Republic of the Philippines
GILLESANIA Engineering Review and Training Center
CEBU

MATHEMATICS, SURVEYING & TRANSPO. ENG'G. **SET A**
Thursday, 22 September 2016 Module 02

SOLUTIONS

Situation 1

1	Q1	C	8.67	5.74	**79.28**	6.31
2	Q2	B	7.43	**73.54**	10.47	8.56
3	Q3	B	22.75	**52.70**	11.94	12.39

r^{th} term = ${}_nC_{r-1}\, a^{n-r+1} b^{r-1}$

1

8^{th} term = ${}_{10}C_{8-1}\,(2x)^{10-8+1}(-y)^{8-1}$
8^{th} term = $120\,(2x)^3(-y)^7$
8^{th} term = $\mathbf{-960x^3 y^7}$

2

Given: a = 23; b = -s; n = 5 and r = 4
4^{th} term = ${}_5C_{4-1}\,(2r)^{5-4+1}(-s)^{4-1}$
$4^{th}\ term\ =\ 10\,(2r)^2(-s)^{4-1}$
4^{th} term = $\mathbf{-40r^2s^3}$

3

The middle (m^{th}) term of binomial expansion with even n exponent is the median of the terms

$$m = \frac{n}{2} + 1 = \frac{10}{2} + 1 = 6$$

6^{th} term = ${}_{10}C_{6-1}\,(a)^{10-6+1}(3b)^{6-1}$
$6^{th}\ term\ = \frac{10!}{5!\,5!}10\,(a)^5(3b)^5$
6^{th} term = $\mathbf{61236}a^5b^5$

Situation 2

4	Q4	B	0.90	**97.07**	1.46	0.56
5	Q5	A	**93.58**	2.36	2.25	1.80
6	Q6	D	4.17	38.74	3.83	**53.27**

4

5

6

Situation 3

7	Q7	B	3.83	**81.31**	10.47	4.28
8	Q8	A	**88.96**	4.95	4.28	1.80
9	Q9	C	6.87	10.02	**67.34**	15.32

The n^{th} term of a geometric sequence is $a_n = a_1 r^{n-1}$

7

$a_n = a_1 r^{n-1}$
$a_8 = 12(1/2)^{8-1}$
$a_8 = 12/128$
$a_8 = 3/32$

8

r = -6/2 = -3
$a_5 = 2(-3)^{5-1}$
$a_5 = \mathbf{162}$

9

a_1 = 10 (beginning of day 1)
$a_2 = 10 \times 2 = 20$ (doubling each day)
r = 2
$a_7 = 10(2)^{7-1} = \mathbf{640}$

Situation 4

10	Q10	A	**78.49**	11.71	6.19	3.38
11	Q11	D	12.50	7.77	16.55	**63.18**
12	Q12	D	15.54	42.57	10.36	**31.42**

The sum of infinite geometric progression is $S = \frac{a_1}{1-r}$

10

$a_1 = 2$; $a_2 = 2/3$; $r = a_2/a_1$

r = (2/3)/2
r = 2/6 = 1/3

S = 2/(1 - 1/3)
S = **3** *letter A*

11

r = 0.75
$a_1 = 24$

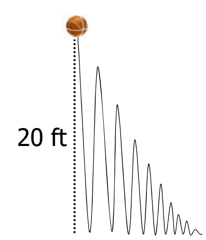
$$S = \frac{24}{1 - 0.75}$$

S = **96 inches**

12

$$S = 2\frac{20}{1 - \frac{4}{5}} - 20$$

S = **180 feet**

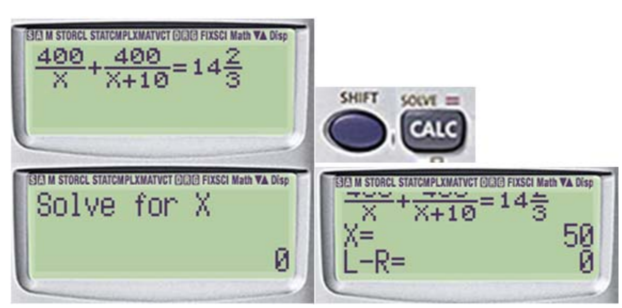


Situation 5

13	Q13	B	7.55	**67.12**	19.26	5.86
14	Q14	B	11.04	**59.57**	21.28	7.55
15	Q15	D	24.66	12.61	5.74	**56.64**

13

$$t = \frac{S}{v} \qquad \frac{400}{v} + \frac{400}{v+10} = 14\frac{2}{3}$$



14

$$t = \frac{S}{v} \qquad t_1 + t_2 = \frac{S}{v}$$

$$\frac{3}{v} + \frac{4}{v-1} = 1\frac{3}{5}$$

v = 5; v – 1 = 4

15 $1 + \frac{3}{5} = \frac{3}{v} + \frac{4}{v-1}$

$$\left(\frac{1}{t-1} + \frac{1}{t}\right) \times 4 = 1$$

$$t = \frac{\sqrt{65}}{2} + \frac{9}{2} = 8.531$$

Situation 6

16	Q16	A	**96.28**	1.24	1.35	1.13
17	Q17	A	**90.20**	1.35	7.32	1.01
18	Q18	C	3.15	6.08	**89.19**	1.46

16

$a^2 + b^2 = 20$

$a \cdot b = 8$

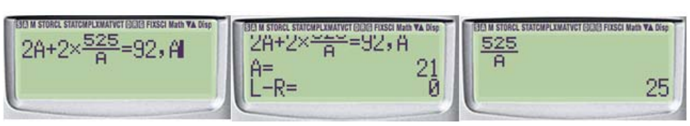
$$a^2 + \left(\frac{8}{a}\right)^2 = 20$$

17

$2a + 2b = 92$

$a \cdot b = 525$

$$2a + 2\left(\frac{525}{a}\right) = 92$$



18

$y = \sqrt{x}$

$x^2 + y^2 = 6$

$x^2 + (\sqrt{x})^2 = 6$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

x = -3 and x = 2

if x = -3

$y = \sqrt{-3}$ (not real, absurd)

if x = 2

$y = \sqrt{2}$ (real)

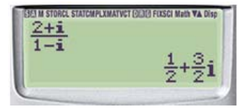
Then the solution is $(\mathbf{2}, \sqrt{\mathbf{2}})$

Situation 7

19	Q19	B	3.94	**90.32**	2.36	3.27
20	Q20	B	10.14	**72.52**	9.01	8.33
21	Q21	D	0.90	0.79	0.68	**97.64**

19

MODE 2



Letter B

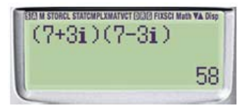
20

Complex, $i^n = i^r$, where r is the remainder of n/4

$11\,^1/_2 = 11\,^2/_4$

Remainder is 2 then $i^2 = -1$ ***(letter B)***

21



letter D

Situation 8

22	Q22	A	**95.72**	0.90	1.35	2.03
23	Q23	A	**62.39**	14.75	12.16	10.70
24	Q24	B	25.45	**44.93**	21.28	8.00

22

$$S = \frac{a + b + c}{2}$$

$$S = \frac{30 + 36 + 18}{2}$$

S = 42 cm

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{42(42-30)(42-36)(42-18)}$

A = **269.4 cm²**

23

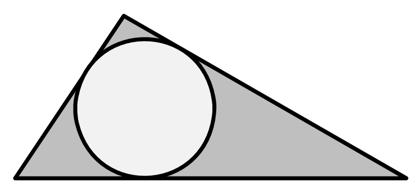
A = rs

269.4 = r(42)

$r = \frac{A}{s}$

$r = \frac{269.4}{42}$

r = **6.41 cm**



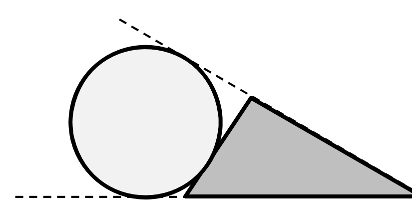
24

$A = (s-b)r_b$

$269.4 = (42 - 36)r_b$

$r_b = \frac{269.4}{42 - 36}$

r_b = 44.9 cm



Situation 9

25	Q25	A	**51.80**	16.89	17.79	13.29
26	Q26	B	6.42	**75.79**	14.19	3.49
27	Q27	D	13.51	19.03	16.10	**50.45**

25

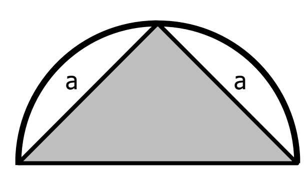
$A = \frac{1}{2}a^2$

$162 = \frac{1}{2}a^2$

a = 18 cm

$r = ½\, a\sqrt{2}$

r = **12.73 cm**



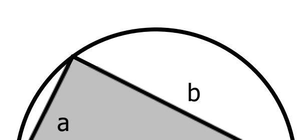
26

$A = \frac{1}{2}ab$

$216 = \frac{1}{2}ab$

ab = 432; b = 432/a

$a^2 + b^2 = 30^2$



$a^2 + \left(\frac{432}{a}\right)^2 = 900$

a = 18

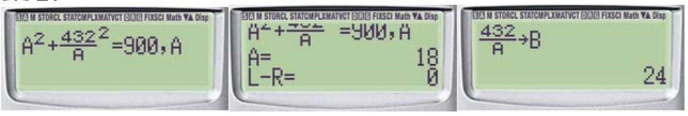
b = 24

c = 30

P = 18 + 24 + 30

P = **72 cm**

📖 **27**



A = 65 in^2

P = 48 in

Radius of inscribed circle, r = A/s

s = 48/2

s = 24

r = 65/24

r = 2.71

d = 2r = 2(2.71)

d = **5.42 inches**

📖 **Situation 10**

28	Q28	B	13.96	59.80	7.55	18.58
29	Q29	B	10.81	70.27	11.71	7.09
30	Q30	D	18.92	27.48	21.06	31.87

📖 **28**

S = (n – 2)×180

720 = (n – 2)×180

n = **6 (Hexagon)**

📖 **29**

$d = \frac{n}{2}(n+3)$

$65 = \frac{n}{2}(n+3)$

$n^2 + 3n - 130 = 0$

$(n + 13)(n - 10) = 0$

n = -13(absurd) or n = 10

n = **10**

📖 **30**

If each interior angle of a regular convex polygon measures α, then

$\alpha = \frac{n-2}{n} \times 180$

$108° = \frac{n-2}{n} \times 180$

108n = 180n – 360

n = 5

$d = \frac{n}{2}(n+3)$

$d = \frac{5}{2}(5+3)$

d = **20 diagonals**

📖 **Situation 11**

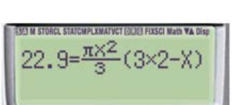
31	Q31	B	22.07	37.73	27.36	12.61
32	Q32	B	22.75	40.65	28.49	7.88
33	Q33	D	20.05	20.27	18.36	41.22

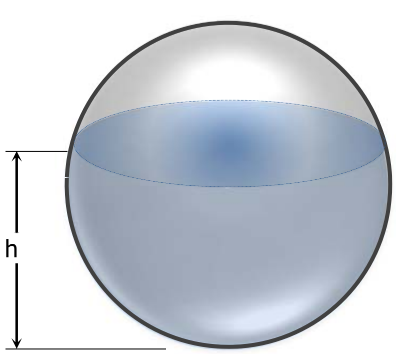
📖 **31**

$V = \frac{\pi h^2}{3}(3r - h)$

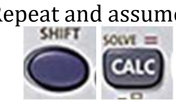
r = ½ d = ½ 4 = 2

$22.9 = \frac{\pi h^2}{3}[3(2) - h]$





5.187316696 is invalid because h must be less than the diameter. Repeat and assume value of x the same as any of the choices.

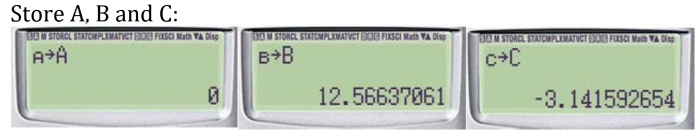


h = 2.5 m = **250 cm**

Alternate way:

MODE 3:3

Store A, B and C:



In integral:

dV = ydx

dV = $(A + Bx + Cx^2)dx$

V = $\int(A + Bx + Cx^2)dx$

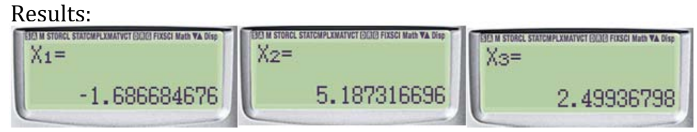
V = $Ax + Bx^2/2 + Cx^3/3$

It will form a cubic equation:

To solve for its roots, use MODE 5:4

Where a = C/3, b = B/2 and c = a; and d = -V

Results:



📖 **32**

$V = \frac{\pi h^2}{3}(3r - h)$

$V_{black} = V_{white}$

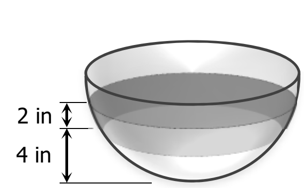
$V_{total} = V_{black} + V_{white}$

$V_{total} = 2V_{white}$

$\frac{\pi h_t^2}{3}(3r - h_t) = 2\left[\frac{\pi h_w^2}{3}(3r - h_w)\right]$

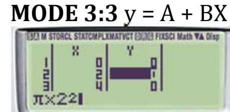
$\frac{\pi(6^2)}{3}[3r - 6] = 2\left[\frac{\pi(4^2)}{3}(3r - h_w)\right]$

r = 7.33 in



📖 **33**

MODE 3:3 y = A + BX + CX



MODE 1

📖 **Situation 12**

34	Q34	D	9.23	12.84	17.23	**60.47**
35	Q35	C	6.53	16.22	**61.71**	15.43
36	Q36	A	**69.14**	8.11	12.61	10.14

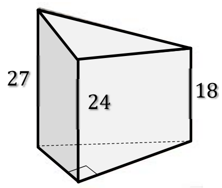
📖 **34**

The base is a right triangle with legs 30 and 40 cm, and hypotenuse 50 cm.
A = ½ 30 × 40 = 600 cm^2

Average altitude,

$$h = \frac{18 + 24 + 27}{3}$$

h = 23
V = 600 × 23 = **13800**



📖 **35**

$$h = \frac{15 + 18 + 15}{3}$$

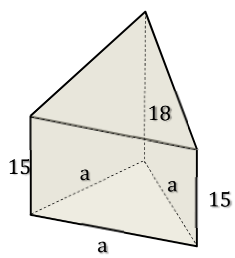
h = 16

6240 = 16A
A = 390

A = ½ a^2 sin 60°

$$.390 = \frac{\sqrt{3}}{4}a^2$$

a = **30.011**

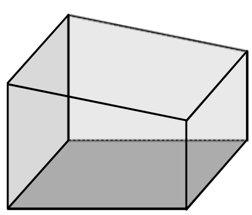


📖 **36**

$$h = \frac{10 + 10 + 7 + 7}{4}$$

h = 8.5

1000 = 8.5A
A = **117.65**



📖 **Situation 13**

37	Q37	A	**44.59**	34.57	14.75	6.08
38	Q38	B	23.76	**54.05**	12.50	9.57
39	Q39	A	**39.53**	17.34	31.19	11.82

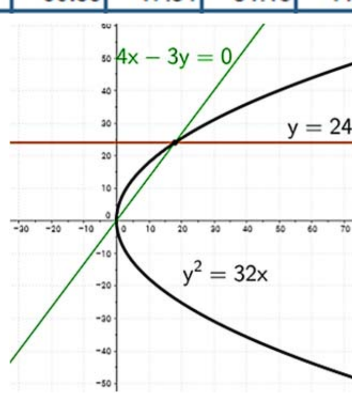
📖 **37**

Equation of the diameter
y – 24 = 0
y = 24

y^2 = 32x

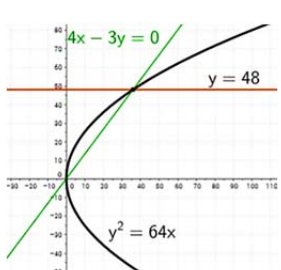
slope at any point is y'
2y y' = 32
y' = 32/2y
y' = 16/y

y' = 16/24
y' = **2/3**



📖 **38**
y^2 = 64x
2y y' = 64
y' = 32/y

m = 2/3



$$\frac{32}{y} = \frac{2}{3}$$

y = **48**

📖 **39**
64x^2 + 25y^2 = 1600
64 (2x) + 25 (2y y') = 0
64x + 25y y' = 0
64x + 25y (1/5) = 0
64x + 5y = 0

📖 **Situation 14**

40	Q40	A	**51.24**	19.03	13.85	15.77
41	Q41	B	9.68	**63.96**	19.48	6.87
42	Q42	A	**27.70**	19.03	24.44	28.49

📖 **40**

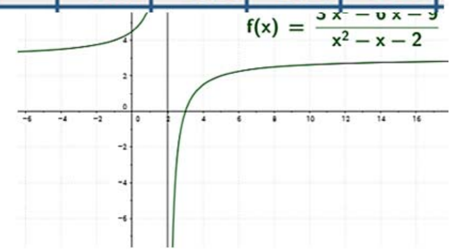
$$f(x) = \frac{3x^2 - 6x - 9}{x^2 - x - 2}$$

Simplify by factoring

$$f(x) = \frac{3(x + 1)(x - 3)}{(x + 1)(x - 2)}$$

$$f(x) = \frac{3(x - 3)}{x - 2}$$

The vertical asymptotes is **x – 2**.



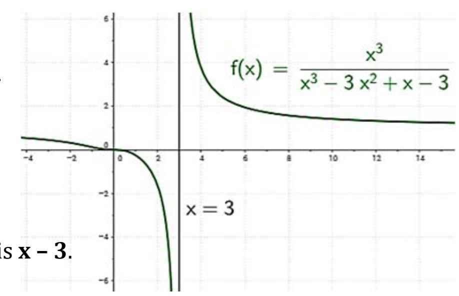
📖 **41**

$$f(x) = \frac{x^3}{x^3 - 3x^2 + x - 3}$$

Simplify by factoring

$$f(x) = \frac{x^3}{(x - 3)(x^2 + 1)}$$

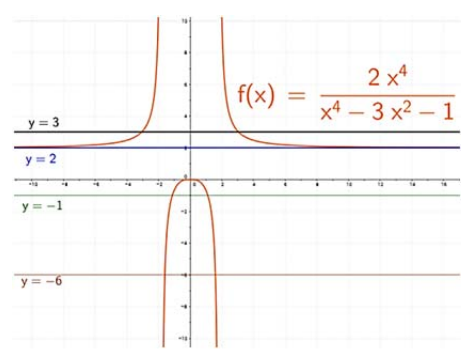
The vertical asymptotes is **x – 3**.



📖 **42**

$$y = \frac{2x^4}{x^4 - 3x^2 - 1}$$

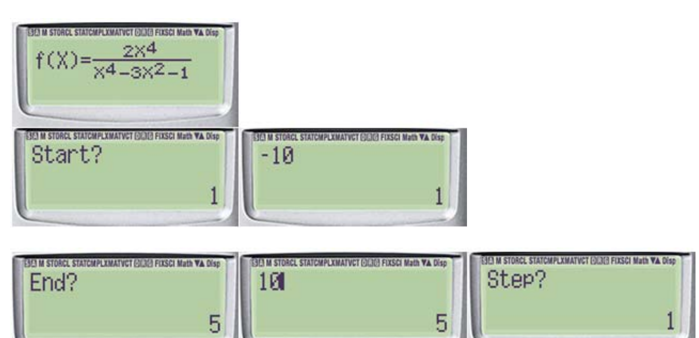
For numerator and denominator with the same degree, the function has horizontal asymptote with an equation y = k, where k is the ratio of the coefficients of highest degree terms of the numerator and denominator.

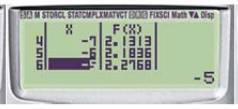


y = 2/1 = 2

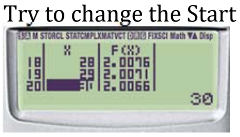
like in the previous problems, the horizontal asymptote for number 40 is y = 3/1 = 3 and for number 41 is y = 1/1 = 1

MODE 7





Try to change the Start to 11 and End to 30.



As X increases, F(x) approaches to 2, then the horizontal asymptote is **y = 2**

🕮 Situation 15

43	Q43	C	18.24	25.45	40.20	15.77
44	Q44	A	12.27	48.99	17.00	21.62
45	Q45	D	16.10	26.80	38.51	18.36

🕮 **43**

The revenue is the product of number of people attending, N, and the admission price per person, P.

R = N × P
If P = 300; N = 6000
If P = 300 – 5; N = 6000 + 200
If P = 300 – 2(5); N = 6000 + 2(200)

Let x be the number of 5-Peso deduction, then the revenue is
R = (300 – 5x)(6000 + 200x)
$R = 180000 + 30000x - 1000x^2$
To maximize R, evaluate dR/dx and equate to zero.

$$\frac{dR}{dx} = 30000 - 2(1000)x = 0$$
x = 15

Price = 300 – 5(15) = **P225**

🕮 **44**

$E \propto v^3$ (energy per hour)
$E = kv^3$

Total energy = energy per hour × total time

Total time of travel is equal to the total distance traveled over speed.

$$t = \frac{S}{v}$$
Since the boat is traveling against the current with 4 kph rate then,
$$t = \frac{S}{v-4}$$
$$E_T = E \times t = kv^3 \frac{S}{v-4}$$
$$E_T = kS\frac{v^3}{v-4}$$

Using maxima minima, $dE_T/dt = 0$
$$\frac{dE_T}{dt} = kS\frac{(v-4)(3v^2) - v^3(1)}{(v-4)^2} = 0$$
$$\frac{dE_T}{dt} = kS\frac{(v-4)(3v^2) - v^3(1)}{(v-4)^2} = 0$$

$3v^3 - 12v^2 - v^3 = 0$
$2v^3 - 12v^2 = 0$
$v^3 - 6v^2 = 0$
$v^2(v - 6) = 0$

v = 0; v = 6
then, v = **6 kph**

🕮 **45**

Cost per hour of fuel, $C_f = kv^3$
If v = 10 knots, $C_f = 20$
k = 0.02

Total distance traveled = S
Total time of travel = S/v

Total Cost,

$C_t = (C_f + 135)\ S/v$
$$C_t = \frac{(0.02v^3 + 135)S}{v}$$
$$C_t = 0.02v^2S + \frac{135}{v}S$$

$$\frac{dC_t}{dv} = 0.02(2v)S + \frac{-135}{v^2}S = 0$$

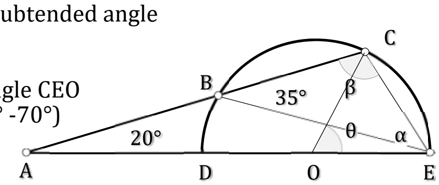
$0.04\ v = 135/v^2$
$v^3 = 135/0.04 = 3375$
v = **15 knots**

🕮 Situation 16

46	Q46	C	11.71	18.36	47.75	22.18
47	Q47	C	28.38	28.04	23.09	20.38
48	Q48	A	27.59	34.12	15.99	22.07

🕮 **46**

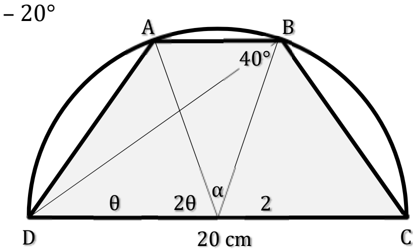
Central angle is twice the subtended angle
θ = 2×35° = 70°



Considering isosceles triangle CEO
α = ½ (180° – θ) = ½ (180° -70°)
α = 55°

Considering triangle ACE
β = 180° – α – 20° = 180° – 55° – 20°
β = **105°**

🕮 **47**

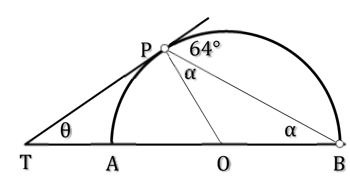


θ = 40°
2θ = 80°

α = 180 – 2(80°)
α = 20°

A = 2 × ½ r² sin (2θ) + ½ r² sin α
A = 10² sin 80° + ½ 10² sin 20°
A = **115.58 cm²**

🕮 **48**



α = 90° - 64°
α = **26°**

θ = 180° - 90° - α - α
θ = **38°**

🕮 Situation 17

49	Q49	A	58.78	15.99	12.05	12.95
50	Q50	A	51.80	7.55	28.49	12.05
51	Q51	C	29.95	9.80	52.25	8.00

🕮 **49**

|5x + 1| + 1 ≤ 10
|5x + 1| ≤ 9

5x + 1 ≤ 9
x ≤ 8/5

5x + 1 ≥ -9
x ≥ -2

[-2, 8/5]

🕮 **50**

$$\left|\frac{2(x+1)}{3}\right| \le 0$$

No value of x that can make the value in the left side less than zero but at x = -1, the equation is zero. **{-1}**

🕮 **51**

|x – 3| ≥ 7
x – 3 ≥ 7 (7, 8, 9, 10, …)

x – 3 ≤ -7 (-7, -8, -9, -10,…)
x ≤ -4

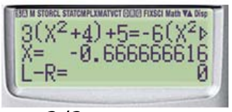
(-∞, -4] U [10, ∞]

📖 **Situation 18**

52	Q52	B	1.58	**94.37**	2.93	1.13
53	Q53	B	10.59	**80.18**	3.60	5.52
54	Q54	B	3.83	**93.47**	1.80	0.90

📖 **52**

3(X² + 4) + 5 = -6(X² + 2X) + 13



x = -2/3

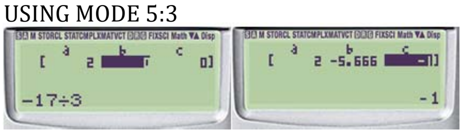
📖 **53**

x³ + 5x² = x + 5

USING MODE 5:4

📖 **54**

USING MODE 5:3



📖 **Situation 19**

55	Q55	D	1.91	2.36	1.69	**93.92**
56	Q56	D	2.48	4.73	4.73	**88.06**
57	Q57	C	12.16	10.47	**68.24**	9.12

📖 **55**

x = kw

k = x/w

5/40 = x/65

x = **8.125**

📖 **56**

P = k/V

k = PV

960(1.4) = P×2.5

P = 537.6 kPa

📖 **57**

$W = kD^4/h^2$

$k = Wh^2/D^4$

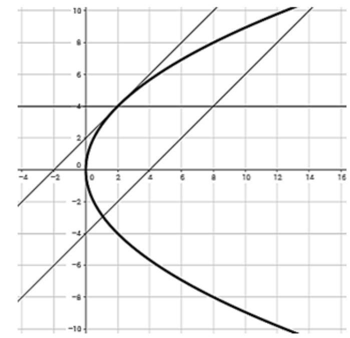
$1 \times 8^2/2^4 = w \times 4^2/1^4$

w = **1/4**

📖 **Situation 20**

58	Q58	B	21.51	**36.26**	22.07	20.05
59	Q59	A	**31.08**	38.06	23.20	7.32
60	Q60	A	**58.22**	21.40	12.84	7.55

📖 **58**



$y^2 = 8x$

$2y\,y' = 8$

$y' = 4/y$

Given equation of chord:

x – y = 4

4/y = 1 (parallel)

y = 4

y - 4 = 0

📖 **59**

Point of tangency

$y^2 = 8x$

$4^2 = 8x$

x = 2

PT (2, 4)

Equation of tangent line:

$y - y_1 = m(x - x_1)$

y – 4 = 1(x – 2)

x – y = -2

x – y + 2 = 0

📖 **60**

$y^2 = 8x$

$2y\,y' = 8$

$y' = 4/y = 1$

y = 4

$4^2 = 8x$; x = 2

Point of tangency is at **(2, 4)**

📖 **Situation 21**

61	Q61	A	**43.13**	20.05	31.08	5.74
62	Q62	B	15.88	**63.74**	15.65	4.62
63	Q63	B	29.17	**33.33**	11.60	25.56

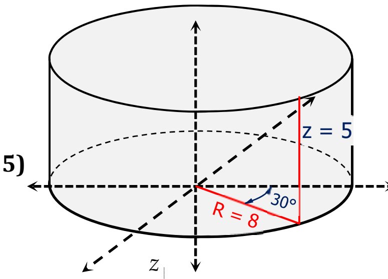
📖 **61**

x = 8 cos 30°

x = 6.928

y = 4

Rectangular coordinate **(6.928, 4, 5)**



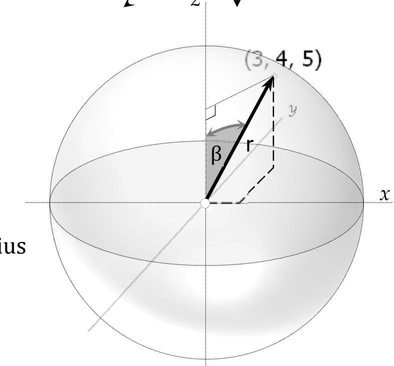
📖 **62**

$r = \sqrt{3^2 + 4^2 + 5^2}$

$r = \sqrt{50}$

$r = \sqrt{5^2(2)}$

$r = 5\sqrt{2}$ units



📖 **63**

β is the angle between the radius and the z-axis.

$\cos\beta = \dfrac{5}{5\sqrt{2}}$

β = **45°**

📖 **Situation 22**

64	Q64	A	**62.50**	11.04	19.26	7.09
65	Q65	B	10.92	**66.78**	14.19	8.11
66	Q66	C	19.26	16.55	**48.09**	15.65

📖 **64**

The equation of a line parallel to line $Ax + By + C_1 = 0$ is
$Ax + By + C_2 = 0$

The equation of a line perpendicular to line $Ax + By + C_1 = 0$ is
$Bx - Ay + C_2 = 0$

The perpendicular line of $x + 5y + 5 = 0$ that passes through (3, 1) is

$5x - y + c = 0$
$c = y - 5x = 1 - 5(3)$
$c = -14$
$\mathbf{5x - y - 14 = 0}$

65
The parallel line of $x + 5y + 5 = 0$ that passes through (4, -2) is

$x + 5y + c = 0$
$c = -x - 5y = -4 - 5(-2)$
$c = 6$
$\mathbf{x + 5y + 6 = 0}$

66
Slope of line that is perpendicular to the given line $x + 5y + 15 = 0$:

$m = 5$
$\theta = \tan^{-1} 5 = 78.69°$

angle of line with the horizontal,
$\delta = \theta - 45°$
$\delta = 78.69° - 45°$
$\delta = 33.69°$
$m_n = \tan 33.69°$
$m_n = 0.66666666667$
$m_n = 2/3$

$y - 1 = (2/3)(x - 3)$
$3y - 3 = 2x - 6$
$\mathbf{2x - 3y - 3 = 0}$

Situation 23

67	Q67	A	**71.51**	12.16	10.59	5.63
68	Q68	B	24.44	**55.07**	12.84	7.55
69	Q69	B	14.86	**45.05**	14.53	25.56

67

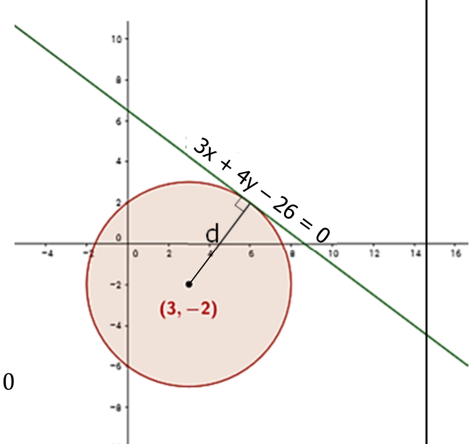
The radius of the circle is the perpendicular distance of the point (3, -2) to the line $3x + 4y - 26 = 0$

Distance of point (x_1, y_1) to a line $Ax + By + C = 0$ is

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

$$d = \left|\frac{3(3) + 4(-2) - 26}{\sqrt{3^2 + 4^2}}\right|$$

d = 5 units



The equation of the circle with center located at (h, k) and radius equal to r is
$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - -2)^2 = 5^2$
$x^2 - 6x + 9 + y^2 + 4y + 4 - 25 = 0$
$\mathbf{x^2 - 6x + y^2 + 4y - 12 = 0}$

68

Slope of the given line:
$3x + 4y - 26 = 0$
$4y = -3x + 26$
$y = -¾ x + 26/4$

The slope is -¾;
slope of normal line, n = -1/m
n = -1/(-¾)
n = 4/3

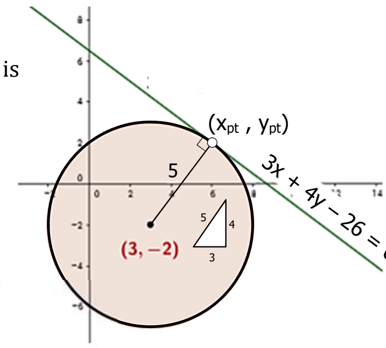
$y - y_1 = m(x - x_1)$
$y - -2 = {}^4/_3(x - 3)$
$3(y + 2) = 4x - 12$
$3y + 6 = 4x - 12$
$\mathbf{4x - 3y = 18}$

69

The perpendicular lines of line $Ax + By + C = 0$ have equations in the form: $Bx - Ay + C_2 = 0$
Given line $3x + 4y - 26 = 0$
A = 3; B = 4

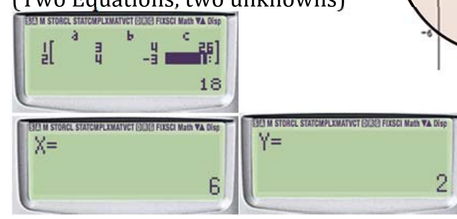
Perpendicular lines:
$4x - 3y + C_2 = 0$; If this line passes through (3, -2), then

$C_2 = -4(3) + 3(-2)$
$C_2 = -18$
Therefore the perpendicular line is
$\mathbf{4x - 3y - 18 = 0}$



Location of point of tangency:
$x_{Pt} = 3 + (3/5)×5 = \mathbf{6}$
$y_{Pt} = -2 + (4/5)×5 = \mathbf{2}$

Best Method is using Mode 5:1
(Two Equations, two unknowns)

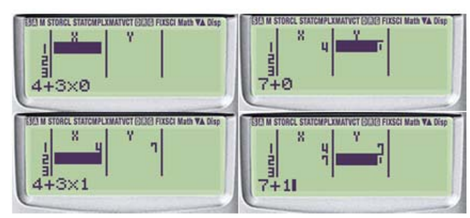


Situation 24

70	Q70	C	10.02	19.37	**62.95**	7.66
71	Q71	D	20.61	22.18	19.14	**37.95**
72	Q72	D	14.08	26.35	14.86	**44.71**

70

Since x and y are functions of t linearly (not t^2 or t^3), then MODE 3:2 can be used. In this mode, the function is y = A + Bx where A is the y-intercept.



A = 17/3

B = 1/3

71

The line has an equation y = 1/3 x + 17/3
Or $x - 3y + 17 = 0$
Distance of origin to the line is

$$d = \frac{17}{\sqrt{1^2 + 3^2}} = 5.38$$

72
Angle with respect to horizontal is θ
$\theta = \tan^{-1} m = \tan^{-1} 1/3$

$\theta = \mathbf{18.435°}$

Situation 25

73	Q73	B	20.05	**49.77**	20.38	9.68
74	Q74	A	**30.41**	21.28	38.29	9.91
75	Q75	C	17.68	22.97	**38.63**	19.93

📖 **73**

$$d = \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}$$

$$d = \frac{4(1) + 5 + 8(-3) + 33}{\sqrt{4^2 + 1^2 + 8^2}}$$

$$d = \frac{18}{9} = 2$$

📖 **74**

Perpendicular distance between two parallel lines:

$$d = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2 + C^2}}$$

$$d = \frac{|-30 - 33|}{\sqrt{4^2 + 1^2 + 8^2}}$$

$$d = \frac{63}{9} = 7$$

📖 **75**

To solve the angle of intersection of two non-parallel planes, represent each in vector form.

A = 4i + j + 8k
B = 2i – 3j + k

A · B = 4(2) +1(-3) +8(1)
4(2) +1(-3) +8(1)
A · B = **13**

Product AB means |A| × |B|
$|A| = \sqrt{4^2 + 1^2 + 8^2} = 9$
$|B| = \sqrt{2^2 + 3^2 + 1^2} = 3.742$

|A|×|B| = 9 × 3.742 = **33.678**

$$\cos\theta = \frac{A \cdot B}{|A||B|}$$

$$\cos\theta = \frac{13}{33.678}$$

θ = 67.29°
θ = **67° 17’** letter C

📖 **Situation 26**

76	Q76	A	**53.72**	20.83	18.58	6.76
77	Q77	B	11.60	**50.45**	24.89	13.06
78	Q78	C	19.26	28.49	**35.14**	16.78

📖 **76**

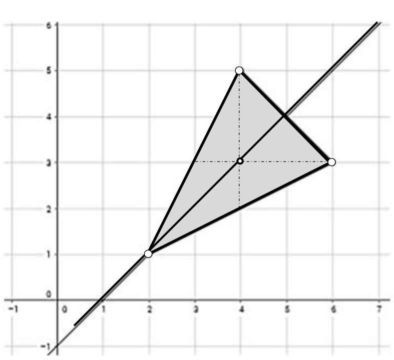
The location of the centroid of n-sided closed polygon with vertices (x_1, y_1), (x_1, y_1), (x_1, y_1), ... , (x_n, y_n) arranged/plotted on Cartesian plane in clockwise or counter clockwise order is located at (x_c, y_c) where x_c and y_c are the average of the x and y coordinates of the vertices.

$$y_c = \frac{1 + 5 + 3}{3} = 3$$

Euler line is the line that passes through centroid, orthocenter and circumcenter of a non-equilateral triangle.

Therefore, line y – x + 1 = 0 passes though the centroid.
If $y_m = 3$; then
$x_m = 3 + 1 = 4$

The centroid is at **(4, 3)**



📖 **77**

$$x_m = \frac{2 + x + 6}{3} = 4$$

x = **4**

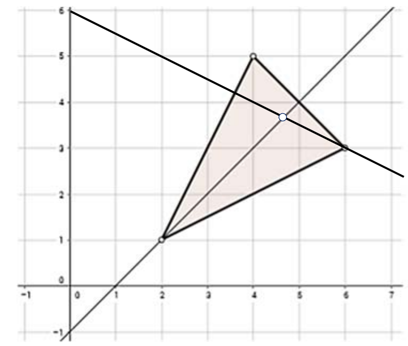
📖 **78**

The orthocenter is located at the point of intersection of the Euler line and the perpendicular line of side $\overline{AB}$ that passes through point C.

Slope of line AB:

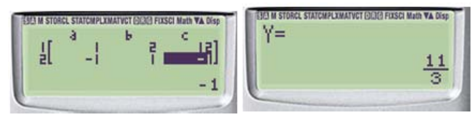
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 1}{4 - 2} = \frac{4}{2} = 2$$



Slope of perpendicular line that passes through C.,
$m_p = -1/2$
Equation of the line
y – 3 = -½ (x – 6)
2y – 6 = -x + 6
x + 2y = 12

Intersection of x + 2y = 12 and Euler line y – x + 1 = 0



Ordinated of orthocenter, y_{oc} = **11/3** letter C

📖 **Situation 27**

79	Q79	C	22.64	29.39	**35.02**	12.95
80	Q80	A	**22.86**	23.99	33.45	19.59
81	Q81	A	**25.68**	24.66	31.19	18.36

📖 **79**

Rectangular form:	Z = a + bi
Trigonometric form:	Z = r cos θ + r i sin θ
Polar form:	Z = r∠θ
Exponential Form:	$Z = re^{i\theta}$

First convert $2e^{j10\pi t} + 2e^{-j10\pi t}$ to trigonometric form:
$2e^{j10\pi t}$ = 2 cos (10π t) + 2j sin (10π t), and
$2e^{-j10\pi t}$ = 2 cos (-10π t)+ 2j sin (-10π t)

Thus, $2e^{j10\pi t} + 2e^{-j10\pi t}$ = 4 cos (10 π t)

$re^{j\theta} + re^{-j\theta} = 2r\cos\theta$

At t = 1
$2e^{j10\pi t} + 2e^{-j10\pi t}$ = 4 cos (10 π × 1) = **4** (letter C)

📖 **80**

10 cos 40πt
r = 10/2 = 5; θ = 40πt
then

10 cos 40πt = $\mathbf{5e^{j40\pi t} + 5e^{-j40\pi t}}$ (letter A)

📖 **81**

f(t) = 10 cos 40π t + 2 cos 80 π t
f(1) = 10 cos (40π × 1) + 2 cos (80 π × 1)
f(1) = 10 × 1 + 2 × 1
f(1) = **12** (letter A)

📖 **Situation 28**

82	Q82	C	3.60	7.09	**86.26**	3.04
83	Q83	D	40.43	22.64	14.53	**22.30**
84	Q84	B	27.03	**36.26**	14.19	22.41

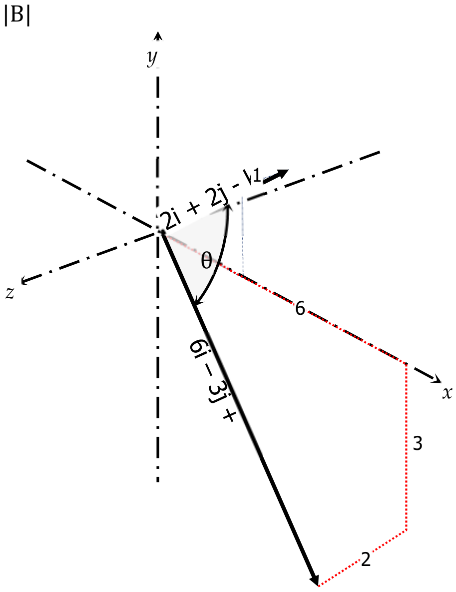
📖 **82**

A · B = 2(6) + 2(-3) – 1(2)
A · B = **4** letter C

83
Product AB means |A| × |B|

$|A| = \sqrt{2^2 + 2^2 + 1^2} = 3$
$|B| = \sqrt{6^2 + 3^2 + 2^2} = 7$

$|A| \times |B| = 3 \times 7 =$ **21**



84

$$\cos\theta = \frac{A \cdot B}{|A||B|}$$

$$\cos\theta = \frac{4}{21}$$

(4/21)
θ = **79.02°**

Situation 29

85	Q85	B	10.92	**66.10**	18.24	4.73
86	Q86	C	10.70	26.80	**47.07**	15.43
87	Q87	C	18.92	17.34	**42.91**	20.72

85
H = kS + (f + c)
H = 100.32(1.15) +0.30
H = **115.668 m**

86
I = kS cos θ + (f + c).
I = 100(2.83) cos 4° 30' + 0.30
I = 282.428 m

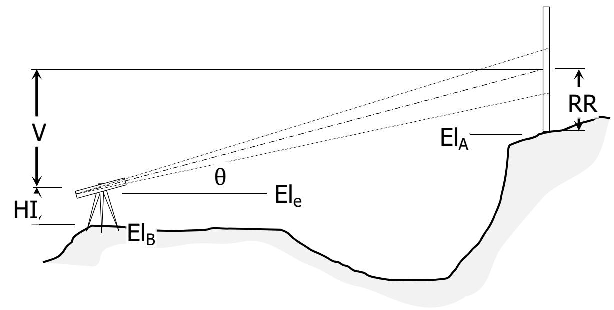
V = 282.428 sin 4° 30'
V = **22.159 m**

87
Elevation of horizontal line of sight of the stadia,
h_e = 134.2 + 1.52 = 135.72

stadia intercept, S = 1.82 m
Rod reading, RR = 2.1 m

I = 99.5(1.82) cos 4° 30'
I = 180.532 m

V = 14.164 m
El_A = 135.72 + 14.164 - 2.1
El_A = **147.784 m**



Situation 30

88	Q88	A	**39.19**	20.50	39.30	1.01
89	Q89	A	**57.09**	10.25	20.95	11.71
90	Q90	D	23.54	22.86	33.90	**19.71**

88
A. Traffic volume
89
A. average speed
90
D. time-mean speed

Situation 31

91	Q91	C	16.33	0.68	**2.03**	80.97
92	Q92	B	31.31	**39.41**	11.71	17.45
93	Q93	B	10.14	**56.31**	21.17	12.27

91
C. Reflectorized markings
92
B. continued across the bridge
93
B. 6 meters

Situation 32

94	Q94	D	4.84	4.73	6.42	**84.01**
95	Q95	B	15.77	**49.32**	18.36	16.55
96	Q96	A	**20.05**	31.31	33.22	15.43
97	Q97	A	**9.23**	7.09	33.56	50.11

94
D. All of the above
95
B. Double unbroken yellow lines
96
A. Single unbroken yellow line
97
A. Chevron markings

Situation 33

98	Q98	A	**29.84**	36.37	7.21	26.58
99	Q99	D	15.20	39.08	8.67	**37.05**
100	Q100	C	6.08	10.81	**75.11**	8.00

98
A. Deep water wave
99
D. Significant wave
100
C. Highest wave

END