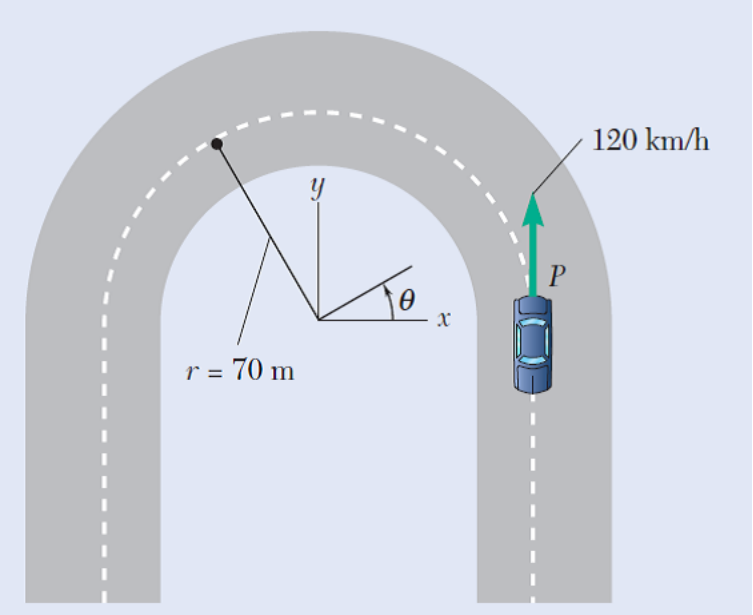
Vector Mechanics for Engineers: Statics and Dynamics (12th Edition)

Problem

A race car enters the circular portion of a track that has a radius of 70 m. When the car enters the curve at point *P*, it is travelling with a speed of 120 km/h that is increasing at 5 m/s^2. Three seconds later, determine the *x* and *y* components of velocity and acceleration of the car.



Step-by-step solution

Step 1 of 7

Write the position of *P* using the polar coordinates.

$$\vec{x} = r\cos\theta i + r\sin\theta j$$

Here, *i* is a unit vector along the *x*-axis and *j* is a unit vector along the *y*-axis.

Differentiate the equation with respect to time *t*.

$$\frac{d\vec{x}}{dt} = \frac{d}{dt}\left(r\cos\theta i + r\sin\theta j\right)$$

$$\vec{v} = -r\dot{\theta}\sin\theta i + r\dot{\theta}\cos\theta j \quad \ldots\ldots (2)$$

Here, *v* is the velocity.

Differentiate equation (2) with respect to time *t*.

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}\left(-r\dot{\theta}\sin\theta i + r\dot{\theta}\cos\theta j\right)$$

$$= -r\left(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right)i + r\left(-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta\right)j$$

Step 2 of 7

At initial state, the particle is at position 1 at which the angle is $0°$ and a velocity of $120\,\text{km/h}$. at final position, let the angle the car makes be θ with the horizontal.

Substitute $0°$ for θ and 70 m for *r* in equation (1).

$$\begin{aligned}\vec{x}_1 &= 70\times\cos 0°\,i + 70\times\sin 0°\,j\\ &= 70i\end{aligned}$$

Substitute $0°$ for θ and 70 m for *r*, and $\dot{\theta}_1$ for $\dot{\theta}$ in equation (2).

$$\begin{aligned}\vec{v}_1 &= -70\times\dot{\theta}_1\sin 0°\,i + 70\times\dot{\theta}_1\cos 0°\,j\\ &= 70\dot{\theta}_1 j\end{aligned}$$

Comment

Step 3 of 7

Calculate the magnitude of velocity at starting point.

$$\begin{aligned}v_1 &= |\vec{v}_1|\\ &= \sqrt{\left(70\dot{\theta}_1\right)^2 + 0^2}\\ &= 70\dot{\theta}_1\end{aligned}$$

Substitute $120\,\text{km/h}$ for v_1.

$$120\,\text{km/h}\times\frac{1000\text{ m}}{1\text{ km}}\times\frac{1\text{ hr}}{3600\text{ s}} = 70\dot{\theta}_1$$

$$\dot{\theta}_1 = 0.4762\,\text{rad/s}$$

Substitute $\dot{\theta}_2$ for $\dot{\theta}$ and 70 m for r in equation (2).

$$\begin{aligned}\vec{v}_2 &= -70\times\dot{\theta}_2\sin\theta i + 70\times\dot{\theta}_2\cos\theta\,j\\ &= -70\dot{\theta}_2\sin\theta i + 70\dot{\theta}_2\cos\theta\,\text{j}\end{aligned}$$

Comment

Step 4 of 7

Calculate the magnitude of velocity at final point.

$$\begin{aligned}v_2 &= |\vec{v}_2|\\ &= \sqrt{\left(-70\dot{\theta}_2\sin\theta\right)^2 + \left(70\dot{\theta}_2\cos\theta\right)^2}\\ &= 70\dot{\theta}_2\end{aligned}$$

Calculate the acceleration of the car from the equation.

$$a = \frac{v_2 - v_1}{t}$$

Substitute $70\dot{\theta}_2$ for v_2.

$$a = \frac{70\dot{\theta}_2 - v_1}{t}$$

Substitute $120\,\text{km/h}$ for v_1, 3 s for t, $5\,\text{m/s}^2$ for a.

Step 5 of 7

Calculate the angular acceleration of the car from the equation.

$$\alpha = \frac{\dot{\theta}_2 - \dot{\theta}_1}{t}$$

Substitute $0.6905\,\text{rad/s}$ for $\dot{\theta}_2$, $0.4762\,\text{rad/s}$ for $\dot{\theta}_1$ and 3 s for t.

$$\alpha = \frac{0.6905 - 0.4762}{3}$$
$$= 0.071433\,\text{rad/s}^2$$

Calculate the final angular coordinate of the car from the equation,

$$\theta_2 = \dot{\theta}_1 t + \frac{1}{2}\alpha t^2$$

Substitute $0.071433\,\text{rad/s}^2$ for α, $0.4762\,\text{rad/s}$ for $\dot{\theta}_1$, and $3\ \text{s}$ for t.

$$\theta_2 = 0.4762 \times 3 + \frac{1}{2} \times 0.071433 \times 3^2$$
$$= 1.75\ \text{radians} \times \frac{180^\circ}{\pi\ \text{radians}}$$
$$= 100.27^\circ$$

Comment

Step 6 of 7

Substitute $70\ \text{m}$ for r, 100.27° for θ, and $0.6905\,\text{rad/s}$ for $\dot{\theta}$ in equation (2).

$$\vec{v} = -r\dot{\theta}\sin\theta i + r\dot{\theta}\cos\theta j$$
$$= -70 \times 0.6905 \times \sin 100.27^\circ i + 70 \times 0.6905 \times \cos 100.27^\circ j$$
$$= -47.56i - 8.62j$$

Therefore, the velocity of the particle is $\boxed{-47.56i - 8.62j}$.

Comment

Step 7 of 7

Substitute $70\ \text{m}$ for r, 100.27° for θ, $0.071433\,\text{rad/s}^2$ for $\ddot{\theta}$, and $0.6905\,\text{rad/s}$ for $\dot{\theta}$ in equation (3).

$$\vec{a} = -r\left(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right)i + r\left(-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta\right)j$$
$$= \begin{pmatrix} -70\left(0.6905^2\cos 100.27 + 0.071433\sin 100.27\right)i \\ +70\left(-0.6905^2\sin 100.27 + 0.071433\cos 100.27\right)j \end{pmatrix}$$
$$= 1.0302i - 33.73j$$

Therefore, the acceleration of the particle is $\boxed{1.0302i - 33.73j}$.

Comment

Recommended solutions for you in Chapter 11

Chapter 11, Problem 186RP

Knowing that slider block A starts from rest and moves to the left with a constant acceleration of 1 ft/s2, determine (a) the...

See solution

Chapter 11, Problem 59P

The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block...

See solution

See more problems in subjects you study