Exercise 4

Reasoning and Problem Solving:

INSTRUCTIONS:

1-2 Give what is being asked

3-10 Answer the problems applying Polya's Technique in Solving problem. Take note that this technique has 4 steps. Some of the steps maybe combined and just show right away the method that you'll use to answer the question. Don't forget to indicate the final answer to the question.

1. Find the 99th term (a 99) of the sequence in which an = 7+3(n−1)

   an=7+3(n−1) ; where n=99

   a99=7+3(99−1) ;Substitute the value

   Answer: a_{99}=301

2. What is the 25th term of the sequence 3, 6, 11, 18, … ?

   f(n)=n^2+2

   f(n)=25^2+2

   Answer: f(n)=627

3. The total seating capacity of an auditorium is 1065. The first row has 21 seats and each row has one seat more than the row in front of it. How many rows of seats are there in the auditorium?

   a) Given: Total capacity: 1065 seats

      First row = 21 seats and the succeeding row it must be more than one than the front row

      Problem: Number of rows of seats occupy in auditorium

   b) Arithmetic Series and Quadratic Equation

   c) a_1=21 d=1 n=? Sn=1065

      Sn =(n/2)[2a_1+(n-1)d]

      1065=(n/2)[(2)(21)+(n-1)1]

      2130=n(42+n-1)

2130=n(41+n)

2130=n^2+41n

n^2+41n-2130=0

(n-30)(n+71)=2130

n-30=0 n+71=0

n=30 n=-71

Since n>0, so the number or rows are 30.

Answer: There are 30 rows of seats.

d.) Checking:

$S_n = (n/2)[2a_1+(n-1)d]$

$S_{(30)}=(30/2)[2(21)+(30-1)1]$

$S_{(30)}=(15)(42)+(29)$

$S_{(30)}=(15)[(42)+(29)]$

$S_{(30)}=1065$

4. Breven is offered Php20,000.00 as starting salary for a job, with a raise of Php2,000.00 at the end of each year of outstanding performance. If he maintains continuous outstanding performance, what will his salary be at the end of 6 years?

a.) Given: Php 20,000 starting salary raised of Php2000 at the end of the year for outstanding performance.

Problem: Salary after 6 years if he maintains continuous performance

b.) Mathematical Modelling

c.) Let r= number of years

Let n= total salary

n=(20,000 x 12)+(2000r)

n=(Php 20,000x12)+(2,000 x 6)

n=Php 252,000

Answer: Therefore, Breven's total salary after 6 years is Php 252,000 if he maintains outstanding performance.

d.) Checking: Manually calculate the salary starting from 20,0000 until you come up with the total salary within 6 years.

5. How many lines are determined by joining dots in a circle if there are 10 dots?

a.)Given: 10 dots in a circle and in forming a line it needs 2 dots

Problem: Number of lines are determined by joining dots

b.)Using the Concept of Combinations

c.) n=10(total number of dots)

r=2 (to form line should have two dots)

$_{10}C_2=10!/(10-2)!2!$

$_{10}C_2=10!/(10-2)!2!$

$_{10}C_2=10!/8!2!$

$_{10}C_2=3628800/(40320)(2)$

$_{10}C_2=45$

Therefore, the number of lines that can be formed are 45.

d.) Manually connect one dots to other dots until it became 45.

6. Use plus signs (+) between digits in 123456789 to have a sum of 90.
   a) Given: 12345678 put (+) in between

      Problem: To come up with the sum of 90

   b) Trial and error
   c)

Trials	Result
Trial 1	1+2+3+4+5+6+7+8+9= 45
Trial 2	12+34+5+6+7+8+9= 81
Trial 3	12+3+45+6+7+8+9=90

Answer: 12+3+45+6+7+8+9=90

7. In a certain year, there were exactly four Fridays and exactly four Mondays in May. On what day of the week did the 21st of May fall that year?

a.) Given: four Mondays and Fridays in May

Problem: To know what day of the week does 21st of May fall on that year

b.) Factors and Multiples

c.)n=31 days, a_{1+1}= Tuesday(Monday+1) and an= Thursday (Friday-1), d=7

n= 1stT(1), 1stW(2),1STTH(3),….5thTH(31)

T=1,8,15,22,29 W=2,9,16,23,30

F=4,11,18,25 S=5, 12, 19, 26

M=7,14,21,28 TH= 3, 10,17,24,31

SU=6,13, 20, 27

Answer: So, May 21st falls on Monday in that year.

d.) Checking:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

8.)To improve her handwriting, Hope practices writing the numbers from 1 to 200. How many times will she have written the digit 1 in all?

a.) Given: 1-200 and digit 1

Problem: Times she write the digit 1 in all

b.) Inductive Reasoning
c.) In a 1digit number, digit 1 occurs once

In a 2 digit number, digit 1 occurs 18 times

In a 3 digit number, digit 1 occurs 121 times

So, 1+18+ 121 = 140

Answer: Therefore, 140 times Hope writes digit 1 from 1- 200.

d.) Manually write down number 1-200 and count the number of times digit 1 occurs.

9. Hanika, a caterer, uses small square tables to make a long table, If each small table can seat only one person on a side, how many tables are needed to accommodate 90 persons?

a.)Given: 90 persons
Problem: Total number of small tables

e.) Using Arithmetic
f.) Since the small table is being attached to one another in order to create a long table which will be used for 90 persons. Probably, the first table can have 3 seats since one of its side will be connected to the other side of another small table and for the succeeding table, it will only cater 2 persons. But the last table must have one more seat same with the first table.

$a_1=3$ $a_n=a_1+(n-1)d$
$d=2$ $a_n=3+(n-1)2$
$n=?$ $a_n=3+2n-2$
$a_n=2n+1$
$89=2n+1$
$89-1=2n$
$\frac{88}{2}=\frac{2n}{2}$
$44=n$

Answer: There are 44 small tables to be used in order to acommodate 90 persons.

g.) $a_n=a_1+(n-1)d$
$a_n=3+(44-1)2$
$a_n=3+(44-1)2$
$a_n=89$

Add 1 person to make it 90 since the last table can occupy 1 more seat same with the first table because only one of its side is being attached to the other.

10. In the backyard, there are goats and chickens. They have a 29 heads and 92 legs. How many goats and chickens are there?

a.)Given: goat + chicken heads=92

goat + chicken legs = 29

Problem: Number of goats and chickens

b.) Using Mathematical Modelling and Equation

c.) Let x = numberof goats

Let y= number of chickens

Each chicken and goat has one head, so: x+y = 29.

Since chickens have 2 legs and goats have 4, so: 4x +2y= 92.

Solve for y: **Value of x=(29-y)**

4x +2y= 92
4(29-y)+2y=92
116-4y+2y=92
116-2y=92
-2y=92-116
$\frac{-2y}{-2}=\frac{-24}{-2}$
y=12

Solve for x: **Value of y=12**

x+y=29
x+12=29
x=29-12
x=17

Answer: Therefore, there are 12 chickens and 17 goats in the backyard.

d.) Checking:

x+y=29
17+12=29(Number of heads)

4x+2y=92
4(17)+2(12)=92
68+24=92 (Number of legs)