parameter lies.

Table 8.2: 100(1-α)% Confidence Interval Estimators for the Population Mean of a Normal Distribution.

Case 1: σ^2 is known	$(\overline{x} - z_{a/2}\frac{\sigma}{\sqrt{n}}, \overline{x} + z_{a/2}\frac{\sigma}{\sqrt{n}})$
Case 2: σ^2 is known (sample size n ≤ 30)	$\overline{x} - {}_{n\text{-}1}t_{a/2}\frac{s}{\sqrt{n}}, \overline{x} + {}_{n\text{-}1}t_{a/2}\frac{s}{\sqrt{n}})$
Case 3: σ^2 is unknown (sample size n > 30)	$(\overline{x} - z_{a/2}\frac{s}{\sqrt{n}}, \overline{x} + z_{a/2}\frac{s}{\sqrt{n}})$

Using the previous example, compute for the 95% confidence interval for the mean score of all applicants admitted in the university.

$$(x - ({}_{df}t_{a/2})\,({}^{s_x}/_{\sqrt{n}}), x + ({}_{df}t_{a/2})\,({}^{s_x}/_{\sqrt{n}})\,)$$

$$(100 - {}_{29}t_{0.025})\,({}^{13}/_{\sqrt{30}}), 100 + {}_{29}t_{0.025})\,({}^{13}/_{\sqrt{30}}))$$

$$(100 - (2.045)(2.37) , 100 + (2.045)(2.37))$$

$$100 - 4.85 < \mu < 100+4.85$$

$$95.15 < \mu < 104.85$$

Notice that the result of the hypothesis test coincides with the computed confidence interval for the mean score. The 95% confidence interval does not include 109, which is the hypothesized value of the mean in the null hypothesis.

At this point, you must have realized that the population mean is different from the sample mean. If the hypothesized mean is 100 and the sample mean happens to be 102, you cannot easily conclude that the population mean is not equal to 100.

There may be random errors in the gathering of the samples or in the scoring of the test. You have to apply the law of probability and the appropriate test statistic in order to arrive at an acceptable conclusion about the population.

HYPOTHESIS TESTING FOR TWO POPULATION MEANS USING INDEPENDENT SAMPLES

In comparing the characteristics of two populations, information about the populations may be obtained using independent or dependent samples.

Comparing the two population means is equivalent to deciding whether the difference between the two means is significantly different from zero. Hence, the null and alternative hypotheses are stated as follows:

$$H_0 : \mu_1 = \mu_0 \text{ or } \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 \neq \mu_2 \text{ or } \mu_1 - \mu_2 \neq 0$$

$$\mu_1 > \mu_2 \text{ or } \mu_1 - \mu_2 > 0$$

$$\mu_1 < \mu_2 \text{ or } \mu_1 - \mu_2 < 0$$

Table 8.3 gives the formulas in computing for the test statistic for different cases.

Table 8.3 Tests of Hypotheses for the Difference of Means Based on Two Independent Samples

Case 1: $\sigma_1{}^2$ and $\sigma_2{}^2$ are known			
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$z = \frac{(\overline{x_1} - \overline{x_2}) - d_0}{\sqrt{\frac{\sigma_{1^2}}{n_1} + \frac{\sigma_{1^2}}{n_1}}}$	$z < -z_a$ $z > z_a$ $\lvert z \rvert < z_{a/2}$

Case 2:			
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t = \frac{\overline{(x_1 - \overline{x_1})} - d_0}{\sqrt{S_p^2\left\{\frac{1}{n_1}+\frac{1}{n_2}\right\}}}$ where $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_1 - 1)S_2^2}{n_1 + n_2 - 2}$	$t < -_{n_1+n_2-2}t_a$ $t > {}_{n_1+n_2-2}t_a$ $\lvert t \rvert > {}_{n_1+n_2-2}t_{a/2}$
Case 3: σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 \neq \sigma_2^2$			
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t = \frac{\overline{(x_1 - \overline{x_1})} - d_0}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}}$	$t < -_{df}t_a$ $t > {}_{df}t_a$ $\lvert t \rvert > {}_{df}t_{a/2}$ $df = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(S_2^2/n_2\right)^2}{n_2 - 1}}$
Case 4: σ_1^2 and σ_2^2 are unknown but $n_1>30$ and $n_2>30$			
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$z = \frac{\overline{(x_1 - \overline{x_1})} - d_0}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}}$	$z < -z_a$ $z > z_a$ $\lvert z \rvert > z_{a/2}$

Consider the problem below.

A researcher claims that there is a significant difference between the scores in a physics diagnostic test (pretest) of teachers with less than four years of teaching experience and those with four or more years of teaching experience. A sample of 30 teachers with less than four years of teaching experience and a sample of 29 teachers with four or more years of teaching experience were given the same diagnostic test. Test the researcher's claim at 0.01 level of significance non-directional.

The variances have a small difference; hence, this is an example of case 2. Use the data below.

	Less than 4 years	4 years or more
n	30	29
$\bar{x}$	43	47
s	8	10

$H_1 : \mu_1 \neq \mu_2$ or $\mu_2 - \mu_2 \neq 0$.

Where

μ_1 is the population mean of teachers with less than four years of teaching experience

μ_2 is the population mean of teachers with four or more years of teaching experience.

The test statistic is computed as follows:

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left[\frac{(n_1-1)(s_1^2)+(n_2-1)(s_2^2)}{n_1+n_2-2}\right]\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$$

$$= \frac{42-47}{\sqrt{\left[\frac{(29)(8^2)+(28)(10^2)}{30+29-2}\right]\left[\frac{1}{30}+\frac{1}{29}\right]}}$$

$$= \frac{-5}{\sqrt{(81.68)(0.07)}}$$

$$= \frac{-5}{2.39}$$

$$= -2.092$$

The critical value is: ${}_{57}t_{0.005} = -2.660$

The rejection area is shown in the figure below.

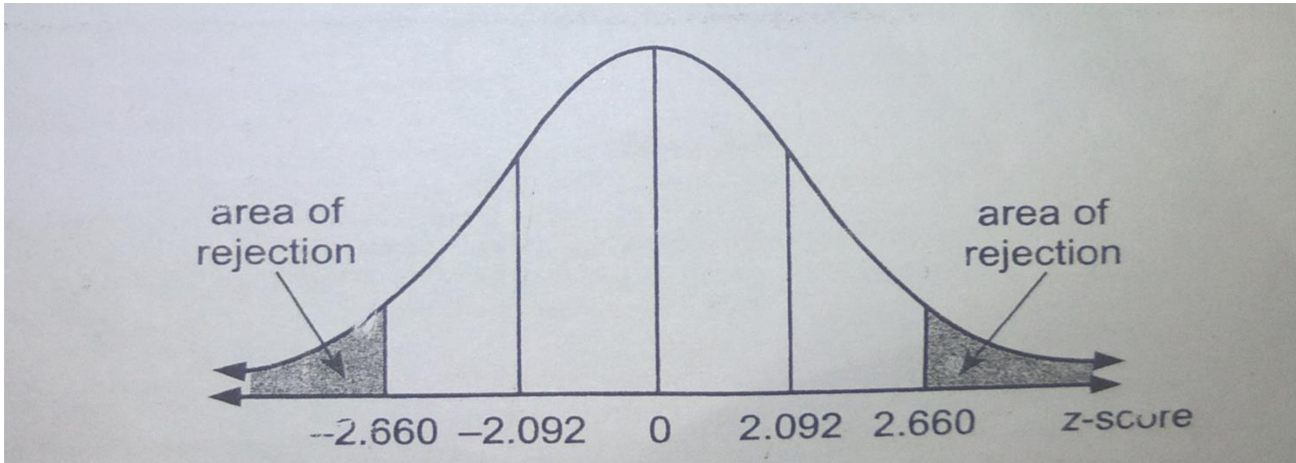

Figure 8.3

Since the test statistic does not lie within the area of rejection, the null hypothesis is rejected. Therefore, there is no significant difference in the mean scores of teachers with less than four years of teaching experience and those with four years of teaching experience.

Table 8.4 Confidence Interval Estimations for $\mu_1 - \mu_2$

Cases	Confidence Interval Estimators
Case 1: σ_2^1 *and* : σ_2^2 *are known*	$[(\overline{x_1}-\overline{x_2}) - z_{\frac{a}{2}}\sqrt{\frac{\sigma_2^1}{n_1}+\frac{\sigma_2^2}{n_2}}\ , (\overline{x_1}-\overline{x_2}) + z_{a/2}\sqrt{\frac{\sigma_2^1}{n_1}+\frac{\sigma_2^2}{n_2}}\ ,$
Case 2: σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma^2$	$\left((\bar{x}_1-\bar{x}_2) - {}_{n_1+n_2-2}t_{\alpha/2}\sqrt{S_P^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},\ (\bar{x}_1-\bar{x}_2) + {}_{n_1+n_2-2}t_{\alpha/2}\sqrt{S_P^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right)$ where $S_P^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$
Case 3: σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 \neq \sigma_2^2$	$\left((\bar{x}_1-\bar{x}_2) - {}_{df}t_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}},\ (\bar{x}_1-\bar{x}_2) + {}_{df}t_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right)$ where $df = \dfrac{\left(S_1^2/n_1 + S_2^2/n_2\right)}{\dfrac{\left(S_1^2/n_1\right)^2}{n_1-1}+\dfrac{\left(S_2^2/n_2\right)^2}{n_2-1}}$
Case 4: σ_1^2 and σ_2^2 are unknown but $n_1 > 30$ and $n_2 > 30$	$\left((\bar{x}_1-\bar{x}_2) - z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}},\ (\bar{x}_1-\bar{x}_2) + z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right)$

Compute for 99% confidence interval for the difference of the means in the given problem.

The endpoints of the range are computed as follows:

$$(\overline{x_1} - \overline{x_2}) \pm {}_{df}t_{a/2} \sqrt{\left[\frac{(n_1-1)(s_1^2)+(n_2-1)(s_2^2)}{n_1+n_2-2}\right]\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}$$

(42-47) ± (2.660)(2.39)

-5 ± 6.36

Hence, the 99% confidence interval is (-11.36, 1.36).

Note that the 99% confidence interval contains the hypothesized difference of the two means which is zero.

If the researcher sets the α level of significance at 0.05, the null hypothesis would have been rejected since the critical value t-value is only 2.000. The study would then have a different finding. That is why the α level ahead of the research is very important.

Notice that n_1 and n_2 are nearly equal. In theory, two nearly equal sample sizes make the test powerful. When designing a study, make the sample size almost equal.

HYPOTHESIS TESTING FOR TWO POPULATION MEANS USING DEPENDENT SAMPLE

In education, dependent samples are used to measure how much the students learned by comparing the results of posttest and pretest. Dependent samples are also used to compare teaching methods.

The test statistic and critical value for comparing two populations means using dependent samples are given below.

Table 8.5. Tests of Hypotheses for the Difference of Means Based on Paired Observations

Null Hypothesis H_0	Alternative Hypothesis H_1	Test Statistic	Region of Rejection
$\mu_D = d_0$	$\mu_D < d_0$	$t = \dfrac{\bar{d}}{S_d/\sqrt{n}}$	$t < -_{n-1}t_{\alpha}$
	$\mu_D > d_0$		$t > _{n-1}t_{\alpha}$
	$\mu_D \neq d_0$		$\lvert t \rvert > _{n-1}t_{\alpha/2}$

Consider the problem below.

The teachers in a certain region participated in a summer program to advance their knowledge in biology. The pretest and posttest scores of 24 randomly selected teachers are shown below. Test whether the mean score in the posttest is significantly different from the mean score in the pretest at $\alpha = 0.01$.

Table 8.6

Participant	1	2	3	4	5	6	7	8	9	10	11	12
Posttest x_2	18	18	20	23	24	26	28	30	24	29	30	29
Pretest x_1	15	17	18	20	21	22	24	25	26	27	29	29
$d_i = x_2 - x_1$	3	1	2	3	3	4	4	5	−2	2	1	0

Participant	13	14	15	16	17	18	19	20	21	22	23	24
Posttest x_2	34	33	30	36	31	34	37	40	40	45	46	47
Pretest x_1	30	30	30	31	32	33	34	35	36	34	45	45
$d_i = x_2 - x_1$	4	3	0	5	−1	1	3	5	4	11	1	2

For convenience, the scores in the posttest are assigned as the minuend so that most of the differences are positive. The computed mean and standard deviation of the differences of paired observations are $\bar{d} = 2.67$ and $s_d = 2.60$, respectively. Thus, the test statistic is

$$t = \frac{2.67}{2.60/\sqrt{24}}$$

$$= 5.03$$

The critical value is ${}_{23}t_{0.01} = 2.50$

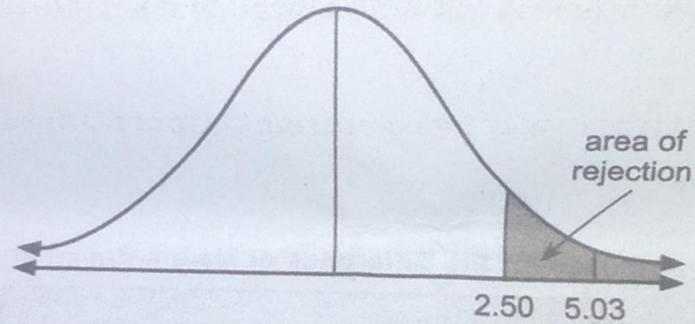

Figure 8.4

Since the test statistic lies within the area of rejection, the null hypothesis is rejected. Thus, the posttest mean score is significantly higher than the pretest mean score.

Such result is important to the sponsors of the summer institute because it will justify the holding of summer programs for advancing the knowledge of others.

HYPOTHESIS TESTING FOR POPULATION PROPORTIONS

There are situations where in the result of hypothesis testing for population proportion is of great importance. The test statistic for testing the population proportion is given in table 8.7.

Table 8.7. Hypothesis Test for Population Proportion

Null Hypothesis H_0	Alternative Hypothesis H_1	Test Statistic	Region of Rejection
$p = p_0$	$p < p_0$	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z < -z_\alpha$
	$p > p_0$		$z > z_\alpha$
	$p \neq p_0$		$\lvert z\rvert > z_{\alpha/2}$

The president of an university is in need of an academic vice president and wants the approval of more than 50% of the faculty for his candidate. He wants statistical significance and absolute value.

Out of 120 randomly selected faculty members, 55 approved. Is this sufficient to appoint the candidate? Use 0.05 level of significance.

Test the following hypotheses.

$H_0 : p = 0.5$

$H_1 : p > 0.5$

With $\hat{p} = \frac{55}{120} = 0.4583 \; and \; p_0 = 0.5$, the test statistic is computed as follows:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{x}}} = \frac{0.4583 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{120}}}$$

$$= \frac{-0.0417}{0.0456}$$

$$= -0.9145$$

The critical z-value at the 0.05 level of significance is ${}_{120}t_{0.050} = 1.658$.

Since the test statistic is less than the critical value, the null hypothesis is not rejected. Thus, there is no significant evidence to conclude that $p>0.5$. this means that more than 55 faculty members should approved the evidence.

HYPOTHESIS TESTING FOR PROPORTIONS USING TWO INDEPENDENT SAMPLES

The test statistic is given by

$$z = \frac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.600 - 0.476}{\sqrt{(0.537)(0.463)\left(\frac{1}{200} + \frac{1}{210}\right)}}$$

$$= \frac{0.124}{0.049}$$

$$= 2.531$$

The critical z-value at the 0.05 level for one-tailed test is ${}_{408}t_{0.05}$ = 1.645. Since the test statistic exceeds the critical z-value, H_0 is rejected.

There is sufficient evidence to conclude that the proportion of registered voters who are in favor of the governor's candidacy for congressman is greater in district 1 than in district II.

It would be wiser for the governor to run for congressman in district 1.

Consider the problem below.

The governor of a certain province with two districts is planning to run as congressman in either of the two districts when his term as governor expires. To decide in which district he will run, he wants to know where he is more popular.

Of the 200 registered voters from district I, 120 are in favor of the governor running for congressman. Of the 210 registered voters from district II, 100 are in favor of his candidacy.

Let $\widehat{p_1}$ *and* $\widehat{p_2}$ be the proportion of registered voters who are in favor of the governor candidacy in districts I and II, respectively.

Test the following hypotheses at 0.05 level of significance.

$H_0 : \widehat{p_1} = \widehat{p_2}$

The alternative hypothesis is

$H_1 : \widehat{p_1} > \widehat{p_2}$,

from the given data,

$$\widehat{p_1} = \frac{120}{200} = 0.60 \; and \; \widehat{p_2} = \frac{100}{210} = 0.476$$

The values of p and q are computed as follows:

$$\hat{p} = \frac{f_1 + f_2}{n_1 + n_2}$$

$$= \frac{120 + 100}{200 + 210}$$

$$= \frac{220}{410}$$

$$= 0.537$$

$$\hat{q} = 1 - 0.537 = 0.463$$

For certain variables, not only the mean but also the variance is very important. Suppose a school has been using standardized final examinations in all subjects. So, the variance σ^2 of these tests are known.

Now, the school decided to change the specifications of these tests and add more easy items and more difficult items. This is to discriminate students who are below average from students who are achieving poorly, and to discriminate those who are very bright from average students.

With these new set of final exams, the variance is expected to be significantly higher than the old one. To verify this claim, hypothesis testing for the variance of the population must be performed.

HYPOTHESIS TESTING FOR POPULATION VARIANCE

The null hypothesis and the alternative hypothesis in testing the variance are stated as follows:

$$H_0 : \sigma^2 = \sigma_0^2$$

$$H_1 : \sigma^2 \neq \sigma_0^2$$

The symbol σ^2 indicates the population variance while σ_0^2 refers to the hypothesized value of the population variance.

The test statistic is given by

$$x^2 = \frac{(n-1)(s^2)}{\sigma_0^2}$$

where s^2 is the sample variance and x^2 has chi-square distribution with n-1 degrees of freedom.

The table below gives the rejection region for the test of hypothesis for population variance.

Null Hypothesis H_0	Alternative Hypothesis H_1	Test Statistic	Region of Rejection
$\sigma^2 = \sigma_0^2$	$\sigma^2 < \sigma_0^2$ $\sigma^2 > \sigma_0^2$ $\sigma^2 \neq \sigma_0^2$	$x^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$x^2 < {}_{n-1}x_{1-a}^2$ $x^2 > {}_{n-1}x_a^2$ $x^2 < {}_{n-1}x_{1-\frac{a}{2}}^2$ $x^2 > {}_{n-1}x_{a/2}^2$
$\sigma = \sigma_0$	$\sigma > \sigma_0$ $\sigma < \sigma_0$ $\sigma \neq \sigma_0$	$x^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$x^2 < {}_{n-1}x_{1-a}^2$ $x^2 > {}_{n-1}x_a^2$ $x^2 < {}_{n-1}x_{1-\frac{a}{2}}^2$ $x^2 > {}_{n-1}x_{a/2}^2$

Consider the problem below.

An educator claims that the new standardized final exam will yield a variance that is higher than the variance of the previous standardized final exam. The new standardized final exam was administered to a random sample of 25 students. The standard deviation is 16, hence, the sample variance is $16^2 = 256$. If the variance of the old test is 100, test the educator's claim at 0.01 level of significance.

To answer the problem, first state the null and alternative hypotheses.

$H_0 : \sigma^2 = 100$

$H_1 : \sigma^2 > 100$

Then compute for the test statistic.

$$x^2 = \frac{(n-1)(s^2)}{\sigma_0^2}$$

$$= \frac{(25-1)(256)}{100}$$

$$= 61.44$$

The critical chi-square value is

$$_{24}x^2_{0.01} = 42.9798$$

Since 61.44 > 42.9798, the null hypothesis is rejected.

Therefore, the new set of test has significantly greater variability than the old test.

Suppose in the same study, the sample size is 36 and the computed X^2 is 150. The X^2 table cannot be used because the maximum degree of freedom is only 30. In this case, use the formula below.

$$z = \sqrt{2X^2} - \sqrt{2df - 1}$$

Thus, we have

$$z = \sqrt{(2)(150)} - \sqrt{2(35) - 1} = 17.32 - 8.31$$

$$= 9.01$$

At 0.01 level of significance, two-tailed or non-directional test, the critical z-value is 2.58. thus, H_0 is again rejected.

Take note again that for a sample size greater than 30, the chi-square value may be converted into a z-test.

HYPOTHESIS TESTING FOR TWO POPULATION VARIANCES USING DEPENDENT SAMPLES

In hypothesis testing for the difference between two means using dependent sample, the example given is the pretest an posttest scores of summer institute participants. Hypothesis testing for the variances of the pretest and posttest can also be performed.

You may expect that the variability in the posttest scores is significantly less than the variance in the pretest because those who obtained low scores in the pretest will have a greater chance to raise their scores in the posttest, whereas those who already scored high in the pretest will find it difficult to raise their scores further due to the so-called ceiling effect.

To verify this claim, hypothesis testing for variances using dependent samples must be performed.

The null hypothesis and alternative hypothesis for testing two population variances are stated as follows:

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2 \; or \; \sigma_1^2 > \sigma_2^2 \; or \; \sigma_1^2 < \sigma_2^2$$

The critical value and the rules for rejecting the null hypothesis are as follows:

Null Hypothesis H_0	Alternative Hypothesis H_1	Region of Rejection
$\sigma_1^2 = \sigma_0^2$	$\sigma_1^2 < \sigma_0^2$	$t < {}_{n-2}t_a$
	$\sigma_1^2 > \sigma_0^2$	$t > {}_{n-2}t_a$
	$\sigma_1^2 \neq \sigma_0^2$	$\lvert t \rvert > {}_{n-2}t_{a/2}$

The test statistic is given by

$$t = \frac{s_1^2 - s_2^2}{\sqrt{\left(\frac{4s_1^2 s_2^2}{n-2}\right)(1 - r_{12}^2)}}$$

where s_1^2 and s_2^2 are the sample variances,

n is the sample size, and

r_{12} is Pearson's r correlation between the two samples.

The test statistic t has student t-distribution with n-2 degree of freedom.

Consider the scores of the participants in pretest and posttest, as shown in table 8.6.

Table 8.6

Participant	1	2	3	4	5	6	7	8	9	10	11	12
Posttest x_2	18	18	20	23	24	26	28	30	24	29	30	29
Pretest x_1	15	17	18	20	21	22	24	25	26	27	29	29
$d_i = x_2 - x_1$	3	1	2	3	3	4	4	5	−2	2	1	0

Participant	13	14	15	16	17	18	19	20	21	22	23	24
Posttest x_2	34	33	30	36	31	34	37	40	40	45	46	47
Pretest x_1	30	30	30	31	32	33	34	35	36	34	45	45
$d_i = x_2 - x_1$	4	3	0	5	−1	1	3	5	4	11	1	2

The following values are computed from the given data.

$r_{12} = 0.950$

$s_2 = 8.30$

$s_1 = 7.76$

Test whether the variance of posttest scores is significantly greater than the variance of pretest scores at 0.05 level of significance.

The hypothesis are stated as follows:

$$H_0 = \sigma_1^2 = \sigma_2^2$$

$$H_1 = \sigma_1^2 < \sigma_2^2$$

The test statistic is computed as follows:

$$t = \frac{(8.30)^2 - (7.76)^2}{\sqrt{\left(\frac{(4)(8.30)^2(7.76)^2}{24-2}\right)(1 - 0.95^2)}}$$

$$= \frac{8.6724}{\sqrt{(754.2528)(0.0975)}}$$

$$= 1.011$$

The critical t-value at the 0.05 level with 22 degrees of freedom is 1.717.

Since the test statistic is less than the critical t-value, the null hypothesis is not rejected. Hence, there is no sufficient evidence to conclude that the variance of posttest scores is significantly greater than the variance of pretest scores.

HYPOTHESIS TESTING FOR TWO POPULATION VARIANCES USING INDEPENDENT SAMPLES

Table 8.19 gives the null and alternative hypotheses, the test statistics, and the rejection region for testing two population variances.

Null Hypothesis H_0	Alternative Hypothesis H_1	Test Statistic	Region of Rejection
$\sigma_1 = \sigma_2 or \frac{\sigma_1}{\sigma_2} = 1$	$\sigma_1 > \sigma_2 or \frac{\sigma_1}{\sigma_2} > 1$	$F = \frac{s_1^2}{s_2^2}$	$F > {}_{(n_1-1,n_2-1)}F_a$
	$\sigma_2 > \sigma_1 or \frac{\sigma_2}{\sigma_1} > 1$	$F = \frac{s_2^2}{s_1^2}$	$F > {}_{(n_2-1,n_1-1)}F_a$
	$\sigma_1 \neq \sigma_2 or \frac{\sigma_1}{\sigma_2} \neq 1$	$F = \frac{s_1^2}{s_2^2}, s_1^2 > s_2^2$	$F > {}_{(n_1-1,n_2-1)}F_{a/2}$

Note that, instead of $H_1 : \sigma_1 < \sigma_2$ or $\frac{\sigma_1}{\sigma_2}$, the alternative hypothesis is stated as $H_1 : \sigma_1 < \sigma_2$ or $\frac{\sigma_2}{\sigma_1} > 1$ so that it will be easy to compare the test statistic with the critical value. The formula for the test statistic is generally of the form.

$$F = \frac{larger\ sample\ variance}{smaller\ sample\ variance}$$

The test statistic follows the F-distribution. Use table D1 to D4 in appendix A for the critical value of the test statistic, taking into account the level of significance. In getting the critical value, locate the degrees of freedom from the sample with larger variance on the first column and the degrees of freedom from the sample with smaller variance on the forst row.

Consider the problem below.

A university has two campuses-one in metro manila and the other in a nearby provine. The university gives the same entrance exam in both campuses and admit almost the same number of applicants. An admission officer claims that the variances of test scores in two campuses are significantly dofferent.

To test this claim, a random sample of 32 students are taken among the Manila examinees. The standard deviation obtained from the scires is 13.5. From the examinees of the other campus, a random sample of 25 students are chosen. The standard deviation is 9.2. Test the claim at 0.05 level of significance.

Let σ_1^2 be the variance of test scores in Manila campus and σ_2^2 be the variance of test scores in the other campus. The hypotheses to be tested are as follows:

$$H_0: \sigma_1^2 = \sigma_2^2 \, or \, \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_1: \sigma_1^2 \neq \sigma_2^2 \, or \, \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

The test statistic is computed as follows:

$$F = \frac{s_1^2}{s_2^2}$$

$$= \frac{13.5^2}{9.2^2}$$

$$= \frac{182.25}{84.64}$$

$$= 2.15$$

The critical value is ${}_{(31,24)}F_{0.025} = 2.21$.

Since the computed F-statisitc is greater than the critical value, reject the null hypothesis at 0.05 level of significance.

Hence, there is a significant difference between the variances of the test scores in two campuses.