Per-unit system

Per-Unit System

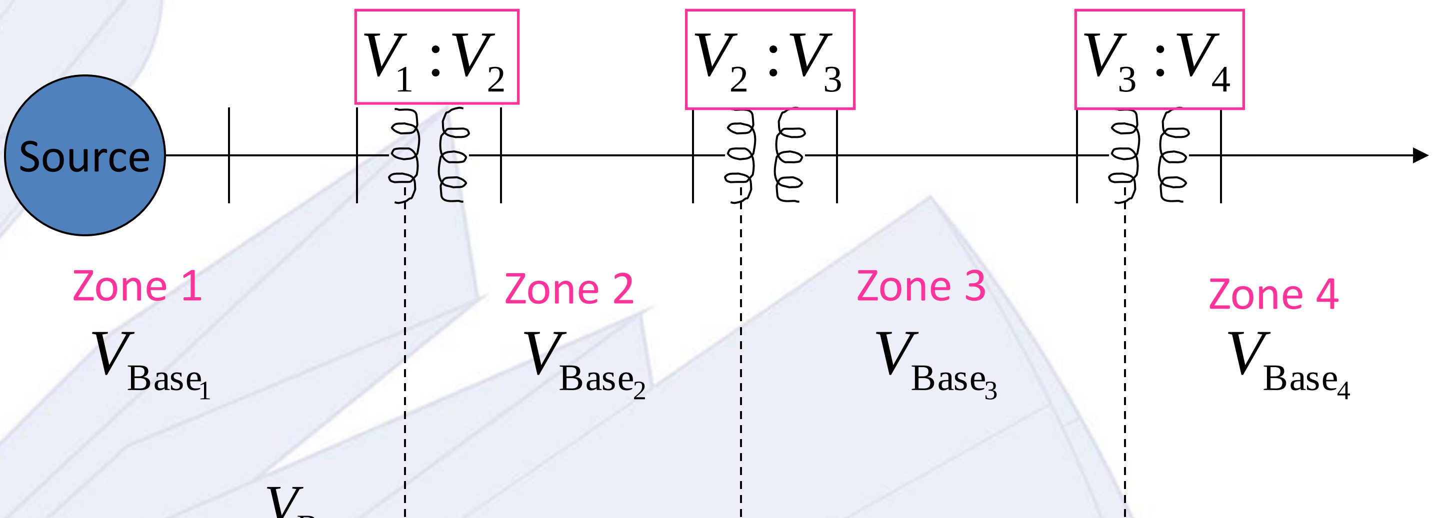
- Expression of power system quantities such as voltage, current power and impedance as a percentage of specified base values
- Properly specified base quantities simplifies transformer equivalent circuit as the value of quantities don't change when referred from one side of transformer to the other
- Per-unit impedances of electrical equipment of similar type usually lie within a narrow range when equipment ratings are used as base values hence p.u. impedance data can be checked rapidly for gross errors
- Manufacturers usually specify the impedances of machines in p.u. or percent of nameplate rating

Procedure for p.u analysis

- Pick S_{base} for the system.
- Pick V_{Base} according to line-to-line voltage.
- Calculate Z_{Base} for different zones.
- Express all quantities in p.u.
- Draw impedance diagram and solve for p.u. quantities.
- Convert back to actual quantities if needed.

How to Choose Base Values ?

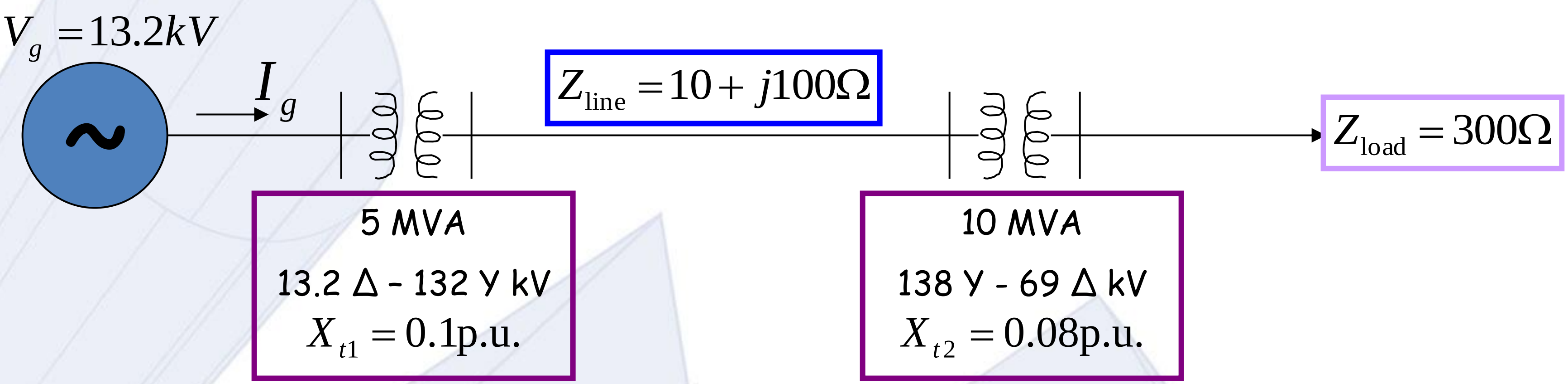
- Divide circuit into zones by transformers.
- Specify two base values out of I_B, V_B, Z_B, S_B; for example S_B and V_B
- Specify voltage base in the ratio of zone line to line voltage.



$$I_{\text{Base}_1} = \frac{|S_{\text{Base}}|}{V_{\text{Base}_1}} \qquad Z_{\text{Base}_1} = \frac{V_{\text{Base}_1}}{I_{\text{Base}_1}}$$

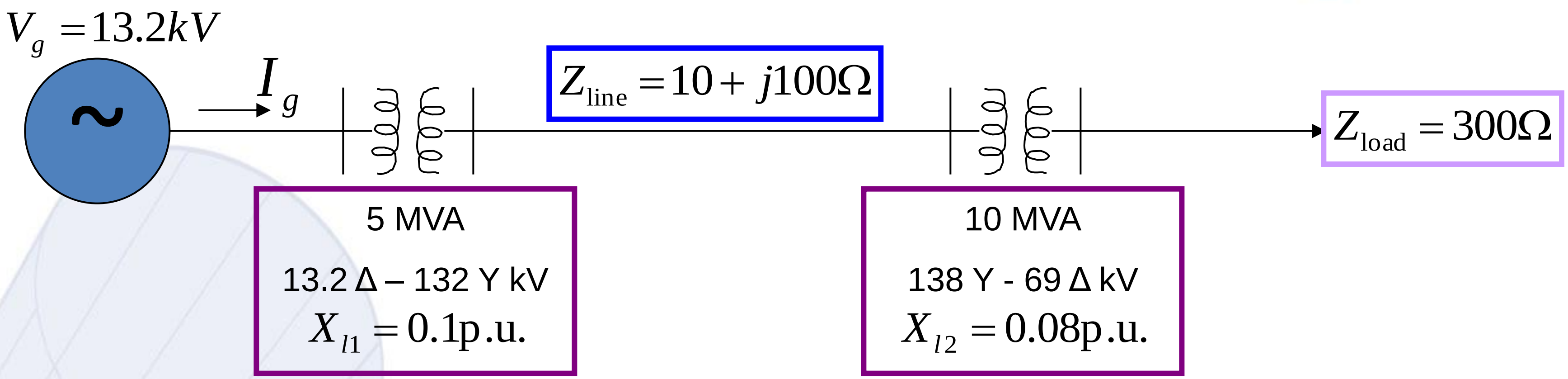
Example 1

- Given a one line diagram,



Find I_g , $I_{\text{t-line}}$, I_{load} , V_{load} and P_{load}

Step 1, 2, and 3: Base Values



$$S_B = 10MVA$$

Zone 1

$$V_{B_1} = 13.8kV$$

$$Z_{B_1} = \frac{\left|V_{B_1}^{1-1}\right|^2}{S_B} = \frac{(13.8k)^2}{10M} = 19.04\Omega$$

$$I_{B_1} = \frac{\left|S_{B_1}^{3\Phi}\right|}{\sqrt{3}\left|V_{B_1}^{1\text{-}1}\right|} = \frac{10M}{\sqrt{3}\cdot 13.8k} = 418.4A$$

Zone 2

$$V_{B_2} = 138kV$$

$$Z_{B_2} = \frac{\left|V_{B_2}^{1-1}\right|^2}{S_B} = \frac{(138k)^2}{10M} = 1904\Omega$$

$$I_{B_2} = \frac{\left|S_{B_2}^{3\Phi}\right|}{\sqrt{3}\left|V_{B_2}^{1\text{-}1}\right|} = \frac{10M}{\sqrt{3}\cdot 138k} = 41.84A$$

Zone 3

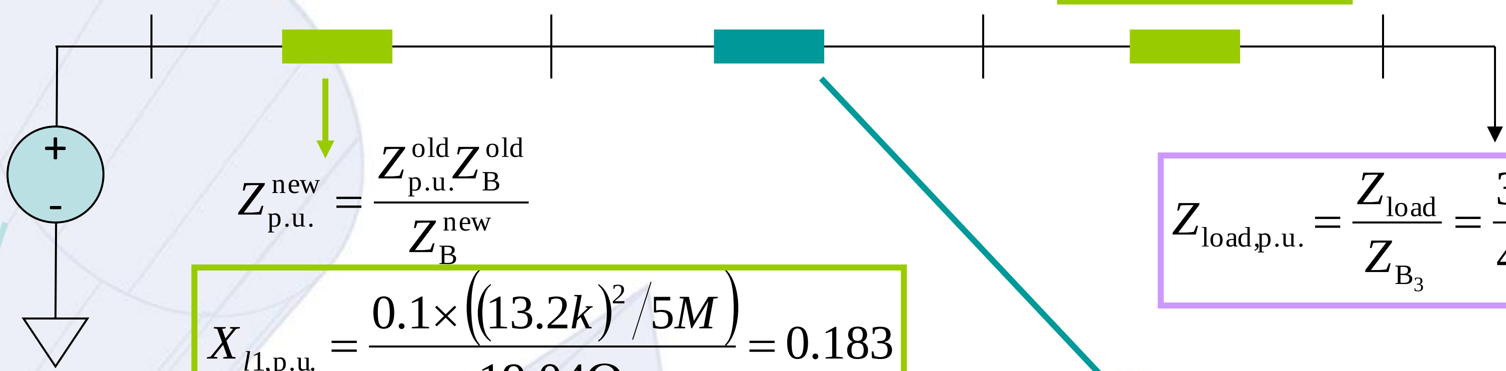
$$V_{B_3} = 69kV$$

$$Z_{B_3} = \frac{\left|V_{B_3}^{1-1}\right|^2}{S_B} = \frac{(69k)^2}{10M} = 476\Omega$$

$$I_{B_3} = \frac{\left|S_{B_3}^{3\Phi}\right|}{\sqrt{3}\left|V_{B_3}^{1\text{-}1}\right|} = \frac{10M}{\sqrt{3}\cdot 69k} = 83.67A$$

Ratio of voltage bases on either side is same as ratio of voltage ratings

Step 4: All in Per Unit Quantities



$$X_{l2} = 0.08\text{p.u.}$$

$$Z_{\text{p.u.}}^{\text{new}} = \frac{Z_{\text{p.u.}}^{\text{old}} Z_{\text{B}}^{\text{old}}}{Z_{\text{B}}^{\text{new}}}$$

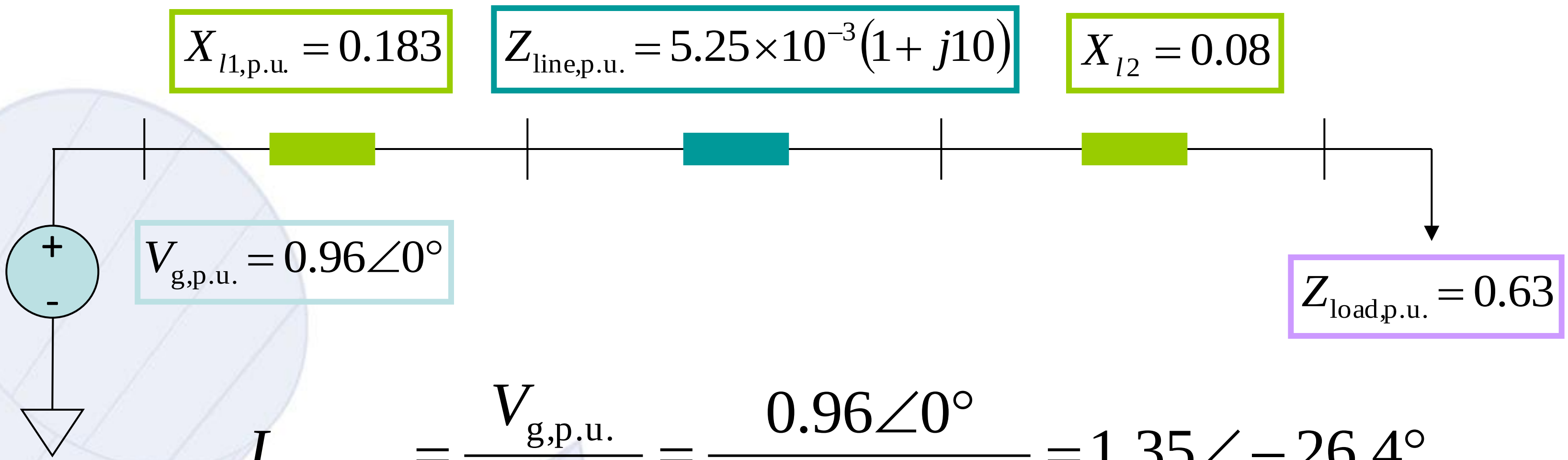
$$X_{l1,\text{p.u.}} = \frac{0.1 \times \left((13.2k)^2 / 5M \right)}{19.04\Omega} = 0.183$$

$$Z_{\text{load,p.u.}} = \frac{Z_{\text{load}}}{Z_{\text{B}_3}} = \frac{300\Omega}{476\Omega} = 0.63$$

$$V_{\text{g,p.u.}} = \frac{V_{\text{g}}}{V_{\text{B}_1}} = \frac{13.2kV}{13.8kV} = 0.96\angle 0^\circ$$

$$Z_{\text{line,p.u.}} = \frac{Z_{\text{line}}}{Z_{\text{B}_2}} = \frac{10 + j100\Omega}{1904} = 5.25 \times 10^{-3}(1 + j10)$$

Step 5: One Phase Diagram & Solve



$$I_{\text{load,p.u.}} = \frac{V_{\text{g,p.u.}}}{Z_{\text{total,p.u.}}} = \frac{0.96\angle 0°}{0.709\angle 26.4°} = 1.35\angle -26.4°$$

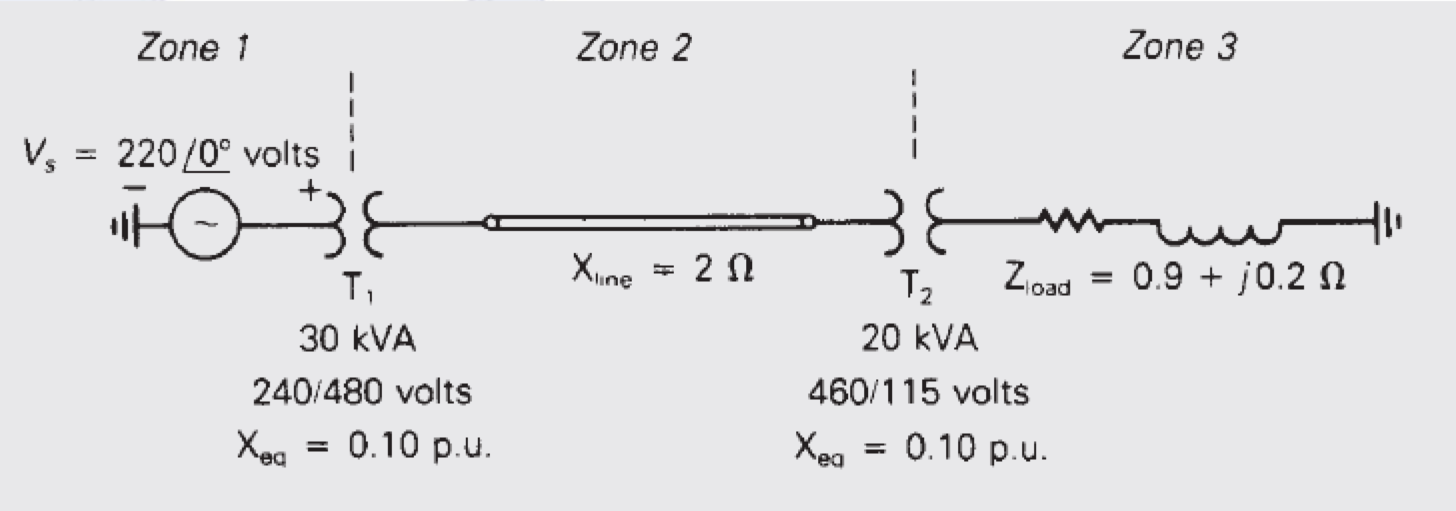
$$V_{\text{load,p.u.}} = I_{\text{load,p.u.}} Z_{\text{load,p.u.}} = 0.8505\angle -26.4°$$

$$S_{\text{load,p.u.}} = V_{\text{load,p.u.}} I^{*}_{\text{load,p.u.}} = 1.148$$

$$I_{\text{g,p.u.}} = I_{\text{t-line,p.u.}} = I_{\text{load,p.u.}} = 1.35\angle -26.4°$$

Example 2

Three zones of a single-phase circuit are identified in Figure below. The zones are connected by transformers T1 and T2, whose ratings are also shown. Using base values of 30 kVA and 240 volts in zone 1, draw the per-unit circuit and determine the per-unit impedances and the per-unit source voltage. Then calculate the load current both in per-unit and in amperes. Transformer winding resistances and shunt admittance branches are neglected.



Solution

$$V_{base2} = \left(\frac{480}{240}\right)(240) = 480 \quad \text{volts}$$

$$V_{base3} = \left(\frac{115}{460}\right)(480) = 120 \quad \text{volts}$$

The base impedances in zones 2 and 3 are

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30{,}000} = 7.68 \quad \Omega$$

and

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30{,}000} = 0.48 \quad \Omega$$

and the base current in zone 3 is

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30{,}000}{120} = 250 \quad A$$

p.u. reactance of T1 is the same as name plate

$$X_{T2p.u.} = (0.10)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

$$Z^{new}_{p.u.} = \frac{Z^{old}_{p.u.} Z^{old}_{B}}{Z^{new}_{B}}$$

Alternatively, using $V_{base3} = 120$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

$$X_{\text{linep.u.}} = \frac{X_{\text{line}}}{Z_{\text{base2}}} = \frac{2}{7.68} = 0.2604 \quad \text{per unit}$$

$$Z_{\text{loadp.u.}} = \frac{Z_{\text{load}}}{Z_{\text{base3}}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \quad \text{per unit}$$

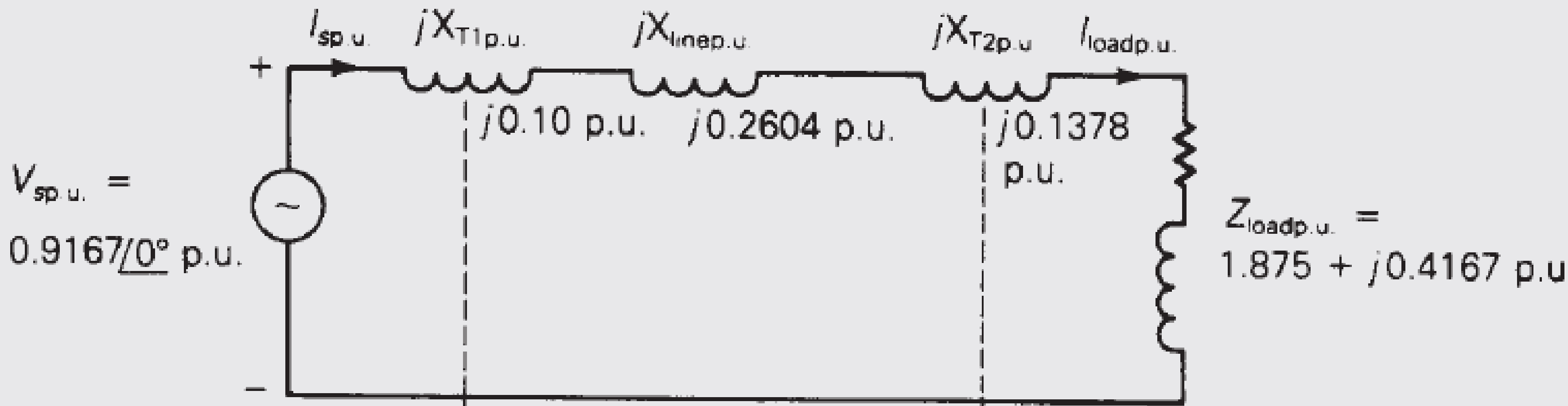
$$I_{\text{loadp.u.}} = I_{\text{sp.u.}} = \frac{V_{\text{sp.u.}}}{j(X_{\text{T1p.u.}} + X_{\text{linep.u.}} + X_{\text{T2p.u.}}) + Z_{\text{loadp.u.}}}$$

$$= \frac{0.9167\angle 0^\circ}{j(0.10 + 0.2604 + 0.1378) + (1.875 + j0.4167)}$$

$$= \frac{0.9167\angle 0^\circ}{1.875 + j0.9149} = \frac{0.9167\angle 0^\circ}{2.086\angle 26.01^\circ}$$

$$= 0.4395\angle -26.01^\circ \quad \text{per unit}$$

$$I_{\text{load}} = (I_{\text{loadp.u.}})I_{\text{base3}} = (0.4395\angle -26.01^\circ)(250) = 109.9\angle -26.01^\circ \quad \text{A}$$



Zone 1	*Zone 2*	*Zone 3*
$V_{base1} = 240$ volts	$V_{base2} = 480$ volts	$V_{base3} = 120$ volts
$Z_{base1} = \frac{(240)^2}{30{,}000} = 1.92\ \Omega$	$Z_{base2} = \frac{(480)^2}{30{,}000} = 7.68\ \Omega$	$Z_{base3} = \frac{(120)^2}{30{,}000} = 0.48\ \Omega$
$S_{base} = 30$ kVA		$I_{base3} = \frac{30{,}000}{120} = 250$ A

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