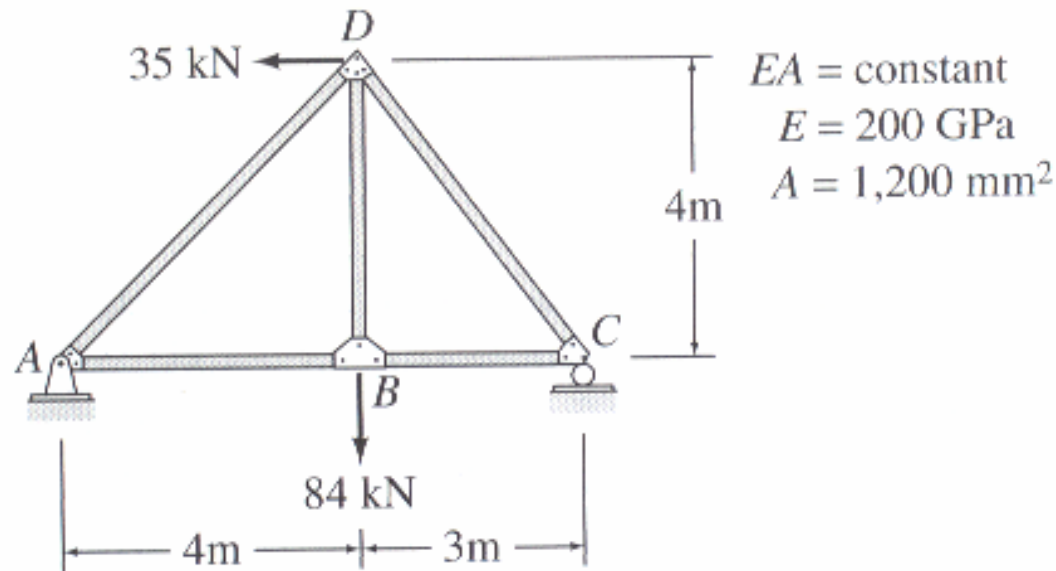


Truss example for dummy load method

- Determine vertical and horizontal deflections at the point B of the truss.



- What makes this truss statically determinate?

Solution

- Need to dust off materials from statics, including free body diagram and method of joints
- Example

Statically indeterminate structures

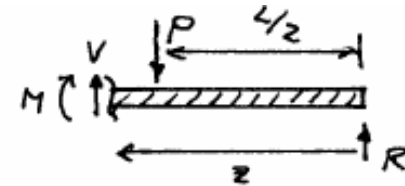
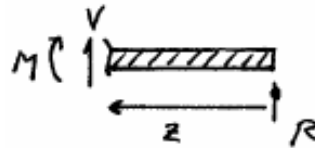
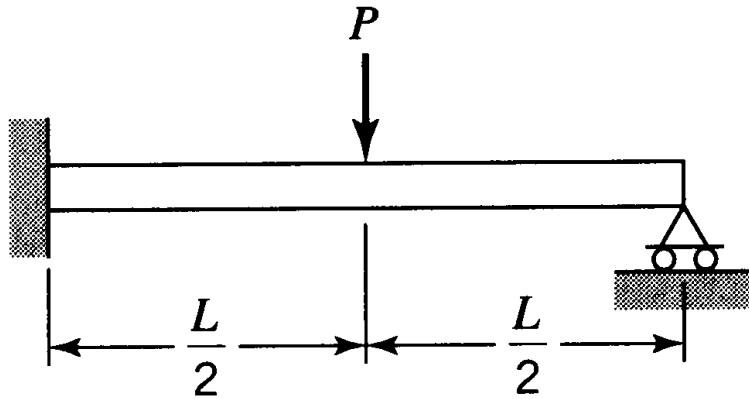
- More internal forces than equations of equilibrium
- Many problems are lightly indeterminate in that the number of equations of equilibrium is almost as high as the number of unknown
- We can replace redundant members by their forces and calculate them using Castigliano's theorem.
- This is called the "force method," and has been attractive to researchers before the advent of widespread use of finite element software
- Automating redundancy selection for general structures has proved to be too difficult, so almost no FE software employs any version of the force method, using instead the "displacement method" with many more unknowns.

Castigliano theorem for solving redundants

- When we have too many boundary conditions, using Castigliano's theorem to do them is intuitive
- We add the force or moment at the support as an additional unknown
- With a displacement or rotation component fixed at the boundary, we can set the derivative of the strain energy with respect to the force to zero

Problem 5.59

- Select right reaction as redundancy



- Bending moment

$$M = Rz \quad (z < L/2) \quad M = Rz - P(z - L/2) \quad z > L/2$$

- Castigliano theorem

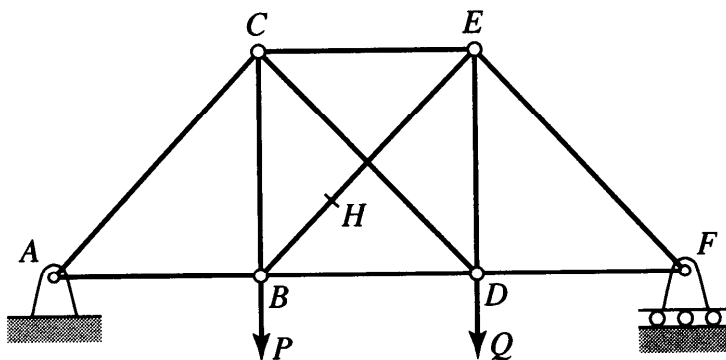
$$0 = \frac{\partial U}{\partial R} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial R} dx \Rightarrow \int_0^{L/2} Rz(z) dz + \int_{L/2}^L (Rz - P[L/2 - z])z dz = 0$$

$$RL^3/3 - 5PL^3/48 = 0 \quad R = 5PL^3/16$$

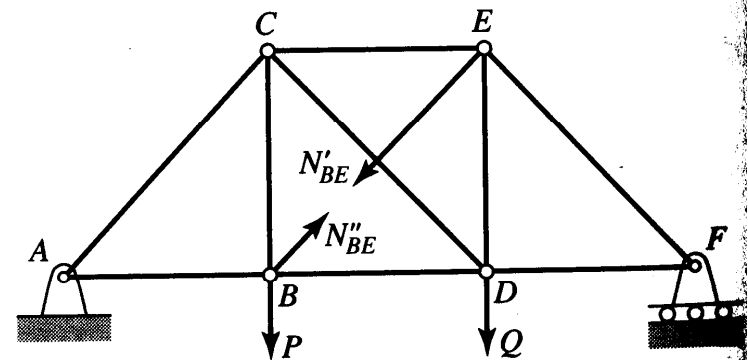
Internal redundancies

- Internal force in member set as unknown
- When set as external force it is replaced by pair of opposite forces
- Textbook shows that derivative of energy is still zero

$$\frac{\partial U}{\partial F} = 0$$



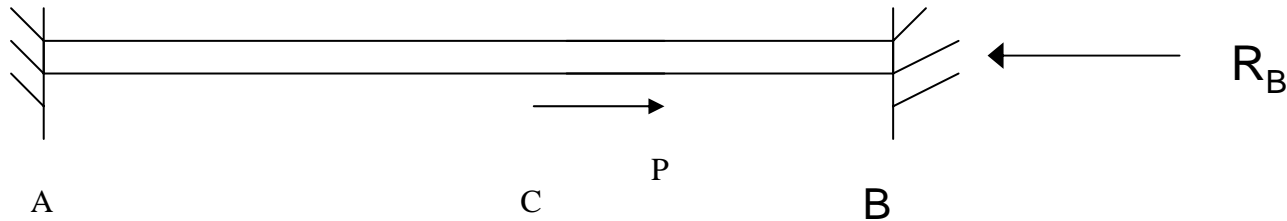
(a)



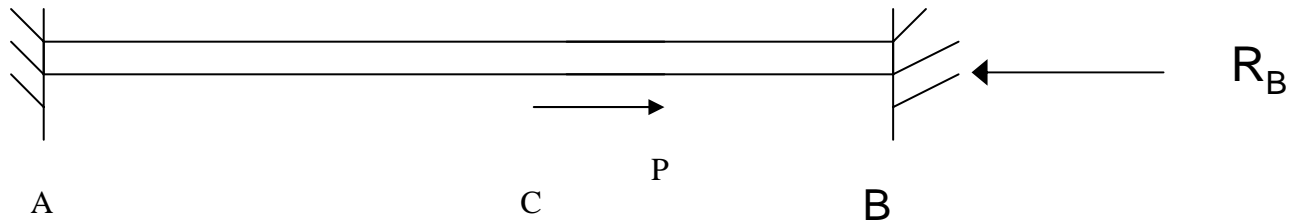
(c)

Previous year exam question

- The bar below has length L , cross-sectional area A , and the axial force P is applied at point C , which is at a distance of $0.6L$ from the left (point A). Young's modulus of the section AC is E , and of the section CB is $2E$.
 - Use Castigliano's theorem to calculate the reaction at B .
 - Use Castigliano's theorem to calculate the displacement at C .
 - Calculate the strain in BC



Solution for R_B



$$\text{Energy in segment BC: } \frac{R_B^2(0.4L)}{2(2EA)} = \frac{0.1R_B^2L}{EA}$$

$$\text{Energy in Segment AC: } \frac{(P-R_B)^2(0.6L)}{2EA} = \frac{0.3(P-R_B)^2L}{EA}$$

$$\text{Zero displacement at B: } \frac{\partial U}{\partial R_B} = \frac{0.2R_B + 0.6(R_B - P)}{EA/L} = 0$$

$$R_B = 0.75P$$

- Why is that reasonable?

Solution for displacement and strain

$$\text{Energy in segment BC: } \frac{R_B^2(0.4L)}{2(2EA)} = \frac{0.1R_B^2L}{EA}$$

$$\text{Energy in Segment AC: } \frac{(P-R_B)^2(0.6L)}{2EA} = \frac{0.3(P-R_B)^2L}{EA}$$

$$\text{Displacement at C: } u_C = \frac{\partial U}{\partial P} = \frac{0.6(P-R_B)}{EA/L} = \frac{0.15PL}{EA}$$

$$\varepsilon = \frac{0 - u_C}{0.4L} = 0.375 \frac{P}{EA}$$

- Alternative way to calculate strain?

Reading assignment

Section 6.1: Question: What are the assumptions we make to solve torsion of prismatic bars of circular cross section (be sure to explain first what is a prismatic bar)

