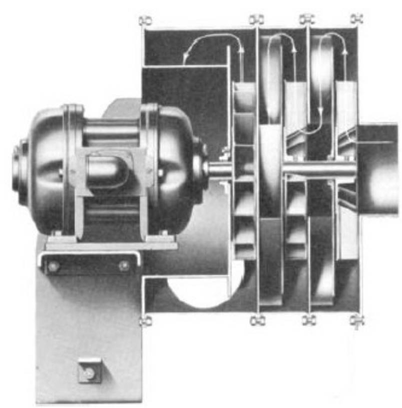
COMPRESSORS

Compressed air has become one of the most important power media used in the industry. Compressed-air is used for machine and tool operation, drilling, painting, soot blowing, pneumatic conveying, food processing, instrument operations, and in situ operations (e.g., underground combustion). Pressures range from 25 psig (172 kPa) to 60,000 psig (413,790 kPa). The largest usage is at 90 to 110 psig which is the normal plant air-pressure range.

Gas compressors are used for refrigeration, air conditioning, heating, pipeline conveying, natural gas gathering, catalytic cracking, polymerization, and in other chemical processes.

A *compressor* is a machine that is used to increase the pressure of a gas or vapor. They can be grouped into two major classifications: *centrifugal* and *positive displacement*.

Centrifugal Compressors



Centrifugal compressors are utilized for low pressure and high capacity applications.

In general, the centrifugal designation is used when the gas flow is radial and the energy transfer is predominantly due to a change in the centrifugal forces acting on the gas. The force utilized by the centrifugal compressor is the same as that utilized by centrifugal pumps.

In a centrifugal compressor, air or gas at atmospheric pressure enters the eye of the impeller. As the impeller rotates, the gas is accelerated by the rotating element within the confined space that is created by the volute of the compressor's casing. The gas is compressed as more gas is forced into the volute by the impeller blades. The pressure of the gas increases as it is pushed through the reduced free space within the volute.

As in centrifugal pumps, there may be several stages to a centrifugal air compressor. In these multi-stage units, a progressively higher pressure is produced by each stage of compression.

Positive Displacement Compressors

Positive-displacement compressors can be divided into two major classifications: rotary and reciprocating.

Rotary Compressors

Rotary compressors are utilized for low capacity and medium pressures applications.

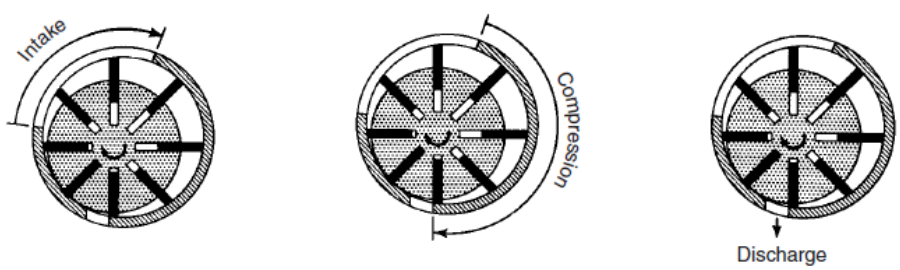
The rotary compressor is adaptable to direct drive by the use of induction motors or multi-cylinder gasoline or diesel engines. These compressors are compact, relatively inexpensive, and require a minimum of operating attention and maintenance. They occupy a fraction of the space and weight of a reciprocating machine having equivalent capacity.

Rotary compressors are classified into three general groups: *sliding vane*, *helical lobe*, and *liquid-seal ring*.

Sliding Vane Compressors

The basic element of the sliding-vane compressor is the cylindrical housing and the rotor assembly. This compressor has longitudinal vanes that slide radially in a slotted rotor mounted eccentrically in a cylinder. The centrifugal force carries the sliding vanes against the cylindrical case with the vanes forming a number of individual longitudinal cells in the eccentric annulus between the case and rotor. The suction port is located where the longitudinal cells are largest. The size of each cell is reduced by the eccentricity of the rotor as the vanes approach the discharge port, thus compressing the gas.

Cyclical opening and closing of the inlet and discharge ports occurs by the rotor's vanes passing over them. The inlet port is normally a wide opening that is designed to admit gas in the pocket between two vanes. The port closes momentarily when the second vane of each air containing pocket passes over the inlet port.



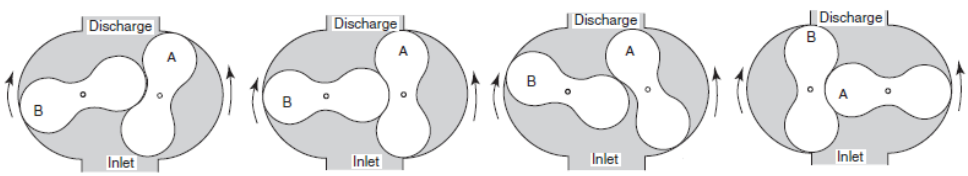
As rotor turns, gas is trap- in pockets formed by vanes. | Gas is gradually compressed as pockets get smaller. | Compressed gas is pushed out through discharge port.

Helical Lobe or Screw Compressors

The helical lobe, or screw, compressor has two or more mating sets of lobe-type rotors mounted in a common housing. The male lobe, or rotor, is usually direct-driven by an electric motor. The female lobe, or mating rotor, is driven by a helical gear set that is mounted on the outboard end of the rotor shafts. The gears provide both motive power for the female rotor and absolute timing between the rotors.

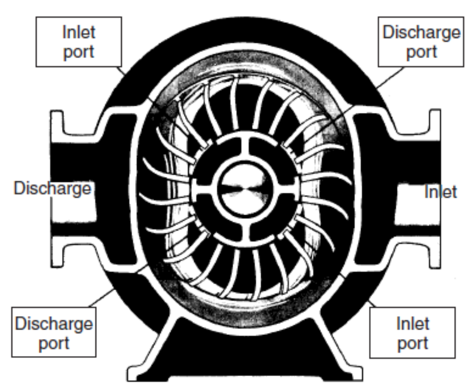
The rotor set has extremely close mating clearance (i.e., about 0.5 mils) but no metal-to-metal contact. Most of these compressors are designed for oil-free operation. In other words, no oil is used to lubricate or seal the rotors. Instead, oil lubrication is limited to the timing gears and bearings that are outside the air chamber. Because of this, maintaining proper clearance between the two rotors is critical.

This type of compressor is classified as a *constant volume, variable-pressure* machine that is quite similar to the vane-type rotary in general characteristics. Both have a built-in compression ratio. Helical-lobe compressors are best suited for base-load applications where they can provide a constant volume and pressure of discharge gas. Helical-lobe compressors are not designed for frequent or constant cycles between load and no-load operation. Each time the compressor unloads, the rotors tend to thrust axially. Even though the rotors have a substantial thrust bearing and, in some cases, a balancing piston to counteract axial thrust, the axial clearance increases each time the compressor unloads.



Liquid-Seal Ring Compressors

The liquid-ring or liquid-piston compressor has a rotor with multiple forward-turned blades that rotate about a central cone that contains inlet and discharge ports. Liquid is trapped between adjacent blades, which drive the liquid around the inside of an elliptical casing. As the rotor turns, the liquid face moves in and out of this space due to the casing shape, creating a liquid piston. Porting in the central cone is built-in and fixed and there are no valves.



Rotation is clockwise

Compression occurs within the pockets or chambers between the blades before the discharge port is uncovered. Since the port location must be designed and built for a specific compression ratio, it tends to operate above or below the design pressure.

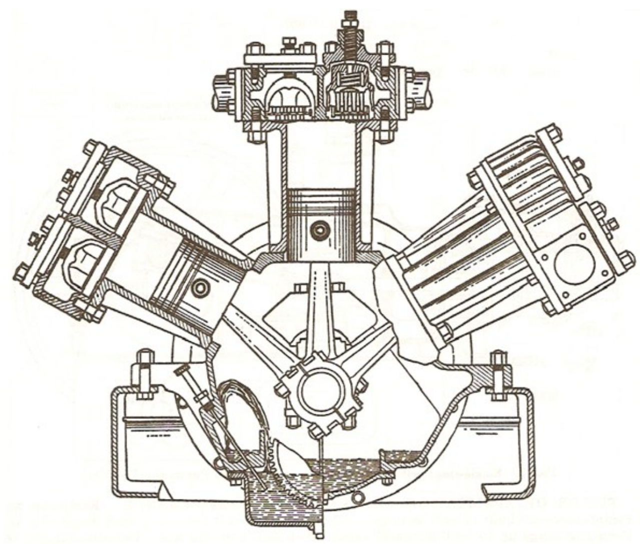
Liquid-ring compressors are cooled directly rather than by jacketed casing walls. The cooling liquid is fed into the casing where it comes into direct contact with the gas being compressed. The excess liquid is discharged with the gas. The discharged mixture is passed through a conventional baffle or centrifugal-type separator to remove the free liquid. Because of the intimate contact of gas and liquid, the final discharge temperature can be held close to the inlet cooling water temperature. However, the discharge gas is saturated with liquid at the discharge temperature of the liquid.

The amount of liquid passed through the compressor is not critical and can be varied to obtain the desired results. The unit will not be damaged if a large quantity of liquid inadvertently enters its suction port.

Lubrication is required only in the bearings, which are generally located external to the casing. The liquid itself acts as a lubricant, sealing medium, and coolant for the stuffing boxes.

Reciprocating Compressors

Reciprocating compressors are utilized for low capacity and high pressure applications.



Reciprocating compressors are widely used by industry and are offered in a wide range of sizes and types. They vary from units requiring less than 1 HP to more than 12,000 HP. Pressure capabilities range from low vacuums at intake to special compressors capable of 60,000 psig or higher.

Reciprocating compressors are classified as constant volume, variable-pressure machines. They are the most efficient type of compressor and can be used for partial load, or reduced-capacity, applications. Because of the reciprocating pistons and unbalanced rotating parts, the unit tends to shake. Therefore, it is necessary to provide a mounting that stabilizes the installation. The extent of this requirement depends on the type and size of the compressor.

Because reciprocating compressors should be supplied with clean gas, inlet filters are recommended in all applications. They cannot satisfactorily handle liquids entrained in the gas, although vapors are no problem if condensation within the cylinders does not take place. Liquids will destroy the lubrication and cause excessive wear.

Reciprocating compressors deliver a pulsating flow of gas that can damage downstream equipment or machinery. This is sometimes a disadvantage, but pulsation dampers can be used to alleviate the problem.

The Operation of a Reciprocating Compressor

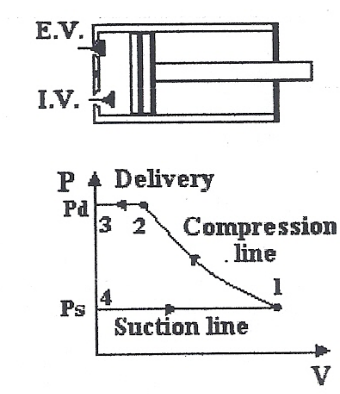


Fig 1: Without clearance

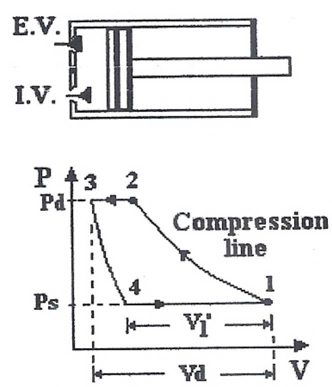


Fig 2. With clearance

In Figure 1 above shows a conventional indicator card for a compressor without a clearance. As the piston starts the stroke 4 – 1, the inlet valve opens and the gas is drawn into the cylinder along the line 4 – 1. At point 1, the piston starts the return stroke all valves being closed and the gas is compressed along the curve 1 – 2. At point 2, the discharge valve opens and the compresses gas is delivered to the receiver.

The events of the diagram with clearance (Figure 2) are the same as those with no clearance, except that since the piston does not force all the gas from the cylinder at the pressure $\mathbf{P_2}$, the remaining gas must re-expand to the intake pressure, process 3 – 4, before intake starts again. Without clearance, the volume of gas taken into the cylinder is equal to the displacement. The volume of gas drawn into the cylinder is $\mathbf{V_1 - V_4 = V'}$ and less than the displacement volume $\mathbf{V_D}$.

Definitions

Adiabatic compression occurs when no heat is transferred to or from the air during the compression process.

Aftercooling refers to the cooling of air after it has been compressed to lower its temperature and to precipitate the condensed vapors.

Boyle's Law states that the absolute pressure of fixed mass of gas varies inversely as the volume, provided the temperature remains constant.

Charles' Law states that the volume of a fixed mass of gas varies directly with the absolute temperature, provided that the pressure remains constant.

Clearance refers to the volume of a reciprocating compressor's cylinder not swept by the movement of the piston. It includes the space between the piston and the head at the end of the compression stroke, and typically expressed as a percentage of the cylinder displacement. The clearance may be different at the two ends of a double-acting cylinder.

Compression efficiency refers to the ratio of the theoretical work required (in a given process) to the actual required to be done to compress and deliver the air. Expressed as a percentage, compression efficiency accounts for fluid-friction losses, leakage, and thermodynamic variations from the theoretical process.

Compression ratio refers to the ratio of the absolute discharge pressure to the absolute suction or intake pressure.

Density is the mass of a unit volume of air, usually taken at standard temperature and pressure.

Displacement refers to the net volume swept by the moving parts of a compressor per unit of time. This term applies only to positive displacement compressors.

Free air is the air at atmospheric conditions at a specific location.

Free air delivery refers to the quantity of compressed air delivered by the compressor at ambient pressure.

Ideal gases follow the perfect gas laws without deviation. However, there are no ideal gases, but they provide a common starting point for calculations and corrections.

Intercooling is the process of cooling air between stages of compression to liquefy condensed vapors and to save power by reducing the temperature of air entering the next stage.

Isothermal compression occurs when the temperature of the air remains constant during compression.

Load factor refers to the ratio of the actual air consumption in plant to the maximum continuous air consumption.

Mechanical efficiency is the ratio of the indicated horsepower to the actual shaft horsepower.

Polytropic compression occurs when heat is transferred to or from the air at a precise rate during compression so that PV^n is constant.

Pumping or ***Surge*** is the reversal of flow within a dynamic compressor. It takes place when insufficient pressure is generated to maintain the flow.

Specific power consumption of a compressor is the ration of the actual power consumption to the quantity of free air delivered.

Specific weight is the weight of a unit volume of air.

Standard pressure and ***temperature*** is generally defined as 68°F (20°C) and 14.7 psia (101.325 kPa absolute), respectively.

Volumetric efficiency is the ratio of the actual volume of air admitted (at a specified temperature and pressure) to the full piston displacement volume. For reciprocating compressors only.

Thermodynamics of Compression

Notations:

A	=	piston area
c	=	clearance ratio
c_n	=	polytropic specific heat
c_p	=	constant pressure specific heat
c_v	=	constant volume specific heat
D	=	bore or diameter
d	=	diameter of piston rod
e	=	eccentricity
g_c	=	gravitational constant
h	=	enthalpy per unit mass
H	=	total enthalpy
k	=	ratio of specific heats
KE	=	kinetic enery
L	=	stroke
m	=	mass
m'	=	actual mass of gas drawn in and delivered
n	=	polytropic exponent
n_v	=	voumetric efficiency
N	=	rotational speed
P	=	pressure
P_D	=	discharge pressure
P_i	=	intercooler pressure
P_x	=	intermediate pressure between 1st and 2nd stages

P_y = intermediate pressure between 2nd and 3rd stages
P_m = mean effective pressure
P_S = suction pressure
PE = potential energy
Q = heat flow
Q_R = heat rejected
R = compression ratio
R = gas constant
t = temperature
T = absolute temperature
v = velocity
V = volume
V' = actual volume of gas delivered corresponding to m'
V_D = volume displacement
W_b = brake work of compressor
W_f = frictional horsepower
W_i = indicated work of compression
W_K = work

subscripts:

1 = refers to the initial condition
2 = refers to the final condition

Thermodynamics of Compression

Most compressors are analyzed using the ideal-gas law and assuming constant specific heat. Real-gas deviations are handled by applying a compressibility factor (also called ***super compressibility***). The ideal-gas law will give satisfactory results for non-hydrocarbon gases for pressures to approximately 1,000 psig (6,900 kPa) at normal temperatures. Most hydrocarbon gases and refrigerants deviate strongly from the ideal-gas laws even at moderate pressures.

The equation of state for an ideal gas is:

$$P_1 V_1 = m_1 R T_1$$ and *eq'n*

$$P_1 V_1' = m_1' R T_1$$ *eq'n 1a*

Properties of Air:

English Units:

k = 1.4

c_P = 0.24 Btu/lb-°R

c_V = 0.1714 Btu/lb-°R

R = 53.342 ft-lb/lb-°R

SI Units:

k = 1.4

c_P = 1.0062 kJ/kg-K

c_V = 0.7186 kJ/kg-K

R = 0.28708 kJ/kg-K

Volume Displacement, V_D

For single-acting reciprocating compressors, the piston displacement is:

$$V_D = \left(\frac{\pi}{4}\right) D^2 \ln \qquad \textit{eq'n 2}$$

For double-acting reciprocating compressors, the piston displacement (neglecting the piston rod diameter) is:

$$V_D = 2\left[\left(\frac{\pi}{4}\right) D^2 \ln\right] \qquad \textit{eq'n}$$

while considering the piston rod diameter, the piston displacement is

$$V_D = \left(\frac{\pi}{4}\right) D^2 \ln + \left(\frac{\pi}{4}\right)\left(D^2 - d^2\right) \ln \qquad \textit{eq'n}$$

Isothermal Compression, ∆*H* = 0

Isothermal compression would occur if the air temperature were kept constant as the pressure increases:

$$P_1 V_1 = P_2 V_2 \qquad \textit{eq'n 3}$$

and the work is

$$W_K = Q = P_1 V_1' \ln\left(\frac{P_1}{P_2}\right) = P_1 V_1' \ln\left(\frac{V_2}{V_1}\right) = m' R T_1 \ln\left(\frac{P_1}{P_2}\right) \qquad \textit{eq'n 4}$$

Adiabatic Compression

Adiabatic compression would occur if there is no heat transferred to or from the outside environment during the compression process. However, in actual practice, true adiabatic compression cannot be attained. The adiabatic equation is:

eq'n 5

$$P_1 V_1^k = P_2 V_2^k$$

where: $k = \frac{c_p}{c_v}$. For dry air, *k* = 1.4.

The first law of thermodynamics for a steady-flow process is (per unit mass):

eq'n

$$Q = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2g} + W$$

or

$$Q = \Delta PE + \Delta KE + \Delta H + W_K \quad \text{or} \quad W_K = Q - \Delta PE - \Delta KE - \Delta H$$ *eq'n 6a*

Thus, if Δ*PE* = 0 and Δ*KE* = 0 equation *6a* becomes

$$W_K = Q - \Delta H$$ *eq'n*

Also

$$-\int vdp = \Delta PE + \Delta KE + W_K \quad \text{or} \quad W_K = -\int vdp - \Delta PE - \Delta KE$$ *eq'n*

Neglecting the kinetic energy of the gas and assuming constant specific heat, for a reversible adiabatic process and substituting $\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ for $\frac{T_2}{T_1}$ and $\frac{k}{k-1}$ for $\frac{c_P}{R}$, equation *6b* becomes

eq'n 7

$$W = -\Delta H = -m c_p (T_2 - T_1) = -m' c_p (T_2 - T_1)$$

For cycle analysis: (from equation *1a*)

$$m' = \frac{P_1 V_1'}{R T_1}$$

and

$$c_p = \frac{kR}{k-1}, \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$ *eq'n 8*

Substituting in and simplifying equation *7* :

$$W_k = \frac{k m' R T_1}{1-k}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right] = \frac{k P_1 V_1'}{1-k}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$ *eq'n*

The adiabatic compression efficiency can be determined from

$$adiabatic\ compression\ efficiency = \frac{isentropic\ work}{actual\ fluid\ work}$$ *eq'n*

The ideal indicated power, W_i , can also be calculated from

eq'n

$W_i = P_m V_D$

Polytropic Compression ($\Delta KE = 0$)

$$W_k = \frac{nm' RT_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{n P_1 V_1'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \qquad \textit{eq'n}$$

Heat Rejected During Compression 1-2: Q_r

$$Q_r = m_1 c_n (T_2 - T_1) \qquad \textit{eq'n 13}$$

The polytropic specific heat can be calculated from

$$c_n = c_v\left[\frac{k-n}{1-n}\right] \qquad \textit{eq'n 14}$$

Volumetric Efficiency, η_v

$$\eta_V = \frac{V_1'}{V_D} = \frac{V_1 - V_4}{V_D} \qquad \textit{eq'n}$$

The volumetric efficiency can also be calculated using the clearance ratio

$$\eta_V = 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \qquad \textit{eq'n}$$

And the clearance ratio is $\frac{V_3}{V_D}$. It is also noted that for isentropic process, use the value *k* of the particular gas.

Mechanical efficiency:

$$\eta_m = \frac{W_i}{W_b} = \frac{\textit{indicated work of compression}}{\textit{brake work of compressor}} \qquad \textit{eq'n}$$

Compression efficiency:

- Ideal work of compression will depend on what process is involved.
- Adiabatic compression efficiency, η_C, is the compression efficiency usually used.

eq'n

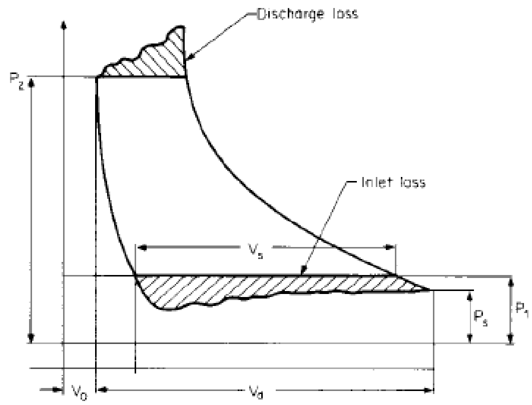
$$\eta_c = \frac{\textit{ideal work of compression}}{\textit{indicated work of compressor}}$$

Over-all compression efficiency, η_O :

$$\eta_O = \eta_m \eta_c = \frac{\textit{ideal work of compression}}{\textit{brake work of compressor}} \qquad \textit{eq'n}$$

Power Losses

The actual power is increased due to losses through the intake and discharge valves or ports as can be seen on the indicator diagram.



Additional losses are caused by turbulence within the compression chamber, preheating of the inlet gas, and leakage from the compression chamber. The isentropic compression efficiency (ratio of isentropic work to actual compression work) can be determined from the indicator diagram or calculated from the isentropic efficiency (ratio of isentropic work to actual work)

$$\eta_c = T_1 \left[\frac{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1}{T_2 - T_1} \right] \qquad \textit{eq'n}$$

provided that there is no liquid injection. In addition to the thermodynamic losses, the mechanical losses must be added to the power requirement.

Drivers

Electric motors are the most widely used compressor drivers for industrial facilities. Steam and other natural gas engines are the other possibilities with limited applications. Internal combustion engines are usually utilized for portable or mobile compressors. In almost every case, the compressor manufacturer provides the driver and compressor as a single assembly according to these general technical considerations: voltage and frequency requirements; any current restrictions particularly, kVA inrush during motor starting to compressor speed, to match torque requirements for starting and running power factor considerations; cycling considerations; proper protective enclosure; ambient temperature range; desired efficiency; and anticipated service factor. Economic considerations involving the motor drive are motor cost, maintenance cost, operating cost, and the most important – an analysis of accessory equipment needed to operate the motor. Along with the starter and motor controls, these include any special transformer requirements and cost additional power lines.

Example Problems:

1. A compressor receives 6 m^3/min of a gas (R = 410 J/kg-K, c_P= 1.03 kJ/kg-K and k= 1.67) at 105 kPa, 27°C and delivers it at 630 kPa; Δ*PE* = 0, Δ*KE* = 0. Find the work if the process is (a) isentropic (b) polytropic with *n* = 1.4 (c) isothermal.

Solution:

We'll use the steady flow energy equation; $Wk=Q-\Delta PE-\Delta KE-\Delta H$, and the students check with

$$vdp-\text{¿}\,\Delta PE-\Delta KE$$
$$Wk=\int \text{¿}$$

a. Isentropic work, Q=0

$$V_1^{'}=6\,m^3/min$$

$$m^{'}=\frac{P_1 V_1{}'}{R\,T_1}=\frac{105(6)}{(0.410)(300\,K)}=5.12\frac{kg}{min}$$

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}=300\left(\frac{630}{105}\right)^{\frac{1.67-1}{1.67}}=615.6\,K\,,then$$

$$Wk=-\Delta H=-m^{'}c_p\left(T_2-T_1\right)=-1{,}664\,kJ/min\text{¿}27.73\,kW$$

b. Polytropic work

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}=300\left(\frac{630}{105}\right)^{\frac{1.4-1}{1.4}}=500.5\,K$$

$$c_v=\frac{C_p}{k}=\frac{1.03}{1.67}=0.617\frac{kJ}{kg-K}$$

$$1-1.4$$
$$\frac{(1.67-1.4)}{\text{¿}}$$

then

$$c_n=c_v\left[\frac{k-n}{1-n}\right]=0.617\,[\text{¿}]=-0.416\frac{kJ}{kg-K}$$

$$c$$
$$(\text{¿}\text{¿}n-c_p)(T_2-T_1)$$
$$Wk=Q-\Delta H=m^{'}c_n\left(T_2-T_1\right)-m^{'}c_p\left(T_2-T_1\right)=m^{'}\text{¿}$$

$$Wk=5.12(-0.416-1.03)(500.5-300)=-1484\,kJ/min\text{¿}24.73\,kW$$

c. Isothermal work: $T_2 = T_1$ and $\Delta H = 0$

$$Wk = P_1 V_1^{'} \ln\left(\frac{P_1}{P_2}\right) = (105)(6)\ln\left(\frac{105}{630}\right) = -1129\,\frac{kJ}{min} \vee 18.82\,kW$$

2. A centrifugal compressor handles 300 CFM of air at 14.7 psia and 80°F. The air is compressed to 30 psia. The initial speed is 35 ft/s and the final speed is 170 ft/s. If the compression is polytropic, with *n* = 1.32, what is the work?

Solution:

$$m^{'} = \frac{P_1 V_1}{R T_1} = \frac{(14.7)(144)(300)}{(53.34)(540)} = 22.05\,\frac{lb}{min}$$

$$¿\,540\left(\frac{30}{14.7}\right)^{\frac{1.32-1}{1.32}} = 641.9\,R$$

$$c_n = c_v \frac{k-n}{1-n} = 0.1714\left(\frac{1.4-1.32}{1-1.32}\right) = -0.0429\,\frac{Btu}{lb-R}$$

$$\Delta H = m^{'} c_p (T_2 - T_1) = (22.05)(0.24)(22.05)(540) = 539.3\,\frac{Btu}{min}$$

$$Q = m^{'} c_n (T_2 - T_1) = (22.05 - 0.0429)(641.9 - 540) = -96.4\,\frac{Btu}{min}$$

$$\Delta KE = \frac{1}{2} m^{'} \left(v_2^2 - v_1^2\right) = \frac{1}{2}\left(\frac{22.05}{32.2}\right)\left[(170)^2 - (35)^2\right]\left(\frac{1}{778}\right) = 12.2\,\frac{Btu}{min}$$

$$Wk = Q - \Delta H - \Delta KE = -96.4 - 539.3 - 12.2 = -647.9\,\frac{Btu}{min} \vee 15.28\,hp$$

3. A single-acting air compressor operated at 150 RPM with initial condition of air at 97.9 kPa and 27 C and discharges the air at 379 kPa to a cylindrical tank. The bore and stroke are 355 mm and 381 mm, respectively, with a percentage clearance of 5%. If the surrounding air at 100 kPa and 20°C while the compression and expansion processes are $PV^{1.3}$ = C. Determine (a) free air in m^3/s, and (b) power needed of the compressor, kW. (**ME Board Exam: Oct. 1986)**

Solution:

a. $$\eta_V = 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.05 - 0.05\left(\frac{379}{97.9}\right)^{\frac{1}{1.3}} = 0.9094$$

$$V_D = V_D = \left(\frac{\pi}{4}\right) D^2 \ln = \left(\frac{\pi}{4}\right) 0.355^2 (0.381)(150) = 5.657\, m^3/min$$

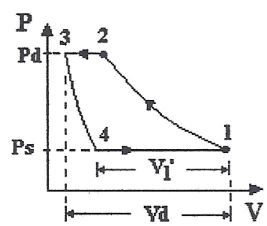
$$\eta_V = \frac{V_1{}'}{V_D}; 0.9094 = \frac{V_1^{'}}{5.657}; V_1^{'} = 5.144\, m^3/min$$

$$V_o = V_1^{'}\left(\frac{P_1}{P_0}\right)\left(\frac{T_o}{T_1}\right) = 5.144\left(\frac{97.9}{100}\right)\left(\frac{293}{300}\right) = 4.198\, m^3/min \vee 0.082\, m^3/s$$

b. For the cycle:

$$W_k = \frac{n P_1 V_1{}'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{(1.3)(97.9)(5.144)}{1-1.3}\left[\left(\frac{379}{97.9}\right)^{\frac{1.3-1}{1.3}} - 1\right] = -800.3\frac{kJ}{min}$$

4. There are compressed 8.48 kg/min of oxygen by a 35.56 cm x 35.56 cm, double-acting motor-driven compressor operating at 100 RPM. These data apply: P_1= 101.325 kPa; t_1=26.7C; and P_2= 310.27 kPa. Compression and expansion are polytropic with n = 1.31. Determine (*a*) the conventional volumetric efficiency, (*b*) heat rejected, (*c*) the work, and (*d*) kW input by the driving motor for an over-all adiabatic efficiency of 70%.



a. $$V_D = \left(\frac{\pi}{4}\right) D^2 \ln = \left(\frac{\pi}{4}\right)(0.2556)^2 (0.3556)(100)(2) = 7.063\, m^3/min$$

$$V_1^{'} = \frac{m^{'} R T_1}{P_1} = \frac{(8.48)(0.2599)(299.7)}{101.35} = 6.517\, m^3/min$$

$$\eta_V = \frac{V_1{}'}{V_D} = \frac{6.517}{7.063} = 0.9227 \vee 92.27\%$$

b. $$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 299.7\left(\frac{310.27}{101.325}\right)^{\frac{1.31-1}{1.31}} = 390.5\, K$$

$$c_n = c_v \frac{k-n}{1-n} = 0.6595\left(\frac{1.395-1.31}{1-1.31}\right) = -0.1808\frac{kJ}{kg-K}$$

$$\eta_V = 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$0.9227=1+c-c\left(\frac{310.27}{101.325}\right)^{\frac{1}{1.31}};\ c=0.0573\vee 5.73\%$$

$$V_1=V_D(1+c)=7.063(1.00573)=7.468\ \frac{m^3}{min}$$

$$m_1=\frac{P_1V_1}{RT_1}=\frac{(101.325)(7.468)}{(0.2599)(299.7)}=9.717\ \frac{kg}{min}$$

$$Q_{1-2}=m^{'}C_n(T_2-T_1)=(9.717)(-0.1801)(390.5-299.7)=-159.5\ \frac{kJ}{min}$$

c. for the cycle

$$W_k=\frac{nP_1V_1'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]=\frac{(1.31)(8.48)(0.2599)(299.7)}{1-1.31}\left[\left(\frac{310.27}{101.325}\right)^{\frac{1.3-1}{1.3}}-1\right]=-846.1\ \frac{kJ}{min}$$

d. Isentropic process with k=1.395, then W_k = - 869.5 kJ/min or 14.49 kW

$$W_B=\frac{W_k}{\eta_o}=\frac{14.49}{0.70}=20.7\ kW$$

Multi-Stage Compression

Temperature rise and mechanical stresses limit the maximum pressure differential across a single stage of any compressor type. The pressure rise across a dynamic compressor stage is further limited by the available polytropic head which the stage can develop. The volumetric efficiency of a reciprocating compressor decreases with increasing pressure ratio placing a practical limit on the maximum ratio per stage.

Multi-staging is simply the compression of gas in two or more cylinders in place of a single cylinder compressor. It is used in the reciprocating compression in order to (1) save power (2) limit the gas discharge temperature (3) limit the pressure differential per cylinder.

In multi-stage compression, the ideal conditions are:

1. no pressure drop in the intercooler
2. there is a perfect intercooling
3. work in all stages are equal

Two-Stage Compression

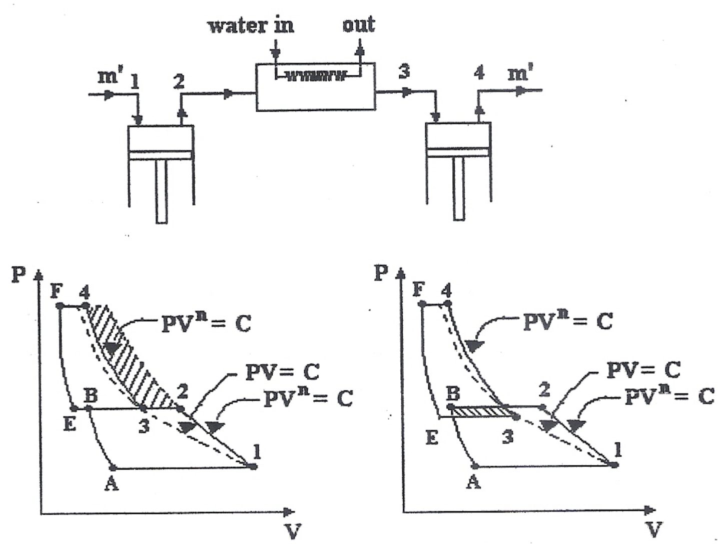


Fig 1: **Without pressure drop** Fig 2: **With pressure drop**

The figures above show the events of the conventional cards of a two-stage machine, with the high-pressure (HP) super-imposed on the low-pressure (LP). Suction in the LP cylinder begins at A and the volume $V_1^{'}$ is drawn in. Compression 1 – 2 occurs and the gas is discharged along 2 – B. The discharged gas passes through the intercooler and is cooled by circulating water through the intercooler tubes. Conventionally, it is assumed that the gas leaving the intercooler and entering the HP cylinder has the same temperature as it had upon entering the LP cylinder ($T_1 = T_3$). The gas is then drawn into the HP cylinder along 3 – 4 and finally discharged from the compressor unit along 4 – F. The residual gas always remains in ach cylinder because of the clearance and must re-expand F – E (for HP cylinder) and B – A (for LP cylinder).

Work of Compression:

$$Wk = Wk\,of\;LP\,cylinder + Wk\,of\;HP\,cylinder \qquad \textit{eq'n 21}$$

$$W_k = \frac{n m' R T_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n m' R T_3}{1-n}\left[\left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}} - 1\right] \qquad \textit{eq'n 22}$$

But it is a common practice to adjust the operation of multi-stage compressors so that approximately equal works are done in the cylinders, a practice that results in minimum total work.

Then for the particular case of $T_1 = T_3$ and pressure $P_2 = P_3 = P_i$, we have the work of LP stage equal to that of the HP stage, or

$$\frac{n m' R T_1}{1-n}\left[\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{n m' R T_3}{1-n}\left[\left(\frac{P_4}{P_i}\right)^{\frac{n-1}{n}} - 1\right] \quad \textit{eq'n 23}$$

where: $P_i = \sqrt{P_1 P_4} = \sqrt{P_D P_S}$

Since the work of each cylinder is the same, the total work is:

$$W_k = \frac{2 n m' R T_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \vee W_k = \frac{2 n m' R T_1}{1-n}\left[\left(\frac{P_4}{P_1}\right)^{\frac{n-1}{2n}} - 1\right] \quad \textit{eq'n 24}$$

A pressure drop, ΔP, in the intercooler could be spread on each side of this ideal value. Then

$$P_2 = P_1 + \frac{1}{2}\Delta P \quad \textit{eq'n 25}$$

also

$$P_3 = P_1 - \frac{1}{2}\Delta P \quad \textit{eq'n}$$

Heat transferred in the intercooler:

$$\int ¿ = m' c_p (T_3 - T_2) \quad \textit{eq'n 26}$$
$$Q_¿$$

Three-stage Compression

The ratio of the pressures is given by

$$\frac{P_x}{P_1} = \frac{P_y}{P_x} = \frac{P_2}{P_y} \quad \textit{eq'n 27}$$

or

$$P_x = \sqrt[3]{P_1{}^2 P_2} \quad \textit{eq'n}$$

Also

$$P_y = \sqrt[3]{P_1 P_2{}^2} \quad \textit{eq'n}$$

The total work is:

eq'n 28

$$W_k=\frac{3nm' RT_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{3n}}-1\right]$$

The heat rejected at each intercooler is

$$Q_r=m' c_p(\Delta T)$$ *eq'n 29*

Summary of Multi-Stage Compression:

Number of Stages	P_x = interstage pressure after first stage	Compressor Work (Power)
2	$P_x=\sqrt{P_1 P_2}$	$W=\frac{2nP_1V_1'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{2n}}-1\right]$
3	$P_x=\sqrt[3]{P_1^2 P_2}$	$W=\frac{3nP_1V_1'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{3n}}-1\right]$
General Formula:		
s	$P_x=\sqrt[s]{P_1^{s-1} P_2}$	$W=\frac{snP_1V_1'}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{sn}}-1\right]$

Example Problem:

1. A two-stage, double-acting compressor operating at 150 RPM takes in air at 14 psia, 80˚F. The LP cylinder is 14 x 15 in, the stroke of HP is 15 in, and the clearance of both cylinders is 4%. Air is discharged at 56 psia from the LP cylinder, passes through the intercooler, and enters the HP cylinder at 53.75 psia, 80˚F; it leaves the HP cylinder at 215 psia. The polytropic exponent *n* = 1.3 for the both cylinders. Neglect the effect of piston rods on the crank end. Environmental atmospheric conditions are 14.7 psia 70˚F. Find (a) volume of free air; (b) heat transferred during compression to cooling water; (c) heat rejected during intercooling; (d) diameter of HP cylinder; and (e) work required for the compressor.

SOLUTION:

a. For LP cylinder, the displacement is:

$$V_D=\left(\frac{\pi}{4}\right)D^2\ln=\left(\frac{\pi}{4}\right)(14)^2(15)(2)\left(\frac{1}{1728}\right)=400.88\,CFM$$

$$V_1{}'=V_D\left[1+c-c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]=400.88\left[1+0.04-0.04\left(\frac{56}{14}\right)^{\frac{1}{1.3}}\right]$$

¿ $370.3357\,cfm$

$$V_o=V_1^{'}\left(\frac{P_1}{P_0}\right)\left(\frac{T_o}{T_1}\right)=(370)\left(\frac{14}{14.7}\right)\left(\frac{530}{540}\right)=346.1692\,cfm$$

b. Heat transferred during compression to cooling water

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}=540(14)^{\frac{1.3-1}{1.3}}=744\,°R$$

$$V_1=V_D(1+c)=401(1+0.04)=417\,cfm$$

$$m_1=\frac{P_1V_1}{RT_1}=\frac{(14)(144)(417)}{(53.3)(540)}=29.2\,\frac{lb}{min}$$

$$Q_{1-2}=m_1c_v\left[\frac{k-n}{1-n}\right](T_2-T_1)=-341\,\frac{Btu}{min}$$

c. Heat rejected in the intercooler corresponds to the volume V_1' = 370 cfm

($m_{1'}$ enters the intercooler excluding the clearance volume from the LP cylinder)

$$m'=\frac{P_1V_1{}'}{RT_1}=\frac{(14)(144)(370)}{(53.3)(540)}=25.9\,\frac{lb}{min}$$

$$Q_{2-3}=\Delta H_{2-3}=m'c_P(T_2-T_1)=-1268\,\frac{Btu}{min}$$

d. Calculating the diameter of the high pressure cylinder for m_1'= 25.9 lb/min, then the volume

$$V_3^{'}=\frac{m^{'}RT}{P_3}=\frac{(25.9)(53.3)(540)}{(53.75)(144)}=96.3\,cfm$$

calculating the diameter:

$$V_{DHP}=\frac{V_3{}'}{\eta_V}=\frac{96.3}{0.924}=104.2\,cfm=\left(\frac{\pi}{4}\right)D^2(15)(2)\left(\frac{1}{1728}\right)$$

thus**,** $D=7.14\in$ ¿ $7\frac{1}{4}\in.$

e. Work required for the compressor: from LP cylinder, the work is,

$$W_k = \frac{n m' R T_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{(1.3)(25.9)(53.34)(540)}{(1-1.3)(33000)}\left[\left(\frac{56}{14}\right)^{\frac{1.3-1}{1.3}} - 1\right] = -37\,hp$$

Then, the total work is Wk= (2)(37) = **74 hp**

Piston Speeds:

The piston speeds may be as much as 350 ft/min for small compressors, say, with a stroke of about 6 in, to more than 700 ft/min in large compressor, say, with a stroke of about 36 in. The piston speed can be determined from

$$v = 2\ln$$ *eq'n 30*

Frictional Horsepower and Piston Displacement:

The frictional horsepower is determined by *eq'n 31*

$$W_f = 0.105\, V_D^{\,0.75}$$

Thus, the brake power is

$$W_B = W_i + W_f$$ *eq'n 32*

Air Receiver

Functions:

1. Remove the pulsating effects of the compressor discharge (reciprocating compressor)
2. Acts as a storage vessel, keeping the fluid to a substantially constant service pressure.
3. Helps to cool the air and thus condense part of its moisture and removes excess oil.

General Considerations:

1. Each compressor should have its own receiver.
2. Receiver should be placed outdoors for most effective cooling and must be equipped with safety valves, pressure gages, and blow-down valves.
3. No shut-off valve should be installed between the compressor and the receiver.
4. Sectionalizing of the system, for maintenance and operation, should be effected by valve beyond the receiver.

Capacity of the Air Receiver:

The air receiver should be large enough to hold all the air delivered by the compressors in one minute. This may be calculated as follows:

$$Capacity\left(ft^3\right)=\left(Volume\ Displacement\ ft^3/min\right)\left(1\ min\right)\left(\frac{P_{intake}}{P_{discharge}}\right)$$

It does no harm to have a receiver larger than above calculated capacity and in some cases, a large capacity is necessary. This is true where occasional momentary sir requirements are greater than the capacity of the compressor. In such instances, the air receiver should have enough capacity to provide all the air required.

Review Problems:

1. A single-acting air compressor with a clearance of 6% takes air at atmospheric pressure and a temperature of 85°F and discharges it at a pressure of 85 psia. The air handled is 0.25 ft^3/cycle measures at discharge pressure. If the compression is isentropic, find (a) piston displacement per cycle (b) air horsepower if RPM is 750. (ME Board: March 1978)

2. An industrial plant has 0.472 m^3/s compressor pumping air to 1.72 MPa. Intake pressure is 101.3 kPa. Find (a) find the minimum capacity of air receiver (b) how long will it take the compressor to fill-up a 5.5 m^3 receiver from 827 kPa to 1700 kPa if the volumetric efficiency is 65%.

3. A single acting compressor has a volumetric efficiency of 87% and operates at 500 RPM. It takes air at 100 kPa and 30°C and discharges it at 600 kPa. The air handled is 6 m^3/min measured at discharge condition. If the compression is isentropic, find (*a*) piston displacement per stroke and (*b*) mean effective pressure, in kPa. (ME Board: April 1983)

4. The cylinders of a single-acting, two-stage tandem air compressor are 55 mm and 215 mm diameter, respectively, and the stroke is 230 mm for all cylinders. It is connected to three air storage bottles of equal sizes of internal dimensions 300 mm x 1.5 m long over-all with hemispherical ends. Taking atmospheric pressure as 1 bar and assuming volumetric efficiency of 88%, calculate the time required to pump up the bottles to a pressure of 31 bar gage from atmospheric conditions when running at 125 rev/min.

5. A two-stage double-acting compressor is to deliver 90 lb/min of air from 14.3 psia and 90°F to a final pressure of 185 psia. The normal barometer is 29.8 in. Hg and the temperature is 80°F. The pressure drop in the intercooler is 3 psi and the temperature of the air at the exit of the intercooler is 90°F, the speed is 210 RPM and $PV^{1.34} = C$ during compression and expansion. The clearance is 5% for both cylinders. The

temperature of the cooling water increases by 18°F. Find (*a*) volume of free air, (*b*) discharge pressure of LP cylinder for minimum work, (*c*) temperature at discharge for both cylinders, (*d*) mass of cooling water to be circulated about each cylinder and through the intercooler, (*e*) the work of compression, (*f*) if for LP cylinder, L/D = 0.68 and if both cylinders have the same stroke, what are the bore and stroke for each cylinder?

6. Methane gas is compressed in a two-stage, double acting compressor which is electrically driven at 165 RPM. The low pressure cylinder (30 cm x 35 cm) receives 6 m^3/min of gas at 96.5 kPa, 43°C and the high-pressure cylinder (20 cm x 35 cm) discharges the gas at 717 kPa. The isothermal over-all efficiency is 75% , Find (*a*) volumetric efficiency and (*b*) the kW output of the motor.

7. A two-stage compressor with first stage piston displacement of 94,390 cm^3/s is driven by a motor. Motor output is 35 HP, suction temperature is 22°C, volumetric efficiency is 85%. Mechanical efficiency is 95 %, intercooler pressure is 30 psig. Air temperature in and out the intercooler is 105°C and 44°C, respectively. Final discharge temperature is 100 psig, suction estimate 14.5 psia. Find the compression efficiency. (ME Board: April 1991)

8. A two cylinder, single-acting air compressor is directly coupled to an electric motor running at 1000 RPM. Other data are as follows:
   Size of each cylinder : 150 mm x 200 mm
   Air molecular mass : 29
   Clearance volume : 10% of displacement
   $PV^{1.6} = C$, both compression and re-expansion
   Calculate (*a*) the volume rate of air delivery in terms of standard air for a delivery pressure of 8 times the ambient pressure under ambient conditions of 300 K and 1 bar and (*b*) shaft power required if the mechanical efficiency 81%. (ME Board: April 1984)

9. A two-stage compressor operated between constant pressure limits 98.6 kPa and 1.03 MPa. The swept volume of the low pressure piston is 0.142 m^3. Due to failure of the cooling water supply to the intercooler, air is passed to the high pressure cylinder without reduction in temperature. Using $PV^{1.2} = C$, determine the percentage increase in power. (ME Board: April 1998)

10. An ideal single air compressor without clearance takes in air at 100 kPa with a temperature of 16°C and delivered it at 413 kPa after isentropic compression. What is the discharge work done in kJ/kg? (ME Board: April 1998)

11. A single air compressor handles 0.454 m^3/s of atmospheric pressure, 27°C and delivers it to a receiver at 652.75 kPa. Its volumetric efficiency

on an isothermal basis is 0.85 and its mechanical efficiency is 0.90. If it operates at 350 RPM, what power in kW is required to drive it. (ME Board: April, 1995)

12. A single-acting air compressor operates at 450 RPM with an initial condition of air at 97.9 kPa and 27°C and discharges the air at 379 kPa to a cylindrical tank. The bore and stroke are 355 mm and 381 mm, respectively, with 5% clearance. If the surrounding air is at 100 kPa and 20°C while the compression and re-expansion processes are $pV^{1.3} = C$, determine

   a. free air capacity, m^3/s

   b. power of the compressor

13. A single-acting, single-cylinder air compressor is rated at 4.25 m^3/min of air. The suction conditions are 1 atm and 27°C and discharge pressure is 1034 kPa. The compression process follows the equation $pV^{1.35} = C$.

   a. Determine the power, in kW, required to compress the air.

   b. What is the engine kW power needed to drive the unit if the combined engine-compressor efficiency is 84%?

   c. If the compressor is to run two-stage at optimum intercooler pressure with perfect intercooling, what will be percentage of power saved?

14. A reciprocating two-stage air compressor takes in air at atmospheric pressure and 27°C. The flash point of the oil used in the air cylinder is 260°C. Safety precautions limit the temperature of the air in the high pressure cylinder to be 28°C below the flash point of the oil. Assuming perfect intercooling, and no pressure drop through the intercooler, what would be the allowable working pressure of this compressor if the compression curve follows the equation $pV^{1.34} = C$?

15. A two-cylinder single-acting air compressor is directly coupled to an electric motor running at 1000 RPM. Other data are as follows:

Size of each cylinder	: 150 mm x 200 mm
Clearance volume	: 10% of displacement
Polytropic exponent, *n*	: 1.36
Air molecular mass	: 29

Calculate the following:

   a. The volume rate of air delivery in terms of standard air for a delivery pressure 8 times ambient pressure under ambient conditions of 300°K and 1 bar.
   b. Shaft power required if mechanical efficiency is 81%

16. A two-stage reciprocating single-acting air compressor has a rated capacity of 80 m^3/hr of free air at 27°C and 1.033 kg/cm^2 abs when running at 600 RPM. The absolute discharge pressure is 30 kg/cm^2 and the air is discharged to an air receiver of 1,250 liters capacity. The compressor has two low-pressure cylinders each 127 mm diameter and one high-pressure cylinder of 69.85 mm diameter, piston stroke is 101.6 mm. The compressor is driven by a 1750 RPM, 3-phase, 60 Hz, 460 V motor through V-belts with transmission efficiency of 95%. Determine:

   a. intercooler pressure, in kg/cm^2

   b. volumetric efficiency

   c. brake horsepower at a efficiency of 85% and polytropic exponent of 1.35

   d. kW rating of the driving motor

17. A three-stage, single-acting, diesel engine-driven reciprocating air compressor is guaranteed to deliver 170 m^3/hr free air at suction conditions of 1.03 kg/cm^2 abs and 27°C and discharge pressure of 35 kg/cm^2 abs. Test results show that the polytropic exponent for both compression and re-expansion processes is 1.34 and the mechanical efficiency of the compressor is 80%. Assuming perfect intercooling with optimum interstage pressures, determine

   a. the interstage pressures, kg/cm^2

   b. brake horsepower of the engine drive

   c. fuel consumption, in kg/hr, of the diesel engine if the brake thermal efficiency is 30% and the fuel used has a heating value of 10,700 kcal/kg

18. A two-stage air compressor with a 5% clearance delivers 90 lb/min of air at 140 psia. At the intake, p_1 = 14.3 psia and t_1 = 60°F. Compression and re-expansion processes follow $pv^{1.31} = C$, and the intercooler cools the air back to 60°F. Determine

   a. the optimum intermediate pressure, psia

   b. the conventional power, HP

   c. What horsepower would be required for isentropic compression in a single-stage compressor?

   d. Calculate the saving due to cooling process.

19. A steady flow compressor compresses 15 lb/min of air from 14.7 psia and 70°F to 5 psig and 110°F.

a. Neglecting kinetic and potential energies and if the process is polytropic with the polytropic exponent n = 1.37, calculate the compressor power, HP

b. Determine the mass of circulating water required if the temperature rise in the cooling water is 6°F.

c. The input horsepower required if the compressor efficiency is 82%.

20. A single-cylinder reciprocating compressor has a 5% clearance and a bore and stroke of 25 cm x 30 cm, respectively. The compressor operates at 500 RPM. The air enters the cylinder at 27°C and 95 kPa and discharges at 2000 kPa. Ambient air conditions are 101.3 kPa and 27° If the polytropic exponent n = 1.3 for compression and re-expansion processes, determine

a. the free air delivery, m^3/hr

b. the power required, in kW, if the compressor efficiency is 82%.