Question: The velocity of a particle is given by v={16t^2i + 4t^3j + (5t +2)k...

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The velocity of a particle is given by v={16t^2i + 4t^3j + (5t +2)k} m/s, where t is in seconds. If the particle is at the origin when t=0, determine the magnitude of the particle's acceleration when t=2s. Also, what is the x,y,z coordinate position of the particle at this instant

Expert Answer

General guidance

Step-by-step

Step 1 of 2

Write the expression of acceleration as,

$a = \frac{dv}{dt}$

Substitute $\left[\left(16t^2\right)\hat{i} + \left(4t^3\right)\hat{j} + (5t+2)\,\hat{k}\right]\frac{\text{m}}{\text{s}}$ for v , and solve for acceleration as,

$$\vec{a} = \frac{d}{dt}\left[\left(16t^2\right)\hat{i} + \left(4t^3\right)\hat{j} + (5t+2)\,\hat{k}\right]\frac{\text{m}}{\text{s}}$$

$$= \left[(32t)\,\hat{i} + \left(12t^2\right)\hat{j} + (5)\,\hat{k}\right]\frac{\text{m}}{\text{s}^2}$$

Now to determine the acceleration at a given time, substitute the given value of time in the equation of acceleration obtained above and solve,

Thus, at $t = 2\text{s}$,

$$\vec{a} = \left[(32\,(2\text{s}))\,\hat{i} + \left(12(2\text{s})^2\right)\hat{j} + (5)\,\hat{k}\right]\frac{\text{m}}{\text{s}^2}$$

$$= \left[64\hat{i} + 48\hat{j} + 5\hat{k}\right]\frac{\text{m}}{\text{s}^2}$$

Now calculate the magnitude of acceleration vector, $\vec{a}$.

$\|\vec{a}\| = \sqrt{x^2 + y^2 + z^2}$

Substitute 64 for x , 48 for y , 5 for z in the above equation and solve,

$$\|\vec{a}\| = \sqrt{(64)^2 + (48)^2 + (5)^2}\,\frac{\text{m}}{\text{s}^2}$$

$$= \sqrt{6425}\,\frac{\text{m}}{\text{s}^2}$$

$$= 80.156\,\frac{\text{m}}{\text{s}^2}$$

Magnitude of the particle's acceleration at $t = 2\text{s}$ is $80.156\frac{\text{m}}{\text{s}^2}$.

Explanation | Common mistakes | Hint for next step

Calculate the acceleration vector by taking derivative of velocity with respect to time and substitute the given time in the acceleration vector thus obtained.

Write the expression of position in terms of velocity as written above.

$s = \int v dt$

Initial condition for displacement is given as,

At $t = 0\text{s}$ and $s = 0\text{m}$,

Substitute $\left[\left(16t^2\right)\hat{i} + \left(4t^3\right)\hat{j} + \left(5t+2\right)\hat{k}\right]\frac{\text{m}}{\text{s}}$ for v, and solve for displacement (or) position by integration within limits of integration from initial condition (that is, $t = 0\text{s}$) to final condition (that is, t),

$$\vec{s} = \int_0^t \left(\left[\left(16t^2\right)\hat{i} + \left(4t^3\right)\hat{j} + \left(5t+2\right)\hat{k}\right]\frac{\text{m}}{\text{s}}\right) dt$$

$$= \int_0^t \left(16t^2\right) dt + \int_0^t \left(4t^3\right)\hat{j}dt + \int_0^t \left(5t+2\right)\hat{k}dt$$

$$= \left[16\left(\frac{t^3}{3}\right)\hat{i} + 4\left(\frac{t^4}{4}\right)\hat{j} + \left(5\frac{t^2}{2} + 2t\right)\hat{k}\right]\text{m}$$

Now calculate the position at $t = 2\text{s}$.

Substitute 2 for t in the expression of s obtained above equation and solve,

$$\vec{s} = \left[16\left(\frac{(2)^3}{3}\right)\hat{i} + 4\left(\frac{(2)^4}{4}\right)\hat{j} + \left(5\frac{(2)^2}{2} + 2\left(2\right)\right)\hat{k}\right]\text{m}$$

$$= \left[42.667\hat{i} + 16\hat{j} + 14\hat{k}\right]\text{m}$$

Thus, coordinate position of the particle is read from the expression of position vector calculated above as,

$x = 42.667\text{m}$

$y = 16\text{m}$

$z = 14\text{m}$

Position of the particle at $t = 2\text{s}$ is $\left(42.667, 16, 14\right)\text{m}$.

Explanation | Common mistakes

Position of a particle at any given time is calculated from the expression of position derived after integration of velocity. Integration is taken within the limits of initial and final state of time.

Answer

Magnitude of the particle's acceleration at $t = 2\text{s}$ is $80.156\frac{\text{m}}{\text{s}^2}$.

Position of the particle at $t = 2\text{s}$ is $\left(42.667, 16, 14\right)\text{m}$.

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A: See answer

Q: The velocity of a particle is given by v = {16t^2i + 4t^3j + (5t+2)k} m/s, where t is in seconds. If the particle is at the origin when t = 0. determine the magnitude of the particle's acceleration when t = 3s.

A: See answer

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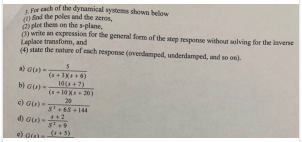

See answer

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Q: Determine the minimum initial velocity and the corresponding angle at which the ball must be kicked in order for it to just cross over the 3 m high fence. Picture shows distance from kicker to the fence (x) is 6m and fence is 3m high (y).

A: See step-by-step answer 94% (34 ratings)

Q: Part A) Neglecting the size of the ball, determine the magnitude vA of the basketball's initial velocity. Part B) Determine the magnitude of the basketball's velocity when it passes through the basket. Part C) Determine the angle at which it passes through the basket, measured clockwise from the x axis.

A: See step-by-step answer 83% (42 ratings)

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