SCE 5441 Hydraulics and Hydrology

----- Assignment 2# -----

1. A rectangular, sharp-crested weir with end contraction is 1.5 m long. How high should it be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?

B = 1.5 m, Q = 0.50 m^3/s

Neglecting the approach velocity head,

$$Q = \frac{2}{3} C_d \sqrt{2g} B' H^{3/2}$$

where B' = B – 0.1nH = 1.5 – 0.1(2)H = 1.5 – 0.2H.

Assuming C_d = 0.62,

$$Q = \frac{2}{3}(0.62)\sqrt{2(9.81)}(1.5 - 0.2H)H^{3/2}$$

$Q = 2.75H^{3/2} – 0.37H^{5/2}$

This equation may be solved using Newton's iteration method. If at iteration j, the head over the weir is H_j, then,

$$Q_j = 2.75H_j^{3/2} - 0.37H_j^{5/2}$$

Let $f(H_j) = Q_j – Q$

$$\frac{df}{dH_j} = \frac{dQ_j}{dH_j} = \frac{3}{2}(2.75)H^{1/2} - \frac{5}{2}(0.37)H^{3/2} = 4.13H^{1/2} – 0.93H^{3/2}$$

$$H_{j+1} = H_j - \frac{f(H_j)}{(df/dH)_j} = H_j - \frac{1 - Q/Q_j}{4.13H_j^{1/2} - 0.93H_j^{3/2}}$$

$$= H_j - \frac{1 - 0.5/Q_j}{4.13H_j^{1/2} - 0.93H_j^{3/2}}$$

Let H_1 = 0.5 m,

$Q_1 = 2.75(0.5)^{3/2} – 0.37(0.5)^{5/2} = 0.91\ m^3/s$.

$$H_2 = 0.50 - \frac{1 - 0.5/0.91}{4.13(0.5)^{1/2} - 0.93(0.5)^{3/2}} = 0.33\ m$$

$Q_2 = 2.75(0.33)^{3/2} – 0.37(0.33)^{5/2} = 0.498\ m^3/s \approx 0.50\ m^3/s$

Hence the head over the weir is 0.33 m and the height of the weir is 2.5 – 0.33 = 2.17 m.

2. The following information was obtained from a discharge measurement on a stream. Determine the discharge.

Distance from bank (ft)	Depth (ft)	Mean velocity (ft)
0	0.0	0.00
12	0.1	0.37
32	4.4	0.87
52	4.6	1.09
72	5.7	1.34
92	4.3	0.71
100	0.0	0.00

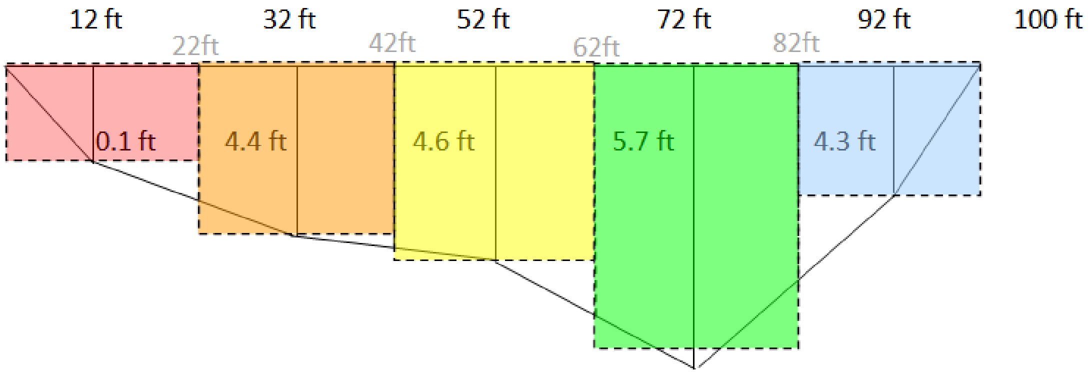
A schematic sketch of the cross-section of the stream is shown in sketch below.

The total discharge, Q, may be approximated as

$$Q = \sum_{i=1}^{5} V_i y_i \Delta w_i$$

=0.37*(0.1)*(22-0) + 0.87(4.4)(42-22) + 1.09(4.6)(62-42) + 1.34(5.7)(82-62) + 0.71(4.3)(100-82)

= 409.79 ft^3/s.



3. An experiment was conducted to determine the hydraulic conductivity of an artesian aquifer. The piezometric heads at two points 150 m apart were found to be 55 m and 48.5 m above a datum. A tracer injected into the first piezometer was observed after 32 hours in the second well. A test on porosity of a sample of the aquifer shows that α = 24%. What is the hydraulic conductivity of the aquifer? Suggest what the aquifer material may be and verify that your solution holds true. Take the subsurface temperature as 15 ℃.

$\Delta L = 150$ m, $h_1 = 55$ m, $h_2 = 48.5$ m, $t = 32$ hrs, $\alpha = 24\% = 0.24$, $T = 15\ ^oC$.

Using Darcy's law,

$$Q = KA\frac{dh}{dL} = KA\frac{\Delta h}{\Delta L} = KA\frac{h_1 - h_2}{\Delta L}$$

$$\frac{Q}{A} = q = K\frac{55 - 48.5}{150} = 0.0433K$$

Velocity of flow, $V = \frac{\Delta L}{t} = \frac{150}{32} = 4.6875$ m/hr = 1.30×10^{-3} m/s.

Using the continuity equation,

$Q = Aq = A_{pore}V$ where A_{pore} is the area of cross-section of the pore spaces

$$q = \frac{A_{pore}}{A}V = \alpha V = 0.24(4.6875) = 1.125 \text{ m/hr}$$

Hence, $0.0433K = 1.125$

$K = 25.96$ m/hr = 7.21×10^{-3} m/s = 7.21 mm/s.

Table 6.1.1 shows that the aquifer is most probably clean sand and less likely gravel or silty sand. The solution is verified by checking the Reynolds number for this problem.

$$R_e = \frac{VD}{\nu}$$

At 15 ^oC, $\nu = 1.141 \times 10^{-6}$ m^2/s (for water)

The range of grain sizes for clean sand is 0.05 mm – 1 mm

Hence, $D = \frac{0.05 + 1}{2} = 0.53$ mm (if the arithmetic mean diameter is taken)

$D = \sqrt{0.05(1)} = 0.22$ mm (if the geometric diameter is taken)

$$R_e = \frac{1.30 \times 10^{-3}(0.53 \times 10^{-3})}{1.141 \times 10^{-6}} = 0.60 \quad \text{(for the arithmetic mean diameter)}$$

$$R_e = \frac{1.30 \times 10^{-3}(0.22 \times 10^{-3})}{1.141 \times 10^{-6}} = 0.25 \quad \text{(for the geometric mean diameter)}$$

In either case, $R_e < 1$. Hence, Darcy's law holds true and the solution procedure is correct.

4. The specific storage of a 45-m thick confined aquifer is 3.0 x 10^{-5} m^{-1}. How much water would the aquifer produce if the piezometric surface is lowered by 10m over an area of 1 km^2?

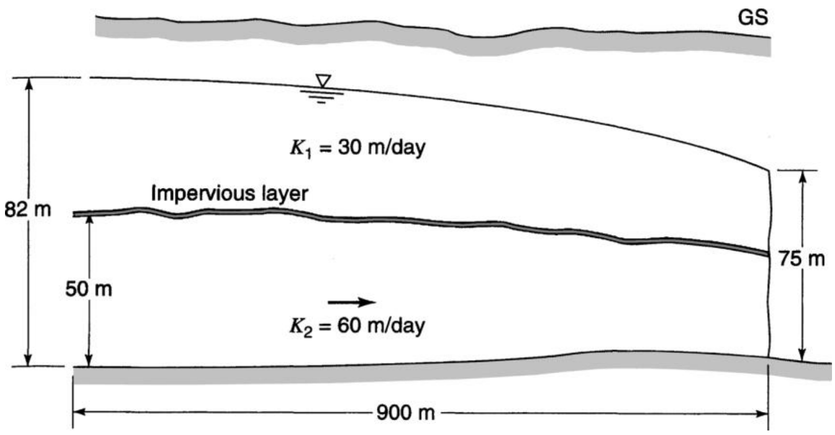
The relationship between specific storage and storage coefficient is

$$S_s = \frac{S}{b} \text{ where } b \text{ is the aquifer thickness.}$$

Thus, $S = S_s b = (3\times10^{-5}\ m^{-1})(45\ m) = 1.35\times10^{-3}$ and the volume of water released is given by

$$V = (A)(\Delta h)(S) = (1\times10^{6}\ m^{2})(10\ m)(1.35\times10^{-3}) = 13{,}500\ m^{3}$$

5. Suppose an unconfined aquifer lies over a confined aquifer as shown in Figure. Deternine the flow out of both aquifers.



K_1 = 30 m/day = 3.47 x 10^{-4} m/s, K_2 = 6.94 x 10^{-4} m/s

Total flow out of the aquifer:

$Q = Q_1 + Q_2$

$$Q_1 = \frac{K_1\left(h_o^2 - h^2\right)}{2L} = \frac{3.47\,x\,10^{-4}\left[(82-50)^2 - (75-50)^2\right]}{2(900)}$$

= 7.69 x 10^{-5} m^3/s (per meter width of the aquifer)

= 6.65 m^3/day

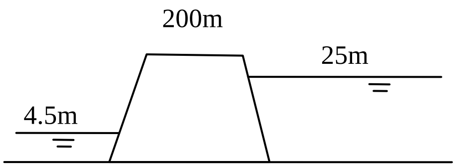
$$Q_2 = \frac{K_2 b\left(h_1 - h_2\right)}{L} = \frac{6.94\,x\,10^{-4}(50)(82-75)}{900}$$

= 2.70 x 10^{-4} m^3/s (per meter width of the aquifer)

$= 23.33\ m^3/day$

$Q = 6.65 + 23.33 = 29.98\ m^3/day$

6. An earthen dam is 200 m across (i.e., the distance from the upstream face to the downstream face) and underlain by an impermeable bedrock. The average hydraulic conductivity of the material of which the dam is constructed is 0.065 m/day. If the water surface elevations in the reservoir and the tail water are 25 m and 4.5 m, respectively, estimate the magnitude of leakage from the reservoir to the tail water per 100-m width of the dam.



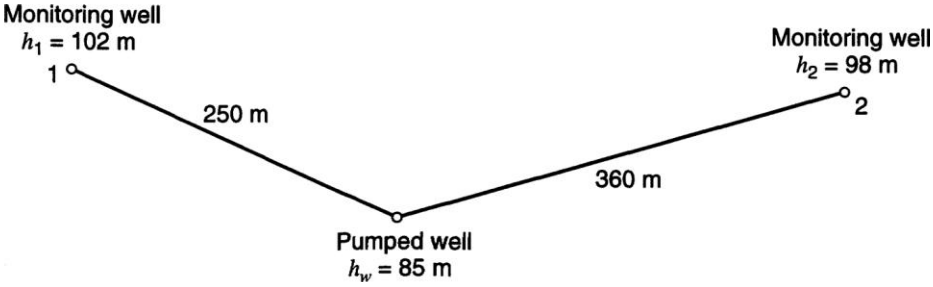
The Dupuit equation is used for estimating the amount of leakage:

$$q = \frac{K}{2x}\left(h_0^2 - h^2\right) = \frac{0.065\,m/day}{2(200\,m)}\left(25^2 - 4.5^2\right) = 0.09827\ m^2/day$$

and the seepage per 100-m width of the dam becomes:

$Q = q \times 100\ m = 0.09827\ m^2/day \times 100\ m = 9.827\ m^3/day$

7. A 50-cm diameter well fully penetrates vertically through a confined aquifer 12 m thick. When the well is pumped at 0.035 m^3/s, the heads in the pumped well and the two other observation wells were found to be as shown in Figure. Does this test suggest that the aquifer material is fairly homogeneous in the directions of the observation wells?



$r_w = 25\ cm = 0.25\ m$, $b = 12\ m$, $Q = 0.035\ m^3/s$.

$$Q = \frac{2\pi Kb(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Considering the first observation well and the pumped well,

$$K_{1\text{-}w} = \frac{Q\ln\left(\frac{r_1}{r_w}\right)}{2\pi b(h_1 - h_w)} = \frac{0.035\ln\left(\frac{250}{0.25}\right)}{2\pi(12)(102-85)} = 1.89 \times 10^{-4} \text{ m/s}$$

Considering the second observation well and the pumped well,

$$K_{2\text{-}w} = \frac{0.035\ln\left(\frac{360}{0.25}\right)}{2\pi(12)(98-85)} = 2.60 \times 10^{-4} \text{ m/s.}$$

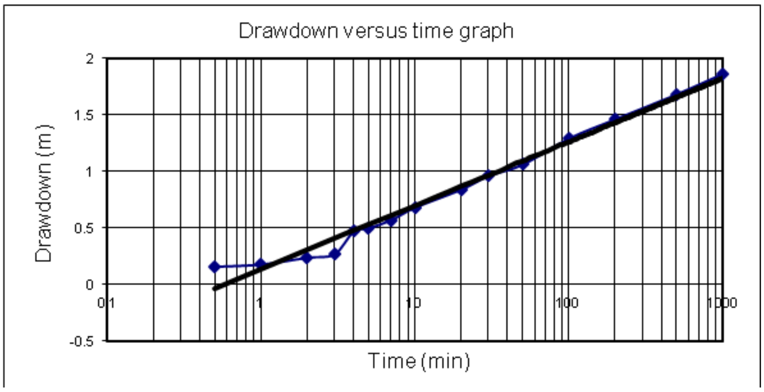
The hydraulic conductivities of the aquifers in both directions are fairly of the same order of magnitude. Hence, the aquifer material may be considered the same in both directions.

8. The following data were obtained in an observation well 80 m away from the pumped well. The discharge from the pumped well was at 2.5 m^3/min. Using Jacob's approximation, determine the aquifer properties. Assume a confined aquifer. Also verify your solution and determine the time after which Jacob's approximation will be valid.

Time (min)	Drawdown (m)
0.5	0.16
1	0.18
2	0.24
3	0.27
4	0.47
5	0.50
7	0.57
10	0.68
20	0.84
30	0.96
50	1.06
100	1.29
200	1.46
500	1.68
1000	1.86

The given data was drawn on a semi-log graph paper as shown in Figure.

From this graph, t_o = 0.75 min, and Δs = 0.6 m for 1 log cycle.



Using equation (6.5.14),

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln u\right) = \frac{0.183}{T}\log\left(\frac{2.25Tt}{r^2 S}\right)$$

$$\text{For } s = 0, \quad \log\left(\frac{2.25Tt_o}{r^2 S}\right) = 0$$

$$\text{Thus,} \quad \frac{2.25Tt_o}{r^2 S} = 1$$

$$S = \frac{2.25Tt_o}{r^2}$$

$$\text{For 1 log cycle,} \quad \Delta s = \frac{0.183Q}{T}\log\left(\frac{2.25Tt_2}{r^2 S}\right) - \frac{0.183Q}{T}\log\left(\frac{2.25Tt_1}{r^2 S}\right)$$

$$= \frac{0.183Q}{T}\log\left(\frac{t_2}{t_1}\right) = \frac{0.183Q}{T}$$

Hence, $T = \dfrac{0.183Q}{\Delta s} = \dfrac{0.183(2.5)}{0.60}$ = 0.76 m^2/min = 1100 m^2/day

$S = \dfrac{2.25(0.76)(0.60)}{80^2}$ = 1.6 x 10^{-4}

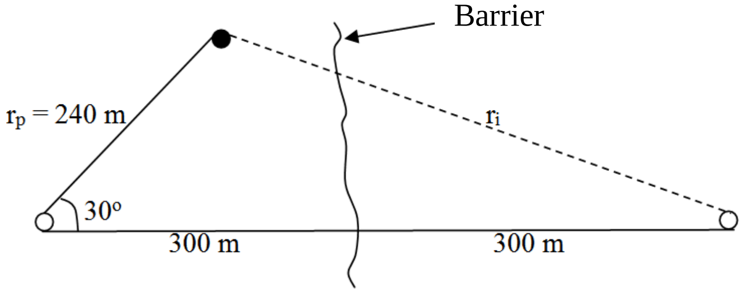
Jacob's approximation method is valid for $u \leq 0.01$

$u = \dfrac{r^2 S}{4Tt} = \dfrac{80^2(1.6 \times 10^{-4})}{4(0.76)(1000)}$ =3.375 x 10^{-4} < 0.01. OK.

Jacob's approximation for this problem is valid for

$t \geq \dfrac{r^2 S}{4T(0.01)} = \dfrac{80^2(1.6 \times 10^{-4})}{4(0.76)(0.01)} \geq 33.68$ min.

9. A well is pumping near a barrier boundary at a rate of 0.03 m^3/s from a confined aquifer 20 m thick. The hydraulic conductivity of the aquifer is 3.2 x 10^{-4} m/s, and its storativity is 3 x 10^{-5}. Determine the drawdown in the observation well after 30 hours of continuous pumping. What is the fraction of the drawdown attributable to the barrier boundary?



Q = 0.03 m^3/s, b = 20 m, K = 3.2 x 10^{-4}, S = 3 x 10^{-5}, t = 10 hrs = 36000 s.

An image well is placed across the boundary at the same distance from the boundary as the pumped well (as shown in Figure).

The drawdown in the observation well is due to the real well and the imaginary well (which accounts for the barrier boundary).

T = Kb = 27.65m/day x 20m = 553 m^2/day

$$s = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i)$$

$$u_p = \frac{r_i^2 S}{4Tt_p} = \frac{240^2 3x10^{-5}}{4(553)(1.25)} = 6.25\ x\ 10^{-4}$$

So $W(u_p) = 6.80$

$r_i^2 = 600^2 + 240^2 - 2(600)(300)\cos 30^o = 168185\ m^2$, so $r_i = 410$ m.

$u_i = 1.82x10^{-3}$ and $W(u_i) = 5.76$

$W(u_p) = 6.80$ for $u_p = 6.25 \times 10^{-4}$ and $W(u_i) = 5.76$ for $u_i = 1.82 \times 10^{-3}$

Q = (0.03 m^3/s)(86400 sec/day) = 2592 m^3/day

$$s = \frac{2592}{4\pi(2764)(20)}[6.80 + 5.76] = 4.68$$

Then s = 4.68 and s_i = 2.15 m (due to boundary)

As a fraction 2.11/4.68 = 0.45 (45 %)

10. Based on the 100-year rainfall records at the Hong Kong Observatory, the rainfall intensities with an annual exceedance probability of 0.1 (i.e., l0-yr return period) of different durations are given in the table below.

Duration, t_d (min)	15	30	60	120
Intensity, i (mm/h)	161	132	103	74

(a) Determine the least-squares estimates of coefficients a and c in the following rainfall intensity-duration equation and the associated R^2 value,

$$i = \frac{a}{(t_d + 4.5)^c}$$

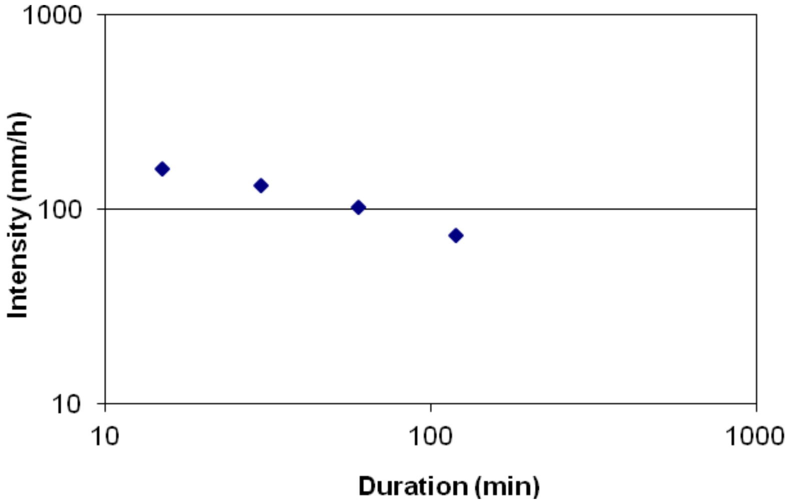
in which i = rainfall intensity (in mm/h) and td = storm duration in (minutes).
(b) Estimate the total rainfall depth (in cm) for a l0-year, 4-hr storm event.

intensity	duration	y=ln(i)	x=ln(td+4.5)	xy	x^2	y-hat
161	15	5.081	2.970	15.094	8.823	5.10416122
132	30	4.883	3.541	17.290	12.538	4.865642363
103	60	4.635	4.167	19.311	17.361	4.604063183
74	120	4.304	4.824	20.764	23.274	4.329133603
	sum =	18.903	15.502	72.459	61.997	
	Avg =	4.726	3.876			

error	y-ybar	error^2	(y-ybar)^2
-0.023	0.356	0.000518	0.126490
0.017	0.157	0.000294	0.024665
0.031	-0.091	0.000940	0.008285
-0.025	-0.422	0.000628	0.177818
0.000	0.000	0.002	0.337

c' = -0.418
c = **0.418**
ln(a) = 6.346
a = **570.182**
R^2 = **0.993**

i-4h = 57.2 mm/hr
d-4h= 229.0 mm = **22.9** cm



11. Consider a rainfall event having 5-min cumulative rainfall record given below:

Time (min)	0	5	10	15	20	25	30
Cumulative rainfall (mm)	0	7	14	23	34	45	58
Time (min)	35	40	45	50	55	60	65
Cumulative rainfall (mm)	70	81	91	100	110	119	125
Time (min)	70	75	80	85	90		
Cumulative rainfall (mm)	131	136	140	140	140		

(a) What is the duration of the entire rainfall event and the corresponding total rainfall amount?
(b) Find the rainfall depth hyetograph (in tabular form) with l0-min time interval for the storm event.
(c) Find the maximum l0-min and 20-min average rainfall intensities (in mm/hr) for the storm event.

(a) Duration of the rainfall event is 80 minutes
(b) Determine the hyetograph

Time	(mm)	(mm/hr)
0-10	14	84
10-20	20	120
20-30	24	144
30-40	23	138
40-50	19	114
50-60	19	114
60-70	12	72
70-80	9	54
80-90	0	0

(c) Highest 10 min intensity is from 25 minute to 35 minutes = 150 mm/hr
Highest 20 minute intensity from 15 to 35 minutes and from 20 to 40 minutes = 141

mm/hr

12. The average weather conditions are net radiation = 40 W/m^2; air temperature = 28 .5 °C; relative humidity = 55 percent; and wind speed = 2.7 m/s at a height of 2 m. Calculate the open water evaporation rate in millimeters per day using the energy method, the aerodynamic method, the combination method, and the Priestley-Taylor method. Assume standard atmospheric pressure is 101 kPa and Z_0 is 0.03 cm.

Energy method

Where l_v = 2.501 x 10^6 -2370(28.5 °C) = 2.433 x 10^6 J/kg

$$E_r = \frac{R_n}{l_v \rho_w}$$

$$E_r = \frac{40\, W/m^2}{\left(2.433 \text{ x } 10^6 \frac{\text{J}}{\text{kg}}\right)(996.3 \frac{\text{kg}}{\text{m}^3})} = 1.65 \text{ x } 10^{-8} \frac{m}{s} = 1.43\, mm/day$$

Aerodynamic method

$$E_a = B(e_{as} - e_a)$$

The vapor transfer coefficient is

$$B = \frac{0.102 u_2}{[\ln(z_2/z_0)]^2}$$

where the wind speed is u_2 = 2.7 m/s, z_2 = 2 m = 200 cm, and z_0 = 0.03 cm. solving B = 0.0036 mm/da·Pa.

The saturation vapor pressure is

$$e_{as} = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$$

With the air temperature T = 28.5 °C, then solving the saturation vapor pressure is e_{as} = 3893 Pa

The ambient vapor pressure is computed using

$$e_a = R_h e_{as}$$

where the R_h = 55 % or 0.55 so e_a = 0.55(3893) = 2141 Pa

$$E_a = B(e_{as} - e_a)$$

$E_a = 0.0036(3893 – 2141) = 6.31$ mm/day

Combined method

$$E = \left(\frac{\Delta}{\Delta+\gamma}\right)E_r + \left(\frac{\gamma}{\Delta+\gamma}\right)E_a$$

Where γ is the psychrometric constant (approximately 66.8 Pa/^{o}C) and Δ is the gradient of the saturated vapor pressure curve $\Delta = de_{as}/dT$ at air temperature T_α

$$\Delta = \frac{4098e_{as}}{(237.3+T_\alpha)}$$

$\Delta = 225.8$ Pa/^{o}C

Using $E_r = 1.43\ mm/day$ and $E_a = 6.31$ mm/day then E = 2.54 mm/day

Priestly-Taylor method

$$E = 1.3\left(\frac{\Delta}{\Delta+\gamma}\right)E_r$$

E = 1.434 mm/day