POWER SYSTEMS LECTURE SERIES

PART II. PER UNIT ANALYSIS

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Introduction

Importance of Electric Circuit Theory

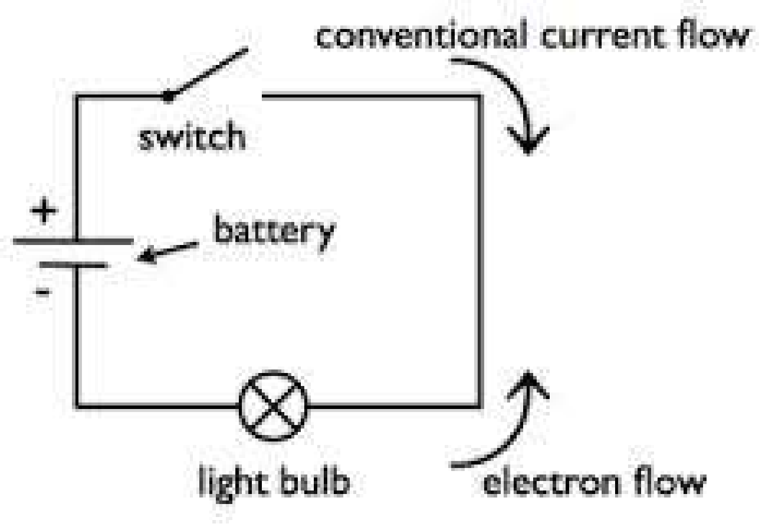
1. Basic Fundamentals of Electrical Engineering.

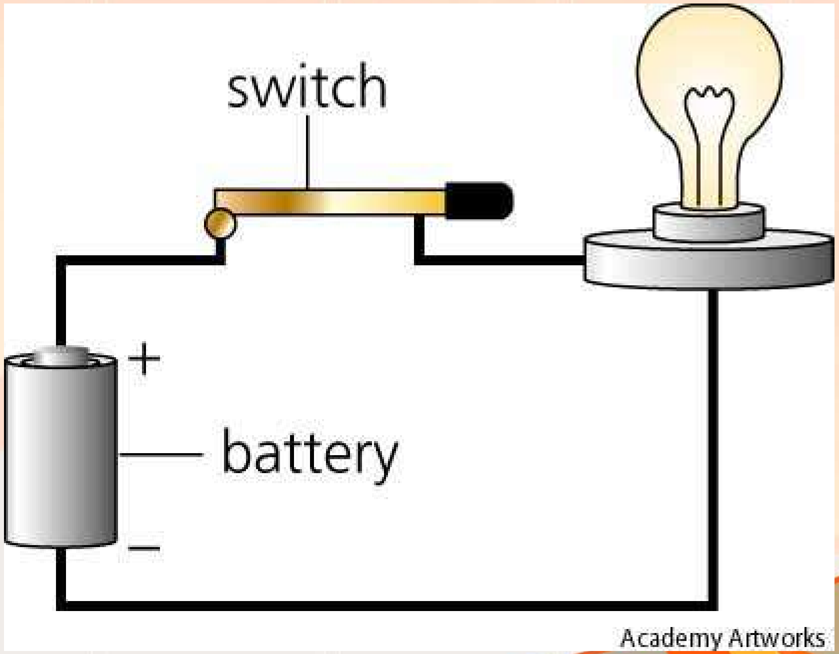
2. Good Model for the study of energy systems.

3. Interconnection of electrical devices.

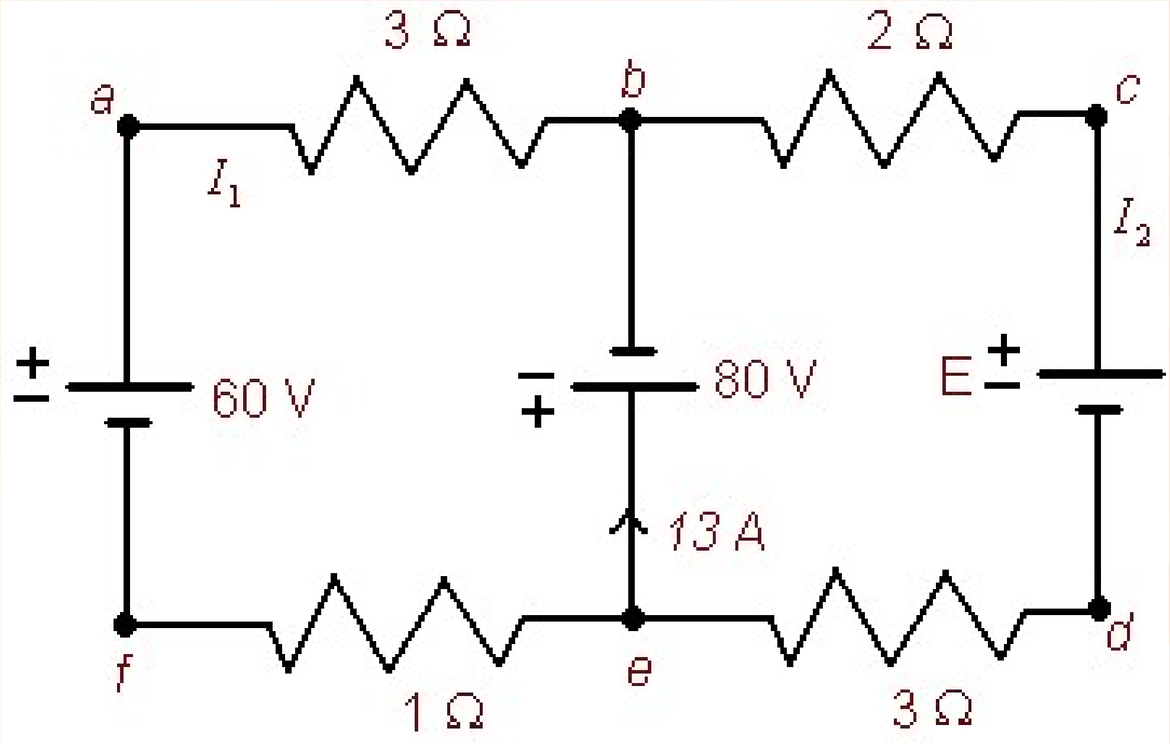
Introduction

Electrical Circuit *– is an interconnection of electrical elements.*





Examples of Electrical Circuits



Examples of Electrical Circuits

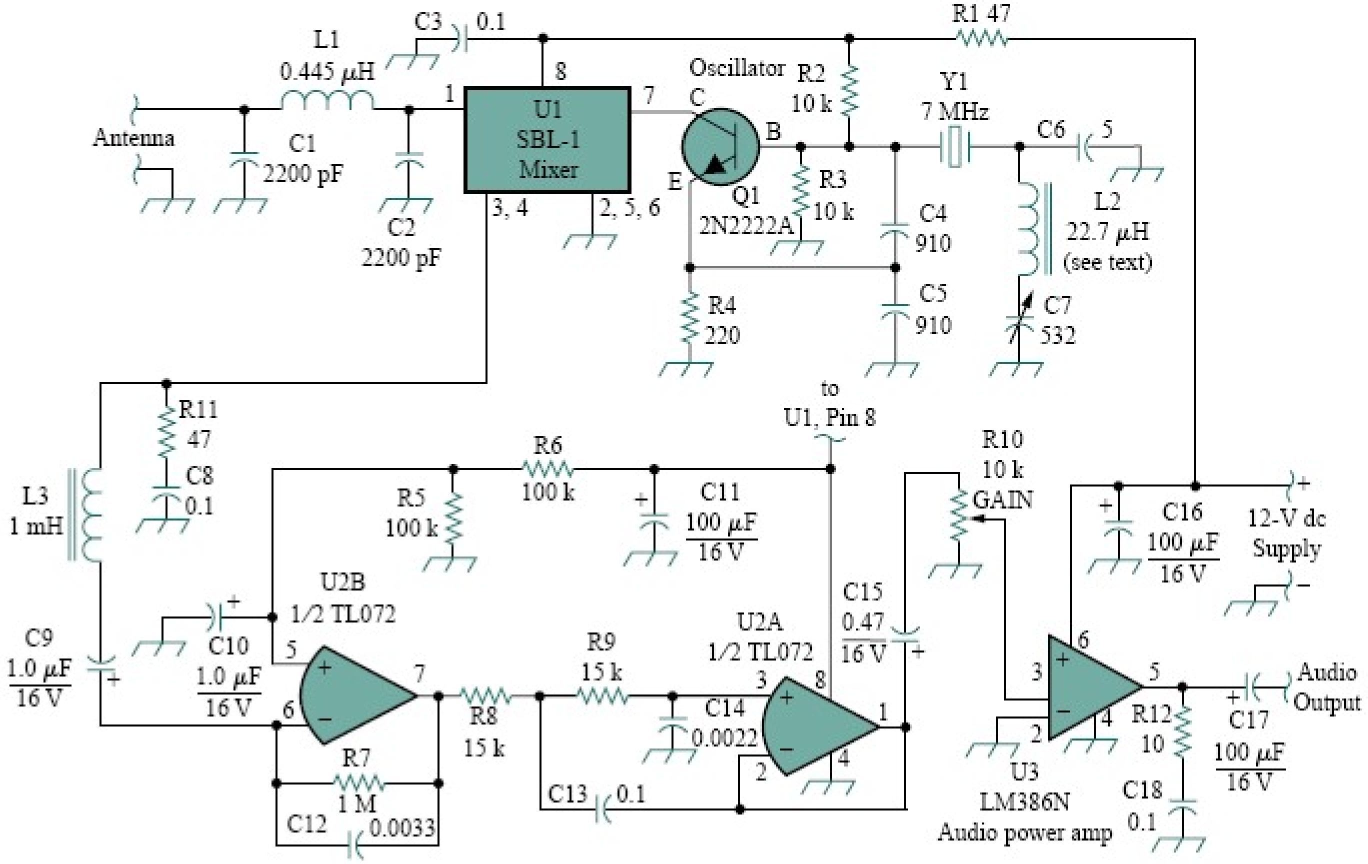
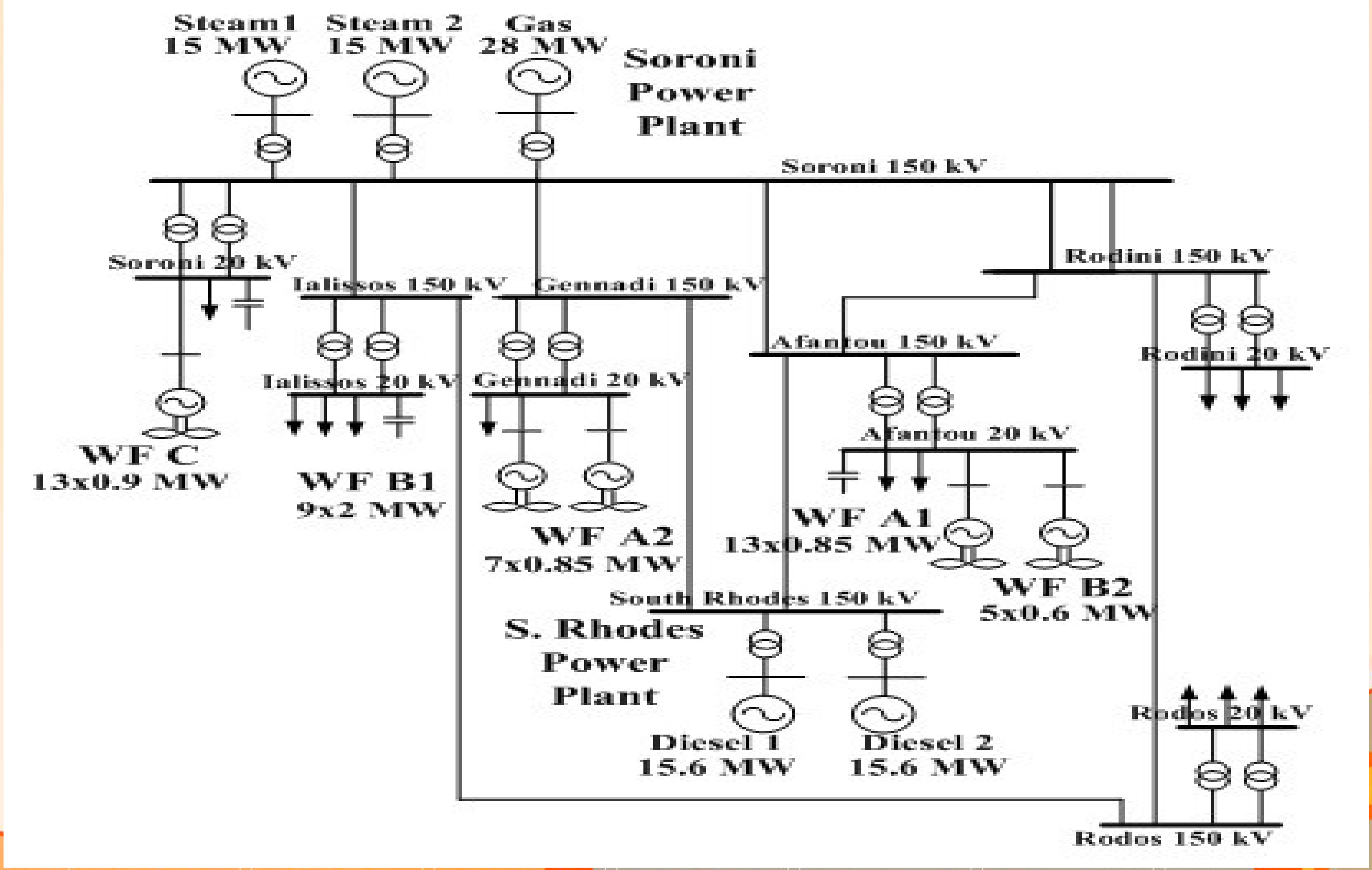
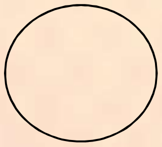
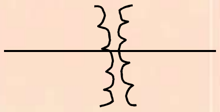
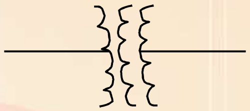


Figure 1.2 Electric circuit of a radio receiver.
(Reproduced with permission from QST, August 1995, p. 23.)

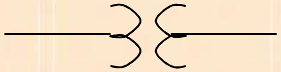
Examples of Electrical Circuits



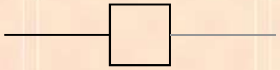
POWER SYSTEMS REPRESENTATIONS

Description	Symbol
Rotating Machine	
Two winding transformer	
Three winding transformer	
Fuse	
Current transformer	

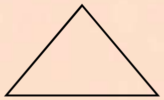
potential transformer



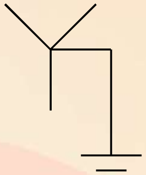
oil (liquid) circuit breaker



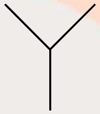
delta connection



wye connection, neutral grounded

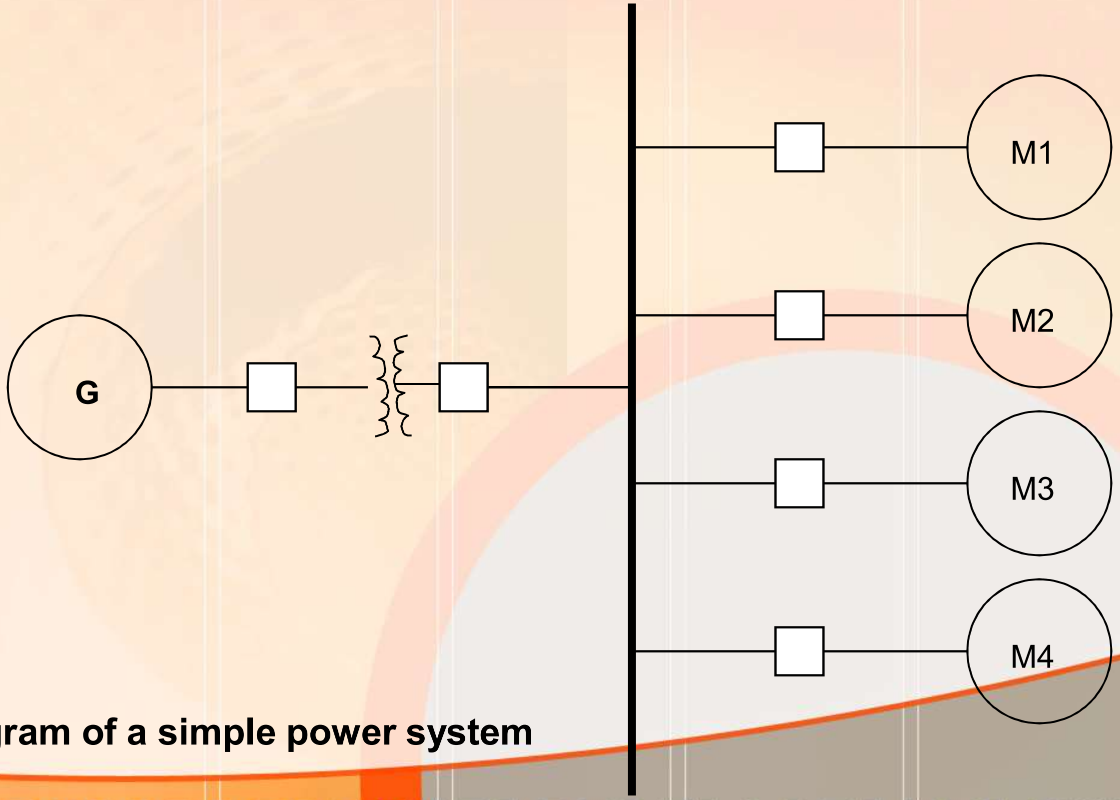


wye connection, neutral ungrounded



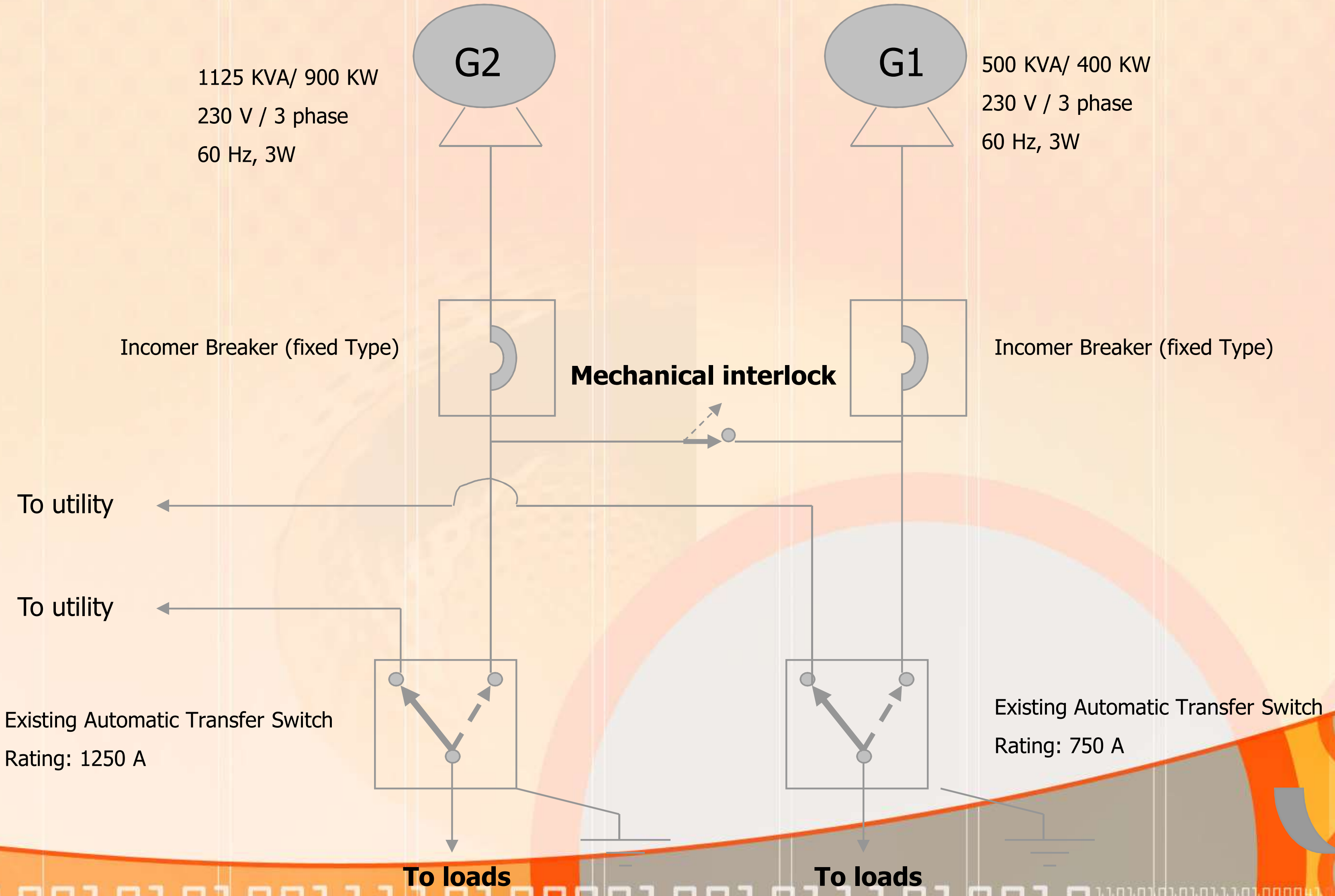
busbar

One line Diagrams *– provides a compact way to represent a great deal of additional information, such as the ratings of machines and transformers, the power being consumed or supplied by all of the loads in the system, and the impedances of the various devices in the system.*



A one line diagram of a simple power system

Example of a Typical Power System



1125 KVA/ 900 KW
230 V / 3 phase
60 Hz, 3W

500 KVA/ 400 KW
230 V / 3 phase
60 Hz, 3W

Incomer Breaker (fixed Type)

Mechanical interlock

Incomer Breaker (fixed Type)

To utility

To utility

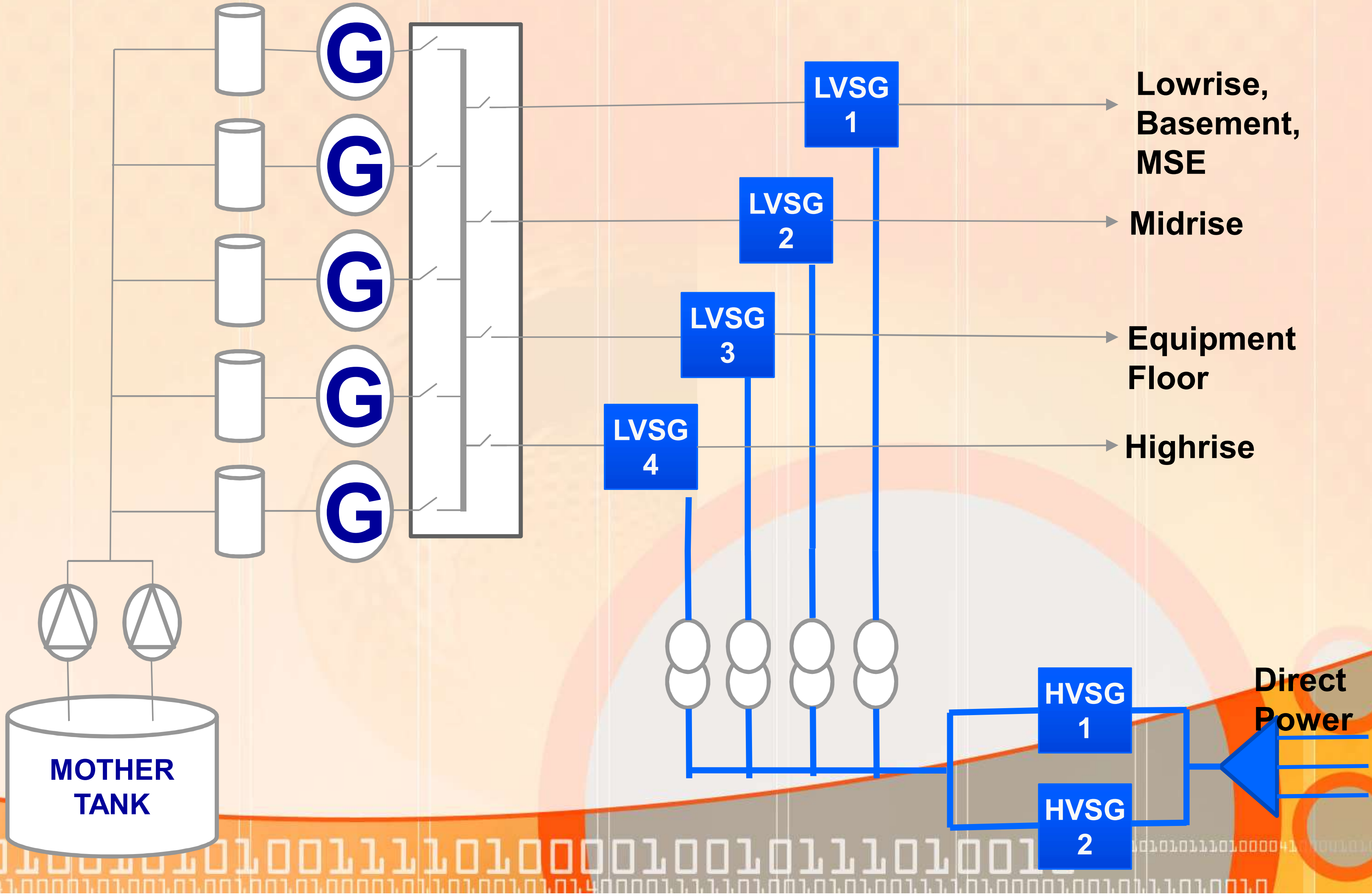
Existing Automatic Transfer Switch
Rating: 1250 A

Existing Automatic Transfer Switch
Rating: 750 A

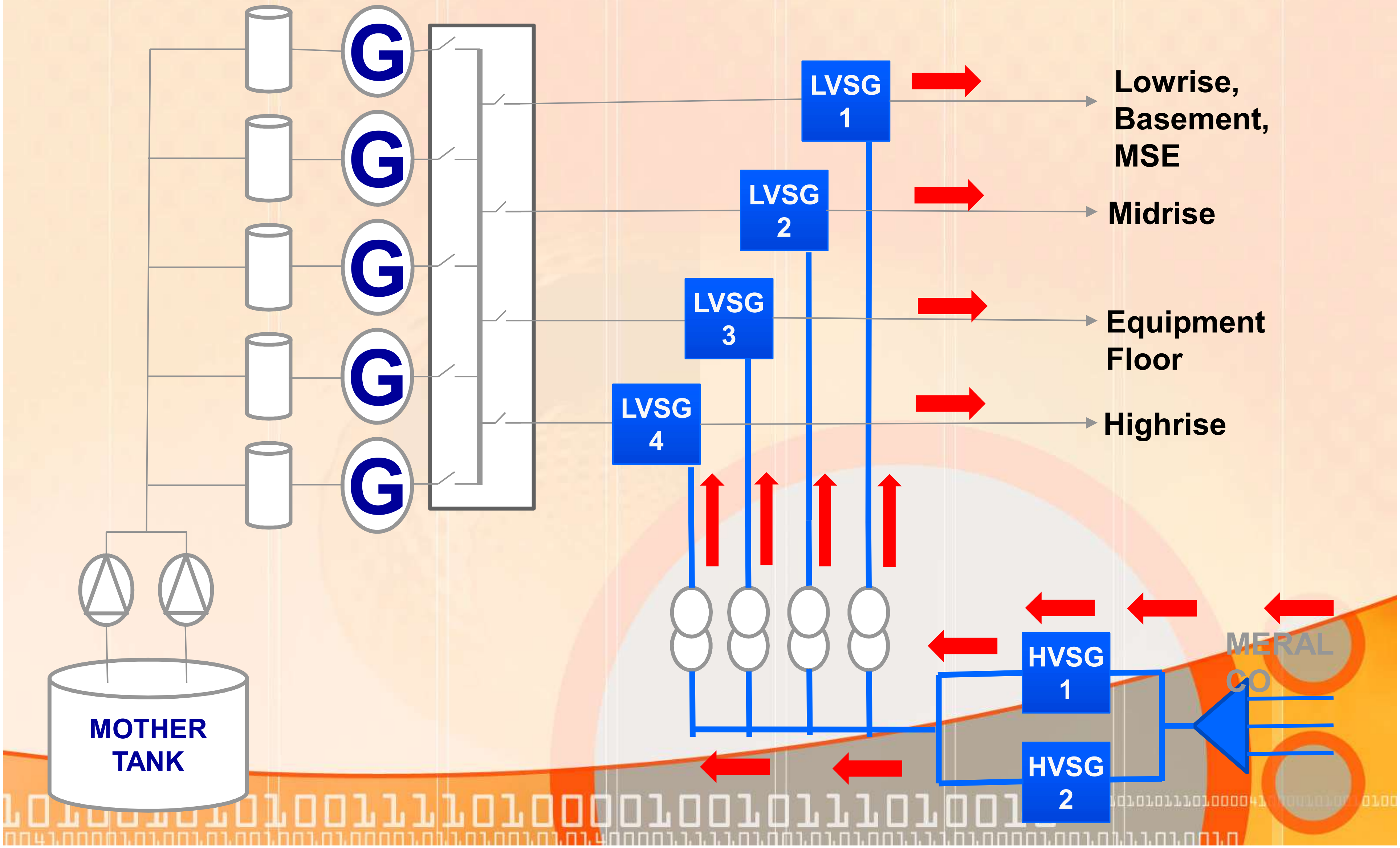
To loads

To loads

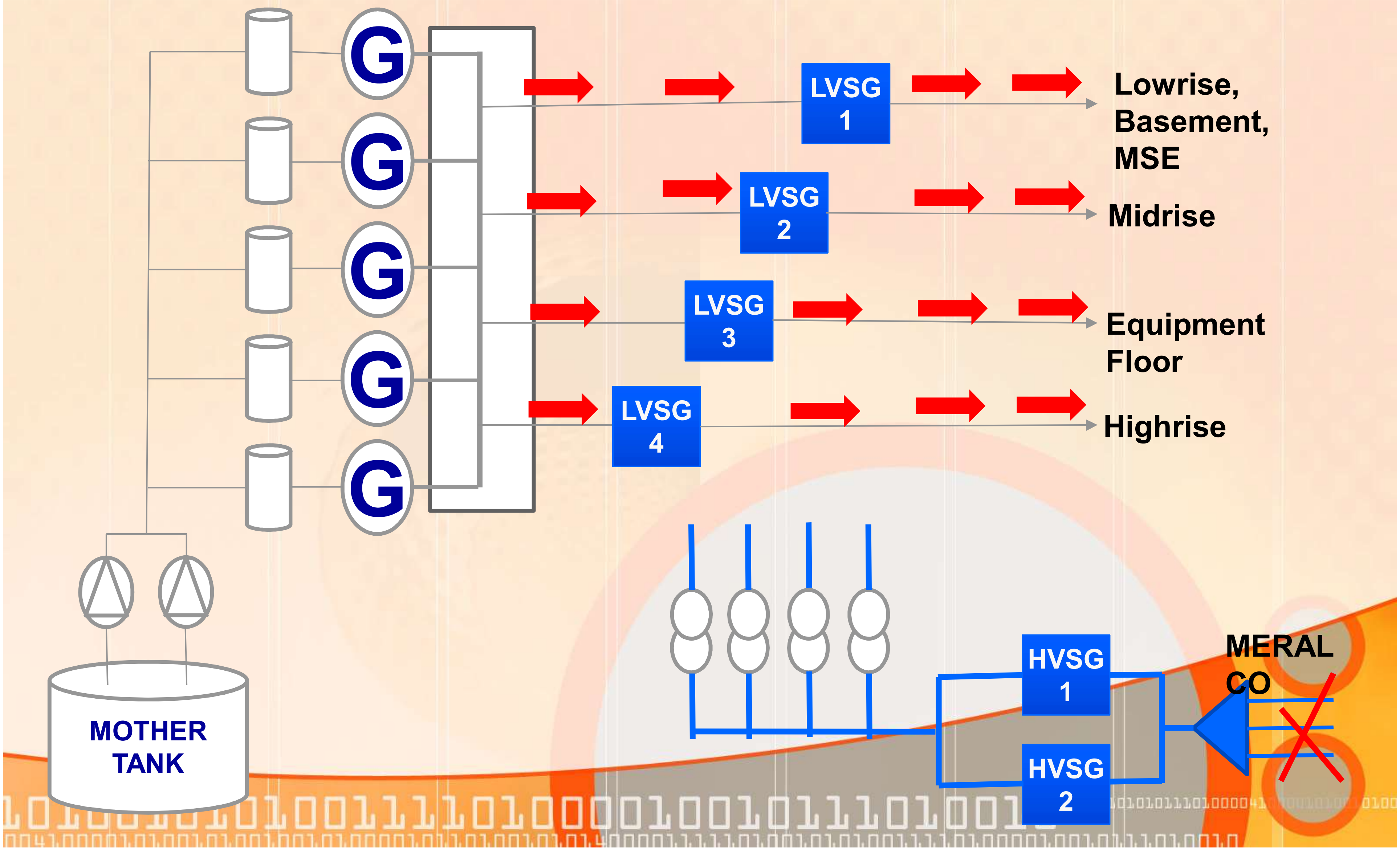
LINE DIAGRAM (SEPS)



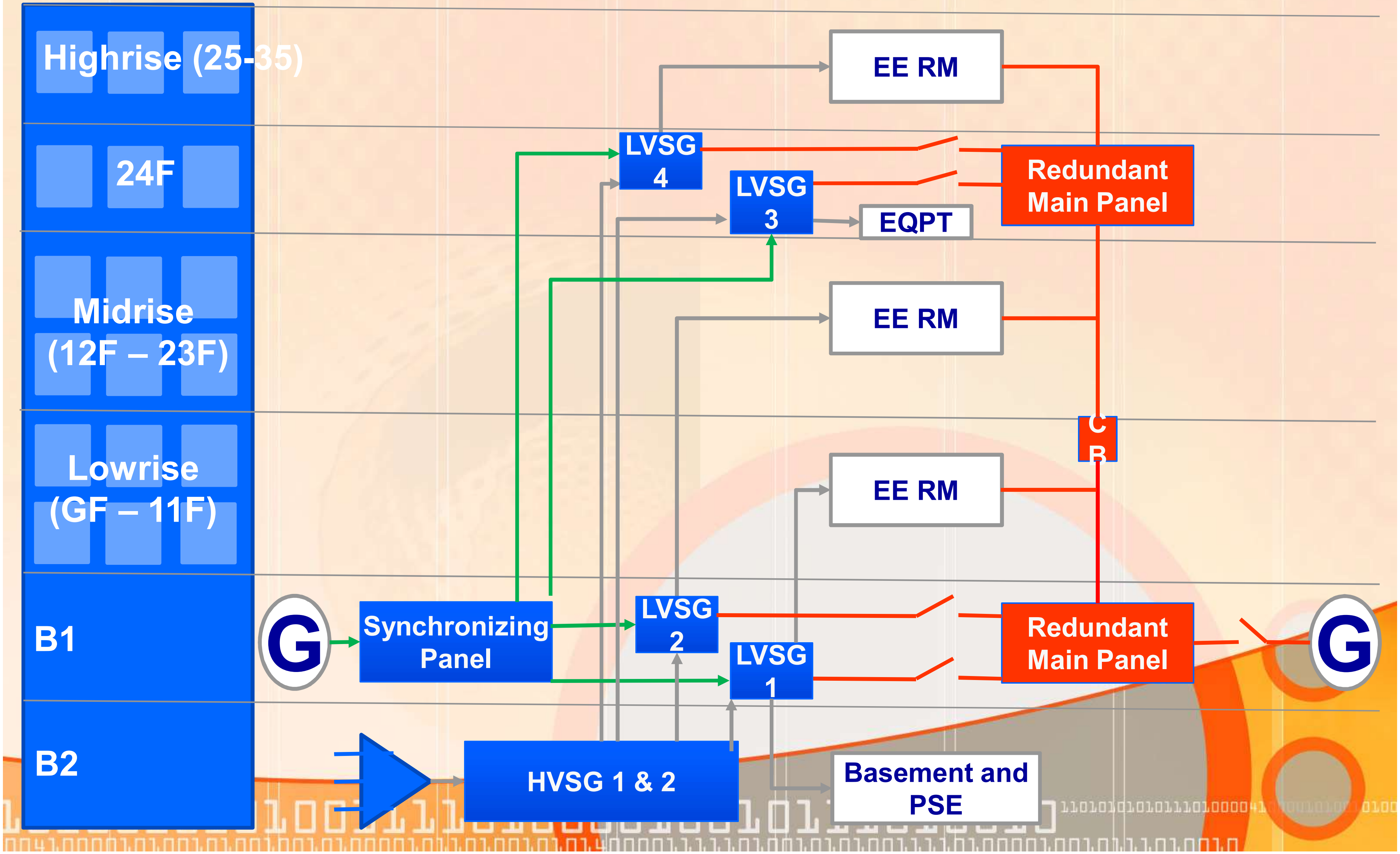
LINE POWER DIAGRAM



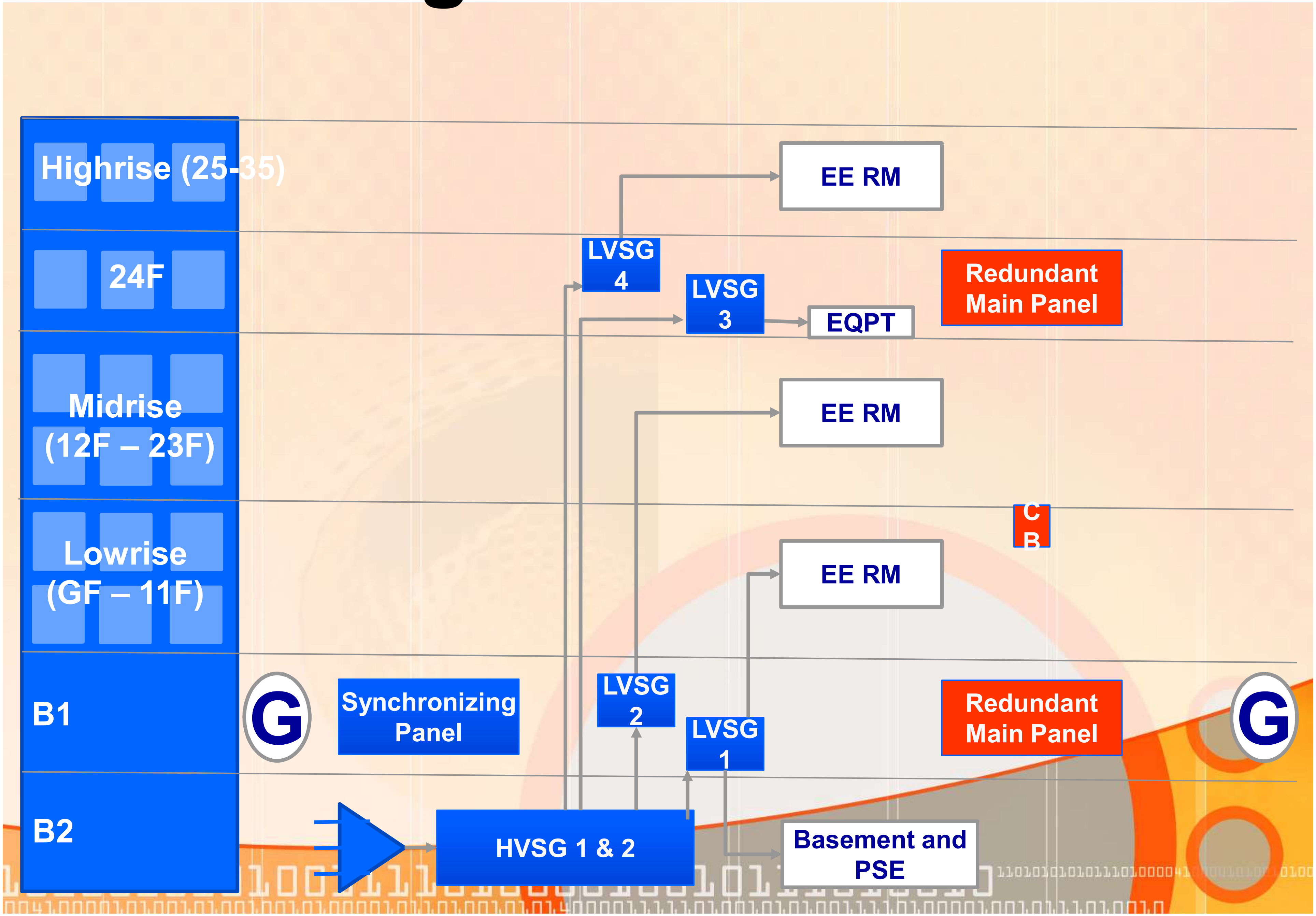
LINE POWER DIAGRAM



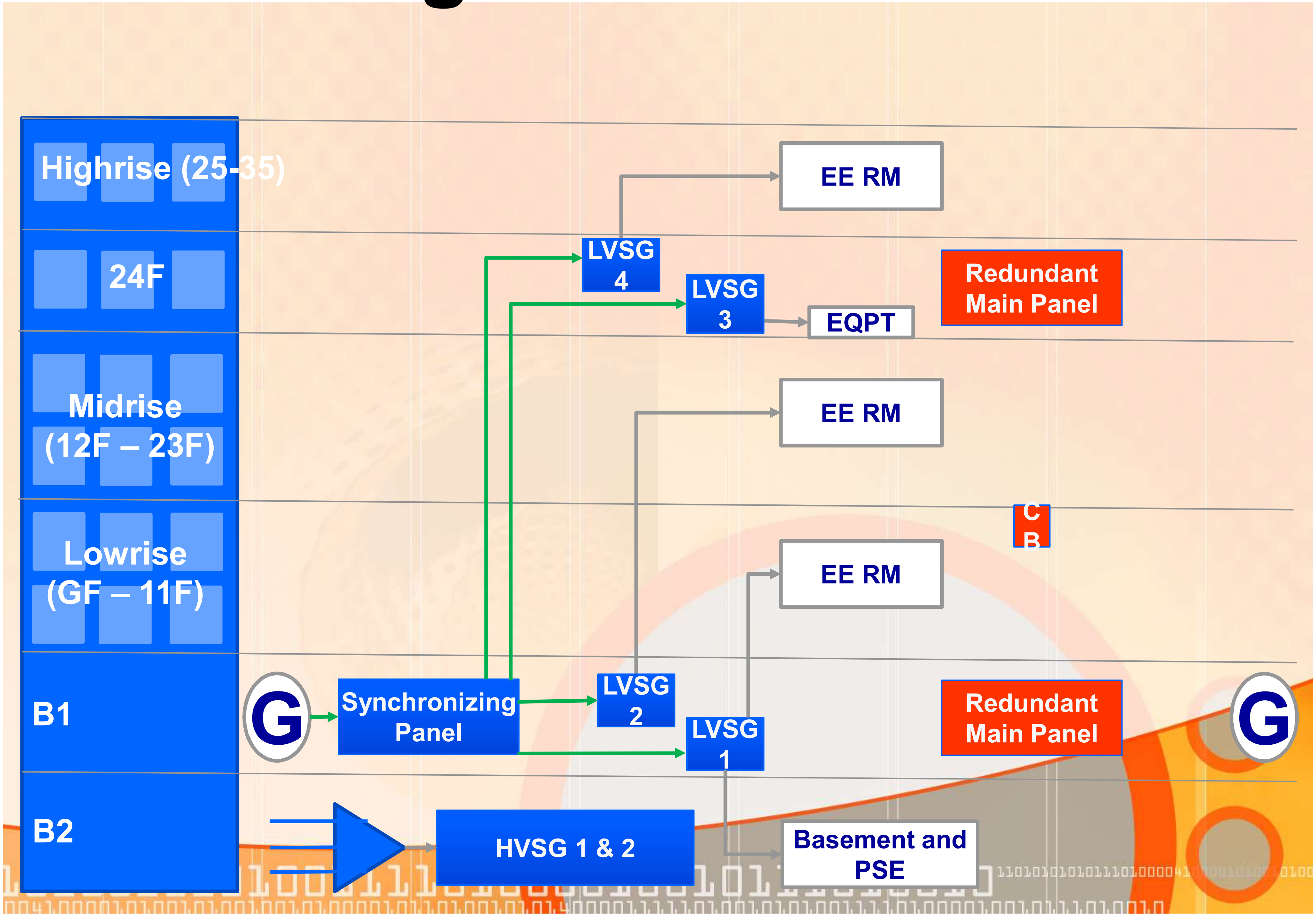
Power Diagram



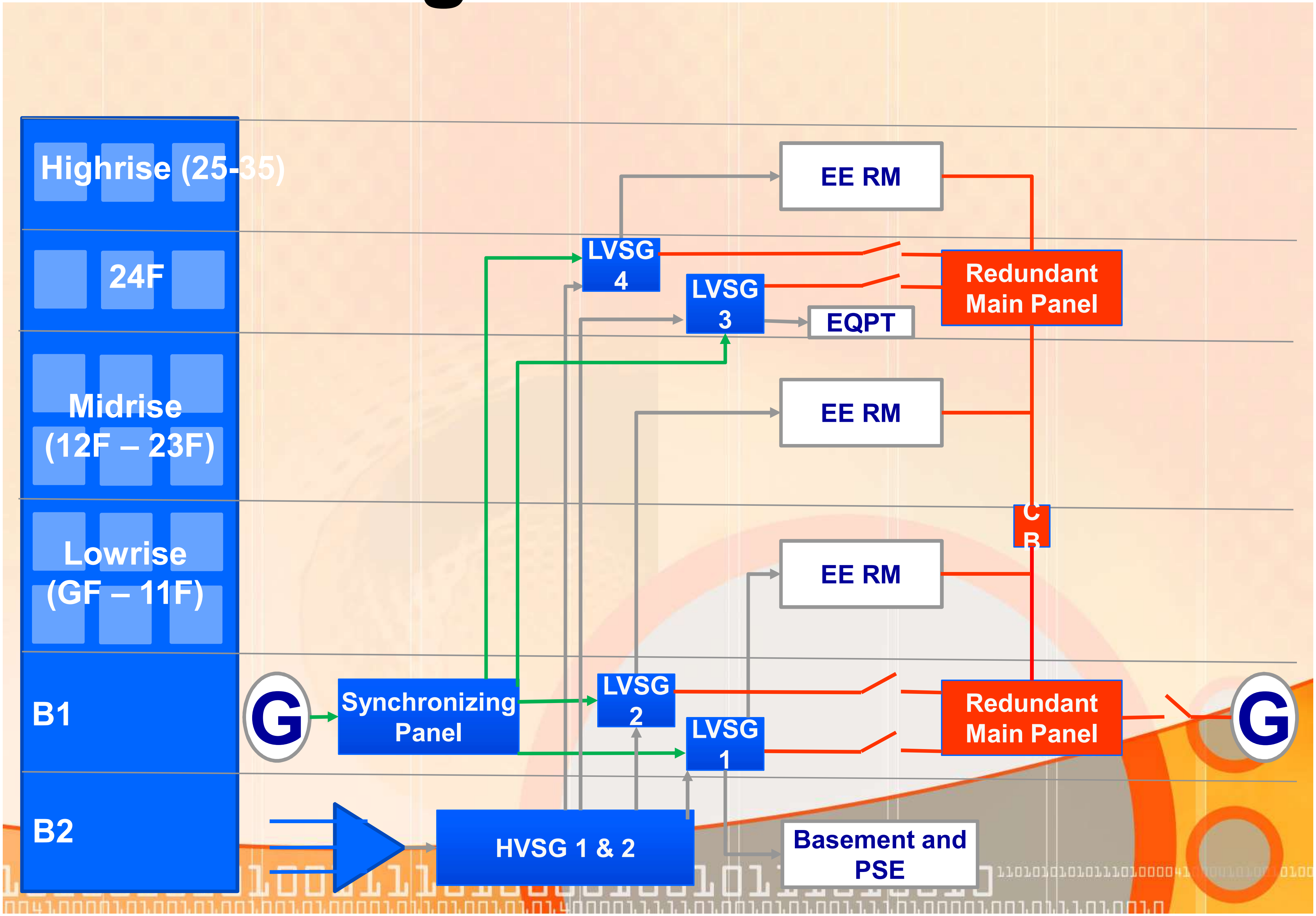
Power Diagram



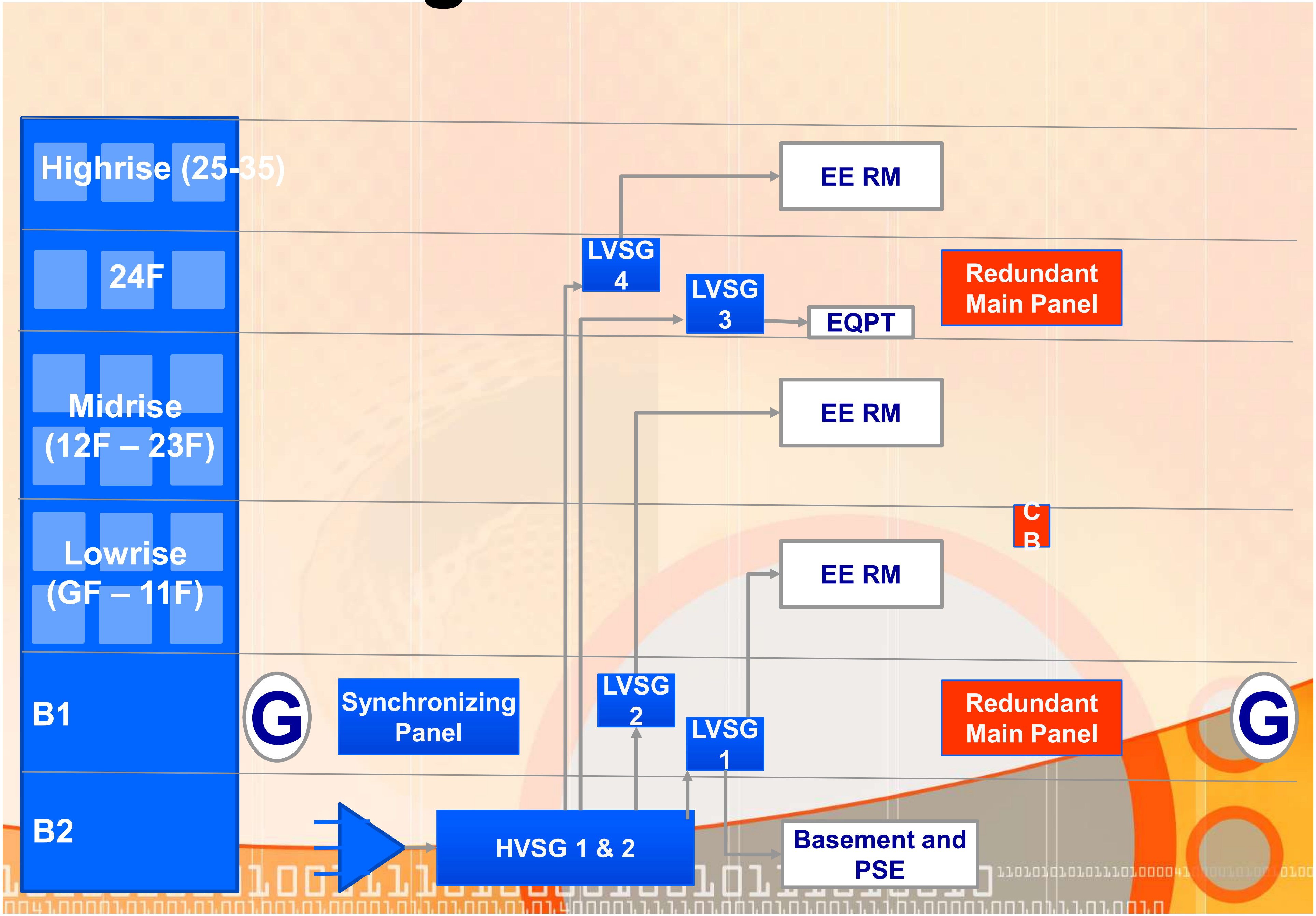
Power Diagram



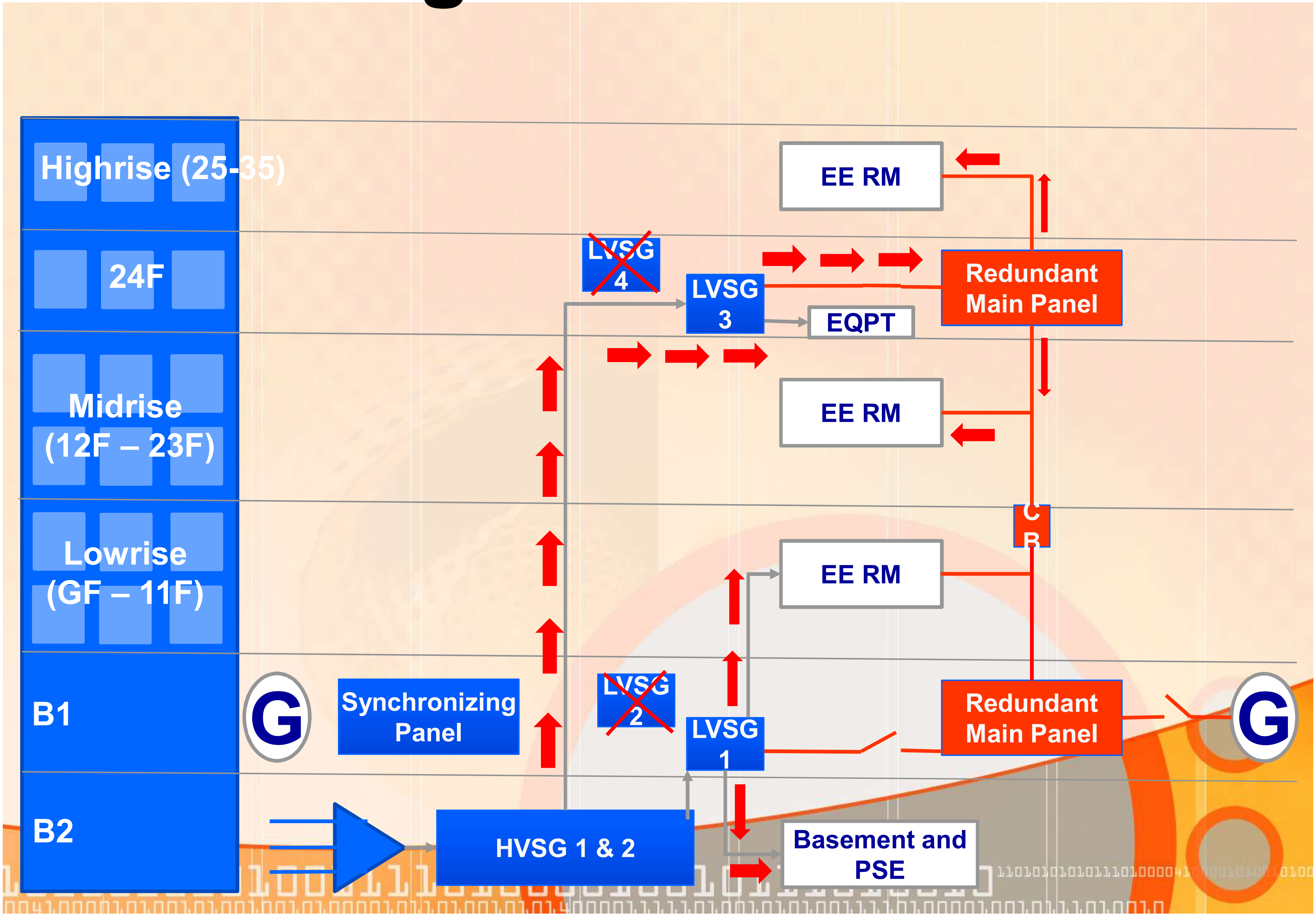
Power Diagram



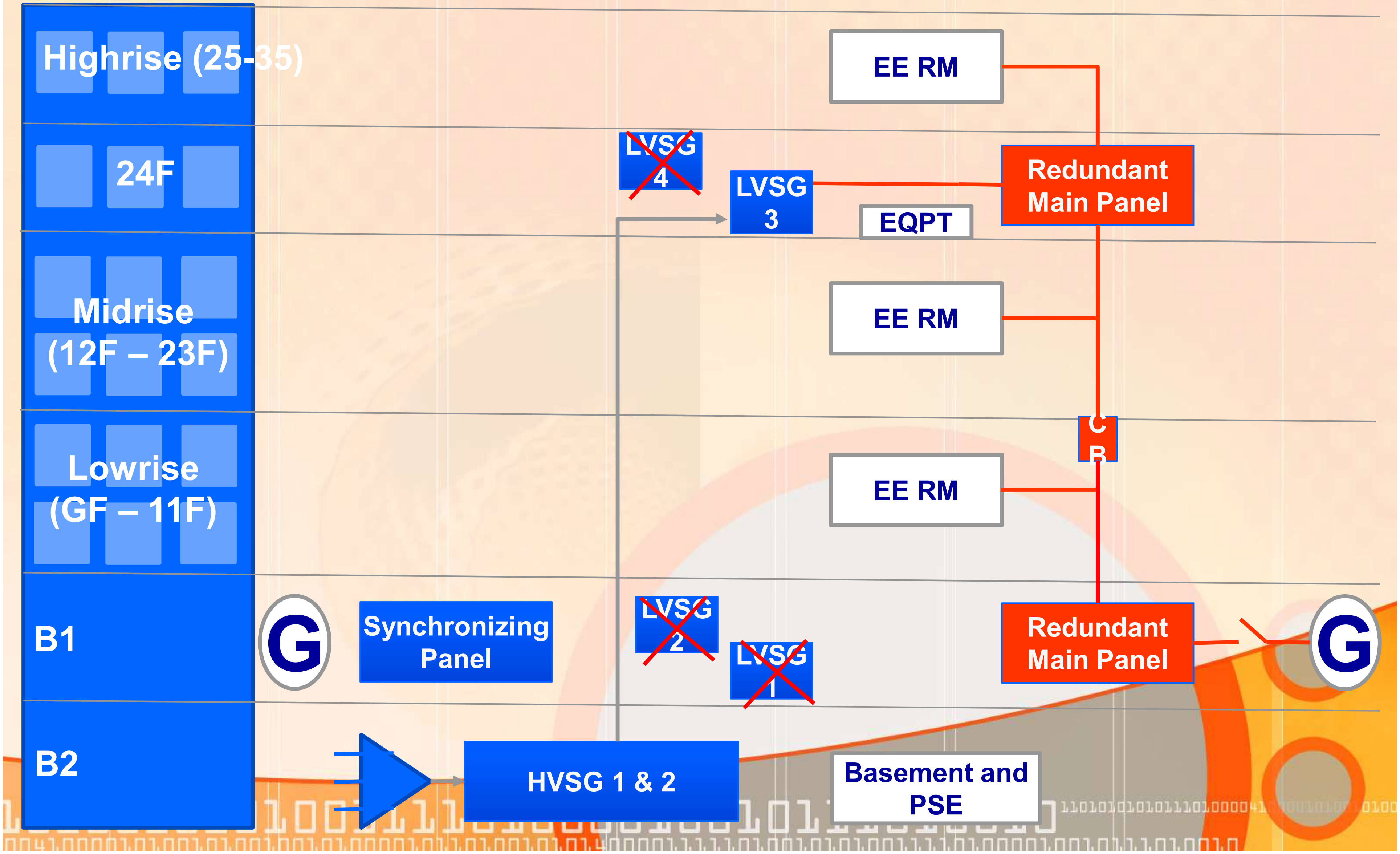
Power Diagram



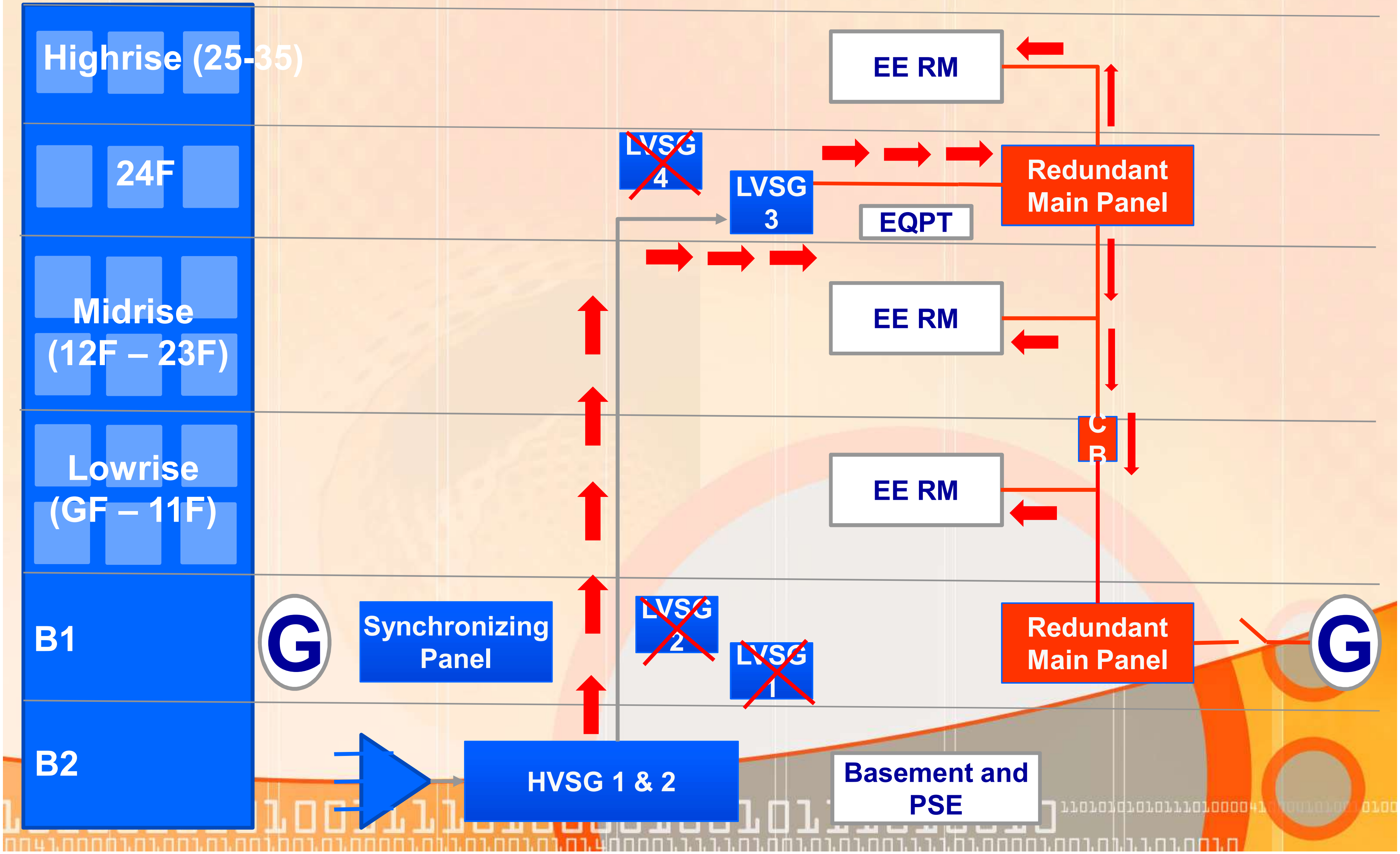
Power Diagram



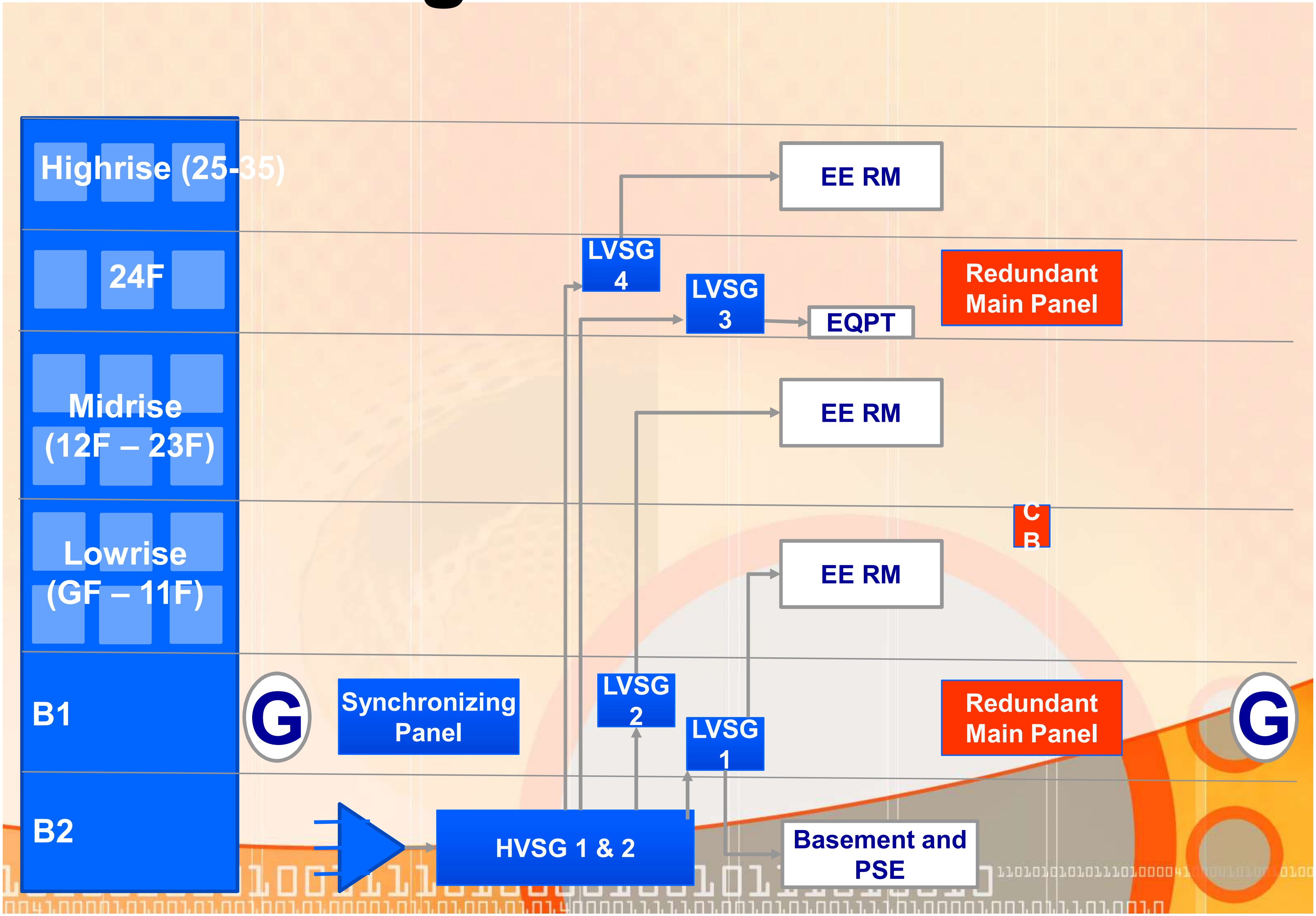
Power Diagram



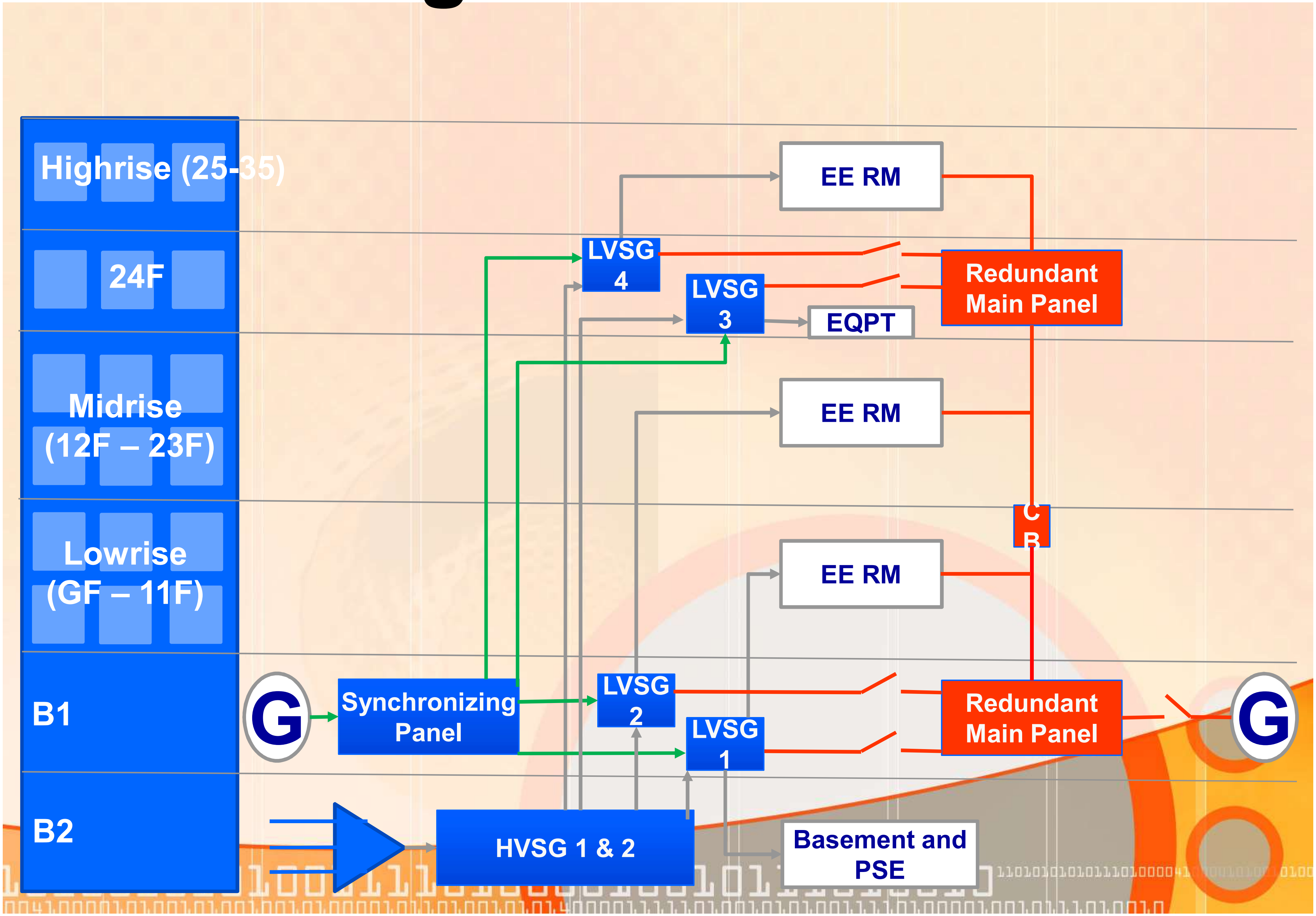
Power Diagram



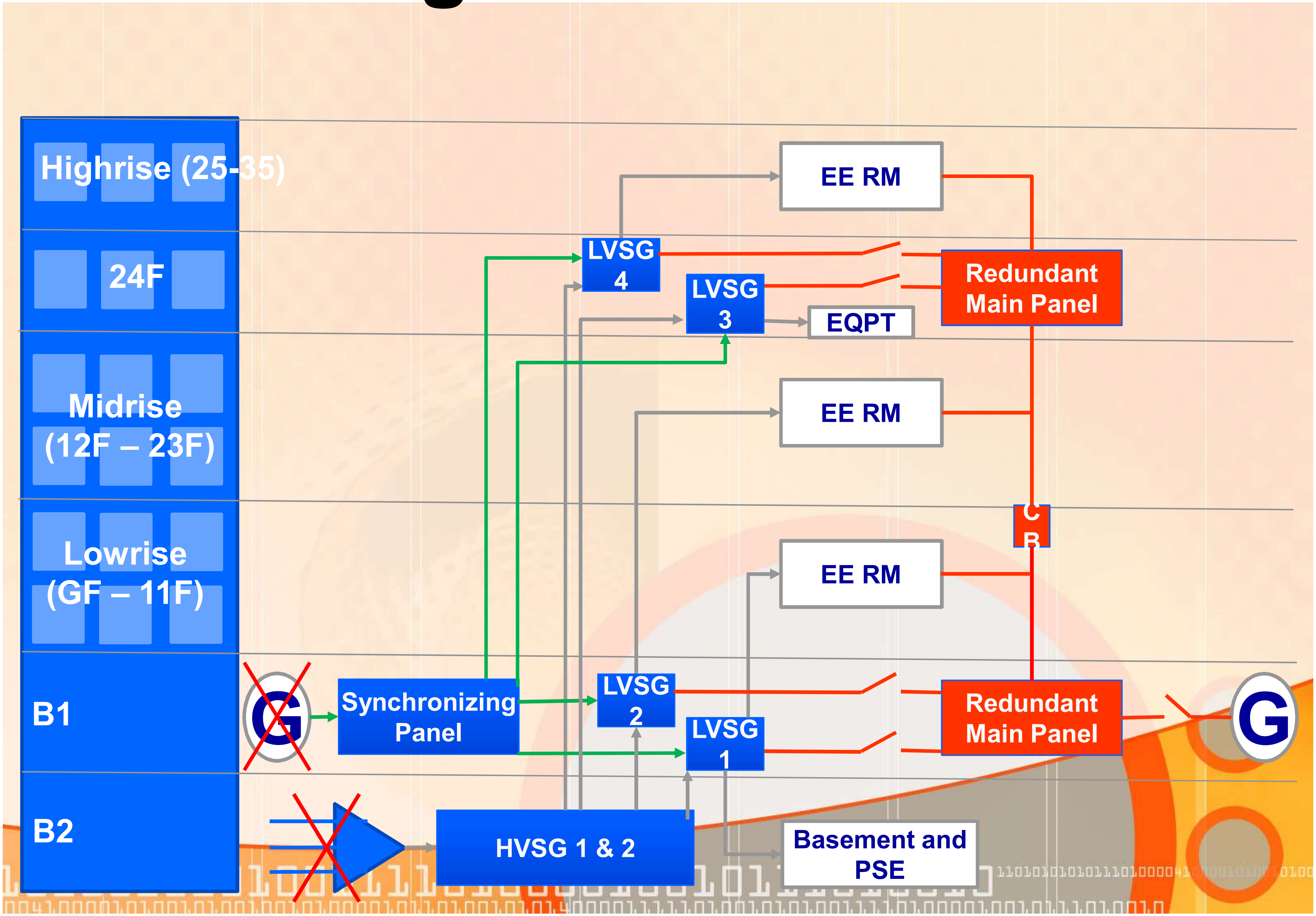
Power Diagram



Power Diagram



Power Diagram



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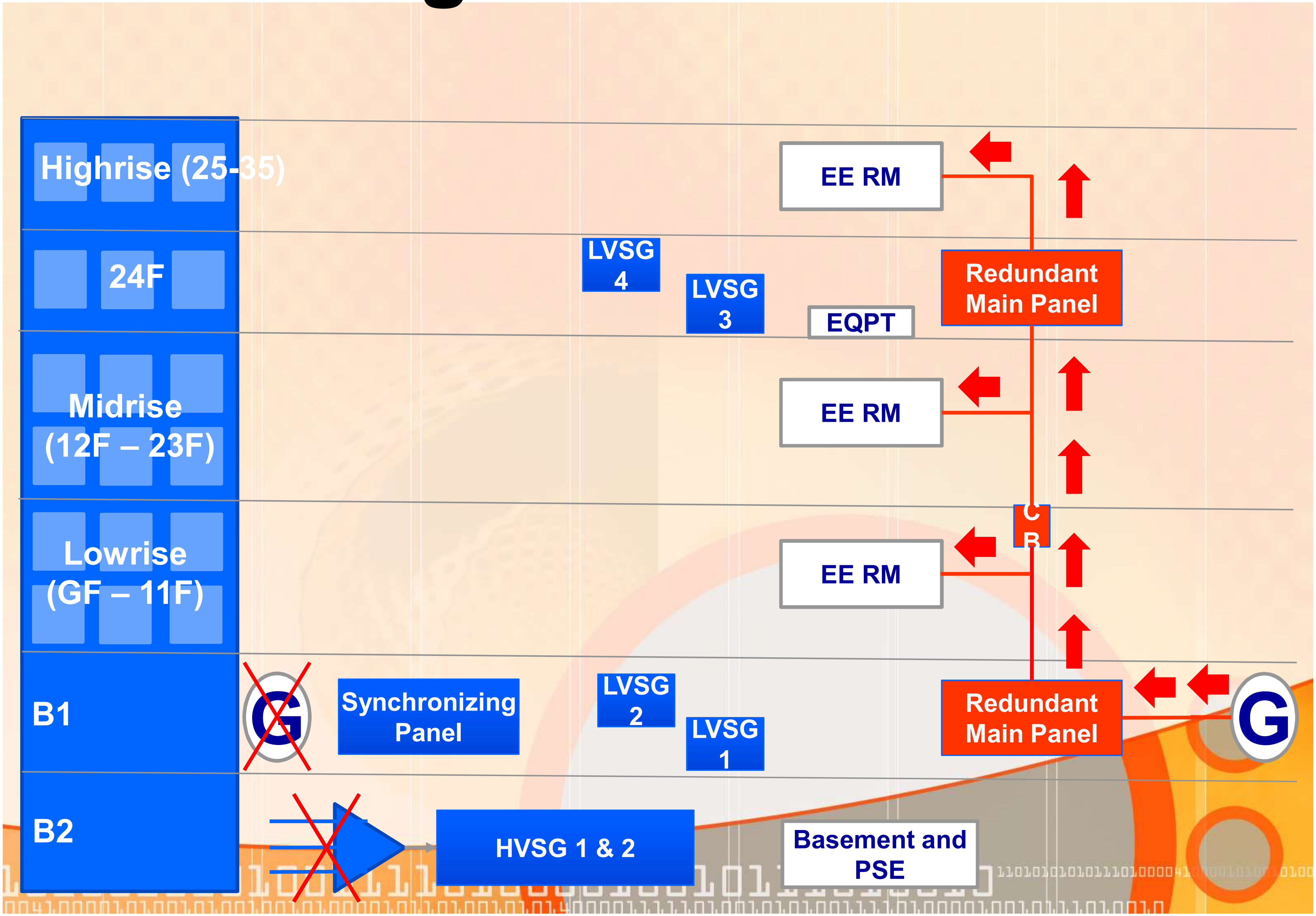
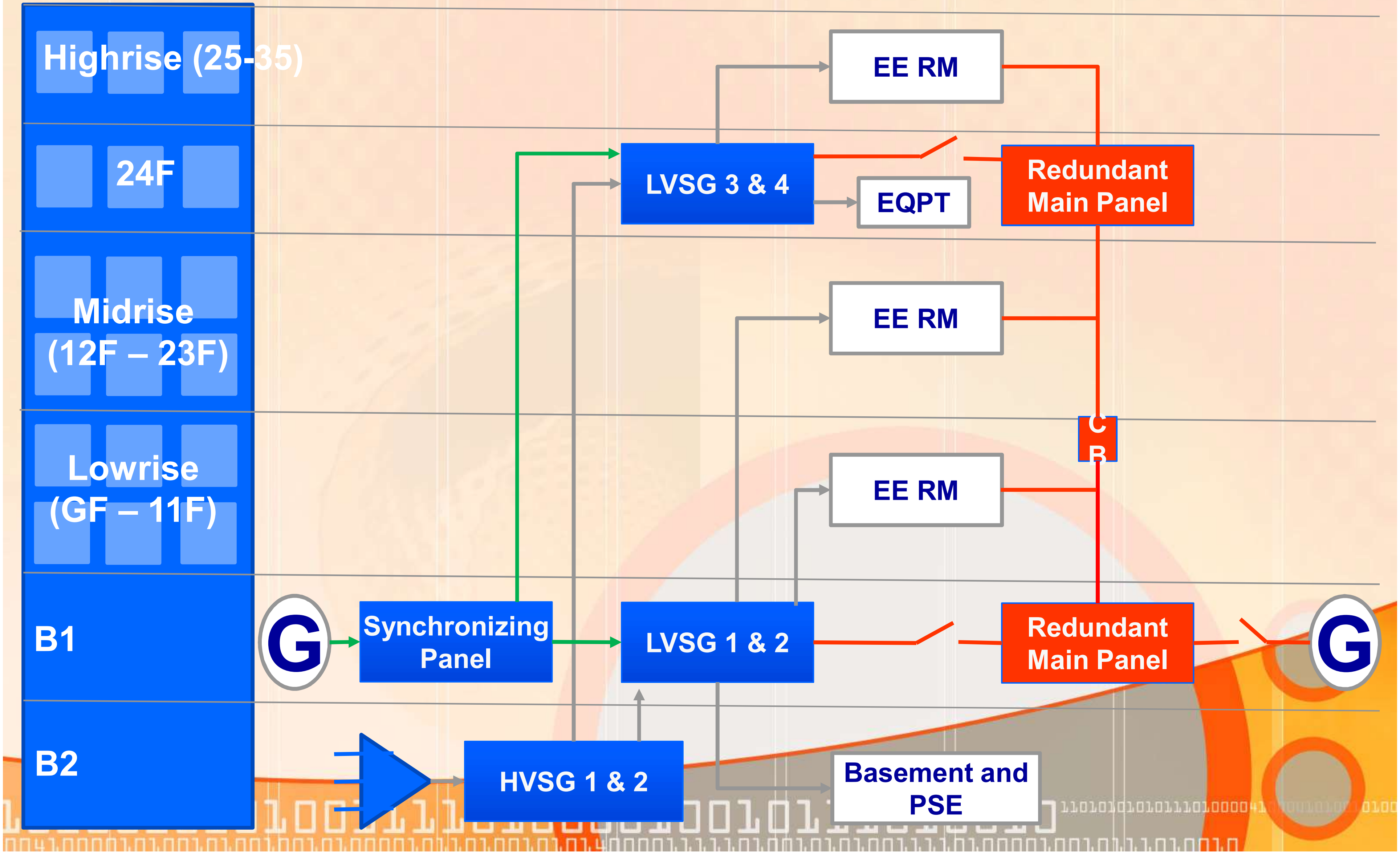


Power Diagram



THE PER UNIT SYSTEM

Per-unit System - the per-unit system has a major advantage in power system analysis, it simplifies the solution of a circuit containing transformers.

- easy to analyze real power that contains a mixture of Y – delta connections, and also containing very, very many transformers.

$$\textbf{Quantity per unit} = \frac{\text{actual value}}{\text{base value of quantity}}$$

- In a **single phase system**, the relationships are:

$$\boxed{P_{base}, Q_{base}, \text{ or } S_{base} = V_{base} I_{base}}$$

$$\boxed{Z_{base} = V_{base} / I_{base}}$$

$$\boxed{Y_{base} = I_{base} / V_{base}}$$

$$\boxed{Z_{base} = \frac{(V_{base})^2}{S_{base}}}$$

where:

Pbase = *base value of the real power*
Qbase = *base value of the reactive power*
Sbase = *base value of the apparent power*
Zbase = *base value of the impedance*
Ibase = *base value of the current*
Vbase = *base value of the voltage*

CHANGE OF BASE FORMULA

$$\text{pu Znew} = \text{pu Zgiven} \left(\text{Vgiven} / \text{Vnew} \right)^2 \left(\text{Snew} / \text{Sgiven} \right)$$

where: Zbase = Rbase = Xbase

CONDITIONS FOR PER UNIT SYSTEMS

- the Sbase and Vbase are selected @ a specific pt. in the system.
- The output power of an ideal transformer is equal to it's input power.

Thus, in a single phase system or three phase systems, the Sbase always remain constant everywhere in the system but the Vbase changes when it goes to a transformer.

Power Conditions for Single Phase and Three Phase Systems

$$S3\theta_{pu} = S3\theta_{act} / S3\theta_{base} = 3S1\theta_{act} / 3S1\theta_{base} = S1\theta_{pu}$$

$$\boldsymbol{S3\theta_{pu} = S1\theta_{pu} = V_{pu} I_{pu}^{*}}$$

Thus, powers in per unit for single phase and three phase systems is unaffected, since in three phase systems the constant ‘3’ is cancelled out.

Sample Problems

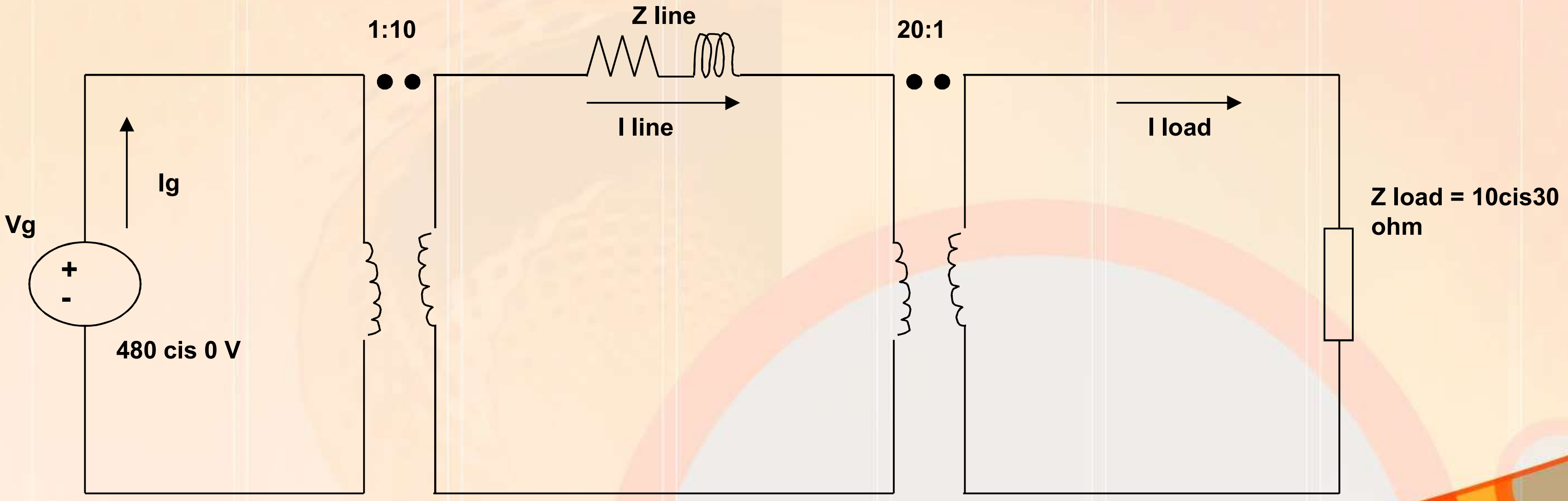
1. A simple power system shown in the figure contains a 480 V generator connected to an ideal 1:10 step up transformer, a transmission line, an ideal 20:1 step down transformer and a load. The impedance of the transmission line is 20 + j 80 ohms, and the impedance of the load is 10cis30. The base values for this system is chosen to be 480V and 10 KVA respectively at the generator terminals.

   a. Find the base voltage, current, impedance and apparent power @ every point in the power system.

   b. Convert this power system to it's per-unit equivalent circuit.

   c. Find the power supplied to the load in this system.

   d. Find the power lost in the transmission line.



SOLUTIONS

TRANSMISSION LINE PARAMETERS

(A Preview)

I. Resistance of the Line

***Resistance** – is the most important cause of power loss in a transmission line.*

$$\boxed{R = \rho\, l / A}$$

R –resistance of the line in Ω

l -length of the line in meter

A – x-sectional area of conductor in m^2

ρ - resistivity of conductor in Ω-m

ρ copper- 1.77 x 10^{-8} Ω-m @ 20°C or 10.66 Ω-cmil / ft

ρ aluminum – 2.83 x 10^{-8} Ω-m @ 20°C or 17 Ω-cmil / ft

Resistance of the line

In a single phase 2 wire dc line, the total resistance is equal to double the resistance of either conductor known as **loop resistance.**

In case of a 3 phase transmission line, the resistance per phase is the resistance of one conductor.

Important Conversions

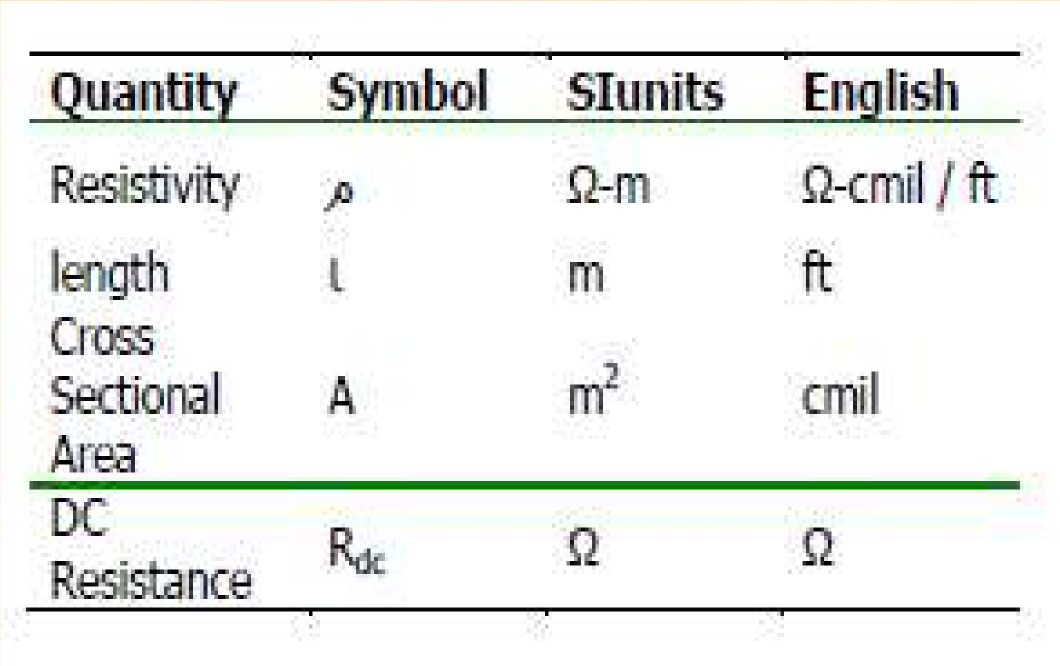
1 inch *= 1000 mils*

1 cmil *= л / 4 sq. mil*

1 MCM *= 1000 CM*

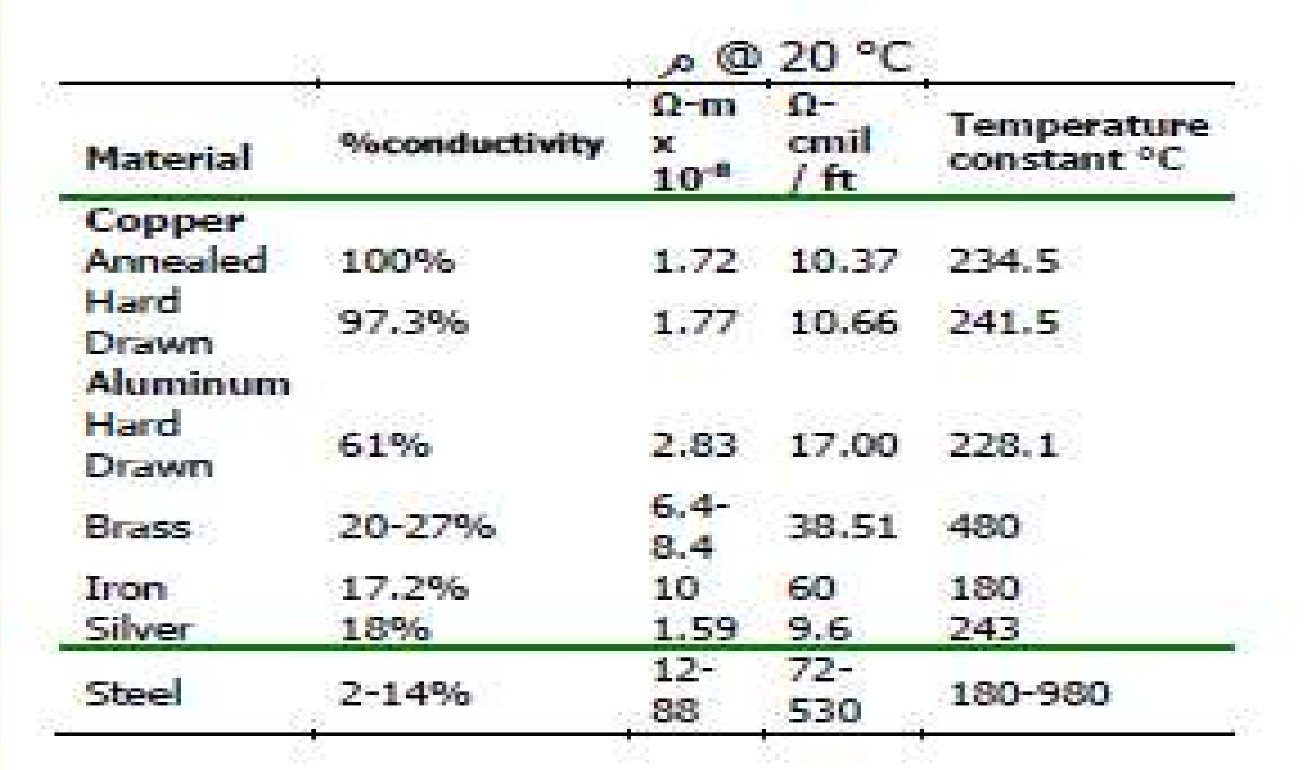
Example: If the diameter of a ACSR wire is 1 inch. Find the area in MCM.

SI Units Vs English Units



Quantity	Symbol	SIunits	English
Resistivity	ρ	Ω-m	Ω-cmil / ft
length	l	m	ft
Cross Sectional Area	A	m^2	cmil
DC Resistance	R_{dc}	Ω	Ω

% Conductivity, resistivity and temperature constants of conductor metals



		ρ @ 20 °C		
Material	%conductivity	Ω-m x 10^{-8}	Ω-cmil / ft	Temperature constant °C
Copper				
Annealed	100%	1.72	10.37	234.5
Hard Drawn	97.3%	1.77	10.66	241.5
Aluminum				
Hard Drawn	61%	2.83	17.00	228.1
Brass	20-27%	6.4-8.4	38.51	480
Iron	17.2%	10	60	180
Silver	18%	1.59	9.6	243
Steel	2-14%	12-88	72-530	180-980

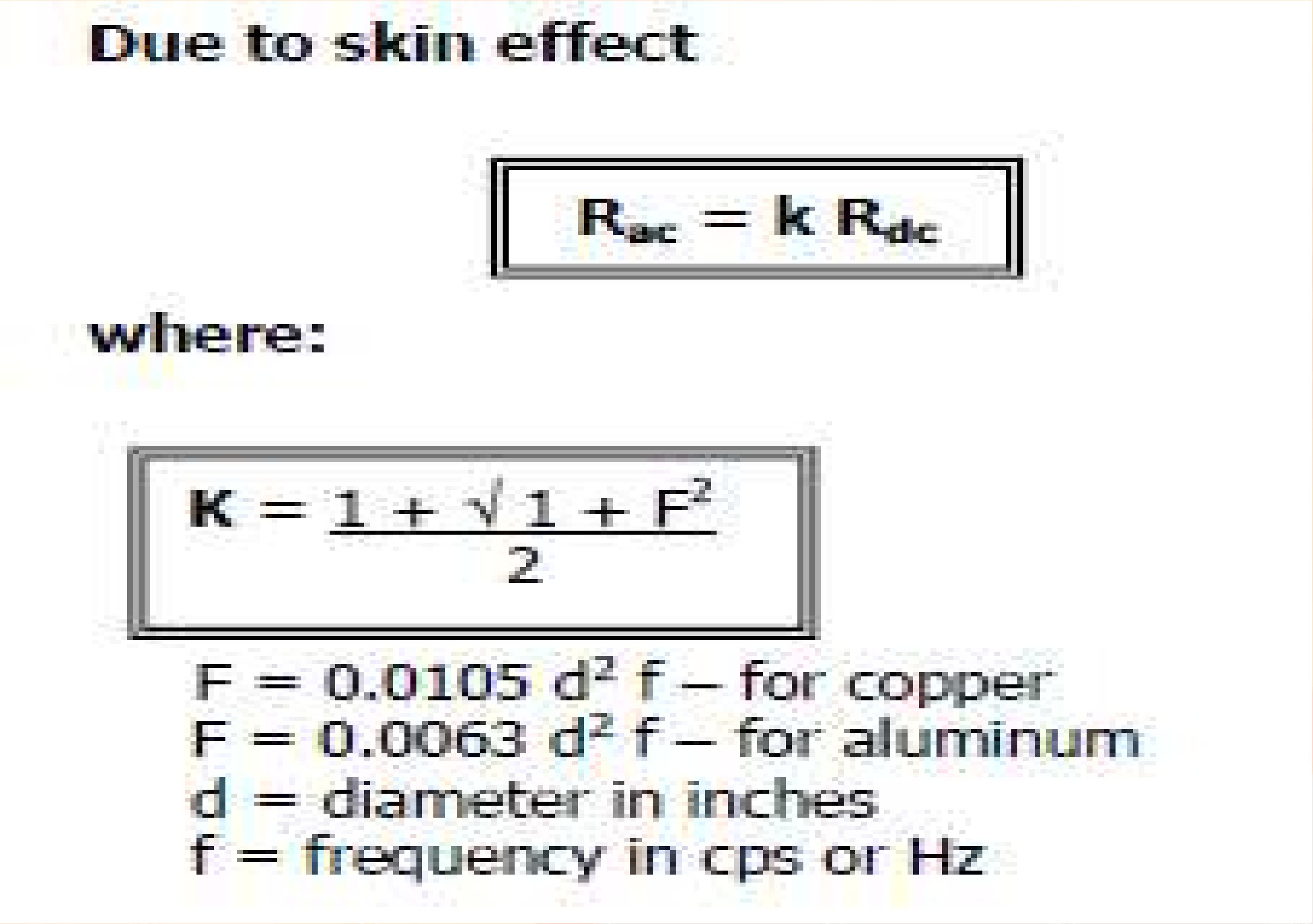
Skin Effect

– the tendency of the alternating current to concentrate near the surface of a conductor. The skin effect depends upon the following factors

- *Nature of the material*
- *Diameter of the wire*
- *Frequency*
- *Shape of the wire*

Note: *skin effect is negligible when the supply frequency is less than 50 Hz and the conductor diameter is less than 1 cm.*

Skin Effect



Due to skin effect

$$\boxed{R_{ac} = k\ R_{dc}}$$

where:

$$\boxed{K = \frac{1 + \sqrt{1 + F^2}}{2}}$$

$F = 0.0105\ d^2\ f$ – for copper
$F = 0.0063\ d^2\ f$ – for aluminum
d = diameter in inches
f = frequency in cps or Hz

Bundled Conductor Resistance

- determine the total area of the conductor first (A_T), then, get the DC resistance using:

$$\boxed{R = \rho\, l / A_T}$$

- remember, as long as the area is increased, the resistance must decrease. The ρ, l are the constants of the line.

Resistance Increase due to Spiralling

For stranded conductors, alternate layers of strands are spiralled in opposite directions to hold the strands together. The strands makes 1% or 2% longer than the actual length. Thus, dc resistance will increase.

1% increase in resistance for 3 strands
2% increase in resistance for concentrically stranded conductors (4 or more).

II. Series Inductance

Besides resistance, the transmission line has got inductance as well as capacitance. The resistance, inductance and capacitance are termed as the “parameters” of the line and are uniformly distributed along the entire length of the line. For single phase line the parameters are usally represented on loop inductance basis and for 3-phase line on per conductor basis.

Series Inductance

for single phase solid lines,

$$\boxed{L = 2 \times 10^{-7} \ln \frac{D}{r'} \quad \textbf{H/m} \text{ (per conductor)}}$$

for three phase lines,

$$\boxed{L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \quad \textbf{H/m} \text{ (per phase)}}$$

where:

L= series inductance of the line

D = distance between conductors

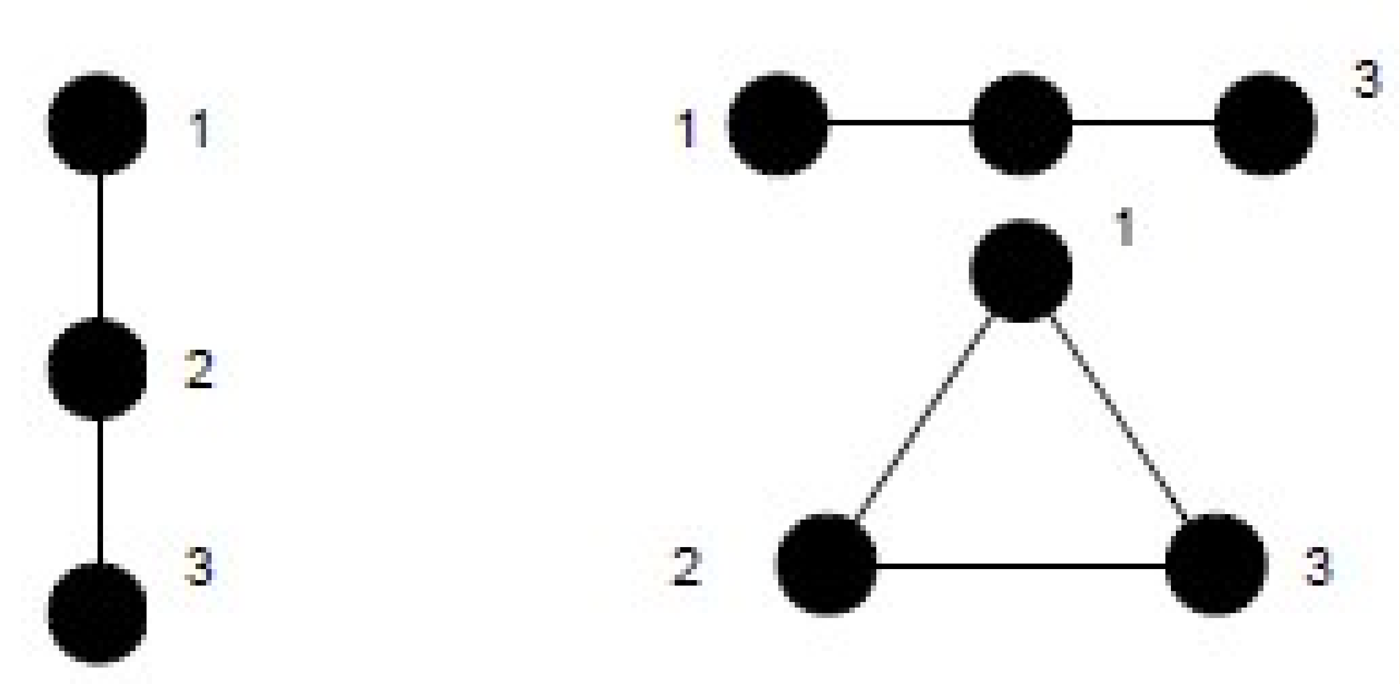
r' = seflf geometric distance of solid conductor

$r' = re^{-1/4} = 0.7788\ r$

D_s = self geometric mean distance of solid conductor or geometric mean radius (GMR) of conductor.

D_{eq} = equivalent spacing of conductors or geometric mean distance (GMD) of conductors.

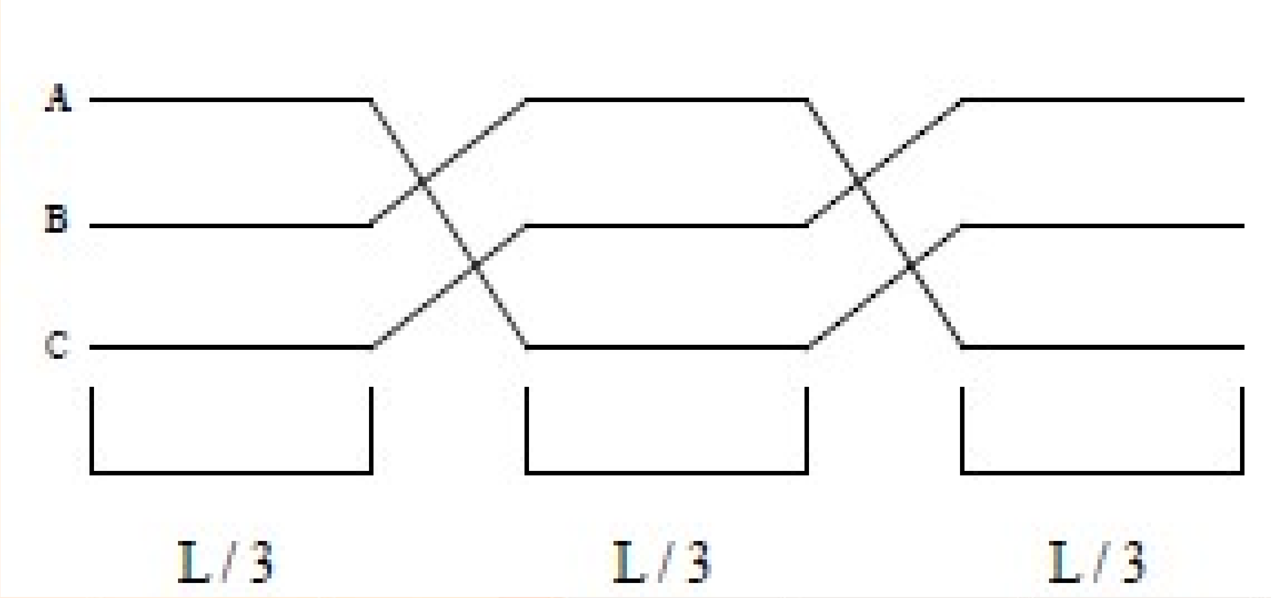
Typical Arrangement of Three Phase Line Conductors



$$\boxed{D_{eq} = \sqrt[3]{D_{12}\, D_{23}\, D_{31}}}$$

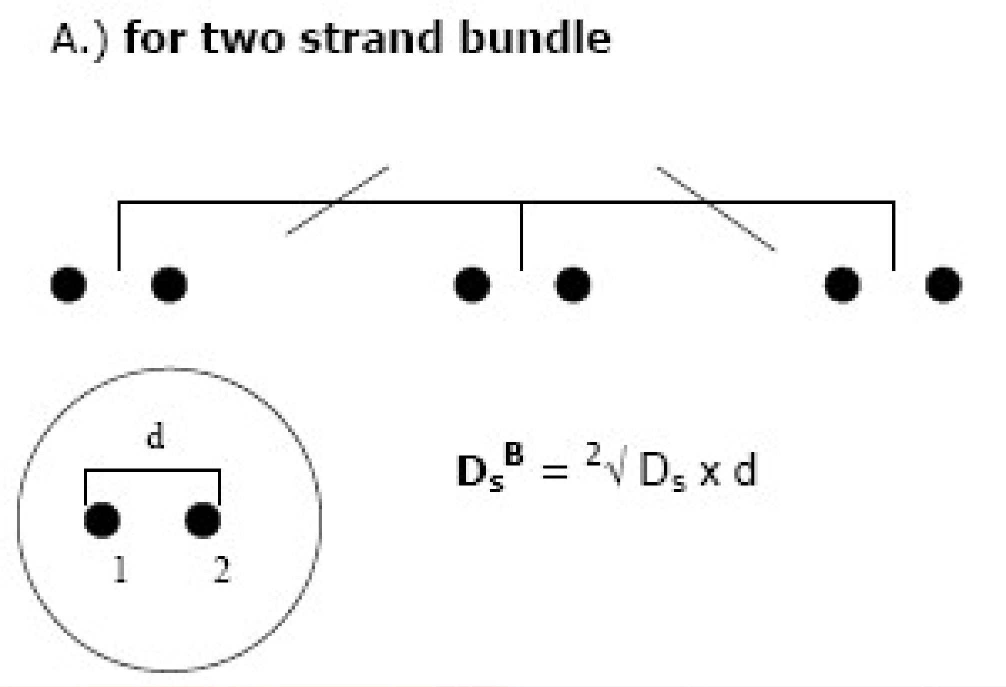
Transposition of the line

- *relative change of the position of conductor to balance the inductance of the line.*



Bundled conductors

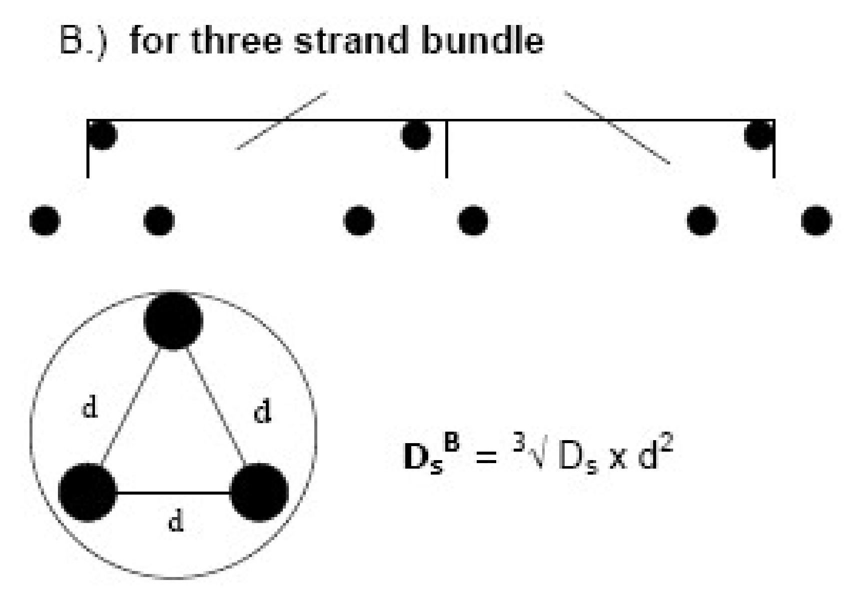
It reduces the reactance of the line and the effect of corona.


A.) for two strand bundle

$D_s^B = \sqrt[2]{D_s \times d}$

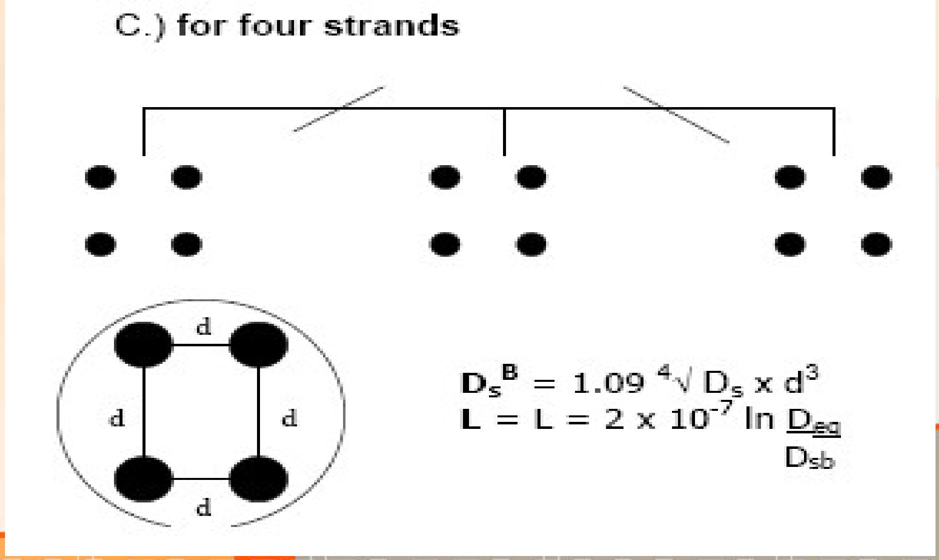
GMR= Ds = $n^2 \sqrt{\text{product of } n^2 \text{ terms}}$

Bundled conductors



B.) for three strand bundle

$$D_s^B = \sqrt[3]{D_s \times d^2}$$



C.) for four strands

$$D_s^B = 1.09 \sqrt[4]{D_s \times d^3}$$

$$L = L = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_{sb}}$$

Sample Problems

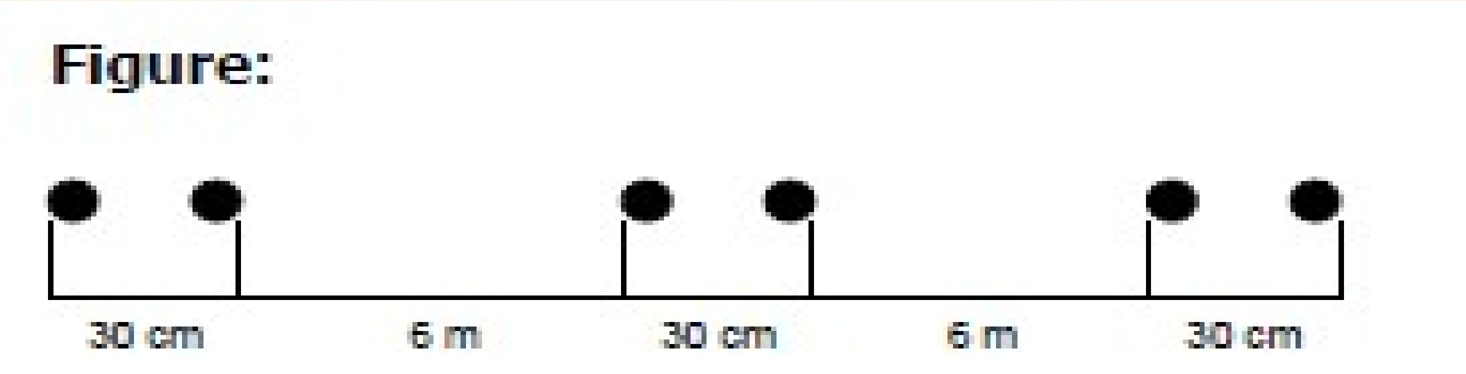
a. Given a 2 bundled conductor diameter of 1 cm per strand and a frequency of 50 Hz, compute for the Rac of the conductor if the length of the line is 1km. Assume an aluminum conductor.

b. A single phase line has two parallel conductors 2 meters apart. The diameter of each conductor is 1.2 cm. Calculate the loop inductance per km of the line.

SOLUTIONS

Sample Problems

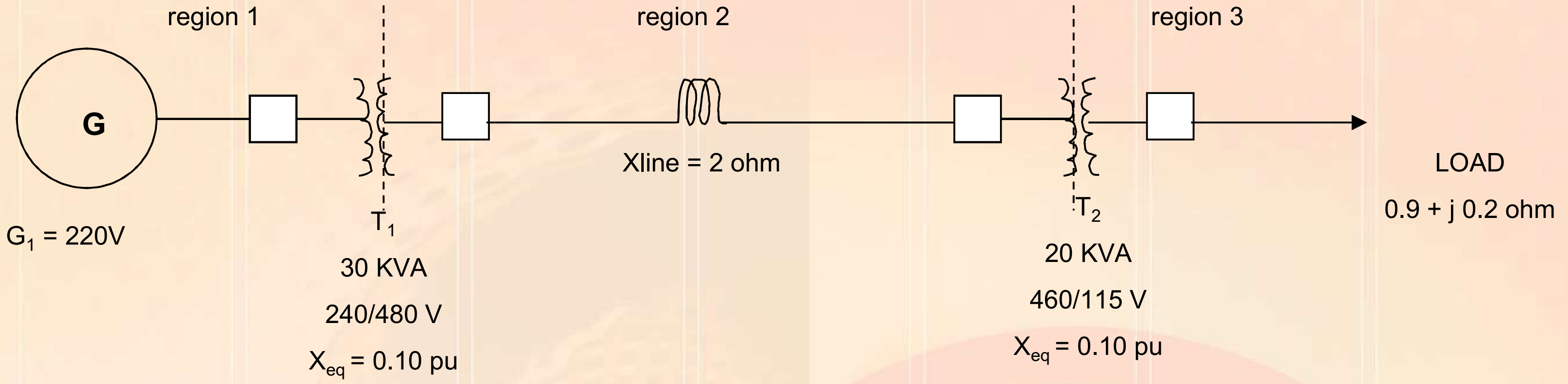
c. A bundled and transposed 3 phase transmission line has a conductor arrangement shown below. The identical conductors have a radius of 0.74 cm. The spacing between phase conductors is 30 cm. Determine the line reactance per phase per mile @ 60 Hz.



SOLUTIONS

Sample Problems

2. Three regions of a single phase circuit are identified in the figure, the regions are connected by transformers T_1 and T_2 whose ratings are also shown. Using the base values of 30 KVA and 240 volts in region 1. Draw the per unit circuit and determine the per unit impedance and per unit voltage. Then also determine the load current both in per unit and amperes.



SOLUTIONS

In **three phase per unit analysis**, the per phase base voltage, current , apparent power of a circuit is related by the following equations in single phase.

$$\boxed{\mathbf{Zbase} = \frac{V^{LN}\,\text{base}}{\text{Ibase}}}$$

$$\boxed{\mathbf{Zbase} = \frac{(\,V^{LN}\,\text{base}\,)^2}{\text{Sbase}}}$$

$$\boxed{\mathbf{Ibase} = \frac{S^{1\theta}\,\text{base}}{V^{LN}\text{base}}}$$

where:

V_{LN} = base voltage from line to neutral

V_{LL} = base voltage from line to line

$S_{3\theta\ \text{base}}$ = base apparent power in three phase

$S_{1\theta\ \text{base}}$ = base apparent power in single phase since:

$S3\theta\ base = 3\ S1\theta\ base$ also $V_{LL} = \sqrt{3}\ V_{LN\ base}$

substituting the above equations to the single phase formulas, the final equations would become:

$$\boxed{Ibase = \frac{S_{3\theta}\ base}{\sqrt{3}\ V_{LL,base}}}$$

$$\boxed{Zbase = \frac{V_{LL,base}}{\sqrt{3}\ Ibase}}$$

Still, the base apparent power and voltage are located at the specified point of the power system

$$\boxed{Zbase = \frac{(V_{LL,base})^2}{S_{3\theta base}}}$$

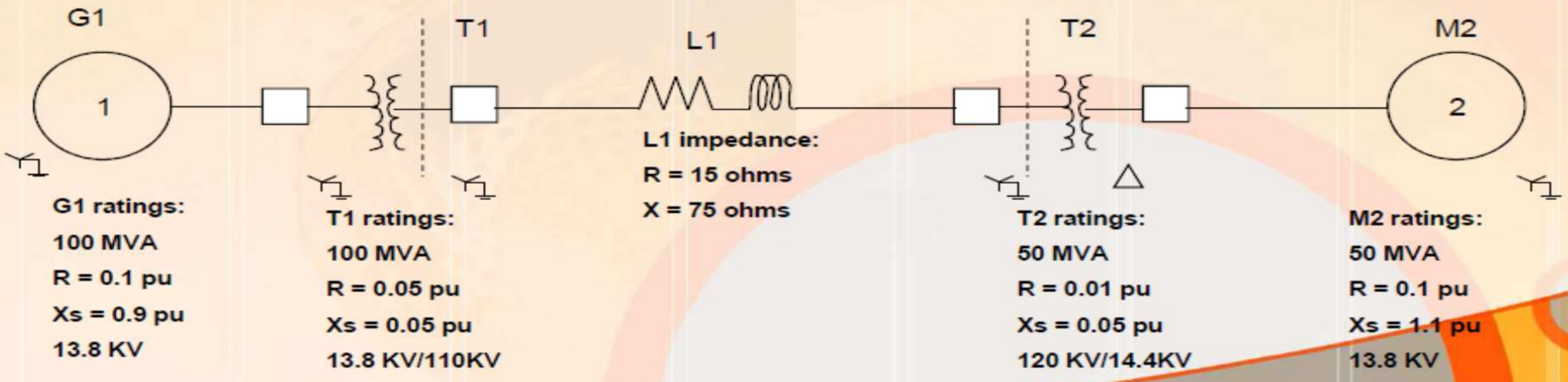
The apparent power is still constant throughout the system

Therefore, the parameters needed for the calculation of three phase per unit analysis is **Stotal, Vline and load impedance per phase!**

3. A 13.8 KV, 100 MVA, 60 Hz three phase synchronous motor has a name plate resistance R of 10% or (0.10 pu) and a reactance of 80%(0.8 pu). These values are specified on the base of the machine's rating. The base quantities of the power system it is connected to are $V_{LL,\ base}$ = 14.4 KV and S_{base} = 500 MVA. Find the pu impedance of the generator on the base of the power system.

4. Given the SLD of a 60 Hz, 3phase, 3 wire system with the following specifications below.

A simple power system shown below consisting of one synchronous generator and one synchronous motor connected by two transformers and a transmission line. Create a per-phase, per-unit equivalent circuit for this power system using base apparent power of 100 MVA and base line voltage @ the generator G_1 of 13.8 KV.



G1 ratings:
100 MVA
R = 0.1 pu
Xs = 0.9 pu
13.8 KV

T1 ratings:
100 MVA
R = 0.05 pu
Xs = 0.05 pu
13.8 KV/110KV

L1 impedance:
R = 15 ohms
X = 75 ohms

T2 ratings:
50 MVA
R = 0.01 pu
Xs = 0.05 pu
120 KV/14.4KV

M2 ratings:
50 MVA
R = 0.1 pu
Xs = 1.1 pu
13.8 KV

Solution to Problem 4

BASE COMPUTATIONS: let: KVAb=100000 KVb=13.8

Region 1

Recall: Zbase = $(KVb)^2$ / MVAb

$Zbase_1 = (13.8)^2 / 100 = 1.9044$ ohms

$Vbase_1 = 13.8$KV

Region 2

$Vbase_2 = Vbase_1 / a_1 = 110$KV

$Zbase_2 = (110)^2 / 100 = 121$ohms

Region 3

$Vbase_3 = Vbase_2 / a_2 = 13.2$ KV

$Zbase_3 = (13.2)^2 / 100 = 1.7424$ohms

Solution to Problem 4

CONVERSION TO PU OF SYSTEM REACTANCES

Recall:

Zpunew = Zpuold * (Vgiven/Vbase)^2 * (Sbase/Sgiven)

for generator:

Xs= j0.9 since Vbase=Vgiven and Sbase=Sgiven

for transformer 1:

Xt_1= j0.05

for transformer 2:

Xt_2= j0.01 (120/110)^2 * (100/50) = j0.0238

Solution to Problem 4

CONVERSION TO PU OF SYSTEM REACTANCES

for motor:

$X_m = j1.1\,(13.8/13.2)^2\,(100/100) = j1.2022$

Solution to Problem 4

CONVERSION TO PU OF PASSIVE AND ACTIVE ELEMENTS

Recall:

pu = actual / base

for active elements

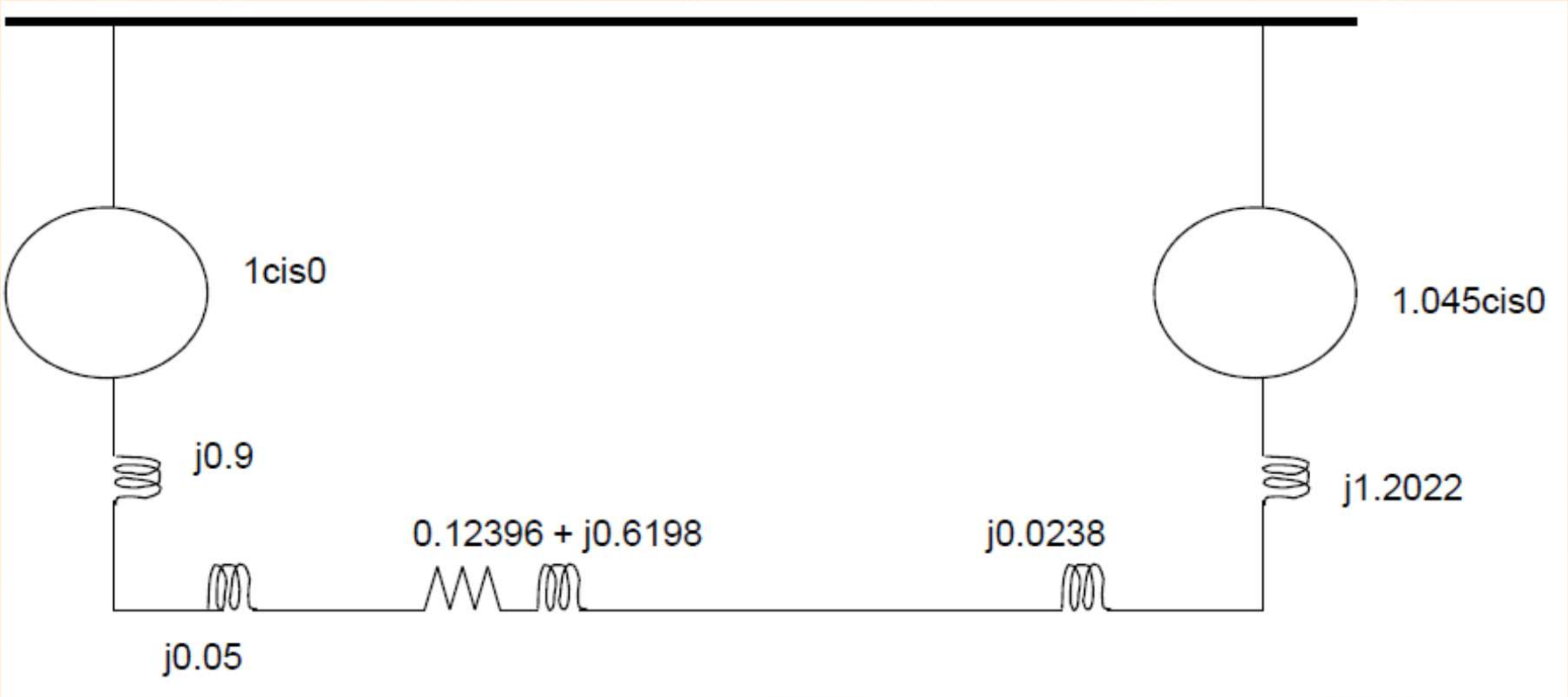
Vgpu = 13.8 / 13.8 = 1cis0

Vmpu = 13.8 / 13.2 = 1.045cis0

for passive elements

Z_{TL} = 15+j75 / 121 = 0.12396 + j0.6198

Solution to Problem 4 (PUEC)



Important things to consider in three phase per unit analysis

1.) since, in pu, the system must be balanced

$V_{LL\ pu=}\ V_{LL\ act} / \sqrt{3} / V_{LL\ base} / \sqrt{3}$ (needed for PPA)

($\sqrt{3}$ will cancel out, thus, no need to get V_{LN})

2.) $Z3\theta_{pu=}\ Z3\theta_{act} / 3 / Z3\theta_{base} / 3$ (needed for PPA)

(3 will cancel out, thus, no need to divide the load by 3)

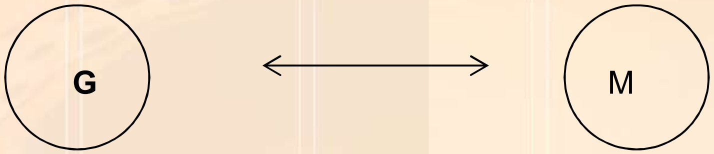
3.) Ilinepu = Iphasepu (since the load is considered Y connected)

4.) Using Sact = Spu*Sbase, the answer is already in total KVA, thus no need to multiply it by 3!

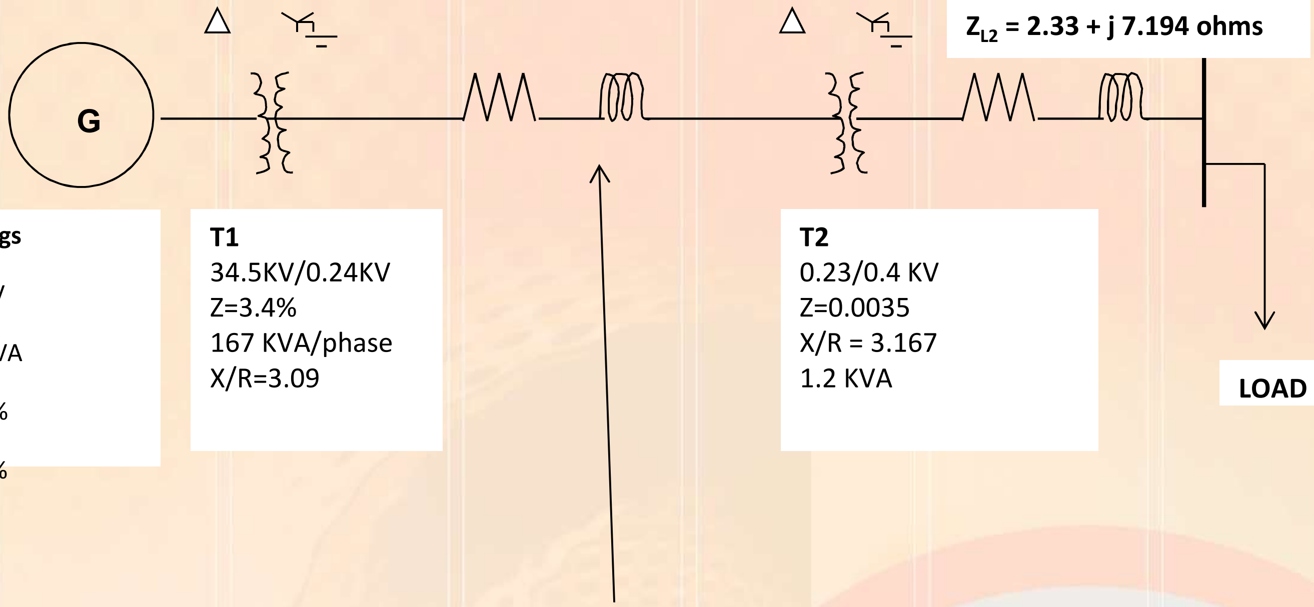
5.) Finally, if the system is already modeled in per unit, consider the phase angles of the voltage sources as zero or reference or otherwise stated.

Important things to consider in three phase per unit analysis

Motor loads also generates back emf (Eb) and thus they are also modeled as generators in per unit analysis.



5. Given the following:



Z_{L2} **= 2.33 + j 7.194 ohms**

G ratings

34.5 KV

100 MVA

R = 10%

X = 40%

T1
34.5KV/0.24KV
Z=3.4%
167 KVA/phase
X/R=3.09

T2
0.23/0.4 KV
Z=0.0035
X/R = 3.167
1.2 KVA

LOAD = 6*30 W

3 sets of 25 mm^2 solid copper wires, horizontally spaced at 5 mm, with 65.352 m length,

a. Develop the PUEC of the problem

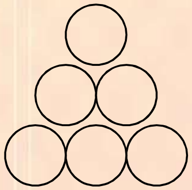
b. Compute for the current flowing at line 2 in amperes.

PROBLEM SET 1 (4th Qtr SY 19-20)

1. *From Lecture Series no.2, Problem 2, repeat the problem using the following bases:*

   a. Sbase = 24KVA, Vbase3=115V

   b. Sbase = 30KVA, Vbase1=240V



2. *Solve for the GMR of a 7 bundled configuration in terms of r .*

Additional Problem

3. If the generator sources on prob. 4 are balanced and having a positive sequence.

   Specs: gen1: Vcb = 2400cis-60

   Determine the voltage Vab on the generator and use this voltage to solve for the new power factor of the generator and motor.

   Use bases Sbase=50KVA and Vbase=110KV at transformer 2.