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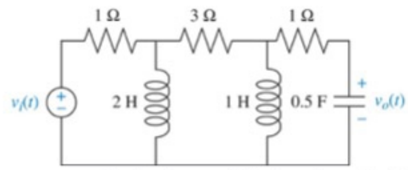
Question: Represent the electrical network shown in Figure P3.1 in state ...

1. Represent the electrical network shown in Figure P3.1 in state space, where $v_o(t)$ is the output



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Expert Answer

Anonymous answered this
1,247 answers

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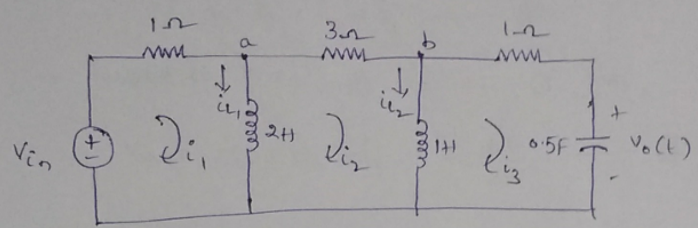
Given circuit



Let us assume the state variables of the System as $i_{L_1} = x_1$, $i_{L_2} = x_2$, $V_C = x_3$

Applying KVL to 1st loop

$$V_{in} = i_1 + L\frac{di_{L_1}}{dt} \quad \text{--- (1)}$$

Applying KVL to 2nd loop

$$3i_2 - 2\frac{di_{L_1}}{dt} + \frac{di_{L_2}}{dt} = 0 \quad \text{--- (2)}$$

[∵ $i_1 - i_2 = i_{L_1}$
$i_2 - i_3 = i_{L_2}$

Applying KVL to 3rd loop

$$i_3 + V_c - \frac{di_{L_2}}{dt} = 0 \quad \text{--- (3)}$$

[∵ $i_2 - i_3 = i_{L_2}$

Applying KCL at node-(a)

$$i_1 = i_{L_1} + i_2 \quad \text{--- (4)}$$

Applying KCL at node-(b)

$$i_2 = i_{L_2} + i_3 \quad \text{--- (5)}$$

(2)

Substituting eq-(5) in eq-(4) gives

$$i_1 = i_{L_1} + i_{L_2} + i_3 \quad \text{--- (6)}$$

Substituting eq-(6) in eq-(1) gives

$$V_{in} = i_{L_1} + i_{L_2} + i_3 + 2\frac{di_{L_1}}{dt} \quad \text{--- (7)}$$

From equation-(2)

$$2\frac{di_{L_1}}{dt} = 3i_2 + \frac{di_{L_2}}{dt}$$

But from eq-(3) $\frac{di_{L_2}}{dt} = i_3 + V_c$

$$\therefore \quad 2\frac{di_{L_1}}{dt} = 3i_2 + i_3 + V_c$$

But from eq-(5) $i_2 = i_{L_2} + i_3$

$$\therefore 2\frac{di_{L_1}}{dt} = 3(i_{L_2} + i_3) + i_3 + V_c$$

$$2\frac{di_{L_1}}{dt} = 3i_{L_2} + 4i_3 + V_c$$

$$\frac{di_{L_1}}{dt} = \frac{3}{2}i_{L_2} + \frac{4}{2}i_3 + \frac{1}{2}V_c \quad \text{—— (8)}$$

Substituting eq —(8) in eq —(7)

$$V_{in} = i_{L_1} + i_{L_2} + i_3 + 2\left[\frac{3}{2}i_{L_2} + \frac{4}{2}i_3 + \frac{1}{2}V_c\right]$$

$$V_{in} = i_{L_1} + i_{L_2} + i_3 + 3i_{L_2} + 4i_3 + V_c$$

$$V_{in} = i_{L_1} + 4i_{L_2} + 5i_3 + V_c \quad (3)$$

$$5i_3 = -i_{L_1} - 4i_{L_2} - V_c + V_{in}$$

But from the circuit i_3 is given as the Current flowing through Capacitor

$$i_3 = C\frac{dV_c}{dt}$$

$$\therefore 5\times 0.5\frac{dV_c}{dt} = -i_{L_1} - 4i_{L_2} - V_c + V_{in}$$

$$\frac{5}{2}\frac{dV_c}{dt} = -i_{L_1} - 4i_{L_2} - V_c + V_{in}$$

Substituting the State variables & I/p variable

$$\frac{5}{2}\dot{x}_3 = -x_1 - 4x_2 - x_3 + u$$

$$\boxed{\therefore \dot{x}_3 = -\frac{2}{5}x_1 - \frac{8}{5}x_2 - \frac{2}{5}x_3 + \frac{2}{5}u}$$

From eq —(3)

$$\frac{di_{L_2}}{dt} = i_3 + V_c$$

But $i_3 = C\frac{dV_c}{dt} = 0.5\frac{dV_c}{dt} = 0.5\dot{x}_3$

$$\therefore \frac{di_{L_2}}{dt} = 0.5\dot{x}_3 + V_c$$

$$\dot{x}_2 = 0.5\left[-\frac{2}{5}x_1 - \frac{8}{5}x_2 - \frac{2}{5}x_3 + \frac{2}{5}u\right] + x_3$$

$$\dot{x}_2 = -\frac{1}{5}x_1 - \frac{4}{5}x_2 - \frac{1}{5}x_3 + \frac{1}{5}u + x_3$$

$$\boxed{\ldots \frac{}{5}x_1 - \frac{4}{5}x_2 + \frac{4}{5}x_3 + \frac{1}{5}u} \quad (4)$$

From eq — (8) we have

$$\frac{di_L}{dt} = \frac{3}{2}i_{L_2} + \frac{4}{2}i_3 + \frac{1}{2}v_c$$

But $i_3 = c\frac{dv_c}{dt} = 0.5\frac{dv_c}{dt} = 0.5\dot{x}_3$

$$\therefore \frac{di_{L_1}}{dt} = \frac{3}{2}i_{L_2} + \dot{x}_3 + \frac{1}{2}v_c \qquad [\because i_3 = 0.5\dot{x}_3$$

$$\therefore \dot{x}_1 = \frac{3}{2}x_2 + \dot{x}_3 + \frac{1}{2}x_3$$

$$= \frac{3}{2}x_2 + \left[-\frac{2}{5}x_1 - \frac{8}{5}x_2 - \frac{2}{5}x_3 + \frac{2}{5}u\right] + \frac{1}{2}x_3$$

$$= -\frac{2}{5}x_1 + \left[\frac{3}{2} - \frac{8}{5}\right]x_2 + \left[\frac{1}{2} - \frac{2}{5}\right]x_3 + \frac{2}{5}u$$

$$\boxed{\dot{x}_1 = -\frac{2}{5}x_1 - \frac{1}{5}x_2 + \frac{1}{5}x_3 + \frac{2}{5}u}$$

The state equations are

$$\dot{x}_1 = -\frac{2}{5}x_1 - \frac{1}{5}x_2 + \frac{1}{5}x_3 + \frac{2}{5}u$$

$$\dot{x}_2 = -\frac{1}{5}x_1 - \frac{4}{5}x_2 + \frac{4}{5}x_3 + \frac{1}{5}u$$

$$\dot{x}_3 = -\frac{2}{5}x_1 - \frac{8}{5}x_2 - \frac{2}{5}x_3 + \frac{2}{5}u$$

The output equation is

$$y = v_o(t) = v_c = x_3$$

$$y = x_3$$

The above State-Space representation can be represented in matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & -\frac{1}{5} & \frac{1}{5} \\ -\frac{1}{5} & -\frac{4}{5} & \frac{4}{5} \\ -\frac{2}{5} & -\frac{8}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \\ \frac{2}{5} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where $A = \begin{bmatrix} -2/5 & -1/5 & 1/5 \\ -1/5 & -4/5 & 4/5 \\ -2/5 & -8/5 & -2/5 \end{bmatrix}$ $B = \begin{bmatrix} 2/5 \\ 1/5 \\ 2/5 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

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