Second Edition

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

R.K. Rajput

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

As per Prescribed Syllabus of "Anna University, Tamil Nadu" and various other Universities

in

S.I. UNITS

For

(First Year B.E. Students of Civil, Mechanical and Technology Branches)

By

R.K. RAJPUT

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PREFACE TO THE SECOND EDITION

This book has been written specifically to meet exhaustively the requirements of the ***latest syllabus*** of the subject **"*Basic Electrical and Electronics Engineering*"** for the B.E. Examination *(common to Civil, Mechanical and Technology branches)* of **Anna University, Tamil Nadu.**

The book consists of *five units* contaning ***ten chapters*** namely : **Unit-I:** 1. *D.C. circuits*, 2. *Single-phase A.C. circuits,* 3. *Three-phase A.C. circuits* and 4. *Measuring instruments,* **Unit-II:** 5. *D.C. machines,* 6. *Single-phase transformer* and 7. *Single-phase induction motors,* **Unit-III:** 8. *Semiconductor devices and applications,* **Unit-IV:** 9. *Digital electronics,* **Unit-V:** 10. *Telecommunication systems.*

The book includes comprehensive treatment of subject matter under wide range of topics mentioned in the syllabus, with a large number of solved examples to support the text. Besides this, *Highlights, Objective Type Questions, Theoretical Questions* and *Unsolved Examples* have been added at the end of each chapter to make the book a comprehensive unit in all respects.

Any suggestions for the improvement of this book will be thankfully acknowledged and incorporated in the next edition.

—Author

PREFACE TO THE FIRST EDITION

This book has been written specifically to meet exhaustively the requirements of the ***latest syllabus***, of the subject ***"Basic Electrical and Electronics Engineering"*** for B.E. Examination (*common to Mechanical, Production, Automobile, Aeronautical and Marine Engineering disciplines*) of **Anna University, Tamil Nadu.**

The book is divided into *two parts* : *Part-I*—**Electrical Engineering** and *Part-II*—**Electronics Engineering. Part-I** consists of Unit-I, Unit-II and Unit-III containing **seven** chapters namely *D.C. circuits, Single-phase A.C. circuits, Three-phase A.C. circuits, Measuring instruments, D.C. machines, Transformers, Single-phase Induction motors,* and *Synchronous machines* ; **Part-II** consists of Unit-IV and Unit-V containing **three** chapters namely *Electronic components, devices and power converters, Digital electronics, Communication systems.*

The book includes comprehensive treatment of subject matter under wide range of topics mentioned in the syllabus, with a large number of solved examples to support the text. Besides this, *Highlights, Objective Type Questions, Theoretical Questions* and *Unsolved Examples* have been added at the end of each chapter to make the book a comprehensive unit in all respects.

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SYLLABUS

ANNA UNIVERSITY, TAMIL NADU

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Basic Electrical and Electronics Engineering

L	T	P	C
4	0	0	4

(*Common to branches under Civil, Mechanical and Technology faculty*)

UNIT-I. ELECTRICAL CIRCUITS AND MEASUREMENTS (12)

Ohm's law—Kirchhoff's laws—Steady state solution of DC circuits—Introduction to AC circuits—Waveforms and RMS value—Power and power factor—Single phase and three phase balanced circuits. Operating principles of moving coil and moving iron instruments (Ammeters and voltmeters), Dynamometer type watt meters and energy meters.

UNIT-II. ELECTRICAL MACHINES (12)

Construction, principle of operation, Basic equations and applications of DC generators, DC motors, Single Phase transformer, Single phase induction motor.

UNIT-III. SEMICONDUCTOR DEVICES AND APPLICATIONS (12)

Characteristics of PN junction diode—Zener effect—Zener diode and its characteristics—Half-wave and full-wave rectifiers—Voltage regulation. Bipolar junction transistor—CB, CE, CC configurations and characteristics—Elementary treatment of small signal amplifier.

UNIT-IV. DIGITAL ELECTRONICS (12)

Binary number system—Logic gates—Boolean algebra—Half and full adders——Flip-flops—Registers and counters—A/D and D/A conversion (single concepts).

UNIT-V. FUNDAMENTALS OF COMMUNICATION ENGINEERING (12)

Types of signals: Analog and digital signals—Modulation and demodulation: Principles of amplitude and frequency modulations. Communication systems: radio, TV, fax, microwave, satellite and optical fibre (Block diagram approach only).

Total = 60 Periods

Introduction to SI Units and Conversion Factors

A. INTRODUCTION TO SI UNITS

SI, the international system of units are divided into three classes :

1. Base units
2. Derived units
3. Supplementary units.

From the scientific point of view division of SI units into these classes is to a certain extent arbitrary, because it is not essential to the physics of the subject. Nevertheless the General Conference, considering the advantages of a single, practical, world-wide system for international relations, for teaching and for scientific work, decided to base the international system on a choice of six well-defined units given in Table 1 below :

Table 1. SI Base Units

Quantity	*Name*	*Symbol*
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol

The second class of SI units contains derived units, *i.e.,* units which can be formed by combining base units according to the algebraic relations linking the corresponding quantities. Several of these algebraic expressions in terms of base units can be replaced by special names and symbols can themselves be used to form other derived units.

Derived units may, therefore, be classified under three headings. Some of them are given in Tables 2, 3 and 4.

Table 2. Examples of SI Derived Units Expressed in terms of Base Units

Quantity	*SI Units*	
	Name	*Symbol*
area	square metre	m^2
volume	cubic metre	m^3
speed, velocity	metre per second	m/s
acceleration	metre per second squared	m/s^2
wave number	1 per metre	m^{-1}
density, mass density	kilogram per cubic metre	kg/m^3
concentration (of amount of substance)	mole per cubic metre	mol/m^3
activity (radioactive)	1 per second	s^{-1}
specific volume	cubic metre per kilogram	m^3/kg
luminance	candela per square metre	cd/m^2

Table 3. SI Derived Units with Special Names

Quantity	*SI Units*			
	Name	*Symbol*	*Expression in terms of other units*	*Expression in terms of SI base units*
frequency	hertz	Hz	—	s^{-1}
force	newton	N	—	$m.kg.s^{-2}$
pressure	pascal	Pa	N/m^2	$m^{-1}.kg.s^{-2}$
energy, work, quantity of heat power	joule	J	N.m	$m^2.kg.s^{-2}$
radiant flux quantity of electricity	watt	W	J/S	$m^2.kg.s^{-3}$
electric charge	coulomb	C	A.s	s.A
electric tension, electric potential	volt	V	W/A	$m^2.kg.s^{-3}.A^{-1}$
capacitance	farad	F	C/V	$m^{-2}.kg^{-1}.s^4$
electric resistance	ohm	Ω	V/A	$m^2.kg.s^{-3}.A^{-2}$
conductance	siemens	S	A/V	$m^{-2}.kg^{-1}.s^3.A^2$
magnetic flux	weber	Wb	V.S.	$m^2.kg.s^{-2}.A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg.s^{-2}.A^{-1}$
inductance	henry	H	Wb/A	$m^2.kg.s^{-2}.A^{-2}$
luminous flux	lumen	lm	—	cd.sr
illuminance	lux	lx	—	$m^{-2}.cd.sr$

Table 4. Examples of SI Derived Units Expressed by means of Special Names

Quantity	*SI Units*		
	Name	*Symbol*	*Expression in terms of SI base units*
dynamic viscosity	pascal second	Pa-s	$m^{-1}.kg.s^{-1}$
moment of force	metre newton	N.m	$m^2.kg.s^{-2}$
surface tension	newton per metre	N/m	$kg.s^{-2}$
heat flux density, irradiance	watt per square metre	W/m^2	$kg.s^{-2}$
heat capacity, entropy	joule per kelvin	J/K	$m^2.kg.s^{-2}.K^{-1}$
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kg.K)	$m^2.s^{-2}.K^{-1}$
specific energy	joule per kilogram	J/kg	$m^2.s^{-2}$
thermal conductivity	watt per metre kelvin	W/(m.K)	$m.kg.s^{-3}.K^{-1}$
energy density	joule per cubic metre	J/m^3	$m^{-1}.kg.s^{-2}$
electric field strength	volt per metre	V/m	$m.kg.s^{-3}.A^{-1}$
electric charge density	coulomb per cubic metre	C/m^3	$m^{-3}.s.A$
electric flux density	coulomb per square metre	C/m^2	$m^{-2}.s.A$
permitivity	farad per metre	F/m	$m^{-3}.kg^{-1}.s^4.A^4$
current density	ampere per square metre	A/m^2	—
magnetic field strength	ampere per metre	A/m	—
permeability	henry per metre	H/m	$m.kg.s^{-2}.A^{-2}$
molar energy	joule per mole	J/mol	$m^2.kg.s^{-2}mol^{-1}$
molar heat capacity	joule per mole kelvin	J/(mol.K)	$m^2.kg.s^{-2}.K^{-1}.mol^{-1}$

The SI units assigned to third class called "Supplementary units" may be regarded either as base units or as derived units. Refer Table 5 and Table 6.

Table 5. SI Supplementary Units

Quantity	*SI Units*	
	Name	*Symbol*
plane angle	radian	rad
solid angle	steradian	sr

Table 6. Examples of SI Derived Units Formed by Using Supplementary Units

Quantity	*SI Units*	
	Name	*Symbol*
angular velocity	radian per second	rad/s
angular acceleration	radian per second squared	rad/s^2
radiant intensity	watt per steradian	W/sr
radiance	watt per square metre steradian	$W\text{-}m^{-2}.sr^{-1}$

Table 7. SI Prefixes

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{12}	tera	T	10^{-1}	deci	d
10^{9}	giga	G	10^{-2}	centi	c
10^{6}	mega	M	10^{-3}	milli	m
10^{3}	kilo	k	10^{-6}	micro	μ
10^{2}	hecto	h	10^{-9}	nano	n
10^{1}	deca	da	10^{-12}	pico	p
			10^{-15}	fasnto	f
			10^{-18}	atto	a

B. DEFINITIONS

The SI *seven* base units and two supplementary units are defined below :

(*i*) **Metre.** The metre is the length equal to 1,650, 76373 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom.

(*ii*) **Kilogram.** One kilogram is equal to the mass of the international prototype of the kilogram.

(*iii*) **Second.** The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

(*iv*) **Ampere.** One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

(*v*) **Kelvin.** The kelvin is the fraction $\frac{1}{273.16}$ of thermodynamic temperature of the triple point of water.

(*vi*) **Candela.** The candela is the luminous intensity, in the perpendicular direction, of a surface of a $\frac{1}{600,000}$ square metre of a black body at a temperature of freezing platinum under a pressure of 101, 325 newton per square metre.

(*vii*) **Mole.** The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg, carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.

(*viii*) **Radian.** The radian is the plane angle between two radii of a circle that cut off on the circle an arc equal in length to the radius.

(*ix*) **Steradian.** The steradian is the solid angle which having its vertex in the centre of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

Some other Definitions

Newton. The newton (N) is a derived unit of force and is defined as the unit of force which when acting on a mass of 1 kilogram gives it an acceleration of one metre per second. Since acceleration due to gravity equals 9.81 m/s^2, one kilogram force equals 9.81 newtons.

Joule. The joule (J) is a derived unit of energy, work or quantity of heat and is defined as the work done when a force of one newton acts so as to cause a displacement of one metre. Energy is defined as the capacity to do work. A unit of energy in nuclear physics is the electron volt (eV) which is defined as the energy gained by an electron in rising through a potential difference of one volt.

$$1 \text{ eV} = 1.6021 \times 10^{-19} \text{ J}.$$

Watt. The watt (W) is a unit of power (*i.e.*, rate of doing work)

$$\text{Power in watts} = \frac{\text{work (or energy) in joules}}{\text{time in seconds}}$$

Thus 1 watt equals 1 Joule/sec.

1 kilo watt-hour (kWh) = 1000 watt-hours = 36000000 joules.

Coulomb. The coulomb (C) is the derived unit of charge. It is defined as *the quantity of electricity passing a given point in a circuit when a current of 1 A is maintained for 1 second.*

$$Q = l.t$$

where Q = charge in coulombs,

l = current in amperes, and

t = time in seconds.

1 coulomb represents 6.24×10^{18} electrons.

Ohm. The ohm (Ω) is the unit of electric resistance and is defined as *the resistance in which a constant current of 1 A generates heat at the rate of 1 watt.*

Siemen. The siemen is a unit of electric conductance (*i.e.*, reciprocal of resistance). If a circuit has a resistance of 5 ohms, its conductance is 0.2 siemens. A more commonly used name for siemen is *mho* (℧).

Volt. The volt is a unit of potential difference and electromotive force. It is defined as *the difference of potential across a resistance of 1 ohm carrying a current of 1 ampere.*

Hertz. The hertz (Hz) is a unit of frequency. 1 Hz = 1 cycle per second.

Horse-power. It is a practical unit of mechanical output. BHP (British horse power or brake horse power) equals 746 watts. The metric horse power equals 735.5 watts. To avoid confusion between BHP and metric horse power, the mechanical output of machines in SI units, is expressed in watts or kilowatts.

C. SALIENT FEATURES OF SI UNITS

The salient features of SI units are as follows :

1. It is a coherent system of units, *i.e.*, product or quotient of any two base quantities results in a unit resultant quantity. For example, unit length divided by unit time gives unit velocity.
2. It is a rationalised system of units, applicable to both, magnetism and electricity.
3. It is a non-gravitational system of units. It clearly distinguishes between the units of mass and weight (force) which are kilogram and newton respectively.
4. All the units of the system can be derived from the base and supplementary units.
5. The decimal relationship between units of same quantity makes possible to express any small or large quantity as a power of 10.
6. For any quantity there is one and only one SI unit. For example, joule is the unit of energy of all forms such as mechanical, heat, chemical, electrical and nuclear. However, kWh will also continue to be used as unit of electrical energy.

Advantages of SI Units :

1. Units for many different quantities are related through a series of simple and basic relationship.
2. Being an absolute system, it avoids the use of factor 'g' *i.e.*, acceleration due to gravity in several expressions in physics and engineering which had been a nuisance in all numericals in physics and engineering.
3. Being a rationalised system, it ensures all the advantages of rationalised MKSA system in the fields of electricity, magnetism, electrical engineering and electronics.
4. Joule is the only sole unit of energy of all forms and watt is the sole unit of power hence a lot of labour is saved in calculations.
5. It is a coherent system of units and involves only decimal co-efficients. Hence it is very convenient and quick system for calculations.
6. In electricity, all the practical units like volt, ohm, ampere, henry, farad, coulomb, joule and watt accepted in industry and laboratories all over the world for well over a century have become absolute in their own right in the SI system, without the need for any more practical units.

Disadvantages :

1. The non-SI time units 'minute' and 'hour' will still continue to be used until the clocks and watches are all changed to kilo seconds and mega seconds etc.
2. The base unit kilogram (kg) includes a prefix, which creates an ambiguity in the use of multipliers with gram.

3. SI units for energy, power and pressure (*i.e.*, joule, watt and pascal) are too small to be expressed in science and technology, and, therefore, in such cases the use of larger units, such as MJ, kW, kPa, will have to be made.
4. There are difficulties with regard to developing new SI units for apparent and reactive energy while joule is the accepted unit for active energy in SI systems.

D. CONVERSION FACTORS

1. Force :

$$1 \text{ newton} = \text{kg-m/sec}^2 = 0.012 \text{ kgf}$$
$$1 \text{ kgf} = 9.81 \text{ N}$$

2. Pressure :

$$1 \text{ bar} = 750.06 \text{ mm Hg} = 0.9869 \text{ atm} = 10^5 \text{ N/m}^2$$
$$= 10^3 \text{ kg/m-sec}^2$$
$$1 \text{ N/m}^2 = 1 \text{ pascal} = 10^{-5} \text{ bar} = 10^{-2} \text{ kg/m-sec}^2$$
$$1 \text{ atm} = 760 \text{ mm Hg} = 1.03 \text{ kgf/cm}^2 = 1.01325 \text{ bar}$$
$$= 1.01325 \times 10^5 \text{ N/m}^2$$

3. Work, Energy or Heat :

$$1 \text{ joule} = 1 \text{ newton metre} = 1 \text{ watt-sec}$$
$$= 2.7778 \times 10^{-7} \text{ kW-h} = 0.239 \text{ cal}$$
$$= 0.239 \times 10^{-3} \text{ kcal}$$
$$1 \text{ cal} = 4.184 \text{ joule} = 1.1622 \times 10^{-6} \text{ kWh}$$
$$1 \text{ kcal} = 4.184 \times 10^3 \text{ joule} = 427 \text{ kgf m}$$
$$= 1.1622 \times 10^{-3} \text{ kWh}$$
$$1 \text{ kWh} = 8.6042 \times 10^5 \text{ cal} = 860 \text{ kcal}$$
$$= 3.6 \times 10^6 \text{ joule}$$
$$1 \text{ kgf-m} = \left(\frac{1}{427}\right) \text{ kcal} = 9.81 \text{ joules}$$

4. Power :

$$1 \text{ watt} = 1 \text{ joule/sec} = 0.860 \text{ kcal/h}$$
$$1 \text{ h.p.} = 75 \text{ m kgf/sec} = 0.1757 \text{ kcal/sec} = 735.3 \text{ watt}$$
$$1 \text{ kW} = 1000 \text{ watts} = 860 \text{ kcal/h}$$

5. Specific heat :

$$1 \text{ kcal/kg-°K} = 0.4184 \text{ joules/kg-K}$$

6. Thermal conductivity :

$$1 \text{ watt/m-K} = 0.8598 \text{ kcal/h-m-°C}$$
$$1 \text{ kcal/h-m-°C} = 1.163 \text{ watt/m-K} = 1.163 \text{ joules/s-m-K.}$$

7. Heat transfer co-efficient :

$$1 \text{ watt/m}^2\text{-K} = 0.8598 \text{ kcal/m}^2\text{-h-°C}$$
$$1 \text{ kcal/m}^2\text{-h-°C} = 1.163 \text{ watt/m}^2\text{-K.}$$

The following conversion factors may be used to convert the quantities in non-SI units into SI units.

To convert	*To*	*Multiply by*
angstroms	m	10^{-10}
atmospheres	kg/m^2	10332
bars	kg/m^2	1.02×10^4
Btu	joules	1054.8
Btu	kwh	2.928×10^{-4}
circular mils	m^2	5.067×10^{-10}
cubic feet	m^2	0.02831
dynes	newtons	10^{-5}
ergs	joules	10^{-7}
ergs	kWh	0.2778×10^{-13}
feet	m	0.3048
foot-pounds	joules	1.356
foot-pounds	kg-m	0.1383
gauss	tesla	10^{-4}
grams (force)	newton	9.807×10^{-3}
horse power (metric)	watts	735.5
lines/sq. inch	tesla	1.55×10^{-5}
Maxwell	webers	10^{-8}
mho	siemens	1
micron	metre	10^{-6}
miles	km	1.609
mils	cm	2.54×10^{-3}
poundals	newton	0.1383
pounds	kilogram	0.494
pounds (force)	newtons	0.448
pounds/sq. ft.	N/m^2	47.878
pounds/sq. inch	N/m^2	6894.43

E. IMPORTANT ENGINEERING CONSTANTS AND EXPRESSIONS

Engineering Constants and Expressions	*M.K.S. System*	*SI Units*
1. Value of g_0	9.81 kg-m/kgf-sec^2	1 kg-m/N-sec^2
2. Universal gas constant	848 kgf-m/kg mole-°K	848 × 9.81 = 8314 J/kg-mole-K (∵ 1 kgf-m = 9.81 joules)
3. Gas constant (R)	29.27 kgf-m/kg-°K for air	$\frac{8314}{29}$ = 287 joules/kg-K for air
4. Specific heats (for air)	c_v = 0.17 kcal/kg-°K c_p = 0.24 kcal/kg-°K	c_v = 0.17 × 4.184 = 0.71128 kJ/kg-K c_p = 0.24 × 4.184 = 1 kJ/kg-K

5. Flow through nozzle-Exit velocity (C_2)	$91.5\sqrt{U}$ where U is in kcal	$44.7\sqrt{U}$ where U is in kJ
6. Refrigeration 1 ton	= 50 kcal/min	= 210 kJ/min
7. **Heat transfer** The Stefan Boltzman Law is given by :	$Q = \sigma T^4$ kcal/m^2-h when $\sigma = 4.9 \times 10^{-8}$ kcal/h-m^2-°K^4	$Q = \sigma T^4$ watts/m^2-h when $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4

F. DIMENSIONS OF QUANTITIES

Different units can be represented dimensionally in terms of units of length L, mass M, time T and current l. The dimentions can be derived as under :

1. Velocity = length/time = $L/T = LT^{-1}$
2. Acceleration = velocity/time = $LT^{-1}/T = LT^{-2}$
3. Force = mass × acceleration = MLT^{-2}
4. Charge (coulomb) = current × time = IT
5. Work or energy = force × distance = ML^2T^{-2}
6. EMF or potential = work/charge
   $= ML^2T^{-2}/IT = ML^2I^{-1}T^{-3}$
7. Power = work/time = $ML^2T^{-2}/T = ML^2T^{-3}$
8. Current density = current/area = $I/L^2 = IL^{-2}$
9. Resistance = emf/current = $ML^2I^{-1}T^{-3}/I = ML^2I^{-2}T^{-3}$
10. Electric flux density = electric flux or charge/area $=IT/L^2 = ITL^2$
11. MMF = current × number of turns = I
12. Conductance = I/resistance = $I/ML^2I^{-2}T^{-3} = I^2T^3M^{-1}L^{-2}$
13. Electric field intensity = volt/metre = $ML^2I^{-1}T^{-3}/L = MLI^{-1}T^{-3}$
14. Resistivity $= \dfrac{\text{resistant} \times \text{area}}{\text{length}}$
    $= (ML^2I^{-2}T^{-3})(L^2)/L$
    $= ML^3I^{-2}T^{-3}$
15. Magnetic field intensity (H) = MMF/length
    $= I/L = IL^{-1}$
16. Magnetic flux = emf × time = $(ML^2I^{-1}T^{-3})\ (T) = ML^2I^{-1}T^{-2}$
17. Magnetic flux intensity = magnetic flux/area
    $= (ML^2I^{-1}T^{-2})/L^2 = MI^{-1}T^{-2}$
18. Impedence = emf/current = $ML^2I^{-1}T^{-3}$
19. Admittance = I/impedence = $I^2T^3M^{-1}L^{-2}$
20. Inductance = magnetic flux/current
    $= ML^2T^{-2}I^{-1}/I = ML^2T^{-2}I^{-2}$
21. Capacitance = electric charge/potential
    $= IT/ML^2T^{-3}l^{-1} = M^{-1}L^{-2}T^4l^2$

UNIT-I : *ELECTRICAL CIRCUITS AND MEASUREMENTS*

Chapters :

1. D.C. Circuits
2. Single-phase A.C. Circuits
3. Three-phase A.C. Circuits
4. Measuring Instruments

1

D.C. Circuits

1. Basic concepts and Ohm's law: Electricity—Electron theory—Electric current—Electromotive force and potential—Resistance—Laws of resistance—Volume resistivity—Conductance (G)—Conductivity (σ)—Temperature coefficient of resistance —Variation of resistivity with temperature—Ohm's law—An electric circuit—Resistances in series—Voltage divider rule—Resistances in parallel—Current divider rule—Superconductivity—Resistors—Insulation resistance of a cable. 2. Definitions of important terms relating network. 3. Limitations of Ohm's law. 4. Kirchhoff's laws. 5. Applications of Kirchhoff's laws: Branch-current method—Maxwell's loop (or mesh) current method—Nodal voltage method. —*Highlights—Objective Type Questions—Theoretical Questions—Exercise*.

1. BASIC CONCEPTS AND OHM'S LAW

1.1. Electricity

It is not easy to define electricity.

- Electricity may be *defined* as a *form of energy*. It involves making and using energy.
- It may also be *defined* as a *way in which materials behave.*

Sometimes people use the term 'electricity' as the name for a material that *flows through a solid wire,* motion of this strange material is called *electric current.*

1.2. Electron Theory

- An *element* is defined as a substance which cannot be decomposed into other substances. The smallest particle of an element which takes part in chemical reaction is known as an *atom.*
- All matter is composed of atoms which are *infinitesimally small*.
- The atom, itself, is composed of *electrons, protons* and *neutrons.* The number and arrangement of these particles determines the type of atom : oxygen, carbon, copper, lead or any other element.
- Weight, colour, density, and all other characteristics of an element are *determined by the structure of the atom.* Electrons from lead would be the same as electrons from any other element.
- The '*electron*' is a *very light particle* that *spins around the centre* of the atom. Electrons move in an orbit. The number of electrons orbiting around the centre or nucleus of the atom varies from element to element. The electron has a *negative* (–) electric charge.
- The '*proton'* is a *very large* and *heavy particle* in relationship to the electron. One or more protons will form the centre or nucleus of the atom. The proton has a *positive* (+) electrical charge.

- The *'neutron'* consists of an electron and proton bound tightly together. Neutrons are located near the centre of the atom. The neutron is electrically neutral ; it has *no electrical charge.*

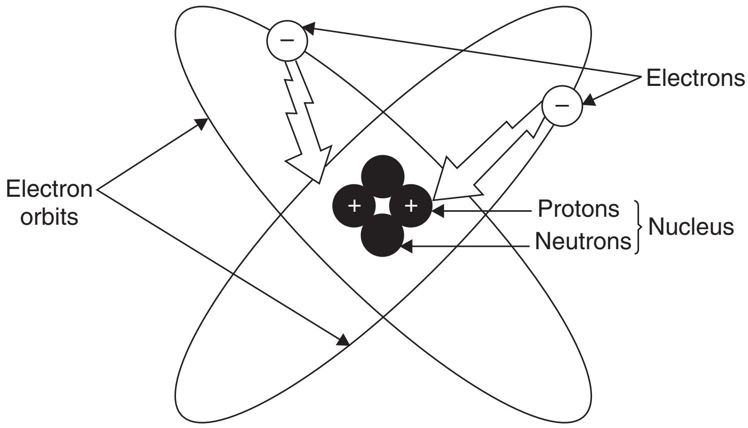


Fig. 1. Atomic structure : Electron, proton and neutron.

- Normally, the atoms are electrically neutral, that, the number of electrons and protons are the same, cancelling out each other's electrical force. Atoms "stay together" because *unlike electrical charges attract each other.* The electrical force of the protons holds the electrons in their orbits. *Like electrical charges repel each other* so negatively charged electrons will not collide with each other.

1.3. Electric Current

- In order to have electric current, *electrons must move from atom to atom* (Fig. 2). There are quite a few substances in which it is relatively easy for an electron to jump out of its orbit and begin to orbit in an adjoining or nearby atom. Substances which permit this movement of electrons are called *conductors of electricity* (*e.g.* copper, aluminium, silver, etc.)

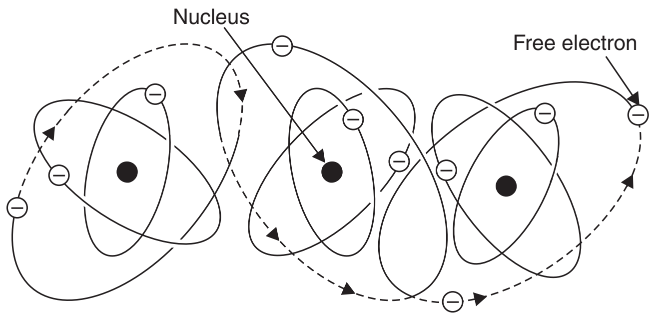


Fig. 2. Current : Flow of electrons within a conductor.

- The controlled *movement of electrons,* (or drift) through a substance is called *current*. Current occurs only when a difference of potential (e.m.f. or voltage) is present. For example, we can get a difference of potential by connecting a battery to the ends of a length of copper wire. The pressure from the battery will then move the electrons. (Fig. 3).

Refer Fig. 3 (a). The electrons of copper are free to drift in random fashion through the copper. If an imaginary line is set up in the copper wire, it will be found that the same number of electrons will cross the line from both the directions. This random movement however will not result in electric current.

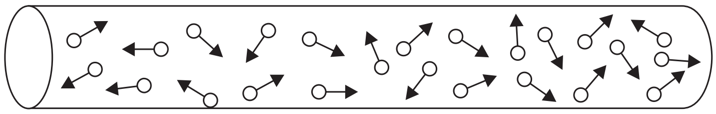


Fig. 3. (*a*) With no voltage applied : Electrons drift aimlessly.

Refer Fig. 3 (b). One end of the wire attracts electrons because it is connected to the battery terminal which has a positive charge (lack of electrons). The electrons in the copper wire drift towards this positive charge. As electrons leave the copper wire and enter the positive terminal, more electrons enter the other end of the copper wire. These electrons are taken from the negative terminal of the battery.

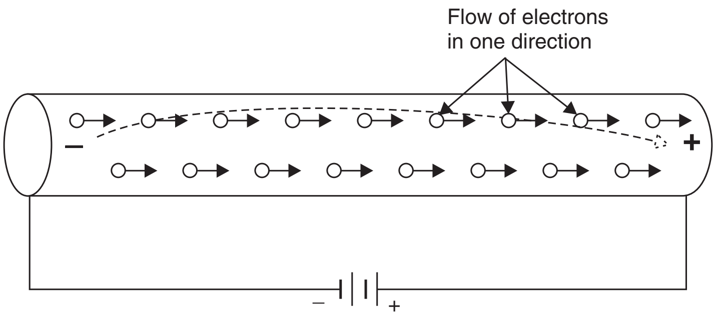


Fig. 3. (*b*) When voltage is applied to the wire :
A controlled drift of electrons takes place.

Current is the rate at which electrons move. One ampere (unit of current) represent 6.28×10^{18} electrons passing a point each second (1 coloumb past a point in 1 second).

A moving stream of positive charges also constitutes an electric current, and in the case of flow through *ionized gases* and *electrolytes* the current consists partly of positively charged particles moving in one direction and partly of negatively charged particles moving in the opposite direction. (Fig. 4). In all other cases the electric current consists of *solely of moving electrons.*

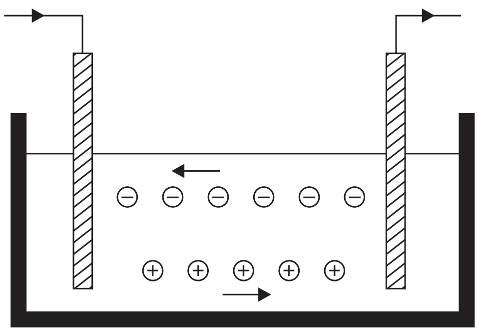


Fig. 4

One ampere is that *constant current, which* if *maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed one metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

Note. Those substances whose atoms have their outermost orbit incomplete act as good conductors of electricity *i.e.*, they permit an easy detachment of their outermost electrons and offer very little hinderance to their flow through atoms. Such substances are known as *good conductors.* But substances whose electrons are rigidly attached to their atoms are termed as *bad conductors*. Materials like germanium, silicon, and silicon carbide whose resistances at ordinary temperatures lie in between those of typical metals and typical insulators are called *semi-conductors.*

1.4. Electromotive Force and Potential

Electromotive force (e.m.f.) *is the force that causes a current* of *electricity to flow.*

- If we have two compressed-air storage tanks, one of which is at a higher pressure than the other, and we connect the two tanks together by means of a pipe, air will flow along this pipe from the higher pressure tank to the low-pressure tank. The force that causes this current of air to flow is the *pressure difference* between the two tanks.
- Similarly, in the zinc-copper voltaic cell, the zinc and copper electrodes both contain vast quantities of electrons, but owing to chemical action the zinc electrode is charged to a greater *electron pressure* than the copper electrode. The *electron-pressure difference* is the *electromotive force* that causes the flow of electrons from the zinc electrode to copper electrode through the external connecting circuit.

The term electron pressure, although descriptive, has no social standing. The word *potential* had been used to express the same idea for several years before the development of the electron theory. Two conducting bodies are said to be at the *same potential* if there is no flow of electric current between them when they are joined together by a conducting wire.

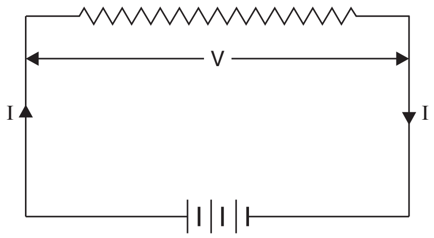


Fig. 5. Current and p.d. in a simple circuit.

The ***potential difference (p.d.)*** *V, between two points in a circuit is the electrical pressure or voltage required to drive the current between them* (Fig. 5).

The volt is unit of potential difference and electromotive force. *It is defined as the difference of potential across a resistance of 1 ohm carrying a current of 1 ampere.*

1.5. Resistance

Some materials have an *abundance of free electrons*, which require a low pressure to move them from atom to atom, and establish a high current. Such materials are known as *good conductors.* Other materials have *few free electrons.* In these the same electric pressure can move only a few electrons from atom to atom, establishing a low current. These are considered *poor conductors.* The progressive motion of free electrons is hindered in all materials, because they collide with atoms of the substance used. *The opposition to flow of electrons* (due to bonds between protons and electrons, as well as to collisions) is *called* ***electrical resistance (R).***

Resistance may also be defined as "*The property of the electric circuit which opposes the flow of current*".

Resistance is analogous in most of its aspects to friction in mechanics or hydraulics and, like friction, results in *heat generation*. The heating of an electric iron or stove is due to the resistance of the heat-unit conductor materials. For safety the material and cross-section of the conductors must be such that the temperature is kept well below a value which would result in damage to the conductor or its protective coating of insulating material.

The practical unit of electric resistance is *ohm* (Ω). It (ohm) is defined *as the resistance in which a constant current of 1 ampere generates heat at the rate of 1 watt. One volt* applied across *1 ohm* will produce *1 ampere.*

1 Mega-ohm (M Ω) = 10^6 Ω
1 kilo-ohm (k Ω) = 10^3 Ω
1 milli-ohm (m Ω) = 10^{-3} Ω
1 micro-ohm (μ Ω) = 10^{-6} Ω .

1.6. Laws of Resistance

The resistance of a conductor, such as a wire, of uniform cross-section depends on the following factors :

(*i*) ***Length (l)*......**Varies *directly* as its length, l.
(*ii*) ***Cross-section (A)*......**Varies *inversely* as the cross-section, A of the conductor.
(*iii*) ***Nature*** *of the material.*
(*iv*) ***Temperature*** *of the conductor.*

Neglecting the last factor (*iv*) for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A} \qquad ...(1)$$

where, l = Length of the conductor,
A = Area of cross-section of the conductor, and
ρ = A constant depending on the nature of the material of the conductor and is known as its *specific resistance* or *resistivity.*

If in eqn. (1), l = 1 metre, A = 1 metre, then $R = \rho$.

Hence *specific resistance* or *resistivity* of a material may be defined as "the *resistance between the opposite faces of a metre cube of that material* " (See Fig. 6).

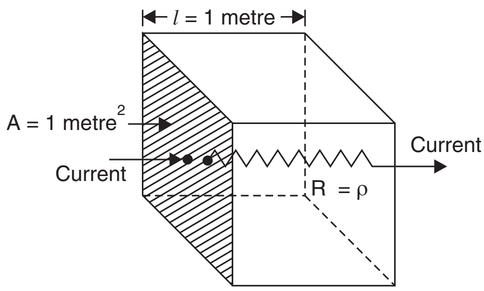


Fig. 6

Unit of resistivity. From eqn. (1), we have

$$\rho = \frac{RA}{l}$$

In S.I. system of units

$$\rho = \frac{R \text{ ohm} \times A \text{ m}^2}{l \text{ m}} = \frac{RA}{l} \text{ ohm-m}$$

Hence the unit of resistivity is *ohm-metre* (Ω-*m*).

1.7. Volume Resistivity

We know that $R = \frac{\rho l}{A}$

$$= \frac{\rho l \times A}{A \times A} = \frac{\rho V}{A^2} \qquad ...(i)$$

$\left[\because V = lA \text{ where } V = \text{volume, } l = \text{length, and } A = \text{uniform cross-sectional area of a conductor.}\right]$

Also, $$R = \frac{\rho \times l \times l}{A \times l} = \frac{\rho l^2}{V} \quad \text{...}(ii)$$

i.e., $$R = \frac{\rho V}{A^2} = \frac{\rho l^2}{V} \quad \text{[from } (i) \text{ and } (ii)\text{]} \quad \text{...}(2)$$

The above eqn. signifies that for a *given volume* :

1. Resistance of a conductor varies *inversely as square of its cross-section*.
2. Resistance varies as *square of its length*.

The values of resistivity and temperature co-efficients for some materials are given in Table 1.

Table 1. Resistivities and Temperature Co-efficients

Material	Resistivity in Ω-m at 20° C	Temperature co-efficient at 20° C
Copper	1.59×10^{-8}	0.00428
Aluminium	2.8×10^{-8}	0.0020
Silver	1.52×10^{-8}	0.00377
Platinum	11×10^{-8}	0.00340
Iron	$(9 \text{ to } 15) \times 10^{-8}$	0.0070
Mercury	19.9×10^{-8}	0.00089
German silver (4 Cu ; 2Ni ; 1 Zn)	20.2×10^{-8}	0.00027
Constantan or Eureka	49×10^{-8}	– 0.00004 to + 0.00001
Carbon	7000×10^{-8}	– 0.0005

1.8. Conductance (G)

Conductance $\left(G = \frac{1}{R} = \frac{A}{\rho l}\right)$ is the *reciprocal of resistance* and therefore be utilised as a measure of the inducement to current flow offered by the circuit. The conductance of circuit is directly proportional to the cross-section of the conductor and inversely proportional to its length.

The unit of conductance is reciprocal ohm, or *mho* (℧).

1.9. Conductivity (σ)

The reciprocal of specific resistance $\left(\sigma = \frac{1}{\rho}\right)$ of a material is called its *conductivity*. The *unit of conductivity* $\left(\sigma = G\,\frac{l}{A}\right)$ is *mho/metre* (℧/*m*).

In practice conductivities are used chiefly to compare one conductor material with another. It is therefore convenient to choose one material as the standard and to compare others with it. Accordingly, *pure annealed copper has been chosen as the standard and its conductivity is taken as 100 percent.*

Example 1. *Find the resistance of copper wire at 20°C whose cross-sectional area is 0.02 cm*^{2} *and length 400 metres. Take resistivity of copper at 20°C as* 1.7×10^{-8} *Ω-m. What is the conductivity of wire ?*

Solution. Resistivity of copper at 20° C, $\rho_{20} = 1.7 \times 10^{-8}$ Ω-m

Cross-sectional area, $A = 0.02 \text{ cm}^2 = 0.02 \times 10^{-4} \text{ m}^2$

Length of wire, $l = 400$ m

Resistance of copper wire at 20° C, R_{20} :

Using the relation, $R = \dfrac{\rho l}{a}$

or $$R_{20} = \frac{\rho_{20} l}{a} = \frac{1.7 \times 10^{-8} \times 400}{0.02 \times 10^{-4}} = \mathbf{3.4\ \Omega}\text{. } \textbf{(Ans.)}$$

Conductivity of wire, σ :

Conductivity (σ) $= \dfrac{1}{\rho} = \dfrac{1}{1.7 \times 10^{-8}}$

$= \mathbf{58.8 \times 10^6}$ **mho/metre. (Ans.)**

Example 2. *Find the specific resistance of a conductor with the following particulars :*

Length .... 250 metres

Volume .... 0.04 m^3

Resistance .... 0.04 Ohm.

Solution. Length, $l = 250$ m, Volume, $V = 0.04 \text{ m}^3$, Resistance, $R = 0.04$ ohm

We know that $R = \dfrac{\rho l^2}{V}$ ...[Eqn. (2)]

Substituting the values of R, l and V in the above expression, we get

$$0.04 = \frac{\rho \times (250)^2}{0.04} \quad \text{or} \quad \rho = \frac{0.04 \times 0.04}{(250)^2}$$

$$= \frac{0.0016}{(250)^2} = 2.56 \times 10^{-8}\ \Omega\ .\ \text{m.}$$

Hence specific resistance of the conductor = 2.56×10^{-8} Ω-m (Ans.)

Example 3. *Find the relative diameters of two wires having resistance 16 Ω and 25 Ω respectively. Assume that they are of equal length and are made up of same material.*

Solution. Resistance of wire 1, $R_1 = 16$ Ω

Resistance of wire 2, $R_2 = 25$ Ω

$\mathbf{\dfrac{d_1}{d_2}}$:

Now, $R = \dfrac{\rho l}{A} = \dfrac{\rho l}{\pi/4\, d^2}$ (where d = diameter of a wire)

As the wires are of equal length and are made up of same material, l and ρ are constant.

∴ $R \propto \dfrac{1}{d^2}$ *i.e.,* $R_1 \propto \dfrac{1}{d_1^2}$ ...(*i*)

and $R_2 \propto \dfrac{1}{d_2^2}$ ...(*ii*)

From (*i*) and (*ii*), we have

$$\frac{R_1}{R_2} = \frac{d_2^2}{d_1^2} \quad \text{or} \quad \left(\frac{d_1}{d_2}\right)^2 = \frac{R_2}{R_1}$$

or $$\frac{d_1}{d_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{25}{16}} = 1.25$$

Hence $$\frac{\mathbf{d_1}}{\mathbf{d_2}} = \mathbf{1.25.} \quad \textbf{(Ans.)}$$

1.10. Temperature Coefficient of Resistance

Consider a metallic conductor having a resistance of R_0 at 0°C. Let it be heated to 1°C and R_t be its corresponding resistance at this temperature. Then it is found that within ordinary temperature ranges ; $R_t - R_0$ depends :

(*i*) directly on its initial resistance,

(*ii*) directly on the temperature rise, and

(*iii*) on the nature of material of conductor.

In other words, $R_t - R_0 \propto R_0 .t$ or $R_t - R_0 = \alpha . R_0 . t$

where α = a constant, known as "*temperature co-efficient of resistance*" of the conductor.

or $$\alpha = \frac{R_t - R_0}{R_0 . t} \qquad ...(3)$$

In case $R_0 = 1\ \Omega$, $t = 1$°C

then $$\alpha = R_t - R_0$$

Hence temperature co-efficient of resistance at 0°C may be defined as :

"*The change in resistance per ohm for change in temperature of 1°C from 0°C.*"

Over *large temperature ranges* the simple formula

$$R_t - R_0 = \alpha R_0 t \quad \text{or} \quad R_t = R_0 (1 + \alpha t) \qquad ...[3(a)]$$

does not completely fit, but a formula of the type

$$R_t = R_0 (1 + \alpha t + \beta t^2) \qquad ...(4)$$

(where β is a smaller co-efficient)

applies.

- At *very low temperatures*, a few degrees above absolute zero, many metals become *superconductors* ; that is, the resistance becomes so close to zero that currents once started circulate for a very long time without generating appreciable heat or needing an external e.m.f. to maintain them.
- In case of many metals it is not too inaccurate for simple rough calculations to suppose that the resistance is directly proportional to absolute temperature. Among pure metals iron and mercury cannot be included in this rough generalisation, which is also, of course, quite inapplicable to alloys.

Effect of temperature on resistance :

The following points are worth noting :

1. The resistance of *metal conductors increases* (α being positive) with rise of temperature ; the rate of increase is very considerable for most pure metals, being as much as about $\frac{1}{150}$ of the total resistance for each centigrade rise in the case of iron ; the effect is smaller in case of alloys, and very small indeed for materials such as manganin and constantan which are therefore very suitable for making *standard resistances.*

2. The resistance of *semiconductors* such as carbon, and all *electrolytes* 'decreases' as the temperature rises (α being negative).

Value of α at different temperatures :

Let R_1, R_2, and R_3 be resistance of a conductor at temperatures t_1, t_2 and t_3 °C respectively and α_1 and α_2 be the temperature co-efficients of resistance at temperatures t_1 and t_2 °C respectively.

Then, we know that,

$$R_2 = R_1 \, [1 + \alpha_1 (t_2 - t_1)] \quad ...(i)$$

$$R_3 = R_1 \, [1 + \alpha_1 (t_3 - t_1)] \quad ...(ii)$$

$$R_3 = R_2 \, [1 + \alpha_2 (t_3 - t_2)] \quad ...(iii)$$

Equation (*iii*) can be rearranged as follows :

$$\frac{R_3}{R_2} = 1 + \alpha_2 \, (t_3 - t_2) \quad ...(iv)$$

Also by dividing (*ii*) by (*i*), we get

$$\frac{R_3}{R_2} = \frac{1 + \alpha_1 (t_3 - t_1)}{1 + \alpha_1 (t_2 - t_1)} \quad ...(v)$$

Comparing (*iv*) and (*v*), we have

$$1 + \alpha_2 \, (t_3 - t_2) = \frac{1 + \alpha_1 (t_3 - t_1)}{1 + \alpha_1 (t_2 - t_1)}$$

or

$$\alpha_2 (t_3 - t_2) = \frac{1 + \alpha_1 (t_3 - t_1)}{1 + \alpha_1 (t_2 - t_1)} - 1$$

$$= \frac{[1 + \alpha_1 (t_3 - t_1)] - [1 + \alpha_1 (t_2 - t_1)]}{1 + \alpha_1 (t_2 - t_1)} = \frac{\alpha_1 (t_3 - t_2)}{1 + \alpha_1 (t_2 - t_1)}$$

or

$$\alpha_2 = \frac{\alpha_1}{1 + \alpha_1 (t_2 - t_1)} \quad ...(5)$$

or

$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)} \quad ...[5(a)]$$

Example 4. *A semi-circular ring of copper has an inner radius of 12 cm, radial thickness 6 cm and axial thickness 8 cm. Find the resistance of the ring at 60° C between its two end faces.*

Assume : Specific resistance of copper at 20° C = 1.724 × 10^{-8} ohm-metre. Temperature co-efficient of resistance of copper at 0°C = 0.0043 / °C.

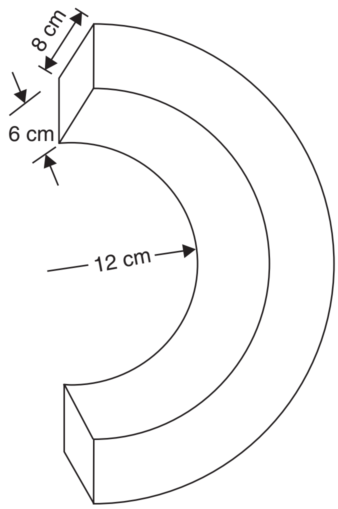


Fig. 7

Solution. Refer Fig. 7.

Mean radius of the ring

$$= \frac{18 + 12}{2} = 15 \text{ cm.}$$

Mean length between the faces 15 π cm

$= 0.47$ m.

Cross-section of the ring

$= 8 \times 6 = 48 \text{ cm}^2 = 48 \times 10^{-4} \text{ m}^2$

Now $\alpha_0 = 0.0043/°$ C ;

$$\alpha_{20} = \frac{\alpha_0}{1 + \alpha_0 (20 - 0)} = \frac{0.0043}{1 + 0.0043 \times 20}$$

$= 0.00396$

$$\rho_{60} = \rho_{20}\,[1 + \alpha_{20}(60 - 20)]$$
$$= 1.724 \times 10^{-8}\,[1 + 0.00396 \times 40] = 1.997 \times 10^{-8}\ \Omega\text{-m.}$$

Now, $$R_{60} = \frac{\rho_{60} \times l}{A} = \frac{1.997 \times 10^{-8} \times 0.47}{48 \times 10^{-4}} = \mathbf{1.955 \times 10^{-6}\ \Omega}.\quad \textbf{(Ans.)}$$

Example 5. *The resistance of a field coil measures 55 ohm at 25° C and 65 ohm at 75° C. Find the temperature co-efficient of conductor at 0° C.*

Solution. $R_1 = 55\ \Omega$, $t_1 = 25°\text{C}$, $R_2 = 65\ \Omega$, $t_2 = 75°\text{C}$

Temperature co-efficient of conductor at 0° C, α_0 :

Now, $$R_1 = R_0\,[1 + \alpha_0 \,.\, t_1\,]$$

i.e., $$55 = R_0\,[1 + \alpha_0 \times 25] \qquad ...(i)$$

and $$R_2 = R_0\,(1 + \alpha_0 \,.\, t_2)$$

i.e., $$65 = R_0\,(1 + \alpha_0 \times 75) \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{65}{55} = \frac{1 + 75\alpha_0}{1 + 25\alpha_0} \quad \text{or} \quad \frac{13}{11} = \frac{1 + 75\alpha_0}{1 + 25\alpha_0}$$

or $$13(1 + 25\,\alpha_0) = 11(1 + 75\,\alpha_o) \quad \text{or} \quad 13 + 325\,\alpha_0 = 11 + 825\,\alpha_o$$

or $$500\,\alpha_0 = 2 \quad \text{or} \quad \alpha_0 = \frac{2}{500} = \frac{1}{250} = 0.004/°\text{C}$$

Hence α_0 = 0.004 per °C rise of temperature. **(Ans.)**

☞ **Example 6.** *A specimen copper wire has a specific resistance of 1.7×10^{-8} ohm-metre at 0°C and has temperature co-efficient of $\frac{1}{254.5}$ at 20°C, find :*

(i) The temperature co-efficient at 70°C.

(ii) The specific resistance at 70°C.

Solution. Specific resistance at 0°C, $\rho_0 = 1.7 \times 10^{-8}\ \Omega\text{-m}$

Temperature co-efficient at 20°C, $\alpha_{20} = \dfrac{1}{254.5}$

(*i*) **Temperature co-efficient at 70°C, α_{70} :**

To find α_{70} using the relation :

$$\alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)} \quad \text{or} \quad \alpha_{70} = \frac{1}{\dfrac{1}{\alpha_{20}} + (70 - 20)} = \frac{1}{254.5 + 50} = \frac{1}{304.5}$$

Hence $$\boldsymbol{\alpha_{70} = \frac{1}{304.5}}.\quad \textbf{(Ans.)}$$

(*ii*) **Specific resistance at 70° C, ρ_{70} :**

Now, $$\rho_{70} = \rho_0(1 + \alpha_0 \times 70) \qquad ...(i)$$

In the above expression ρ_{70} can be calculated only if α_0 is known, α_0 will be calculated as follows :

$$\alpha_1 = \frac{1}{\dfrac{1}{\alpha_0} + (t_1 - 0)}$$

$$\frac{1}{254.5} = \frac{1}{\frac{1}{\alpha_0} + 20} \quad \text{or} \quad 254.5 = \frac{1}{\alpha_0} + 20 \quad \text{or} \quad \alpha_0 = \frac{1}{234.5}$$

By putting the value of α_0 in expression (*i*), we get

$$\rho_{70} = 1.7 \times 10^{-8} \left(1 + \frac{1}{234.5} \times 70\right) = 2.21 \times 10^{-8} \ \Omega\text{-m.}$$

Hence $\boldsymbol{\rho_{70} = 2.21 \times 10^{-8}}$ **Ω-m. (Ans.)**

Example 7. *The specific resistance of platinum at 0°C is 10.3 × 10^{-8} Ω-m. How long must a wire of No. 32 S.W.G. (diameter = 0.0274 cm) platinum be to have a resistance of 4 Ω at 0°C.*

What will be resistance of a wire at 100°C, if the temperature co-efficient of platinum is 0.0038/°C ?.

Solution. Specific resistance of platinum at 0°C, $\rho_0 = 10.3 \times 10^{-8}$ Ω-m.

Diameter of the platinum wire, d = 0.0274 cm = 0.000274 m.

∴ Area of the platinum wire, $A = \pi/4 \times 0.000274^2 = 5.896 \times 10^{-8}$ m^2

Resistance of the wire at 0°C, R_0 = 4 Ω

Now, using the relation :

$$R = \frac{\rho l}{A}$$

$$\therefore \quad l = \frac{RA}{\rho} = \frac{4 \times 5.896 \times 10^{-8}}{10.3 \times 10^{-8}} = 2.289 \text{ m}$$

Hence length of the wire **= 2.289 m. (Ans.)**

Resistance of the wire at 100°C, R_{100} :

$$R_{100} = R_0 (1 + \alpha_0 \times 100)$$
$$= 4 (1 + 0.0038 \times 100) = 5.52 \ \Omega$$

Hence $\mathbf{R_{100}}$ **= 5.52 Ω . (Ans.)**

Example 8. *A potential difference of 200 V is applied to a copper field coil at a temperature of 15° C and the current is 10A. What will be the mean temperature of the coil when current has fallen to 5 A, the applied voltage being the same as before ?*

$$\alpha = \frac{1}{234.5} \text{ per °C at 0°C.}$$

Solution. Potential difference, V = 200 V

$$R_{15} = \frac{200\,(\text{V})}{10\,(\text{I})} = 20 \ \Omega$$

$$R_{t_2} = \frac{200}{5} = 40 \ \Omega$$

where t_2 is the temperature when current falls to 5 A

$$\alpha_0 = \frac{1}{234.5} \qquad \text{...(Given)}$$

Temperature co-efficient of resistance at 15° C,

$$\alpha_{15} = \frac{1}{\frac{1}{\alpha_0} + (15 - 0)} = \frac{1}{234.5 + 15} = \frac{1}{249.5} /°\text{C}$$

Also, $R_{t_2} = R_{15}\,[1 + \alpha_{15}\,(t_2 - 15\,)]$

$$40 = 20\left[1 + \frac{1}{249.5}(t_2 - 15)\right]$$

$$2 = 1 + \frac{1}{249.5}(t_2 - 15)$$

$$1 = \frac{t_2 - 15}{249.5}$$

i.e., $t_2 = 264.5°$ C.

Hence mean temperature of the coil when current has fallen to 5 A

= 264.5° C. (Ans.)

☞ **Example 9.** *The moving coil of a permanent magnet instrument has a temperature co-efficient of resistance 0.004 / ° C. A swamping resistance of temperature coefficient of resistance 0.0002 / ° C is to be added such that the overall temperature coefficient of resistance is 0.0006 / °C. Assuming that all measurements are given for 0°C, find the value of resistance if the coil resistance is 2 ohms.*

Solution. Let R_c = Resistance of the coil,

R_s = Swamping resistance,

R_T = Total resistance,

α_c = Temperature co-efficient of resistance for coil,

α_s = Temperature co-efficient of resistance for swamping resistance, and

α_T = Overall temperature co-efficient of resistance.

Here, α_c (at 0° C) = 0.004/°C

α_s (at 0° C) = 0.0002/° C

α_T (at 0° C) = 0.0006/° C

Total resistance at 0°C $= R_c + R_s$

Total resistance at any temperature t° C,

$$R_T = R_c\,(1 + \alpha_c\,.\,t) + R_s\,(1 + \alpha_s\,.\,t) \qquad ...(i)$$

Also, $$R_T = (R_c + R_s)\,(1 + \alpha_T\,.\,t) \qquad ...(ii)$$

Comparing eqns. (*i*) and (*ii*), we get

$$R_c(1 + \alpha_c\,.\,t) + R_s\,(1 + \alpha_s\,.\,t) = (R_c + R_s)\,(1 + \alpha_T\,.\,t)$$

$$R_c\,(1 + 0.004\,t) + R_s\,(1 + 0.0002\,t) = (R_c + R_s)\,(1 + 0.0006t)$$

$$R_c + 0.\,004 + R_c\,.\,t + R_s + 0.0002\,R_s\,.\,t = R_c + 0.0006\,R_c.\,t + R_s + 0.0006\,R_s.t$$

$$0.0034\,R_c = 0.0004\,R_s$$

$$R_s = \frac{0.0034\,R_c}{0.0004} = 8.5\,R_c.$$

i.e., Swamping resistance = 8.5 × coil resistance

= 8.5 × 2 = 17 Ω

Hence swamping resistance **= 17 Ω . (Ans.)**

☞ **Example 10.** *Calculate the resistance at the working temperature of 75° C of a 4-pole, lap connected armature winding from the following data :*

Number of slots 100
Conductors per slot 12
Mean length of turn .... 3 m
Cross-section of each conductor .... 1.5 cm × 0.2 cm
Specific resistance of copper of 20° C .... 1.72 × 10^{-8} ohm metre
Temperature co-efficient of resistivity at 0°C .... 0.00427/° C.

Solution. Total number of conductors, $Z = 100 \times 12 = 1200$

Length of each conductor, $l = \dfrac{3}{2} = 1.5$ m

Cross-sectional area of each conductor = 1.5 cm × 0.2 cm = 0.3 cm^2 = 0.3 × 10^{-4} m^2

Specific resistance of copper at 20° C, $\rho_{20} = 1.72 \times 10^{-8}$ ohm-metre.

Temperature co-efficient of resistivity at 0°C, $\alpha_0 = 0.00427$/°C

If '*a*' is the number of parallel paths,

Resistance of each parallel path, $R_0 = \dfrac{\rho_0 lZ}{A \times a}$

As there are such '*a*' paths in parallel, hence equivalent resistance,

$$R_0 = \frac{\rho_0 l \,.\, Z}{A \times a \times a} = \frac{\rho_0 l \,.\, Z}{A \times a^2} \qquad ...(i)$$

Also, $$\rho_t = \rho_0 (1 + \alpha_0 \,.\, t)$$

$$1.72 \times 10^{-8} = \rho_0 (1 + 0.00427 \times 20) = 1.0854\, \rho_0$$

or $$\rho_0 = \frac{1.72 \times 10^{-8}}{1.0854} = 1.585 \times 10^{-8}\ \Omega\text{-m}$$

Now by substituting the different values in eqn. (*i*), we get,

$$R_0 = \frac{1.585 \times 10^{-8} \times 1.5 \times 1200}{0.3 \times 10^{-4} \times 4 \times 4} = 0.0594\ \Omega$$

Resistance of winding at 75° C,

$$R_{75} = R_0 (1 + \alpha_0 \times 75)$$
$$= 0.0594 (1 + 0.00427 \times 75) = 0.0784\ \Omega$$

Hence resistance of winding at 75° C **= 0. 0784 Ω.** (**Ans.**)

Example 11. *The base of an incandescent lamp is marked 120 V, 50 W. Measurement of resistance of the lamp on a Wheatstone bridge indicates 20 ohms at an ambient temperature of 20° C. What is the normal operating temperature of incandescent of the filament, if the resistance temperature co-efficient of tungsten is 5 × 10^{-3}/° C at 20° C ?*

Solution. Marked voltage of incandescent lamp, $V = 120$

Marked wattage of incandescent lamp, $W = 50$

∴ Current, $I = \dfrac{W}{V} = \dfrac{50}{120} = \dfrac{5}{12}$ A and, Resistance of lamp at operating temperature (t),

$$R_t = \frac{V}{I} = \frac{120}{5/12} = 288\ \Omega$$

$$R_{20} = 20\ \Omega \quad \text{(given)}$$

$$\alpha_{20} = 5 \times 10^{-3}/°\ C \quad \text{(given)}$$

Also,
$$R_t = R_{20}\ [1 + \alpha_{20}\ (t - 20)]$$

$$288 = 20\ [1 + 5 \times 10^{-3}\ (t - 20)]$$

$$14.4 = 1 + 5 \times 10^{-3}\ (t - 20)$$

$$13.4 = 5 \times 10^{-3}\ (t - 20)$$

i.e.,
$$t = \frac{13.4}{5 \times 10^{-3}} + 20 = 2680 + 20 = 2700°\ C$$

Hence normal operating temperature **= 2700° C. (Ans.)**

Example 12. *A coil has a resistance of 21.6 ohms when its mean temperature is 20°C and of 24 ohms when its mean temperature is 50° C. Find its mean temperature rise, when its resistance is 25.2 ohms and the surrounding temperature is 16° C.*

Solution. *Given :* $R_1 = 21.6\ \Omega$, $t_1 = 20°C$; $R_2 = 24\ \Omega$; $t_2 = 50°C$; $R_3 = 25.2\ \Omega$, $t_{surr.} = 16°C$.

Mean temperature rise :

Now,
$$R_2 = R_1\ [1 + \alpha_1\ (t_2 - t_1)]$$

or
$$24 = 21.6\ [\ 1 + \alpha_1(50 - 20)\ ]$$

$\therefore$
$$\alpha_1 = \frac{1}{270}\ \text{per° C} \quad \text{(The temperature coefficient of resistance at 20°C)}$$

Again,
$$R_3 = R_1\ [1 + \alpha_1\ (t_3 - t_1)]$$

$$25.2 = 21.6\left[1 + \frac{1}{270}(t_3 - 20)\right]$$

$\therefore$
$$t_3 = 65°\ C$$

$\therefore$ Temperature rise $= t_3 - t_{surr.} = 65 - 16 =$ **49° C. (Ans.)**

Example 13. *A potential difference of 250 V is applied to a copper field coil at a temperature of 15° C, and the current is 5A. What will be the mean temperature of the coil when the current has fallen to 4A, the applied voltage being the same as before ? Assume* $\alpha_0 = \frac{1}{234.5}$ *per°C.*

Solution. *Given* : $V = 250$ V ; $t_1 = 15°C$, $I_1 = 5$ A ; $I_2 = 4$ A ;

$$\alpha_0 = \frac{1}{234.5}\ \text{per° C}$$

At temperature 15° C,
$$R_1 = \frac{V}{I_1} = \frac{250}{5} = 50\ \Omega$$

Let, t_2 be the temperature when the current is 4 A.

$\therefore$
$$R_2 = \frac{V}{I_2} = \frac{250}{4} = 62.5\ \Omega$$

We know that,
$$R_2 = R_1\ [1 + \alpha_1(t_2 - t_1)]$$

$$62.5 = 50\ [1 + \alpha_{15}\ (t_2 - 15)] \quad ...(i)$$

Also, $$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 \, . \, t_1}$$

$\therefore$ $$\alpha_{15} = \frac{\frac{1}{234.5}}{1 + \frac{1}{234.5} \times 15} = \frac{1}{249.5} \text{ per °C}$$

Inserting the value of α_{15} in eqn. (1), we get

$$62.5 = 50 \left[1 + \frac{1}{249.5} (t_2 - 15) \right]$$

$\therefore$ $\mathbf{t_2 = 77.37\ °C.}$ **(Ans.)**

1.11. Variation of Resistivity with Temperature

Like resistance, the specific resistance or resistivity of a metallic conductor increases with rise in temperature and *vice versa*. Within normal ranges of temperatures ρ_t and ρ_0 follow the relation given below :

$$\rho_t = \rho_0 \, (1 + \alpha_0 \, . \, t) \quad ...(6)$$

where ρ_t and ρ_0 are the resistivities at $t°$ and 0° C respectively.

1.12. Ohm's Law

Ohm's law can be stated as follows :

"*The ratio of potential difference between any two points of a conductor to the current flowing through it, is constant provided the physical conditions (i.e., temperature etc.) do not change.*

In other words, Ohm's law states that in a closed circuit (Fig. 8), when a voltage V is applied across a conductor, then current I flowing through it is directly proportional to the applied voltage,

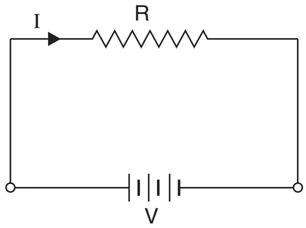


Fig. 8

i.e., $$I \propto V \quad \text{or} \quad \frac{V}{I} = \text{constant}$$

$\therefore$ $$\frac{V}{I} = R \text{ ohms} \quad ...[7] \qquad \text{or} \qquad V = IR \text{ volts} \quad ...[7\,(a)]$$

or $$I = \frac{V}{R} \text{ amp.} \quad ...[7\,(b)]$$

where, V = Voltage between two points,

I = Current flowing in the circuit, and

R = The resistance of the conductor.

The units of voltage, current and resistance are volt (V), ampere (A) and ohm (Ω) respectively.

- The *linear relationship ($I \propto V$) does not apply to all non-metallic conductors.* For example, for silicon carbide, the relationship is given by :

$$V = k\, I^x$$

where, k and x are *constants* and x is *less than unity.*

1.12.1. Conditions for applying Ohm's law. Ohm's law is applicable under the following *conditions* :

1. Ohm's law can be applied either to the entire circuit or to a part of the circuit.

2. When Ohm's law is applied to a part of a circuit, part resistance and the potential difference across that part resistance should be used.

3. Ohm's law can be applied to both D.C. and A.C. circuits. However, in case of A.C. circuits, impedance Z, is used in place of resistance.

Then, $$I = \frac{V}{Z} = \frac{\text{Applied voltage}}{\text{Impedance in the circuit}}.$$

- ***Electrical power.*** It *is the rate of doing work.* In other words, the *amount of work done in one second is called "power".*

or $$P = \frac{W}{t} \qquad ...(8)$$

where P = The power in watts,

W = The work done in joules, and

t = Time in second.

Power is equal to the product of voltage (V) and current (I) in a particular circuit

i.e., $$^*P = V \times I \qquad ...(9)$$

The following relations hold good :

(*i*) $$^*P = VI = I^2 R = \frac{V^2}{R}$$

where, P = power in watts,
V = voltage in volts,
I = current in amperes, and
R = resistance in ohms.

(*ii*) $$I = \frac{P}{V} = \sqrt{\frac{P}{R}}$$

(*iii*) $$R = \frac{P}{I^2} = \frac{V^2}{P}$$

(*iv*) $$V = \frac{P}{I} = \sqrt{PR}$$

Power is expressed in terms of kW (kilowatt = 1000 W) or MW (megawatt = 1000 kW or 10^6 W).

- ***Electrical energy.*** It *is the total amount of work done in an electric circuit.*

In other words, it is measured by the product of *power and time.*

i.e., $W = P \times t$ ...(From eqn. (8)]

or $W = VI\,t$

$= VQ$ joules

where Q = The quantity of electricity passing through the circuit in coulombs.

The unit of electrical energy is *joules or watt-sec.*

It is expressed in *kWh* (kilowatt hours)

1 *kWh* (commercial unit) = 1 kW × 1 hour = 1000 watt-hours

= 1000 × 60 × 60 watt-sec

= 3.6×10^6 watt-sec. or joules.

1.12.2. Linear and non-linear resistors. A ***linear resistor*** is *one which obeys ohm's law.* A circuit which contains only linear components is called a *linear circuit.*

*Such elements in which V/I (volt-ampere) plots are not straight lines but curves are called **non-linear resistors*** or ***non-linear elements***.

Examples of non-linear elements : Filaments of incandescent lamps, diodes, thermistors and varistors.

"Varistor (Non-linear resistor)" :

- It is a voltage-dependent metal-oxide material whose resistance decreases sharply with increasing voltage.
- The *zinc oxide-based varistors* are primarily used for protecting solid-state power supplies from low and medium size voltage in the supply line.
- *Silicon carbide varistors* provide protection against high-voltage surges caused by lightning and by discharge of electromagnetic energy stored in the magnetic fields of large coils.

1.12.3. Limitations of Ohm's law. Ohm's law *does not apply under the following conditions* :

1. Electrolytes where enormous gases are produced on either electrode.
2. Non-linear resistors like vacuum radio valves, semiconductors, gas filled tubes etc.
3. Arc lamps.
4. Metals which get heated up due to flow of current.
5. Appliances like metal rectifiers, crystal detectors, etc. in which operation depends on the direction of current.

1.13. An Electric Circuit

An "*electric circuit*" is a *conducting path through which either an electric current flows, or is intended to flow.* It can be divided into *four* categories :

(*i*) Closed circuit (*ii*) Open circuit

(*iii*) Short circuit (*iv*) Earth or leakage circuit.

(*i*) **Closed circuit.** It is the *complete path* for flow of electric current through the load.

Example. The glowing of a bulb.

(*ii*) **Open circuit.** In case any one of the supply wires is disconnected or the fuse burns out, then the current will *not flow* through the bulb, which is an *example of open circuit*.

(*iii*) **Short circuit.** If the supply mains are *connected directly* by *a piece of wire without any load,* then the value of current will be much greater than that in closed circuit. Hence, the fuse gets blown off and this circuit is known as *short circuit.*

(*iv*) **Earth or leakage circuit.** If any wire of supply mains *touches the 'body of an appliance'*, then it is known as earth or leakage circuit.

Depending upon the type of current flowing in it, an electric circuit may be further classified as :

(*i*) D.C. circuit (*ii*) A.C. circuit.

1.14. Resistances in Series

Fig. 9 shows three resistance connected in series. Obviously *current flowing through each resistance will be same* but *voltage drop across* each of them will *vary* as per value of individual resistance.

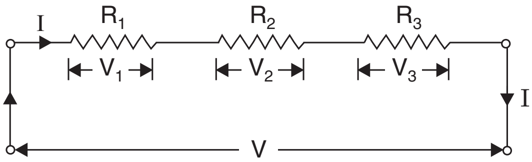


Fig. 9. Resistances in series.

Also the sum of all the voltage drops ($V_1 + V_2 + V_3$) is *equal to* the applied voltage (V).

i.e., $$V = V_1 + V_2 + V_3$$

$$IR = IR_1 + IR_2 + IR_3 \quad \text{(Using Ohm's law : } V = IR\text{)}$$

i.e., $$R = R_1 + R_2 + R_3 \quad ...(10)$$

where R is the *equivalent resistance* of series combination.

Also $$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} \quad ...[10(a)]$$

As seen from above, the *main characteristics of a series circuit* are :

1. Same current flows through all parts of the circuit.
2. Different resistors have their individual voltage drops.
3. Voltage drops are additive.
4. Applied voltage equals the sum of different voltage drops.
5. Resistances are additive.
6. Powers are additive.

1.14.1. Voltage divider rule. Since in a series circuit, same the same current flows in each of the given resistors, voltage drop varies directly with its resistance. Fig. 10 shows a 24 V battery connected across a series combination of three resistors.

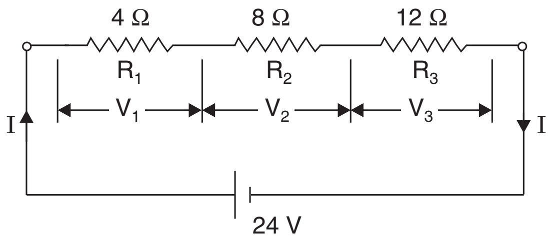


Fig. 10

Total resistance, $R = R_1 + R_2 + R_3 = 4 + 8 + 12 = 24\ \Omega$

According to **voltage divider rule**, various voltage drops are :

$$V_1 = V \times \frac{R_1}{R} = 24 \times \frac{4}{24} = 4\ \text{V}$$

$$V_2 = V \times \frac{R_2}{R} = 24 \times \frac{8}{24} = 8 \text{ V}$$

$$V_3 = V \times \frac{R_3}{R} = 24 \times \frac{12}{24} = 12 \text{ V}$$

1.15. Resistances in Parallel

Refer Fig. 11. In this case *voltage across each resistance will be same but current will be different* depending upon the value of the individual resistance.

i.e., $I = I_1 + I_2 + I_3$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{...(11)}$$

where R is the *equivalent resistance* of the parallel combination.

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \quad \text{...[11(a)]}$$

$$G = G_1 + G_2 + G_3 \quad \text{...(12)}$$

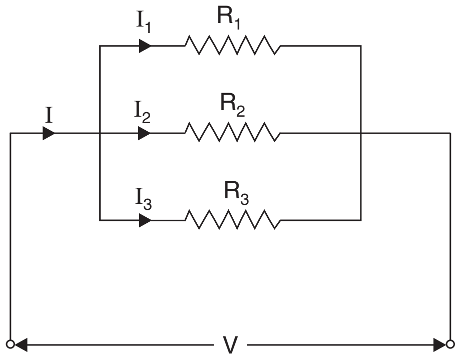


Fig. 11. Resistances in parallel.

The *main characteristics of a parallel circuit* are :

1. Same voltage acts across all parts of the circuit.
2. Different resistors have their individual current.
3. Branch currents are additive.
4. Conductances are additive.
5. Powers are additive.

1.15.1. Division of current in parallel circuits–Current divider rule. Fig. 12 shows two resistances R_1 and R_2 connected in parallel across a voltage V. According to Ohm's law, the current in each branch is given as :

$$I_1 = \frac{V}{R_1} \; ; \quad I_2 = \frac{V}{R_2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_2}{R_1} \quad \text{...(13)}$$

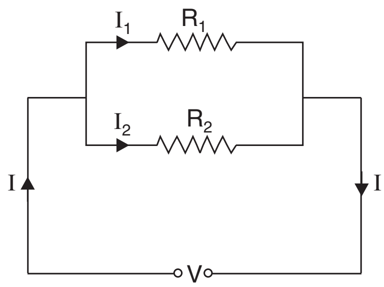


Fig. 12

Hence, the division of current in the branches of a parallel circuit is *inversely proportional to their resistances.*

This Current Divider Rule has direct application in solving electric circuits by Norton's theorem. We may also express the branch currents in terms of the total circuit current as follows :

Now, $I_1 + I_2 = I$

$\therefore$ $I_2 = I - I_1$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_1}{I - I_1} = \frac{R_2}{R_1}$$

or $$I_1 R_1 = R_2 (I - I_1) = IR_2 - I_1 R_2$$

or $$I_1(R_1 + R_2) = IR_2$$

or $$I_1 = I \times \frac{R_2}{R_1 + R_2} \quad \text{...(14)}$$

Similarly, $$I_2 = \frac{R_1}{R_1 + R_2} \quad \text{...(15)}$$

Refer Fig. 11 where three resistors R_1, R_2 and R_3 are connected in parallel across a voltage V, we have :

$$I_1 = I \times \left(\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) \quad \text{...(16)}$$

$$I_2 = I \times \left(\frac{R_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}\right) \quad \text{...(17)}$$

$$I_3 = I \times \left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) \quad \text{...(18)}$$

1.16. Superconductivity

Equation $R_t = R[1 + \alpha(t - 20)]$ holds good for temperature below 20°C. But at *very low temperature*, *some metals acquire zero electrical resistance and zero magnetic induction ; the property known as* ***superconductivity.***

Superconducting elements. Zinc, cadmium, mercury, lead.

Typical superconducting compounds and alloys : PbAu, $PbTl_2$, SnSb, CuS, NbN, NbB, ZrC.

The superconductivity will *disappear* if

(*i*) *The temperature of the material is raised above its critical temperature,*

Or

(*ii*) *a sufficiently strong magnetic field or current density is employed.*

1.17. Resistors

A resistor entails the following two main characteristics :

(*i*) Its *resistance (R)* in ohms. ...... The resistors are available from a fraction of an ohm to many mega ohms.

(*ii*) The *wattage rating*. ...... The power rating may be as high as several hundred watts or as low as $\frac{1}{10}$ watt. *Power rating indicates the maximum wattage the resistor can dissipate without excessive heat* (Too much heat can make the resistor burn open).

Classification of resistors :

The resistors are classified as follows :

1. *Fixed resistors.* The fixed resistor is the simplest type of resistor. Fixed means that the unit is so constructed that its resistance value is *constant* and *unchangeable*. These are made of a *carbon composition* and have a cover of black or brown hard plastics.

2. *Tapped resistors.* A tapped resistor is a resistor which has a tap, or connection, somewhere along the resistance material. These resistors are usually wire wound type. If they have more than one tap, they will have a separate terminal for each.

3. *Variable resistors.* A variable resistor has a movable contact that is used to adjust or select the resistance value between two or more terminals. A variable resistor is commonly called a *control.*

4. *Special resistors*. The most common type of special resistors is the fusible type. A *fusible resistor* have a definite resistance value and it protects the circuit much like a fuse. Another special resistor is the *temperature compensating unit.* Such resistors are used to provide special control of circuit that must be extremely stable in their operation.

Schematic symbols for various resistors are shown in Fig. 13.

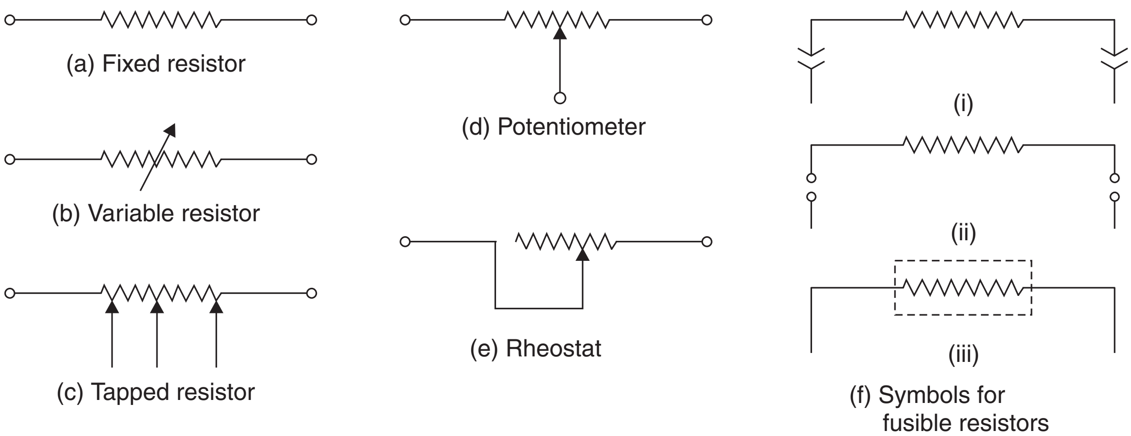


Fig. 13. Schematic symbols for various resistors.

The following types of resistors are used in electrical circuits :

(*i*) Carbon resistors.

(*ii*) Wire-wound resistors on ceramic or plastic forms (as in case of rheostats etc.)

(*iii*) Deposited carbon resistors on ceramic base.

(*iv*) Deposited metal resistors on ceramic base.

(*v*) Printed, painted or etched circuit resistors.

Example 14. *Three resistors are connected in series across a 12 V battery. The first resistance has a value of 2 Ω, second has a voltage drop of 4 V and third has a power dissipation of 12 W. Calculate the value of the circuit current.*

Solution. *Given* : $V = 12$ V ; $R_1 = 1\ \Omega$, $V_{R_2} = 4$ V ; $W_{R_3} = 12$ W

Let R_2, R_3 = Unknown resistances.

Circuit current, I :

$$I^2 R_3 = 12 \qquad ...(i)$$

$$IR_2 = 4 \quad \text{or} \quad I = \frac{4}{R_2} \qquad ...(ii)$$

and $$I^2 R_2^2 = 16 \qquad ...(iii)$$

∴ $$R_3 = \frac{3}{4} R_2^2 \qquad \text{[From (i) and (ii)]}$$

Also, $$I = \frac{4}{R_2} \quad ...(iv)$$

Now, $$I\,(R_1 + R_2 + R_3) = 12 \quad ...(v)$$

Inserting the values of I and R_3, we get

$$\frac{4}{R_2}\left(1 + R_2 + \frac{3}{4}R_2^{\,2}\right) = 12$$

or $$4 + 4R_2 + 3R_2^{\,2} = 12R_2 \quad \text{or} \quad 3R_2^{\,2} - 8R_2 + 4 = 0$$

or $$R_2 = \frac{8 \pm \sqrt{64 - 48}}{6} = \frac{8 \pm 4}{6} \quad \text{or} \quad 2\ \Omega, \frac{2}{3}\ \Omega$$

$\therefore$ $$R_3 = \frac{3}{4}R_2^{\,2} = \frac{3}{4} \times 2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4} \times \left(\frac{2}{3}\right)^2 = \frac{1}{3}\ \Omega$$

$\therefore$ $$I = \frac{12}{R_1 + R_2 + R_3} = \frac{12}{1 + 2 + 3} = \mathbf{2\ A.} \quad \textbf{(Ans.)}$$

or $$I = \frac{12}{1 + \frac{2}{3} + \frac{1}{3}} = \mathbf{6\ A.} \quad \textbf{(Ans.)}$$

Example 15. *In the Fig. 14 is shown a combination of resistances. If the voltage across L and M is 75 volts find :*

(*i*) *The effective resistance of the circuit.* (*ii*) *Voltage drop across each resistance.*

Solution. Refer Fig. 14.

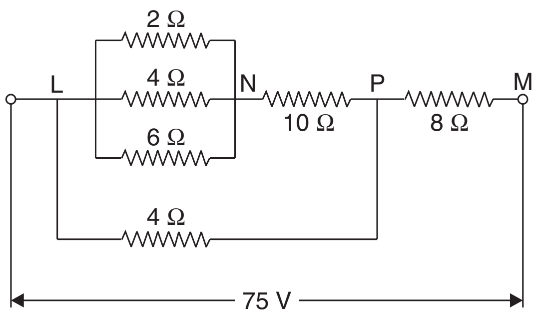


Fig. 14

(*i*) **Effective resistance of the circuit :**

Resistance between L and N,

$$\frac{1}{R_{LN}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6 + 3 + 2}{12} = \frac{11}{12}\ \Omega$$

$\therefore$ $$R_{LN} = \frac{12}{11}\ \Omega \quad ...\text{(Fig. 14)}$$

Resistance of branch LNP,

$$R_{LNP} = \frac{12}{11} + 10 = \frac{122}{11} = 11.09\ \Omega \quad ...\text{(Fig. 15)}$$

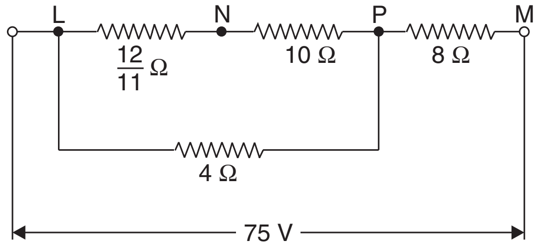


Fig. 15

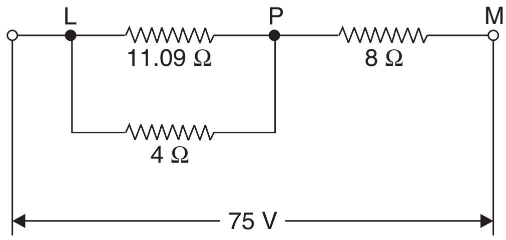


Fig. 16

Resistance between L and P

$$\frac{1}{R_{LP}} = \frac{1}{11.09} + \frac{1}{4}$$ [As there are two parallel paths between points L and P of resistances 11.09 Ω and 4 Ω]

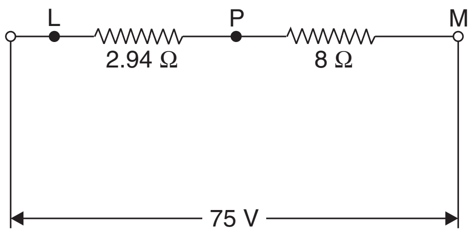


Fig. 17

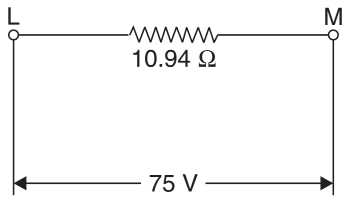


Fig. 18

$$= \frac{4 + 11.09}{44.36}$$

or $$R_{LP} = \frac{44.36}{15.09} = 2.94\ \Omega$$ ...(Fig. 16)

Effective resistance of the circuit,

$$R_{LM} = R_{LP} + R_{PM}$$
$$= 2.94 + 8 = 10.94\ \Omega$$

Hence effective resistance of the circuit **= 10.94 Ω. (Ans.)**

Total current through the circuit $= \frac{V}{R_{LM}} = \frac{75}{10.94} = 6.85$ A.

Current through 4 Ω resistor $= 6.85 \times \frac{11.09}{11.09 + 4} = 5.03$ A

Current in branch *LNP* $= 6.85 \times \frac{4}{11.09 + 4} = 1.82$ A

∴ *Voltage drop across 2, 4 and 6 Ω resistors*

$= 1.82 \times R_{LN} = 1.82 \times \frac{12}{11}$ = **1.98 V. (Ans.)**

Voltage drop across 10 Ω resistor = 1.82 × 10 = **18.2 V. (Ans.)**

Voltage drop across 4 Ω resistor = 5.03 × 4 = **20.12 V. (Ans.)**

Voltage drop across 8 Ω resistor = 6.85 × 8 = **54.8 V. (Ans.)**

Example 16. *In the series-parallel circuit shown in the Fig. 19, find :*

(*i*) *The total resistance of the circuit.*

(*ii*) *The total current flowing through the circuit.*

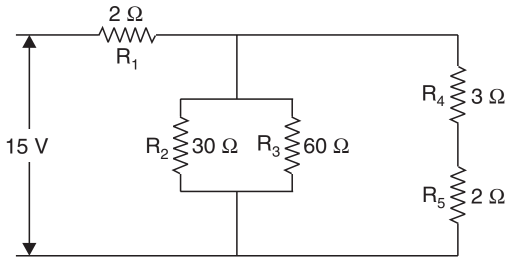


Fig. 19

Solution. (*i*) **Total resistance R_T :**

Resistors R_4 and R_5 are connected in series and have a net value of 5 Ω ($R_4 + R_5 = 3 + 2 = 5$) ...(Fig. 19)

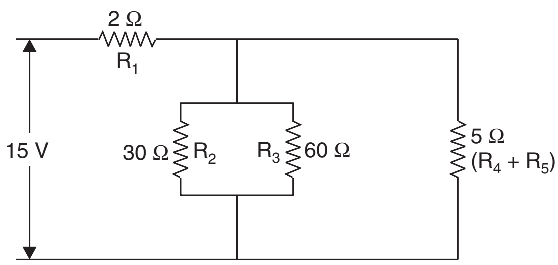


Fig. 20

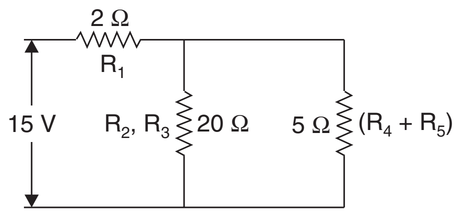


Fig. 21

The equivalent resistance of resistor R_2 and R_3 (connected in parallel)

$$= \frac{R_2 \times R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \Omega \qquad \text{...(Fig. 20)}$$

To simplify the circuit even more, we can combine the 20 Ω equivalent of R_2 and R_3, and 5 Ω equivalent of R_4 and R_5 (Fig. 21) :

$$R_{eq} = \frac{20 \times 5}{20 + 5} = \frac{100}{25} = 4\ \Omega$$

We now find the simple two-resistor series circuit shown in Fig. 22. This circuit includes a single resistance (4 Ω) which is equivalent of the series and parallel arrangement shown in Fig. 21 and R_2, R_3, R_4 and R_5. The total resistance of the circuit, R_T can now be found by addition :

$$R_T = R_1 + R_{eq} = 2 + 4 = \mathbf{6\ \Omega}. \quad \textbf{(Ans.)}$$

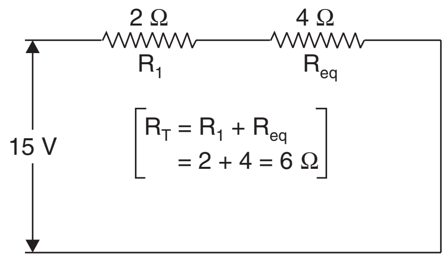


Fig. 22

(*ii*) **Total current, I_T :**

$$I_T = \frac{V}{R_T} = \frac{15}{6} = 2.5\ \text{A}$$

Hence total current **= 2.5 A. (Ans.)**

Example 17. *A resistance of 20 ohms is connected in series with a combination of two resistances arranged in parallel each of value 40 ohms. Determine the resistance R_3 which should be shunted across the parallel combination so that the total current drawn by the circuit is 1.5 A with applied voltage of 40 V.*

Solution. Refer Fig. 23.

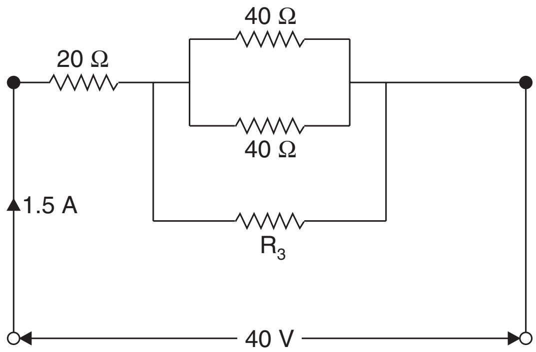


Fig. 23

Total resistance of the circuit, $R_T = \frac{V}{I} = \frac{40}{1.5} = 26.66$ ohms.

Series resistance $R = 20$ ohms

∴ Resistance of the parallel combination of resistances

$$R_T - R = 26.66 - 20 = 6.66 \text{ ohms}$$

Also resistance of parallel combination is given by :

$$\frac{1}{40} + \frac{1}{40} + \frac{1}{R_3} = \frac{1}{6.66} = 0.15$$

∴ $$\frac{1}{R_3} = 0.15 - \frac{1}{40} - \frac{1}{40} = 0.1$$

i.e., $\mathbf{R_3 = 10}$ **ohms. (Ans.)**

Example 18. *For the circuit shown in Fig. 24, find V_{CE} and V_{AG}.*

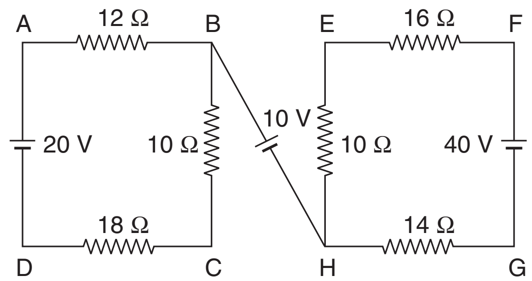


Fig. 24

Solution. Refer Fig. 24.

- Let us consider the two battery circuits separately.

Current in 20 V battery circuit $ABCD = \dfrac{20}{(12 + 10 + 18)} = 0.5$ A

Current in 40 V battery circuit $EFGH = \dfrac{40}{10 + 16 + 14} = 1$ A

Voltage drops over different resistors can be found by using Ohm's law.

- In order to find V_{CE} *i.e.*, voltage of point C w.r.t. point E, we shall start from E and go to C via points H and B. We shall find the algebraic sum of the voltage drops met on the way from point E to C.

$$V_{CE} = -10 \times 1 + 10 - 10 \times 0.5 = \mathbf{-5\ V.} \quad \textbf{(Ans.)}$$

The negative sign shows that point C *is negative w.r.t. point E.*

$$V_{AG} = 14 \times 1 + 10 + 12 \times 0.5 = \mathbf{30\ V.} \quad \textbf{(Ans.)}$$

The positive sign shows that point A is at a positive potential of 30 V w.r.t. point G.

Example 19. *Determine the current in and the voltage drop across each one of the resistors in Fig. 25.*

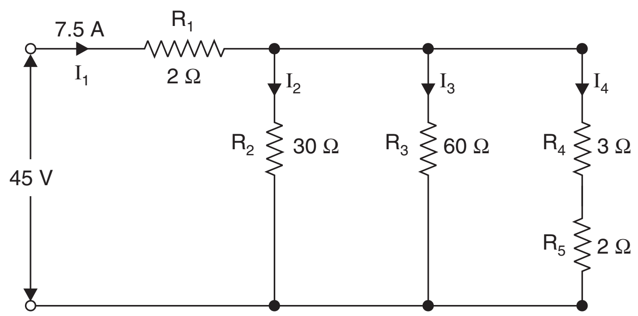


Fig. 25

Solution. The current from the source is *7.5 A* (I_1). This is the amount of current present in resistor R_1. The voltage drop across resistor R_1 is **15 V** (7.5 × 2). **(Ans.)**

- Now, subtract this voltage (15 V) from the supply voltage (45 V) to determine the voltage across the three parallel branches on the right side of resistor R_1. This is of course 30 V (45 – 15).
- Knowing the voltage across the parallel branches, we can determine the currents present in resistors R_2 and R_3 as follows :

$$I_2 = \frac{V}{R_2} = \frac{30}{30} = \mathbf{1\ A.} \quad \textbf{(Ans.)}$$

$$I_3 = \frac{V}{R_3} = \frac{30}{60} = \mathbf{0.5\ A.} \quad \textbf{(Ans.)}$$

The total current that is present in the parallel combination of R_2 and R_3 is

1.5 (1 + 0.5 = 1.5).

- The current present in the series combination $R_4 + R_5$ is

$$I_4 = \frac{V}{R_4 + R_5} = \frac{30}{3+2} = \mathbf{6\ A.} \quad \textbf{(Ans.)}$$

Note that the sum of the three currents (three parallel branches) is equal the supply current of 7.5 A (1 + 0.5 + 6 = 7.5).

- When the series current in the $R_4 + R_5$ combination is known, it is possible to determine the voltage drop across each resistor as follows :

$$V_{R_4} = I_4 \times R_4 = 6 \times 3 = \mathbf{18\ V.} \quad \textbf{(Ans.)}$$

$$V_{R_5} = I_5 \times R_5 = 6 \times 2 = \mathbf{12\ V.} \quad \textbf{(Ans.)}$$

Check the sum of the two voltage drops to be sure they equal 30 V (18 + 12 = 30).

1.18. Insulation Resistance of a Cable

Fig. 26 shows a single-core cable of inner and outer radii r_1 and r_2 respectively and length l. Consider an annular ring of radius 'x' and radial thickness 'dx'. If the volume resistivity of the material is ρ, then resistance of this narrow ring (dR) will be :

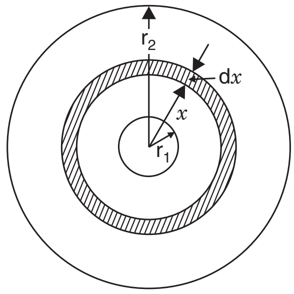


Fig. 26. Single-core cable.

$$dR = \frac{\rho dx}{2\pi x l}$$

and the insulation resistance of the whole cable will be :

$$\int dR = \int_{r_1}^{r_2} \frac{\rho dx}{2\pi x l}$$

or $$R = \frac{\rho}{2\pi l} \left| \log_e x \right|_{r_1}^{r_2}$$

or $$R = \frac{\rho}{2\pi l} . \log_e \frac{r_2}{r_1}\ \Omega \qquad ...(19)$$

$$= \frac{2.3\rho}{2\pi l} . \log_{10} \frac{r_2}{r_1} \qquad ...[19\ (a)]$$

In the above equation it may be noted that insulation resistance varies *inversely as the length* (whereas the *conductor resistance varies directly as the length*).

Example 20. *A single cable has a conductor (copper) diameter of 1 cm and an overall diameter of 2 cm. If the resistivity of the insulating material is 3 × 10⁶ M Ω-m, calculate the insulation resistance of 3 km length of the cable.*

What will be the conductor resistance of this cable length at a temperature of 20°C if the resistivity of copper is 1.73 × 10⁻⁸ Ω-m at 20°C ?

Solution. Inner radius of the cable, $r_1 = \frac{1}{2} = 0.5$ cm = 0.005 m.

Outer radius of the cable, $r_2 = 2/2 = 1$ cm = 0.01 m.

Length of the cable, $l = 3$ km = 3000 m.

Resistivity of insulating material, $\rho_i = 3 \times 10^6$ MΩ-m.

Resistivity of copper, $\rho_c = 1.73 \times 10^{-8}$ Ω-m (at 20°C)

Insulation resistance, R_i :

$$\mathbf{R_i} = \frac{\rho_i}{2\pi l} \log_e \frac{r_2}{r_1}$$

$$= \frac{3 \times 10^6}{2\pi \times 3000} \log_e \left(\frac{0.01}{0.005}\right) = \mathbf{110.3\ M\ \Omega.} \quad \textbf{(Ans.)}$$

Conductor resistance at 20°C, R :

$$R = \frac{\rho l}{a} = \frac{1.73 \times 10^{-8} \times 3000}{\pi/4 \times (0.01)^2} = \mathbf{0.66\ \Omega.} \quad \textbf{(Ans.)}$$

☞ **Example 21.** *At 20°C, the insulation resistance of a cable, 600 metres in length is 480 mega-ohms and copper conductor resistance of the cable is 0.7 ohm. If the insulating material has its resistance halved by a temperature rise of 10°C, find for 1600 metres length of the cable at 36°C.*

(*i*) *The insulation resistance.*

(*ii*) *The resistance of copper conductor. The temperature co-efficient of copper at 20°C = 0.0043 / °C.*

Solution. (*i*) **Insulation resistance at 36°C :**

Insulation resistance of 1600 metres long cable at 20°C,

$$R_1 = \frac{480 \times 600}{1600} = 180\ \text{M}\Omega$$

(∵ Insulation resistance is *inversely* proportional to *length*)

Now, $t_1 = 20°\text{C}$ and $t_2 = 36°\text{C}$

The insulation resistance R_2 at temperature t_2°C can be found out by using the expression :

$$\log_e R_2 = \log_e R_1 - k(t_2 - t_1) \quad \text{...}(i)$$

where $k = \dfrac{\log_e m}{t'}$

and t' = Rise of temperature required to reduce the insulation resistance to $\dfrac{1}{m}$ of the resistance at initial temperature (*i.e.*, $m = 2$ in this case)

Here $t' = 10°\text{C}$

and the value of k becomes : $k = \dfrac{\log_e 2}{10} = 0.0693$

Now, substituting the various values in eqn. (*i*), we get

$$\log_e R_2 = \log_e 180 - 0.0693\ (36 - 20) = 5.193 - 1.1088$$

i.e., $\mathbf{R_2 = 59.4\ \Omega.}$ **(Ans.)**

(*ii*) **Resistance of copper conductor at 36°C :**

Resistance of conductor for 1600 metres of its length at 20°C,

$$R_{20} = \frac{0.7 \times 1600}{600}$$

(∵ Conductor resistance is *directly* proportional to the *length*)

$$= 1.866\ \Omega$$

Resistance at 36°C, $R_{36} = R_{20}\ [1 + \alpha_{20}\ (360 - 20)]$

$$= 1.866\ [1 + 0.0043 \times 16] = \mathbf{1.99\ \Omega.} \quad \textbf{(Ans.)}$$

2. DEFINITIONS OF IMPORTANT TERMS RELATING NETWORK

1. **Circuit.** A *conducting path* through which an electric current either flows or is intended to flow is called a *circuit*. The various elements of an electric circuit are called *parameters (e.g.* resistance, inductance and capacitance). These parameters may be *distributed* or *lumped*.

2. **Linear circuit.** The circuit whose parameters are *constant* (*i.e.*, they do not change with voltage or current) is called a *linear circuit*.

3. **Non-linear circuit.** The circuit whose parameters *change* with voltage or current is called a *non-linear circuit*.

4. **Unilateral circuit.** A unilateral circuit is one whose properties or characteristics change with the direction of its operation (*e.g. diode rectifier*).

5. **Bilateral circuit.** It is that circuit whose properties or characteristics are same in either direction (*e.g. transmission line*).

6. **Electric network.** An electric network arises when a number of parameters or electric elements coexist or combine in any manner or arrangement.

7. **Active network.** An *active network* is one which contains one or more than one sources of *e.m.f.*

8. **Passive network.** A *passive network* is one which does not contain any source of e.m.f.

9. **Node.** A *node* is a junction in a circuit where two or more circuit elements are connected together.

10. **Branch.** The part of a network which lies between two junctions is called **branch.**

3. LIMITATIONS OF OHM'S LAW

In a series circuit or in any branch of a simple parallel circuit the calculation of the current is easily effected by the direct application of Ohm's law. But such a simple calculation is not possible if one of the branches of a parallel circuit contains a source of e.m.f., or if the current is to be calculated in a part of a network in which sources of e.m.f. may be present in several meshes or loops forming the network. The treatment of such cases is effected by the application of fundamental principles of electric circuits. These principles were correlated by Kirchhoff many years ago and enunciated in the form of *two laws*, which can be considered as the foundations of circuit analysis. Other, later, methods have been developed, which when applied to special cases considerably shorten the algebra and arithmetic computation compared with the original Kirchhoff's method.

4. KIRCHHOFF'S LAWS

For complex circuit computations, the following two laws first stated by Gutsav R. Kirchhoff (1824–87) are indispensable.

***(i)* Kirchhoff's Point Law or Current Law (KCL).** It states as follows :

The sum of the currents entering a junction is equal to the sum of the currents leaving the junction. Refer Fig. 27.

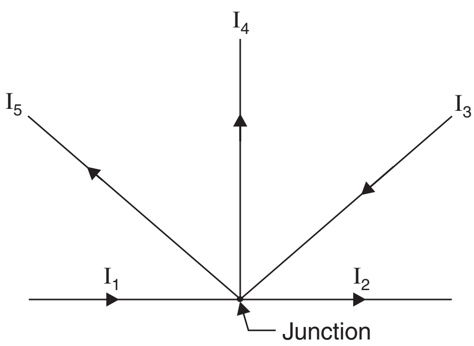


Fig. 27

If the currents *towards* a junction are considered *positive* and those *away* from the same junction *negative*, then this law states that the *algebraic sum of all currents meeting at a common junction is zero*.

i.e., Σ Currents entering = Σ Currents leaving

$$I_1 + I_3 = I_2 + I_4 + I_5 \quad ...[20\ (a)]$$

or

$$I_1 + I_3 - I_2 - I_4 - I_5 = 0 \quad ...[20\ (b)]$$

***(ii)* Kirchhoff's Mesh Law or Voltage Law (KVL).** It states as follows :

The sum of the e.m.fs (rises of potential) around any closed loop of a circuit equals the sum of the potential drops in that loop.

Considering a rise of potential as positive (+) and a drop of potential as negative (–), the *algebraic sum* of potential differences (voltages) around a closed loop of a circuit is zero :

$$\Sigma E - \Sigma IR \text{ drops} = 0 \text{ (around closed loop)}$$

i.e., $$\Sigma E = \Sigma IR \qquad ...[21\ (a)]$$

or $$\Sigma \text{ Potential rises} = \Sigma \text{ Potential drops} \qquad ...[21\ (b)]$$

To apply this law in practice, assume an arbitrary current direction for each branch current. The end of the resistor through which the current enters, is then positive, with respect to the other end. *If the solution for the current being solved turns out negative, then the direction of that current is opposite to the direction assumed.*

In tracing through any single circuit, whether it is by itself or a part of a network, the following **rules** must be applied :

1. A *voltage drop exists* when tracing through a resistance *with or in the same direction as the current*, or through a battery or generator against their voltage, that is from *positive (+) to negative (–)*. Refer Fig. 28.

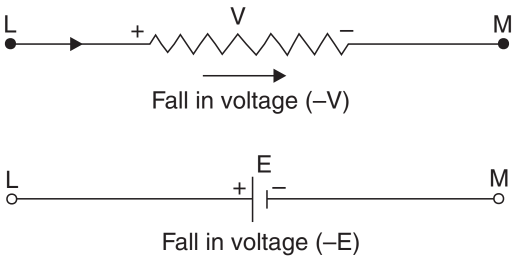


Fig. 28

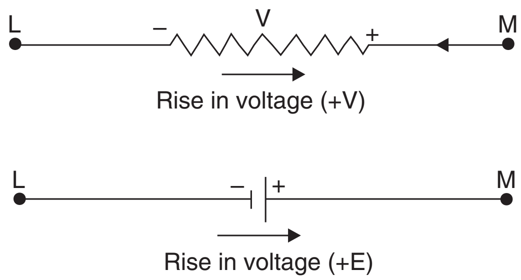


Fig. 29

2. A *voltage rise exists* when tracing through a resistance *against* or *in opposite direction to the current* or through a battery or a generator with their voltage that is from *negative (–) to positive (+)*. Refer Fig. 29.

Illustration. Consider a circuit shown in Fig. 30.

Considering the loop **ABEFA**, we get

$$-I_1R_1 - I_3R_3 + E_1 = 0$$

or $$E_1 = I_1R_1 + I_3R_3 \quad \text{(where } I_3 = I_1 + I_2\text{)} \quad ...(i)$$

Considering the loop **BCDEB**, we have

$$I_2R_2 - E_2 + I_3R_3 = 0 \qquad ...(ii)$$

or $$E_2 = I_2R_2 + I_3R_3$$

If E_1, E_2, R_1, R_2 and R_3 are known, then I_1, I_2 and I_3 can be calculated from eqns. (*i*) and (*ii*).

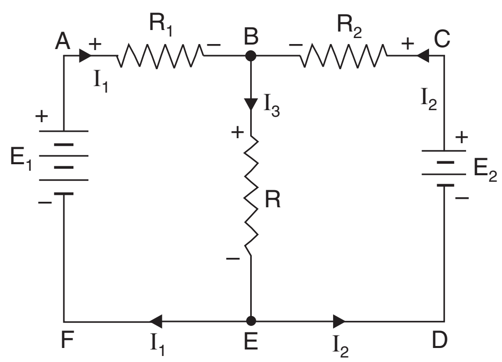


Fig. 30

5. APPLICATIONS OF KIRCHHOFF'S LAWS

Kirchhoff's laws may be employed in the following methods of solving networks :

1. Branch-current method
2. Maxwell's loop (or mesh) current method
3. Nodal voltage method.

5.1. Branch-Current Method

For a multi-loop circuit the following ***procedure*** is adopted for writing equations :

1. Assume currents in different branch of the network.

2. Write down the smallest number of voltage drop loop equations so as to include all circuit elements ; these loop equations are independent.

If there are n nodes of three or more elements in a circuit, then write $(n - 1)$ equations as per current law.

3. Solve the above equations simultaneously.

The assumption made about the directions of the currents initially is arbitrary. In case the actual direction is *opposite to the assumed one*, it will be reflected as a negative value for that current in the answer.

The branch-current method (the most primitive one) involves more labour and is not used *except for very simple circuits.*

Example 22. *In the circuit of Fig. 31, find the current through each resistor and voltage drop across each resistor.*

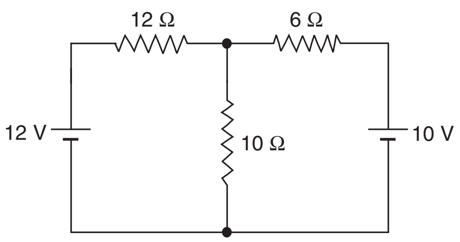


Fig. 31

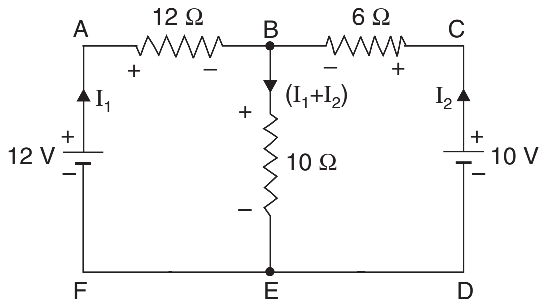


Fig. 32

Solution. Let the currents be as shown in Fig. 32.

Applying Kirchhoff's voltage law to the circuit **ABEFA**, we get

$$-12I_1 - 10(I_1 + I_2) + 12 = 0$$

$$-22I_1 - 10I_2 + 12 = 0$$

or

$$+11I_1 + 5I_2 - 6 = 0 \quad ...(i)$$

Circuit **BCDEB** gives,

$$6I_2 - 10 + 10(I_1 + I_2) = 0$$

$$10I_1 + 16I_2 - 10 = 0$$

$$5I_1 + 8I_2 - 5 = 0 \quad ...(ii)$$

Multiplying eqn. (*i*) by 5 and eqn. (*ii*) by 11 and subtracting, we get

$$55I_1 + 25I_2 - 30 = 0$$
$$55I_1 + 88I_2 - 55 = 0$$
$$- \quad - \quad +$$

$$-63I_2 + 25 = 0$$

i.e., $$I_2 = 0.397 \text{ A}$$

Substituting this value in eqn. (*i*), we get

$$11I_1 + 5 \times 0.397 - 6 = 0$$

i.e., $$I_1 = 0.365 \text{ A}$$

Hence, **Current through 12 Ω resistor, I_1 = 0.365 A. (Ans.)**

Current through 6 Ω resistor, I_2 = 0.397 A. (Ans.)

Current through 10 Ω resistor, $I_1 + I_2$ = 0.762 A. (Ans.)

The voltage drop across :

12 Ω resistor = 0.365 × 12 = **4.38 V. (Ans.)**

6 Ω resistor = 0.397 × 6 = **2.38 V. (Ans.)**

10 Ω resistor = 0.762 × 10 = **7.62 V. (Ans.)**

Example 23. *Find the magnitude and direction of currents in each of the batteries shown in Fig. 33.*

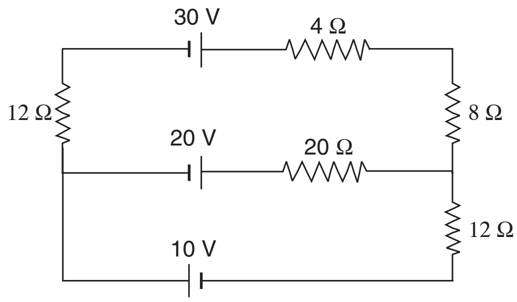


Fig. 33

Solution. Let the directions of currents I_1, I_2 and I_3 in the batteries be as shown in Fig. 34.

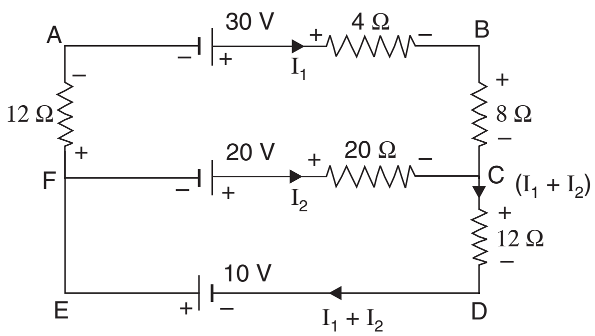


Fig. 34

Applying Kirchhoff's voltage law to the circuit **ABCFA**, we get

$$30 - 4I_1 - 8I_1 + 20I_2 - 20 - 12I_1 = 0$$

$$-24I_1 + 20I_2 + 10 = 0$$

or $$12I_1 - 10I_2 - 5 = 0 \quad ...(i)$$

Circuit **ECDEF** gives,

$$20 - 20I_2 - 12(I_1 + I_2) + 10 = 0$$

$$20 - 20I_2 - 12I_1 - 12I_2 + 10 = 0$$

$$-12I_1 - 32I_2 + 30 = 0$$

or $$6I_1 + 16I_2 - 15 = 0 \quad ...(ii)$$

Multiplying eqn. (*ii*) by 2 and subtracting it from (*i*), we get

$$-42I_2 + 25 = 0$$

i.e., $$I_2 = 0.595 \text{ A}$$

Substituting this value of I_2 in eqn. (*i*), we get

$$12I_1 - 10 \times 0.595 - 5 = 0$$

or $$I_1 = 0.912 \text{ A}$$

Hence current through,

30 V battery, I_1 = **0.912 A. (Ans.)**

20 V battery, I_2 = **0.595 A. (Ans.)**

10 V battery, $(I_1 + I_2)$ = **1.507 A. (Ans.)**

Example 24. *The terminal resistances of batteries A and B are 2.5 Ω and 2 Ω respectively. The battery has an e.m.f. of 20 volts. A resistance of 10 Ω is connected across the battery terminals. Calculate :*

(*i*) *The discharge current of battery A, the discharge current of battery B being 1.75 A.*

(*ii*) *The e.m.f. of battery B.*

(*iii*) *The energy dissipated in 10 Ω resistance in 40 minutes.*

Solution. Refer Fig. 35.

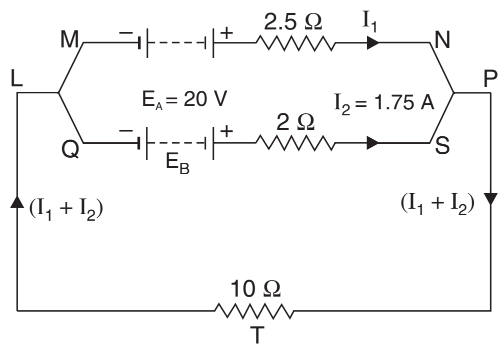


Fig. 35

(*i*) **I_1 :**

Applying Kirchhoff's voltage law to the circuit **LMNPSQL**

$$20 - 2.5I_1 + 1.75 \times 2 - E_B = 0$$

i.e., $$E_B + 2.5I_1 - 23.5 = 0 \quad ...(i)$$

Circuit **LMNPTL** gives,

$$20 - 2.5I_1 - 10(I_1 + I_2) = 0$$

i.e., $$20 - 2.5I_1 - 10(I_1 + 1.75) = 0$$

or $$20 - 12.5I_1 - 17.5 = 0$$

or $$I_1 = 0.2 \text{ A.}$$

Hence discharge current of battery A = $\mathbf{I_1}$ **= 0.2 A. (Ans.)**

(*ii*) $\mathbf{E_B}$ **:**

Substituting the value of I_1 in eqn. (*i*), we get

$$E_B + 2.5 \times 0.2 - 23.5 = 0$$

$\therefore$ $$E_B = 23 \text{ V}$$

Hence e.m.f. of battery B, E_B **= 23 V. (Ans.)**

(*iii*) **Energy dissipated in 10 Ω resistor :**

Energy dissipated $= I^2Rt$

$= (I_1 + I_2)^2 \times 10 \times (40 \times 60)$ joules

$= (0.2 + 1.75)^2 \times 10 \times (40 \times 60)$ joules

= **91260 joules. (Ans.)**

Example 25. *A battery having an e.m.f. of 110 V and an internal resistance of 0.2 Ω is connected in parallel with another battery with e.m.f. of 100 V and a resistance of 0.25 Ω. The two in parallel are placed in series with a regulating resistance of 5 ohms and connected across 220 V mains. Calculate :*

(*i*) *The magnitude and direction of the current in each battery.*

(*ii*) *The total current taken from the mains supply.*

Solution. Refer Fig. 36.

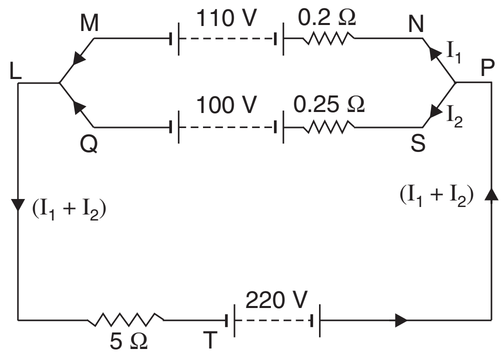


Fig. 36

(*i*) $\mathbf{I_1}$ **:,** $\mathbf{I_2}$ **:**

Let the directions of flow of currents I_1 and I_2 be as shown in Fig. 36.

Applying Kirchhoff's voltage law to **LMNPSQL**, we get

$$110 + 0.2I_1 - 0.25I_2 - 100 = 0$$

$$0.2I_1 - 0.25I_2 = -10$$

or $$I_1 - 1.25I_2 = -50 \quad ...(i)$$

Circuit **LMNPTL** gives,

$$110 + 0.2I_1 - 220 + 5(I_1 + I_2) = 0$$

$$5.2I_1 + 5I_2 = 110$$

$$I_1 + 0.96I_2 = 21.15 \qquad ...(ii)$$

Subtracting (*ii*) from (*i*), we get

$$-2.21I_2 = -71.15$$

$\therefore$ $\mathbf{I_2 = 32.19}$ **A. (Ans.)**

and $\mathbf{I_1 = -9.75}$ **A. (Ans.)**

Since I_1 turns out to be negative, its actual direction of flow is *opposite* to that shown in Fig. 36. In other words it *is not a charging current but a discharging one*. However, I_2 is a *charging current*.

(*ii*) $\mathbf{(I_1 + I_2)}$ **:**

The total current taken from the mains supply,

$$I_1 + I_2 = -9.75 + 32.19 = \mathbf{22.44\ A.\ (Ans.)}$$

Example 26. *In the circuit shown in the Fig. 37 determine :*

(*i*) *All the currents in the network.*

(*ii*) *Voltages between the points.*

Solution. Refer Fig. 37.

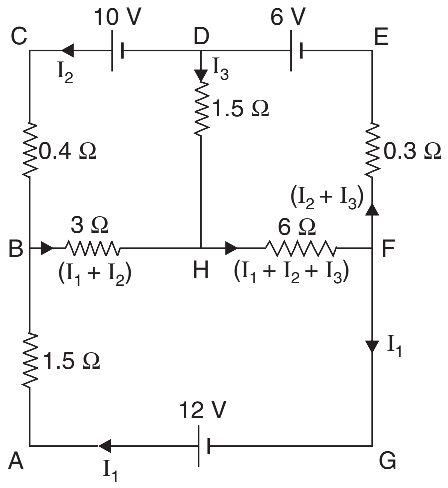


Fig. 37

(*i*) **All the currents in the network :**

Let the directions of the currents be as shown in Fig. 37.

Applying Kirchhoff's voltage law to the circuit **BCDHB**, we get

$$0.4I_2 - 10 - 1.5I_3 + 3(I_1 + I_2) = 0$$

i.e., $$3I_1 + 3.4I_2 - 1.5I_3 = 10 \qquad ...(i)$$

Circuit **HDEFH** gives,

$$1.5I_3 - 6 + 0.3(I_2 + I_3) + 6(I_1 + I_2 + I_3) = 0$$

$$6I_1 + 6.3I_2 + 7.8I_3 = 6 \qquad ...(ii)$$

Circuit **ABHFGA** gives,

$$-1.5I_1 - 3(I_1 + I_2) - 6(I_1 + I_2 + I_3) + 12 = 0$$

$$-10.5I_1 - 9I_2 - 6I_3 + 12 = 0$$

i.e., $$10.5I_1 + 9I_2 + 6I_3 = 12 \quad ...(iii)$$

Multiplying eqn. (*ii*) by 2 and subtracting eqn. (*ii*) from eqn. (*i*), we get

$$0.5I_2 - 10.8I_3 = 14 \quad ...(iv)$$

Multiplying eqn. (*ii*) by 10.5 and eqn. (*iii*) by 6 and subtracting eqn. (*iii*) from eqn. (*ii*), we get

$$12.15I_2 + 45.9I_3 = -9 \quad ...(v)$$

Multiplying eqn. (*iv*) by 12.15 and eqn. (*v*) by 0.5 and subtracting eqn. (*v*) from eqn. (*iv*), we get

$$-154.17\ I_3 = 174.6$$

$\therefore$ $$I_3 = -1.132 \text{ A}$$

Substituting the value of I_3 in eqn. (*iv*), we get

$$0.5I_2 - 10.8 \times (-1.132) = 14$$

i.e., $$I_2 = 3.549 \text{ A}$$

Substituting the values of I_2 and I_3 in eqn. (*iii*), we get

$$10.5I_1 + 9 \times 3.549 + 6 \times (-1.132) = 12$$

i.e., $$I_1 = -1.252 \text{ A}$$

Hence the directions of I_1 and I_3 are actually *opposite to the assumed directions*.

The *current between B and H* = $I_1 + I_2 = -1.252 + 3.549 =$ **2.297 A. (Ans.)**

The *current between H and F* = $I_1 + I_2 + I_3 = -1.252 + 3.549 - 1.132 =$ **1.165 A. (Ans.)**

The current between H and F (and E) = $I_1 + I_3 = 3.549 - 1.132 =$ **2.417 A. (Ans.)**

(*ii*) **Voltage between the points :**

Voltage across BH = 2.297 × 3 = **6.891 V. (Ans.)**

Voltage across HF = 1.165 × 6 = **6.99 V. (Ans.)**

Voltage across CE = 12 – (– 1.252 × 1.5) = **13.878 V. (Ans.)**

Example 27. *Determine the magnitude and direction of flow of current in the branch MN for the circuit shown in Fig. 38.*

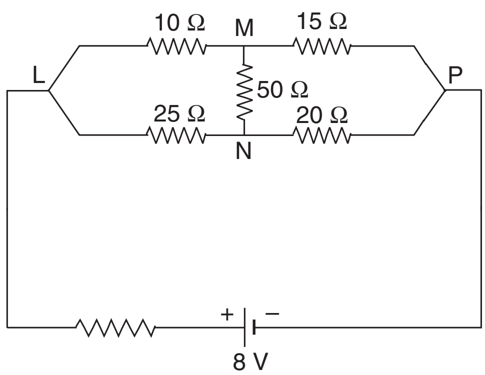


Fig. 38

Solution. Refer Fig. 39.

Let the directions and magnitudes of the currents flowing in the various circuits be as shown in Fig. 39.

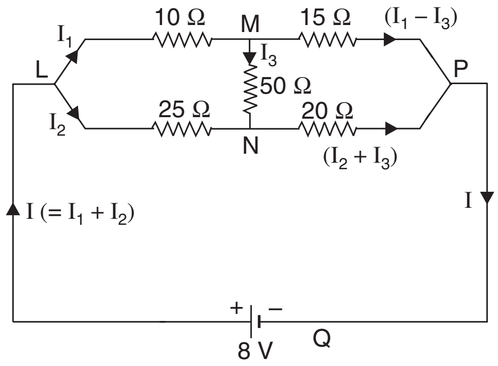


Fig. 39

Applying Kirchhoff's voltage law to the circuit **LMNL**, we get

$$-10I_1 - 50I_3 + 25I_2 = 0$$

or $$I_1 - 2.5I_2 + 5I_3 = 0 \quad ...(i)$$

Circuit **MPNM** gives,

$$-15(I_1 - I_3) + 20(I_2 + I_3) + 50I_3 = 0$$

$$-15I_1 + 20I_2 + 85I_3 = 0$$

or $$I_1 - 1.33I_2 - 5.66I_3 = 0 \quad ...(ii)$$

Circuit **LNPQL** gives,

$$-25I_2 - 20(I_2 + I_3) + 8 = 0$$

$$-45I_2 - 20I_3 + 8 = 0$$

or $$I_2 + 0.44I_3 = 0.177 \quad ...(iii)$$

Subtracting eqn. (*ii*) from eqn. (*i*), we get

$$-1.17I_2 + 10.66I_3 = 0$$

or $$I_2 - 9.11I_3 = 0 \quad ...(iv)$$

Subtracting eqn. (*iv*) from eqn. (*iii*), we get

$$9.55I_3 = 0.177$$

$$I_3 = 0.0185 \text{ A.}$$

Hence magnitude of current (I_3) flowing through the branch MN = 0.0185 A (from M to N). (Ans.)

Example 28. *Determine the current in the 4 Ω resistance of the circuit shown in Fig. 40.*

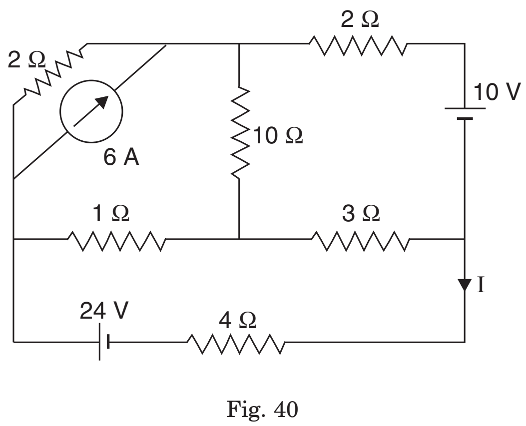


Fig. 40

Solution. Refer Fig. 41.

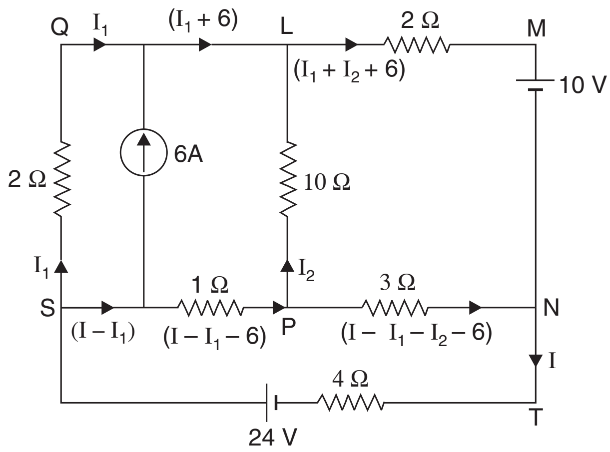


Fig. 41

Let the directions of various currents in different circuits be as shown in Fig. 41.

Applying Kirchhoff's voltage law to the circuit **SQLPS**, we get

$$-2I_1 + 10I_2 + 1(I - I_1 - 6) = 0$$

$$I - 3I_1 + 10I_2 = 6 \quad ...(i)$$

Circuit **LMNPL** gives,

$$-2(I_1 + I_2 + 6) - 10 + 3(I - I_1 - I_2 - 6) - 10I_2 = 0$$

$$3I - 5I_1 - 15I_2 = 40 \quad ...(ii)$$

Circuit **SPNTS** gives,

$$-1(I - I_1 - 6) - 3(I - I_1 - I_2 - 6) - 4I + 24 = 0$$

$$-8I + 4I_1 + 3I_2 = -48$$

or

$$8I - 4I_1 - 3I_2 = 48 \quad ...(iii)$$

Multiplying eqn. (*i*) by 3 and subtracting eqn. (*ii*) from eqn. (*i*), we get

$$-4I_1 + 45I_2 = -22$$

or

$$I_1 - 11.25I_2 = 5.5 \quad ...(iv)$$

Multiplying eqn. (*i*) by 8 and subtracting eqn. (*iii*) from eqn. (*i*), we get

$$-20I_1 + 83I_2 = 0 \quad ...(v)$$

Multiplying eqn. (*iv*) by 20 and adding eqn. (*v*), we get

$$-142I_2 = 110$$

∴

$$I_2 = -0.774 \text{ A}$$

and

$$I_1 = -3.212 \text{ A}$$

Substituting the values of I_1 and I_2 in eqn. (*i*), we get

$$I - 3 \times (-3.212) + 10 \times (-0.774) = 6$$

i.e., **I = 23.37 A. (Ans.)**

Example 29. *Determine the current in each of the resistors of the network shown in Fig. 42.*

Solution. Refer Fig. 42.

Let the current directions be as shown in Fig. 42.

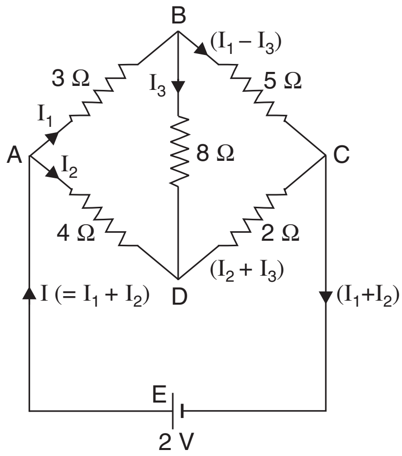


Fig. 42

Applying Kirchhoff's voltage law to the circuit **ABDA**, we get

$$-3I_1 - 8I_3 + 4I_2 = 0$$

or $$3I_1 - 4I_2 + 8I_3 = 0 \quad ...(i)$$

Circuit **BCDB** gives,

$$-5(I_1 - I_3) + 2(I_2 + I_3) + 8I_3 = 0$$

$$-5I_1 + 2I_2 + 15I_3 = 0$$

$$5I_1 - 2I_2 - 15I_3 = 0 \quad ...(ii)$$

Circuit **ADCEA** gives,

$$-4I_2 - 2(I_2 + I_3) + 2 = 0$$

$$-6I_2 - 2I_3 = -2$$

or $$3I_2 + I_3 = 1 \quad ...(iii)$$

Multiplying eqn. (*i*) by 5 and eqn. (*ii*) by 3 and subtracting (*ii*) from (*i*), we get

$$15I_1 - 20I_2 + 40I_3 = 0$$

$$15I_1 - \ \ 6I_2 - 45I_3 = 0$$

$$- \qquad + \qquad +$$

$$-14I_2 + 85I_3 = 0$$

or $$14I_2 - 85I_3 = 0 \quad ...(iv)$$

Multiplying eqn. (*iii*) by 14 and eqn. (*iv*) by 3 and subtracting (*iv*) from (*iii*), we get

$$42I_2 + \ \ 14I_3 = 14$$

$$42I_2 - 255I_3 = 0$$

$$- \qquad +$$

$$269I_3 = 14$$

∴ $I_3 = 0.052$ A

From eqn. (*iv*), $I_2 = 0.316$ A

From eqn. (*i*), $I_1 = 0.283$ A

Hence, Current through 3 Ω resistor = $\mathbf{I_1}$ **= 0.283 A. (Ans.)**

Current through 4 Ω resistor = $\mathbf{I_2}$ **= 0.316 A. (Ans.)**

Current through 8 Ω resistor = $\mathbf{I_3}$ **= 0.052 A. (Ans.)**

Current through 5 Ω resistor = $I_1 - I_3$ = **0.231 A. (Ans.)**

Current through 2 Ω resistor = $I_2 + I_3$ = **0.368 A. (Ans.)**

Example 30. *Determine the branch currents in the network of Fig. 43.*

Solution. Refer Fig. 43. Let the current directions be as shown.

Applying Kirchhoff's voltage law to the circuit **ABDA**, we get

$$5 - I_1 \times 1 - I_3 \times 1 + I_2 \times 1 = 0$$

or $$I_1 - I_2 + I_3 = 5 \quad ...(i)$$

Circuit **BCDB** gives,

$$-(I_1 - I_3) \times 1 + 5 + (I_2 + I_3) \times 1 + I_3 \times 1 = 0$$

$$-I_1 + I_2 + 3I_3 = -5$$

or $$I_1 - I_2 - 3I_3 = 5 \quad ...(ii)$$

Circuit **ADCEA** gives,

$$-I_2 \times 1 - (I_2 + I_3) \times 1 + 10 - (I_1 + I_2) \times 1 = 0$$

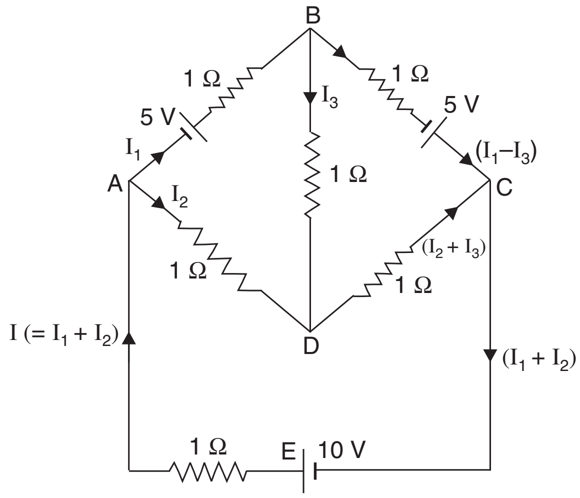


Fig. 43

$$-I_1 - 3I_2 - I_3 = -10$$

or $$I_1 + 3I_2 + I_3 = 10 \quad ...(iii)$$

Subtracting (*ii*) from (*i*), we get $I_3 = 0$

From (*i*), we have $I_1 - I_2 = 5 \quad ...(iv)$

and from (*iii*), we have $I_1 + 3I_2 = 10 \quad ...(v)$

By solving (*iv*) and (*v*), we get $I_2 = 1.25$ A

and $I_1 = 6.25$ A

Hence, Current in branch *AB* = **6.25 A. (Ans.)**

Current in branch *BC* = **6.25 A. (Ans.)**

Current in branch *BD* = **0. (Ans.)**

Current in branch *AD* = **1.25 A. (Ans.)**

Current in branch *DC* = **1.25 A. (Ans.)**

Current in branch *CA* = **7.5 A. (Ans.)**

Example 31. *Find the current in the galvanometer arm of the wheatstone bridge shown in Fig. 44.*

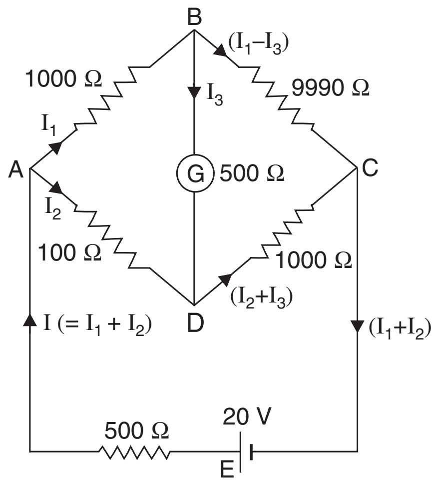


Fig. 44

Solution. Refer Fig. 44. Let the current directions be as shown.

Applying Kirchhoff's voltage law to the circuit **ABDA**, we get

$$-1000I_1 - 500I_3 + 100I_2 = 0$$

$$I_1 - 0.1I_2 + 0.5I_3 = 0 \quad ...(i)$$

Circuit **BCDB** gives,

$$-9990(I_1 - I_3) + 1000(I_2 + I_3) + 500I_3 = 0$$

$$-9990I_1 + 1000I_2 + 11490I_3 = 0$$

$$I_1 - 0.1001I_2 - 1.15I_3 = 0 \quad ...(ii)$$

Circuit **ADCEA** gives,

$$-100I_2 - 1000(I_2 + I_3) + 20 - 500(I_1 + I_2) = 0$$

$$-500I_1 - 1600I_2 - 1000I_3 = -20$$

$$I_1 + 3.2I_2 + 2I_3 = 0.04 \quad ...(iii)$$

Subtracting (*ii*) from (*i*), we get

$$0.0001I_2 + 1.65I_3 = 0 \quad ...(iv)$$

Subtracting (*iii*) from (*ii*), we get

$$+3.3001I_2 + 3.15I_3 = +0.04 \quad ...(v)$$

Solving (*iv*) and (*v*), we get $I_3 = 0.735 \times 10^{-6}$ A

Hence current in the galvanometer arm = 0.735 μA. (Ans.)

Example 32. *Determine the current supplied by the battery in the circuit shown in Fig. 45.*

Solution. Refer Fig. 45.

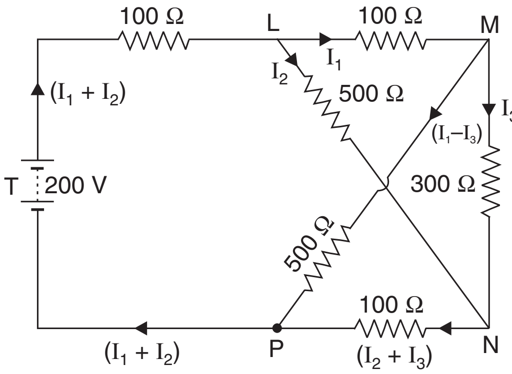


Fig. 45

Applying Kirchhoff's voltage law to the circuit **LMNL**, we get

$$-100I_1 - 300I_3 + 500I_2 = 0$$

$$I_1 - 5I_2 + 3I_3 = 0 \quad ...(i)$$

Circuit **MNPM** gives,

$$-300I_3 - 100(I_2 + I_3) + 500(I_1 - I_3) = 0$$

$$500I_1 - 100I_2 - 900I_3 = 0$$

$$I_1 - 0.2I_2 - 1.8I_3 = 0 \quad ...(ii)$$

Circuit **LMPTL** gives,

$$-100I_1 - 500(I_1 - I_3) + 200 - 100(I_1 + I_2) = 0$$

$$-700I_1 - 100I_2 + 500I_3 = -200$$

$$I_1 + 0.143I_2 - 0.714I_3 = 0.286 \quad ...(iii)$$

Multiplying (*i*) by 1.8 and (*ii*) by 3 and adding, we get

$$1.8I_1 - 9I_2 + 5.4I_3 = 0$$

$$3I_1 - 0.6I_2 - 5.4I_3 = 0$$

$$4.8I_1 - 9.6I_2 = 0$$

or $$I_1 - 2I_2 = 0 \quad ...(iv)$$

Multiplying (*ii*) by 0.714 and (*iii*) by 1.8 and subtracting, we get

$$0.714I_1 - 0.143I_2 - 1.285I_3 = 0$$

$$1.8I_1 + 0.257I_2 - 1.285I_3 = 0.515$$

$$- \quad - \quad + \quad -$$

$$-1.086I_1 - 0.4I_2 = -0.515$$

or $$I_1 + 0.368I_2 = 0.474 \quad ...(v)$$

Subtracting (*v*) from (*iv*), we get $2.368I_2 = 0.474$

i.e., $$I_2 = 0.2 \text{ A}$$

and $$I_1 = 0.4 \text{ A}$$

$\therefore$ **Current supplied by the battery** $= I_1 + I_2 = 0.2 + 0.46 =$ **0.6 A. (Ans.)**

5.2. Maxwell's Loop (or Mesh) Current Method

The method of *loop* or *mesh* currents is generally used in solving networks having some degree of complexity. Such a degree of complexity already begins for a network of three meshes. It might even be convenient at times to use the method of loop or mesh currents for solving a two-mesh circuit.

The *mesh-current method* is preferred to the general or branch-current method because the unknowns in the initial stage of solving a network are equal to the number of meshes, *i.e.*, the mesh currents. The necessity of writing the node-current equations, as done in the general or branch-current method where branch currents are used, is *obviated*. There are as many mesh-voltage equations as these are independent loop or mesh, currents. Hence, the M-mesh currents are obtained by solving the M-mesh voltages or loop equations for M unknowns. After solving for the mesh currents, only a matter of resolving the confluent mesh currents into the respective branch currents by very simple algebraic manipulations is required.

This method eliminates a great deal of tedious work involved in branch-current method and is best suited when energy sources are voltage sources rather than current sources. This method can be used only for *planar circuits*.

The **procedure** for writing the equations is as follows :

1. Assume the smallest number of mesh currents so that at least one mesh current links every element. As a matter of convenience, all mesh currents are assumed to have a *clockwise direction*.

The *number of mesh currents is equal to the number of meshes in the circuit*.

2. For each mesh write down the Kirchhoff's voltage law equation. Where more than one mesh current flows through an element, the algebraic sum of currents should be used. The algebraic sum of mesh currents may be sum or the difference of the currents flowing through the element depending on the direction of mesh currents.

3. Solve the above equations and from the mesh currents find the branch currents.

Fig. 46 shows two batteries E_1 and E_2 connected in a network consisting of three resistors. Let the loop currents for two meshes be I_1 and I_2 (both clockwise-assumed). It is obvious that current through R_3 (when considered as a part of first loop) is $(I_1 - I_2)$. However, when R_3 is considered part of the second loop, current through it is $(I_2 - I_1)$.

Applying Kirchhoff's voltage law to the *two loops*, we get

$$E_1 - I_1R_1 - R_3(I_1 - I_2) = 0$$

or $$E_1 - I_1(R_1 + R_3) + I_2R_3 = 0 \quad \text{...Loop 1}$$

Similarly, $$-I_2R_2 - E_2 - R_3(I_2 - I_1) = 0$$

$$-I_2R_2 - E_2 - I_2R_3 + I_1R_3 = 0$$

or $$I_1R_3 - I_2(R_2 + R_3) - E_2 = 0 \quad \text{...Loop 2}$$

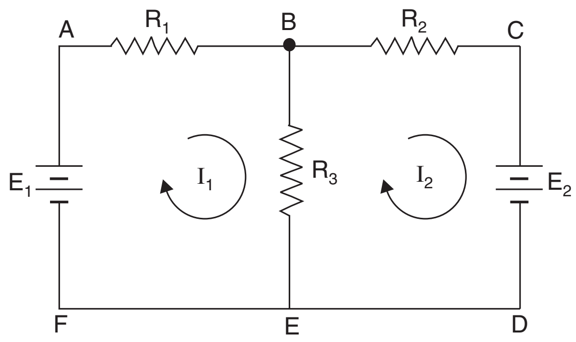


Fig. 46

The above two equations can be solved not only to find loop currents but branch currents as well.

Example 33. *Determine the currents through various resistors of the circuit shown in Fig. 47 using the concept of mesh currents.*

Solution. Refer Fig. 47.

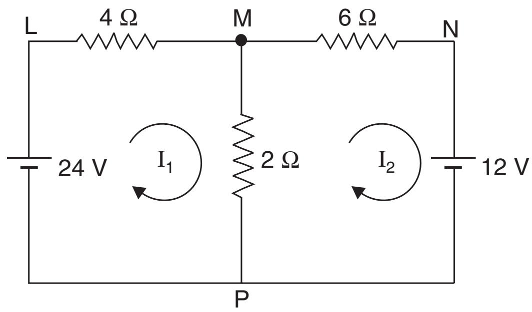


Fig. 47

Since there are two meshes, let the loop currents be as shown.

Applying Kirchhoff's law to *loop 1*, we get

$$24 - 4I_1 - 2(I_1 - I_2) = 0$$

$$-6I_1 + 2I_2 + 24 = 0$$

or $$3I_1 - I_2 = 12 \quad \text{...}(i)$$

For *loop 2*, we have

$$-2(I_2 - I_1) - 6I_2 - 12 = 0$$

$$2I_1 - 8I_2 - 12 = 0$$

$$I_1 - 4I_2 = 6 \quad \text{...}(ii)$$

Solving (*i*) and (*ii*), we get $I_1 = \frac{42}{11}$ A

and $$I_2 = -\frac{6}{11} \text{ A}$$

Hence Current through 4 Ω resistor = $\mathbf{\frac{42}{11}}$ **A (from L to M). (Ans.)**

Current through 6 Ω resistor = $\mathbf{\frac{6}{11}}$ **A (from N to M). (Ans.)**

Current through 2 Ω resistor = $\frac{42}{11} - \left(-\frac{6}{11}\right) = \mathbf{\frac{48}{11}}$ **A (from M to P). (Ans.)**

Example 34. *Determine the current supplied by each battery in the circuit shown in Fig. 48.*

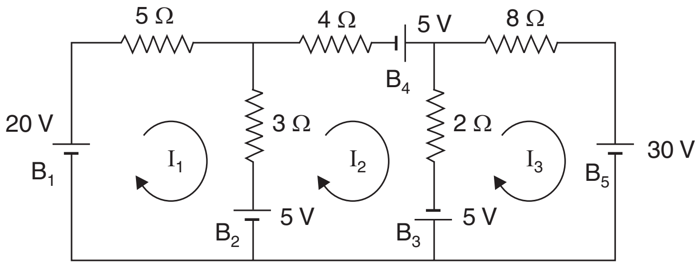


Fig. 48

Solution. Refer to Fig. 48.

As there are three meshes, let the three loop currents be as shown.

Applying Kirchhoff's law to *loop 1*, we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$

or $$8I_1 - 3I_2 = 15 \qquad ...(i)$$

For *loop 2*, we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$

$$3I_1 - 9I_2 + 2I_3 = -15 \qquad ...(ii)$$

For *loop 3*, we have

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$$

$$2I_2 - 10I_3 = 35 \qquad ...(iii)$$

Eliminating I_1 from (*i*) and (*ii*), we get

$$63I_2 - 16I_3 = 165 \qquad ...(iv)$$

Solving (*iii*) and (*iv*), we get

$$I_2 = 1.82 \text{ A} \quad \text{and } I_3 = -3.15 \text{ A}$$

(– ve sign means direction of current is *counter-clockwise*)

Substituting the value of I_2 in (*i*), we get

$$I_1 = 2.56 \text{ A}$$

Current through battery B_1 (discharging current) $= I_1 =$ **2.56 A. (Ans.)**

Current through battery B_2 (charging current) $= I_1 - I_2 = 2.56 - 1.82 =$ **0.74 A. (Ans.)**

Current through battery B_3 (discharging current) $= I_2 + I_3 = 1.82 + 3.15 =$ **4.97 A. (Ans.)**

Current through battery B_4 (discharging current) $= I_2 =$ **1.82 A. (Ans.)**

Current through battery B_5 (discharging current) $= I_3 =$ **3.15 A. (Ans.)**

Example 35. *Determine the currents through the different branches of the bridge circuit shown in Fig. 49.*

Solution. Refer to Fig. 49.

The three mesh currents are assumed as shown.

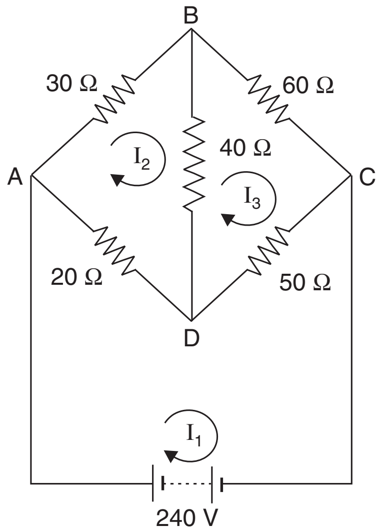


Fig. 49

The equations for the three meshes are :

For loop 1 : $240 - 20(I_1 - I_2) - 50(I_1 - I_3) = 0$

or $-70I_1 + 20I_2 + 50I_3 = -240$

or $70I_1 - 20I_2 - 50I_3 = 240$ ...(*i*)

For loop 2 : $-30I_2 - 40(I_2 - I_3) - 20(I_2 - I_1) = 0$

or $20I_1 - 90I_2 + 40I_3 = 0$

or $2I_1 - 9I_2 + 4I_3 = 0$ ...(*ii*)

For loop 3 : $-60I_3 - 50(I_3 - I_1) - 40(I_3 - I_2) = 0$

$50I_1 + 40I_2 - 150I_3 = 0$

$5I_1 + 4I_2 - 15I_3 = 0$ ...(*iii*)

Solving these equations, we get

$$I_1 = 6.10 \text{ A}, I_2 = 2.56 \text{ A}, I_3 = 2.72 \text{ A}$$

Current through 30 Ω resistor = I_2

= **2.56 A (A to B). (Ans.)**

Current through 60 Ω resistor = I_3 = **2.72 A (B to C). (Ans.)**

Current through 20 Ω resistor = $I_1 - I_2 = 6.10 - 2.56$ = **3.54 A (A to D). (Ans.)**

Current through 50 Ω resistor = $I_1 - I_3 = 6.10 - 2.72$ = **3.38 A (D to C). (Ans.)**

Current through 40 Ω resistor = $I_3 - I_2 = 2.72 - 2.56$ = **0.16 A (D to B). (Ans.)**

Example 36. *Determine the current in the 8 Ω branch in the circuit shown in Fig. 50.*

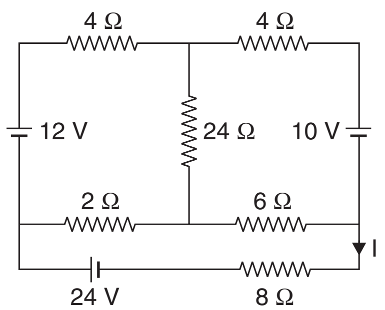


Fig. 50

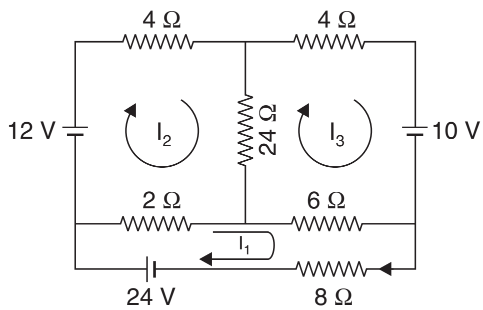


Fig. 51

Solution. The three loop currents are shown in Fig. 51 ; for these loops we have

Loop-1 : $-2(I_1 - I_2) - 6(I_1 - I_3) - 8I_1 + 24 = 0$

$-2I_1 + 2I_2 - 6I_1 + 6I_3 - 8I_1 + 24 = 0$

or $16I_1 - 2I_2 - 6I_3 = 24$

or $8I_1 - I_2 - 3I_3 = 12$ ...(*i*)

Loop-2 : $12 - 4I_2 - 24(I_2 - I_3) - 2(I_2 - I_1) = 0$

or $12 - 4I_2 - 24I_2 + 24I_3 - 2I_2 + 2I_1 = 0$

or $2I_1 - 30I_2 + 24I_3 = -12$

or $I_1 - 15I_2 + 12I_3 = -6$ ...(*ii*)

Loop-3 : $-24(I_3 - I_2) - 4I_3 - 10 - 6(I_3 - I_1) = 0$

or $-24I_3 + 24I_2 - 4I_3 - 10 - 6I_3 + 6I_1 = 0$

or $6I_1 + 24I_2 - 34I_3 = 10$

or $3I_1 + 12I_2 - 17I_3 = 5$ ...(*iii*)

Multiplying eqn. (*i*) and 15 and subtracting eqn. (*ii*) from (*i*), we get

$$\begin{array}{r} 120I_1 - 15I_2 - 45I_3 = 180 \\ I_1 - 15I_2 + 12I_3 = -6 \\ - \quad + \quad - \quad\quad + \\ \hline 119I_1 \quad\quad - 57I_3 = 186 \end{array}$$ ...(*iv*)

Multiplying eqn. (*ii*) by 12 and eqn. (*iii*) by 15 and adding eqn. (*iii*) and eqn. (*ii*), we get

$$\begin{array}{r} 12I_1 - 180I_2 + 144I_3 = -72 \\ 45I_1 + 180I_2 - 255I_3 = 75 \\ \hline 57I_1 \quad - 111I_3 = 3 \end{array}$$ ...(*v*)

Multiplying eqn. (*iv*) by 111 and eqn. (*v*) by 57 and subtracting eqn. (*v*) from eqn. (*iv*), we get

$$\begin{array}{r} 13209I_1 - 6327I_3 = 20646 \\ 3249I_1 - 6327I_3 = 171 \\ - \quad\quad + \quad\quad - \\ \hline 9960I_1 \quad\quad = 20475 \end{array}$$

∴ $\mathbf{I_1}$ (Current through 8 Ω resistance) $= \dfrac{20475}{9960} =$ **2.06 A. (Ans.)**

5.3. Nodal Voltage Method

Under this method the following ***procedure*** is adopted :

1. Assume the voltages of the different independent nodes.
2. Write the equations for each mode as per Kirchhoff's current law.
3. Solve the above equations to get the node voltages.
4. Calculate the branch currents from the values of node voltages.

Let us consider the circuit shown in the Fig. 52. L and M are the two independent nodes ; M can be taken as the reference node. Let the voltage of node L (with respect to M) be V_L.

Using Kirchhoff's law, we get

$$I_1 + I_2 = I_3 \quad ...(22)$$

Ohm's law gives $I_1 = \dfrac{V_1}{R_1} = \dfrac{(E_1 - V_L)}{R_1}$

$$I_2 = \frac{V_2}{R_2} = \frac{(E_2 - V_L)}{R_2} \quad ...(23)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_L}{R_3}$$

$$\frac{E_1 - V_L}{R_1} + \frac{E_2 - V_L}{R_2} = \frac{V_L}{R_3} \quad ...(24)$$

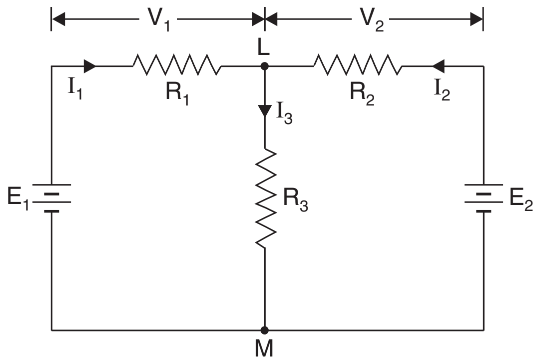


Fig. 52

Rearranging the terms, we get

$$V_L\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0 \quad ...(25)$$

It may be noted that the above nodal equation contains the following terms :

(*i*) The node voltage multiplied by the sum of all conductances connected to that anode. This term is *positive*.

(*ii*) The node voltage at the other end of each branch (connected to this node) multiplied by the conductance of branch. These terms are *negative*.

— In this method of solving a network the *number of equations required for the solution is one less than the number of independent nodes in the network*.

— In general the nodal analysis *yields similar solutions*.

— The nodal method is very suitable for *computer work*.

Example 37. *For the circuit shown in Fig. 53, find the currents through the resistances R_3 and R_4.*

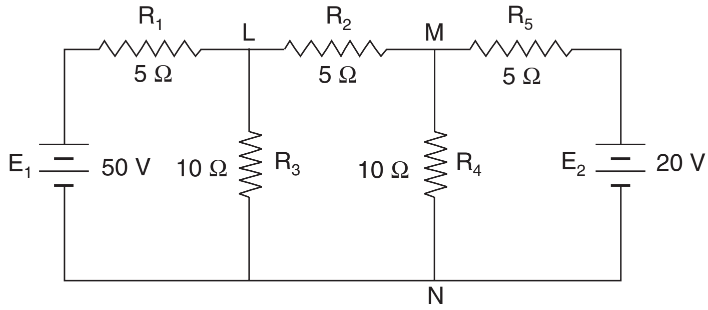


Fig. 53

Solution. Refer Fig. 53.

Let L, M and N = Independent nodes, and

V_L and V_M = Voltages of nodes L and M with respect to node N.

The nodal equations for the nodes L and M are :

$$V_L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{V_M}{R_2}=0 \qquad ...(i)$$

$$V_M\left[\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right]-\frac{E_2}{R_5}-\frac{V_L}{R_2}=0 \qquad ...(ii)$$

Substituting the values in (*i*) and (*ii*) and simplifying, we get

$$V_L\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{10}\right)-\frac{50}{5}-\frac{V_M}{5}=0$$

or $$2.5V_L - V_M - 50 = 0 \qquad ...(iii)$$

and $$V_M\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{5}\right)-\frac{20}{5}-\frac{V_L}{5}=0$$

or $$2.5V_M - V_L - 20 = 0$$

or $$-V_L + 2.5V_M - 20 = 0 \qquad ...(iv)$$

Solving (*iii*) and (*iv*), we get

$$V_L = 27.6 \text{ V}, V_M = 19.05 \text{ V}$$

Current through $\mathbf{R_3}$ $= \dfrac{V_L}{R_3} = \dfrac{27.8}{10}$ **= 2.76 A. (Ans.)**

Current through $\mathbf{R_4}$ $= \dfrac{V_M}{R_4} = \dfrac{19.05}{10}$ **= 1.905 A. (Ans.)**

Example 38. *Using Node voltage method, find the current in 6 Ω resistance for the network shown in Fig. 54.*

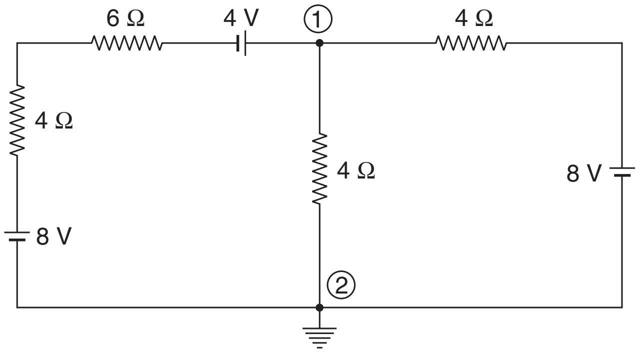


Fig. 54

Solution. Refer Fig. 54. Considering node 2 as the reference node and using node voltage method, we have

$$V_1\left[\frac{1}{(6+4)}+\frac{1}{4}+\frac{1}{4}\right]-\frac{8}{4}-\left(\frac{8+4}{10}\right)=0$$

(The reason for adding the two battery voltages of 4 V and 8 V is because they are connected in additive series).

or $V_1\,(0.1 + 0.25 + 0.25) - 2 - 1.2 = 0$

$\therefore$ $V_1 = 5.33$ V

The current flowing through the 6 Ω resistance towards node 1 is

$$= \frac{12-5.33}{6+4} = \mathbf{0.667\ A.}\ \textbf{(Ans.)}$$

Example 39. *Find the branch currents in the circuit of Fig. 55 by nodal analysis.*

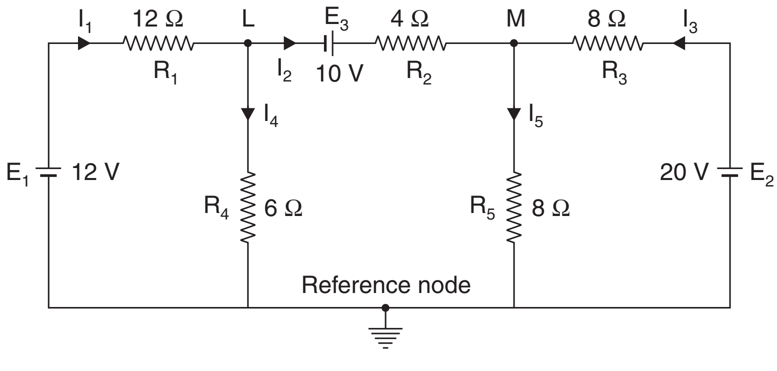


Fig. 55

Solution. Node L :

$$V_L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right]-\frac{E_1}{R_1}-\frac{V_M}{R_2}+\frac{E_3}{R_2}=0$$

Substituting the given data, we get

$$V_L\left[\frac{1}{12}+\frac{1}{4}+\frac{1}{6}\right]-\frac{12}{12}-\frac{V_M}{4}+\frac{10}{4}=0$$

$$V_L\left[\frac{1+3+2}{12}\right]-1-\frac{V_M}{4}+\frac{10}{4}=0$$

or $$0.5V_L - 1 - 0.25V_M + 2.5 = 0$$

or $$2V_L - V_M = -6 \qquad ...(i)$$

Node M :

$$V_M\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]-\frac{E_2}{R_3}-\frac{V_L}{R_2}-\frac{E_3}{R_2}=0$$

$$V_M\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{8}\right]-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

or $$V_M\left[\frac{2+1+1}{8}\right]-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

or $$\frac{V_M}{2}-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

$$V_M-\frac{V_L}{2}=10 \qquad ...(ii)$$

From eqn. (*i*) and (*ii*), we get

$$V_L=\frac{8}{3}\,V;\quad V_M=\frac{34}{3}\,V$$

$$\mathbf{I_1}=\frac{E_1-V_L}{R_1}=\frac{12-\frac{8}{3}}{12}=\mathbf{\frac{7}{9}}\ \textbf{A. (Ans.)}$$

$$\mathbf{I_2}=\frac{V_L+E_1-V_M}{R_2}=\frac{\frac{8}{3}+10-\frac{34}{3}}{4}=\mathbf{\frac{1}{3}}\ \textbf{A. (Ans.)}$$

$$\mathbf{I_3}=\frac{E_2-V_M}{R_3}=\frac{20-\frac{34}{3}}{8}=\mathbf{\frac{13}{12}}\ \textbf{A. (Ans.)}$$

$$\mathbf{I_4}=\frac{V_L}{6}=\frac{\frac{8}{3}}{6}=\mathbf{\frac{4}{9}}\ \textbf{A. (Ans.)}$$

$$\mathbf{I_5}=\frac{V_M}{R_5}=\frac{\frac{34}{3}}{8}=\mathbf{\frac{17}{12}}\ \textbf{A. (Ans.)}$$

HIGHLIGHTS

1. **Electricity** may be defined as a form of energy. It involves making and using energy.
2. The controlled movement of electrons (or drift) through a substance is called *current.* Current is the rate at which electrons move.
3. *Electromotive force* (e.m.f.) is the force that causes a current of electricity to flow. The *volt* is a unit of potential difference and electromotive force.

4. The opposition to flow of electrons (due to bonds between protons and electrons, as well as to collisions) is called *electrical resistance*. Resistance may also be defined as "the property of the electric circuit which opposes the flow of current".

5. The resistance (R) of a conductor is given by :

$$R = \frac{\rho l}{A}$$

where, l = Length of the conductor,

A = Area of cross-section of the conductor, and

ρ = A constant depending on the nature of material of the conductor is known as its specific resistance or resistivity.

6. Conductance (G) is the reciprocal of resistance. Conductivity (σ) is the reciprocal of specific resistance (ρ).

7. The temperature co-efficient of resistance at 0°C may be defined as :

"The change in resistance per ohm for change in temperature of 1°C from 0°C.

Also
$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$

where, α_1 = Temperature co-efficient of resistance at temperature t_1°C.

α_2 = Temperature co-efficient of resistance at temperature t_2°C

8. Ohm's law may be stated as :

"For a fixed metal conductor, the temperature and other conditions remaining constant, the current (I) through it is proportional to the potential difference (V) between its ends".

9. A linear resistor is one which obeys ohm's law. Such elements in which the V/I (volt-ampere) plots are not straight lines but curves are called non-linear resistors or non-linear elements.

10. In a series combination : $R = R_1 + R_2 + R_3$

where R_1, R_2 and R_3 are the resistance in series and R is the equivalent resistance of the combination.

11. In a parallel combination : $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

where R_1, R_2 and R_3 are the resistances connected in parallel and R is the equivalent resistance of the combination.

12. The insulation resistance (R) of a cable is given by :

$$R = \frac{\rho}{2\pi l} \log_e{}^{r_2/r_1} \ \Omega$$

where r_1 = Inner radius of the cable (single core),

r_2 = Outer radius of the cable,

l = Length of the cable, and

ρ = Resistivity of the material.

13. Kirchhoff's law :

First law : Σ Currents entering = Σ Currents leaving

Second law : Σ potential rises = Σ potential drops.

14. Applications of Kirchhoff's laws include the following :

(*i*) Branch-current method

(*ii*) Maxwell's loop (or mesh) current method

(*iii*) Nodal voltage method.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answers :

1. The flow of current in solids is due to
(*a*) electrons (*b*) atoms (*c*) electrons and ions.

2. An atom is composed of
(*a*) electrons and protons (*b*) electrons and neutrons
(*c*) electrons, protons and neutrons.

3. The proton is a .... particle.
(*a*) very small and heavy (*b*) very very large and light
(*c*) very large and heavy.

4. An electron has a .... charge
(*a*) negative (*b*) positive (*c*) neutral.

5. Those substances whose atoms have their outermost orbit incomplete act as :
(*a*) good conductors of electricity (*b*) insulators
(*c*) semiconductors.

6. The property of the electric circuit which opposes the flow of current is called
(*a*) resistivity (*b*) resistance (*c*) conductance (*d*) conductivity.

7. The unit of conductivity is
(*a*) Ω (*b*) ℧ (*c*) ℧/m.

8. If resistances R_1 and R_2 are connected in series then equivalent resistance R will be given by the relation
(*a*) $R = R_1 + R_2$ (*b*) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (*c*) $R = \frac{1}{R_1 + R_2}$.

9. The resistivity of insulators .... rapidly with the rise of temperature.
(*a*) increases (*b*) decreases (*c*) first increases then decreases.

10. The insulation resistance varies .... as the length.
(*a*) directly (*b*) indirectly (*c*) three times.

11. Kirchhoff's current law states that
(*a*) net current flow at the junction is positive
(*b*) algebraic sum of the currents meeting at the junction is zero
(*c*) no current can leave the junction without some current entering it
(*d*) total sum of currents meeting at the junction is zero.

12. According to Kirchhoff's voltage law, the algebraic sum of all *IR* drops and e.m.fs. in any closed loop of a network is always
(*a*) negative (*b*) positive
(*c*) determined by battery e.m.fs. (*d*) zero.

13. Kirchhoff's current law is applicable to only
(*a*) junction in a network (*b*) closed loops in a network
(*c*) electric circuits (*d*) electronic circuits.

14. Kirchhoff's voltage law is related to
(*a*) junction voltages (*b*) battery e.m.fs. (*c*) *IR* drops (*d*) both (*a*) and (*b*)
(*e*) none of the above.

Answers

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)
6. (*b*)	**7.** (*c*)	**8.** (*a*)	**9.** (*b*)	**10.** (*b*)
11. (*b*)	**12.** (*d*)	**13.** (*a*)	**14.** (*d*).	

THEORETICAL QUESTIONS

1. Define the term 'electricity'.
2. Write a short-note on 'Electron theory'.
3. What is an electric current ?
4. What is the difference between electromotive force and potential difference ?
5. What do you mean by the term 'resistance' ? On what factors does it depend ?
6. Define the terms 'conductance' and 'conductivity'.
7. What is temperature co-efficient of resistance ?
8. Derive the relation $\alpha_2 = \dfrac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)}$ where α_1 and α_2 are the temperature co-efficients of resistance at temperatures t_1 and t_2°C respectively.
9. State and explain Ohm's law.
10. What is the difference between linear and non-linear resistors ?
11. Derive an expression for the equivalent resistance in the following cases :
(*a*) When the resistances are connected in series.
(*b*) When the resistances are connected in parallel.
12. How does a conductor differ from an insulator ?
13. Derive an expression for insulation resistance of a cable.
14. Define the following terms :
Circuit, Electrical network, Active network, Node and Branch.
15. What are the limitations of Ohm's law ?
16. State and explain Kirchhoff's laws.
17. Discuss briefly application of Kirchhoff's laws.
18. Explain the nodal voltage method for solving networks. How are the nodal equations written ?
19. Explain Cramer's rule used for solving equations by determinants.

EXERCISE

1. Calculate the resistance of a manganin wire having the following particulars :
Length of the wire = 100 m
Uniform cross-sectional area = 0.1 mm^2
Resistivity of the material = 50×10^{-8} Ω m. [**Ans.** 500 Ω]
2. Calculate the area of cross-section and diameter of a copper wire to have a resistance of 0.13 ohm per kilometre ; given that resistivity for copper is 1.7×10^{-8} Ω-m. [**Ans.** 12.9 mm]
3. Find the area of cross-section of a cable of 1 km length to transmit 500 amperes so that total drop in voltage along the cable may not exceed 25 volts.
Assume : $\rho = 1.7 \times 10^{-8}$ Ω-m. [**Ans.** 3.4 cm^2]
[**Hint.** $R = \dfrac{V}{I} = \dfrac{25}{500} = 0.05\ \Omega$.]
4. Two wires, each of the same length and of the same metal, have resistances of 25 and 36 ohms respectively. What are the relative diameters of the wires ? [**Ans.** 1.2 : 1]
5. A semicircular ring of copper has an inner radius of 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end faces. Assume resistivity of copper at 20°C = 1.724×10^{-8} Ω-m. and temperature co-efficient of resistance of copper at 0°C = 0.0043/°C. [**Ans.** 3.75×10^{-6} Ω]

6. A coil has a resistance of 18 ohms when its mean temperature is 25°C and of 20 ohms when its mean temperature is 55°C. Find its mean temperature rise when its resistance is 21 ohms and the surrounding temperature is 20°C. Assume that temperature co-efficient of resistance is constant. [**Ans.** 50°C]

7. It is required to construct a resistance of 100 ohms having a temperature co-efficient of 0.001/°C. Wires of two materials of suitable cross-sectional area are available. For material *A*, the resistance is 97 ohms per 100 metres and for material *B*, the resistance is 40 ohms per 100 metres. The temperature co-efficient of resistance for material *A* is 0.003/°C and for material *B* is 0.0005/°C. Determine the suitable lengths of wires of materials *A* and *B*. [**Ans.** l_A = 19.4 m ; l_B = 200 m]

8. The resistance of a copper field coil is 171 ohms at 16°C. What is the average temperature of the coil when the resistance is 128 ohms ? Take the temperature co-efficient of copper as 0.00427 at 0°C.

[**Ans.** 55.5°C]

9. The resistance of the field coils with copper conductors of a dynamo is 120 ohms at 25°C. After working for six hours on full load, the resistance of the coil increases to 240 ohms. Calculate the mean temperature rise of the field coils. [**Ans.** 39.8°C]

10. The resistance of the winding of a motor is 3.42 ohms at 20°C. After extended operation at full load the resistance is 4.2 ohms. Find the temperature rise.

α = 0.00393 at 20°C. [**Ans.** 59.52°C]

11. Find the resistance between the points *A* and *B* in the series-parallel net-work shown in Fig. 56.

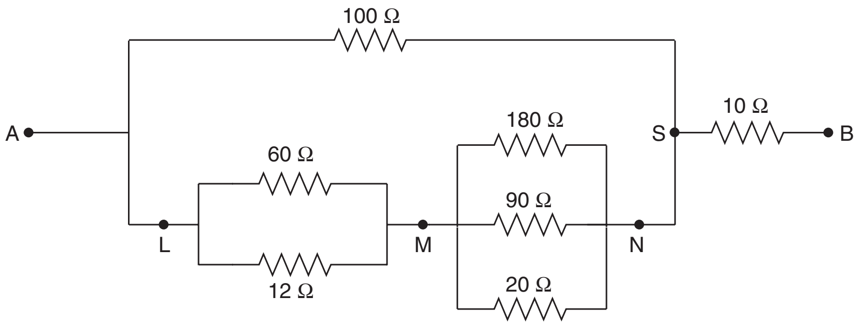


Fig. 56

12. A resistance of 10 ohms is connected in series with a combination of two resistances arranged in parallel each of value 20 ohms. Determine the resistance R_3 which should be shunted across the parallel combination so that current drawn by the circuit is 1.5 A with applied voltage of 20 V. [**Ans.** 5 ohms]

13. A series parallel resistor circuit is shown in Fig. 57. Determine

(*i*) Equivalent resistance across the battery terminals.
(*ii*) Current supplied by the battery.
(*iii*) Current through 4 Ω resistor.
(*iv*) Equivalent resistance of the circuit with an open circuit at point *L*.
(*v*) Equivalent resistance of the circuit with an open circuit at point *M*.
(*vi*) Equivalent resistance of the circuit with points *N* and *P* short circuited.
(*vii*) Equivalent resistance of the circuit with points *L* and *N* short circuited.

[**Ans.** (*i*) 12 Ω (*ii*) 2 A (*iii*) 1 A (*iv*) Infinite (*v*) 12.66 Ω (*vi*) 8 Ω (*vii*) 10 Ω]

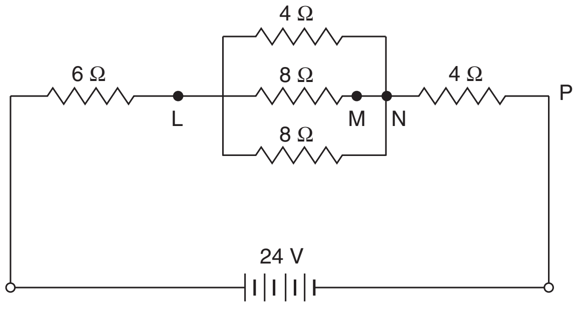


Fig. 57

14. Find the resistance of the circuit shown in Fig. 58. [**Ans.** 4.2 Ω]

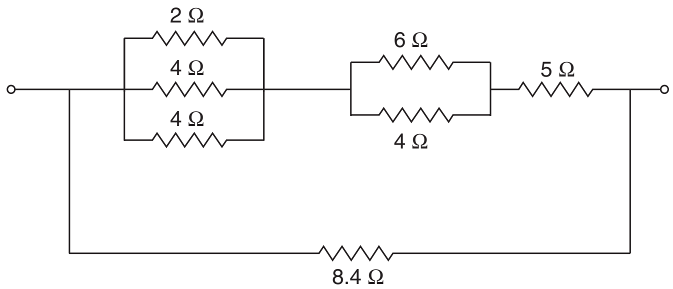


Fig. 58

15. Find the resistance of the circuit shown in Fig. 59. [**Ans.** 2.2 Ω]

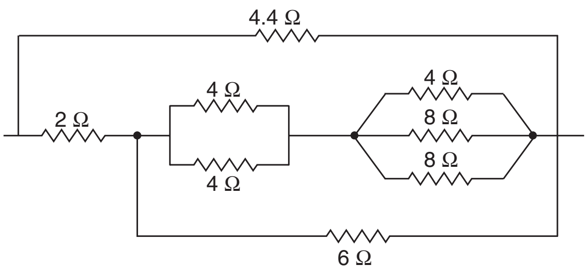


Fig. 59

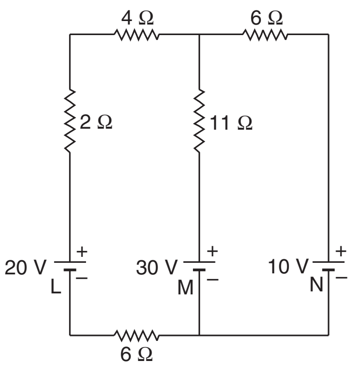


Fig. 60

16. Determine the magnitude and direction of the current in each of the batteries *L*, *M* and *N* shown in the Fig. 60.

17. Two batteries are connected in parallel. The e.m.f. and internal resistance of one are 110 V and 6 Ω respectively and the corresponding values for other are 130 V and 4 Ω respectively. A resistance of 20 Ω is connected across the parallel combination. Calculate :

(*i*) The value and direction of the current in each battery.

(*ii*) The terminal voltage. [**Ans.** (*i*) 0.1786 A, 5.2678 A (*ii*) 108.928 V]

[**Hint.** Terminal voltage = $(I_1 + I_2)R$.]

18. Two cells A and B are connected in parallel, unlike poles being joined together. The terminals of the cells are then joined by two resistors of 4 Ω and 2 Ω in parallel. The e.m.f. of A is 2 V, its internal resistance is 1 Ω ; the e.m.f. of B is 1 V, its internal resistance is 2 Ω. Find the current in each of the four branches of the circuit. [**Ans.** 4/3 A, 5/6 A, 1/6 A, 1/3 A]

19. What is the equivalent resistance of the network shown in Fig. 61 ? [**Ans.** 600 Ω]

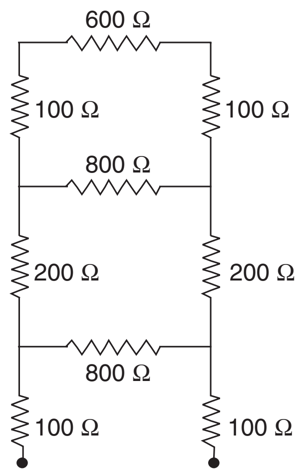


Fig. 61

20. Determine the currents in the three batteries (L, M and N) in the network shown in Fig. 62 and show their values and direction of flow on the diagram. Neglect the internal resistance of batteries. [**Ans.** 2 A, 1 A, 0 A]

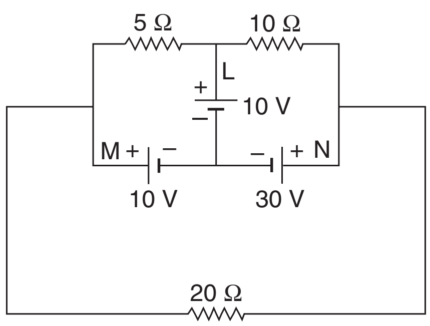


Fig. 62

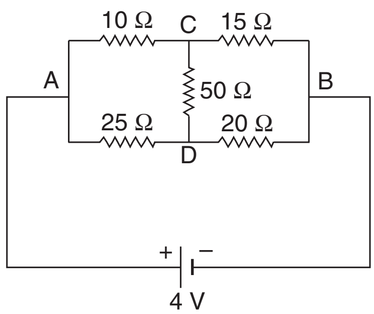


Fig. 63

21. Determine the magnitude and direction of flow of current in the branch CD for the circuit shown in the above Fig. 63. [**Ans.** 7/755 A from C to D]

22. For the lattice-type network shown in Fig. 64, calculate the current in each branch of the network.

[**Ans.** $I_{ab} = \frac{1}{50}$ A ; $I_{bed} = \frac{1}{100}$ A ; $I_{bc} = \frac{1}{100}$ A ; $I_{ad} = \frac{1}{100}$ A ; $I_{dc} = \frac{1}{50}$ A ; $I_{cfa} = \frac{3}{100}$ A]

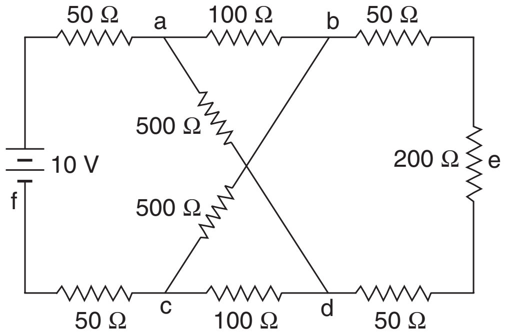


Fig. 64

23. Two storage batteries, *A* and *B*, are connected in parallel to supply a load the resistance of which is 1.2 Ω. Calculate :

(*i*) The current in this load.

(*ii*) The current supplied by each battery if the open-circuit e.m.f. of *A* is 12.5 V and that of *B* is 12.8 V, the internal resistance of *A* being 0.05 Ω and that of *B* 0.08 Ω.

[**Ans.** (*i*) 10.25 A (*ii*) 4 A (*A*), 6.25 A (*B*)]

24. A load having a resistance of 0.1 Ω is fed by two storage batteries connected in parallel. The open circuit e.m.f. of one battery (*A*) is 12.1 V and that of the other battery (*B*) is 11.8 V. The internal resistances are 0.03 Ω and 0.04 Ω respectively. Calculate :

(*i*) The current supplied to the load (*ii*) The current in each battery

(*iii*) The terminal voltage of each battery. [**Ans.** (*i*) 102.2 A, (*ii*) 62.7 A (*A*), 39.5 A (*B*), (*iii*) 10.22 V]

25. A battery having an e.m.f. of 110 V and an internal resistance of 0.2 Ω is connected in parallel with another battery having an e.m.f. of 100 V and internal resistance 0.25 Ω. The two batteries in parallel are placed in series with a regulating resistance of 5 Ω and connected across 200 V mains. Calculate :

(*i*) The magnitude and direction of the current in each battery.

(*ii*) The total current taken from the supply mains.

[**Ans.** (*i*) 11.96 A (discharge) ; 30.43 A (charge) (*ii*) 18.47 A]

26. A wheatstone bridge *ABCD* is arranged as follows : *AB* = 50 ohms, *BC* = 100 ohms and *CD* = 101 ohms. A galvanometer of 1000 ohms resistance is connected between *B* and *D*. A 2-volt battery having negligible resistance is connected across *A* and *C*. Estimate the current flowing through the galvanometer.

[**Ans.** 4.14 μA from *D* to *B*]

27. Determine the current through the galvanometer *G* in the wheatstone bridge network of the given Fig. 65. [**Ans.** 0.52 mA]

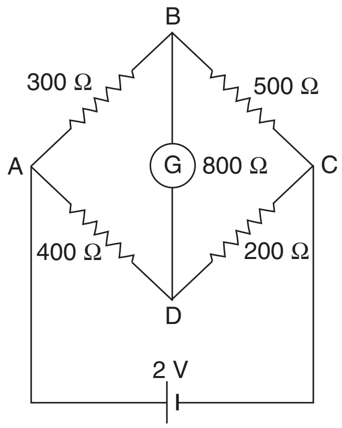


Fig. 65

28. Determine the currents through various resistors of the circuit shown in Fig. 66.

$$\left[\textbf{Ans. } \frac{42}{11}\text{ A}(L \text{ to } M)\,;\, \frac{6}{11}\text{ A}(N \text{ to } M)\,;\, \frac{48}{11}\text{ A}(M \text{ to } P)\right]$$

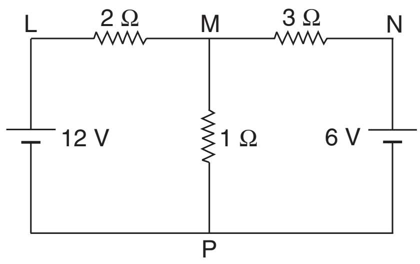


Fig. 66

29. Find I_1, I_2 and I_3 in the network shown in Fig. 67, using loop-current method. [**Ans.** 1 A, 2 A, 3 A]

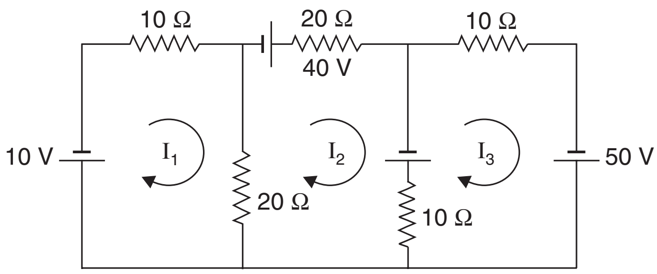


Fig. 67

30. Find the currents in the branches of network of Fig. 68 using nodal voltage method.

[**Ans.** I_1 = 1.42 A, I_2 = 1.68 A, I_3 = 0.26 A, I_4 = 1.1 A, I_5 = 1.36 A, I_6 = 0.32 A]

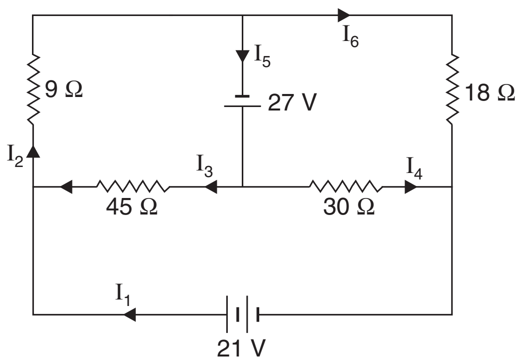


Fig. 68

2

Single-phase A.C. Circuits

1. Introduction to alternating current. 2. Generation and equations of alternating voltages and currents. 3. Alternating voltage and current. 4. Single phase circuits: A.C. through pure ohmic resistance alone—A.C. through pure inductance alone—A.C. through pure capacitance alone—Phasor algebra—A.C. series circuits—R-L circuit—R-C. circuit—R-L-C circuit—A.C. parallel circuits. *—Highlights—Objective Type Questions—Theoretical Questions—Exercise.*

1. INTRODUCTION TO ALTERNATING CURRENT

A.C. means **alternating current—***The current or voltage which alternates its direction and magnitude every time*. Now a days 95% of the total energy is produced, transmitted and distributed in A.C. supply.

The *reasons* are the following :

(*i*) More voltage can be generated (upto 33000 V) than D.C. (650 V only).

(*ii*) A.C. voltage can be increased and decreased with the help of a static machine called the '*transformer*'.

(*iii*) A.C. transmission and distribution is more economical as line material (say copper) can be saved by transmitting power at higher voltage.

(*iv*) A.C. motors for the same horse power as of D.C. motors are cheaper, lighter in weight, require less space and require lesser attention in operation and maintenance.

(*v*) A.C. can be converted to D.C. (direct current) easily, when and where required but D.C. cannot be converted to A.C. so easily and it will not be economical.

However, D.C. entails the following *merits* and hence finds wide applications.

(*i*) D.C. series motors are most suitable for traction purposes in tramway, railways, crains and lifts.

(*ii*) For electroplating, electrolytic and electrochemical processes (battery charging etc.), D.C. is required.

(*iii*) Arc lamps for search lights and cinema projectors work on D.C.

(*iv*) Arc welding is better than on A.C.

(*v*) Relay and operating time switches, etc., and circuit-breakers, D.C. works more efficiently.

(*vi*) In rolling mills, paper mills, colliery winding, etc., where fine speed control of speeds in both directions is required, D.C. motors are required.

2. GENERATION AND EQUATIONS OF ALTERNATING VOLTAGES AND CURRENTS

Generation of Alternating Voltages and Currents

Alternating voltages may be generated in the following two ways :

1. By *rotating a coil in a stationary magnetic field,* as shown in Fig. 1.

2. By *rotating a magnetic field within a stationary coil,* as shown in Fig. 2.

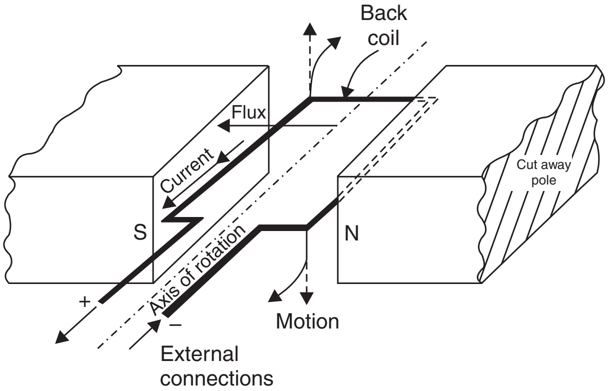


Fig. 1. Rotating a coil in a stationary magnetic field.

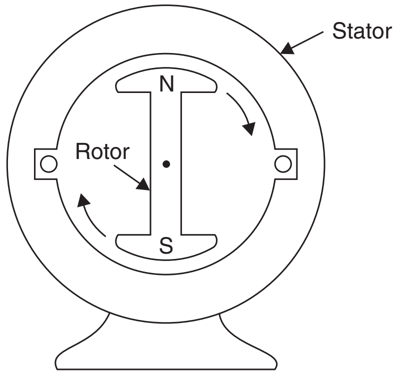


Fig. 2. Rotating a magnetic field within a stationary coil.

The value of the voltage generated in each case depends upon the following factors :

(*i*) The number of turns in the coils ;

(*ii*) The strength of the field ;

(*iii*) The speed at which the coil or magnetic field rotates.

- Out of the above two methods the *rotating-field method is mostly used in practice.*

Equations of Alternating Voltages and Currents

Fig. 3 shows a rectangular coil of N turns rotating clockwise with an angular velocity ω radians per second in a uniform magnetic field.

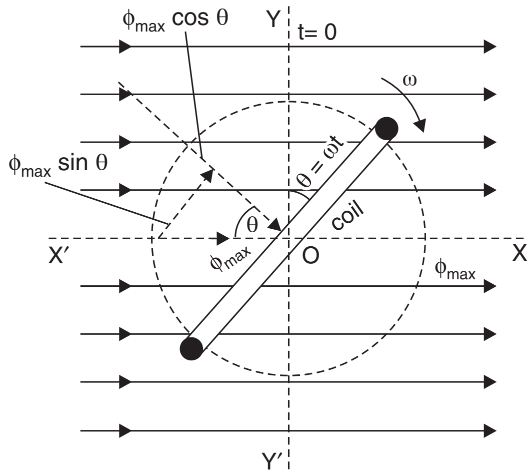


Fig. 3. A coil rotating in a magnet field.

Since by Faraday's law, the voltage is proportional to the rate at which the conductor its across the magnetic field or to the rate of change of flux linkages, the shape of the wave of voltage applied to the external circuit will be determined by the *flux distribution in the air gap*. For a uniform field between the poles it is evident that *maximum* flux will link with the coil when its plane is in *vertical position i.e.*, perpendicular to the direction of flux between the poles. Also it is obvious that when the plane of coil is *horizontal no flux will link with the coil*.

If the position of the coil with reference to the vertical axis be denoted by θ the flux linking with the coil at any instant, as the coil rotates may be determined from the relation,

$$\phi = \phi_{max} \cos \theta$$
$$= \phi_{max} \cos \omega t \qquad \text{...}(i) \quad (\because \ \theta = \omega t)$$

where, ϕ_{max} = Maximum flux which can link with the coil, and

t = Time taken by the coil to move through an angle θ from vertical position.

Using Faraday's law to eqn. (*i*), in order to determine the voltage equation,

$$e = -N \frac{d\phi}{dt}$$ (where e is the *instantaneous* value of the induced e.m.f.)

$$= -N \frac{d}{dt} (\phi_{max} \cos \omega t) = \omega N \phi_{max} \sin \omega t$$

or $$e = \omega N \phi_{max} \sin \theta \qquad ...(ii)$$

As the value of e will be maximum when $\sin \theta = 1$,

$$\therefore \qquad E_{max} = \omega N \phi_{max}$$

The eqn. (*ii*) can be written in simpler form as

$$e = E_{max} \sin \theta \qquad ...(iii)$$

Similarly the equation of induced alternating current (instantaneous value) is

$$i = I_{max} \sin \theta \qquad \text{(if the load is resistive)} \qquad ...(iv)$$

Waveforms. A waveform (or wave-shape) is the shape of a curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa.

Fig. 4 (*a, b, c, d, e*) shows irregular waveforms, but each cycle of current/voltage is an exactly replica of the previous one. Alternating e.m.fs and currents produced by machines usually both have positive and negative half waves, the same shape as shown. Fig. 4(*f*) represents a sine wave of A.C. This is the simplest possible waveform, and alternators are designed to give as nearly as possible a sine wave of e.m.f.

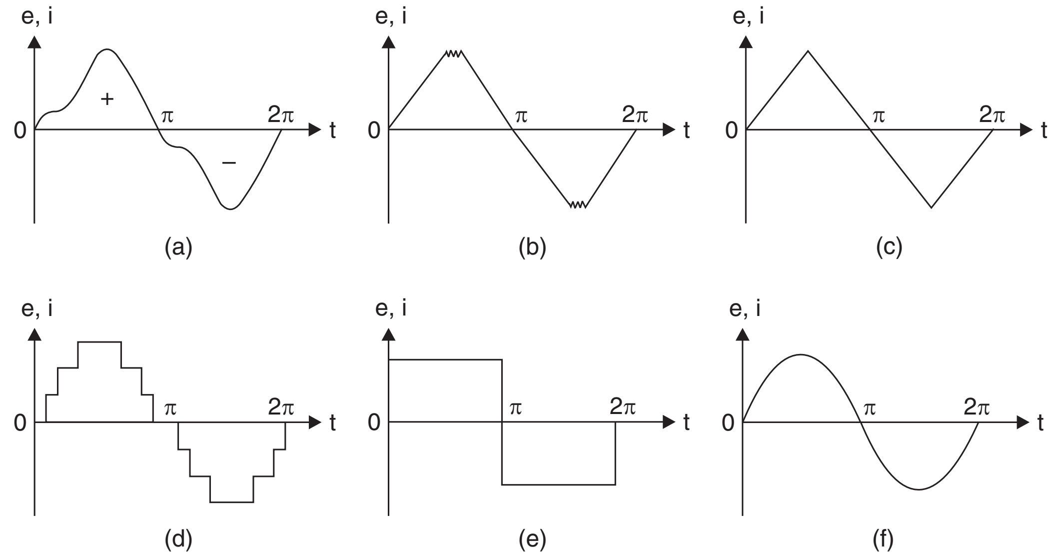


Fig. 4. Waveforms.

- In general, an *alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.*
- The waves deviating from the standard sine wave are termed as *distorted waves.*
- *Complex waves* are those which depart from the ideal sinusoidal form. All alternating complex waves, which are periodic and have equal positive and negative half cycles can be shown to be made up of a number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the *fundamental* (or first harmonic). These waves of higher frequencies are called *harmonics.*

3. ALTERNATING VOLTAGE AND CURRENT

Modern alternators produce an e.m.f. which is for all practical purposes sinusoidal (*i.e.*, a sine curve), the equation between the e.m.f. and time being

$$e = E_{max} \sin \omega t \qquad ...(1)$$

where, e = Instantaneous voltage ; E_{max} = Maximum voltage ;

ωt = Angle through which the armature has turned from neutral.

Taking the frequency as f hertz (cycles per second), the value of ω will be $2\pi f$, so that the equation reads

$$e = E_{max} \sin (2\pi f)t.$$

The graph of the voltage will be as shown in Fig. 5.

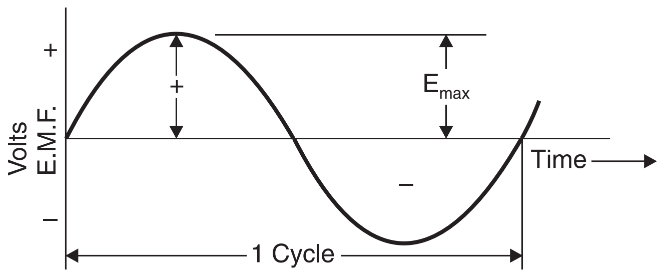


Fig. 5. The graph of the sinusoidal voltage.

1. Cycle. One complete set of positive and negative values of an alternating quantity is known as a *cycle*. A cycle may also sometimes be specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radians.

2. Amplitude. The maximum value, positive or negative, of an alternating quantity, is known as its *amplitude*.

3. Frequency (f). The number of cycles/second is called the frequency of the alternating quantity.

Its unit is *hertz* (Hz).

4. Time Period (T). The time taken by an alternating quantity to complete the cycle is called its *time period*. For example, a 50 hertz (Hz) alternating current has a time period of $\frac{1}{50}$ second.

Time period is reciprocal of frequency,

i.e., $$T = \frac{1}{f} \left(\text{or } f = \frac{1}{T} \right) \qquad ...(2)$$

5. Root mean square (R.M.S.) value. The r.m.s. (or effective) value of an alternating current is given by that steady (D.C.) current *which when flowing through a given circuit for a given time produces the same* **heat** *as produced by the alternating current when flowing through the same circuit for the same time*.

R.M.S. value is the value which is taken for power purposes of any description. This value is obtained by finding the square root of the mean value of the squared ordinates for a cycle or half-cycle (See Fig. 5).

This is the value which is used for all power, lighting and heating purposes, as in these cases the power is proportional to the square of the voltage.

Refer to Fig. 5.

The equation of sinusoidal alternating current is given as :

$$i = I_{max} \sin \theta$$

The mean of squares of the instantaneous values of current over half cycle is

$$I^2 = \int_0^{\pi} \frac{i^2 \, d\theta}{(\pi - 0)}$$

$$I^2 = \frac{1}{\pi} \int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi} \int_0^{\pi} (I_{max} \sin \theta)^2 \, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I^2_{max} \sin^2 \theta \, d\theta = \frac{I^2_{max}}{\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta$$

$$= \frac{I^2_{max}}{2\pi} \int_0^{\pi} (1 - \cos 2\theta) \, d\theta = \frac{I^2_{max}}{2\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{\pi}$$

$$= \frac{I^2_{max}}{2\pi} \times \pi = \frac{I^2_{max}}{2} \quad \text{or} \quad I = \sqrt{\frac{I^2_{max}}{2}} = \frac{I_{max}}{\sqrt{2}}$$

or $\quad I = \mathbf{0.707\ I_{max}}$. ...(3)

Note. While solving problems, the values of given current and voltage should always be taken as the r.m.s. values, unless indicated otherwise.

6. Average or mean value. The average value of an alternating current is expressed by *that steady current which transfers across any circuit the same* **charge** *as is transferred by that alternating current during the same time*.

The *mean value is only of use in connection with processes where the results depend on the current only, irrespective of the voltage, such as electroplating or battery charging*.

Refer to Fig. 6.

The value of instantaneous current is given by

$$i = I_{max} \sin \theta$$

Refer to Fig. 6. The value of instantaneous current is given by :

$$i = I_{max} \sin \theta \qquad [\theta = \omega t]$$

$$I_{av} = \frac{1}{(\pi - 0)} \int_0^{\pi} i d\theta$$

[Limits are taken from 0 to π, since only first half cycle is considered. For whole cycle, the average value of sine wave is zero.]

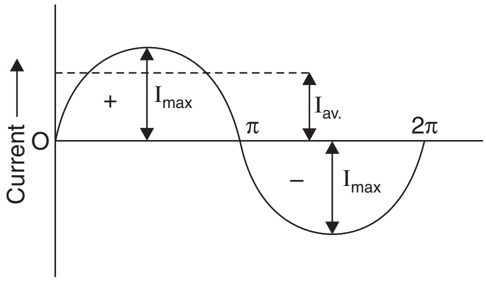


Fig. 6

$$= \frac{1}{\pi} . \int_0^{\pi} I_{max} . \sin \theta \, d\theta = \frac{1}{\pi} . I_{max} \left| - \cos \theta \right|_0^{\pi}$$

$$= \frac{1}{\pi} . I_{max} [1 - (-1)] = \frac{2}{\pi} . I_{max}$$

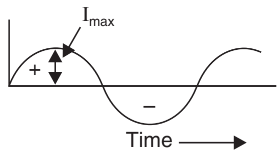
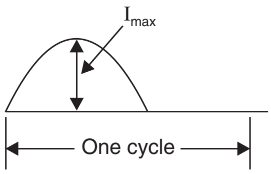
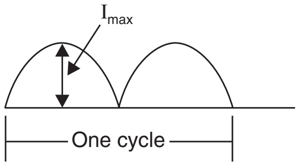
or $\quad \mathbf{I_{av} = 0.637\ I_{max}}$. ...(4)

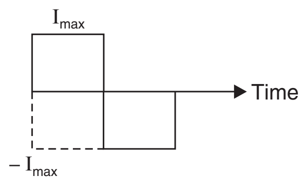
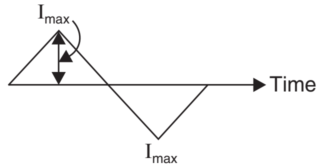
Note. In case of unsymmetrical alternating current *viz.* half-wave rectified current the average value must always be taken over the whole cycle.

7. Form and Peak Factors

Form factor. *The ratio of r.m.s. (or effective) value to average value is the form factor (K_f) of the wave form*. It has use in voltage generation and instrument correction factors.

Peak factor. *The ratio of maximum value to the r.m.s. value is the peak factor (K_p) of the wave form.*

S. No.	*Wave form*	*Form factor* (K_f) $= \frac{r.m.s.\ value}{average\ value}$	*Peak factor* (K_p) $= \frac{max.\ value}{r.m.s.\ value}$
1.	**Sine wave :**  Fig. 7 R.M.S. value $= \frac{I_{max}}{\sqrt{2}} = 0.707\ I_{max}$ Average value $= \frac{2}{\pi} I_{max} = 0.637\ I_{max}$	$K_f = \frac{0.707\ I_{max}}{0.637\ I_{max}} = 1.11$	$K_p = \frac{I_{max}}{0.707\ I_{max}} = 1.41$
2.	**Half wave rectified sine wave :**  Fig. 8 R.M.S. value $= \frac{I_{max}}{2} = 0.5\ I_{max}$ Average Value $= \frac{1}{\pi} I_{max} = 0.318\ I_{max}$	$K_f = \frac{0.5\ I_{max}}{0.318\ I_{max}} = 1.57$	$K_p = \frac{I_{max}}{0.5\ I_{max}} = 2.0$
3.	**Full wave rectified sine wave :**  Fig. 9 R.M.S. value $= \frac{I_{max}}{\sqrt{2}} = 0.707\ I_{max}$ Average value $= \frac{2}{\pi} I_{max} = 0.637\ I_{max}$	$K_f = \frac{0.707\ I_{max}}{0.637\ I_{max}} = 1.11$	$K_p = \frac{I_{max}}{0.707\ I_{max}} = 1.41$

S. No.	*Wave form*	*Form factor* (K_f) $= \dfrac{r.m.s.\ value}{average\ value}$	*Peak factor* (K_p) $= \dfrac{max.\ value}{r.m.s.\ value}$
4.	**Rectangular wave :**  Fig. 10 R.M.S. value = I_{max} Average value = I_{max}	$K_f = 1$	$K_p = 1$
5.	**Triangular wave :**  Fig. 11 R.M.S. = $\dfrac{I_{max}}{\sqrt{3}} = 0.578\ I_{max}$ Average value = $\dfrac{I_{max}}{2} = 0.5\ I_{max}$	$K_f = \dfrac{0.578\ I_{max}}{0.5\ I_{max}} = 1.16$	$K_p = \dfrac{I_{max}}{0.578\ I_{max}} = 1.73$

Reasons for using alternating current (or voltage) of sinusoidal form :

An alternating current (or voltage) of sinusoidal form is normally used because of the following reasons :

(*i*) Mathematically, it is quite simple.

(*ii*) Its integrals and differentials both are sinusoidal.

(*iii*) It lends itself to vector representation.

(*iv*) A complex waveform can be analysed into a series of sine waves of various frequencies, and each such component can be dealt with separately.

(*v*) This waveform is desirable for power generation, transmission and utilisation.

8. Phase and phase angle. The **'phase'** of an A.C. wave may be defined as *its position with respect to a reference axis or reference wave* and **'phase angle'** as *the angle of lead or lag with respect to the reference axis or with respect to another wave.*

Examples. The *phase* of current at point *L* is $\dfrac{T}{4}$ second where *T* is the time period or expressed in terms of angle θ, it is $\dfrac{\pi}{2}$ radian (Fig. 12). Similarly

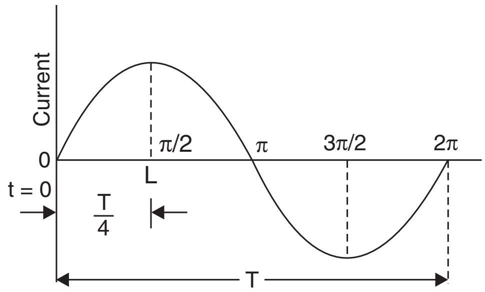


Fig. 12

phase of the rotating coil at the instant shown in Fig. 13 is ωt which is therefore called its *phase angle*.

The e.m.fs. induced in both the coils (Fig. 13) will be of the same frequency and of sinusoidal shape, although the values of instantaneous e.m.f. induced will be different. However, the alternating e.m.fs. would reach their maximum and zero values at the *same time* as shown in Fig. 13 (*b*). Such alternating voltages or curve are said to *in phase* with each other.

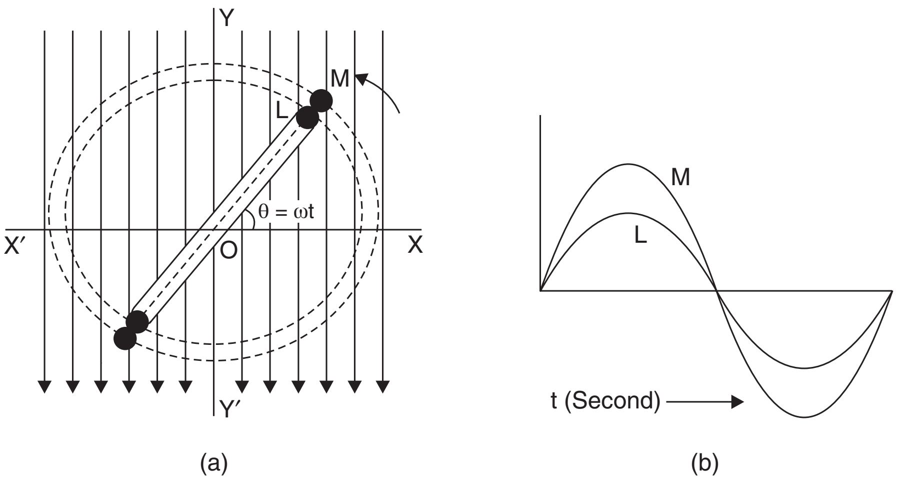


Fig. 13

Refer to Fig. 14. *M lags* behind *L* by β and *N lags* behind *L* by (α + β) because they reach their maximum later.

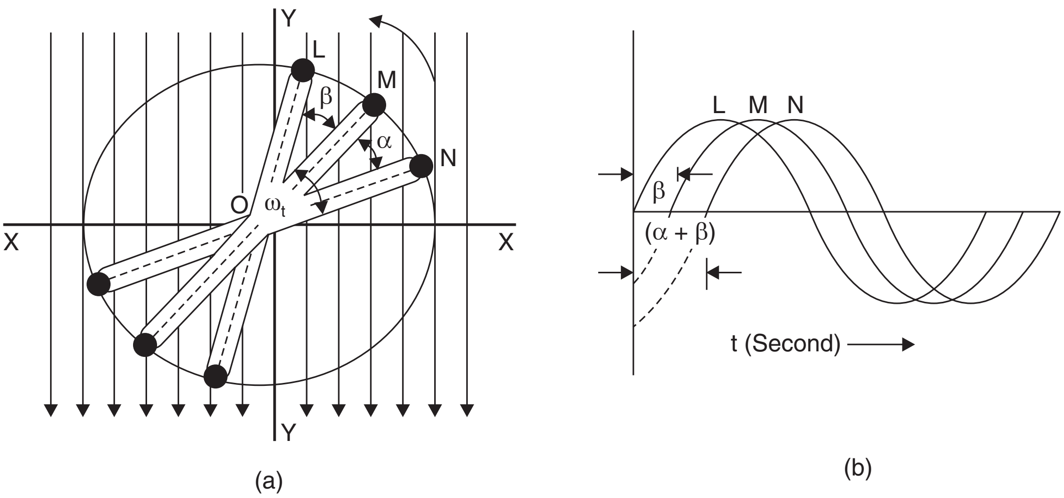


Fig. 14

Example 1. (*a*) *What is the equation of a 25 cycle current sine wave having r.m.s. value of 30 amps ?*

(*b*) *A 60 cycle engine-driven alternator has a speed of 1200 r.p.m. How many poles are there in the alternator ?*

Solution. (*a*) We know that, $i = I_{max} \sin \omega t$

$= I_{max} \sin 2\pi ft$ $(\because \quad \omega = 2\pi f)$

$= 30 \times \sqrt{2} \,.\, \sin (2\pi \times 25 \times t)$ $\left(\because \quad \frac{\text{R.M.S. value}}{\text{Max. value}} = \frac{1}{\sqrt{2}}\right)$

= 42.42 sin 157 t. (Ans.)

(*b*) Using the relation, $f = \frac{Np}{120}$

where, f = frequency, N = speed in r.p.m., and

p = No. of poles

$\therefore \quad 60 = \frac{1200\, p}{120}$ or p = **6. (Ans.)**

Example 2. *An alternating current varying sinusoidally with a frequency of 50 Hz has an r.m.s. value of 40 A. Find :*

(*i*) *The instantaneous value 0.0025 seconds after passing through maximum positive value, and*

(*ii*) *The time measured from a maximum value when the instantaneous current is 14.14 A.*

Solution. $I_{max} = \sqrt{2} \times 40 = 56.56$ A

$\omega = 2\pi f = 2\pi \times 50 = 100\pi$ radians

(*i*) $i = I_{max} \cos \omega t$ ... after +ve maximum value

$= 56.56 \cos 100\, \pi t$

$= 56.56 \cos (100\pi \times 0.0025)$ ...(t = 0.0025 s ...given)

$= 56.56 \cos 45°$ (Taking $\pi = 180°$)

= 40 A. (Ans.)

(*ii*) $14.14 = 56.56 \cos (100 \times 180 \times t)$

or $\frac{14.14}{56.56} = \cos (100 \times 180 \times t)$

or $\cos^{-1} (0.25) = 100 \times 180 \times t$

$75.5° = 100 \times 180 \times t$

$\therefore$ t = **0.00419 s. (Ans.)**

Example 3. *A sinusoidal alternating voltage of 50 Hz has an r.m.s. value of 200 V. Write down the equation for the instantaneous value and find this value 0.0125 sec. after passing through a positive maximum value. At what time measured from a positive maximum value will the instantaneous voltage be 141.4 volts ?*

Solution. Refer to Fig. 15.

$V_{max} = \sqrt{2} \times 200 = 282.2$ volts

$\omega = 2\pi f = 2\pi \times 50 = 100\, \pi$ rad/sec.

$\therefore$ Equation for the instantaneous voltage.

$V = V_{max} \sin \omega t$ (with reference to point O)

$= 282.2 \sin 100\, \pi t$ ...(*i*)

Since the time (0.0125 sec.) is given from the point L (*i.e.*, from positive maximum value) the equation (*i*) when referred to point L can be written as

$v = 282.2 \sin (90° + 100\, \pi t)$

$= 282.2 \cos 100\, \pi t$

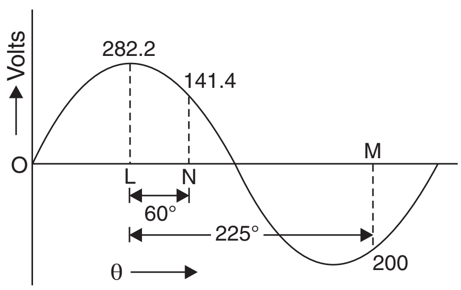


Fig. 15

Hence instantaneous value of the voltage 0.0125 sec. after passing through + ve maximum value,

$$v = 282.2 \cos (100\, \pi \times 0.0125) \quad \text{...(angle in radians)}$$

$$= 282.2 \cos (100 \times 180 \times 0.0125) \quad \text{...(angle in degrees)}$$

$$= 282.2 \cos 225° = 280.2 \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \mathbf{-\,200\ V\ (point\ M).\ (Ans.)}$$

Also $v = 141.4$ V

$\therefore$ $141.4 = 282.2 \cos (100 \times 180 \times t)$

or $0.5 = \cos (100 \times 180 \times t)$

or $\cos^{-1} (0.5) = 100 \times 180 \times t$

or $60° = 100 \times 180 \times t$

or $t = \mathbf{\frac{1}{300}}$ **sec. (point N). (Ans.)**

Example 4. (*a*) *What is the peak value of a sinusoidal alternating current of 4.78 r.m.s. amps ?*

(*b*) *What is the r.m.s. value of a rectangular wave with an amplitude of 9.87 volts ?*

(*c*) *What is the average value over half a cycle of a sinusoidal alternating current whose r.m.s. value is 31 A ?*

Solution. (*a*) Peak value, $I_{max} = \sqrt{2} \times 4.78 =$ **6.76 A. (Ans.)**

(*b*) Refer to Fig. 16. If the first half-cycle is divided into *n* equal parts each of value *V*, then

$$\text{r.m.s. value} = \left(\frac{V^2 + V^2 + V^2 + ...}{n}\right)^{1/2}$$

$$= V = \mathbf{9.87\ volts.\ (Ans.)}$$

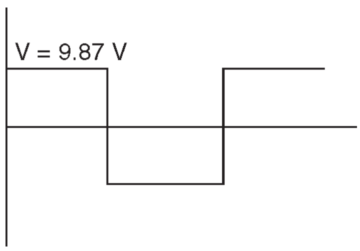


Fig. 16

(*c*) $I_{r.m.s.} = 31$ A

$$I_{av} = \frac{I_{r.m.s.}}{\text{form factor}}$$

$$= \frac{31}{1.11} = \mathbf{27.93\ A.\ (Ans.)}$$

Example 5. *The graph in Fig. 17 shows the variation of voltage with time. Use the graph to calculate the average and r.m.s. value of the voltage. What is the frequency of the voltage ? What would be the r.m.s. value of sine wave having the same peak value ?*

Solution. Refer to Fig. 17.

As the graph is symmetrical about time axis, considering only the positive half cycle.

Average value, $V_{av} = \frac{0 + 10 + 20 + 40 + 100 + 120 + 100 + 40 + 20 + 10}{10}$

= **46 V. (Ans.)**

R.M.S. value, $V = \sqrt{\frac{0^2 + 10^2 + 20^2 + 40^2 + 100^2 + 120^2 + 100^2 + 40^2 + 20^2 + 10^2}{10}}$

$$= \sqrt{\frac{0 + 100 + 400 + 1600 + 10000 + 14400 + 10000 + 1600 + 400 + 100}{10}}$$

$$= \sqrt{\frac{38600}{10}} = \sqrt{3860} = \mathbf{62.1\ V.} \quad \textbf{(Ans.)}$$

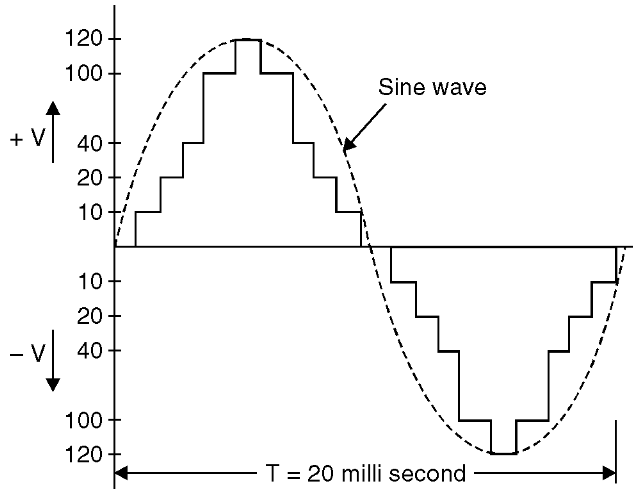


Fig. 17

Since the time period 'T' is 20 millisec.

$\therefore$ Frequency $\quad$ $'f' = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = \mathbf{50\ Hz.}$ **(Ans.)**

R.M.S. value of a sine wave of the same peak value

$$= 0.707 \times 120 = \mathbf{84.84\ V.} \quad \textbf{(Ans.)}$$

Example 6. *Prove that if a D.C. current of $I_{amps.}$ is superposed in a conductor by an A.C. current of max. value I amps, the r.m.s. value of the resultant is* $\sqrt{\frac{3}{2}}\,I$.

Solution. Let the *A.C. current* be $i = I \sin \theta$ where i is the instantaneous value of the A.C. current and I the *D.C. current.*

The r.m.s. value of $(I + i)$ over one complete cycle is,

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I + I \sin \theta)^2 \, d\theta}$$

$$= I \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I + 2 \sin \theta + \sin^2 \theta)}$$

$$= I \sqrt{\left\{ \frac{1}{2\pi} \int_0^{2\pi} 1 + 2 \sin \theta + \left(\frac{1 - \cos 2\theta}{2} \right) \right\}}$$

$$= I \sqrt{\frac{1}{2\pi} \left| \theta - 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right|_0^{2\pi}}$$

$$= I \sqrt{\frac{1}{2\pi} (2\pi - 2 + \pi + 2)}$$

$$= I \cdot \sqrt{\frac{3}{2}} \ . \quad \textbf{(Ans.)}$$

Example 7. *A resultant current wave is made up of two components : a 4A D.C. component and a 50 Hz A.C. component, which is of sinusoidal waveform and which has a maximum value of 4A.*

(*i*) *Sketch the resultant wave.*

(*ii*) *Write an analytical expression for the current wave, reckoning t = 0 at a point where the A.C. component is at zero value and where di/dt is positive.*

(*iii*) *What is the average value of the resultant current over a cycle ?*

(*iv*) *What is the effective or r.m.s. value of the resultant current ?*

Solution. (*i*) **Sketch of the resultant wave :**

The two current components and the resultant current wave are shown in Fig. 18. **(Ans.)**

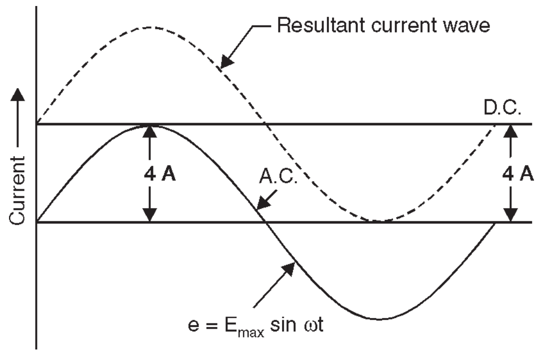


Fig. 18

(*ii*) **Analytical expression.** The instantaneous value of the resultant current is given by

$$i = (4 + 4 \sin \omega t) = (\mathbf{4 + 4 \sin} \theta). \quad \textbf{(Ans.)}$$

(*iii*) **Average value.** Since the average value of the alternating current over one complete cycle is zero, hence the *average value of the resultant current is equal to the value of D.C. component i.e.,* **4A (Ans.)**

(*iv*) **Effective or r.m.s. value :**

Mean value of i^2 over complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi}\int_0^{2\pi} (4 + 4 \sin \theta)^2 \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (16 + 32 \sin \theta + 16 \sin^2 \theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[16 + 32 \sin \theta + 16\left(\frac{1 - \cos 2\theta}{2}\right)\right] d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (24 + 32 \sin \theta - 8 \cos 2\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left(24\,\theta - 32 \cos \theta - 8 \times \frac{\sin 2\,\theta}{2}\right)_0^{2\pi}$$

$$= \frac{1}{2\pi}\,[(48\pi - 32 \cos 2\pi - 4 \sin 4\pi) - (-32)] = \frac{48\pi}{2\pi} = 24 \text{ A}$$

∴ R.M.S. value, $\quad I = \sqrt{24}$ = **4.9 A. (Ans.)**

Example 8. *Determine the average and effective values of the saw-tooth waveform shown in Fig. 19.*

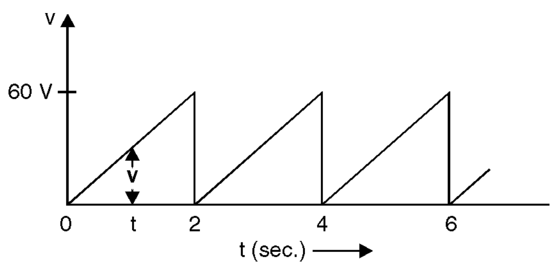


Fig. 19

Solution. Refer to Fig. 19.

Since the voltage increases linearly, therefore,

$$V_{av} = \frac{60 + 0}{2} = \mathbf{30\ V.} \quad \textbf{(Ans.)}$$

The slope, for the internal $0 < t < 2$ is given by :

$$\text{Slope} = \frac{60}{2} = 30$$

$\therefore$ Instantaneous voltage, $v = 30t$ volts

The r.m.s. or effective value of the voltage,

$$V^2_{\text{r.m.s.}} = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{2}\int_0^2 (30t)^2\, dt = 450\int_0^2 t^2\, dt = 450\left|\frac{t^3}{3}\right|_0^2 = 1200$$

or $$V_{\textbf{r.m.s}} = \mathbf{34.64\ V.} \quad \textbf{(Ans.)}$$

Example. 9. *Determine the r.m.s. and average values of the waveform shown in Fig. 20.*

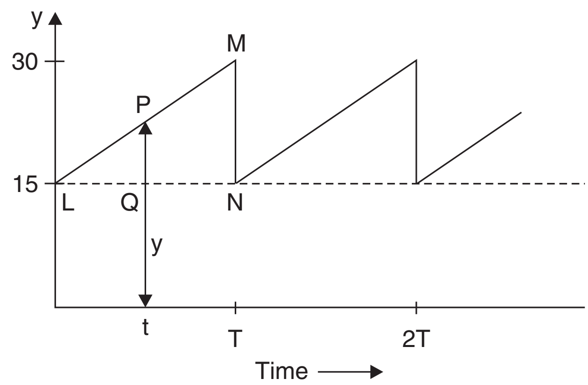


Fig. 20

Solution. Refer to Fig. 20.

The slope the curve $\quad LM = \frac{MN}{LN} = \frac{30 - 15}{T} = \frac{15}{T}$

Now consider the function y at any time 't'. We have,

$$\therefore \quad \frac{PQ}{LQ} = \frac{MN}{LN} = \frac{15}{T}$$

or $$\frac{y - 15}{t} = \frac{15}{T} \quad \text{or} \quad y = 15 + \left(\frac{15}{T}\right)t$$

This gives the equation for the function for one cycle.

$$Y_{av} = \frac{1}{T}\int_0^T \left[15 + \left(\frac{15}{T}\right)t\right] dt = \frac{1}{T}\int_0^T \left[15\, dt + \frac{15}{T}\,.\,t\,.\,dt\right]$$

or $$Y_{av} = \frac{1}{T}\left[15\,t + \frac{15t^2}{2T}\right]_0^T = \frac{1}{T}\left[15\,T + 7.5\,T\right] = \mathbf{22.5.} \quad \textbf{(Ans.)}$$

Mean square value $$= \frac{1}{T}\int_0^T y^2 dt = \frac{1}{T}\int_0^T \left\{15 + \left(\frac{15}{T}\right)t\right\}^2 dt$$

$$= \frac{1}{T}\int_0^T \left(225 + \frac{225t^2}{T^2} + \frac{450}{T}\,.\,t\right) dt$$

$$= \frac{1}{T}\left| 225t + \frac{225t^3}{3T^2} + \frac{450t^2}{2T} \right|_0^T$$

$$= \frac{1}{T}\ |\ 225\ T + 75\ T + 225\ T\ |\ = 525$$

∴ R.M.S. value $= \sqrt{525}$ = **22.9. (Ans.)**

Example 10. *Find the r.m.s. and average value of the trapezoidal current wave-form shown in the Fig. 21.*

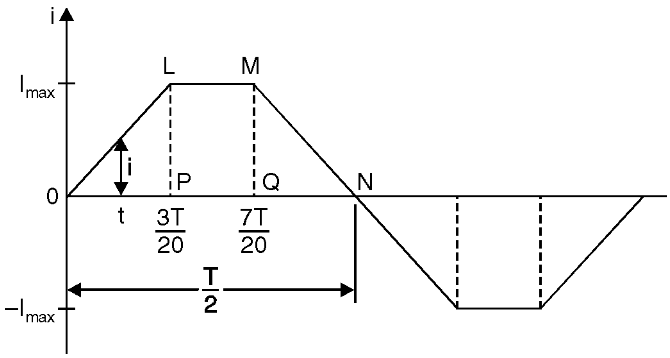


Fig. 21

Solution. Refer to Fig. 21. The equation of the current for $0 < t < \frac{3T}{20}$ can be found from the relation,

$$\frac{i}{t} = \frac{I_{max}}{(3T/20)} \quad \text{or} \quad i = \frac{20\ I_{max}}{3T}\,.\,t$$

The equation of the current for $\frac{3T}{20} < t < \frac{7T}{20}$ is given by $i = I_{max}$. Remembering that ΔOLP is identical with MQN,

R.M.S. value of current, $I_{r.m.s.} = \sqrt{\frac{1}{(T/2)}\left[2\int_0^{3T/20} i^2 dt + \int_{3T/20}^{7T/20} I_{max}^2\ dt\right]}$

$$= \sqrt{\frac{2}{T}\left[2\left\{\left(\frac{20\ I_{max}}{3T}\right)^2 \int_0^{3T/20} t^2\ dt\right\} + I_{max}^2 \int_{3T/20}^{7T/20} dt\right]}$$

$$= \sqrt{\frac{2}{T}\left[2\left\{\left(\frac{20\ I_{max}}{3T}\right)^2 \left|\frac{t^3}{3}\right|_0^{3T/20}\right\} + I_{max}^2 \left| t \right|_{3T/20}^{7T/20}\right]}$$

$$= \sqrt{\frac{2}{T}\left[2 \times \frac{400\ I_{max}^2}{9T^2} \times \frac{27}{3 \times 8000} T^3 + I_{max}^2 \times \left(\frac{7T}{20} - \frac{3T}{20}\right)\right]}$$

$$= \sqrt{\frac{2}{T}\left(0.1\, I_{max}^2\, T + 0.2\, I_{max}^2\, T\right)} = \sqrt{0.6\, I_{max}^2} = \mathbf{0.775\ I_{max}}.\quad \textbf{(Ans.)}$$

Average value of current,

$$I_{av} = \frac{1}{T/2}\left[2\int_0^{3T/20} i dt + \int_{3T/20}^{7T/20} I_{max}\, dt\right] = \frac{2}{T}\left[2\int_0^{3T/20}\left(\frac{20\, I_{max}}{3T}\right) t\, dt + I_{max}\int_{3T/20}^{7T/20} dt\right]$$

$$= \frac{2}{T}\left[2\left(\frac{20\, I_{max}}{3T}\right)\left|\frac{t^2}{2}\right|^{3T/20} + I_{max}\left| t \right|_{3T/20}^{7T/20}\right]$$

$$= \frac{2}{T}\left[2\left(\frac{20\, I_{max}}{3T}\right)\times\frac{1}{2}\left(\frac{3T}{20}\right)^2 + I_{max}\left(\frac{7T}{20} - \frac{3T}{20}\right)\right]$$

$$= \frac{2}{T}\left[2\left(\frac{20\, I_{max}}{3T}\right)\times\frac{1}{2}\left(\frac{9T}{400}\right)^2 + I_{max}\times\frac{T}{5}\right]$$

$$I_{av} = \frac{2}{T}\left[0.15\, I_{max}\times T + 0.2\, I_{max}\times T\right] = \mathbf{0.7\ I_{max}}.\ \textbf{(Ans.)}$$

Example 11. *A half wave single anode rectifier has a voltage given by 100 sin ωt applied to it. Estimate the average value on the d.c. side.*

Solution. The wave form on the d.c. side is as shown in Fig. 22.

Mean/Average d.c. voltage will be :

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} 100 \sin\theta\, .\, d\theta$$

$$= \frac{100}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

= **31.83 V. (Ans.)**

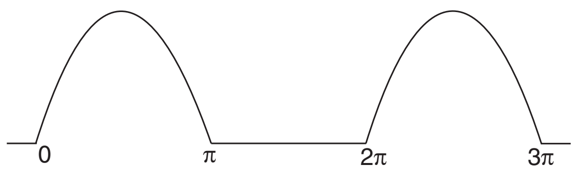


Fig. 22

Example 12. *Calculate from the first principles, the reading which would be indicated by a hot-wire ammeter in a circuit whose current waveform is given by 10 sin ωt + 3 sin 3ωt + 2 sin 5ωt.*

Solution. The expression for instantaneous current is :

$$i = 10 \sin \omega t + 3 \sin 3\omega t + 2 \sin 5\omega t$$

The hot-wire ammeter will read the "r.m.s. value" of the wave form.

Now $i^2 = (10 \sin \omega t + 3 \sin 3\omega t + 2 \sin 5\omega t)^2$

∴ *R.M.S. value of the current* $= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$

$$= \left[\frac{1}{2\pi}\int_0^{2\pi}\left[(10 \sin \omega t + 3 \sin 3\omega t + 2 \sin 5\omega t)^2\, d(\omega t)\right]\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\int_0^{2\pi}\{10^2 \sin^2 \omega t + 3^2 \sin^2 3\omega t + 2^2 \sin^2 5\omega t + 2\times 10\times 3 \sin \omega t \sin 3\omega t\right.$$

$$\left. + 2\times 10\times 2 \sin \omega t \sin 5\omega t + 2\times 3\times 2 \sin 3\omega t\, .\, \sin 5\omega t\}\, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi} \int_0^{2\pi} \left\{ \frac{10^2 (1 - \cos 2\omega t)}{2} + \frac{3^2}{2} (1 - \cos 6\omega t) + \frac{2^2}{2} (1 - \cos 10\omega t) + {}^* etc. \right\} d(\omega t) \right]^{1/2}$$

*All the terms containing the product of two sines when integrated over the range 0 to 2π disappear. This is easily seen by splitting such terms into the difference of two cosines.

$$= \left[\frac{1}{2\pi} \frac{(10^2 + 3^2 + 2^2)}{2} . 2\pi \right]^{1/2} = \left(\frac{10^2 + 3^2 + 2^2}{2} \right)^{1/2} = \left(\frac{113}{2} \right)^{1/2}$$

$= (56.5)^{1/2} =$ **7.52. (Ans.)**

Example 13. *Four branches A, B, C, D in an A.C. circuit meet at a junction point P. The currents in branches A, B, C flow towards P while the current in branch D flows away from P. The currents in branches A, B and C are*

$$i_A = 20 \sin 628t$$
$$i_B = 15 \sin (628t - \pi/6)$$
$$i_C = 25 \sin (628t + \pi/3)$$

Find an expression for the instantaneous value of current in branch D, and calculate (i) its frequency, and (ii) the heat (watts) that it would produce when flowing in a resistance of 5 ohms.

Solution.

Analytical method. Let the current flowing in the branch D be, $i_D = I_D \sin (628t + \phi)$

$$\Sigma H = I_{Amax} \cos \phi_1 + I_{Bmax} \cos \phi_2 + I_{Cmax} \cos \phi_3 + I_{Dmax} \cos \phi$$
$$= 20 \cos 0° + 15 \cos (-\pi/6) + 25 \cos \pi/3 + I_{Dmax} \cos \phi$$
$$= 20 + 15 \times \frac{\sqrt{3}}{2} + 25 \times \frac{1}{2} + I_{Dmax} \cos \phi$$
$$= 20 + 13 + 12.5 + I_{Dmax} \cos \phi = 45.5 + I_{Dmax} \cos \phi$$
$$\Sigma V = I_{Amax} \sin \phi_1 + I_{Bmax} \sin \phi_2 + I_{Cmax} \sin \phi_3 + I_{Dmax} \sin \phi$$
$$= 20 \sin 0° + 15 \sin (-\pi/6) + 25 \sin \pi/3 + I_{Dmax} \sin \phi$$
$$= 0 - 15 \times \frac{1}{2} + 25 \times \frac{\sqrt{3}}{2} + I_{Dmax} \sin \phi$$
$$= -7.5 + 21.65 + I_{Dmax} \sin \phi = 14.15 + I_{Dmax} \sin \phi$$

Since all the currents are meeting at point P,

$\therefore$ $\Sigma H = 0$

i.e., $45.5 + I_{Dmax} \cos \phi = 0$ or $I_{Dmax} \times \cos \phi = -45.5$ ...(*i*)

and $\Sigma V = 0$

i.e., $14.15 + I_{Dmax} \sin \phi = 0$ or $I_{Dmax} \times \sin \phi = -14.15$ ...(*ii*)

From (*i*) and (*ii*), $I_{Dmax} = (-45.5)^2 + (-14.15)^2 = 47.6$ A.

$$\phi = \tan^{-1} \frac{-14.15}{-45.5} = 197° \text{ or } 3.44°$$

Hence the current in branch D follows the relation,

$$i_D = 47.6 \sin (678\, t + 3.44). \textbf{ (Ans.)}$$

(*i*) **Frequency** $= \frac{\omega}{2\pi} = \frac{628}{2\pi} = 100$ Hz. **(Ans.)**

(*ii*) **Heat produced** $= \left(\frac{I_{Dmax}}{\sqrt{2}} \right)^2 \times R = \left(\frac{47.6}{\sqrt{2}} \right)^2 \times 5 = 5620$ W. **(Ans.)**

4. SINGLE PHASE CIRCUITS

The study of circuits involves three basic types of units (*R, L, C i.e.,* resistance, reactance and capacitance respectively) and four possible series combination of them. The latter, in turn, may be arranged in many kinds of parallel, series-parallel, parallel-series or other complex circuits.

4.1. A.C. Through Pure Ohmic Resistance Alone

The circuit containing a pure resistance R is shown in Fig. 23 (a). Let the applied voltage be given by the equation,

$$v = V_{max} \sin \theta = V_{max} \sin \omega t \quad ...(i)$$

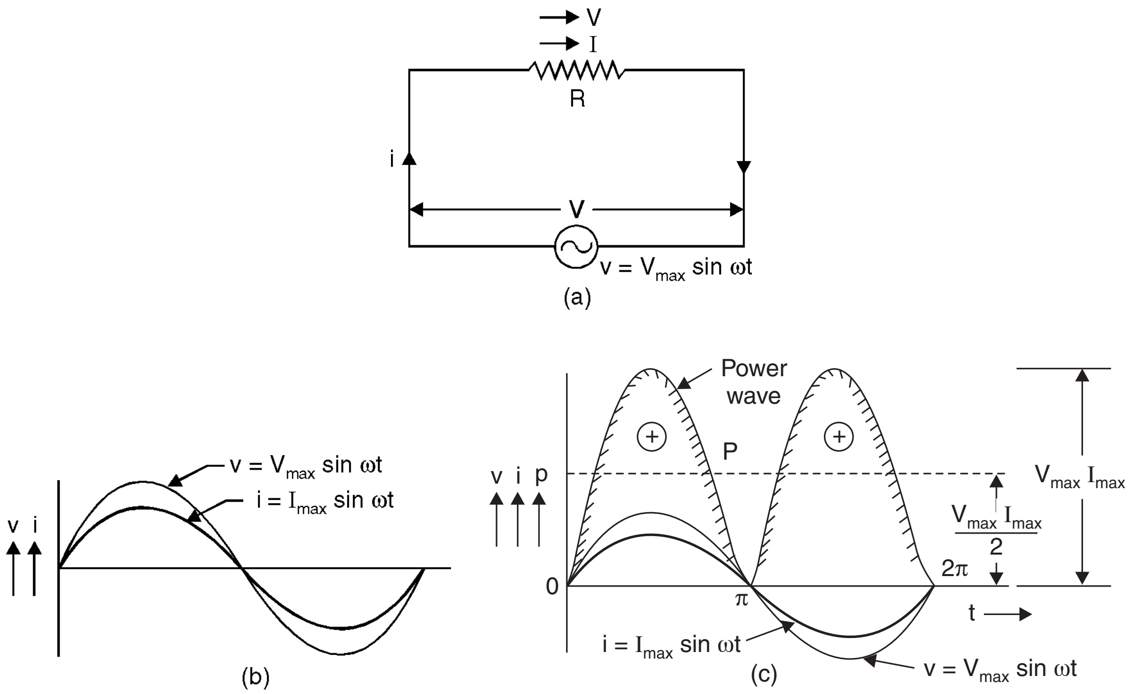


Fig. 23. A.C. through pure ohmic resistance alone.

Then the instantaneous value of current flowing through the resistance R will be,

$$i = \frac{v}{R} = \frac{V_{max} \sin \omega t}{R} \quad ...(ii)$$

The value of current will be maximum

when $\sin \omega t = 1$ or ($\omega t = 90°$)

$$\therefore \quad I_{max} = \frac{V_{max}}{R}$$

Substituting this value in eqn. (ii), we get

$$i = I_{max} \sin \omega t \quad ...(iii)$$

Comparing (i) and (iii), we find that alternating *voltage and current are in phase with each other* as shown in Fig. 23 (b), also shown vectorially in Fig. 23 (c).

Power. Refer to Fig. 23 (*c*)

Instantaneous power,

$$p = vi = V_{max} \sin \omega t \times I_{max} \sin \omega t = V_{max} I_{max} \sin^2 \omega t$$

$$= \frac{V_{max} I_{max}}{2} \times 2 \sin^2 \omega t = \frac{V_{max} I_{max}}{2} (1 - \cos 2 \omega t)$$

$$= \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} - \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos 2\omega t$$

(Constant part) (Fluctuating part)

For a complete cycle the average of $\frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos 2 \omega t$ *is zero.*

Hence, power for the whole cycle,

$$P = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = V_{r.m.s} \cdot I_{r.m.s.}$$

or $$P = VI \text{ watt}$$

where V = R.M.S. value of applied voltage, and

I = R.M.S. value of the current.

It may be observed from the Fig. 23 (*c*) that no part of the power cycle at any time becomes negative. In other words the power in a purely resistive circuit *never becomes zero*.

Hence in ***pure resistive circuit*** we have :

1. Current is in phase with the voltage.

2. Current $I = \frac{V}{R}$ where I and V are r.m.s. values of current and voltage.

3. Power in the circuit, $P = VI = I^2R$.

4.2. A.C. Through Pure Inductance Alone

Fig. 24 (*a*) shows the circuit containing a pure inductance of L henry.

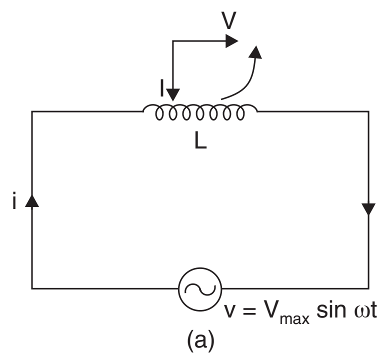


Fig. 24 (*a*)

Let the alternating voltage applied across the circuit be given by the equation,

$$v = V_{max} \sin \omega t \qquad ...(i)$$

Whenever an alternating voltage is applied to a purely inductive coil, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall of the current through the coil. Since there is no ohmic drop in this case, therefore, the applied voltage has to overcome this induced e.m.f. only. Thus at every step,

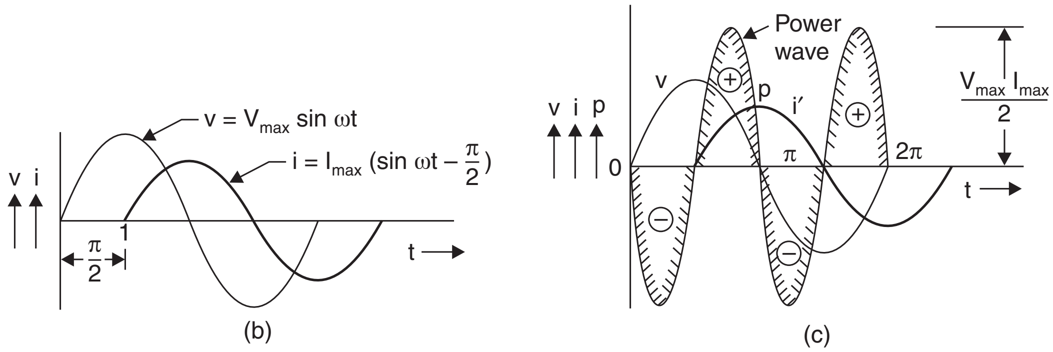


Fig. 24 (*b*), (*c*). A.C. through pure inductance alone. Resultant power zero.

$$v = L \frac{di}{dt}$$

or $$V_{max} \sin \omega t = L \frac{di}{dt}$$

or $$di = \frac{V_{max}}{L} \sin \omega t \; dt$$

Integrating both sides, we get

$$\int di = \int \frac{V_{max}}{L} \sin \omega t \; dt$$

or $$i = \frac{V_{max}}{L}\left(-\frac{\cos \omega t}{\omega}\right) = \frac{V_{max}}{\omega L} \sin\left[\omega t - \frac{\pi}{2}\right]$$

or $$i = \frac{V_{max}}{X_L} \sin\left[\omega t - \frac{\pi}{2}\right] \qquad ...(ii)$$

where $X_L = \omega L$ (opposition offered to the flow of alternating current by a pure inductances) and is called **Inductive reactance.** It is given in ohms if L is in henry and ω is in radian/second.

The value of current will be maximum when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

∴ $$I_{max} = \frac{V_{max}}{X_L}$$

Substituting this value in eqn. (*ii*), we get

$$i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(iii)$$

Power. Refer to Fig. 24 (*c*)

Instantaneous power, $$p = vi = V_{max} \sin \omega t \times I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= - V_{max} I_{max} \sin \omega t \, . \cos \omega t$$

$$= - \frac{V_{max} I_{max}}{2} \times 2 \sin \omega t \cos \omega t$$

$$= - \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \, . \sin 2\omega t$$

∴ Power for the whole cycle, $P = -\frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}} \int_0^{2\pi} \sin 2\omega t = 0$

Hence *average power consumed in a pure inductive circuit is* **zero.**

Hence in a ***pure inductive circuit,*** we have :

1. Current $I = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi f L}$ amp.
2. Current always *lags* behind the voltage by *90°*.
3. Average power consumed is *zero.*

Variation of X_L and f :

Since $X_L = \omega L = 2\pi f L$, and here if L is constant, then

$$X_L \propto f$$

Fig. 25, shows the variation. As frequency is increased X_L increases and the current taken by the circuit decreases.

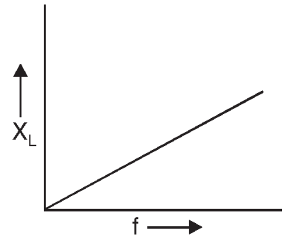


Fig. 25. Variation of X_L with f.

4.3. A.C. Through Pure Capacitance Alone

The circuit containing a pure capacitor of capacitance C farad is shown in Fig. 26 (*a*). Let the alternating voltage applied across the circuit be given by the equation,

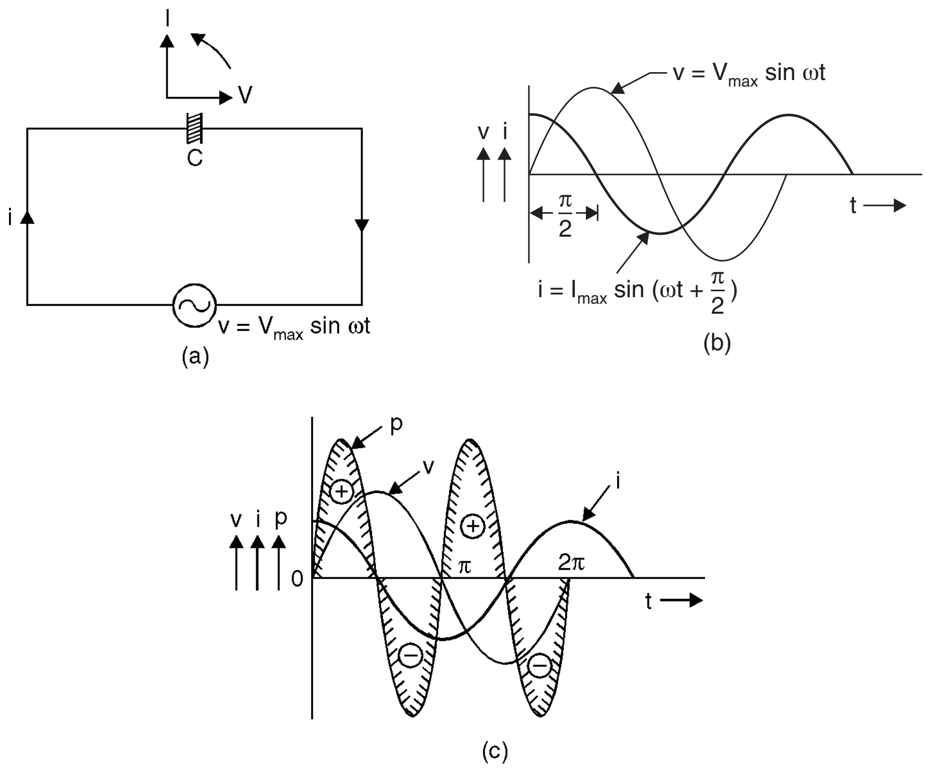


Fig. 26. A.C. through pure capacitance alone. Resultant power is zero.

$$v = V_{max} \sin \omega t \qquad ...(i)$$

Charge on the capacitor at any instant,

$$q = C\,v$$

Current through the circuit,

$$i = \frac{dq}{dt} = \frac{d}{dt}\,(CV_{max} \sin \omega t) = \omega\, CV_{max} \cos \omega t$$

or
$$i = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

∴
$$i = \frac{V_{max}}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(ii)$$

The denominator $X_C = \frac{1}{\omega C}$ (opposition offered to the flow of alternating current by a pure capacitor) is known as *capacitive reactance.*

It is given in ohms if C is in farad and ω in radian/second.

The value of current will be maximum when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

∴
$$I_{max} = \frac{V_{max}}{X_C}$$

Substituting this value in eqn. (*ii*), we get

$$i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(iii)$$

Power. Refer to Fig. 26 (*c*)

Instantaneous power,

$$p = vi = V_{max} \sin \omega t \times I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_{max}\, I_{max} \sin \omega t \cos \omega t = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}}\ \sin 2\omega t$$

Power for the whole cycle $= \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \int_0^{2\pi} \sin 2\,\omega t = 0$

This fact is graphically illustrated in Fig. 26 (*c*). It may be noted that, during the first quarter cycle, what so ever power or energy is supplied by the source is stored in the electric field set-up between the capacitor plates. During the next quarter cycle, the electric field collapses and the power or energy stored in the field is returned to the source. The process is repeated in each alternation and this circuit does not absorb any power.

Hence in a ***pure capacitive circuit,*** we have

1. $I = \frac{V}{X_C} = V \times 2\,\pi f C$ amps.

2. Current always leads the applied voltage by 90°.

3. Power consumed is *zero*.

Variation of X_C and f :

Since $X_C = \frac{1}{2\pi f C}$ and if C is kept constant, than

$$X_C \propto \frac{1}{f}$$

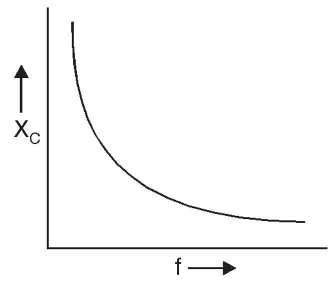


Fig. 27

Fig. 27 shows the variation. As the frequency increases X_C decreases, so the current increases.

4.4. Phasor Algebra

The following are the methods of representing vector quantities :

1. Symbolic notation 2. Trigonometrical form
3. Exponential form 4. Polar form.

A vector as shown in Fig. 28 may be described in the above forms as follows :

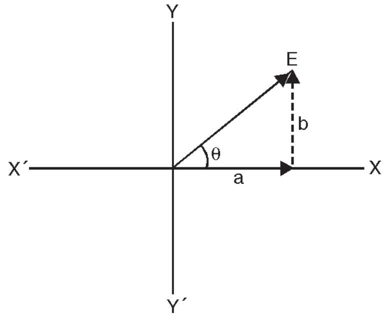


Fig. 28

1. *Symbolic notation* :

$$E = a + jb$$

2. *Trigonometrical form* :

$$E = \sqrt{a^2 + b^2}\ (\cos\theta + j\sin\theta)$$

$$= \sqrt{a^2 + b^2}\ (\cos\theta \pm j\sin\theta) \quad \text{......in general}$$

3. *Exponential form* : $E = \sqrt{a^2 \times b^2}\ e^{+j\theta}$

$$= \sqrt{a^2 + b^2}\ e^{\pm j\theta} \quad \text{......in general}$$

4. *Polar form* : $E = \sqrt{a^2 + b^2}\ \angle\theta$

$$= \sqrt{a^2 + b^2}\ \angle \pm\theta \quad \text{......in general}$$

Significance of operator j. The letter j used in the above expressions is a symbol of an operation. It is used to indicate the counter-clockwise rotation of a vector through 90°. It is assigned a value of $\sqrt{-1}$. The double operation of j on a vector rotates it counter-clockwise (*CCW*) through 180° and hence reverses its sense because, $j \times j = j^2 = \sqrt{(-1)^2} = -1$.

In general, each successive multiplication of j, rotates the phasor further by 90° as given below (Refer to Fig. 29)

$$j = \sqrt{-1}$$

...90° CCW rotation from OX-axis

$$j^2 = \left(\sqrt{-1}\right)^2 = -1$$

...180° CCW rotation from OX-axis

$$j^3 = \left(\sqrt{-1}\right)^3 = -\sqrt{-1} = -j$$

...270° CCW rotation from OX-axis

$$j^4 = \left(\sqrt{-1}\right)^4 = +1$$

...360° CCW rotation from X-axis

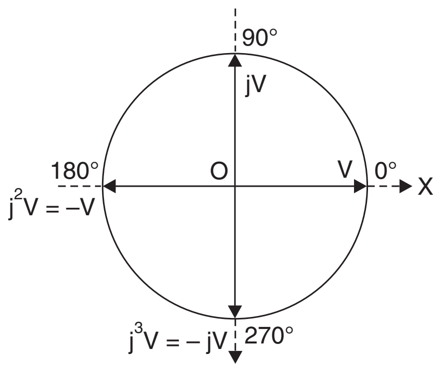


Fig. 29

It should also be noted that,

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j.$$

Example 14. *Write the equivalent potential and polar forms of vector 6 + j8. Also illustrate the vector by means of diagram.*

Solution. Refer to Fig. 30.

Magnitude of the vector $= \sqrt{6^2 + 8^2} = 10,\ \tan\theta = \frac{8}{6}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.1°$

$\therefore$ **Exponential form = 10 e$^{j\,53.1°}$. (Ans.)**

The angle may also be *expressed in radians*.

Polar form = 10 ∠ 53.1°. (Ans.)

The vector is illustrated by means of diagram in Fig. 30.

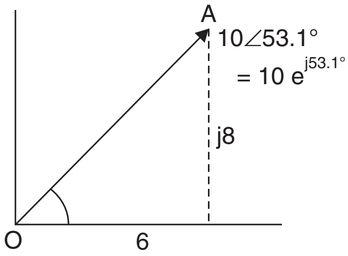


Fig. 30

Example 15. *A vector is represented by $30e^{-j2\pi/3}$. Write down the various equivalent forms of the vector and illustrate by means of a vector diagram, the magnitude and position of the above vector.*

Solution. Refer to Fig. 31. Draw the vector in a direction making an angle of $\frac{2\pi}{3} = \frac{2 \times 180}{3} = 120°$ in the *clockwise* direction (since the angle is negative).

(*i*) **Rectangular form :**

$$a = 30 \cos(-120°) = -15$$

$$b = 30 \sin(-120°) = -25.98$$

$\therefore$ Expression is $=$ **(– 15 – j 25.98). (Ans.)**

(*ii*) **Polar form = 30 ∠ – 120°. (Ans.)**

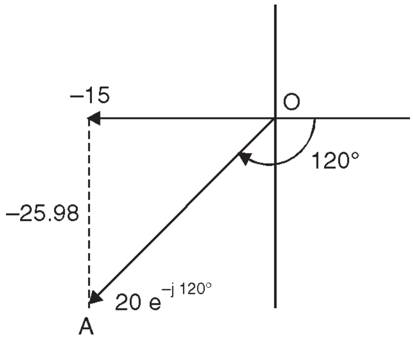


Fig. 31

Addition and subtraction of vector quantities :

For addition and subtraction of vector quantities rectangular form is best suited. Consider two voltage phasors represented as :

$$\overline{V_1} = a_1 + jb_1 \text{ and } \overline{V_2} = a_2 + jb_2$$

Addition. $\overline{V} = \overline{V_1} + \overline{V_2} = (a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j\,(b_1 + b_2)$

The magnitude of the resultant vector $\quad \overline{V} = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

The position of $\overline{V}$ with respect to X-axis is $\quad \theta = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$

Subtraction. $\overline{V} = \overline{V_1} - \overline{V_2} = (a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$

The magnitude of the resultant vector $\quad \overline{V} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

The position of $\overline{V}$ with respect to X-axis is $\quad \theta = \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right)$

Multiplication and division of vector quantities :

It the vectors are represented in the polar exponential form, their multiplication and division becomes very easy and simple.

Consider two voltage phasors represented as

$$\overline{V_1} = a_1 + jb_1 = V_1\angle\theta_1, \text{ where } \theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

$$\overline{V_2} = a_2 + jb_2 = V_2\angle\theta_2, \text{ where } \theta_2 = \tan^{-1}\left(\frac{b_2}{a_2}\right)$$

Multiplication. When the phasor quantities are represented in polar form, while multiplying their magnitudes are multiplied and their *angles added algebraically*,

i.e, $$\overline{V_1} \times \overline{V_2} = V_1\angle\theta_1 \times V_2\angle\theta_2 = V_1V_2\angle(\theta_1 + \theta_2)$$

Division. In this case, the magnitudes of phasor quantities (expressed in polar from) are divided and their *angles subtracted algebraically*

i.e. $$\frac{\overline{V_1}}{\overline{V_2}} = \frac{V_1\angle\theta_1}{V_2\angle\theta_2} = \frac{V_1}{V_2}\angle(\theta_1 - \theta_2).$$

Example 16. *Perform the following operation and express the final result in polar form :*

$$10\angle 30° + 16\angle -30°.$$

Solution.

$$10\angle 30° = 10(\cos 30° + j\sin 30°) = 8.66 + j5$$

$$16\angle -30° = 16[\cos(-30°) + j\sin(-30°)] = 13.86 - j8$$

$$\therefore \quad 10\angle 30° + 16\angle -30° = (8.66 + j5) + (13.86 - j8) = 22.52 - j3$$

$$= (\sqrt{22.52^2 + 3^2})\tan^{-1}(-3/22.52)$$

$$= 22.72\tan^{-1}(-3/22.52) = \mathbf{22.72\angle -7.6°}. \quad \textbf{(Ans.)}$$

Example 17. *Subtract the following given vectors from one another.*

$$\overline{A} = 15 + j26 \text{ and } \overline{B} = -19.75 - j7.18.$$

Solution. $\overline{A} - \overline{B} = \overline{C} = (15 + j26) - (-19.75 - j7.18) = 34.75 + j33.18$

$$\therefore \quad \text{Magnitude of } \overline{C} = \sqrt{34.75^2 + 33.18^2} = \mathbf{48}$$

$$\text{Slope of } \overline{C} = \tan^{-1}(33.18/34.75) = 43.68°$$

$$\therefore \quad \overline{C} = \mathbf{48\angle 43.68°}. \quad \textbf{(Ans.)}$$

Example 18. *Perform the operation* $\frac{\overline{A}\overline{B}}{\overline{C}}$ *and express the final result in polar form for the vectors given below :*

$$\overline{A} = 10 + j10;\ \overline{B} = 15\angle -120°;\ \overline{C} = 5 + j0°.$$

Solution. Rearranging vectors $\overline{A}$ and in polar form, we have

$$\overline{A} = 10 + j10 = \sqrt{10^2 + 10^2}\tan^{-1}(10/10) = 14.14\angle 45°$$

$$\overline{C} = 5 + j0 = \sqrt{5^2 + 0^2}\tan^{-1}(0/5) = 5\angle 0°$$

$$\therefore \quad \frac{\overline{A}\overline{B}}{\overline{C}} = \frac{14.14\angle 45° \times 15\angle -120°}{5\angle 0°} = \frac{14.14 \times 15}{5}\angle(45° - 120° - 0°)$$

$$= \mathbf{42.42\angle -75°}. \quad \textbf{(Ans.)}$$

Example 19. *The instantaneous values of two currents* i_1 *and* i_2 *are given as :*

$$i_1 = 5\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and } i_2 = 2.5\cos\left(\omega t - \frac{\pi}{2}\right)$$

Find the r.m.s. value of $i_1 + i_2$ *using complex number representation.*

Solution. *Given :* $i_1 = 5 \sin\left(\omega t + \frac{\pi}{4}\right)$,

$$i_2 = 2.5 \cos\left(\omega t - \frac{\pi}{2}\right) = 2.5 \sin\left[90° + \left(\omega t - \frac{\pi}{2}\right)\right] = 2.5 \sin \omega t$$

$\therefore$ $I_{1(max)} = 5 (\cos 45° + j \sin 45°) = (3.53 + j3.53)$

$I_{2(max)} = 2.5 (\cos 0° + j \sin 0°) = (2.5 + j0)$

The maximum value of resultant current is

$$I_{max} = (3.53 + j3.53) + (2.5 + j0) = 6.03 + j3.53 = 6.987 \angle 30.34°$$

$\therefore$ R.M.S. value $= \frac{6.987}{\sqrt{2}}$ = **4.94 A. (Ans.)**

Conjugate complex numbers

Two numbers are said to be conjugate if they *differ only in the algebraic sign of their quadrature components.* Accordingly, the numbers $(a + jb)$ and $(a - jb)$ are conjugate.

- The *sum* of two conjugate numbers gives in-phase or *active* component and their *difference* gives quadrature or *reactive* component.

i.e. $(a + jb) + (a - jb) = 2a$ (*i.e.,* active component), and

$(a + jb) - (a - jb) = j2b$ (*i.e.,* reactive component).

The resultant is the sum of two vertical components only.

- The resultant arising out of the *multiplication* of two conjugate numbers contains *no quadrature component*.

i.e., $(a + jb) \times (a - jb) = a^2 - j^2b^2 = a^2 + b^2$

The conjugate of a complex number is used to determine the *apparent power* of an A.C. circuit in complex form.

Power and roots of vectors/phasors

The *powers* and *roots* of vectors can be found conveniently in *polar form*. If the vector are not, in polar form, these should be converted into polar form before carrying out the algebraic operations, as mentioned below.

Powers. Consider a vector phasor quantity represented in polar form as $\overline{A} = A\angle\theta$,

Then $(\overline{A})^n = A^n\angle(n \times \theta)$

Example. Suppose it is required to find cube of the vector $4\angle 12°$

Then, $(4\angle 12°)^3 = (4)^3 \angle (3 \times 12°) = 64 \angle 36°$

Roots. Consider a vector (phasor) quantity represented in polar form as $(\overline{A})^{1/n} = (A)^{1/n} \angle\theta/n$.

Example. Suppose it is required to find cube root of $125 \angle 60°$

Then, $(125 \angle 60°)^{1/3} = (125)^{1/3} \angle 60°/3 = 5\angle 20°$.

The 120° operator

In case of 3-phase work, where voltage vectors are displaced by 120° from one another (Fig. 32) it is convenient to use an operator, which rotates a vector/phasor through 120° toward or backwards without altering its length. This operator is '*a*' and any operator which is multiplied by '*a*' remains unaltered in magnitude but is rotated in CCW (counter-clockwise) direction by 120°.

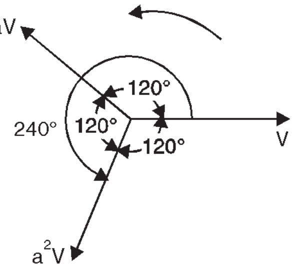


Fig. 32

$\therefore$ $a = 1 \angle 120°$

In cartesian form, $a = \cos 120° + j \sin 120°$

$= -0.5 + j\, 0.866$

Similarly, $a^2 = 1 \angle 120° \times 1 \angle 120° = 1 \angle 240° = \cos 240° + j \sin 240° = -0.5 - j\,0.866$

Hence the operator '*a*' will rotate the in CCW by 240° which is the same as rotating the vector in CW(clock-wise) direction by 120°.

$\therefore$ $a^2 = 1 \angle -120°$

Similarly $a^3 = 1 \angle 360° = 1$ (Numerically, *a* is equivalent to the cube root of unity.)

4.5. A.C. Series Circuits

Under this heading we shall discuss *R-L, R-C* and *R-L-C* series circuits.

4.5.1. R-L circuit (Resistance and inductance in series)

Fig. 33 (*a*) shows a pure resistance *R* and a pure inductive coil of inductance *L* connected in series. Such a circuit is known as *R-L* circuit (usually met a cross in practice).

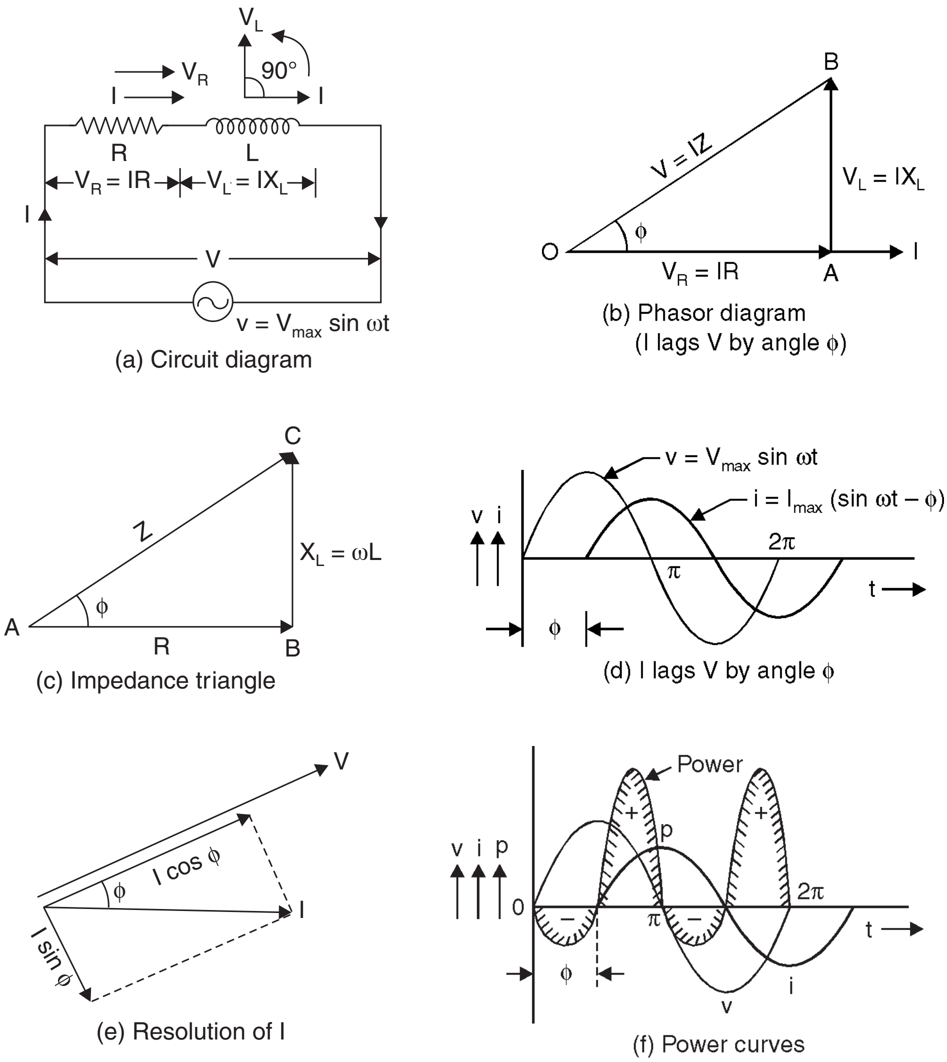


Fig. 33. *R-L* circuit (Resistance and inductance in series).

Let V = R.M.S. value of the applied voltage,

I = R.M.S. value of the resultant current,

$V_R = IR$ = Voltage drop across R(*in phase* with I), and

$V_L = IX_L$ = Voltage drop across L (coil), *ahead* of I by 90°.

The voltage drop V_R and V_L and shown in voltage triangle OAB in Fig. 33 (*b*), I being taken as the reference vector in the phasor diagram. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. Vector OB represents the applied voltage V which is the vector sum of the two (*i.e.,* V_R and V_L).

$$\therefore \quad V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R^2 + X_L^2)}$$

or

$$I = \frac{V}{\sqrt{R^2 + (X_L)^2}} = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$ (total opposition offered to the flow of alternating current by R-L series circuit) is known as **impedance** of the circuit.

As seen from the "*impedance triangle*" ABC [Fig. 33 (*c*)],

$$Z^2 = R^2 + X_L^2$$

i.e., (Impedance)^2 = (Resistance)^2 + (Inductive reactance)^2

From Fig. 33 (*b*) it is evident that voltage V *leads* the current by an angle ϕ such that,

$$\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{Inductive reactance}}{\text{Resistance}}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

The same is illustrated graphically in Fig. 33 (*d*).

In other words I *lags* V by an angle ϕ.

Power factor, $\cos \phi = \frac{R}{Z}$ [From Fig. 33 (*c*)]

Thus, if the applied voltage is given by $v = V_{max} \sin \omega t$, then current equation is given as,

$$i = I_{max} \sin (\omega t - \phi),$$

where

$$I_{max} = \frac{V_{max}}{Z}$$

In the Fig. 33 (*e*), I has been shown resolved into two components, $I \cos \phi$ along V and $I \sin \phi$ in quadrature (*i.e.,* perpendicular) with V.

Mean power consumed by the circuit

$= V \times I \cos \phi$ (*i.e.,* component of I which is in *phase with V*

i.e., $P = V I \cos \phi$ (= r.m.s. voltage × r.m.s. current × $\cos \phi$)

The term '$\cos \phi$' is called the ***power factor*** $\left(= \frac{R}{Z}\right)$ of the circuit

It may noted that :

— In A.C. circuit the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (*i.e., VA*) and *not true power in watts*. True power (W) = volt-amperes (VA) × power factor

or Watts = VA (Apparent power) × $\cos \phi$

— The *power consumed is due to ohmic resistance only* since pure inductance consumes no power.

i.e. $$P = VI \cos\phi = VI \times \frac{R}{Z} = \frac{V}{Z}\, IR = I \times IR = I^2R, \text{ watts}$$

$$\left(\because \quad \cos\phi = R/Z \text{ and } \frac{V}{Z} = I\right)$$

This shows that power is actually consumed in *resistance only ;* the *inductor does not consume any power.*

The power consumed in *R-L* circuit is shown graphically in Fig. 33 (*f*).

Thus in ***R-L circuit*** we have :

1. Impedance, $Z = \sqrt{R^2 + X_L^2}$ (where $X_L = \omega L = 2\pi \times fL$)

2. Current, $I = \frac{V}{Z}$

3. Power factor, $\cos\phi = \frac{R}{Z}\left(= \frac{\text{True power}}{\text{Apparent power}} = \frac{\text{W}}{\text{VA}}\right)$

[or angle of lag, $\phi = \cos^{-1}(R/Z)$]

4. Power consumed, $P = VI\cos\phi \left(= IZ \times I \times \frac{R}{Z} = I^2 R\right)$

Symbolic Notation :

$$Z = R + jX_L$$

The numerical value of impedance vector $= \sqrt{R^2 + X_L^2}$

The phase angle with the reference axis, $\phi = \tan^{-1}(X_L/R)$.

In *polar form :* $\bar{Z} = Z\angle\phi°$.

Apparent, Active (True or real) and Reactive Power :

Every circuit current has two components : (*i*) Active component and (*ii*) Reactive component.

"Active component" consumes power in the circuit while "*reactive component*" is responsible for the field which lags or leads the main current from the voltage.

In Fig. 34. active component is $I_{\text{active}} = I\cos\phi$, and reactive component is $I_{\text{reactive}} = I\sin\phi$

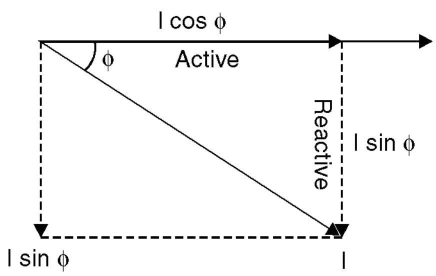


Fig. 34. Active and reactive components of circuit current I.

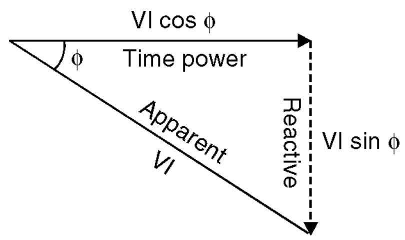


Fig. 35. Apparent, true and reactive power.

So, $$I = \sqrt{(I_{\text{active}})^2 + (I_{\text{reactive}})^2}$$

Refer to Fig. 35.

(*i*) **Apparent power (S).** It *is given by the product of r.m.s. values of applied voltage and circuit current.*

$$\therefore \quad S = VI = (I \times Z) \,.\, I = I^2 Z \text{ volt-amperes } (VA)$$

(*ii*) **Active or true or real power (P or W).** It *is the power which is actually dissipated in the circuit resistance.*

$$P = I^2 R = VI \cos \phi \text{ watts}$$

(*iii*) **Reactive power (Q).** A pure inductor and a pure capacitor do not consume any power, since in a half cycle what so ever power is received from the source by these components the same is returned to the source. This power which flows back and forth (*i.e.,* in both directions in the circuit) or *reacts upon itself* is called *"reactive power."*

It may be noted that the *current in phase with the voltage produces active or true or real power while the current 90° out of phase with the voltage contributes to* reactive power.

In a R-L circuit , reactive power which is the power developed in the inductive reactance of the circuit, is given as :

$$Q = I^2 X_L = I^2 Z \sin \phi = I \,.\, (IZ) \sin \phi$$
$$= VI \sin \phi \text{ volt-amperes-reactive } (VAR)$$

These three powers are shown in Fig. 35

Relation between VA, W and VR

$$W = VA \cos \phi \quad \text{...}(i)$$

$$VAR = VA \sin \phi \quad \text{...}(ii)$$

$$\therefore \quad VA = \frac{W}{\cos \phi} \quad \text{...[From } (i)]$$

and,

$$VA = \frac{VAR}{\sin \phi} \quad \text{...[From } (ii)]$$

$$\text{Power factor } (p.f.) = \frac{W}{VA} = \frac{\text{True power}}{\text{Apparent power}}$$

The larger bigger units of apparent, true and reactive power are *kVA* (or *MVA*), *kW*(or *MW*) and *kVAR* (or *MVAR*) respectively.

The power factor depends on the reactive power component. If it is made equal to the active power component, the power factor becomes unity.

Example 20. *A coil takes 2.5 amps. when connected across 200 volt 50 Hz mains. The power consumed by the coil is found to be 400 watts. Find the inductance and the power factor of the coil.*

Solution. Current taken by the coil, $I = 2.5$ A

Applied voltage, $V = 200$ volts

Power consumed, $P = 400$ W

We know that $P = VI \cos \phi$

or

$$400 = 200 \times 2.5 \times \cos \phi \text{ or } \cos \phi = \frac{400}{200 \times 2.5} = 0.8$$

Hence power factor of coil is **0.8. (Ans.)**

Impedance of the coil, $Z = \frac{V}{I} = \frac{200}{2.5} = 80\ \Omega$

Also $\frac{X_L}{Z} = \sin \phi$

$\therefore \quad X_L = Z \sin \phi$

$$= 80 \sin \phi = 80 \sqrt{1 - \cos^2 \phi}$$

$$= 80 \sqrt{1 - 0.8^2} = 80 \times 0.6 = 48\ \Omega$$

But $X_L = 2\pi f L$

$\therefore$ $L = \dfrac{X_L}{2\pi f} = \dfrac{48}{2\pi \times 50}$ = **0.1529 H (henry). (Ans.)**

Example 21. *A 100 V, 80 W lamp is to be operated on 230 volts, 50 Hz A.C. supply. Calculate the inductance of the choke required to be connected in series with lamp for its operation. The lamp can be taken as equivalent to a non inductive resistance.*

Solution. Current through the lamp when connected across 100 V supply,

$$I = \frac{W}{V} = \frac{80}{100} = 0.8\ \text{A}$$

Resistance of the lamp, $R = \dfrac{V}{I} = \dfrac{100}{0.8} = 125\ \Omega$

If a choke of inductance L henry is connected in series with the lamp to operate it on 230 V, the current through the choke will also be 0.8 A.

The impedance of the circuit when choke is connected in series with the lamp,

$$Z = \frac{V}{I} = \frac{230}{0.8} = 287.5\ \Omega$$

Reactance of choke coil, $X_L = \sqrt{Z^2 - R^2} = \sqrt{287.5^2 - 125^2} = 258.5\ \Omega$

But $X_L = 2\pi f L$

or $L = \dfrac{X_L}{2\pi f} = \dfrac{258.5}{2\pi \times 50} = 0.825$ H

Hence inductance of choke coil, **L = 0.825 H. (Ans.)**

Example 22. *A coil has a resistance of 5 Ω and an inductance of 31.8 mH. Calculate the current taken by the coil and power factor when connected to 200 V, 50 Hz supply.*

Draw the vector diagram.

If a non-inductive resistance of 10 Ω is then connected in series with coil, calculate the new value of current and its power factor.

Solution. $R = 5\ \Omega$

$L = 31.8$ mH or 0.0318 H

$\therefore$ $X_L = 2\pi f L$

$= 2\pi \times 50 \times 0.0318 = 10\ \Omega$

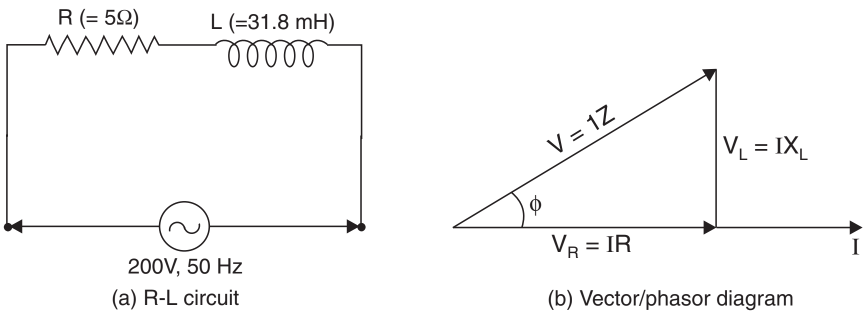


Fig. 36

Impedance of the coil, $Z = \sqrt{R^2 + X_L{}^2}$

$= \sqrt{5^2 + 10^2} = 11.18\ \Omega$

$\therefore$ Current taken by the coil, $I = \frac{V}{Z} = \frac{200}{11.18} =$ **17.9 A. (Ans.)**

Power factor, $\cos\phi = \frac{R}{Z} = \frac{5}{11.18} =$ **0.4475. (Ans.)**

Fig. 36 (*b*) shows the vector diagram.

When non-inductive resistance of 10 Ω is connected in series with the coil :

Total resistance in the circuit, $R' = 5 + 10 = 15\ \Omega$

Reactance in the circuit, $X_L' = X_L = 10\ \Omega$

Impedance of the circuit, $Z' = \sqrt{R'^2 + X'_L{}^2} = \sqrt{15^2 + 10^2} = 18\ \Omega$

Current through the circuit, $I' = \frac{V}{Z'} = \frac{200}{18} =$ **11.11 A. (Ans.)**

Power factor of the circuit, $\cos\phi = \frac{R'}{Z'} = \frac{15}{18} =$ **0.833. (Ans.)**

Example 23. *A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate :*

(*i*) *Impedance, reactance and resistance of the coil,*

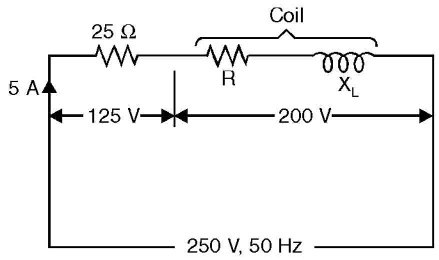
(*ii*) *The power absorbed by the coil,*

(*iii*) *The total power.*

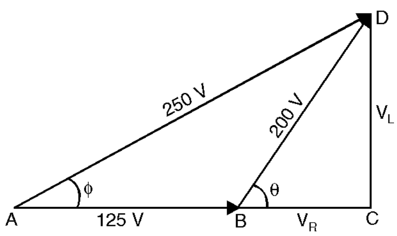
Draw the vector diagram.

Solution. Non-inductive resistance connected in series with coil $= \frac{125}{5} = 25\ \Omega$

Refer to Fig. 37 (*b*).



(a) Series circuit



(b) Vector diagram

Fig. 37

$$BC^2 + CD^2 = (200)^2 = 40000 \quad ...(i)$$

$$(125 + BC)^2 + CD^2 = (250)^2 = 62500 \quad ...(ii)$$

Subtracting eqn. (*i*) from eqn. (*ii*), we get

$$(125 + BC)^2 - BC^2 = 62500 - 40000 = 22500$$

$$15625 + BC^2 + 250BC - BC^2 = 22500$$

$\therefore$ $BC = 27.5\ V\ ;\ CD = \sqrt{(200)^2 - (27.5)^2} = 198.1$ V (*i*) **Coil impedance,** $Z = \frac{200}{5} = \mathbf{40}\ \Omega.$ **(Ans.)**

$V_R = IR = BC = 27.5$ or $5R = 27.5$

$\mathbf{R} = \frac{27.5}{5} = \mathbf{5.5}\ \Omega$ **(Ans.)**

Also, $V_L = IX_L = CD = 198.1$

$\therefore$ $\mathbf{X_L} = \frac{198.1}{5} = \mathbf{39.62}\ \Omega.$ **(Ans.)**

(*ii*) **Power absorbed by the coil,**

$P = I^2R = 5^2 \times 5.5 = \mathbf{137.5\ W.}$ **(Ans.)**

(*iii*) **The total power** $= VI \cos \phi = 250 \times 5 \times \frac{AC}{AD}$

$= 250 \times 5 \times \frac{(125 + 27.5)}{250} = \mathbf{762.5\ W.}$ **(Ans.)**

The vector diagram is shown in Fig. 37 (*b*). **(Ans.)**

Example 24. *An iron-cored coil has a D.C. resistance of 6 ohms. When it is connected to 230 V, 50 Hz mains, the current taken is 3.5 A at a power factor of 0.5. Determine :*

(*i*) *Effective resistance of the coil.*

(*ii*) *Inductance of the coil.*

(*iii*) *Resistance which represents the effect of the iron loss.*

Solution. *Given :* D.C. resistance (True resistance), R = 6 Ω ; supply voltage = 230 V, f = 50 Hz, I = 3.5 A ; *p.f.* = 0.5.

(*i*) **Effective resistance of the coil, $\mathbf{R_e}$:**

Total power consumed by the iron-cored choke coil,

P = Power loss in ohmic resistance + Iron loss in core = $I^2R + P_i$

or $\frac{P}{I^2} = R + \frac{P_i}{I^2}$, where $\frac{P}{I^2}$ is known as *effective resistance* of the coil.

$\therefore$ Effective resistance, $R_e = \frac{P}{I^2} = \frac{VI \cos \phi}{I^2} = \frac{230 \times 3.5 \times 0.5}{(3.5)^2} = \mathbf{32.86}\ \Omega.$ **(Ans.)**

(*ii*) **Inductance of the coil, L :**

Impedance of the coil, $Z = \frac{V}{I} = \frac{230}{3.5} = 65.7\ \Omega$

Inductive reactance of the coil,

$$X_L = \sqrt{Z^2 - R_e^2} = \sqrt{(65.7)^2 - (32.86)^2} = 56.9\ \Omega$$

$\therefore$ $L = \frac{X_L}{2\pi f} = \frac{56.9}{2\pi \times 50} = \mathbf{0.1811\ H.}$ **(Ans.)**

(*iii*) **Resistance representing iron loss :**

Since $\frac{P}{I^2} = R + \frac{P_i}{I^2}$

Effective resistance, R_e = True resistance + Resistance representing iron loss

32.86 = 6 + Resistance representing iron loss

$\therefore$ Resistance representing iron loss = 32.86 – 6 = **26.86 Ω.** **(Ans.)**

Example 25. *Three coils connected in series across a 100 V, 50 Hz supply have the following parameters :*

$R_1 = 18\ \Omega,\ L_1 = 0.012\ H\ ;\ R_2 = 12\ \Omega,\ L_2 = 0.036\ H\ ;\ R_3 = 3.6\ \Omega,\ L_3 = 0.072\ H$

Determine the potential drop and phase angle for each coil.

Solution. Fig. 38. shows the circuit diagram.

Total resistance in the circuit, $R = R_1 + R_2 + R_3 = 18 + 12 + 3.6 = 33.6\ \Omega$

Total inductance in the circuit, $L = L_1 + L_2 + L_3 = 0.012 + 0.036 + 0.072 = 0.12$ H

Impedance of coil-1, $Z_1 = \sqrt{R_1^2 + (2\pi f L_1)^2} = \sqrt{(18)^2 + (2\pi \times 50 \times 0.012)^2} = 18.39\ \Omega$

Impedance of coil-2, $Z_2 = \sqrt{R_2^2 + (2\pi f_2 L_2)^2} = \sqrt{(12)^2 + (2\pi \times 50 \times 0.036)^2} = 16.49\ \Omega$

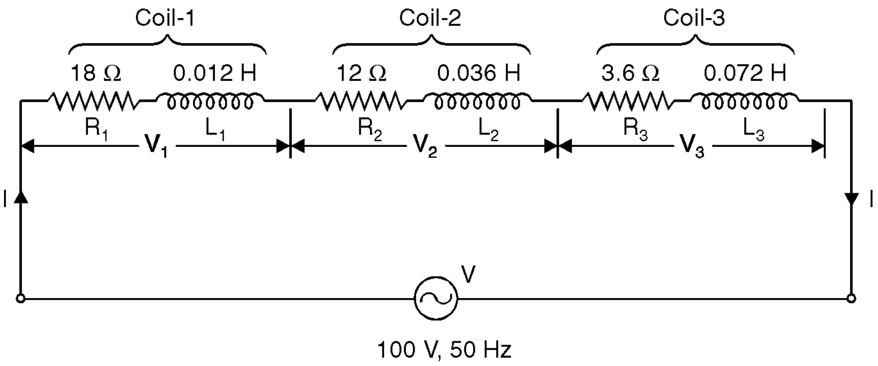


Fig. 38. Circuit diagram.

Impedance of coil–3, $Z_3 = \sqrt{R_3^2 + (2\pi f L_3)^2} = \sqrt{(3.6)^2 + (2\pi \times 50 \times 0.072)^2} = 22.90\ \Omega$

Impedance of the whole circuit, $Z = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(33.6)^2 + (2\pi \times 50 \times 0.12)^2} = 50.5\ \Omega$

Current through the circuit, $I = \dfrac{V}{Z} = \dfrac{100}{50.5} = 1.98\ A$

Potential drop across coil–1, $\mathbf{V_1} = IZ_1 = 1.98 \times 18.39 =$ **36.41 V (Ans.)**

Potential drop across coil–2, $\mathbf{V_2} = IZ_2 = 1.98 \times 16.49 =$ **32.65 V (Ans.)**

Potential drop across coil–3, $\mathbf{V_3} = IZ_3 = 1.98 \times 22.90 =$ **45.34 V (Ans.)**

Phase angle of coil–1, ϕ_1, $\cos^{-1}(R_1/Z_1) = \cos^{-1}(18/18.39) =$ **11.82°. (Ans.)**

Phase angle of coil–2, $\phi_2 = \cos^{-1}(R_2/Z_2) = \cos^{-1}(12/16.49) =$ **43.3°. (Ans.)**

Phase angle of coil–3, $\phi_3 = \cos^{-1}(R_3/Z_3) = \cos^{-1}(3.6/22.90) =$ **80.96°. (Ans.)**

Example 26. *An alternating voltage of (176 + j132) is applied to a circuit and the current in the circuit is given by (6.6 + j 8.8) A. Determine :*

(*i*) *Values of elements of the circuit.*

(*ii*) *Power factor of the circuit.*

(*iii*) *Power consumed.*

Solution. Given : Supply voltage, $\bar{V} = 176 + j132 = 220\ \angle 36.87°$

Circuit current, $\bar{I} = 6.6 + j\,8.8 = 11\ \angle 53.13°$

(*i*) **Values of elements of the circuit, R, C :**

Circuit impedance, $\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{220 \angle 36.87°}{11 \angle 53.13°} = 20 \angle -16.26°$

$= 20 [\cos (-16.26°) + \sin (-16.26°)]$

$= 20 (0.96 - j\, 0.28) = (19.2 - j5.6)\ \Omega$

$\therefore \quad R = \mathbf{19.2\ \Omega}$ **(Ans.)**

$X_C = 5.6\ \Omega$ or $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 5.6}$ F = $\mathbf{568.4\ \mu F}$. **(Ans.)**

(*ii*) **Power factor of the circuit ; cos φ :**

$$\cos \phi = \frac{R}{Z} = \frac{19.2}{20} = \mathbf{0.96\ (leading)}. \quad \textbf{(Ans.)}$$

(*iii*) **Power (true) consumed, P :**

Apparent power, $\overline{S} = \overline{V} \times \overline{I}$

$= 220 \angle 36.87° \times 11 \angle -53.13° = 2420 \angle -16.26°$

$= 2323.2 - j\, 677.6$

∴ True power, P = **2323.2 W. (Ans.)**

(Alternatively : $P = VI \cos \phi = 220 \times 11 \times 0.96 = 2323.2$ W).

Example 27. *In a circuit, the equations for instantaneous voltage and current are given by,*

$$v = 141.4 \sin \left(\omega t - \frac{2\pi}{3}\right), \text{ volt and}$$

$$i = 7.07 \sin \left(\omega t - \frac{\pi}{2}\right), \text{ amp, where } \omega = 314 \text{ rad/sec.}$$

(*i*) *Sketch a neat phasor diagram for the circuit*

(*ii*) *Use polar notation to calculate impedance with phase angle.*

(*iii*) *Calculate average power.*

(*iv*) *Calculate the instantaneous power at the instant t = 0*

Solution. *Given :* $v = 141.4 \sin \left(\omega t - \frac{2\pi}{3}\right)$, and $i = 7.07 \sin \left(\omega t - \frac{\pi}{2}\right)$, where $\omega = 314$ rad/s.

(*i*) **Phasor diagram :**

From the voltage equation, it is seen that the voltage lags behind the reference quantity by $\frac{2\pi}{3}$ rad or $2 \times \frac{180}{3} = 120°$. Similarly, current lags behind the reference quantity by $\frac{\pi}{2}$ rad or $\frac{180}{2} = 90°$. Between themselves, voltage lags behind the current by $(120° - 90°) = 30°$ as shown in Fig. 39 (*b*).

(*ii*) **Impedance with phase angle (polar notation)**

$$V = \frac{V_{max}}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100\ V\ ;$$

$$I = \frac{I_{max}}{\sqrt{2}} = \frac{7.07}{\sqrt{2}} = 5A.$$

$\therefore \quad V = 100 \angle -120°$ and $I = 5 \angle -90°$

$\therefore \quad Z = \frac{V}{I} = \frac{100 \angle -120°}{5 \angle -90°} = \mathbf{20 \angle -30°\ \Omega}$ **(Ans.)**

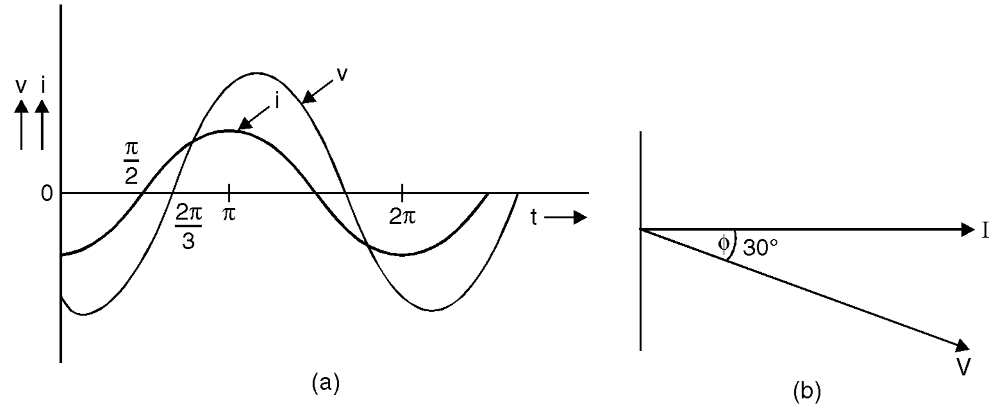


Fig. 39

(*iii*) **Average power :**

$$\text{Average power} = VI \cos \phi$$
$$= 100 \times 5 \times \cos 30° = \mathbf{433\ W.} \quad \textbf{(Ans.)}$$

(*iv*) **Instantaneous power at t = 0 :**

At $t = 0$; $v = 141.4 \sin (0 - 120°) = - 122.45\ V$

$i = 7.07 \sin (0 - 90°) = - 7.07\ A$

∴ Instantaneous power at $t = 0$,

$$p = v_i = (- 122.45) \times (- 7.07) = \mathbf{865.7\ W.} \quad \textbf{(Ans.)}$$

Example 28. *A voltage e(t) = 100 sin 314 t is applied to a series circuit consisting of 10 ohms resistance, 0.0318 henry inductance and a capacitor of 63.6 μF. Determine :*

(*i*) *Expression for i(t).*

(*ii*) *Phase angle between voltage and current.*

(*iii*) *Power factor.*

(*iv*) *Active power consumed.*

(*v*) *Peak value of pulsating energy.*

Soution. *Given :* $e(t) = 100 \sin 314\, t$, $R = 10\ \Omega$, $L = 0.0318$ H, $C = 63.6\ \mu F = 63.6 \times 10^{-6}\ F$.

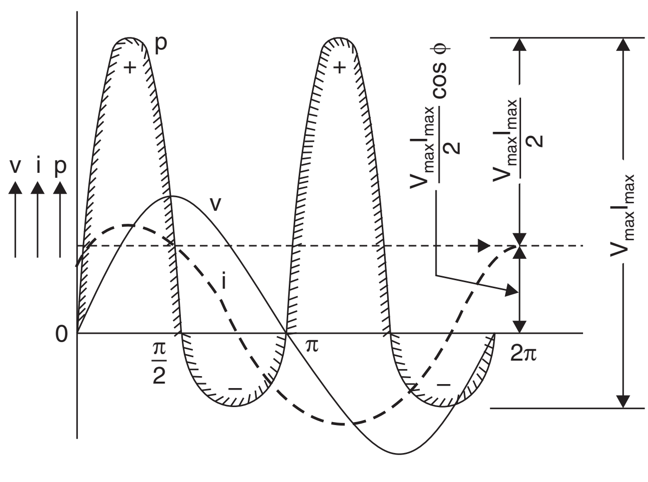


Fig. 40

Here $\omega = 314$ rad/s ; $X_L = \omega L = 314 \times 0.0318 = 10\ \Omega$; $X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 63.6 \times 10^{-6}} = 50\ \Omega$;

$$X = X_L - X_C = 10 - 50 = -40\ \Omega \text{ (capacitive)}$$

$$\overline{Z} = 10 - j40 = 41.2 \angle -76° \; ; \; \overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{(100/\sqrt{2}) \angle 0°}{41.2 \angle -76°} = 1.716 \angle 76°$$

$$I_{max} = I \times \sqrt{2} = 1.716 \times \sqrt{2} = 2.43 \text{ A}$$

(*i*) **Expression for i(t) :**

$i(t) = 2.43 \sin (314\, t + 76°)$. **(Ans.)**

(*ii*) **Phase angle between voltage and current,** ϕ :

ϕ **= 76°** with current leading.

(*iii*) **Power factor, cos** ϕ **:**

$\cos \phi = \cos 76° =$ **0.24 (lead). (Ans.)**

(*iv*) **Active power consumed, P :**

$P = VI \cos \phi = (100/\sqrt{2})\,(2.43/\sqrt{2}) \times 0.24 =$ **29.16 W. (Ans.)**

(*v*) **Peak value of pulsating energy :**

Refer to Fig. 40. The peak value of pulsating energy

$$= \frac{V_{max}\, I_{max}}{2} + \frac{V_{max}\, I_{max}}{2} \cos \phi$$

$$= \frac{V_{max}\, I_{max}}{2} (1 + \cos \phi) = \frac{100 \times 2.43}{2} (1 + 0.24) = \mathbf{150.66\ W.} \quad \textbf{(Ans.)}$$

4.5.2. R-C circuit (Resistance and capacitance in series)

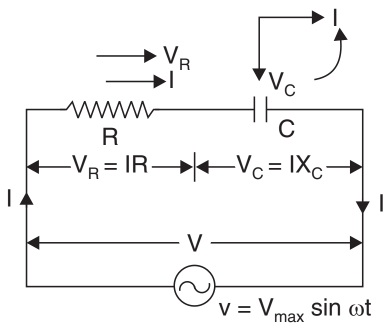
Fig. 41 (*a*) shows a pure resistance *R* (ohms) and a pure capacitor of capacitance *C* (farads) connected in series. Such a circuit is known as *R-C* circuit.

Let, V = R.M.S. value of the applied voltage,

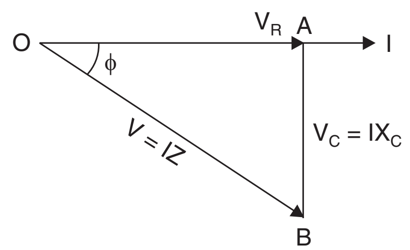
I = R.M.S. value of the resultant current,

$V_R = IR$ = Voltage drop across *R* (in phase with *I*), and

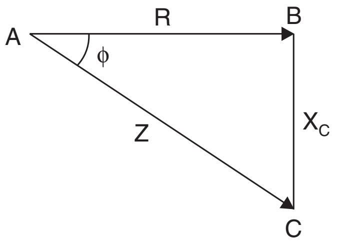
$V_C = IX_C$ = Voltage drop across *C*, lagging *I* by 90°.



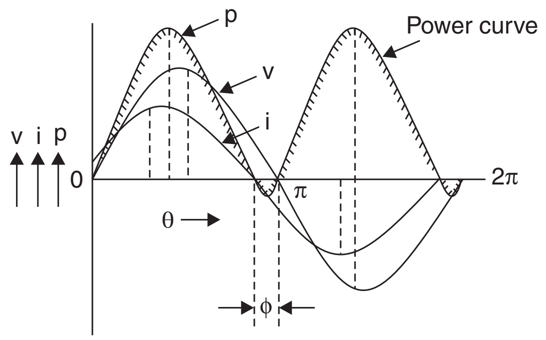
(a) Circuit diagram



(b) Phasor diagram
(I leads V by angle φ)



(c) Impedance triangle



(d) Power curve

Fig. 41. *R-C* circuit (Resistance and capacitance in series).

Voltage drops V_R and V_L are shown in voltage triangle *OAB* in Fig. 41 (*b*) *I* being taken as the reference vector in the phasor diagram. Vector *OA* represents ohmic drop V_R and *AB* represents the capacitive drop V_C. Vector *OB* represents the applied voltage *V*, which is the vector sum of the V_R and V_C)

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

or

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_C^2}$ (total opposition offered to the flow of alternating current by *R-C* series circuit) is known as the **impedance** of the circuit.

As seen from the *"impedance triangle" ABC* [Fig. 41 (*c*)],

$$Z^2 = R^2 + X_C^2$$

i.e., (Impedance)^2 = (Resistance)^2 + (Capacitive reactance)^2

From Fig. 41 (*b*) it is evident that *I* leads the voltage *V* by an angle ϕ such that,

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{(1/\omega C)}{R} = \frac{\text{Capacitive reactance}}{\text{Resistance}}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

The same is illustrated graphically in Fig. 41 (*d*).

In other words I leads V_R by an angle ϕ.

Power factor, $\cos\phi = \dfrac{R}{Z}$ [From Fig. 41 (*c*)]

Power. Refer to Fig. 41 (*d*),

Instantaneous power,

$$p = vi = V_{max} \sin \omega t \times I_{max} \sin(\omega t + \phi)$$

$$= \frac{V_{max}\, I_{max}}{2} \times 2 \sin(\omega t + \phi) \sin \omega t$$

$$= \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} [\cos\phi - \cos(2\,\omega t + \phi)]$$

Average power consumed in the circuit over a complete cycle,

$$P = \text{Average of } \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos\phi - \text{Average of } \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos(2\omega t + \phi)$$

or $$P = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos \phi - \text{zero},$$

or $$P = V_{r.m.s.} \times I_{r.m.s.} \cos \phi = VI \cos \phi$$

where cos ϕ is the *power factor* of the circuit :

Alternatively, $$P = VI \cos \phi = IZ \times I \times \frac{R}{Z} = I^2R$$

This shows that *power is actually consumed in resistance only* ; the *capacitor does not consume any power.*

Thus in ***R-C circuit***, we have :

1. Impedance, $Z = \sqrt{R^2 + X_C^2}$ $\left(\text{where } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f}, C \text{ being in farad}\right)$

2. Current, $I = \frac{V}{Z}$

3. Power factor, $\cos \phi = \frac{R}{Z} \left(= \frac{\text{True power}}{\text{Apparent power}} = \frac{W}{VA}\right)$

[or angle of lead, $\phi = \cos^{-1} (R/Z)$]

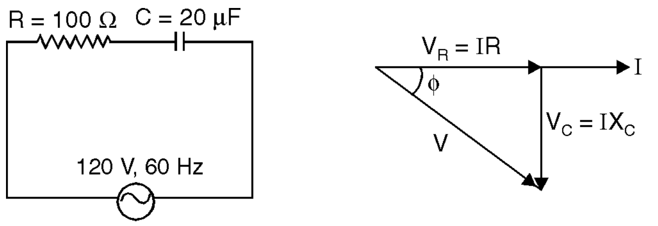
4. Power consumed, $P = VI \cos \phi$ (= I^2R).

Example 29. *A capacitance of 20 μF and a resistance of 100 ohms are connected in series across 120 V, 60 Hz mains. Determine the average power expended in the circuit.*

Also draw the vector diagram.

Solution. $R = 100\ \Omega$

$C = 20\ \mu F = 20 \times 10^{-6}$ F (farad)



(*a*) R-C circuit (*b*) Vector/phasor diagram

Fig. 42

Capacitive reactance, $$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}} = 159\ \Omega$$

Impedance of the circuit, $$Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 150^2} = 188\ \Omega$$

Current through the circuit $$I = \frac{V}{Z} = \frac{120}{188} = 0.638 \text{ A}$$

Power factor, $$\cos \phi = \frac{R}{Z} = \frac{100}{188} = 0.532$$

Average power expended in the circuit,

$$P_{av} = VI \cos \phi$$
$$= 120 \times 0.638 \times 0.532 = \textbf{40.75 W.} \quad \textbf{(Ans.)}$$

Fig. 42 (*b*) shows the vector/phasor diagram.

Example 30. *A voltage v = 100 sin 314t – 50 cos 314t, is applied to a circuit having R = 20 Ω in series with C = 100 μF. Obtain expression for instantaneous current, r.m.s. value of current and the power in the circuit.*

Solution. *Given* : $v = 100 \sin 314t - 50 \cos 314t$; $R = 20\ \Omega$; $C = 100\ \mu F$.

The R-C circuit and the phasor diagram for the given instantaneous voltage are shown in Figs. 43 and 44 respectively.

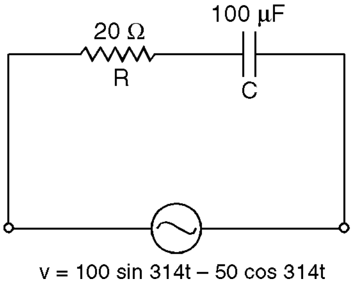


Fig. 43

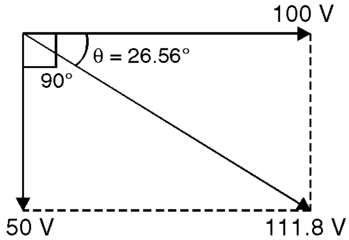


Fig. 44

Resultant voltage, $V_{max(R)} = \sqrt{(100)^2 + (50)^2} = 111.8$ V

Phase angle with the horizontal, $\theta = \tan^{-1}\left(-\dfrac{50}{100}\right) = -26.56°$

$\therefore \quad v = 111.8 \sin (314t - 26.56°)$

Now, $\omega t = 314\, t$ or $\omega = 314$

$\therefore$ Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 100 \times 10^{-6}} = 31.85\ \Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C{}^2} = \sqrt{(20)^2 + (31.85)^2} = 37.6\ \Omega$

Maximum value of current, $I_{max} = \dfrac{V_{max}}{Z} = \dfrac{111.8}{37.6} = 2.97$ A

Phase difference between voltage and current,

$$\phi = \cos^{-1}\left(\frac{R}{Z}\right) = \cos^{-1}\left(\frac{20}{37.6}\right) = 57.86° \text{ (leading)}$$

$\therefore$ *Instantaneous value of current*

$$i = 2.97 \sin (314t - 26.56° + 57.86°) = 2.97 \sin (314t + 31.3°)$$

$$i = 2.97 [\sin 314t \,.\, \cos 31.3° + \cos 314t \,.\, \sin 31.3°]$$

or **i = 2.54 sin 314t + 1.54 cos 314t. (Ans.)**

RMS value of the current, $\mathbf{I} = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{2.97}{\sqrt{2}} = \mathbf{2.1\ A.}$ **(Ans.)**

Power in the circuit, $P = VI \cos \phi$

$$= \frac{V_{max(R)}}{\sqrt{2}} \times I \cos \phi = \frac{111.8}{\sqrt{2}} \times 2.1 \times \cos (57.86°) = \mathbf{88.32\ W}\ \textbf{(Ans.)}$$

Example 31. *A two element series circuit is connected across an A.C. source e = $200\sqrt{2}$ sin (ωt + 20°) V. The current in the circuit then is found to be i = $10\sqrt{2}$ cos (314 t – 25°) A. Determine parameters of the circuit.*

Solution. Given :

$$e = 200\sqrt{2} \sin (\omega t + 20°)$$

$$i = 10\sqrt{2} \cos (314\, t - 25°)$$

Parameters of the circuit, cos ϕ, R, X_C and C :

The current i can be written as,

$$i = 10\sqrt{2}\ \sin(314\,t - 25° + 90°) = 10\sqrt{2}\ \sin(314\,t + 65°)$$

It is seen that applied voltage leads by 20° and current leads by 65° with regards to the reference quantity, their mutual difference is 65° – 20° = 45°.

Hence, p.f. $\cos\phi = \cos 45° =$ **0.707 (lead). (Ans.)**

Now, $V_{max} = 200\sqrt{2}$ and $I_{max} = 10\sqrt{2}$

$\therefore$ $$Z = \frac{V_{max}}{I_{max}} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20\ \Omega$$

$\therefore$ $R = Z\cos\phi = 20 \times 0.707 =$ **14.14 Ω. (Ans.)**

$X_C = Z\sin\phi = 20 \times 0.707 =$ **14.14 Ω. (Ans.)**

Also, $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 14.14$$

where $$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

$\therefore$ $$X_C = 14.14 = \frac{1}{2\pi \times 50 \times C}$$

or $$C = \frac{1}{14.14 \times 2\pi \times 50} \text{ F} = \mathbf{225.1\ \mu F.}\quad \textbf{(Ans.)}$$

Hence the *given circuit is an R-C circuit.*

4.5.3. R-L-C circuit (Resistance, inductance and capacitance in series)

Fig. 45 shows a *R-L-C* circuit.

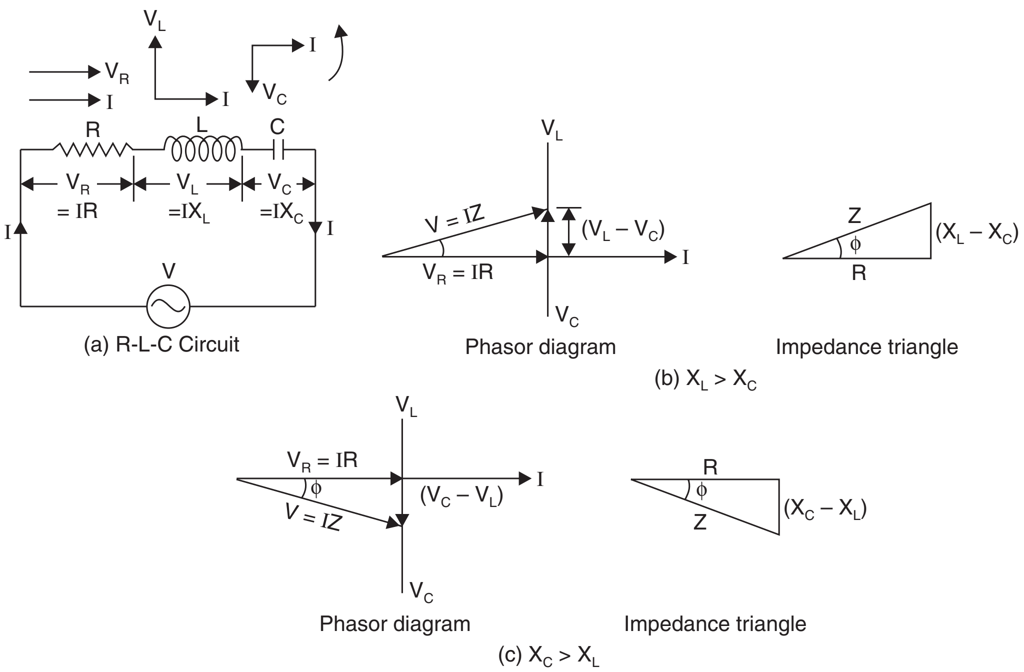


Fig. 45. Resistance, inductance and capacitance in series.

Important formulae :

1. Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\left[\text{where } X_L = 2\pi fL, L \text{ in } henry \text{ and } X_C = \frac{1}{2\pi fC}, C \text{ in farad}\right]$

2. Current, $I = \frac{V}{Z}$

3. Power factor, $\cos\phi = \frac{R}{Z}$

$\left[\text{angle of lag (when } X_L > X_C\text{) or lead (when } X_C > X_L\text{)}, \phi = \cos^{-1}\frac{R}{Z}\right]$

4. Power consumed $= VI\cos\phi\ (= I^2R)$

Resonance in R-L-C circuits

Refer to Fig. 45 (*a*).

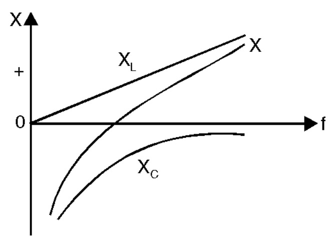


Fig. 46. Reactance (X) v/s frequency (*f*).

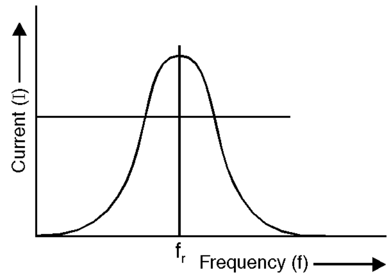


Fig. 47. Current in *R-L-C* circuit v/s frequency.

The frequency of the voltage which gives the maximum value of the current in the circuit is called ***resonant frequency,*** and the circuit is said to be ***resonant.***

At resonance, $X_L = X_C$ (*i.e., Z = R*)

i.e., $2\pi f_r L = \frac{1}{2\pi f_r C}$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad ...(5)$$

where f_r = Resonance frequency in Hz ; L = Inductance in henry ; and C = Capacitance in farad.

Fig. 46 shows variation of X_L, X_C and X (total reactance = $X_L - X_C$) with variation of frequency f.

Fig. 47 shows the variation of current (I) with frequency (f).

At **series resonance,** it is seen that :

1. Net reactance of the circuit is zero *i.e.,* $X_L - X_C = 0$ or $X = 0$.

2. The impedance of the circuit is *minimum* and equal to the resistance (R) of the circuit $\left(i.e., I = \frac{V}{R}\right)$. Consequently *circuit admittance is maximum*.

3. The current drawn is maximum (*i.e.,* $I_r = I_{max}$).

4. The phase angle between the current and voltage is zero ; the *power factor is unity*.

5. The resonant frequency is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$; if the *frequency is below* the resonant frequency the net reactance in the circuit is *capacitive* and if the frequency is above the resonant frequency, the net reactance in the circuit is *inductive*.

6. Although $V_L = V_C$, yet V_{coil} is greater than V_C because of its resistance.

Half power frequencies, Bandwidth and Selectivity

Half power (cut-off) frequencies :

The half power frequencies *are those frequencies at which the power dissipation in the circuit is half of the power dissipation at resonant frequency f_r*. They are the corresponding frequencies f_1 and f_2 at the value of current $I = I_r/\sqrt{2}$; where I_r is the current at resonance in *R-L-C* series circuit (Refer to Fig. 48).

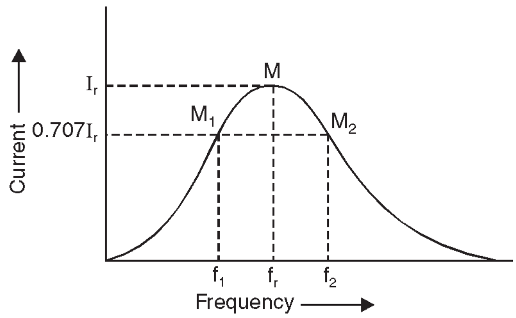


Fig. 48

Hence power, P_r drawn by the circuit at the *resonance* is

$$P_r = I_r^2 R \quad \text{...(6)}$$

Power in the circuit at $f_1 = \left(\frac{I_r}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_r^2 R$ (= half the power at resonance)

Power in the circuit at $f_2 = \left(\frac{I_r}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_r^2 R$ (= half the power at resonance)

Also,
$$f_1 = f_r - \frac{R}{4\pi L}$$
$$f_2 = f_r + \frac{R}{4\pi L}$$
$$f_2 - f_1 = \frac{R}{2\pi L}$$

Bandwidth and Selectivity :

The difference $(f_2 - f_1)$ is called the *bandwidth* (B_{hp}) of the resonant network.

The ratio of the bandwidth to the resonance frequency is defined as the ***selectivity*** of the circuit.

When frequency is varied in *R-L-C* circuit, the selectivity becomes

$$\frac{*(f_2 - f_1)}{f_r} = \frac{1}{Q} \quad \text{...(7)}$$

where Q_r is the *quality factor* of the resonant circuit.

***Relation between bandwidth and quality factor in series resonant conditions :**

A series *R-L-C* circuit is considered. The resonant frequency and angular frequency are expressed by f_r and ω_r respectively. In the above circuit, the current (I) can be described as follows :

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

where *V, R, L* and *C* are the source voltage, resistance, inductance and capacitance of the circuit respectively.

The current, at a power, half of the maximum power developed at resonant frequency, is $\frac{1}{\sqrt{2}} I_r$, where I_r is the series resonant current *i.e.* $\frac{V}{R}$.

According to the definition of bandwidth,

$$\frac{1}{\sqrt{2}} \frac{V}{R} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

or $$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \quad \text{or} \quad R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

or $$\omega L - \frac{1}{\omega C} = \pm R.$$

According to Fig. 48

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad ...(i) \qquad \omega_1 L - \frac{1}{\omega_1 C} = -R \quad ...(ii)$$

Adding equations (*i*) and (*ii*), we get

$$(\omega_1 + \omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

or $$(\omega_1 + \omega_2)L - \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right) = 0$$

Since $$\omega_1 + \omega_2 \neq 0,\ \omega_1\omega_2 = \frac{1}{LC}.$$

Again $\omega_r^2 = \frac{1}{LC}$, [$\omega_r$ is the angular frequency at resonant condition.]

Subtracting equations (*i*) and (*ii*), we have

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_2\omega_1}\right) = 2R$$

or $$(\omega_2 - \omega_1) + \frac{1}{LC}\frac{(\omega_2 - \omega_1)}{\omega_2\omega_1} = \frac{2R}{L}$$

or $$(\omega_2 - \omega_1) + \omega_1\omega_2 \cdot \frac{(\omega_2 - \omega_1)}{\omega_1\omega_2} = \frac{2R}{L} \qquad \left[\because\ \omega_1\omega_2 = \frac{1}{LC}\right]$$

or $$(\omega_2 - \omega_1) = \frac{R}{L}$$

or $$\frac{\omega_2 - \omega_1}{\omega_0} = \frac{R}{\omega_0 L}$$

[ω_0 is the angular frequency at resonant condition]

or $$\omega_2 - \omega_1 = \omega_0 \times \frac{1}{\frac{\omega_0 L}{R}}$$

or $$\omega_2 - \omega_1 = \frac{\omega_0}{Q} \qquad \left[\because\ Q = \frac{\omega_1 L}{R}\right]$$

or $$f_2 - f_1 = \frac{f_r}{Q}$$ $[\because \quad \omega = 2\pi f]$

or *Bandwidth* at series resonant condition $= \dfrac{f_r}{Q}$.

$$\left[Q = \frac{\omega_r}{\text{bandwidth}} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{R/L} = \frac{\omega_r L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \right]$$

Q-factor of a resonant series circuit :

The Q-factor of an R-L-C series circuit can be defined in the following different ways :

(*i*) ***Q-factor*** is *defined as the voltage magnification in the circuit at the time of resonance.*

Since at resonance current is maximum *i.e.,* $I_r = \dfrac{V}{R}$, the voltage across either coil or capacitor $= I_r X_{Lr}$ or $I_r X_{Cr}$ and supply voltage, $V = I_r R$.

$\therefore$ $$\text{Voltage magnification} = \frac{V_{Lr}}{V} = \frac{I_r X_{Lr}}{I_r R} = \frac{X_{Lr}}{R} = \frac{\omega_r L}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$

or $$\frac{V_{Cr}}{V} = \frac{I_r \times C_r}{I_r R} = \frac{X_{Cr}}{R} = \frac{\text{Reactance}}{\text{Resistance}} = \frac{1}{\omega_r CR}$$

$\therefore$ $$Q\text{-factor} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \tan\theta \qquad ...[8(a)]$$

where θ is the circuit *power factor angle of the coil*.

(At resonance, circuit phase angle $\theta = 0$, and $Q = \tan\theta = 0$)

(*ii*) The ***Q-factor*** may also be *defined* as under :

$$Q\text{-factor} = 2\pi \, \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}} \qquad \text{...... in the circuit}$$

$$= 2\pi \frac{\frac{1}{2} L I_r^2}{I^2 R T_r} = 2\pi \frac{\frac{1}{2} L(\sqrt{2} I)^2}{I^2 R (1/f_r)} = \frac{I^2 \times 2\pi f_r L}{I^2 R} = \frac{\omega_r L}{R} \left(= \frac{1}{\omega_r CR} \right) \qquad \left(\because \; T_r = \frac{1}{f_r} \right)$$

But resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ or $2\pi f_r = \dfrac{1}{\sqrt{LC}}$

Putting this value in eqn. 8(*a*), we get

$$Q\text{-factor} = \frac{2\pi f_r L}{R} = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad ...[8(b)]$$

In series resonance, higher quality factor *i.e.,* Q-factor means higher voltage magnification as well as higher selectivity of the tuning coil.

Example 32. *A resistance 12 Ω, an inductance of 0.15 H and a capacitance of 100 μF are connected in series across a 100 V, 50 Hz supply. Calculate :*

(*i*) *The current.*

(*ii*) *The phase difference between current and the supply voltage.*

(*iii*) *Power consumed.*

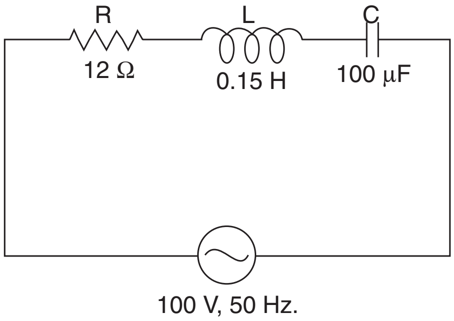
Draw the vector diagram of supply voltage and the line current.

Solution. *Given* : $R = 12\ \Omega$, $L = 0.15$ H or $X_L = 2\pi f L$

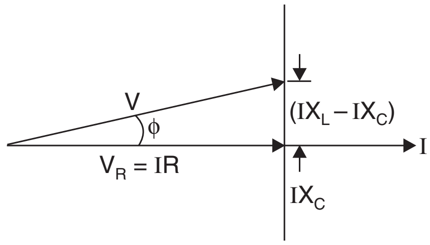
$= 2\pi \times 50 \times 0.15 = 47.1\ \Omega$

$C = 100\ \mu F = 100 \times 10^{-6} F$

or $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.8\ \Omega$$



(*a*) R-L-C circuit



(*b*) Vector/phasor diagram

Fig. 49

(*i*) **The current, I :**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{12^2 + (47.1 - 31.8)^2} = 19.43\ \Omega$$

Current,
$$I = \frac{V}{Z} = \frac{100}{19.43} = \mathbf{5.15\ A.\ (Ans.)}$$

(*ii*) **Phase difference, ϕ :**

$$\phi = \cos^{-1}\frac{R}{Z}\left[\text{or } \tan^{-1}\frac{X_L - X_C}{R}\right]$$
$$= \cos^{-1}\frac{12}{19.43}\left[\text{or } \tan^{-1}\frac{15.3}{12.0}\right] = 52° \text{ (lag)}$$

Hence current lags **supply voltage by 52°. (Ans.)**

(*iii*) **Power consumed, P :**

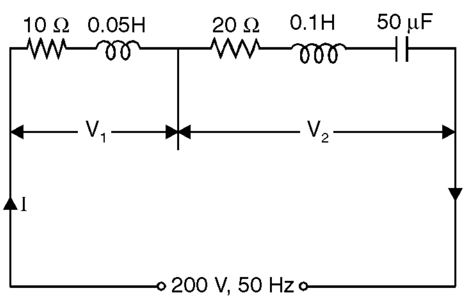
$$P = VI\cos\phi$$
$$= 100 \times 5.15 \times \cos 52° = \mathbf{371.1\ W.\quad (Ans.)}$$

Fig. 49 (*a*), (*b*) show the circuit and vector/phasor diagrams respectively.

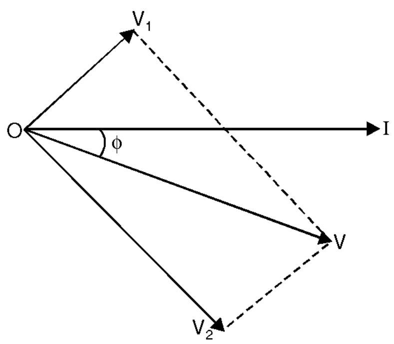
Example 33. *For the circuit shown in Fig. 50 find the values of (i) current I, (ii) V_1 and V_2 and (iii) p.f.*

Draw the vector diagram.

Solution. Refer to Fig. 50



(*a*) Series circuit



(*b*) Vector diagram

Fig. 50

$$R = 10 + 20 = 30\ \Omega$$
$$L = 0.05 + 0.1 = 0.15\ \text{H}$$
$$\therefore \quad X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.1\ \Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10^{-6} \times 50} = 63.7\ \Omega$$
$$X = 47.1 - 63.7 = -16.6\ \Omega$$
$$Z = \sqrt{R^2 + X^2} = \sqrt{(30)^2 + (-16.6)^2} = 34.3\ \Omega$$
(*i*) $$I = \frac{V}{Z} = \frac{200}{34.3} = \mathbf{5.83\ A.\ (Ans.)}$$
(*ii*) $$X_{L1} = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$$
$$Z_1 = \sqrt{10^2 + 15.7^2} = 18.6\ \Omega$$
$$\therefore \quad V_1 = IZ_1 = 5.83 \times 18.6 = \mathbf{108.4\ V.\ (Ans.)}$$
$$\phi_1 = \cos^{-1}(10/18.6) = 57.5° \text{ (lag)}$$
$$X_{L2} = 2\pi \times 50 \times 0.1 = 31.4\ \Omega$$
$$X = 31.4 - 63.7 = -32.3\ \Omega$$
$$Z_2 = \sqrt{20^2 + (-32.3)^2} = 38\ \Omega$$
$$\therefore \quad V_2 = IZ_2 = 5.83 \times 38 = \mathbf{221.5\ V.\ (Ans.)}$$
$$\phi_2 = \cos^{-1}(20/38) = 58.2° \text{ (lead)}$$
(*iii*) Combined p.f. $$= \cos\phi = \frac{R}{Z} = \frac{30}{34.3} = \mathbf{0.875\ (lead)}$$

Vector diagram is shown in Fig. 50 (*b*).

Example 34. *For the circuit shown in Fig. 51. Calculate :*

(*i*) *Current ;* (*ii*) *Voltage drops V_1, V_2 and V_3 ;*

(*iii*) *Power absorbed by each importance ;* (*iv*) *Total power absorbed by the circuit.*

Take voltage vector along the reference axis.

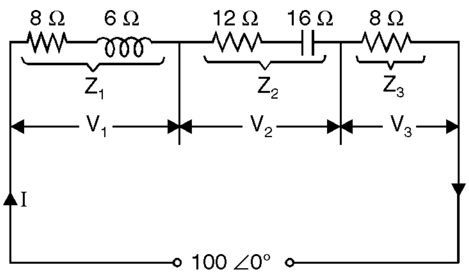


Fig. 51

Solution. $$Z_1 = (8 + j6)\ \Omega\ ;\ Z_2 = (12 - j16)\ \Omega\ ;\ Z_3 = (8 + j0)$$
$$Z = Z_1 + Z_2 + Z_3 = (8 + j6) + (12 - j16) + (8 + j0) = (28 - j10)\ \Omega$$
Taking $$V = V\angle 0° = 200\angle 0° = (200 + j0)$$
$$\therefore \quad I = \frac{V}{Z} = \frac{100}{(28 - j10)} = \frac{200(28 + j10)}{(28 - j10)(28 + j10)} = \frac{100(28 + j10)}{(28)^2 + (10)^2}$$
$$= \frac{200(28 + j10)}{884} = 3.17 + j1.13$$

(*i*) Magnitude of current $= \sqrt{(3.17)^2 + (1.13)^2}$ = **3.36 A. (Ans.)**

(*ii*)

$$V_1 = IZ_1 = (3.17 + j1.13)(8 + j6)$$
$$= 3.17 \times 8 + 3.17 \times j6 + 8 \times j1.13 + j1.13 \times j6$$
$$= 25.36 + j19.02 + j9.04 - 6.78 = \mathbf{18.58 + j28.06.\ (Ans.)}$$
$$V_2 = IZ_2 = (3.17 + j1.13)(12 - j16)$$
$$= 38.04 - j50.72 + j13.56 + 18.08 = \mathbf{56.12 - j37.16.\ (Ans.)}$$
$$V_3 = IZ_3 = (3.17 + j1.13)(8 + j0) = \mathbf{25.36 + j9.04.\ (Ans.)}$$
$$[V = V_1 + V_2 + V_3 = (18.58 + j28.06) + (56.12 - j37.16) + (25.36 + j9.04) = 100 + j0 \text{ (check)}]$$

Example 35. *Fig. 52 shows a circuit connected to a 230 V, 50 Hz supply. Determine the following :*

(*i*) *Current drawn* (*ii*) *Voltages V_1 and V_2*

(*iii*) *Power factor.*

Draw also the phasor diagram.

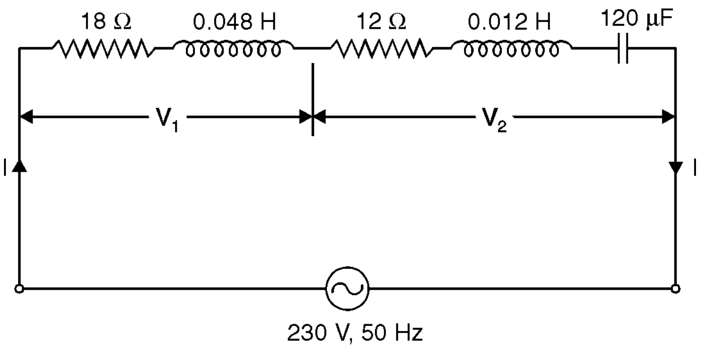


Fig. 52

Solution. Refer to Fig. 52.

Given : $R_1 = 18\ \Omega$, $L_1 = 0.048$ H ; $R_2 = 12\ \Omega$, $L_2 = 0.012\ H$; $C = 120\ \mu F = 120 \times 10^{-6}$ F.

$\therefore$ $X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times 0.048 = 15.08\ \Omega$; $X_{L2} = 2\pi f L_2 = 2\pi \times 50 \times 0.012 = 3.77\ \Omega$

$$X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$$

Impedance, $\bar{Z}_1 = R_1 + jX_{L1} = 18 + j15.08 = 23.48\ \angle 39.96°$

Impedance, $\bar{Z}_2 = R_2 + jX_{L2} - jX_C = 12 + j3.77 - j26.53 = 12 - j22.76 = 25.73\ \angle -62.2°$

Total impedance, $\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = (18 + j15.08) + (12 - j22.76) = 30 - j7.68 = 30.97\ \angle -14.36°$

Current drawn, I :

Taking supply voltage as reference vector, $\bar{V} = V\angle 0° = 230\angle 0°$.

Current, $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{230\angle 0°}{30.97\angle -14.36°}$ = **7.43 ∠14.36°A. (Ans.)**

Voltages V_1 and V_2 :

Voltage, $\overline{V_1} = \overline{IZ_1} = 7.43 \angle 14.36° \times 23.48 \angle 39.96°$

$= 7.43 \times 23.48 \angle(14.36° + 39.96°)$

$= \mathbf{174.46 \angle 54.32° \ V.}$ **(Ans.)**

Voltage $\overline{V_2} = \overline{IZ_2} = 7.43 \angle 14.36° \times 25.73 \angle -62.2°$

$= 7.43 \times 25.73 \angle(14.36° - 62.2°)$

$= \mathbf{191.2 \angle -47.84°.}$ **(Ans.)**

Phase angle between supply voltage and current *i.e.,* V and I, $\phi = 14.36°$ (lead)

Power factor, cos φ :

$\cos \phi = \cos (14.36°)$

$= \mathbf{0.9687}$ **(leading). (Ans.)**

$[\overline{V} = \overline{V_1} + \overline{V_2} = 174.46 \angle 54.32° + 191.2 \angle -47.84° = 230 \angle 0°$ (check)]

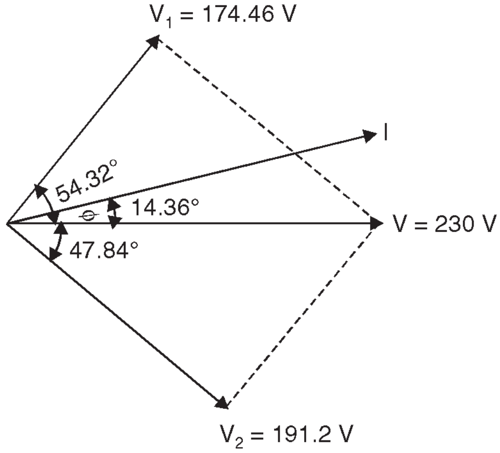


Fig. 53

Example 36. *A circuit consisting of a coil having an inductance of 0.25 H and a resistance of 3 Ω is arranged in series with a capacitor of capacitance 20 μF. Calculate at what frequency resonance will take place and current flow if an alternating voltage of 40 V at the resonant frequency is applied to the circuit. Find also the voltage across the capacitor.*

Solution. Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$ [L in henry, C in farad]

$$= \frac{1}{2\pi\sqrt{0.25 \times 20 \times 10^{-6}}} = \mathbf{71.2\ Hz.\ (Ans.)}$$

At the resonant frequency, I_r (or I_{max}) $= \frac{V}{R} = \frac{40}{3} = \mathbf{13.33\ A.\ (Ans.)}$

Voltage across the capacitor, $V_c = IX_c$

$$= \frac{13.33}{2\pi fC} \text{ (where } C \text{ is in farad)}$$

$$= \frac{13.33}{2\pi \times 71.2 \times 20 \times 10^{-6}} = \mathbf{1489.8\ V.\ (Ans.)}$$

Example 37. *A coil of inductance 0.64 H and resistance 40 Ω is connected in series with a capacitor of capacitance 12 μF.*

Estimate :

(*i*) *The frequency at which resonance will occur.*

(*ii*) *The voltage across the coil and capacitor, respectively and also the supply voltage when a current of 1.5 A at the resonant frequency is flowing.*

(*iii*) *The three voltages in (ii) with a current of 1.5 A flowing at a frequency of 50 Hz.*

Solution. (*i*) $f_r = \frac{1}{2\pi\sqrt{LC}}$ [L is in henry, C is in farad]

$$= \frac{1}{2\pi\sqrt{0.64 \times 12 \times 10^{-6}}} = \mathbf{57.4\ Hz.\ (Ans.)}$$

(*ii*) At resonance the supply voltage $= IR$

$= 1.5 \times 40 = \mathbf{60\ V.\ (Ans.)}$

Voltage across the coil $= I\sqrt{R^2 + X_L{}^2}$

$= 1.5\sqrt{40^2 + (2\pi \times 57.4 \times 0.64)^2}$ = **351.4 V. (Ans.)**

Voltage across the capacitor $= IX_C$

$$= I \times \frac{1}{2\pi f C} \qquad [C \text{ in } \mu F]$$

$$= \frac{1.5}{2\pi \times 57.4 \times 12 \times 10^{-6}} = \textbf{346.6 V. (Ans.)}$$

(*iii*) **At 50 Hz :**

Voltage across the coil $= I\sqrt{R^2 + X_L{}^2}$

$= 1.5\sqrt{40^2 + (2\pi \times 50 \times 0.64)^2}$ = **307.5 V. (Ans.)**

Voltage across the capacitor $= IX_C$

$$= \frac{1.5}{2\pi \times 50 \times 12 \times 10^{-6}} \simeq \textbf{398 V. (Ans.)}$$

Voltage across the entire circuit,

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$= 1.5\sqrt{(40)^2 + \left\{(2\pi \times 50 \times 0.64) - \left(\frac{1}{2\pi \times 50 \times 12 \times 10^{-6}}\right)\right\}^2}$$

$$= 1.5\sqrt{(40)^2 + (-64.2)^2} = \textbf{113.5 V. (Ans.)}$$

Example 38. *A coil having an inductance of 50 mH and resistance 10 Ω is connected in series with a 25 μF capacitor across a 200 V A.C. supply. Calculate :*

(*i*) *Resonance frequency of the circuit ;*

(*ii*) *Current flowing at resonance ;*

(*iii*) *Value of Q by using different data.*

Solution. (*i*)
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50 \times 10^{-3} \times 25 \times 10^{-6}}} = \textbf{142.3 Hz. (Ans.)}$$

(*ii*)
$$I_{max} = \frac{V}{R} = \frac{200}{10} = \textbf{20 A. (Ans.)}$$

(*iii*)
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50 \times 10^{-3}}{25 \times 10^{-6}}} = \textbf{4.47. (Ans.)}$$

Example 39. *A series R-L-C series circuit consists of R = 800 Ω, L = 80 mH and C = 8 pico-farad. The applied voltage across the circuit is 100 V. Determine :*

(*i*) *Resonant frequency of the circuit.*

(*ii*) *Q-factor of the circuit at the resonant frequency.*

(*iii*) *Bandwidth of the resonant circuit.*

(*iv*) *Frequencies at which the half power points occur.*

(*v*) *Bandwidth of the circuit.*

Solution. *Given :* $R = 800\ \Omega$; $L = 80$ mH $= 0.08$ H ; $C = 8 \times 10^{-12}$ F ; $V = 100$ volts.

(*i*) **Resonant frequency of the circuit, f_r :**

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.08 \times 8 \times 10^{-12}}} = \textbf{198.94 kHz.} \quad \textbf{(Ans.)}$$

(*ii*) **Q-factor at the resonant frequency :**

$$(Q\text{-factor})_{\text{resonant frequency}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{800}\sqrt{\frac{0.08}{8 \times 10^{-12}}} = \mathbf{125.} \quad \textbf{(Ans.)}$$

(*iii*) **Bandwidth of the resonant circuit ; B_{hp} ;**

$$B_{hp} = \frac{R}{2\pi L} = \frac{800}{2\pi \times 0.08} = \mathbf{1591.5\ Hz.} \quad \textbf{(Ans.)}$$

Also, $$B_{hp} = \frac{f_r}{Q} = \frac{198.94\text{ kHz}}{125} = 1.5915\text{ kHz} = 1591.5\text{ Hz} \quad \text{...as above}$$

(*iv*) **Frequencies at which the half power points occur, ω_1, ω_2 :**

$$f_1 = f_r - \frac{R}{4\pi L} = 198.94\ (\text{kHz}) - \frac{800}{4\pi \times 0.08} \times \frac{1}{1000}\ (\text{kHz}) = \mathbf{198.14\ kHz.} \quad \textbf{(Ans.)}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 198.94 + \frac{800}{4\pi \times 0.08} \times \frac{1}{1000} = \mathbf{199.74\ kHz.} \quad \textbf{(Ans.)}$$

(*v*) **Bandwidth of the resonant circuit :**

Bandwidth $= f_2 - f_1 = 199.74 - 198.14 =$ **1.6 kHz. (Ans.)**

Example 40. *A series R-L-C circuit consists of R = 20 Ω, L = 20 mH and C = 0.5 μF. If the circuit is connected to a 20 V variable frequency supply calculate the following :*

(*i*) *Resonant frequency f_r .*

(*ii*) *Resonance circuit Q-factor using L/C ratio.*

(*iii*) *Half-power bandwidth, using f_r and Q-factor.*

(*iv*) *Half-power bandwidth using the general formula for any bandwidth.*

(*v*) *Half-power bandwidth using the given component values.*

(*vi*) *Maximum power dissipated at f_r.*

Solution. *Given :* $R = 20\ \Omega$; $L = 20$ mH $= 0.02$ H ; $C = 0.5 \times 10^{-6}$ F ; V = 20 volts.

(*i*) **Resonant frequency,** $$\mathbf{f_r} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.02 \times 0.5 \times 10^{-6}}} = \mathbf{1591\ Hz.} \quad \textbf{(Ans.)}$$

(*ii*) **Q-factor** $$= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{0.02}{0.5 \times 10^{-6}}} = \mathbf{10.} \quad \textbf{(Ans.)}$$

(*iii*) **Half-power bandwidth** $\left(\textit{using } \frac{L}{C} \textit{ ratio}\right)$, $$\mathbf{B_{hp}} = \frac{f_r}{Q\text{-factor}} = \frac{1591}{10} = \mathbf{159.1\ Hz.} \quad \textbf{(Ans.)}$$

(*iv*) **Half-power bandwidth** *(using the general formula),*

$$B_{hp} = \frac{f_r Q}{(Q)_{\text{resonance}}} = \frac{1591 \times \tan 45°}{10} = \mathbf{159.1\ Hz.} \quad \textbf{(Ans.)}$$

($\because$ At half power points, $Q = \tan\theta = \tan 45° = 1$)

(*v*) **Half-power bandwidth** *(using component values),*

$$B_{hp} = \frac{R}{2\pi L} = \frac{20}{2\pi \times 0.02} = \mathbf{1591.5.} \quad \textbf{(Ans.)}$$

(*vi*) **Maximum power dissipated at $\mathbf{f_r}$,**

$$P_r = I_r^2 R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R} = \frac{(20)^2}{20} = \mathbf{20\ W.} \quad \textbf{(Ans.)}$$

4.6. A.C. Parallel Circuits

4.6.1. Introduction

Now-a-days, owing to multiple system of transmission and distribution, we come across parallel circuits (*i.e.*, impedances joined in parallel) more often. Practically all lighting and power circuits are constant voltage circuits with the loads connected in parallel. In a parallel A.C. circuit (like parallel D.C. circuit) the *voltage is the same across each branch.*

4.6.2. Methods for solving A.C. parallel circuits

The following *three* methods are available to solve such circuits :

1. Phasor or vector method
2. Admittance method
3. Vector algebra (symbolic method or *j*-method)

1. *Vector or phasor method*

Consider a parallel circuit consisting of two branches of impedances $Z_1(R_1, L)$ and $Z_2(R_2, C)$ respectively, and connected in parallel across an alternating voltage V volts (r.m.s.), as shown in Fig. 54 (*a*). Since the two branches are connected in parallel therefore, the voltage across each branch is the same and equal to supply voltage V but currents through them will be different.

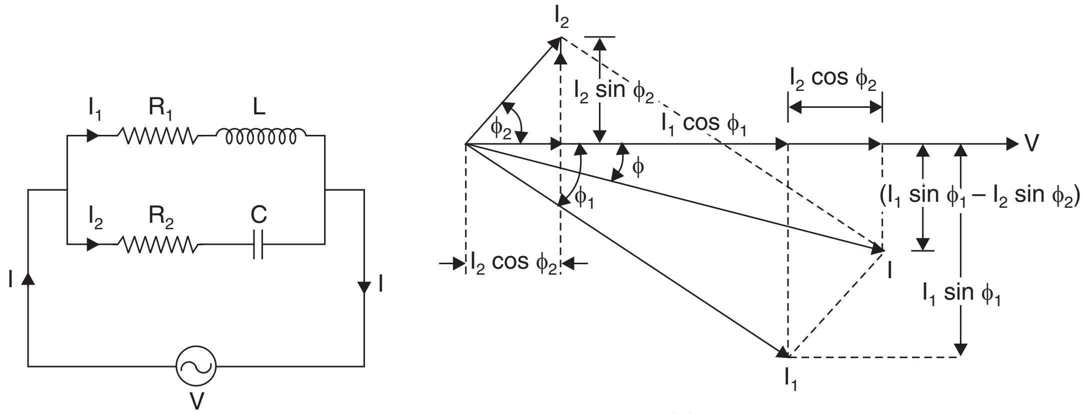


Fig. 54. Single-phase parallel circuit—Phasor method.

Branch-1 Impedance, $Z_1 = \sqrt{R_1^2 + X_L^2}$

Current, $I_1 = \dfrac{V}{Z_1}$;

Power factor, $\cos \phi_1 = \dfrac{R_1}{Z_1}$ or $\phi_1 = \cos^{-1}\left(\dfrac{R_1}{Z_1}\right)$;

Current, I_1 lags behind the applied voltage by ϕ_1.

Branch-2 Impedance, $Z_2 = \sqrt{R_2^2 + X_C^2}$;

Current, $I_2 = \dfrac{V}{Z_2}$;

Power factor, $\cos \phi_2 = \dfrac{R_2}{Z_2}$ or $\phi_2 = \cos^{-1}\left(\dfrac{R_2}{Z_2}\right)$

Current I_2 leads V by ϕ_2 [Fig. 54 (*b*)]

Resultant current I, which is phasor sum of I_1 and I_2, can be determined either by using parallelogram law of phasors, as shown in Fig. 54 (*b*) or by resolving branch currents I_1 and I_2 along X-axis and Y-axis and then determining the resultant of these components analytically.

Component of resultant current I along X-axis

= Sum of components of branch currents I_1 and I_2 along X-axis

or $$I \cos \phi = I_1 \cos \phi_2 + I_2 \cos \phi_2$$

Similarly, component of resultant current I along Y-axis

= Sum of components of branch currents I_1 and I_2 along Y-axis

or $$I \sin \phi = - I_1 \sin \phi_1 + I_2 \sin \phi_2$$

$$\therefore \quad I = \sqrt{(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (I_2 \sin \phi_2 - I_1 \sin \phi_1)^2} \quad \text{...(9)}$$

and $$\tan \phi = \frac{I_2 \sin \phi_2 - I_1 \sin \phi_1}{I_1 \cos \phi_1 + I_2 \cos \phi_2} = \frac{Y\text{-component}}{X\text{-component}} \quad \text{...[9(a)]}$$

If tan ϕ is +ve, then current I will lead the applied voltage V, if ϕ is –ve current I will lag behind the applied voltage V.

Power factor of the whole circuit is given by,

$$\cos \phi = \frac{I_1 \cos \phi_1 + I_2 \cos \phi_2}{I} = \frac{X\text{-component}}{I} \quad \text{...[9(b)]}$$

2. *Admittance method*

Admittance (denoted by symbol Y) of a circuit is defined as the *reciprocal of its impedance.*

$$\therefore \quad Y = \frac{1}{Z} = \frac{I}{V} \quad \text{or} \quad Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

The unit of admittance is *siemens* (S). The old unit was mho (℧).

As the impedance Z of a circuit has two components R and X (See Fig. 55), similarly, shown in Fig. 56, admittance Y also has two components G (*conductance*-X-component) and B (*susceptance*-Y-component).

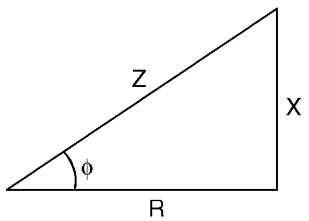


Fig. 55

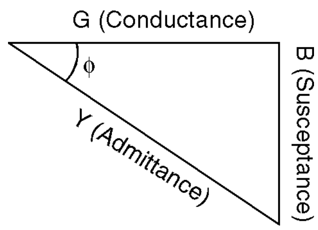


Fig. 56

Obviously, $$G = Y \cos \phi = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

Similarly, $$B = Y \sin \phi = \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}$$

$\therefore$ Admittance, $$Y = \sqrt{G^2 + B^2} \text{ just as } Z = \sqrt{R^2 + X^2}$$

The units of G, B and Y are in Siemens. Here, we shall consider capacitive susceptance as +ve and inductive capacitance as –ve.

Application of admittance method in solution of single-phase parallel circuits :

Refer to Fig. 57. Determine conductance and susceptance of individual branches from the relations

$$G = \frac{R}{Z^2} \text{ and } B = \frac{X}{Z^2}$$

Taking B as +ve if X is capacitive and as –ve if X is inductive. Let the conductances of the three branches of circuit shown in Fig. 57 be G_1, G_2 and G_3 respectively and susceptances be B_1, B_2 and B_3 respectively. Total conductance is found by merely adding the conductances of three branches. Similarly, total susceptance is found by algebraically adding the individual susceptances of different branches.

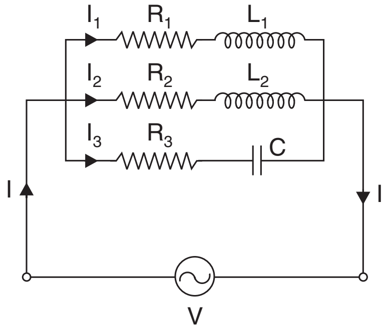


Fig. 57. Admittance method.

∴ Total conductance, $G = G_1 + G_2 + G_3$

and, total susceptance, $B = B_1 + B_2 + B_3$

∴ Total admittance $Y = \sqrt{G^2 + B^2}$ ...(10)

Total current, $I = VY$ ...(11)

Power factor, $\cos \phi = \frac{G}{Y}$ ...(12)

3. *Complex or Phasor algebra*

Consider the parallel circuit shown in Fig. 58 in which two impedances $\overline{Z}_1$ and $\overline{Z}_2$, being in parallel, have the same potential difference across them

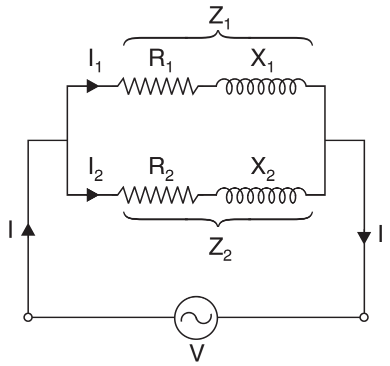


Fig. 58

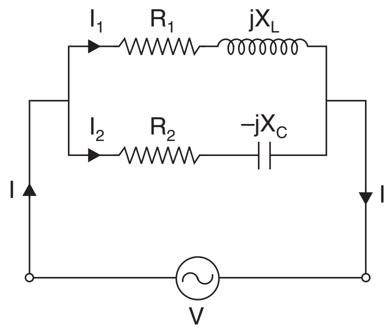


Fig. 59

Now, $\overline{I}_1 = \frac{\overline{V}}{\overline{Z}_1}$ and $\overline{I}_2 = \frac{\overline{V}}{\overline{Z}_2}$

and current $\overline{I} = \overline{I}_1 + \overline{I}_2 = \frac{\overline{V}}{\overline{Z}_1} + \frac{\overline{V}}{\overline{Z}_2}$

$$= \overline{V}\left(\frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}\right) = \overline{V}(\overline{Y} + \overline{Y}_2) = \overline{V}\overline{Y} \quad ...(13)$$

where $\overline{Y}$ (= total admittance) = $\overline{Y}_1 + \overline{Y}_2$ ...(14)

It may be noted that the *admittances* are *added in parallel branches*, whereas *impedances are added for branches in series*.

It is most important to remember that *admittances and impedances being complex quantities must be added in complex form.*

Let us now consider the two parallel branches shown in Fig. 59, we have

$$\overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{R_1 + jX_L} = \frac{R_1 - jX_L}{(R_1 + jX_L)(R_1 - jX_L)}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2} = \frac{R_1}{R_1^2 + X_L^2} - j\,\frac{X_L}{R_1^2 + X_L^2} = G_1 - jB_1$$

where $G_1 = \dfrac{R_1}{R^2 + X_L^2}$ ...Conductance of upper branch,

$B_1 = -\dfrac{X_L}{R_1^2 + X_L^2}$ ...Susceptance of upper branch.

Similarly, $$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{R_2 - jX_C} = \frac{(R_2 + jX_C)}{(R_2 - jX_C)(R_2 + jX_C)}$$

$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\,\frac{X_C}{R_2^2 + X_C^2} = G_2 + jB_2$$

Total admittance $\overline{Y} = \overline{Y}_1 + \overline{Y}_2 = (G - jB_1) + (G_2 + jB_2) = (G_1 + G_2) - j(B_1 - B_2) = G - jB$

$$\overline{Y} = \sqrt{(G_1 + G_2)^2 + (B_1 - B_2)^2} \quad ...(15)$$

and $$\phi = \tan^{-1}\left(\frac{B_1 - B_2}{G_1 + G_2}\right) \quad ...(16)$$

For admittance the *polar form* is :

$\overline{Y} = Y\angle\phi°$, where ϕ is as given above

$$Y = \sqrt{G^2 + B^2}\ \angle \tan^{-1}\left(\frac{B}{G}\right)$$

Total current $\overline{I} = \overline{V}\,\overline{Y}$; $I_1 = \overline{V}\,\overline{Y}_1$ and $I_2 = \overline{V}\,\overline{Y}_2$

If $\overline{V} = V\angle 0°$ and $\overline{Y} = Y\angle\phi$ then $\overline{I} = \overline{V}\,\overline{Y} = V\angle 0° \times Y\angle\phi = VY\angle\phi$

In general, *i.e.,* $\overline{V} = V\angle\alpha$ and $\overline{Y} = Y\angle\beta$, then

$$\overline{I} = \overline{V}\,\overline{Y} = V\angle\alpha \times Y\angle\beta = VY\angle(\alpha + \beta) \quad ...(17)$$

Thus, it is worth noting that when vector algebra is multiplied by admittance either in complex (rectangular) or polar form, the result is vector current in its proper phase relationship with respect to the voltage, irrespective of the axis to which the voltage may have been referred to.

Example 41. *A resistance of 60 Ω, an inductance of 0.18 H and a capacitance of 120 μF are connected in parallel across a 100 V, 50 Hz supply. Calculate :*

(*i*) *Current in each path.*

(*ii*) *Resultant current.*

(*iii*) *Phase angle between the resultant current and the supply voltage.*

(*iv*) *Power factor of the circuit.*

Solution. *Given :* $R = 60\ \Omega$; $L = 0.18$ H ; $C = 120\ \mu F = 120 \times 10^{-6}$ F, $V = 100$ volts, 50 Hz.

∴ Inductance reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 0.18 = 56.55\ \Omega$, and

capacitance reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$

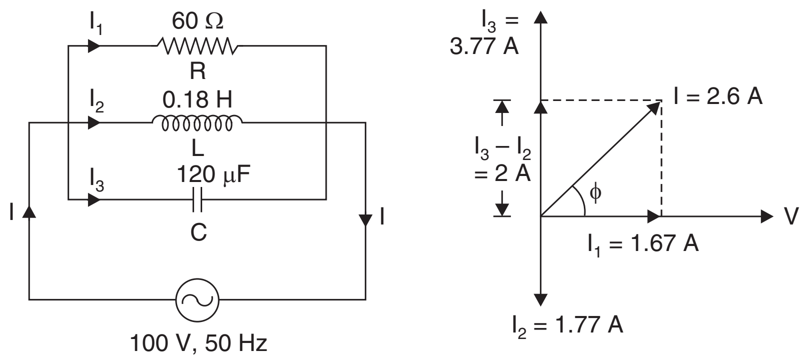
(*i*) **Current in each path :**

Current through reactance,

$$\mathbf{I_1} = \frac{V}{R} = \frac{100}{60} = \textbf{1.67 A in phase with voltage V.} \quad \textbf{(Ans.)}$$

Current through inductance,

$$\mathbf{I_2} = \frac{V}{X_L} = \frac{100}{56.55} = \textbf{1.77 A lagging behind voltage V by 90°.} \quad \textbf{(Ans.)}$$



(a) Circuit diagram (b) Phasor diagram

Fig. 60

Current through capacitance,

$$\mathbf{I_3} = \frac{V}{X_C} = \frac{100}{26.53} = \textbf{3.77 A leading the voltage V by 90°.} \quad \textbf{(Ans.)}$$

The circuit and phasor diagrams are shown in Fig. 60 (*a*) and (*b*) respectively.

(*ii*) **Resultant current, I :**

Resultant current, $I = \sqrt{(I_1)^2 + (I_3 - I_2)^2} = \sqrt{(1.67)^2 + (3.77 - 1.77)^2}$ = **6 A. (Ans.)**

(*iii*) **Phase angle between the resultant current and the supply voltage, φ :**

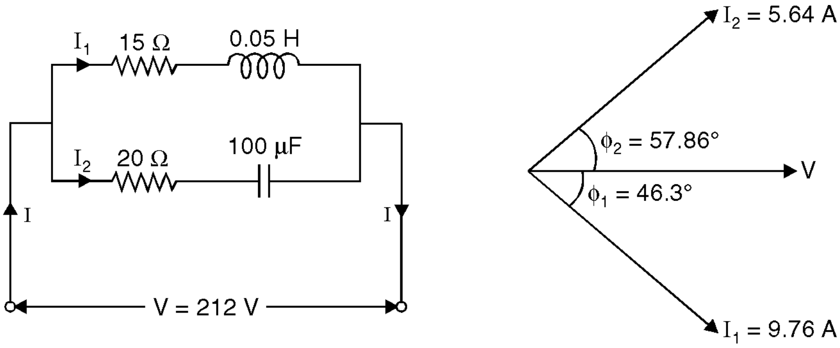
$$\phi = \tan^{-1}\left(\frac{I_3 - I_2}{I_1}\right) = \tan^{-1}\left(\frac{2}{1.67}\right) = \textbf{50.14° (lead).} \quad \textbf{(Ans.)}$$

(*iv*) **Power factor of the circuit, cos φ :**

$\cos\phi = \cos 50.14° =$ **0.641 (lead). (Ans.)**

Example 42. *Determine the r.m.s. value of current in each branch and total current of the circuit shown in Fig. 61. Draw the phasor diagram.*

Solution. Refer to Fig. 61 (*a*).



(*a*) Parallel circuit (*b*) Phasor diagram

Fig. 61

Branch No. 1 :

$$R_1 = 15\ \Omega,\ X_1 = 2\pi \times f \times L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$$

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{15^2 + 15.7^2} = 21.71\ \Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{212}{21.71} = 9.76\ \text{A}$$

$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{15}{21.71} = 0.691\ [\text{or } \phi_1 = \cos^{-1}(0.691) = 46.3°\ (\textit{lagging})]$$

$$\sin\phi_1 = \sin(46.3°) = 0.723$$

Branch No. 2 :

$$R_2 = 20\ \Omega,\ X_2 = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{20^2 + 31.83^2} = 37.59\ \Omega$$

$$I_2 = \frac{V}{Z_2} = \frac{212}{37.59} = 5.64\ \text{A}$$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{20}{37.59} = 0.532 \qquad [\text{or } \phi_2 = \cos^{-1}(0.532) = 57.86°\ (\textit{leading})]$$

$$\sin\phi_2 = \sin(57.86°) = 0.847$$

The phasor diagram is shown in Fig. 60 (*b*)

X-components of I_1 and $I_2 = I_1 \cos\phi_1 + I_2 \cos\phi_2$

$= 9.76 \times 0.691 + 5.64 \times 0.532 = 9.74$ A

Y-components of I_1 and $I_2 = -I_1 \sin\phi_1 + I_2 \sin\phi_2$

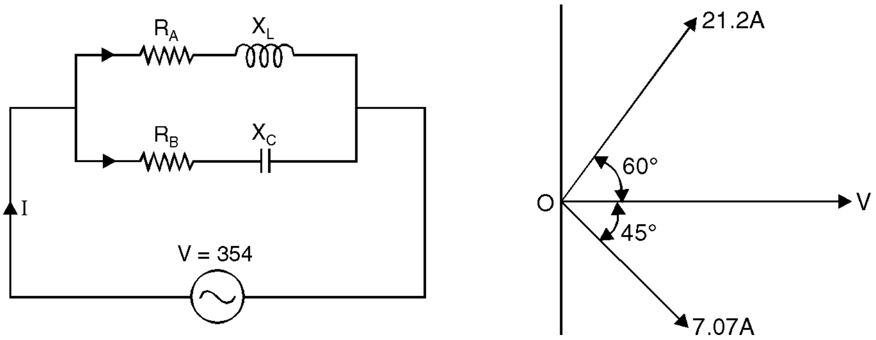
$= -9.76 \times 0.723 + 5.64 \times 0.847 = -2.28$ A

Total current, $I = \sqrt{(9.74)^2 + (2.28)^2}$ = **10 A. (Ans.)**

Example 43. *The currents in each branch of a two-branched parallel circuit are given by the expressions $i_A = 7.07 \sin(314t - \pi/4)$ and $i_B = 21.2 \sin(314t + \pi/3)$. The supply voltage is given by the expression $v = 354 \sin 314t$. Derive a similar expression for the supply current and calculate the ohmic value of the components, assuming two pure components in each branch. State whether the reactive components are inductive or capacitive.*

Solution. From the given expressions of currents, we find that :

- i_A lags the voltage by π/4 radian or 45° and i_B leads it by π/3 radian or 60°. Hence branch *A* consists of a resistance in series with a pure inductive reactance. Branch *B* consists of a resistance in series with a pure capacitive reactance as shown in Fig. 62



(*a*) Parallel circuit (*b*) Phasor diagram

Fig. 62

- Maximum value of current in branch *A* is 7.07 A and in branch *B* is 21.2 A.

The resultant current can be found as follows :

As seen from the vector diagram [Fig. 62 (*b*)],

X-component = 21.2 cos 60° + 7.07 cos 45° = 15.6 A

Y-component = 21.2 sin 60° – 7.07 sin 45° = 13.36 A

Maximum value of the resultant current

$$= \sqrt{(15.6)^2 + (13.36)^2} = 20.54 \text{ A}$$

$$\phi = \tan^{-1} (13.36/15.6) = 40.6° \text{ (lead)}$$

Hence the expression for the supply current is :

$$\mathbf{i = 20.54 \sin (314t + 40.6°)} \textbf{ (Ans.)}$$

$$Z_A = \frac{354}{7.07} = \mathbf{50}\ \Omega \text{ ; } \cos \phi_A = \cos 45° = 0.707, \sin \phi_A = \sin 45° = 0.707$$

$$R_A = Z_A \cos \phi_A = 50 \times 0.707 = \mathbf{35.4}\ \Omega$$

$$X_L = Z_A \sin \phi_A = 50 \times 0.707 = \mathbf{35.4}\ \Omega.$$

$$Z_B = \frac{354}{21.2} = \mathbf{16.7}\ \Omega. \textbf{ (Ans.)}$$

$$R_B = Z_B \cos 60° = 16.7 \times \cos 60° = \mathbf{8.35}\ \Omega$$

$$X_B = Z_B \sin 60° = 16.7 \times \sin 60° = \mathbf{14.46}\ \Omega. \textbf{ (Ans.)}$$

Example 44. *Two impedances given by $Z_1 = (10 + j5)$ and $Z_2 = (8 + j6)$ are joined in parallel and connected across a voltage of v = 200 + j0. Calculate the circuit current, its phase and the branch currents. Draw the phasor diagram.*

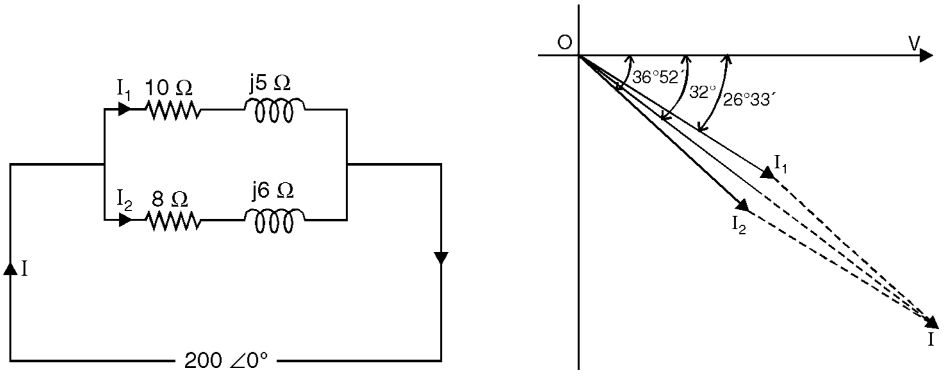
Solution. Refer to Fig. 63 (*a*).

Branch A,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{(10 + j5)}$$

$$= \frac{1}{(10 + j5)} \times \frac{10 - j5}{(10 - j5)} = \frac{10 - j5}{100 + 25} = \frac{10 - j5}{125}$$

$$= (0.08 - j0.04) \text{ siemens}$$



(*a*) Parallel circuit (*b*) Phasor diagram (not to scale)

Fig. 63

Branch B, $Y_2 = \dfrac{1}{Z_2} = \dfrac{1}{(8 + j6)}$

$= \dfrac{1}{(8 + j6)} \times \dfrac{(8 - j6)}{(8 - j6)} = \dfrac{8 - j6}{64 + 36} = \dfrac{8 - j6}{100}$

$= (0.08 - j0.06)$ siemens

$\therefore$ $Y = Y_1 + Y_2 = (0.08 - j0.04) + (0.08 - j0.06)$

$= (0.16 - j0.1)$ siemens

[*Alternatively :*

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{Z_1 + Z_2}{Z_1 Z_2}$$

$\therefore$ $$Y = \frac{Z_1 + Z_2}{Z_1 Z_2} = \frac{(10 + j5) + (8 + j6)}{(10 + j5)(8 + j6)} = \frac{(18 + j11)}{50 + j100}$$

Rationalising the above expression, we get

$$Y = \frac{(18 + j11)(50 - j100)}{(50 + j100)(50 - j100)} = \frac{2000 - j1250}{12500}$$

$= 0.16 - j0.1$ (same as before)]

Now $V = 200\angle 0°$

$\therefore$ $I = VY = (200 + j0)(0.16 - j0.1)$

$= 32 - j20 =$ **$37.74 \angle -32°$** ...polar form. **(Ans.)**

It lags behind the applied voltage by 32°.

Power factor $= \cos 32° =$ **0.848. (Ans.)**

$I_1 = VY_1 = (200 + j0)(0.08 - j0.04)$

$= 16 - j8 =$ **$17.89 \angle -26°\ 33'$. (Ans.)**

It lags behind the applied voltage by 26° 33′.

$I_2 = VY_2 = (200 + j0)(0.08 - j0.06)$

$= 16 - j12 = 20\angle -36°\ 52'$

It lags behind the applied voltage by 36° 52′.

The phasor diagram is shown in Fig. 63 (*b*).

Example 45. *Fig. 64 shows a parallel circuit in which the values of the parameters are as given below :*

R_1 = 70 Ω (non-inductive) ; Coil : R_C = 30 Ω, L_C = 0.5 H ; R_2 = 100 Ω ; X_C = 157 Ω (at 50 Hz).

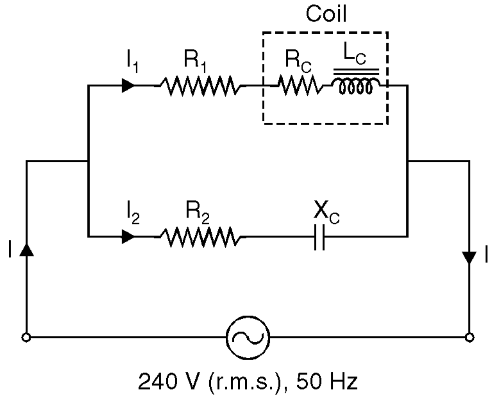


Fig. 64

(*i*) *Determine the branch currents and the total current.*

(*ii*) *Draw the phasor diagram indicating the currents and voltages across coil and condenser.*

(*iii*) *If the A.C. source is replaced by an equivalent D.C. source, what current would be drawn by the circuit ?*

Solution. *Given :* R_1 = 70 Ω ; R_C = 30 Ω, L_C = 0.5 H ; R_2 = 100 Ω ; X_C = 157 Ω,

Applied voltage = 240 V (r.m.s.), 50 Hz

(*i*) **The branch currents (I_1, I_2) and the total current (I) :**

Resistance of the inductive branch $= R_1 + R_C = 70 + 30 = 100\ \Omega$

Reactance of the inductive branch $= 2\pi f L_C = 2\pi \times 50 \times 0.5 = 157\ \Omega$

Conductance of inductive branch,

$$G_1 = \frac{R_1 + R_C}{(R_1 + R_C)^2 + (2\pi f L_C)^2} = \frac{100}{(100)^2 + (157)^2} = 0.00288 \text{ S (siemens)}$$

Susceptance of inductive branch,

$$B_1 = \frac{2\pi f L_C}{(R_1 + R_C)^2 + (2\pi f L_C)^2}$$

$$= \frac{157}{(100)^2 + (157)^2} = -\ 0.00453 \text{ S (Being inductive)}$$

Conductance of capacitive branch,

$$G_2 = \frac{R_2}{{R_2}^2 + {X_C}^2} = \frac{100}{(100)^2 + (157)^2} = 0.00288 \text{ S}$$

Susceptance of capacitive branch,

$$B_2 = \frac{X_C}{{R_2}^2 + {X_C}^2} = \frac{157}{(100)^2 + (157)^2} = 0.00453 \text{ S (Being capacitive)}$$

Total conductance of the circuit,

$$G = G_1 + G_2 = 0.00288 + 0.00288 = 0.00576 \text{ S}$$

Total susceptance of the circuit,

$$B = B_1 + B_2 = -\ 0.00453 + 0.00453 = 0$$

Total admittance of the circuit,

$$Y = \sqrt{G^2 + B^2} = \sqrt{(0.00576)^2 + 0^2} = 0.00576 \text{ S}$$

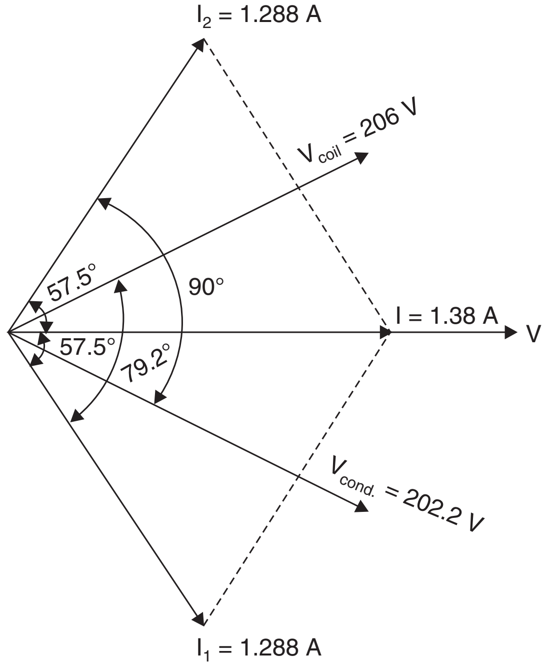


Fig. 65. Phasor diagram.

Current in inductive branch, $\mathbf{I_1} = V \times Y_1 = V\sqrt{G_1^{\,2} + B_1^{\,2}}$

$$= 240\sqrt{(0.00288)^2 + (-0.00453)^2} = \mathbf{1.288\ A.} \quad \textbf{(Ans.)}$$

Phase angle, $\quad \phi_1 = \tan^{-1}\left(\dfrac{B_1}{G_1}\right) = \tan^{-1}\left(\dfrac{-0.00453}{0.00288}\right) = -\mathbf{57.5°.} \quad$ **(Ans.)**

Current in capacitive branch, $\mathbf{I_2} = V \times Y_2 = V\sqrt{G_2^{\,2} + B_2^{\,2}}$

$$= 240\sqrt{(0.00288)^2 + (0.00453)^2} = \mathbf{1.288\ A.} \quad \textbf{(Ans.)}$$

Phase angle, $\quad \phi_2 = \tan^{-1}\left(\dfrac{B_2}{G_2}\right) = \tan^{-1}\left(\dfrac{0.00453}{0.00288}\right) = \mathbf{57.5°.} \quad$ **(Ans.)**

Total current, $\quad \mathbf{I} = V \times Y = 240 \times 0.00576 = \mathbf{1.38\ A.} \quad$ **(Ans.)**

Phase angle, $\quad \phi = \tan^{-1}\left(\dfrac{B}{G}\right) = \tan^{-1}\left(\dfrac{0}{0.00576}\right) = 0°$

Voltage across condenser

$= I_2X_C = 1.288 \times 157 =$ **202.2 V** *lagging behind I_2 by 90°.* **(Ans.)**

Voltage across coil $= I_1 \times \sqrt{(R_C)^2 + (2\pi f L_C)^2} = 1.288\sqrt{(30)^2 + (157)^2} = 206 \text{ V}$

Phase angle with current $I_1 = \tan^{-1}\left(\dfrac{157}{30}\right) = \mathbf{79.2°.} \quad$ **(Ans.)**

(*ii*) **Phasor diagram :**

The phasor diagram indicating the currents and voltages across coil and condenser is shown in Fig. 65.

(*iii*) **Current drawn by the circuit when D.C. source is used :**

When energised by equivalent D.C. source (*i.e.,* 240 V D.C.), the capacitive branch will be open and current drawn by the circuit

= Current drawn by inductive branch

$$= \frac{V}{R_1 + R_C} = \frac{240}{70 + 30} = \mathbf{2.4\ A.}\quad \textbf{(Ans.)}$$

Example 46. *Fig. 66 shows two impedances (18 + j24) and (15 – j30) Ω connected in parallel ; across the combination is applied a voltage of 200∠53°8′. Determine :*

(*i*) *kVA, kVAR and kW in each branch.*

(*ii*) *The power factor of the whole circuit.*

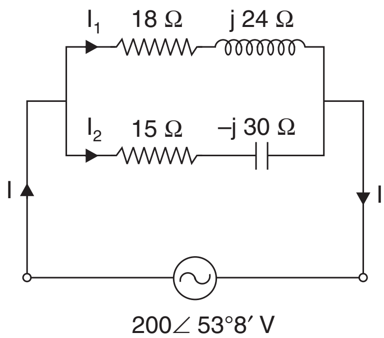


Fig. 66

Solution. Refer to Fig. 66

$$\bar{Y}_1 = \frac{1}{18 + j24} = \frac{18 - j24}{(18 + j24)(18 - j24)}$$

$$= \frac{18 - j24}{900} = (0.02 - j0.0266)\ \text{S}$$

$$\bar{Y}_2 = \frac{1}{15 - j30} = \frac{15 + j30}{(15 + j30)(15 - j30)} = \frac{15 + j30}{1125}$$

$$= (0.0133 + j0.0266)\ \text{S}$$

Now, $V = 200\angle 53°8' = 200\,(\cos 53°8' + j \sin 53°8')$

$= 200(0.6 + j0.8) = (120 + j160)$ volts

$$\bar{I}_1 = \bar{V}\bar{Y}_1 = (120 + j160)(0.02 - j0.0266)$$

$= 2.4 + j3.2 - j3.2 + 4.26 = (6.66 + j0)$ ...(along the reference axis)

$$\therefore \quad I_2 = \bar{V}\bar{Y}_2 = (120 + j160)(0.0133 + j0.0266)$$

$= 1.6 + j3.2 + j2.13 - 4.26 = -\,2.66 + j5.33$ (leading)

(*i*) **kVA, kVAR and kW in each branch :**

Power calculations can be calculated by the method of conjugates.

Branch 1 (*Inductive branch*)

The current conjugate of $(6.66 + j0)$ is $(6.66 - j0)$

$$\therefore \quad \bar{V}\bar{I}_1 = (120 + j160)(6.66 - j0) = 800 + j1066 \quad \therefore \quad \textbf{kW} = \frac{800}{1000} = \mathbf{0.8.}\quad \textbf{(Ans.)}$$

$$\therefore \quad \textbf{kVAR} = \frac{1066}{1000} = \mathbf{1.066.}\quad \textbf{(Ans.)}$$

The fact that it is positive merely shows that reactive *VA* (volt-amps) are due to lagging current

$$\textbf{kVA} = \sqrt{(kW)^2 + (kVAR)^2} = \sqrt{(0.8)^2 + (1.066)^2} = \mathbf{1.33.}\quad \textbf{(Ans.)}$$

Branch 2 (*Capacitive branch*) :

The current conjugate of $(-\,2.66 + j5.33)$ is $(-\,2.66 - j5.33)$

$$\therefore \quad \bar{V}\bar{I}_2 = (120 + j160)(-\,2.66 - j5.33) = 533.6 - j1065.2$$

$$\therefore \quad \textbf{kW} = \frac{533.6}{1000} = \mathbf{0.5336.}\quad \textbf{(Ans.)} \quad ; \quad \textbf{kVAR} = \frac{-\,1065.2}{1000} = \mathbf{-\,1.065.}\quad \textbf{(Ans.)}$$

The negative sign merely indicates the reactive volt-amps are due to leading current.

$$\therefore \qquad \textbf{kVA} = \sqrt{(0.5336)^2 + (-1.065)^2} = \textbf{1.191.} \quad \textbf{(Ans.)}$$

(*ii*) **The power factor of the whole circuit, cos φ :**

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 = (0.02 - j0.0266) + (0.0133 + j0.0266) = 0.0333 + j0$$

$$\bar{I} = \bar{V}\bar{Y} = (120 + j160)(0.0333 + j0) = 4 - j5.33 = 6.66 \angle 53°8'$$

or

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = (6.66 + j0) + (-2.66 + j5.33) = 4 - j5.33 \quad \text{(Same as above)}$$

Power factor of the circuit, $\cos \phi = \cos 0° =$ **1. (Ans.)**

(∵ Current is in phase with voltage)

Example 47. *For the circuit of Fig. 67, calculate the current supplied by the voltage source and the voltage across the current source.*

Solution. Refer to Figs. 67 and 68.

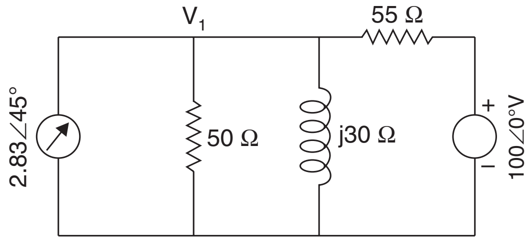


Fig. 67

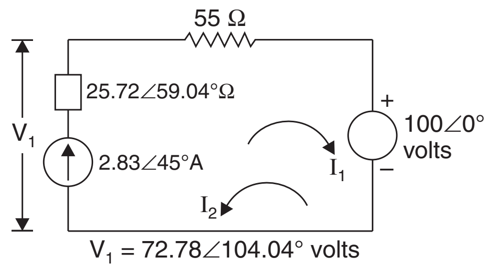


Fig. 68

Let the impedance of the current source be Z_i, then

$$\frac{1}{Z_i} = \frac{1}{50\angle 0°} + \frac{1}{30\angle 90°} = \frac{30\angle 90° + 50\angle 0°}{1500\angle 90°} = \frac{50 + j30}{1500\angle 90°} = \frac{58.31\angle 30.96°}{1500\angle 90°}$$

$$Z_i = \frac{1500\angle 90°}{58.31\angle 30.96°} = 25.72 \angle 59.04° = (13.23 + j22.05)\ \Omega$$

Voltage across current source,

$$V_1 = IZ_i = 2.83 \angle 45° \times 25.72 \angle 59.04° = \textbf{72.78} \angle \textbf{104.04° volts.} \quad \textbf{(Ans.)}$$

Current supplied by one source,

$$I_1 = \frac{V_1}{Z_i + R} = \frac{72.78\angle 104.04°}{13.25 + j22.05 + 55} = \frac{72.78\angle 104.04°}{68.25 + j22.05}$$

$$= \frac{72.78\angle 104.04°}{71.72\angle 17.91°} = 1.015 \angle 86.13° = (0.0685 + j1.01)\ \text{A}$$

Current supplied by the other source,

$$I_2 \frac{V_2}{Z_i + R} = \frac{100\angle 0°}{71.72\angle 17.91°} = 1.394 \angle (-17.91°) = 1.326 - j0.428$$

Current supplied by the voltage source,

$$I = I_2 - I_1 = 1.326 - j0.428 - 0.0685 - j1.01 = 1.2575 - j1.438$$

$$= \textbf{1.91} \angle \textbf{– 48.83 amp. (Ans.)}$$

4.6.3. Series-Parallel Circuits

Series-parallel circuits may be solved by the following methods :

1. Admittance method.
2. Symbolic method.

1. *Admittance method*

In series-parallel circuits, the parallel circuit is first reduced to an equivalent series circuit and then combined with the rest of the circuit as usual.

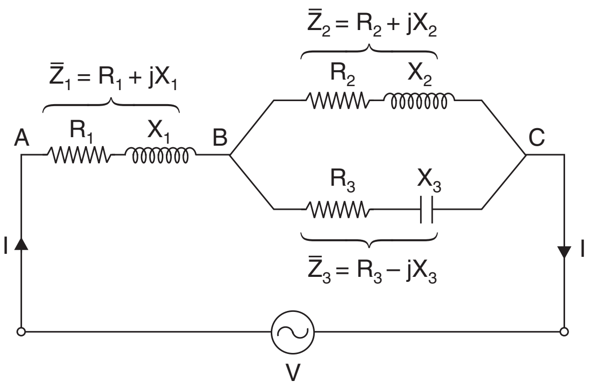


Fig. 69. Series-parallel circuit.

For a parallel circuit,

Equivalent series resistance, $R_{eq} = Z \cos \phi = \dfrac{1}{Y} \cdot \dfrac{G}{Y} = \dfrac{G}{Y^2}$

Equivalent series reactance, $X_{eq} = Z \sin \phi = \dfrac{1}{Y} \cdot \dfrac{B}{Y} = \dfrac{B}{Y^2}$

2. *Symbolic method*

Refer series-parallel circuit shown in Fig. 69. First calculate the equivalent impedance of parallel branches and then add it to the series impedance to get the total impedance of the circuit. Then current flowing through the circuit is found as follows :

$$\bar{Y}_2 = \frac{1}{R_2 + jX_2} \quad \text{and} \quad \bar{Y}_3 = \frac{1}{R_3 - jX_3}$$

$$\therefore \quad \bar{Y}_{23} = \frac{1}{R_2 + jX_2} + \frac{1}{R_3 - jX_3}$$

$$\therefore \quad \bar{Z}_{23} = \frac{1}{\bar{Y}_{23}} \; ; \; \bar{Z}_1 = R_1 + jX_1$$

$$\bar{Z} = \bar{Z}_{23} + \bar{Z}_1$$

$$\therefore \quad \bar{I} = \frac{\bar{V}}{\bar{Z}}.$$

Example 48. *For the circuit shown in Fig. 70 calculate :*

(*i*) *Currents I_A, I_B and I_C.*

(*ii*) *Total power factor for the whole circuit.*

Draw the complete phasor diagram.

Solution. Refer to Fig. 70.

Given : $\overline{Z}_A = 2 + j1.5 = 2.5\angle 36.9°$; $\overline{Z}_B = 5 - j3.5 = 6.1\angle -35°$, $\overline{Z}_C = 3 + j2.5 = 3.9\angle 39.8°$

(*i*) **Currents $\mathbf{I_A}$, $\mathbf{I_B}$ and $\mathbf{I_C}$:**

$$\frac{1}{\overline{Z}_{AB}} = \frac{1}{\overline{Z}_A} + \frac{1}{\overline{Z}_B} \quad \text{or} \quad \overline{Z}_{AB} = \frac{\overline{Z}_A \overline{Z}_B}{\overline{Z}_A + \overline{Z}_B} = \frac{2.5\angle 36.9° \times 6.1\angle -35°}{(2 + j1.5) + (5 - j3.5)}$$

$$= \frac{15.25\angle 1.9°}{7 - j2} = \frac{15.25\angle 1.9°}{7.28\angle -16°} = 2.095\angle 17.9° \simeq 2 + j0.65$$

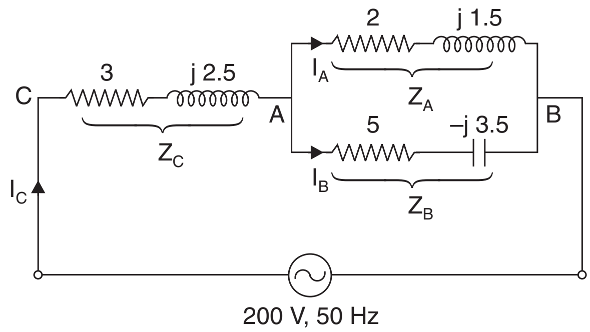


Fig. 70. Circuit diagram.

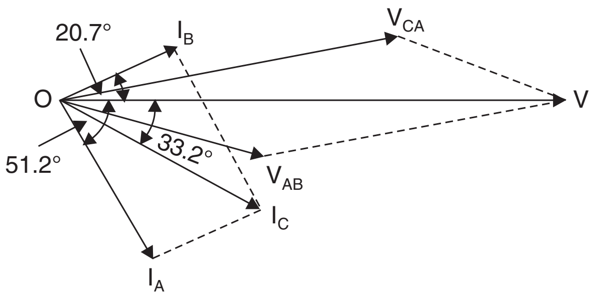


Fig. 71. Phasor diagram.

$$\overline{Z} = \overline{Z}_C + \overline{Z}_{AB} = (3 + j2.5) + (2 + j0.65) = 5 + j3.15 = 5.9\angle 32.2°$$

Let $\overline{V} = 200\angle 0°$,

Then, $\overline{\mathbf{I}}_\mathbf{C} = \frac{\overline{V}}{\overline{Z}} = \frac{200\angle 0°}{5.9\angle 32.2°} = \mathbf{33.9\angle -32.2°.}$ **(Ans.)**

$$\overline{\mathbf{I}}_\mathbf{A} = \overline{I}_C \times \frac{\overline{Z}_B}{\overline{Z}_A + \overline{Z}_B} = 33.9\angle -32.2° \times \frac{6.1\angle -35°}{7.28\angle -16°} = \mathbf{28.4\angle -51.2°.} \quad \textbf{(Ans.)}$$

$$\overline{\mathbf{I}}_\mathbf{B} = \overline{I}_C \times \frac{\overline{Z}_A}{\overline{Z}_A + \overline{Z}_B} = 33.9\angle -32.2° \times \frac{2.5\angle 36.9°}{7.28\angle -16°} = \mathbf{11.64\angle 20.7°.} \quad \textbf{(Ans.)}$$

(*ii*) **Total power factor in the whole circuit :**

The phase angle between V and total circuit current I_C is 32.2°.

∴ Power factor for the whole circuit ; cos ϕ = cos 32.2° = **0.846 (lag). (Ans.)**

For drawing the phasor diagram of Fig. 71, V_{CA} and V_B have to be calculated.

$$\overline{V}_{CA} = \overline{I}_C \overline{Z}_C = 33.9\angle -32.2° \times 3.9\angle 39.8° = 132.2\angle 7.6°$$

$$\overline{V}_{AB} = \overline{I}_C \overline{Z}_{AB} = 33.9\angle -32.2° \times 2.095\angle 17.9° = 71\angle -14.3°$$

Fig. 71 shows the complete phasor diagram.

Example 49. *Determine the current drawn by the following circuit (Fig. 72) when a voltage of 200 V is applied across the same. Draw the phasor diagram.*

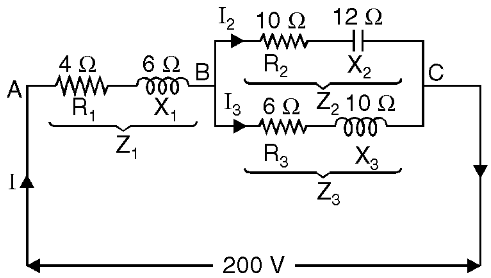


Fig. 72. Series parallel circuit

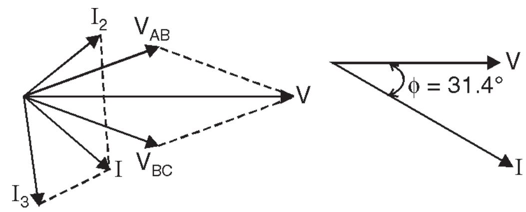


Fig. 73. Phasor diagram

Solution. Refer to Fig. 72.

$$Z_1 = (4 + j6) = 7.2\ \angle 56.3° \ ; Z_2 = (10 - j12) = 15.6\ \angle -50.2°$$

$$Z_3 = (6 + j10) = 11.7\ \angle 59°$$

$$Z_{BC} = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(10 - j12)(6 + j10)}{(10 - j12) + (6 + j10)} = \frac{(10 - j12)(6 + j10)}{(16 - j2)} = \frac{(180 + j28)}{(16 - j2)}$$

$$= \frac{(180 + j28)(16 + j2)}{(16 - j2)(16 + j2)} = \frac{2824 + j808}{260} = 10.9 + j3.1 = 11.3\ \angle 15.9°$$

$$Z = Z_1 + Z_{BC} = (4 + j6) + (10.9 + j3.1) = (14.9 + j9.1) = 17.5\angle 31.4°$$

Assuming $V = 200\angle 0° \ ; I = \dfrac{V}{Z} = \dfrac{200\angle 0°}{17.5\angle 31.4°} = 11.4\ \angle -31.4°$

For drawing the phasor diagram, let us find the following quantities :

$$V_{AB} = IZ_1 = 11.4\ \angle -31.4° \times 7.2\ \angle 56.3° = 82.08\ \angle 24.9°$$

$$V_{BC} = IZ_{BC} = 11.4\ \angle -31.4° \times 11.3\ \angle 15.9° = 128.8\ \angle -15.5°$$

$$I_2 = \frac{V_{BC}}{Z_L} = \frac{128.8\angle -15.5°}{15.6\angle -50.2°} = 8.26\ \angle 34.7°$$

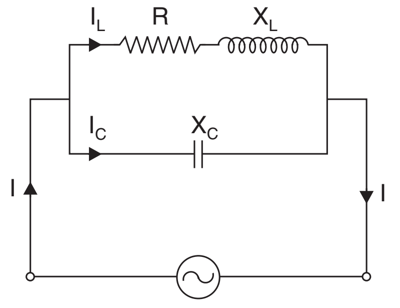
$$I_3 = \frac{V_{BC}}{Z_3} = \frac{128.8\angle -15.5°}{11.7\angle 59°} = 11\ \angle -74.5°$$

Various currents and voltages are shown in their phase relationship in Fig. 73.

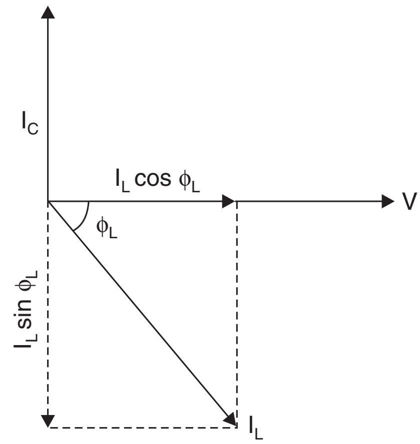
4.6.4. Resonance in parallel circuits

In case of a series circuit consisting of R (resistance), L (inductance) and C (capacitance), resonance takes place when $V_L = V_C$ *i.e.,* when $X_L = X_C$. In other words, resonance takes place when the power factor of the circuit approaches *unity*. The basic condition for resonance, *i.e.*, power factor of the entire circuit being unity, remains the same for parallel circuits also. Thus, *resonance in a parallel circuit will occur, when the power factor of the entire circuit becomes unity.*

A parallel circuit consisting of an inductive coil in parallel with a capacitor is shown in Fig. 74 (*a*). The phasor diagram of this circuit with applied voltage as the reference phasor is shown in Fig. 74 (*b*). The current drawn by the inductive branch lags the applied voltage by an angle ϕ_L. The current drawn by the capacitive branch leads the applied voltage by 90°.



(a) Circuit diagram (Resonant circuit)



(b) Phasor diagram (Resonant circuit)

Fig. 74. Resonance in a parallel circuit.

The power factor of the current becomes unity when the total current drawn by the entire circuit is in phase with the applied voltage. This will happen only when the current drawn by the capacitive branch I_C equals the active component of the current of the inductive branch I_L [Fig. 74 (*b*)].

Hence for *resonance* in parallel circuit.

$$I_C = I_L \sin \phi_L \quad \text{...(18)}$$

Now, $I_L = \dfrac{V}{Z}$; $\sin \phi_L = \dfrac{X_L}{Z}$ and $I_C = \dfrac{V}{X_C}$

Hence, condition for resonance becomes

$$\frac{V}{X_C} = \frac{V}{Z} \times \frac{X_L}{Z} \quad \text{or} \quad Z^2 = X_L \times X_C$$

Now, $X_L = \omega L,\ X_C = \dfrac{1}{\omega C}$

$$\therefore \quad Z^2 = \frac{\omega L}{\omega C} = \frac{L}{C} \quad \text{...(19)}$$

or $$R^2 + X_L{}^2 = R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

or $$(2\pi f_r L)^2 = \frac{L}{C} - R^2 \quad \text{or} \quad 2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

or $$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{...(20)}$$

This is the resonant frequency and is given in Hz if R is in ohm, L is in henry and C in farad. If R is negligible, then

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{...same as for series resonance.}$$

Current at resonance

Refer Fig. 74 (*b*). Since wattless component is zero, the circuit current is given as :

$$I = I_L \cos \phi_L = \frac{V}{Z} \times \frac{R}{Z} = \frac{VR}{Z^2}$$

Putting the value of $Z^2 = \frac{L}{C}$ from eqn. (19), we get

$$I = \frac{VR}{L/C} = \frac{V}{L/CR} \quad \text{...(21)}$$

Thus, the *impedance* offered by a resonant parallel circuit = $\frac{L}{CR}$.

This impedance is purely resistive and generally termed as *equivalent* or *dynamic impedance* of the circuit. As the resultant current drawn by a resonant parallel circuit is minimum, the circuit is normally called ***rejector*** circuit. Such a type of circuit is quite useful in *radio work*.

The phenomenon of resonance in *parallel circuits* is normally termed as ***"current resonance"*** whereas it is termed ***"voltage resonance"*** in *series circuit*.

Resonance characteristics :

Fig. 75 shows the characteristics of the parallel circuit consisting of an inductance L and capacitance C in parallel plotted against frequency, the voltage applied being constant.

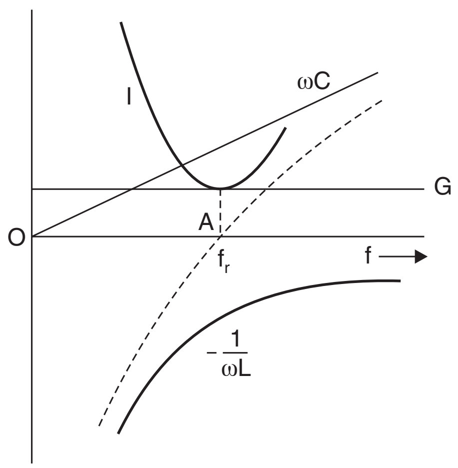


Fig. 75. Resonance characteristics.

Inductive susceptance, $b = -\frac{1}{X_L} = -\frac{1}{\omega L} = -\frac{1}{2\pi f L}$

Thus inductive susceptance is *inversely proportional* to the *frequency* and is represented by rectangular hyperbola in the fourth quadrant (because it is assumed –ve).

Capacitive susceptance, $b = \frac{1}{X_C} \omega C = 2\pi f C$

Thus capacitive susceptance is *directly proportional to the frequency* and is represented by a straight line passing through the origin.

Net susceptance, B is the difference of the two susceptances and is represented by the dotted hyperbola. The net susceptance is zero at point A, hence *admittance is minimum* and is equal to G. Thus, at point A, *line current is minimum.* The frequency at which the total current becomes minimum is the resonance frequency f_r.

Evidently, below the resonant frequency, the inductive susceptance predominates, thus making the circuit current to be lagging, whereas beyond f_r capacitive susceptance predominates and the current leads the applied voltage. At resonant frequency f_r, the current is in phase with the applied voltage.

Hence at ***parallel resonance*** it is seen that :

1. The admittance of the circuit is *minimum* and is *equal to* the conductance of the circuit.
2. The *current drawn is minimum.*
3. The phase angle between the current and voltage is zero, the *power factor is unity.*
4. The resonant frequency is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$ if the *resistance* in the inductance and capacitance branches is *negligible.*

4.6.5. Comparison of series and parallel resonant circuits

S. No.	*Aspects*	*Series circuit (R-L-C)*	*Parallel circuit (R-L and C)*
1.	*Impedance at resonance*	Minimum	Maximum
2.	*Current at resonance*	Maximum = $\frac{V}{R}$	Minimum = $V/(L/CR)$
3.	*Effective impedance*	R	L/CR
4.	*Power factor at resonance*	Unity	Unity
5.	*Resonant frequency*	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}-\frac{R^2}{L^2}\right)}$
6.	*It magnifies*	Voltage	Current
7.	*Magnification is*	$\frac{\omega L}{R}$	$\frac{\omega L}{R}$

4.6.6. Q-factor of a parallel circuit

It is defined as the *ratio of the current circulating between its two branches to the line current drawn from the supply or simply, as the current magnification.*

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \text{...(22)}$$

- It may be noted that in *series circuits, Q-factor gives the voltage magnification, whereas in parallel circuits it gives the current magnification.*

Bandwidth of a parallel resonant circuit :

The bandwidth of a parallel circuit is defined in the same way as that for a series circuit. This circuit also has upper and lower half-power frequencies where power dissipated is half of that at resonant frequency.

The net susceptance B, at bandwidth frequencies, equals conductance. Hence,

At f_2 : $\quad B = B_{C2} - B_{L2}$

At f_1 : $\quad B = B_{L1} - B_{C1}$

Hence $Y = \sqrt{G^2 + B^2} = \sqrt{2}G$ and $\phi = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}(1) = 45°$

However, at off-resonant frequencies, $Y > G$ and $B_C \neq B_L$ and phase angle is greater than zero.

Example 50. *An inductive circuit of resistance 2 ohms and inductance 0.01 H is connected to a 250 V, 50 Hz supply.*

(*i*) *What capacitance placed in parallel will produce resonance ?*

(*ii*) *Determine also the total current taken from the supply and the currents in the branch circuit.*

Solution. *Given :* $R = 2\ \Omega$; $L = 0.01$ H, Supply voltage = 250 V, 50 Hz.

(*i*) **Value of capacitance which will produce resonance, C :**

Now, $X_L = 2\pi f L = 2\pi \times 50 \times 0.01 = 3.14\ \Omega$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{2^2 + 3.14^2} = 3.72\ \Omega$$

We know that, $Z^2 = \dfrac{L}{C}$ or $C = \dfrac{L}{Z^2}$

$\therefore$ $C = \dfrac{0.01}{(3.72)^2} = 722.6 \times 10^{-6}$ F or **722.6 μF. (Ans.)**

(*ii*) **Total current and currents in the branch circuits, I, I_L, I_C :**

$$\mathbf{I_{R\text{-}L}} = \frac{V}{Z} = \frac{250}{3.72} = \mathbf{67.2\ A.}\quad \textbf{(Ans.)}$$

$$\tan \phi_L = \frac{3.14}{2} = 1.57 \quad \text{or} \quad \phi_L = \tan^{-1}(1.57) = \mathbf{57.5°}\quad \textbf{(Ans.)}$$

Hence, current in *R-L* branch lags the applied voltage by 57.5°.

$$\mathbf{I_C} = \frac{V}{X_C} = \frac{V}{1/\omega C} = \omega VC = 2\pi \times 50 \times 250 \times 722.6 \times 10^{-6} = \mathbf{56.75}\quad \textbf{(Ans.)}$$

This current leads the applied voltage by 90°.

Total current, $\mathbf{I} = I_{R\text{-}L} \cos \phi = 67.2 \cos 57.5° =$ **36.1 A. (Ans.)**

$$\left[\text{or } I = \frac{V}{L/CR} = \frac{VCR}{L} = \frac{250 \times 722.6 \times 10^{-6} \times 2}{0.01} = 36.1\,\text{A}\right]$$

HIGHLIGHTS

1. Modern alternators produce an e.m.f. which is for all practical purposes sinusoidal (*i.e.* a sine curve), the equation between the e.m.f. and time being

$$e = E_{max} \sin \omega t$$

where e = instantaneous voltage ; E_{max} = maximum voltage;

ωt = angle through which the armature has turned from neutral.

2. The *r.m.s. value* of an alternating current is given by that steady (D.C.) current which when flowing through a given circuit produces the same *heat* as is produced by the alternating current when flowing through the same circuit for the same time.

$$I_{rms} = 0.707\, I_{max}.$$

3. The *average or mean value* of an alternating current is expressed by that steady current which transfers across any circuit the same **charge** as is transferred by that alternating current during the same time.

$$I_{av} = 0.637\, I_{max}$$

Form factor is the ratio of r.m.s. value to average value of the wave form.

Peak factor is the ratio of maximum value to the r.m.s. value of the wave form.

4. The frequency (series circuit) of the voltage which gives the maximum value of the current in the circuit is called *resonant frequency* (f_r) and the circuit is said to be *resonant*.

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

5. Q-factor of a series circuit is defined as equal to the voltage magnification in the circuit at resonance.

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

6. Q-factor of a parallel circuit is defined as the ratio of the current circulating between its two branches to the line current drawn from the supply or simply, as the current magnification.

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. A sine wave has a frequency of 50 Hz. Its angular frequency is ...... radian/second.
(*a*) 100 π (*b*) 50 π
(*c*) 25 π (*d*) 5 π.

2. The reactance offered by a capacitor to alternating current of frequency 50 Hz is 20 Ω. If frequency is increased to 100 Hz, reactance becomes ...... ohms.
(*a*) 2.5 (*b*) 5
(*c*) 10 (*d*) 15.

3. The period of a wave is
(*a*) the same as frequency (*b*) time required to complete one cycle
(*c*) expressed in amperes (*d*) none of the above.

4. The form factor is the ratio of
(*a*) peak value to r.m.s. value (*b*) r.m.s. value to average value
(*c*) average value to r.m.s. value (*d*) none of the above.

5. The period of a sine wave is $\frac{1}{50}$ seconds. Its frequency is
(*a*) 20 Hz (*b*) 30 Hz
(*c*) 40 Hz (*d*) 50 Hz.

6. An A.C. current is given by $i = 200 \sin 100\,\pi t$. It will achieve a value of 100 A after ...... second.
(*a*) $\frac{1}{900}$ (*b*) $\frac{1}{800}$
(*c*) $\frac{1}{700}$ (*d*) $\frac{1}{600}$.

7. A heater is rated as 230 V, 10 kW, A.C. The value 230 V refers to
(*a*) average voltage (*b*) r.m.s. voltage
(*c*) peak voltage (*d*) none of the above.

8. If two sinusoids of the same frequency but of different amplitudes and phase angles are subtracted, the resultant is
(*a*) a sinusoid of the same frequency (*b*) a sinusoid of half the original frequency
(*c*) a sinusoid of double the frequency (*d*) not a sinusoid.

9. The peak value of a sine wave is 200 V. Its average value is
(*a*) 127.4 V (*b*) 141.4 V
(*c*) 282.8 V (*d*) 200 V.

10. If two sine waves of the same frequency have a phase difference of π radians, then
(*a*) both will reach their minimum values at the same instant
(*b*) both will reach their maximum values at the same instant
(*c*) when one wave reaches its maximum value, the other will reach its minimum value
(*d*) none of the above.

11. The r.m.s. value of a sine wave is 100 A. Its peak value is
(*a*) 70.7 A (*b*) 141.4 A
(*c*) 150 A (*d*) 282.8 A.

12. If two waves are expressed as $e_1 = E_{m_1} \sin(\omega t + \alpha_1)$ and $e_2 = E_{m_2} \sin(\omega t + \alpha_2)$, then
(*a*) e_1 is leading e_2 by $\angle(\alpha_2 - \alpha_1)$ (*b*) e_2 is leading e_1 by $\angle(\alpha_2 - \alpha_1)$
(*c*) e_2 is leading e_1 by $\angle(\alpha_1 - \alpha_2)$ (*d*) e_1 is in phase with e_2.

13. The voltage of domestic supply is 220 V. This figure represents
(*a*) mean value (*b*) r.m.s. value
(*c*) peak value (*d*) average value.

14. Two waves of the same frequency have opposite phase when the phase angle between them is
(*a*) 360° (*b*) 180°
(*c*) 90° (*d*) 0°.

15. The power consumed in a circuit element will be least when the phase difference between the current and voltage is
(*a*) 180° (*b*) 90°
(*c*) 60° (*d*) 0°.

16. The r.m.s. value and mean value is the same in the case of
(*a*) triangular wave (*b*) sine wave
(*c*) square wave (*d*) half wave rectified sine wave.

17. For the same peak value which of the following wave will have the highest r.m.s. value ?
(*a*) square wave (*b*) half wave rectified sine wave
(*c*) triangular wave (*d*) sine wave.

18. For the same peak value, which of the following wave has the *least* mean value ?
(*a*) half wave rectified sine wave (*b*) triangular wave
(*c*) sine wave (*d*) square wave.

19. For a sine wave with peak value I_{max} the r.m.s. value is
(*a*) 0.5 I_{max} (*b*) 0.707 I_{max}
(*c*) 0.9 I_{max} (*d*) 1.414 I_{max}.

20. Form Factor is the ratio of
(*a*) average value/r.m.s. value (*b*) average value/peak value
(*c*) r.m.s. value/average value (*d*) r.m.s. value/peak value.

21. Form factor for a sine wave is
(*a*) 1.414 (*b*) 0.707
(*c*) 1.11 (*d*) 0.637.

22. For a sine wave with peak value E_{max} the average value is
(*a*) 0.636 E_{max} (*b*) 0.707 E_{max}
(*c*) 0.434 E_{max} (*d*) 1.414 E_{max}.

23. The current in a circuit is given by : $i = 100 \sin 314\, t$ amperes
The maximum value and frequency of current are
(*a*) $50\sqrt{2}$ A, 100 Hz (*b*) $100\sqrt{2}$ A, 100 Hz
(*c*) 100 A, 50 Hz (*d*) 70.7A, 50 Hz.

24. For a frequency of 200 Hz, the time period will be
(*a*) 0.05 s (*b*) 0.005 s
(*c*) 0.0005 s (*d*) 0.5 s.

25. The phase difference between voltage and current wave through a circuit element is given as 30°. The essential condition is that
(*a*) both waves must have same frequency (*b*) both waves must have identical peak values
(*c*) both waves must have zero value at the same time
(*d*) none of the above.

26. An A.C. voltage of 50 Hz has a maximum value of 50 V. Its value after 1/600 second after the instant the current is zero, will be
(*a*) 5 V (*b*) 12.5 V
(*c*) 25 V (*d*) 43.3 V.

27. When two waves are in phase they have peak values at an interval of
(*a*) 180° (*b*) 120°
(*c*) 90° (*d*) none of the above.

28. For 200 V r.m.s. value triangular wave, the peak voltage will be
(*a*) 200 V (*b*) 222 V
(*c*) 282 V (*d*) 346 V.

29. A sine wave of voltage varies from zero to maximum of 200 V. How much is the voltage at the instant of 30° of the cycle ?
(*a*) 50 V (*b*) 82.8 V
(*c*) 100 V (*d*) 173.2 V.

30. How much r.m.s. current does a 300 W 200 V bulb take from the 200 V, 50 Hz power line ?
(*a*) 0.5 A (*b*) 1.5 A
(*c*) 2 A (*d*) 3 A.

31. Two sinusoidal currents are given by $i_1 = 100 \sin (\omega t + \pi/3)$, and $i_2 = 150 \sin (\omega t - \pi/4)$
The phase difference between them is ...... degrees.
(*a*) 15 (*b*) 50
(*c*) 60 (*d*) 105.

32. The r.m.s. value of a half-wave rectified current is 100 A. Its value for full-wave rectification would be ...... amperes.
(*a*) 141.4 (*b*) 200
(*c*) 200/π (*d*) 40/π.

33. From the two voltages equations $e_1 = E_{max} \sin 100\pi t$, and $e_2 = E_{max} \sin (100\,\pi t + \pi/6)$, it is obvious that
(*a*) 1 leads 2 by 30° (*b*) 2 lags behind 1
(*c*) 2 achieves its maximum value $\frac{1}{600}$ second before 1 does
(*d*) 1 achieves its zero value $\frac{1}{600}$ second before 2.

34. The r.m.s. value of a sinusoidal A.C. current is equal to its value at an angle of ...... degrees.
(*a*) 90 (*b*) 60
(*c*) 45 (*d*) 30.

35. Capacitive reactance is more when
(*a*) capacitance is less and frequency of supply is less
(*b*) capacitance is less and frequency of supply is more
(*c*) capacitance is more and frequency of supply is less
(*d*) capacitance is more and frequency of supply is more.

36. Time constant of a capacitive circuit increases with the
(*a*) increase of capacitance and decrease of resistance
(*b*) increase of capacitance and increase of resistance

(*c*) decrease of capacitance and decrease of resistance
(*d*) decrease of capacitance and increase of resistance.

37. In a series circuit on resonance, following will occur
(*a*) $V = V_R$ (*b*) $X_L = X_C$
(*c*) $Z = R$ (*d*) $V_L = V_C$
(*e*) all above.

38. In a series resonant circuit, the impedance of the circuit is
(*a*) minimum (*b*) maximum
(*c*) zero (*d*) none of the above.

39. Power factor of an electrical circuit is equal to
(*a*) R/Z
(*b*) cosine of phase angle difference between current and voltage
(*c*) kW/kVA (*d*) ratio of useful current to total current I_W/I
(*e*) all above.

40. The best place to install a capacitor is
(*a*) very near to inductive load (*b*) across the terminals of the inductive load
(*c*) far away from the inductive load (*d*) any where.

41. Poor power factor
(*a*) reduces load handling capability of electrical system
(*b*) results in more power losses in the electrical system
(*c*) overloads alternators, transformers and distribution lines
(*d*) results in more voltage drop in the line (*e*) results in all above.

42. Capacitors for power factor correction are rated in
(*a*) kW (*b*) kVA
(*c*) kV (*d*) kVAR.

43. In series resonant circuit, increasing inductance to its twice value and reducing capacitance to its half value
(*a*) will change the maximum value of current at resonance
(*b*) will change the resonance frequency
(*c*) will change the impedance at resonance frequency
(*d*) will increase the selectivity of the circuit.

44. Pure inductive circuit
(*a*) consumes some power on average
(*b*) does not take power at all from a line
(*c*) takes power from the line during some part of the cycle and then returns back to it during other part of the cycle
(*d*) none of the above.

45. Inductance affects the direct current flow
(*a*) only at the time of turning off (*b*) only at the time of turning on
(*c*) at the time of turning on and off (*d*) at all the time of operation.

46. Inductance of a coil varies
(*a*) directly as the cross-sectional area of magnetic core
(*b*) directly as square of number of turns (*c*) directly as the permeability of the core
(*d*) inversely as the length of the iron path (*e*) as (*a*) to (*d*).

47. All the rules and laws of D.C. circuit also apply to A.C. circuit containing
(*a*) capacitance only (*b*) inductance only
(*c*) resistance only (*d*) all above.

48. Time constant of an inductive circuit
(*a*) increases with increase of inductance and decrease of resistance
(*b*) increases with the increase of inductance and the increase of resistance

(*c*) increases with decrease of inductance and decrease of resistance
(*d*) increases with decrease of inductance and increase of resistance.

49. Power factor of an inductive circuit is usually improved by connecting capacitor to it in
(*a*) parallel (*b*) series
(*c*) either (*a*) or (*b*) (*d*) none of the above.

50. In a highly capacitive circuit the
(*a*) apparent power is equal to the actual power (*b*) reactive power is more than the apparent power
(*c*) reactive power is more than the actual power (*d*) actual power is more than its reactive power.

51. Power factor of the following circuit will be zero
(*a*) resistance (*b*) inductance
(*c*) capacitance (*d*) both (*b*) and (*c*).

52. Power factor of the following circuit will be unity
(*a*) inductance (*b*) capacitance
(*c*) resistance (*d*) both (*a*) and (*b*).

53. Power factor of the system is kept high
(*a*) to reduce line losses
(*b*) to maximise the utilization of the capacities of generators, lines and transformers
(*c*) to reduce voltage regulation of the line
(*d*) due to all above reasons.

54. The time constant of the capacitance circuit is defined as the time during which voltage
(*a*) falls to 36.8% of its final steady value (*b*) rises to 38.6% of its final steady value
(*c*) rises to 63.2% of its final steady value (*d*) none of the above.

55. In the *R-L-C* containing $R = 4.5\ \Omega$, $L = 0.06$ H, $C = 0.6\ \mu F$ the power factor will be
(*a*) zero (*b*) lagging
(*c*) leading (*d*) unity.

ANSWERS

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)	**6.** (*d*)
7. (*b*)	**8.** (*a*)	**9.** (*a*)	**10.** (*c*)	**11.** (*b*)	**12.** (*b*)
13. (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*c*)	**17.** (*a*)	**18.** (*a*)
19. (*b*)	**20.** (*c*)	**21.** (*c*)	**22.** (*a*)	**23.** (*c*)	**24.** (*b*)
25. (*a*)	**26.** (*c*)	**27.** (*d*)	**28.** (*d*)	**29.** (*c*)	**30.** (*b*)
31. (*d*)	**32.** (*a*)	**33.** (*c*)	**34.** (*c*)	**35.** (*a*)	**36.** (*b*)
37. (*e*)	**38.** (*a*)	**39.** (*e*)	**40.** (*b*)	**41.** (*e*)	**42.** (*d*)
43. (*d*)	**44.** (*c*)	**45.** (*c*)	**46.** (*e*)	**47.** (*c*)	**48.** (*a*)
49. (*a*)	**50.** (*c*)	**51.** (*d*)	**52.** (*c*)	**53.** (*d*)	**54.** (*c*)
55. (*c*).					

THEORETICAL QUESTIONS

1. Define the following terms :
   Circuit, Electrical network, Active network, Node and Branch.
2. What are the limitations of ohm's law ?
3. State and explain Kirchhoff's laws.
4. Discuss briefly application of Kirchhoff's laws.
5. Explain the nodal voltage method for solving networks. How are the nodal equations written ?
6. Explain Cramer's rule used for solving equations by determinants.
7. State and explain Superposition theorem.

8. State Norton's theorem. List the steps for finding the current in a branch of a network with the help of this theorem.

9. State Thevenin's theorem.

10. State the maximum power transfer theorem and explain its importance.

11. Define the following terms as applied to an alternating current :
Cycle, frequency, time period, amplitude.

12. What do you mean by the term "Phase difference" ?

13. Explain the following terms relating alternating current :
(*i*) R.M.S. value (*ii*) Average value
(*iii*) Form factor (*iv*) Peak factor.

14. Explain briefly the following as applied to A.C. series and parallel circuits :
(*i*) Resonance frequency (*ii*) Q-factor.

15. What do you mean by transient disturbances ?

16. Define single energy and double energy transients.

EXERCISE

1. An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive, find :
(*i*) The instantaneous value after $\frac{1}{360}$ second ;
(*ii*) The time taken to reach 96 A for the first time. [**Ans.** 103.9 A, 0.00245 second]

2. An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate :
(*i*) Its value 1/600 second after the instant the current is zero and its value decreasing thereafter wards.
(*ii*) How many seconds after the instant the current is zero (increasing thereafter wards) will the current attain the value of 86.6 A ? [**Ans.** – 50 A, 1/300 s]

3. Calculate the r.m.s. value, the form factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, 5, 10 V etc. What would be the r.m.s. value of sine wave having the same peak value ?
[**Ans.** 31 V ; 23 V ; 1.35 ; (app.) ; 42.2 V]

4. A sinusoidally varying alternating current has an average value of 127.4 A. When its value is zero, then its rate of change is 62,800 A/s. Find the analytical expression for the sine wave.
[**Ans.** $i = 200 \sin 100\,\pi t$]

5. A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150 μF capacitor across a 200 V, 50 Hz supply. Calculate (*i*) the inductive reactance, (*ii*) the capacitive reactance, (*iii*) the impedance (*iv*) the current, (*v*) the power factor, (*vi*) the voltage across the coil and the capacitor respectively.
[**Ans.** (*i*) 31.4 Ω, (*ii*) 21.2 Ω, (*iii*) 14.3 Ω, (*iv*) 14 A, (*v*) 0.7 lag (*vi*) 460 V, 297 V]

6. A circuit is made up of 10 Ω resistance, 12 mH inductance and 281.5 μF capacitance in series. The supply voltage is 100 V (constant). Calculate the value of the current when the supply frequency is (*i*) 50 Hz and (*ii*) 150 Hz. [**Ans.** 8 A leading ; 8 A lagging]

7. A coil having a resistance of 10 Ω and an inductance of 0.2 H is connected in series with a capacitor of 50.7 μF. The circuit is connected across a 100 V, 50 Hz A.C. supply. Calculate (*i*) the current flowing (*ii*) the voltage across the capacitor (*iii*) the voltage across the coil. Draw a vector diagram to scale.
[**Ans.** (*i*) 10 A (*ii*) 628 V (*iii*) 635 V]

8. A coil is in series with a 20 μF capacitor across a 230 V, 50 Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is (*i*) leading and (*ii*) lagging.
Sketch a vector diagram for each condition and calculate the coil power factor in each case.
[**Ans.** 0.415 H ; 0.597 H ; 0.0238 ; 0.0166]

9. An A.C. series circuit has a resistance of 10 Ω, an inductance of 0.2 H and a capacitance of 60 μF. Calculate :
(*i*) the resonant frequency (*ii*) the current and
(*iii*) the power at resonance.
Given that the applied voltage is 200 V. [**Ans.** 46 Hz ; 20 A ; 4 kW]

10. A circuit consists of an inductor which has a resistance of 10 Ω and an inductance of 0.3 H, in series with a capacitor of 30 μF capacitance. Calculate :
(*i*) The impedance of the circuit to currents of 40 Hz ;
(*ii*) The resonant frequency ;
(*iii*) The peak value of stored energy in joules when the applied voltage is 200 V at the resonant frequency.
[**Ans.** 58.31 Ω ; 53 Hz ; 120 J]

11. A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 240 V, 50 Hz supply, the maximum current given by varying the inductance is 0.5 A. At this current, the voltage across the capacitor is 250 V. Calculate the values of the following :
(*i*) The resistance ; (*ii*) The capacitance ;
(*iii*) The inductance.
Neglect the resistance of the inductor. [**Ans.** 480 Ω, 6.36 μF ; 1.59 H]

12. A resistance, a capacitor and a variable inductance are connected in series across a 200 V, 50 Hz supply. The maximum current which can be obtained by varying the inductance is 314 mA and the voltage across the capacitor is then 300 V. Calculate the capacitance of the capacitor and the values of the inductance and resistance. [**Ans.** 3.33 μF, 3.04 H, 637 Ω]

13. A circuit consisting of a coil of resistance 12 Ω and inductance 0.15 H in series with a capacitor of 12 μF is connected to a variable frequency supply which has a constant voltage of 24 V.
Calculate : (*i*) The resonant frequency, (*ii*) The current in the circuit at resonance, (*iii*) The voltage across the capacitor and the coil at resonance. [**Ans.** (*i*) 153 Hz, (*ii*) 2 A, (*iii*) 224 V]

14. A resistance of 24 Ω, a capacitance of 150 μF and an inductance of 0.16 H are connected in series with each other. A supply at 240 V, 50 Hz is applied to the ends of the combination. Calculate (*i*) the current in the circuit (*ii*) the potential differences across each element of the circuit (*iii*) the frequency to which the supply would need to be changed so that the current would be at unity power-factor and find the current at this frequency. [**Ans.** (*i*) 6.37 A (*ii*) V_R = 152.8 V, V_C = 320 V, V_L = 123.3 V (*iii*) 32 Hz ; 10 A]

15. A coil-A of inductance 80 mH and resistance 120 Ω is connected to a 230 V, 50 Hz single-phase supply. In parallel with it is a 16 μF capacitor is series with a 40 Ω non-inductive resistor *B*. Determine
(*i*) The power factor of the combined circuit,
(*ii*) The total power taken from the supply. [**Ans.** (*i*) 0.945 lead (*ii*) 473 W]

16. A choking coil of inductance 0.08 H and resistance 12 ohm, is connected in parallel with a capacitor of 120 μF. The combination is connected to a supply at 240 V, 50 Hz. Determine the total current from the supply and its power factor. Illustrate your answers with a phasor diagram. [**Ans.** 3.94 A, 0.943 lag]

17. A choking coil having a resistance of 20 Ω and an inductance of 0.07 henry is connected with a capacitor of 60 μF capacitance which is in series with a resistor of 50 Ω. Calculate the total current and the phase angle when this arrangement is connected to 200 V, 50 Hz mains. [**Ans.** 7.15 A, 24° 39′ lag]

18. A coil of resistance of 15 Ω and inductance 0.05 H is connected in parallel with a non-inductive resistance of 20 Ω. Find (*i*) the current in each branch ; (*ii*) the total current (*iii*) the phase angle of whole arrangement for an applied voltage of 200 V at 50 Hz. [**Ans.** 9.22 A ; 10 A ; 22.1°]

19. A sinusoidal 50 Hz voltage of 200 V (r.m.s.) supplies the following three circuits which are in parallel : (*i*) a coil of inductance 0.03 H and resistance 3 Ω ; (*ii*) a capacitor of 400 μF in series with a resistance of 100 Ω ; (*iii*) a coil of inductance 0.02 H and resistance 7 Ω in series with a 300 μF capacitor. Find the total current supplied and draw a complete vector diagram. [**Ans.** 29.4 A]

20. In a series-parallel circuit, the two parallel branches *A* and *B* are in series with *C*. The impedances are $Z_A = (10 - j8)$ Ω, $Z_B = (9 - j6)$ Ω and $Z_C = (100 + j0)$. Find the currents I_A and I_B and the phase difference between them. Draw the phasor diagram. [**Ans.** $I_A = 12.71 \angle -30° 58'$ $I_B = 15 \angle -35° 56'$; 4° 58′]

3

Three-Phase A.C. Circuits

1. INTRODUCTION

- Generation, transmission and heavy-power utilisation of A.C. electric energy almost invariably involve a type of system or circuit called a *polyphase system* or *polyphase circuit*. In such a system, each voltage source consists of a group of voltages having relative magnitudes and phase angles. Thus, a *m-phase system will employ voltage sources which, conventionally, consist of m voltages substantially equal in magnitude and successively displaced by a phase angle of 360°/m*.
- A *3-phase system* will employ voltage sources which, conventionally, consist of *three voltages substantially equal in magnitude* and *displaced by phase angles of 120°*. Because it *possesses definite economic and operating advantages, the 3-phase system is by far the most common,* and consequently emphasis is placed on 3-phase *circuits*.

2. ADVANTAGES OF POLYPHASE SYSTEMS

The advantages of polyphase systems *over single-phase systems* are :

1. A polyphase transmission line requires less conductor material than a single-phase line for transmitting the same amount power at the same voltage.

2. For a given frame size a polyphase machine gives a higher output than a single-phase machine. For example, output of a 3-phase motor is 1.5 times the output of single-phase motor of same size.

3. Polyphase motors have a uniform torque where most of the single-phase motors have a pulsating torque.

4. Polyphase induction motors are self-starting and are more efficient. On the other hand single-phase induction motors are not self-starting and are less efficient.

5. Per unit of output, the polyphase machine is very much cheaper.

6. Power factor of a single-phase motor is lower than that of polyphase motor of the same rating.

7. Rotating field can be set up by passing polyphase current through stationary coils.

8. Parallel operation of polyphase alternators is simple as compared to that of single-phase alternators because of pulsating reaction in single-phase alternator.

It has been found that the above advantages are *best realised in the case of three-phase systems*. Consequently, the electric power is generated and transmitted in the form of three-phase system.

3. GENERATION OF THREE-PHASE VOLTAGES

- Let us consider an elementary 3-phase 2-pole generator as shown in Fig. 1. On the armature are three coils, *ll′*, *mm′*, and *nn′* whose axes are displaced 120° in space from each other.

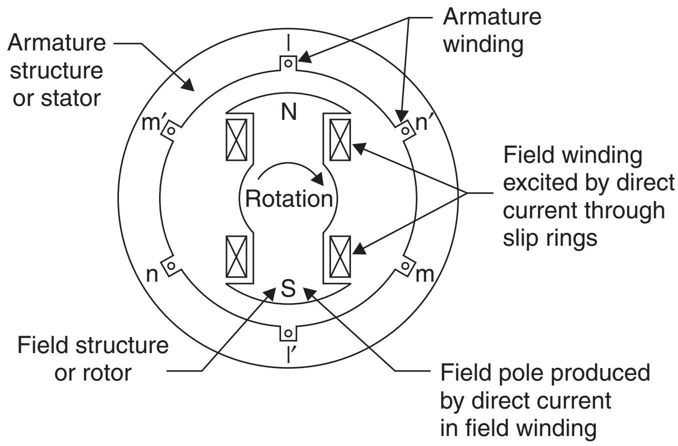


Fig. 1. Elementary 3-phase 2-pole generator.

- When the field is excited and rotated, voltages will be generated in the three phases in accordance with Faraday's law. If the field structure is so designed that the flux is distributed sinusoidally over the poles, the flux linking any phase will vary sinusoidally with time and sinusoidal voltages will be induced in three-phases. These three waves will be displaced 120 electrical degrees (Fig. 2) in time as a result of the phases being displaced 120° in space. The corresponding phasor diagram is shown in Fig. 3. The equations of the instantaneous values of the three voltages (given by Fig. 2) are :

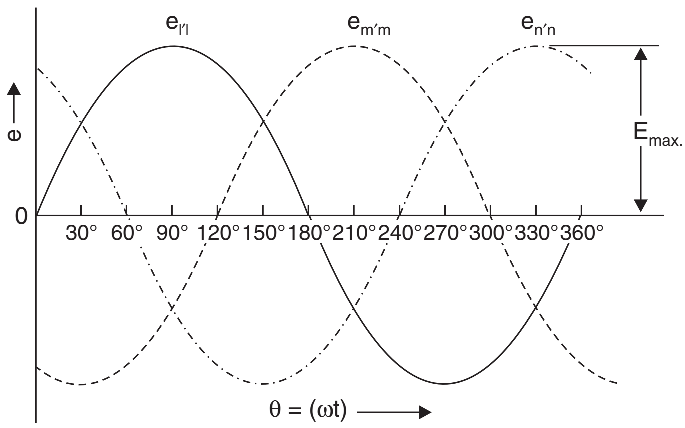


Fig. 2. Voltage waves generated in windings of Fig. 1.

$$e_{l'l} = E_{max.} \sin \omega t$$
$$e_{m'm} = E_{max.} \sin (\omega t — 120°)$$
$$e_{n'n} = E_{max.} \sin (\omega t — 240°)$$

The sum of the above three e.m.fs. is always zero as shown below :

Resultant instantaneous e.m.f.

$$= e_{l'l} + e_{m'm} + e_{n'n}$$
$$= E_{max.} \sin \omega t + E_{max.} \sin (\omega t - 120°) + E_{max.} \sin (\omega t - 240°)$$
$$= E_{max.} [\sin \omega t + (\sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 240° - \cos \omega t \sin 240°)]$$
$$= E_{max.} [\sin \omega t + (- \sin \omega t \cos 60° - \cos \omega t \sin 60° - \sin \omega t \cos 60° + \cos \omega t \sin 60°)]$$
$$= E_{max.} (\sin \omega t - 2 \sin \omega t \cos 60°)$$
$$= E_{max.} (\sin \omega t - \sin \omega t) = 0.$$

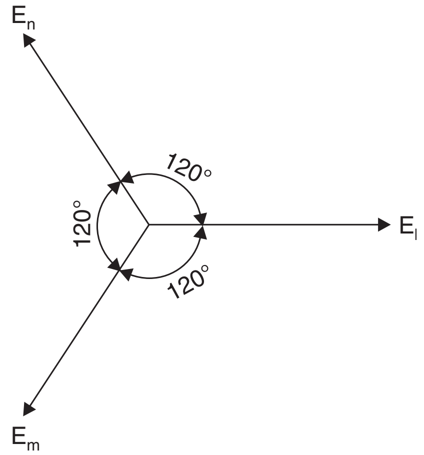


Fig. 3. Phasor diagram of generated voltages.

4. PHASE SEQUENCE AND NUMBERING OF PHASES

- *By phase sequence is meant the order in which the three phases attain their peak or maximum.*

In the generation of three-phase e.m.fs. in Fig. 2 clockwise rotation of the field system in Fig. 1 was assumed. This assumption made the e.m.f. of phase '**m**' lag behind that of '**l**' by 120° and in a similar way, made that of '**n**' lag behind that of '**m**' by 120° (or that of **l** by 240°). Hence, the order in which the e.m.fs. of phases **l, m** and **n** attain their maximum value is **lmn**. It is called the *phase order* or phase sequence **l** → **m** → **n**. If now the rotation of field structure of Fig. 1 is reversed *i.e.,* made counter-clockwise, then the order in which three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become **l** → **n** → **m**. This means that e.m.f. of phase '**n**' would now lag behind that of phase '**l**' by 120° instead of 240° as in the previous case.

The phase sequence of the voltages applied to a load, in general, is determined by the order in which the 3-phase lines are connected. The *phase sequence can be reversed* by *interchanging any pair of lines*. (In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation).

- The three-phases may be *numbered* **l, m, n** or 1, 2, 3 or they may be given three *colours* (as is customary).

The colours used commercially are *red, yellow* (or sometimes white) and *blue*. In this case sequence is RYB.

Evidently in any three-phase system, there are two possible sequences, in which three coils or phase voltages may pass through their maximum value *i.e.,* red → yellow → blue (RYB) or red → blue → yellow (RBY).

By convention :

RYB ...... taken as *positive*.

RBY ...... taken as *negative*.

5. INTER-CONNECTION OF THREE PHASES

Each coil of three phases has two terminals [one 'start' (*S*) and another 'finish' (*F*)] and if individual phase is connected to a separate load circuit, as shown in Fig. 4, we get a non-interlinked 3-phase system. In such a system each circuit will require two conductors, therefore, 6 conductors in all. This makes the whole system *complicated* and *expensive*. Hence, the *three phases are generally interconnected which results in substantial saving of copper*.

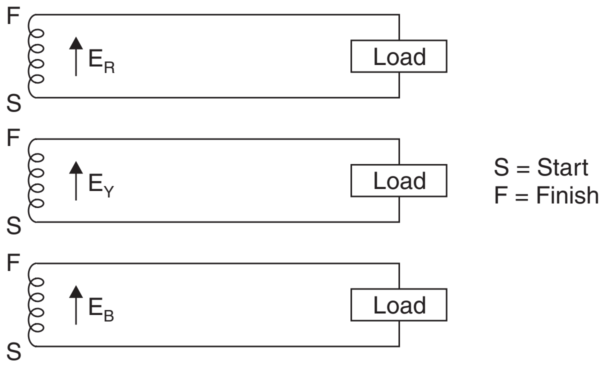


Fig. 4. Non-interlinked 3-phase system.

The general method of inter-connections are :

1. Star or Wye (*Y*) connection.
2. Mesh or delta (Δ) connection.

6. STAR OR WYE (Y) CONNECTION

- In this method of inter-connection the similar ends either the 'start' or 'finish' are joined together at point *N*. This common point *N* [Fig. 5 (*a*)] is called *star point* or *neutral point*. Ordinarily only three wires are carried to the external circuit giving 3-phase, 3-wire start connected system but sometimes a *fourth-wire*, known as *neutral wire* is carried to the neutral point of the external load circuit giving 3-phase, 4 wire star connected system.
- The *voltage between any line and the neutral point* (*i.e.,* voltage across the phase winding) is called the **'phase voltage' (E_{ph})** ; while the *voltage available between any pair of terminals* (or outers) is called the **'line voltage' (E_L).**
- In star connection, as is evident in Fig. 5 (*a*) there are two-phase windings between each pair of terminals, but since their *similar* ends have been joined together, they are in *opposition*. Obviously, the instantaneous value of potential difference between any two terminals is the *arithmetic difference* of the two-phase e.m.fs. concerned. However, the r.m.s. value of this potential difference is given by the *vector difference* of the two-phase e.m.fs.
- Fig. 5 (*b*) shows the vector diagram for phase voltages and currents in a star connection where a *balanced system has been assumed*. [A balanced system is one in which (*i*) the voltages in all phases are equal in magnitude and differ in phase from one another by equal angles, in this case, the angle = 360/3 = 120°, (*ii*) the currents in the three phases are equal in magnitude and also differ in phase from one another by equal angles.

A three-phase balanced load is that in which the loads connected across the three-phases are *identical*]. Thus, we have

$E_R = E_Y = E_B = E_{ph}$ (phase e.m.f.)

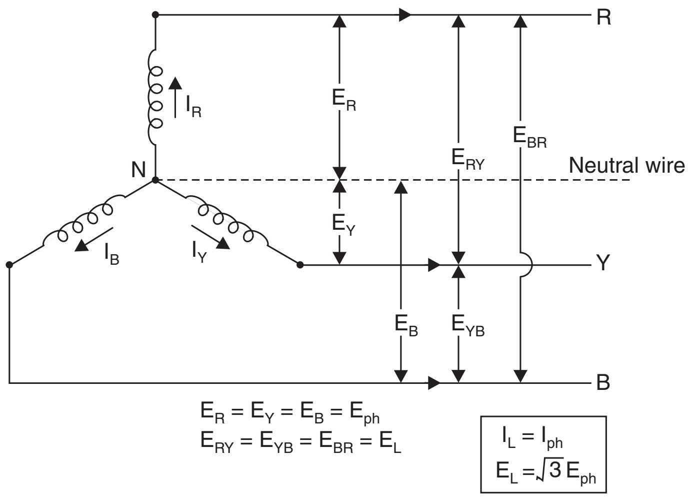


Fig. 5 (*a*)

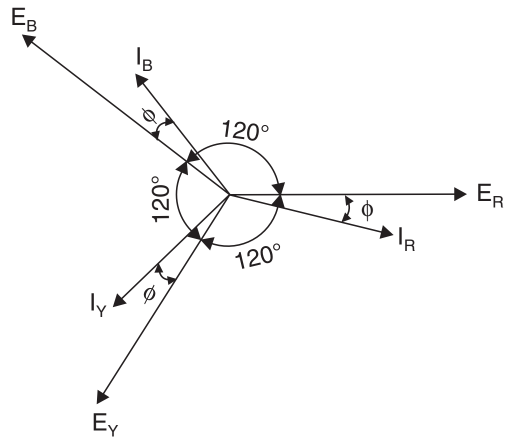


Fig. 5 (*b*) Star-connected three-phase network.

Line voltage, E_{RY} (= E_L) = Vector difference of E_R and E_Y

$= E_R - E_Y$

Line voltage, $E_{YB} = E_Y - E_B$

Line voltage, $E_{BR} = E_B - E_R$.

(*a*) **Relation between Line Voltages and Phase Voltages.** Refer Fig. 6.

The potential difference between outers *R* any *Y* is

$$E_{RY} = E_R - E_Y \quad \text{[vector difference]}$$

or $$E_{RY} = E_R + (-E_Y) \quad \text{[vector sum]}$$

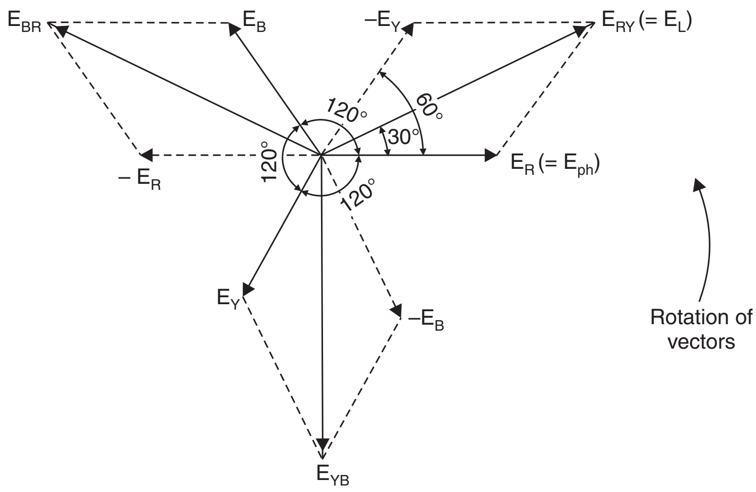


Fig. 6. Vector diagram for star connected network.

Hence, E_{RY} is found by compounding E_R and E_Y reversed and its value is given by the diagonal of the parallelogram (Fig. 6). Obviously the angle between E_R and E_Y reversed is 60° and the value of

$$E_{RY} \text{ (or } E_L) = \sqrt{E_R^{\ 2} + E_Y^{\ 2} + 2E_R E_Y \cos 60°}$$

$$= \sqrt{E_{ph}^{\ 2} + E_{ph}^{\ 2} + 2E_{ph} \times E_{ph} \times \frac{1}{2}} = \sqrt{3}\, E_{ph}$$

Similarly $E_{YB}(= E_L) = E_Y - E_B = \sqrt{3}\, E_{ph}$

and $E_{BR}(= E_L) = E_B - E_R = \sqrt{3}\, E_{ph}$

i.e., $E_{RY} = E_{YB} = E_{BR} = E_L = \sqrt{3}\, E_{ph}$

Hence, $E_L = \sqrt{3}\, E_{ph}$ ...(1)

(*i.e., Line voltage* = $\sqrt{3}$ *phase voltage*).

(*b*) **Relation between Line Currents and Phase Currents.** Since in star-connected system each line conductor is connected to separate phase, so the current flowing through the line and phase are same.

Current in outer (or line) $R = I_R$

Current in outer $Y = I_Y$

Current in outer $B = I_B$

Since $I_R = I_Y = I_B$ = say, I_{ph}—the phase current

∴ Line current, $I_L = I_{ph}$ ...(2)

(*c*) **Power.** If the phase current has a phase difference of ϕ with phase voltage,

Power per phase $= E_{ph} I_{ph} \cos \phi$

Total power (true), $P = 3 \times$ power per phase

$P = 3 \times E_{ph} I_{ph} \cos \phi$ ...(3)

Now, $$E_{ph} = \frac{E_L}{\sqrt{3}} \text{ and } I_{ph} = I_L$$

Hence, in terms of line values, the above expression becomes

$$P = 3 \times \frac{E_L}{\sqrt{3}} I_L \cos \phi$$

or $$P = \sqrt{3} E_L I_L \cos \phi \quad ...(4)$$

(Apparent power = $\sqrt{3} E_L I_L$).

In a *balanced star-connected network* the following points are worthnoting :

(*i*) *Line voltages are* $\sqrt{3}$ *times the phase voltages.*

(*ii*) *Line currents are equal to phase currents.*

(*iii*) Line voltages are 120° apart.

(*iv*) Line voltages are 30° ahead of the respective phase voltages.

(*v*) The angle between line currents and the corresponding line voltages is (30° ± ϕ) + for lagging currents – ve for leading currents.

(*vi*) True power = $\sqrt{3}\ E_L I_L \cos \phi$, where ϕ is the angle between respective phase current and phase voltage, not between the line current and line voltage.

(*vii*) Apparent power = $\sqrt{3} E_L I_L$.

(*viii*) In balance system, the potential of the neutral or star point is zero.

∴ Potential at neutral (or star) point $= E_{NR} + E_{NY} + E_{NB} = 0$.

Example 1. *A balanced star connected load of (8 + j6) Ω/phase is connected to a 3-phase, 230 volts, 50 Hz supply. Find the current, p.f., power, volt ampere and reactive power. Draw the phasor diagram for the above circuit.*

Solution. *Given* : $R = 8\ \Omega$; $X_L = 6\ \Omega$; $E_L = 230$ volts, $f = 50$ Hz.

The circuit is shown in Fig. 7 (*a*).

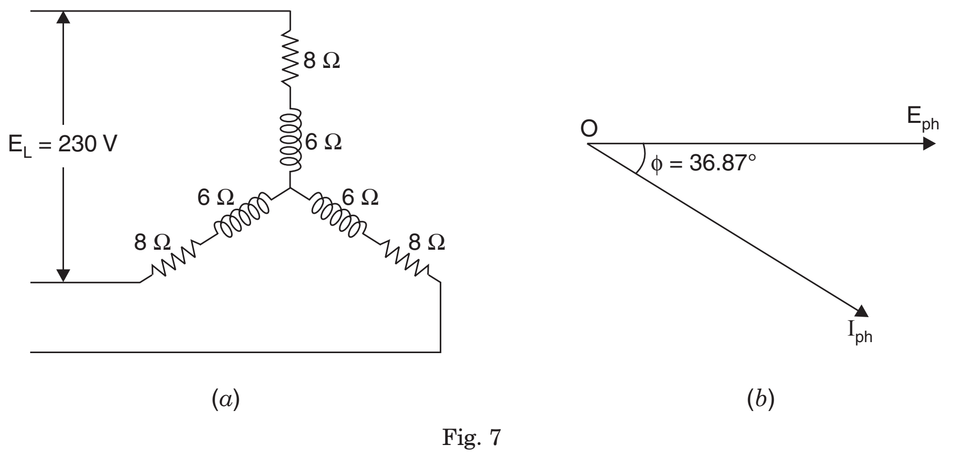


Fig. 7

Phase voltage, $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8 \text{ V}$

Impedance, $Z = \sqrt{R^2 + {X_L}^2} = \sqrt{8^2 + 6^2} = 10 \ \Omega$

Current, $I_{ph} = I_L = \frac{E_{ph}}{Z} = \frac{132.8}{10} =$ **13.28 A. (Ans.)**

Power factor, $\cos \phi = \frac{R}{Z} = \frac{8}{10} =$ **0.8. (Ans.)** $(\because \ \phi = \cos^{-1}(0.8) = 36.87°)$

Power, $P = \sqrt{3} E_L I_L \cos \phi$

$= \sqrt{3} \times 230 \times 13.28 \times 0.8 =$ **4342.3 W. (Ans.)**

Apparent power $= \sqrt{3} E_L I_L = \sqrt{3} \times 230 \times 13.28 =$ **5290.4 VA. (Ans.)**

Reactive power $= \sqrt{3} E_L I_L \sin \phi = \sqrt{3} \times 230 \times 13.28 \times \sin(36.87°)$

$=$ **3174 VAR. (Ans.)**

The phasor diagram is shown in Fig. 7 (*b*).

Example 2. *Three equal impedances each having a resistance of 25 Ω and reactance of 40 Ω are connected in star to a 400 V, 3-phase, 50 Hz system. Calculate :*

(*i*) *The line current* (*ii*) *Power factor, and*

(*iii*) *Power consumed.*

Solution. Resistance per phase, $R_{ph} = 25 \ \Omega$

Reactance per phase, $X_{ph} = 40 \ \Omega$

Line voltage, $E_L = 400 \text{ V}$

Line current, I_L :

Power factor, $\cos \phi$:

Power consumed, P :

Refer to Fig. 8.

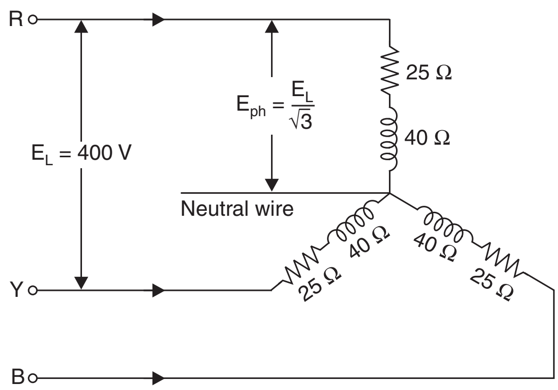


Fig. 8

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2}$

$\therefore$ $Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\ \Omega$

Phase voltage, $E_{ph} = \dfrac{E_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231$ V

Phase current, $I_{ph} = \dfrac{E_{ph}}{Z_{ph}} = \dfrac{231}{47.17} = 4.9$ A (app.)

(*i*) **Line current**, I_L = phase current, I_{ph}

$\therefore$ $\mathbf{I_L = 4.9\ A.}$ **(Ans.)**

(*ii*) **Power factor**, $\cos\phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{95}{47.17} = \mathbf{0.53}$ **(lag). (Ans.)**

(*iii*) **Power consumed,** $P = \sqrt{3}\,E_L I_L \cos\phi = \sqrt{3} \times 400 \times 4.9 \times 0.53$

$= \mathbf{1800\ W}$ **(app.). (Ans.)**

[or $P = 3I_{ph}^2 R_{ph} = 3 \times 4.9^2 \times 25 = 1800$ W].

Example 3. *Three identical coils are connected in star to a 400 V (line voltage), 3-phase A.C. supply and each coil takes 300 W. If the power factor is 0.8 (lagging). Calculate :*

(*i*) *The line current,* (*ii*) *Impedance, and*

(*iii*) *Resistance and inductance of each coil.*

Solution. Line voltage, $E_L = 400$ V

Power taken by each coil, $P_{ph} = 300$ W

Power factor, $\cos\phi = 0.8$ (lagging)

$\mathbf{I_L}$ **;** $\mathbf{Z}$ **;** $\mathbf{R_{ph}}$ **;** $\mathbf{L_{ph}}$ **:**

Phase voltage, $E_{ph} = \dfrac{E_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}$ V

Also $P_{ph} = E_{ph} I_{ph} \cos\phi$

$300 = \dfrac{400}{\sqrt{3}} \times I_{ph} \times 0.8$

$\therefore$ $I_{ph} = \dfrac{300 \times \sqrt{3}}{400 \times 0.8} = 1.62$ A.

(*i*) **Line current,** I_L = phase current, I_{ph}

$\therefore$ $\mathbf{I_L = 1.62\ A.}$ **(Ans.)**

(*ii*) **Coil impedance**, $Z_{ph} = \dfrac{E_{ph}}{I_{ph}} = \dfrac{\frac{400}{\sqrt{3}}}{1.62} = 142.5\ \Omega$

$\therefore$ $\mathbf{Z_{ph} = 142.5\ \Omega.}$ **(Ans.)**

(*iii*) **Resistance and inductance of each coil,**

$R_{ph} = Z_{ph} \cos\phi = 142.5 \times 0.8 = 114\ \Omega$

Coil reactance, $X_{ph} = Z_{ph} \sin\phi = 142.5 \times 0.6 = \mathbf{85.5\ \Omega.}$ **(Ans.)**

But $X_{ph} = 2\pi f L_{ph}$

$\therefore$ $L_{ph} = \frac{X_{ph}}{2\pi f} = \frac{85.5}{2\pi \times 50} = 0.272$ H.

Hence, $\mathbf{R_{ph} = 114\ \Omega.}$ **(Ans.)**

and $\mathbf{L_{ph} = 0.272\ H.}$ **(Ans.)**

Example 4. *In a 3-phase, 3-wire system with star-connected load the impedance of each phase is (3 + j4) Ω. If the line voltage is 230 V, calculate :*

(*i*) *The line current, and* (*ii*) *The power absorbed by each phase.*

Solution. Line voltage, $E_L = 230$ V

Resistance per phase, $R_{ph} = 3\ \Omega$

Reactance per phase, $X_{ph} = 4\ \Omega$

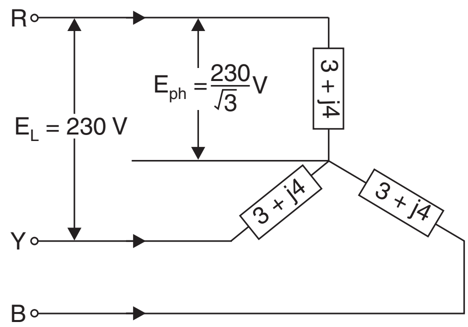


Fig. 9

(*i*) $\mathbf{I_L\ ;\ P_{ph}}$ **:**

Phase voltage, $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}}$ V

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2} = \sqrt{3^2 + 4^2} = 5\ \Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6$

Phase current, $I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{230}{\sqrt{3} \times 5} = 26.56$ A

Line current, $\mathbf{I_L = I_{ph} = 26.56\ A.}$ **(Ans.)**

(*ii*) **Power absorbed by each phase :**

$$\mathbf{P_{ph}} = E_{ph} I_{ph} \cos\phi = \frac{230}{\sqrt{3}} \times 26.56 \times 0.6 = \mathbf{2116\ W.}\quad \textbf{(Ans.)}$$

$$[\text{or } P_{ph} = I_{ph}^2 R_{ph} = 26.56^2 \times 3 = 2116 \text{ W}]$$

Solution by Symbolic Notation. In Fig. 10 E_R, E_Y and E_B are the phase voltages whereas I_R, I_Y and I_B are phase currents.

Taking E_R *as the reference vector,* we get

$$E_R = \frac{230}{\sqrt{3}} \angle 0° = 133 \angle 0° = 133 + j0 \text{ volt}$$

$$E_Y = 133 \angle -120° = 133\,(-0.5 - j0.866) = (-66.5 - j115) \text{ volts}$$

$$E_B = 133 \angle 120° = 133\,(-0.5 + j0.866) = (-66.5 + j115) \text{ volts}$$

$$Z = 3 + j4 = 5 \angle 53° 8'$$

$$I_R = \frac{E_R}{Z} = \frac{133 \angle 0°}{5 \angle 53° 8'} = 26.6 \angle -53° 8'$$

This current lags behind the reference voltage (E_R) by 53° 8′ (Fig. 10).

$$I_Y = \frac{E_Y}{Z} = \frac{133 \angle -120°}{5 \angle 53° 8'} = 26.6 \angle -173° 8'$$

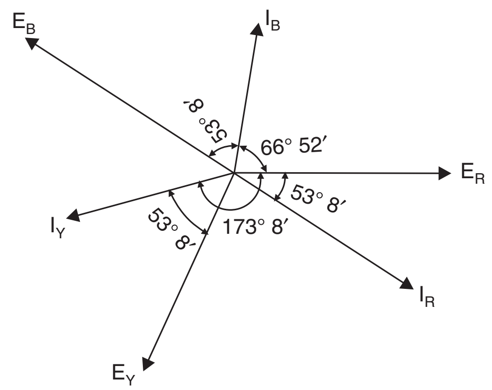


Fig. 10

It lags the reference vector *i.e.*, E_R by 173° 8′ which amounts to *lagging* behind its phase voltage E_Y by 53° 8′.

$$I_B = \frac{E_B}{Z} = \frac{133 \angle 120°}{5 \angle 53° 8'} = 26.6 \angle 66° 52'$$

This current leads E_R by 66° 52′ which is the same as *lagging* behind its phase voltage by 53° 8′.

Let us consider *R*-phase for calculation of power

$$E_R = (133 + j0)\ ;\ I_R = 26.6\,(0.6 - j0.8) = (15.96 - j21.28)$$

Using method of conjugates, we get

$$P_{VA} = (133 - j0)\,(15.96 - j21.28) = 2116 - j2830$$

∴ Real power absorbed/phase = 2116 W ...(As before)

Example 5. *A star-connected, 6000 V, 3-phase alternator is supplying 4000 kW at a power factor of 0.8. Calculate the active and reactive components of the current in each phase.*

Solution. Line voltage, $E_L = 6000$ V

Power supplied, $P = 4000$ kW

Power factor, $\cos \phi = 0.8$

Active and reactive components of current :

We know that, $P = \sqrt{3}\ E_L I_L \cos\phi$

$$4000 \times 1000 = \sqrt{3} \times 6000 \times I_L \times 0.8$$

i.e., $$I_L = \frac{4000 \times 1000}{\sqrt{3} \times 6000 \times 0.8} = 481 \text{ A}$$

$\therefore$ $I_{ph} = I_L = 481$ A

Active component $= I_{ph} \cos\phi = 481 \times 0.8 =$ **384.8 A. (Ans.)**

Reactive component $= I_{ph} \sin\phi = 481 \times 0.6 =$ **288.6 A. (Ans.)**

Example 6. *A balanced 3-phase star connected load of 100 kW takes a leading current of 80 A, when connected across a 3-phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.*

Solution. *Given* : $P = 100$ kW ; I_{ph} $(= I_L) = 80$ A ; $E_L = 1100$ V ; $f = 50$ Hz

Circuit constants of the load per phase, R, C :

As the 3-ϕ load is balanced and star connected, line or phase current,

$$I_L = (I_{ph}) = \frac{P}{\sqrt{3}\ E_L \cos\phi} = \frac{100 \times 10^3}{\sqrt{3} \times 1100 \times \cos\phi}$$

or $$\cos\phi = \frac{100 \times 10^3}{\sqrt{3} \times 1100 \times 80} = 0.656$$

Load impedance, $$Z = \frac{E_{ph}}{I_{ph}} = \frac{(1100/\sqrt{3})}{80} = 7.94\ \Omega$$

$\therefore$ $\mathbf{R} = Z \cos\phi = 7.94 \times 0.656 =$ **5.2 Ω. (Ans.)**

Now, $X_C = \dfrac{1}{2\pi fC}$ as the current given is *leading* current.

$\therefore$ $$C = \frac{1}{2\pi f X C}$$

But $X_C = Z \sin\phi = 7.94 \times 0.755 = 5.99\ \Omega$

$\therefore$ $$\mathbf{C} = \frac{1}{2\pi \times 50 \times 5.99} \text{ F}$$

$$= \frac{10^6}{2\pi \times 50 \times 5.99}\ \mu\text{F} = \mathbf{531.4\ \mu F.}\ \ \textbf{(Ans.)}$$

Example 7. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 8, 10 and 20 Ω respectively in three phases, calculate :*

(*i*) *The current flowing in each phase,* (*ii*) *The neutral current, and*

(*iii*) *The total power absorbed.*

Solution. Refer to Figs. 11 (*a*) and (*b*).

Phase voltage, $E_{ph} = 230$ V

(*i*) Current in 8 Ω resistor $= \dfrac{230}{8} =$ **28.75 A. (Ans.)**

Current in 10 Ω resistor $= \dfrac{230}{10} =$ **23 A. (Ans.)**

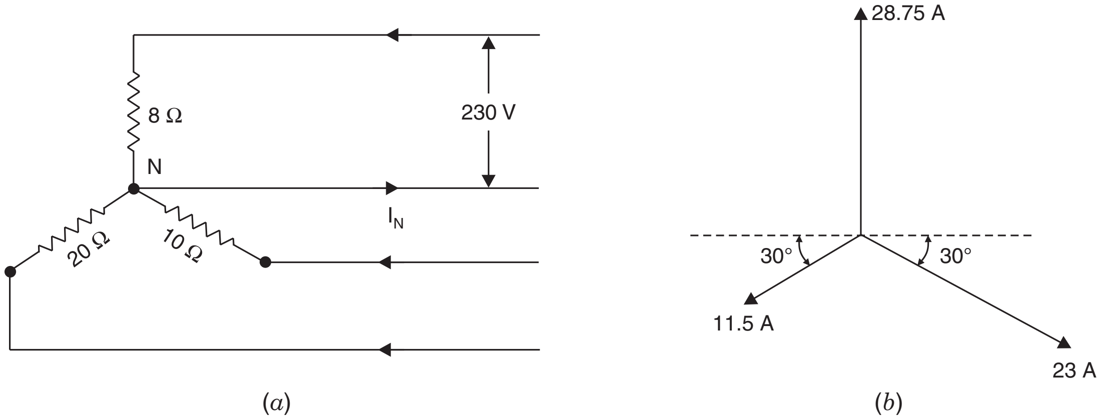


Fig. 11

Current in 12 Ω resistor $= \frac{230}{20}$ = **11.5 A. (Ans.)**

(*ii*) The above currents are mutually displaced by 120°. The neutral current I_N is the vector sum of these three currents.

I_N can be found by splitting up these three-phase currents into their *X*-components and *Y*-components and then by combining them together.

Σ*X*-components = 23 cos 30° – 11.5 cos 30° = 11.5 cos 30° = 9.96 A

Σ*Y*-components = 28.75 – 23 sin 30° – 11.5 sin 30° = 28.75 – 34.5 sin 30° = 11.5 A

∴ **Neutral current,** $I_N = \sqrt{(9.96)^2 + (11.5)^2}$ = **15.21 A. (Ans.)**

(*iii*) **Total power absorbed,**

$$P = 230\ (28.75 + 23 + 11.5) = \mathbf{14547.5\ W.}\quad \textbf{(Ans.)}$$

7. DELTA (Δ) OR MESH CONNECTION

In a delta or mesh connection the *dissimilar* ends of the three-phase windings are joined together *i.e.,* the '*starting*' end of one phase is joined to the '*finishing*' end of the other phase and so on as shown in Fig. 12. In other words, the three windings are joined in series to form a closed mesh. Three leads are taken out from the junctions as shown and outward directions are taken as positive.

(*a*) **Relation between line voltages and phase voltages :**

Since in delta or mesh connected system, only one phase is included between any pair of line outers, therefore, potential difference between the line outers, called the *line voltage*, is equal to phase voltage.

i.e., *Line voltage*, E_L = *phase voltage*, E_{ph}.

(*b*) **Relation between line currents and phase currents :**

From Fig. 12 it is obvious that line current is the vector difference of phase currents of two phases concerned.

Thus, line current, $I_R = I_{YR} - I_{RB}$ (Vector difference)

$= I_{YR} + (-I_{RB})$ (Vector sum)

Similarly, $I_Y = I_{BY} - I_{YR}$ and $I_B = I_{RB} - I_{BY}$

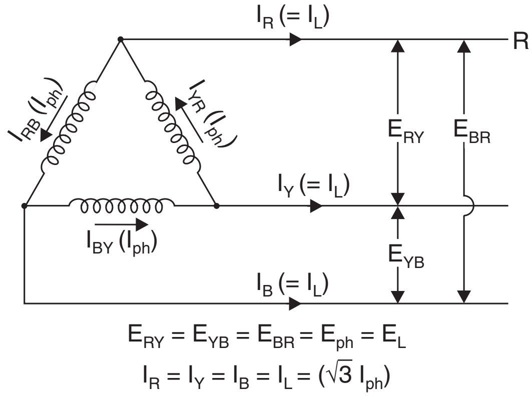


Fig. 12. Delta or mesh connected diagram.

Refer to Fig. 13. Since phase angle between phase current I_{YR} and $-I_{RB}$ is 60°,

$$\therefore \qquad I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_{YR}I_{RB}\cos 60°}$$

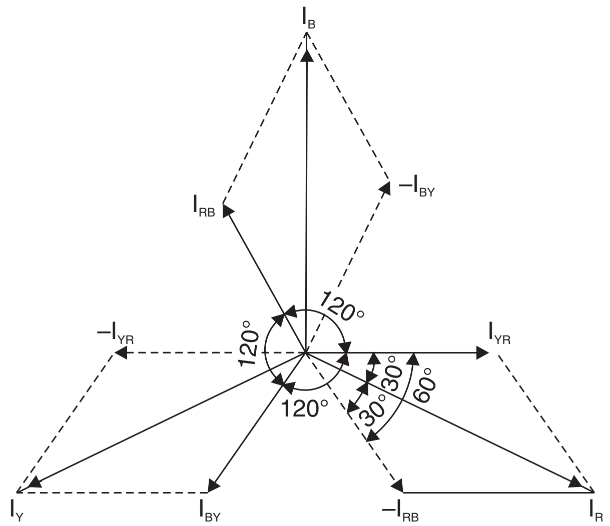


Fig. 13. Vector diagram for delta connected network.

Assuming the delta connected system or network be balanced, the phase current in each winding is equal and let each be equal to I_{ph} (*i.e.*, $I_{YR} = I_{BY} = I_{RB} = I_{ph}$)

$$\therefore \qquad I_R (= I_L) = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60°} = \sqrt{2I_{ph}^2 + 2I_{ph}^2 \times \frac{1}{2}} = \sqrt{3}\, I_{ph}$$

Similarly, $\quad I_Y = I_B = \sqrt{3}\, I_{ph}$

Hence, $\quad I_L = \sqrt{3}\ I_{ph} \qquad$ ...(5)

(*i.e.*, *line current* = $\sqrt{3}$ *phase current*)

(*c*) **Power :**

Power/phase $\quad = E_{ph}I_{ph}\cos\phi$

Total power (true) $\quad P = 3E_{ph}I_{ph}\cos\phi$

But $$E_{ph} = E_L \text{ and } I_{ph} = \frac{I_L}{\sqrt{3}}$$

Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times E_L \times \frac{E_L}{\sqrt{3}} \cos \phi$$

or $$P = \sqrt{3}\ E_L I_L \cos \phi \quad \text{...(6)}$$

where ϕ = the phase power factor angle.

(*Apparent power* = $\sqrt{3} E_L I_L$)

In case of delta or mesh connected system the *following points are worthnoting* :

(*i*) *Line voltages are equal to phase voltages.*

(*ii*) Line currents are $\sqrt{3}$ times phase currents.

(*iii*) Line currents are 120° apart.

(*iv*) Line currents are 30° behind their respective phase currents.

(*v*) The angle between line currents and corresponding line voltages is $(30° \pm \phi)$ as in the star system.

(*vi*) True power = $\sqrt{3}\ E_L I_L \cos \phi$, where ϕ is the phase angle between respective phase current and phase voltage.

(*vii*) Apparent power = $\sqrt{3}\ E_L I_L$.

(*viii*) In balanced system, the resultant e.m.f. in the closed circuit will be zero.

i.e., $E_{RY} + E_{YB} + E_{BR} = 0$.

Hence, there will no circulating current in the mesh if no-load is connected to the lines.

8. COMPARISON BETWEEN STAR AND DELTA SYSTEMS

The comparison between star and delta connected systems is given below :

Star connected system	*Delta connected system*
1. *Similar ends* are joined together.	1. *Dissimilar ends* are joined.
2. Phase voltage = $\frac{1}{\sqrt{3}}$ line voltage $\left(i.e., E_{ph} = \frac{E_L}{\sqrt{3}}\right)$.	2. Phase voltage = line voltage (*i.e.*, $E_{ph} = E_L$).
3. Phase current = line current (*i.e.*, $I_{ph} = I_L$).	3. Phase current = $\frac{1}{\sqrt{3}}$ × line current $\left(i.e., I_{ph} = \frac{I_L}{\sqrt{3}}\right)$.
4. Possible to carry neutral to the load.	4. Neutral wire not available.
5. Provides 3-phase 4-wire arrangement.	5. Provides 3-phase 3-wire arrangement.
6. Can be used for lighting as well as power load.	6. Can be used for power loads only.
7. Neutral wire of a star connected alternator can be connected to earth, so relays and protective devices can be provided in the star connected alternators for safety.	7. Not possible. Delta connected system is mostly used in transformer for running of small low voltage 3-phase motors and *best suited for rotary converters*.

Example 8. *Three identical coils connected in delta across 400 V, 50 Hz, 3-phase supply take a line current of 15 A at a power factor 0.8 lagging. Calculate :*

(*i*) *The phase current, and*

(*ii*) *The impedance, resistance and inductance of each winding.*

Solution. Line voltage, $E_L = 400$ V

Line current, $I_L = 15$ A

Power factor, $\cos\phi = 0.8$ lagging

I_{ph} ; Z_{ph} ; R_{ph} ; L :

Phase voltage, $E_{ph} = E_L = 400$ V

(*i*) **Phase current,** $I_{ph} = \dfrac{I_L}{\sqrt{3}} = \dfrac{15}{\sqrt{3}} =$ **8.66 A. (Ans.)**

(*ii*) **Impedance of each phase,** $\mathbf{Z_{ph}} = \dfrac{E_{ph}}{I_{ph}} = \dfrac{400}{8.66} =$ **46.19 Ω. (Ans.)**

Resistance of each phase, $\mathbf{R_{ph}} = Z_{ph}\cos\phi = 46.19 \times 0.8 =$ **36.95 Ω. (Ans.)**

Reactance of each phase, $X_{ph} = Z_{ph}\sin\phi = 46.19\sqrt{(1-\cos^2\phi)}$

$$= 46.19\sqrt{1-(0.8)^2} = 27.71\ \Omega$$

∴ **Inductance,** $\mathbf{L} = \dfrac{X_{ph}}{2\pi f} = \dfrac{27.71}{2\pi \times 50} =$ **0.088 H. (Ans.)**

Example 9. *A 220 V, 3-phase voltage is applied to a balanced delta-connected 3-phase load of phase impedance (6 + j8).*

(*i*) *Find the phasor current in each line.*

(*ii*) *What is the power consumed per phase ?*

(*iii*) *What is the phasor sum of the three line currents ? What does it have this value ?*

Solution. Refer to Fig. 14.

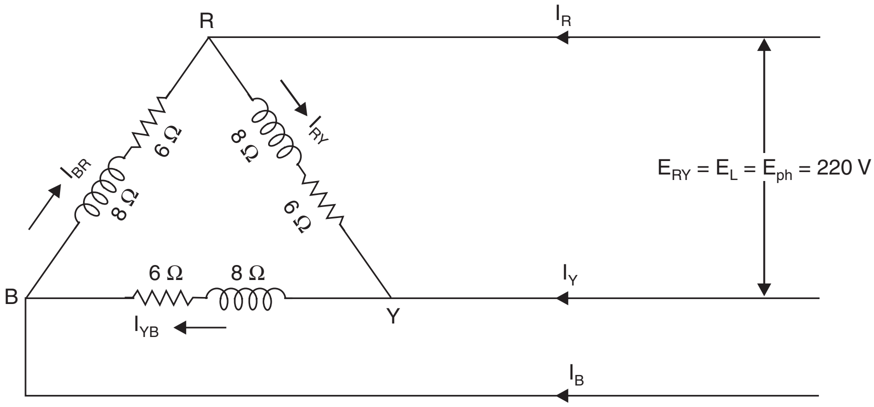


Fig. 14

Resistance per phase, $R_{ph} = 6\ \Omega$

Reactance per phase, $X_{ph} = 8\ \Omega$

$E_L = E_{ph} = 220$ V

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}{}^2 + X_{ph}{}^2} = \sqrt{6^2 + 8^2} = 10\ \Omega$

(*i*) Phase current, $I_{ph} = \dfrac{E_{ph}}{Z_{ph}} = \dfrac{220}{10} = 22$ A

∴ **Line current,** $I_L = \sqrt{3} \times 22 =$ **38.1 A. (Ans.)**

(*ii*) **Power consumed per phase,**

$P_{ph} = I_{ph}{}^2 \times R_{ph} = 22^2 \times 6 =$ **2904 W. (Ans.)**

(*iii*) Phasor sum would be zero because the three currents are equal in magnitudes and have a mutual phase difference of 120°.

Solution by Symbolic Notation. Let E_{RY} is taken as a reference vector (Fig. 15).

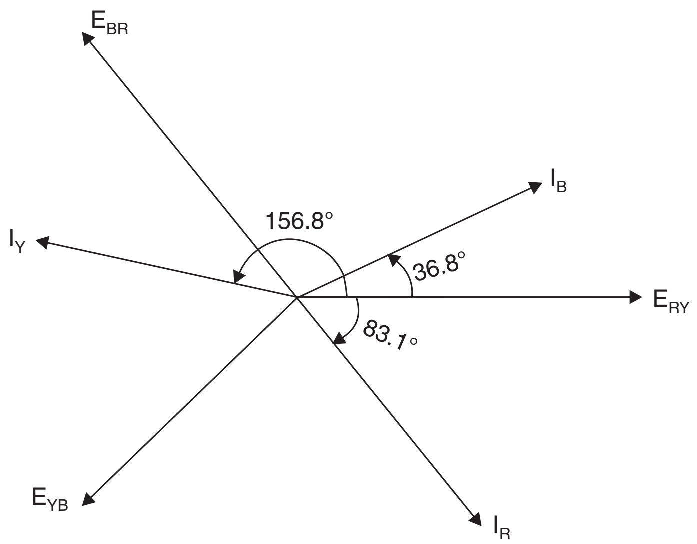


Fig. 15

$E_{RY} = 220\angle 0°$, $E_{YB} = 220\angle -120°$

$E_{BR} = 220\angle 120°$, $Z = 6 + j8 = 10\angle 53°\ 8'$,

$$I_{RY} = \frac{E_{RY}}{Z} = \frac{220\ \angle 0°}{10\ \angle 53°8'} = 22\angle -53°8' = (13.22 - j17.6)\ \text{A}$$

$$I_{YB} = \frac{E_{YB}}{Z} = \frac{220\ \angle -120°}{10\ \angle 53°8'} = 22\angle -173°\ 8' = (-21.84 - j2.63)$$

$$I_{BR} = \frac{E_{BR}}{Z} = \frac{220\ \angle 120°}{10\ \angle 53°8'} = 22\angle\ 66°\ 52' = (8.64 + j20.23)$$

(*i*) **Current in each line :**

$I_R = I_{RY} - I_{BR} = (13.22 - j17.6) - (8.64 + j20.23) = 4.58 - j37.83 =$ **35.1∠83.1°. (Ans.)**

$I_Y = I_{YB} - I_{RY} = (-21.84 - j2.63) - (13.22 - j17.6)$

$= -21.84 - j2.63 - 13.22 + j17.60 = -35.06 + j14.97 =$ **38.12∠156.8°. (Ans.)**

$I_B = I_{BR} - I_{YB} = (8.64 + j20.23) - (-21.84 + j2.63) = 8.64 + j20.23 + 21.84 + j2.63$

$= 30.48 + j22.86 =$ **36.8∠36.8°. (Ans.)**

(*ii*) **Power consumed per phase :**

Using conjugate of voltage, we get for R-phase

$$P_{VA} = E_{RY} \cdot I_{RY} = (220 - j0)(13.22 - j17.6) = (2908.4 - j3872) \text{ volt ampere}$$

True power per phase = **2.908 kW. (Ans.)**

(*iii*) **Phase sum of the three line currents**

$$= I_R + I_Y + I_B$$
$$= (4.58 - j37.83) + (-35.06 + j14.96) + (30.48 + j22.86) = 0$$

Hence, **the phasor sum of three line currents drawn by a 'balanced load' is zero. (Ans.)**

Example 10. *A delta-connected balanced 3-phase load is supplied from a 3-phase, 400 V supply. The line current is 30 A and the power taken by the load is 12 kW. Find :*

(*i*) *Impedance in each branch ; and*

(*ii*) *The line current, power factor and power consumed if the same load is connected is star.*

Solution. Delta-connection :

$$E_{ph} = E_L = 400 \text{ V}$$
$$I_L = 30 \text{ A}$$
$$\therefore \quad I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 17.32 \text{ A.}$$

(*i*) **Impedance per phase**

$$\mathbf{Z_{ph}} = \frac{E_{ph}}{I_{ph}} = \frac{400}{17.32} = \mathbf{23.09\ \Omega.} \quad \textbf{(Ans.)}$$

Now $$P = \sqrt{3} E_L I_L \cos \phi$$
$$12000 = \sqrt{3} \times 400 \times 30 \times \cos \phi.$$

or $$\cos \phi \text{ (power factor)} = \frac{12000}{\sqrt{3} \times 400 \times 30} = 0.577$$

(*ii*) **Star-connection**

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\mathbf{I_L} = I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{231}{23.09} = \mathbf{10\ A.} \quad \textbf{(Ans.)}$$

Power factor, $\cos \phi = 0.577$ (since impedance is same)

Power consumed = $\sqrt{3}\ E_L I_L \cos \phi = \sqrt{3} \times 400 \times 10 \times 0.577 =$ **3997.6 W. (Ans.)**

Example 11. *Three 50 Ω non-inductive resistances are connected in* (*i*) *star,* (*ii*) *delta across a 400 V, 50 Hz., 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting opened, what would be the value of the total power taken from the mains in each of the two cases.*

Solution. Star connection :

Phase voltage, $$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Phase current, $$I_{ph} = \frac{E_{ph}}{R_{ph}} = \frac{231}{50} = 4.62 \text{ A}$$

Power consumed, $P = 3I_{ph}^2 R_{ph} = 3 \times 4.62^2 \times 50 =$ **3200 W. (Ans.)**

[or $P = \sqrt{3} E_L I_L \cos\phi = \sqrt{3} \times 400 \times 4.62 \times 1 = 3200$ W]

Delta connection :

Phase voltage, $E_{ph} = E_L = 400$ V

Phase current, $I_{ph} = \dfrac{E_{ph}}{R_{ph}} = \dfrac{400}{50} = 8$ A

Power consumed, $P = 3I_{ph}^2 R_{ph} = 3 \times 8^2 \times 50 =$ **9600 W. (Ans.)**

When one of the resistances is disconnected :

(*i*) **Star connection.** Refer to Fig. 16.

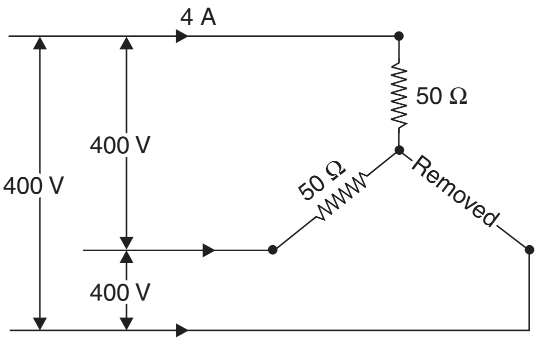


Fig. 16

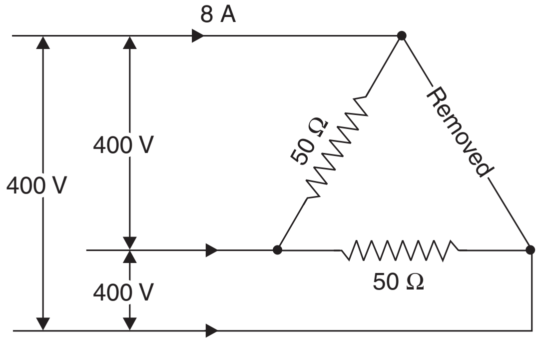


Fig. 17

When one of the resistances is disconnected, the circuit is no longer 3-phase but converted into single-phase circuit, having two resistances each of 50 ohm connected in series across supply of 400 V.

Hence line current, $I_L = \dfrac{E_L}{2R_{ph}} = \dfrac{400}{2 \times 50} = 4$ A

Power consumed, $P = 4^2 (50 + 50) =$ **1600 W. (Ans.)**

[or $P = VI \cos\phi = 400 \times 4 \times 1 = 1600$ W].

(*ii*) **Delta connection.** Refer Fig. 17.

Potential difference across each resistance, $E_L = 400$ V

Current in each resistance $= \dfrac{400}{50} = 8$ A

Power consumed in both resistances $= 2 \times 8^2 \times 50 =$ **6400 W. (Ans.)**

[or $P = 2 \times E_{ph} I_{ph} \cos\phi = 2 \times 400 \times 8 \times 1 = 6400$ W].

Example 12. *The secondary of a 3-phase star-connected transformer, which has a phase voltage of 230 V feeds a 3-phase delta connected load ; each phase of which has a resistance of 30 Ω and an inductive reactance of 40 Ω. Draw the circuit diagram of the system and calculate :*

(*i*) *The voltage across each phase of load,*

(*ii*) *The current in each phase of load,*

(*iii*) *The current in the transformer secondary windings, and*

(*iv*) *The total power taken from the supply and its power factor.*

Solution. Refer to Fig. 18.

Resistance per phase, $R_{ph} = 20\ \Omega$

Reactance per phase, $X_{ph} = 40\ \Omega$

Phase voltage across transformer secondary,

$$E_{ph} = 230 \text{ V}$$

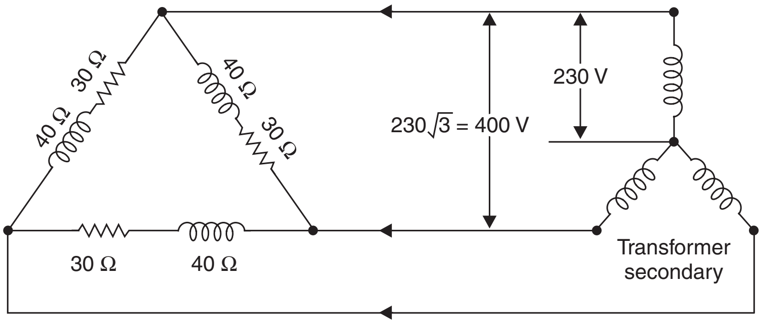


Fig. 18

Line voltage across delta connected load = line voltage across transformer secondary

$$= \sqrt{3} \times 230 = 400 \text{ V (app.)}$$

(*i*) **Voltage across each phase of the load,**

$$E_{ph} = E_L = \mathbf{400\ V.} \quad \textbf{(Ans.)}$$

(*ii*) **Current in each phase of the load,**

$$I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{400}{\sqrt{R_{ph}^{\ 2} + X_{ph}^{\ 2}}} = \frac{400}{\sqrt{30^2 + 40^2}} = \frac{400}{50} = \mathbf{8\ A.} \quad \textbf{(Ans.)}$$

(*iii*) **Current in the transformer secondary**

$$= \text{line current of load} = \sqrt{3} \times I_{ph} = \sqrt{3} \times 8 = \mathbf{13.86\ A.} \quad \textbf{(Ans.)}$$

(*iv*) **Power factor,** $\cos \phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{30}{50} = \mathbf{0.6.} \quad \textbf{(Ans.)}$

Total power consumed, $P = \sqrt{3}\, E_L I_L \cos \phi = \sqrt{3} \times 400 \times 13.86 \times 0.6 = \mathbf{5761.5\ W.} \quad \textbf{(Ans.)}$

9. MEASUREMENT OF POWER IN 3-PHASE CIRCUIT

- The power in 3-phase load can be measured by using the following methods :

1. Three-wattmeters method.
2. Two-wattmeters method.
3. One-wattmeter method.

- A wattmeter consists of two coils : Refer Fig. 19.

1. *Current coil*—possesses a *low resistance*.

2. *Pressure or potential coil*—possesses a *high resistance*.

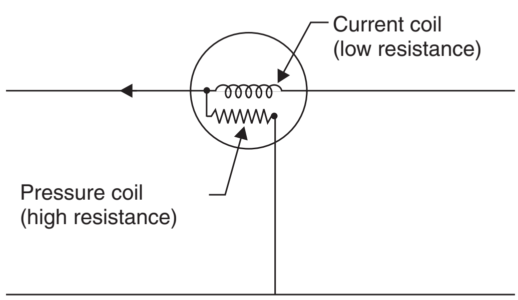


Fig. 19. Connections of a wattmeter.

The '*current coil*' is connected in *series with the line carrying the current* and the '*pressure coil*' *is* *connected across the two points whose potential difference is to be measured*. A wattmeter shows a reading which is proportional to the *product* of the *current* through its current coil, the *potential difference* across its pressure coil and *cosine of the angle* between this voltage and current.

9.1. Three-wattmeters method

Figs. 20 and 21 show the connection diagram for star-connected and delta-connected loads respectively. As indicated in the figures three wattmeters are connected in each of the three phases of the load whether star or delta connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase-voltage of the phase. Hence, *each wattmeter measures the power in a single phase. The total power in the load is given by the algebraic sum of the readings of the three wattmeters*.

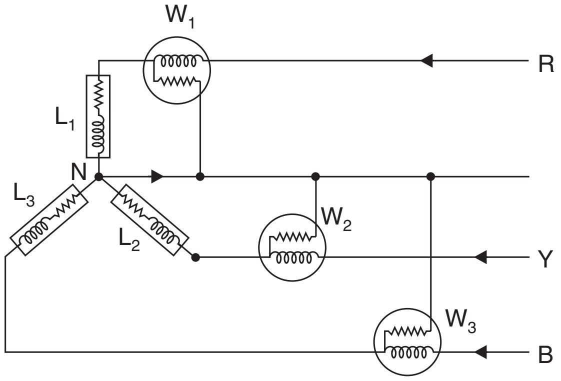


Fig. 20. Star-connected load.

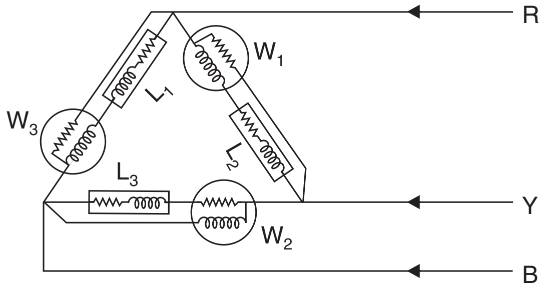


Fig. 21. Delta-connected load.

While using this method following *difficulty* is met with :

- In case of star-connected load it is not always possible to get at neutral point which is required for connections (Fig. 20).
- In case of delta-connected load, under ordinary conditions it is not generally feasible to break into the phases of the load.

To measure power it is not necessary to use three wattmeters, two wattmeters can be used for the purpose as explained in the Article 9.2.

9.2. Two-wattmeters method

Balanced or unbalanced load, Figs. 22 and 23 show connection diagrams for star-connected and delta-connected loads respectively. In this method the current coils of the two wattmeters are inserted in *any two* lines and the pressure (or potential) coil of each joined to the *third line*.

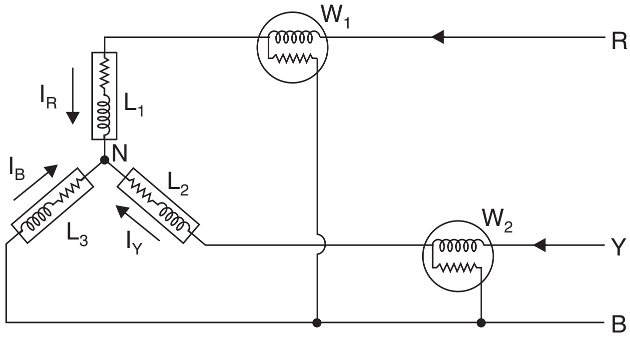


Fig. 22. Star-connected load.

It can be proved that the sum of the instantaneous powers indicated by W_1 and W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 and L_3. Let us consider a star-connected load (although it can be equally applied to a delta-connected load which can always be replaced by an equivalent star-connected load).

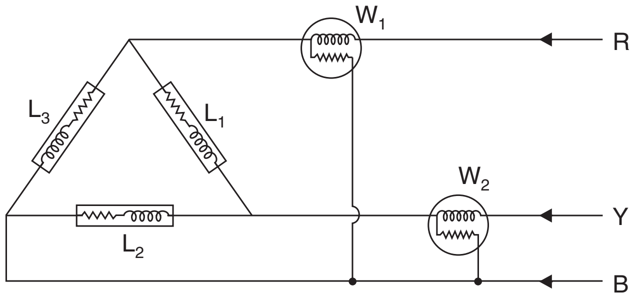


Fig. 23. Delta-connected load.

Keeping in mind that it is important to take *the direction of the voltage through the circuit as the same as that taken for the current when establishing the readings of the two wattmeters*.

Instantaneous current through $W_1 = i_R$

Instantaneous potential difference across $W_1 = e_{RB} = e_R - e_B$

Instantaneous power read by $W_1 = i_R(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous potential difference across $W_2 = e_{YB} = e_Y - e_B$

Instantaneous power read by $W_2 = i_Y(e_Y - e_B)$

$\therefore \quad W_1 + W_2 = i_R(e_R - e_B) + i_Y(e_Y - e_B) = i_R e_R + i_Y e_Y - e_B(i_R + i_Y)$

Now, according to Kirchhoff's point law

$$i_R + i_Y + i_B = 0$$

$\therefore \quad i_R + i_Y = - i_B$

$\therefore \quad W_1 + W_2 = i_R e_R + i_Y e_Y + i_B e_B = p_1 + p_2 + p_3$

where p_1 = power absorbed by load L_1, p_2 = power absorbed by load L_2, and

p_3 = power absorbed by load L_3.

$\therefore \quad W_1 + W_2$ = total power absorbed.

This proof is true whether the load is *balanced* or *unbalanced*.

In case the load is star-connected, then it should have no neutral connection (*i.e.,* 3-phase, 3-wire connected) and if it has a neutral connection (*i.e.,* 3-phase, 4-wire connected) that it should be exactly balanced so that in each case there is no neutral current i_N otherwise Kirchhoff's point law will give $i_R + i_Y + i_B + i_N = 0$.

In the above derivation we have considered the *instantaneous* readings. In fact the moving system wattmeter, due to its inertia, cannot quickly follow the variations taking place in cycle, hence it indicates the *average power.*

$\therefore$ Total power $= W_1 + W_2$

$$= \frac{1}{T}\int_0^T i_R e_{RB}\, dt + \frac{1}{T}\int_0^T i_Y e_{YB}\, dt. \qquad ...(7)$$

Two-Wattmeter Method—Balanced load. The total power consumed by a *balanced load* can be found by using two wattmeters (Figs. 22 and 23). When load is assumed inductive in Fig. 20, the vector diagram for such a balanced star-connected load is shown in Fig. 24.

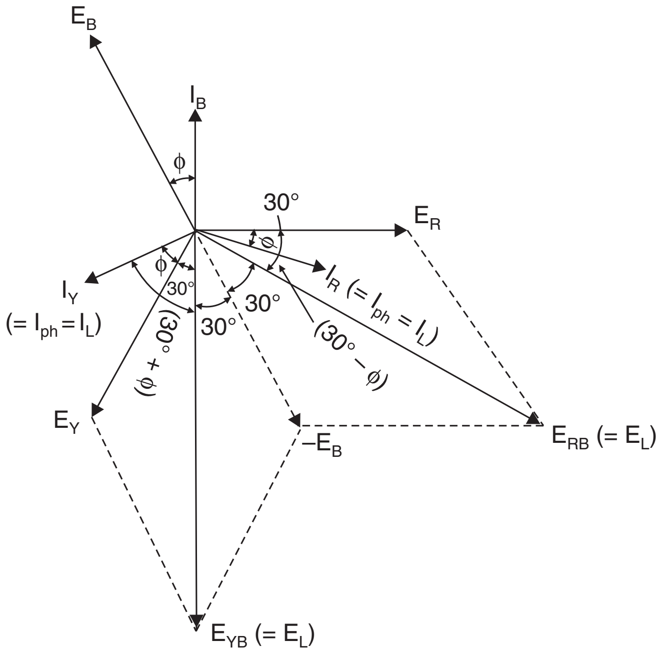


Fig. 24. Vector diagram—two wattmeters method.

Let us consider the problem in terms of r.m.s. values (instead of instantaneous values).

Let E_R, E_Y, E_B = r.m.s. values of the three phase voltages,

and I_R, I_Y, I_B = r.m.s. values of the currents.

Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by ϕ.

Refer Fig. 24.

Current through wattmeter $W_1 = I_R$

Potential difference across pressure coil of wattmeter

$$W_1 = E_{RB} = E_R - E_B \quad \text{(Vectorially)}$$

The value of E_{RB} is found by compounding E_R and E_B reversed as shown in Fig. 24. It may be observed that phase difference between E_{RB} and I_R = (30° − ϕ).

∴ Reading of wattmeter $W_1 = E_{RB} I_R \cos (30° - \phi)$ ...(8)

Similarly, current through wattmeter $W_2 = I_Y$

Potential difference across pressure coil of wattmeter

$$W_2 = E_{YB} = E_Y - E_B \quad \text{(Vectorially)}$$

The value of E_{YB} is found by compounding E_Y and E_B reversed as shown in Fig. 24. The phase difference between E_{YB} and I_Y = (30° + ϕ).

∴ Reading of wattmeter $W_2 = E_{YB} I_Y \cos (30° + \phi)$ ...(9)

Since the load is balanced, $E_{RB} = E_{YB} = E_L$ (Line voltage)

and $I_R = I_Y = I_L$ (Line current)

$\therefore$ $W_1 = E_L I_L \cos(30° - \phi)$

and $W_2 = E_L I_L \cos(30° + \phi)$

$\therefore$ Total power, $P = W_1 + W_2$

$= E_L I_L \cos(30° - \phi) + E_L I_L \cos(30° + \phi)$

$= E_L I_L [\cos(30° - \phi) + \cos(30° + \phi)]$

$= E_L I_L [\cos 30° \cos \phi + \sin 30° \sin \phi + \cos 30° \cos \phi - \sin 30° \sin \phi]$

$= E_L I_L (2 \cos 30° \cos \phi) = E_L I_L \times 2 \times \dfrac{\sqrt{3}}{2} \cos \phi = \sqrt{3} E_L I_L \cos \phi$

i.e., $P = \sqrt{3} E_L I_L \cos \phi$

Hence, the *sum of the readings of the two wattmeters give the total power consumption in the 3-phase load*.

It is worth noting that in the above case the phase sequence of *RYB* has been assumed, the readings of the two wattmeters will change if the phase sequence is reversed.

Variations in wattmeter readings. As shown above that for a *lagging power factor*

$W_1 = E_L I_L \cos(30° - \phi)$

and $W_2 = E_L I_L \cos(30° + \phi)$

From above it is evident that individual readings of the wattmeters *not only depend on the load but also upon its power factor*. Let us take up the following cases :

(*i*) When $\phi = 0$

i.e., power factor is *unity* (*i.e.,* load is *resistive*)

Then $W_1 = W_2 = E_L I_L \cos 30°$

The reading of each wattmeter will be equal and opposite (*i.e.,* up-scale reading).

(*ii*) When $\phi = 60°$

i.e., power factor = 0.5 (lagging)

Then $W_2 = E_L I_L \cos(30° + 60°) = 0$

Hence, the power is measured by W_1 above.

(*iii*) When $90° > \phi > 60°$

i.e., $0.5 > \text{p.f.} > 0$

Then W_1 is still *positive* but reading of W_2 is *reversed*. For a leading p.f., conditions are just the opposite of this. In that case, W_1 will read *negative* because the phase angle between the current and voltage is *more than* 90°. For getting the total power, the reading of W_2 is to be *subtracted* from that of W_1. Under this condition, W_2 will read '*down scale' i.e., backwards*. Hence, *to obtain a reading on W_2, it is necessary to reverse either its pressure coil or current coil, usually the former*.

All readings taken after reversal of pressure coil are to be taken as negative.

(*iv*) When $\phi = 90°$

(*i.e.,* p.f. = 0 *i.e.,* pure inductive or capacitive load)

Then $W_1 = E_L I_L \cos(30° - 90°) = E_L I_L \sin 30°$

and $W_2 = E_L I_L \cos(30° + 90°) = -E_L I_L \sin 30°$

These two readings are equal in magnitude but opposite in sign

$\therefore \qquad W_1 + W_2 = 0.$

So, far we have considered *lagging angles* (taken as *positive*). Now let us discuss how the readings of wattmeters change when the power factor is *leading* one.

— For $\phi = +60°$ (lag) : $W_2 = 0$

— For $\phi = -60°$ (lead) : $W_1 = 0$

Thus, we find that for angles of lead the readings of the two wattmeters are interchanged.

Hence, when the power is *leading* :

$$W_1 = E_L I_L \cos(30° + \phi)$$
$$W_2 = E_L I_L \cos(30° - \phi).$$

Power Factor—When the load is 'balanced'. When load is balanced with a *lagging power factor* and the voltage and currents are sinusoidal :

$$W_1 + W_2 = E_L I_L \cos(30° - \phi) + E_L I_L \cos(30° + \phi) = \sqrt{3} E_L I_L \cos\phi \qquad ...(10)$$

Similarly, $W_1 - W_2 = E_L I_L \cos(30° - \phi) - E_L I_L \cos(30° + \phi)$

$$= E_L I_L (2 \sin\phi \sin 30°) = E_L I_L \sin\phi \qquad ...(11)$$

Dividing (11) by (12), we get

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} \qquad ...(12)$$

For a *leading power*, this expression becomes

$$\tan\phi = -\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} \qquad ...(13)$$

After finding $\tan\phi$, hence ϕ, the value of power factor $\cos\phi$ can be found (from trigonometrical tables).

One important point which must be kept in mind is that if W_2 reading has been taken after reversing the pressure coil *i.e.*, if W_2 is *negative*, then the eqn. (12) becomes

$$\tan\phi = \sqrt{3}\,\frac{W_1 - (-W_2)}{W_1 + (-W_2)}$$

or
$$\tan\phi = \sqrt{3}\,\frac{W_1 + W_2}{W_1 - W_2} \qquad ...(14)$$

The power factor may also be expressed in terms of ratio of the readings of the two wattmeters.

Let $\qquad \dfrac{\text{Smaller reading}}{\text{Larger reading}} = \dfrac{W_2}{W_1} = \alpha$

Then from eqn. (12) above, we have

$$\tan\phi = \frac{\sqrt{3}\left[1 - \left(\dfrac{W_2}{W_1}\right)\right]}{\left[1 + \left(\dfrac{W_2}{W_1}\right)\right]} = \frac{\sqrt{3}\,(1 - \alpha)}{(1 + \alpha)}$$

We know that, $\sec^2 \phi = 1 + \tan^2 \phi$

or $$\frac{1}{\cos^2 \phi} = 1 + \tan^2 \phi \text{ or } \cos^2 \phi = \frac{1}{1 + \tan^2 \phi}$$

$$\cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} = \frac{1}{\sqrt{1 + \left[\frac{\sqrt{3}\,(1-\alpha)}{(1+\alpha)}\right]^2}}$$

or $$= \frac{1}{\sqrt{1 + 3\left(\frac{(1-\alpha)}{(1+\alpha)}\right)^2}} = \frac{1+\alpha}{\sqrt{(1+\alpha)^2 + 3\,(1-\alpha)^2}}$$

$$= \frac{1+\alpha}{\sqrt{1+\alpha^2+2\alpha+3\,(1+\alpha^2-2\alpha)}} = \frac{1+\alpha}{\sqrt{4+4\alpha^2-4\alpha}}$$

$$= \frac{1+\alpha}{2\sqrt{1-\alpha+\alpha^2}}$$

i.e., $$\cos \phi = \frac{1+\alpha}{2\sqrt{1-\alpha+\alpha^2}} \qquad ...(15)$$

If a curve is plotted between α and $\cos \phi$, then the curve obtained will be as shown in Fig. 25, this curve is called **watt-ratio curve.**

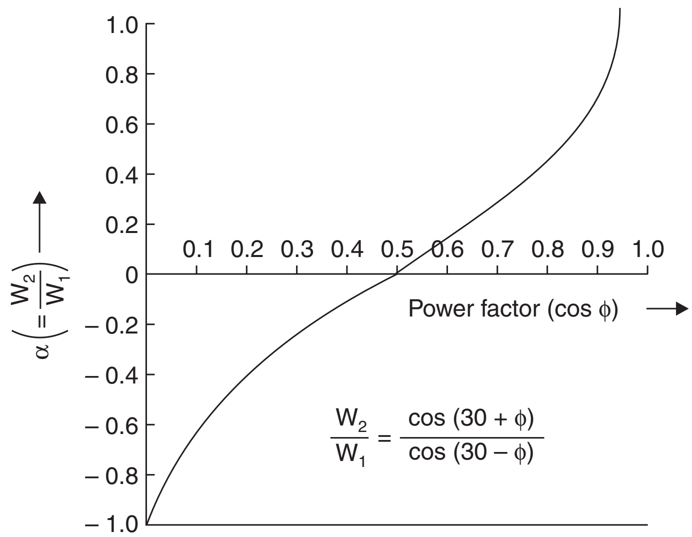


Fig. 25. Watt-ratio curve.

Reactive volt amperes (*with two wattmeters*)

We know that, $$\tan \phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

As the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of reactive power to the true power (Fig. 26). Hence, in case of a balanced load, the reactive power is given by $\sqrt{3}$ *times the difference of the readings of the two wattmeters* used to measure the power of a 3-phase circuit by two wattmeter method.

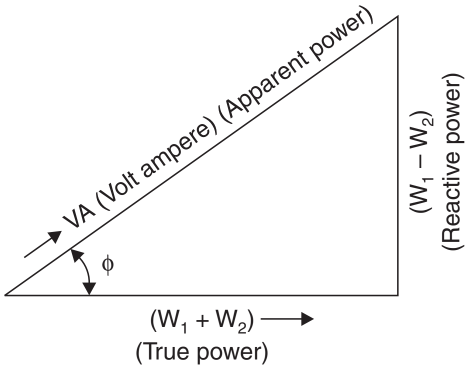


Fig. 26

Mathematical proof is as follows :

$$\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\,[(E_L I_L \cos(30° - \phi) - E_L I_L \cos(30° + \phi)]$$

$$= \sqrt{3}\,E_L I_L\,[(\cos 30° \cos\phi + \sin 30° \sin\phi) - (\cos 30° \cos\phi - \sin 30° \sin\phi)]$$

$$= \sqrt{3}\,E_L I_L\,[\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\phi + \sin 30° \sin\phi]$$

$$= \sqrt{3}\,E_L I_L\,(2 \sin 30° \sin\phi) = \sqrt{3}\,E_L I_L \sin\phi$$

i.e., $\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\,E_L I_L \sin\phi.$

9.3. One-wattmeter method

In this method the current coil is connected in anyone line and the pressure coil is connected alternately between this and the other two lines as shown in Fig. 27. The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method.

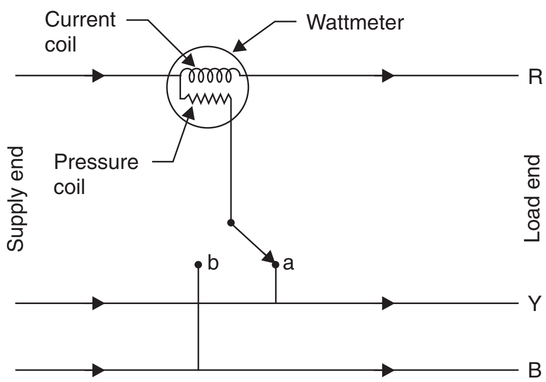


Fig. 27. One-wattmeter method.

This method is not of as much universal application as the two wattmeter method because it is restricted to *fairly balanced loads only*. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor *in order to check the load upon the motor*.

10. MEASUREMENT OF REACTIVE VOLT AMPERES

In order to measure reactive power in a single phase circuit a *compensated wattmeter* is used. In this wattmeter the voltage applied to the pressure coil is *90° out of phase with the actual voltage* and hence it will read $VI \cos(90° - \phi)$ *i.e.,* $VI \sin\phi$, reactive power.

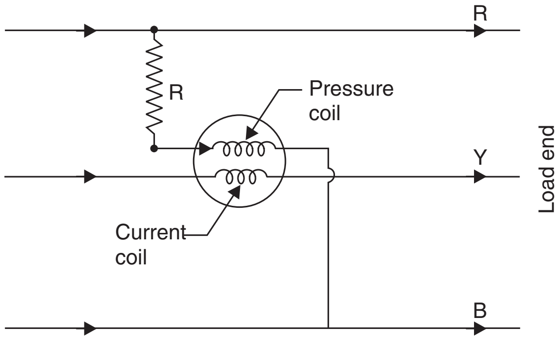


Fig. 28. Measurement of reactive volt-amperes.

In balanced three-phase circuit the reactive power can be determined by using one wattmeter. The necessary connections are shown in Fig. 28. The current coil is inserted in one line and the pressure coil is connected across the other two lines.

The current following through the current coil of the wattmeter

$= I_Y = I_{ph}$ (say)

The potential difference across the potential coil of wattmeter,

$E = E_R - E_B$

$= \sqrt{3}\ E_{ph}$ leading vector E_Y by 90°

$= \sqrt{3}\ E_{ph}$ leading the vector of current I_Y by (90° + ϕ)

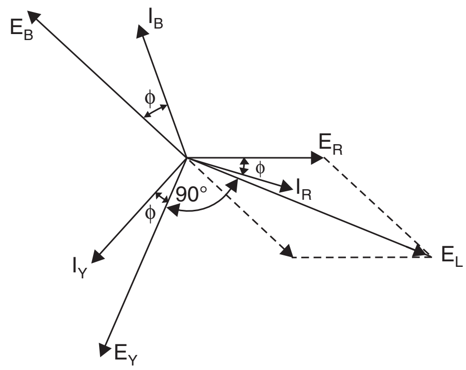


Fig. 29

$\therefore$ Reading of the wattmeter (Fig. 29)

$$= \sqrt{3}\ E_{ph} I_{ph} \cos (90° + \phi)$$

$$= -\sqrt{3}\ E_{ph} I_{ph} \sin \phi = -W_r$$

Total reactive power of the circuit,

$$= 3\ E_{ph} I_{ph} \sin \phi = -\sqrt{3}\ W_r$$

Reactive power (as earlier started) can also be determined from two wattmeter readings connected for measurement of power.

$$W_1 - W_2 = E_L I_L \cos (30° - \phi) - E_L I_L \cos (30° + \phi)$$

or $\quad W_1 - W_2 = E_L I_L \times 2 \sin 30° \sin \phi$

or $\quad E_L I_L \sin \phi = W_1 - W_2$

Reactive power of load circuit,

$$W_r = \sqrt{3}\ E_L I_L \sin \phi = \sqrt{3}\ (W_1 - W_2).$$

Example 13. *The power input to a 3-phase induction motor is read by two wattmeters. The readings are 920 W and 300 W. Calculate the power factor of the motor.*

Solution. Reading of wattmeter, $W_1 = 920$ W

Reading of wattmeter, $W_2 = 300$ W

Power factor of the motor, cos φ :

Using the relation, $\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}\,(920 - 300)}{(920 + 300)} = \frac{\sqrt{3} \times 620}{1220} = 0.88$

∴ $\phi = \tan^{-1} 0.88 = 41.35°$

Power factor of the motor,

$\cos\phi = \cos 41.35° =$ **0.75 (lag). (Ans.)**

Example 14. *While performing a load test on a 3-phase wound-rotor induction motor by two wattmeters method, the readings obtained on two wattmeters were + 14.2 kW and – 6.1 kW and the line voltage was 440 V. Calculate :*

(*i*) *True power drawn by the motor* (*ii*) *Power factor, and*

(*iii*) *Line current.*

Solution. Reading of wattmeter, $W_1 = 14.2$ kW

Reading of wattmeter, $W_2 = -6.1$ kW

Line voltage, $E_L = 440$ V

(*i*) **True power drawn by the motor**

= 14.2 – 6.1 = **8.1 kW. (Ans.)**

(*ii*) $\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}\,[14.2 - (-6.1)]}{[14.2 + (-6.1)]} = \frac{\sqrt{3} \times 20.3}{8.1} = 4.34$

∴ $\phi = \tan^{-1} 4.34 = 77°$

∴ **Power factor,** $\cos\phi = \cos 77° =$ **0.2249 (lag). (Ans.)**

(*iii*) **Line current, I_L :**

Using the relation, $P = \sqrt{3}\, E_L I_L \cos\phi$

$8.1 \times 1000 = \sqrt{3} \times 440 \times I_L \times 0.2249$

∴ $I_L = \frac{8.1 \times 1000}{\sqrt{3} \times 440 \times 0.2249} = 47.26$ A

Hence, **line current = 47.26 A. (Ans.)**

Example 15. *A 3-phase, 440 V motor load has a power factor of 0.6. Two wattmeters connected to measure the power show the input to be 25 kW. Find the reading on each instrument.*

Solution. Input power = 25 kW

Line voltage, $E_L = 440$ V

Power factor of the motor load, cos φ = 0.6

W_1 ; W_2 :

Using the relation : $\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$ ...(*i*)

Now, $\cos\phi = 0.6$, ∴ $\phi = 53.13°$

∴ $\tan\phi = \tan 53.13° = 1.333$

Also $W_1 + W_2 = 25$ kW (*given*) ...(*ii*)

Substituting these values in (*i*), we get

$$1.333 = \frac{\sqrt{3}\,(W_1 - W_2)}{25}$$

$\therefore \qquad W_1 - W_2 = \dfrac{1.333 \times 25}{\sqrt{3}} = 19.24 \text{ kW} \qquad ...(iii)$

From (*ii*) and (*iii*), we get **W_1 = 22.12 kW**

and **W_2 = 2.88 kW. (Ans.)**

Example 16. *In a 3-phase circuit two wattmeters used to measure power indicate 1200 W and 600 W respectively. Find the power factor of the circuit :*

(*i*) *When both wattmeter readings are positive.*

(*ii*) *When the latter is obtained by reversing the current coil connections.*

Solution. (*i*) **When both wattmeter readings are positive :**

Reading of wattmeter, $W_1 = 1200$ W

Reading of wattmeter, $W_2 = 600$ W

We know that,
$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\,(1200 - 600)}{(1200 + 600)} = 0.577$$
$$\phi = \tan^{-1} 0.577 = 30°$$

Power factor, cos φ = cos 30° = 0.866 (lag). (Ans.)

(*ii*) **When the reading of wattmeter W_2 is obtained by reversing the coil connection :**

Reading of wattmeter, $W_1 = 1200$ W

Reading of wattmeter, $W_2 = -600$ W

We know that,
$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\,[1200 - (-600)]}{[(1200 + (-600)]} = \frac{\sqrt{3} \times 1800}{600} = 5.196$$

or
$$\phi = \tan^{-1} 5.196 = 79.1°$$

Hence, **power factor, cos φ = cos 79.1° = 0.1889. (Ans.)**

Example 17. *In order to measure the power input and the power factor of an over-excited synchronous motor two wattmeters are used. If the meters indicate (– 3.5 kW) and (+ 8.0 kW) respectively. Calculate :*

(*i*) *Power factor of the motor.* (*ii*) *Power input to the motor.*

Solution. (*i*) *Since an over-excited synchronous motor runs with a leading power factor*, we should use the relation,
$$\tan\phi = -\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

Moreover it is W_1 that gives negative reading and not W_2. Hence, $W_1 = -3.5$ kW.
$$\therefore \qquad \tan\phi = -\frac{\sqrt{3}\,(-3.5 - 8)}{(-3.5 + 8)} = -\frac{\sqrt{3} \times 11.5}{5.5} = 3.62$$
$$\therefore \qquad \phi = \tan^{-1} 3.62 = 74.6° \text{ (lead)}$$

$\therefore$ **Power factor, cos φ = cos 74.6° = 0.2655 (lead). (Ans.)**

(*ii*) **Power input** = $W_1 + W_2 = -3.5 + 8$ = **4.5. W. (Ans.)**

Example 18. *Two wattmeters are used to measure power input to a 1.5 kV, 50 Hz, 3-phase motor running on full-load at an efficiency of 85 per cent. Their readings are 250 kW and 80 kW respectively. Calculate :*

(*i*) *Input,* (*ii*) *Power factor,*

(*iii*) *Line current, and* (*iv*) *Output.*

Solution. *Since the motor is running at full-load, its power factor must be greater than 0.5. Hence, W_2 reading is positive*

$\therefore$ $W_1 = +\,250$ kW and $W_2 = +\,80$ kW

(*i*) **Input** $= W_1 + W_2 = 250 + 80 =$ **330 kW. (Ans.)**

(*ii*)
$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}\,(250 - 80)}{(250 + 80)} = 0.892$$

$\therefore$ $\phi = \tan^{-1} 0.892 = 41.74°$

and **power factor** $= \cos\phi = \cos 41.74° =$ **0.746 (lag). (Ans.)**

(*iii*) **Power,** $P = \sqrt{3}\;E_L I_L \cos\phi$

$$\therefore \quad I_L = \frac{P}{\sqrt{3}\,E_L \cos\phi} = \frac{330 \times 1000}{\sqrt{3} \times 1.5 \times 1000 \times 0.746} = 170.27$$

Hence, **line current = 170.27 A. (Ans.)**

(*iv*) **Output** = input × efficiency = 330 × 0.85 = **280.5 kW. (Ans.)**

☞ **Example 19.** *Two wattmeters are used to measure power input to a synchronous motor. Each of them indicates 60 kW. If the power factor be changed to 0.866 leading, determine the readings of the two wattmeters, the total input power remaining the same. Draw the vector diagram for the second condition of the load.*

Solution. Reading of wattmeter, $W_1 = 60$ kW

Reading of wattmeter, $W_2 = 60$ kW.

First case. In the first case, *both wattmeters read equal and positive. Hence, motor must be running at unity power factor.*

Second case. *Power factor is 0.866 leading.*

In this case : $W_1 = E_L I_L \cos(30° + \phi)$

$W_2 = E_L I_L \cos(30° - \phi)$

$\therefore$ $W_1 + W_2 = \sqrt{3}\;E_L I_L \cos\phi$

$W_1 - W_2 = -\,E_L I_L \sin\phi$

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

Since $\cos\phi = 0.866$ (*given*)

$\therefore$ $\phi = \cos^{-1} 0.866 = 30°$

and
$$\tan\phi = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \frac{1}{\sqrt{3}} = -\frac{\sqrt{3}\,(W_1 - W_2)}{120}$$

[$\because$ $W_1 + W_2 = 120$ kW (*given*)]

$$\therefore \quad W_1 - W_2 = -\frac{120}{3} = -\,40 \qquad ...(i)$$

But $W_1 + W_2 = 120$ ...(*ii*)

From (*i*) and (*ii*), we get **W_1 = 40 kW, W_2 = 80 kW. (Ans.)**

For connection diagram, please refer to Fig. 23. The vector or phasor diagram is shown in Fig. 30.

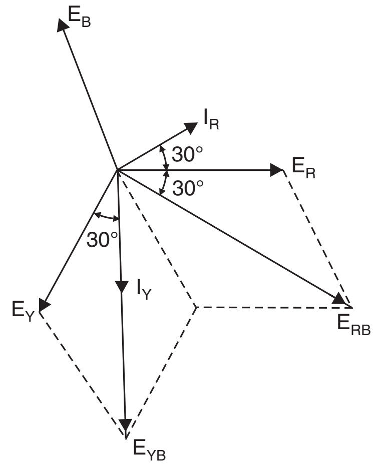


Fig. 30

Example 20. *Two wattmeters connected to read the total power in a 3-phase system supplying a balanced load read 10.5 kW and – 2.5 kW respectively. Calculate the total power and power factor.*

Also, explain the significance of (*i*) *equal wattmeter readings and* (*ii*) *a zero reading on one wattmeter*.

Solution. *Given* : $W_1 = 10.5$ kW ; $W_2 = -2.5$ kW.

For two wattmeter method,

$$\textbf{total power} = W_1 + W_2 = 10.5 + (-2.5) = \textbf{8.0 kW.} \quad \textbf{(Ans.)}$$

We know that, $$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\,[10.5 - (-2.5)]}{[10.5 + (-2.5)]} = 2.8145$$

or $$\phi = 70.44°$$

∴ **Power factor,** $\cos\phi = \cos 70.44° = \textbf{0.335.}$ **(Ans.)**

(*i*) For readings of the two wattmeters to be equal,

$$W_1 = W_2$$

or $$\sqrt{3}\;E_L I_L \cos(30° - \phi) = \sqrt{3}\;E_L I_L \cos(30° + \phi)$$

or $$\cos(30° - \phi) = \cos(30° + \phi)$$

∴ $$\phi = 0° \text{ or } \cos\phi = 1$$

i.e., for *unity power factor, the readings of two wattmeters are equal*. **(Ans.)**

(*ii*) As readings of wattmeters are

$$W_1 = \sqrt{3}\,E_L I_L \cos(30° - \phi), \text{ and}$$

$$W_2 = \sqrt{3}\;E_L I_L \cos(30° + \phi),$$

for *reading of one of the wattmeters to be zero* ϕ *must be 60°, which makes* $W_2 = \sqrt{3}\;E_L I_L \cos 90° = \mathbf{0}$. **(Ans.)**

HIGHLIGHTS

1. In a star-connected system $E_{ph} = \dfrac{E_L}{\sqrt{3}}$

$$I_{ph} = I_L$$

$$P = \sqrt{3}\;E_L I_L \cos\phi.$$

2. In a delta-connected system

$$E_{ph} = E_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = \sqrt{3}\,E_L I_L \cos\phi.$$

3. Two-wattmeter method is generally used to measure 3-phase power. In this method, the current coils of the two wattmeters are connected in any two lines and their potential coils to the remaining third line. The sum of the two wattmeters readings gives the total power in the circuit. If the load is balanced, then its power factor can also be calculated from these two readings. The readings of the two wattmeters are :

(*i*)

$$\left.\begin{aligned} W_1 &= E_L I_L \cos(30° - \phi) \\ W_2 &= E_L I_L \cos(30° + \phi) \\ \tan\phi &= \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} \end{aligned}\right\} \text{Lagging power factor}$$

(*ii*)

$$\left.\begin{aligned} W_1 &= E_L I_L \cos(30° + \phi) \\ W_2 &= E_L I_L \cos(30° - \phi) \\ \tan\phi &= -\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} \end{aligned}\right\} \text{Leading power factor}$$

4. In a single-phase as well as in a three-phase system, the kVA is directly proportional to the current I. The disadvantage of a lower power factor is that the current required for a given power is very high, which fact leads to many undesirable results. The power factor may be improved by the following :

— Static capacitors
— Phase advancers
— Synchronous capacitors
— Capacitor boosters
— High power factor motors.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The power in a 3-phase system is given by $\sqrt{3}\, V_L I_L \cos\phi$ where ϕ is the phase angle between
(*a*) line-voltage and line current (*b*) phase voltage and phase current
(*c*) line voltage and phase current (*d*) phase voltage and line current.

2. Which of the following statements associated with 3-phase delta connected circuits is *true* ?
(*a*) Line voltage is equal to phase voltage (*b*) Line current is equal to phase current
(*c*) Line voltage is $\sqrt{3}$ time of phase voltage (*d*) Line currents are 60° apart.

3. In a 3-phase supply, floating neutral is undesirable because it may result in ...... across the load.
(*a*) unequal line voltages (*b*) high voltage
(*c*) low voltage.

4. Phase reversal in a 4-wire unbalanced load supplied from a balanced 3-phase supply causes change in
(*a*) the power consumed (*b*) magnitude of phase currents
(*c*) only the magnitude of the neutral current
(*d*) magnitude as well as phase angle of the neutral current.

5. A 3-phase star connected symmetrical load consumes P watts of power from a balanced supply. If the same load is connected in delta to the same supply, the power consumption will be
(*a*) P (*b*) $\sqrt{3}\,P$
(*c*) $3P$ (*d*) not determined from the given data.

6. Three unequal impedances are connected in star to a 3-phase system. The sum of three line currents will be
(*a*) equal to the each line current (*b*) zero
(*c*) none of these.

7. Three equal impedances are first connected in delta across a 3-ϕ balanced supply. If the same impedances are connected in star across the same supply
(*a*) phase currents will be one-third (*b*) line currents will be one-third
(*c*) power consumed will be one-third (*d*) none of the above.

8. Which of the following is a four wire system ?

(*a*) Delta (*b*) Star

(*c*) Both delta and star (*d*) Neither delta nor star.

9. In a 3-phase balanced star-connected load, neutral current is equal to

(*a*) zero (*b*) I_P

(*c*) I_L (*d*) unpredictable.

10. Which of the following equations is valid for a 3-phase 4-wire balanced star-connected load ?

(*a*) $I_R + I_Y + I_B = I_N = 0$ (*b*) $I_R + I_Y - I_B = I_N$

(*c*) $I_R - I_Y + I_B = I_N$ (*d*) $\frac{V_B + V_R + V_Y}{Z} = I_N$.

11. Three unequal impedances are connected in delta to a 3-phase, 3-wire system.

(*a*) The voltages across the three phases will be different

(*b*) Both of the phase currents and line currents will be unbalanced

(*c*) Phase currents will be unbalanced but the line currents will be balanced

(*d*) None of the above.

12. The relationship between the line and phase voltages of a delta-connected circuit is given by

(*a*) $V_L = V_P$ (*b*) $V_L = \sqrt{3}\, V_P$

(*c*) $V_L = \frac{V_P}{\sqrt{2}}$ (*d*) $V_L = \frac{2}{\pi} V_P$.

13. In case of a delta connected load, if one resistor is removed, the power will become

(*a*) zero (*b*) one-third

(*c*) two-third (*d*) none of the above.

14. Readings of 1154 and 577 watts are obtained when two wattmeters method was used on a balanced load. The delta connected load impedance for a system of 100 V will be

(*a*) $15 \angle \pm 30°$ (*b*) $15 \angle + 30°$

(*c*) $15 \angle - 30°$ (*d*) $15 \angle + 90°$.

15. Which of the following statements is true about two-wattmeters method for power measurement in 3-phase circuit ?

(*a*) Power can be measured using two wattmeters method only for star-connected 3-phase circuits

(*b*) When two wattmeters show identical readings, the power factor is 0.5

(*c*) When power factor is unity, one of the wattmeter reads zero

(*d*) When the readings of the two wattmeters are equal but of opposite sign, the power factor is zero.

16. The minimum number of wattmeters to measure power in a 3-phase unbalanced star-connected load is

(*a*) one (*b*) two

(*c*) three (*d*) four.

17. While measuring power in a three phase load by two-wattmeters method, the readings of two wattmeters will be equal and opposite when

(*a*) *pf* is unity (*b*) load is balanced

(*c*) phase angle is between 60° and 90° (*d*) the load is purely inductive.

18. In the measurement of 3-phase power by two-wattmeters method, if the two wattmeter readings are equal the power factor of the circuit is

(*a*) 0.8 lagging (*b*) 0.8 leading

(*c*) zero (*d*) unity.

19. Which of the following are the necessary conditions for an entire 3-ϕ system to be balanced ?

1. The line voltages are equal in magnitude
2. The phase differences between successive line voltages are equal
3. The impedances in each of the phases are identical.

Select the correct answer using the codes given below :

Codes :

(*a*) 1, 2 and 3 (*b*) 1 and 3

(*c*) 1 and 2 (*d*) 2 and 3.

20. The power delivered to a 3-phase load can be measured by the use of 2 wattmeters only when the

(*a*) load is balanced (*b*) load is unbalanced

(*c*) 3-phase load is connected to the source through 3-wires

(*d*) 3-phase load is connected to the source through 4-wires.

21. The ratio of the readings of two wattmeters connected to measure power in a balanced 3-phase load is 5 : 3 and the load is inductive. The power factor of the load is

(*a*) 0.917 lead (*b*) 0.917 lag

(*c*) 0.6 lead (*d*) 0.6 lag.

22. In two phase system phase voltages differ by

(*a*) 60° (*b*) 90°

(*c*) 120° (*d*) 180°.

23. W_1 and W_2 are the readings of two wattmeters used to measure power of a 3-phase balanced load. The reactive power drawn by the load is

(*a*) $W_1 + W_2$ (*b*) $W_1 - W_2$

(*c*) $\sqrt{3}\,(W_1 + W_2)$ (*d*) $\sqrt{3}\,(W_1 - W_2)$.

24. In a two wattmeters method of measuring power in a 3-phase system one of the wattmeters reads negative implying

(*a*) wattmeter connection is faulty (*b*) load is unbalanced

(*c*) power flow is in the reverse direction (*d*) power factor is less than 0.5.

25. v_{RN}, v_{YN} and v_{BN} are the instantaneous line to neutral voltages and i_R, i_Y and i_B are instantaneous line currents in a balanced three phase circuit, the computation $v_{RN}(i_Y - i_B) - (v_{YN} - v_{BN})i_R$ will yield a quantity proportional to the

(*a*) active power (*b*) power factor

(*c*) reactive power (*d*) complex power.

26. If the *R*-phase of a 3-phase star connected alternator becomes reversely connected by mistake, it will affect

(*a*) E_{RY} and E_{YB} (*b*) E_{YB} and E_{BR}

(*c*) E_{RY} and E_{BR} (*d*) E_{RY}, E_{YB} and E_{BR}.

27. When phase sequence at the 3-phase load is reversed

(*a*) phase currents are changed in magnitude

(*b*) phase currents change in phase angle but not in magnitude

(*c*) total power consumed is changed

(*d*) phase powers are changed.

28. Lamp load is being supplied from a 3-phase, 4-wire, 230/400 V ac supply system. If a 3-phase motor is now switched on across the same supply, then neutral current will

(*a*) increase (*b*) decrease

(*c*) remain unchanged (*d*) unpredictable.

29. Lamps of 40 W, 100 W and 200 W ratings are connected in different phases of a 3-phase, 4-wire supply. If the neutral wire breaks, then the lamp likely to fuse first will be

(*a*) 200 W (*b*) 100 W

(*c*) 40 W (*d*) all lamps.

30. The instantaneous values of currents in phases *R* and *Y* of a 3-phase system are 25 A each. For a phase sequence of *BRY*, the instantaneous value of current in phase *B* is

(*a*) 25 A (*b*) 50 A

(*c*) 12.5 A (*d*) 43.3 A.

31. The power measurement in balanced 3-ϕ circuit can be done by

(*a*) one-wattmeter method only (*b*) two-wattmeter method only

(*c*) three-wattmeter method only (*d*) any one of the above.

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*b*)	**7.** (*c*)
8. (*b*)	**9.** (*a*)	**10.** (*a*)	**11.** (*b*)	**12.** (*a*)	**13.** (*c*)	**14.** (*b*)
15. (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*d*)	**19.** (*a*)	**20.** (*c*)	**21.** (*b*)
22. (*b*)	**23.** (*d*)	**24.** (*d*)	**25.** (*c*)	**26.** (*c*)	**27.** (*b*)	**28.** (*c*)
29. (*c*)	**30.** (*b*)	**31.** (*d*).				

THEORETICAL QUESTIONS

1. State the advantages of A.C. polyphase supply system over single-phase system.
2. Why is the number of phases in a polyphase system always three rather than any other number ?
3. Explain clearly what is meant by 'phase sequence' of 3-phase voltages.
4. What are the two systems in which three-phases can be connected ? What are the advantages and disadvantages of each system ?
5. What are the advantages of inter-connecting the 3-phases ?
6. Derive an expression for power in a 3-phase star-connected system in terms of (*i*) phase values and (*ii*) line values of voltages and currents.
7. Derive the numerical relationship between line and phase currents for a balanced 3-phase delta-connected load.
8. Deduce an expression for power in a 3-phase balanced load circuit. Show that it is the same irrespective of the load being connected in star or delta.
9. Compare the star and delta connections in a 3-phase system.
10. Enumerate various methods for 3-phase power measurement, and describe in detail two-wattmeter method for 3-phase power measurement.
11. Prove that the power in a balanced 3-phase circuit can be deduced from the readings of two-wattmeters. Draw the relevant connection diagram and vector diagram.
12. Discuss two-wattmeter method for power measurement in 3-phase system and obtain a relation for the power factor.
13. Derive an expression for star-connected impedances equivalent to three delta-connected impedances.

EXERCISE

Star/Delta Connections

1. Three equal impedances each having a resistance of 20 Ω and reactance of 15 Ω are connected in star to a 400 V, 3-phase, 50 Hz system. Calculate :

(*i*) The line current, (*ii*) The power factor, and

(*iii*) The power consumed. [**Ans.** 9.24 A ; 0.8 (lag) ; 5120 W]

2. Three equal impedances are star-connected to a 3-phase, 50 Hz supply. If the resistance and reactance of each branch are 25 Ω and 38 Ω respectively, calculate :

(*i*) The line current, and (*ii*) The power consumed. [**Ans.** 5.28 A ; 2086 W]

3. Three resistances of 20 Ω each are connected in star across the 400 V, 3-phase A.C. supply. Calculate :
(*i*) The line and phase currents, (*ii*) Phase voltages, and
(*iii*) Total power taken. [**Ans.** (*i*) 11.55 A ; (*ii*) 231 V ; (*iii*) 8.0 kW]

4. A star-connected, 3-phase load consists of three similar impedances. When the load is connected to a 3-phase, 500 V, 50 Hz supply, the line current is 28.85 A and the power factor is 0.8 lagging, calculate :
(*i*) The total power taken by the load, and (*ii*) The resistance of each phase of the load.
[**Ans.** 20 kW ; 8 Ω]

5. Three non-inductive resistances each of 50 Ω are connected in star across 400 V, 3-phase A.C. supply. Calculate the current through each. Calculate the current if they were connected in delta across the same supply. [**Ans.** 4.62 A ; 8 A]

6. Three identical coils are connected in star to a 200 V (line voltage), 3-phase, A.C. supply and each coil takes 200 W. The power factor is 0.8 (lagging), calculate :
(*i*) The line current, (*ii*) Impedance, and
(*iii*) Resistance and inductance of each coil. [**Ans.** 2.165 A ; 53.334 Ω, 42.662 Ω, 0.102 H]

7. In a three-phase, 3-wire system with star-connected load the impedance of each phase is $(6 + j8)$ Ω. If the line voltage is 230 V, calculate :
(*i*) The line current, and (*ii*) The power absorbed by each phase.
[**Ans.** 13.3 A ; 1067 W]

8. A balanced star-connected load of $(8 + j6)$ Ω per phase is connected to 3-phase, 230 V supply. Find :
(*i*) Line current, (*ii*) Power factor,
(*iii*) Power, (*iv*) Reactive volt-amperes, and
(*v*) Total volt-amperes. [**Ans.** 13.28 A ; 0.8 (lag) ; 4.232 W ; 3174 ; 5290]

9. A star-connected, 5000 V, 3-phase alternator is supplying 3,000 kW at power factor of 0.8. Calculate the active and reactive components of the current in each phase. [**Ans.** 346.2 A ; 260 A]

10. In a 3-phase, 4-wire system, two phases have currents of 10 A and 6 A in lagging power factors of 0.8 and 0.6 respectively, while the third phase is open-circuited. Calculate the current in the neutral and sketch the vector diagram. [**Ans.** $7 \angle -73° 24'$]

11. In a star-connected load each phase consists of a resistance of 100 Ω in parallel with a capacitor of capacitance 31.8 μF. When it is connected to a 416 V, 3-phase, 50 Hz supply, calculate :
(*i*) The line current, (*ii*) The power factor,
(*iii*) The power absorbed, and (*iv*) The total kVA.
[**Ans.** 3.39 A ; 0.707 (leading) ; 1.728 kW ; 2.443 kVA]

12. Three identical coils connected in delta across 400 V, 50 Hz, 3- phase supply takes a line current of 17.32 A at power factor 0.5 lagging. Determine :
(*i*) The current in each phase, and
(*ii*) Resistance, inductance and impedance of each phase winding. [**Ans.** 10 A ; 20 Ω ; 0.11 H, 40 Ω]

13. A 220 V, 3-phase voltage is applied to a balanced delta-connected 3-phase load of phase impedance $(15 + j20)$ Ω. Find :
(*i*) The phasor current in each line. (*ii*) What is the power consumed per phase ?
(*iii*) What is the phasor sum of the three line currents ? Why does it have this value ?
[**Ans.** 15.24 A ; 1161.6 W ; zero]

14. Calculate (*i*) line current and (*ii*) the total power absorbed when three coils each having a resistance of 10 Ω and reactance of 7 Ω are connected (*a*) in star and (*b*) in delta across a 400 V, 3-phase supply.
[**Ans.** 18.93 A, 10750 W ; 56.7 A, 32250 W]

15. A delta-connected balanced 3-phase load is supplied from a 3-phase 400 V supply. The line current is 20 A and power taken by the load is 10000 W. Find :
(*i*) Impedance in each branch ; and
(*ii*) The line current, power factor and power consumed if the same load is connected in star.
[**Ans.** 34.6 Ω, 20/3 A, 0.7217, 3330 W]

16. A balanced 3-phase load consists of resistances of 4 Ω each. Determine the total power when the resistances connected to 400 V supply are :
(*i*) Star connected (*ii*) Delta connected. [**Ans.** 40 kW ; 120 kW]

17. Three non-inductive resistances, each of 100 Ω are connected in star across 400 V supply. Calculate the current through each. What would be the current through each if they are connected in delta across the same supply. [**Ans.** 2.31 A ; 4 A]

18. Three 100 Ω non-inductive resistances are connected in (*i*) star (*ii*) delta across a 400 V, 50 Hz, 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances opened, what would be the value of the total power taken from the mains in each of the two cases. [**Ans.** 1600 W ; 4800 W]

19. A 3-phase delta connected load ; each phase of which has an inductive reactance of 40 Ω and a resistance of 25 Ω, is fed from the secondary of a 3-phase star-connected transformer, which has phase voltage of 240 V. Draw the circuit diagram of the system and calculate :

(*i*) The current in each phase of the load, (*ii*) The voltage across each phase of the load,

(*iii*) The current in the transformer secondary windings, and

(*iv*) The total power taken from the supply and its power factor.

[**Ans.** 8.8 A ; 415.7 V ; 15.3 A ; 5820 W]

20. Three similar resistors are connected in star across 400 V, 3-phase supply. The line current is 5 A. Calculate the value of each resistor. To what value line voltage be changed to obtain the same current with the resistors connected in delta ? [**Ans.** 46.2 Ω ; 133.3 V]

21. A 3-phase, star-connected system with 230 V between each phase and neutral has a resistance of 4, 5 and 6 Ω respectively in three phases. Calculate :

(*i*) The current flowing in each phase, (*ii*) The neutral current, and

(*iii*) The total power absorbed. [**Ans.** 57.5 A, 46 A, 38.3 A ; 16.71 A ; 32610 W]

22. In a 3-phase, 4 wire system there is a balanced 3-phase motor load taking 20 kW at a power factor of 0.8 lagging while lamps connected between phase conductors and the neutral are taking 5, 4 and 10 kW respectively. Voltage between line conductors is 430 V. Calculate the current in each conductor and in the neutral wire of the feeder supplying these loads. [**Ans.** 51.2 A ; 47.5 A ; 80 A ; 22.4 A]

Power Measurement in 3-phase Circuits

23. The power input to a 3-phase induction motor is read by two wattmeters. The readings are 860 W and 240 W. What is the power factor of the motor ? [**Ans.** 0.7155 (lag)]

24. While performing a load test on a 3-phase wound rotor induction motor by two wattmeters method, the readings obtained on two wattmeters were + 12.5 kW and – 4.8 kW and the line voltage was 440 V. Calculate :

(*i*) Power drawn by the motor, (*ii*) Power factor, and

(*iii*) Line current. [**Ans.** 7.7 kW ; 0.2334 ; 43.3 A]

25. The input power to a 3-phase delta-connected motor was measured by two wattmeters method. The readings were 20.8 kW and – 6.8 kW and the line voltage was 400 V. Calculate :

(*i*) Input power, (*ii*) Power factor, and

(*iii*) Line current. [**Ans.** 14 kW ; 0.281 ; 72 A]

26. A 3-phase, 500 V motor load has a power factor of 0.4. Two wattmeters connected to measure the power show the input to be 30 kW. Find the reading on each instrument. [**Ans.** 35 kW ; – 5 kW]

27. The power in a 3-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter ? For what power factor will one of the wattmeters read zero ? [**Ans.** 17.15 kW ; 82.85 kW ; p.f. = 0.5]

28. Two wattmeters used to measure the power input in a 3-phase circuit indicate 1000 W and 500 W respectively. Find the power factor of the circuit

(*i*) when both wattmeter readings are positive ; and

(*ii*) when the latter is obtained by reversing the current coil connections.

[**Ans.** 0.866 (lag) ; 0.1889 (lag)]

29. Two wattmeters connected to read the total power in a 3-phase system supplying a balanced load read 10.5 kW and – 2.5 kW respectively. Calculate the total power and the power factor. Draw suitable phasor diagram, explain the significance of (*i*) equal wattmeter readings ; and (*ii*) a zero reading on one wattmeter.

[**Ans.** 8 kW ; 0.3348 (lag) (*i*) unity p.f., (*ii*) 0.5 p.f.]

30. The power input to a l000 V, 3-phase induction motor running on full-load is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Determine :

(*i*) Input, (*ii*) Line current,

(*iii*) Power factor, and (*iv*) Output if the efficiency of the motor is 92%.

[**Ans.** 400 kW ; 303.5 A ; 0.756 (lag) ; 368 kW]

31. Two wattmeters are used for measuring the power input and the power factor of an over-excited synchronous motor. If the readings of the meters are – 2.0 kW and + 7.0 kW respectively, calculate the input and power factor of the motor. [**Ans.** 5 kW ; 0.3057 (leading)]

32. The power input to a 2-kV, 50 Hz, 3-phase motor running on full-load at an efficiency of 90 per cent is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate :

(*i*) Input, (*ii*) Power factor,

(*iii*) Line current, and (*iv*) Output.

[**Ans.** 400 kW ; 0.756 (lag) ; 153 A ; 360 kW]

33. In a balanced three-phase system power is measured by two-wattmeters method and the ratio of the two wattmeter readings is 2 : 1. Find the power factor of the system. [**Ans.** 0.866 (log)]

34. The power input of a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the power factor of the motor be changed to 0.866 leading, determine the readings of the two wattmeters, the total input power remaining the same. Draw the vector diagram for the second condition of load.

[**Ans.** 33.33 kW ; 66.67 kW]

35. Each phase of a 3-phase delta-connected load consists of an impedance $Z = 20 \angle 60° \ \Omega$. The line voltage is 440 V at 50 Hz. Calculate the power consumed by each phase impedance and the total power. What will be the readings of the two wattmeters connected ? [**Ans.** 4840 W ; 14520 W ; 14520 W. zero]

36. A 3-phase 3-wire, 415 V supplies a balanced load of 20 A at power factor 0.8 lagging. Two wattmeters are used to measure power. Calculate :

(*i*) Power, (*ii*) Reading of wattmeter No 1, and

(*iii*) Reading of wattmeter No. 2. [**Ans.** 11.5 kW ; 8240 W ; 3260 W]

37. A balanced load is supplied from a 3-phase, 400 V, 3-wire system whose power is measured by two wattmeters. If the total power supplied is 26 kW at 0.75 power factor lagging, find the readings of each of the two wattmeters. [**Ans.** 19.62 kW ; 6.38 kW]

38. A 3-phase 4-wire, star-connected system supplies only non-inductive load. The current in line *R* is 8 A, the current in line *Y* is 10 A and the current in line *B* is 6 A. The voltage from each line to neutral is 120 V. Find :

(*i*) Wattage shown by each of the three wattmeters, and

(*ii*) Power taken by the lighting load. [**Ans.** 960 W, 1200 W, 720 W ; 2880 W]

39. Three identical coils, each having a resistance of 20 Ω and reactance of 20 Ω are connected in (*i*) star ; and (*ii*) delta across 440 V, 3-phase supply. Calculate for each method of connection the line current and reading on each of the two wattmeters connected to measure power.

[**Ans.** 8.98 A ; 3817.5 W, 1022.5 W ; 26.95 A ; 12452.5 W ; 3067.5 W]

40. A balanced star-connected load, each having a resistance of 10 Ω and inductive reactance of 30 Ω is connected to a 400 V, 50 Hz supply. The phase sequence is RYB. Two wattmeters connected to read total power have their current coils connected in the red and blue lines respectively. Calculate the reading of each wattmeter. Draw the circuit and vector diagrams. [**Ans.** W_1 = – 585 W ; W_2 = 2185 W]

4

Measuring Instruments

1. INTRODUCTION AND CLASSIFICATION

The instruments used for all electrical measurements are called *measuring instruments*. They include *ammeters, voltmeters, wattmeters,* energy meters etc. The various electrical instruments may broadly be divided into two groups :

1. **Absolute instruments.** Absolute instruments are those instruments which indicate the quantity to be measured in terms of the *constants of the instrument* (dimensions, turns etc.) and in order to find out the quantity in the practical units it is necessary to *multiply such deflections with an instrument constant.* No previous calibration or comparison is necessary in this case. The most common absolute instrument is *tangent* galvanometer which gives the measured current in terms of tangent of the deflected angle, the radius and the number of turns of the galvanometer. Such instruments are rarely used (the use being merely confined within laboratories as standardizing instruments).

2. **Secondary instruments.** Secondary instruments are those in which the value of electrical quantity to be measured can be determined from the deflection of instrument only when they have been *pre-calibrated* by comparison with an absolute instrument. The deflection of the instrument gives directly the quantity to be measured. These instruments are most generally used in everyday work.

Secondary instruments may also be *classified* as follows :

1. **Indicating instruments.** Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured at the time at which it is being measured. Their indications are given by pointers moving over calibrated chief.

Example. *Ammeters, voltmeters* and *wattmeters*.

2. **Recording instruments.** Recording instruments are those which give a *continuous record* of the variations of an electrical quantity over a selected period of time. The pointer in these types of instruments is an infed pen which leaves a trace on a paper put over a moving drum.

3. **Integrating instruments.** Integrating instruments are those which measure the total quantity of electricity delivered in a particular time.

Example. *Ampere-hour and watt-hour meters*.

Electrical measuring instruments may also be *classified* as follows :

1. **According to the quantity being measured :**

Ammeters. ...... for measuring the magnitude of current.

Voltmeters. ...... for measuring voltages.

Ohmmeters and resistance bridges. ...... for measuring resistances.

Wattmeters. ...... for power measurements.

Watt-hour meters. ...... for energy measurements.

Frequency meters. ...... for frequency measurements.

Power factor meters. ...... for power-factor measurements.

2. **According to the kind of current :**

Instruments are classified into *D.C.*, *A.C.* and *A.C./D.C. instruments*.

3. **According to accuracy limits :**

4. **According to the principle of operation :**

Instruments are grouped into :

— Moving coil
— Moving iron
— Electrodynamic
— Induction
— Hot-wire
— Thermo-electric
— Rectifier types.

5. **According to the type of indication :**

— Instruments may be :
— Indicating type
— Recording type.

6. **According to application :**

— Switchboard
— Portable.

2. ELECTRICAL PRINCIPLES OF OPERATION

All electrical measuring instruments depend for their action on any of many physical effects of electric current or potential. The following are the effects generally used in the manufacture :

(*i*) **Magnetic effect.** ...... Voltmeters, ammeters, wattmeters, power factor meters etc.

(*ii*) **Thermal effect.** ...... Ammeters, voltmeters, maximum demand meters etc.

(*iii*) **Chemical effect.** ...... D.C. ampere hour meters (integrating meters).

(*iv*) **Electrostatic effect.** ...... Voltmeters which can indirectly be used as ammeters and wattmeters.

(*v*) **Electro-magnetic induction effect.** ...... Voltmeters, ammeters, wattmeters and integrating meters used in A.C. only.

3. ELECTRICAL INDICATING INSTRUMENTS

Almost invariably an indicating instrument is fitted with a pointer which indicates on a scale the value of the quantity being measured. The moving system of such an instrument is usually carried by a *spindle of hardened steel*, having its ends tapered and highly polished to form pivots which rest in hollow-ground bearings, usually of saphine, set in steel screws. In some instruments, the moving system is attached to *thin ribbons of spring material* such as beryllium-copper alloy, held taut by tension springs mounted on the frame of movement. This arrangement *eliminates pivot friction* and the instrument *is less susceptible to damage by shock or vibration*.

3.1. Essential Features

Indicating instruments possess **three essential features**.

1. **Deflecting device.** ...... whereby a mechanical force is produced by the electric current, voltage or power.

2. **Controlling device.** ...... whereby the value of deflection is dependent upon the magnitude of the quantity being measured.

3. **Damping device.** ...... to prevent oscillation of the moving system and enable the latter to reach its final position quickly.

1. Deflecting Device. A deflecting device produces a deflecting torque which is caused by anyone of the previously mentioned effects (*i.e.*, thermal effect, chemical effect, electrostatic effect etc.) ; with the help of this deflecting torque the needle or the pointer moves from zero position to the final position. The arrangement of the deflecting device with each type of instrument will be discussed individually.

2. Controlling Devices. There are two types of controlling devices :

(*i*) Spring control

(*ii*) Gravity control.

(*i*) **Spring control.** Fig. 1 shows a commonly used spring control arrangement. It utilises two spiral hair springs, 1 and 2, the inner ends of which are attached to the spindle *S*. The outer end of spring 2 is fixed while that of 1 is attached to a lever, the adjustment of which gives zero adjustments. The two springs 1 and 2 are wound in *opposite directions* so that when the moving system is deflected, one spring winds up while the other unwinds, and the controlling torque is due to the *combined torsions* of the springs.

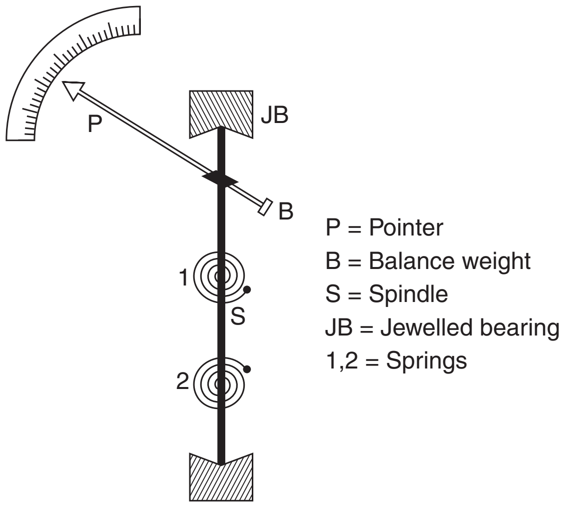


Fig. 1. Spring control.

Since the torsional torque of a spiral spring is proportional to the angle of twist, the controlling torque (T_c) is directly proportional to the angular deflection of the pointer (θ).

i.e., $$T_c \propto \theta.$$

The spring material should have the following **properties :**

(*i*) It should be non-magnetic.

(*ii*) It must be of low temperature co-efficient.

(*iii*) It should have low specific resistance.

(*iv*) It should not be subjected to fatigue.

(*ii*) **Gravity control.** With gravity control, weights L and M are attached to the spindle S [Fig. 2 (*a*)], the function of L being to *balance* the weight of the pointer P. Weight M therefore provides the controlling torque. When the pointer is at zero, M hangs vertically downwards. When P is deflected through angle θ, the controlling torque is equal to (weight of M × distance d) and is therefore proportional to the *sine* of the angular deflection [Fig. 2. (*b*)],

i.e., $$T_c \propto \sin \theta.$$

The degree of control is adjusted by screwing the weight up or down the carrying system.

It may be seen from Fig. 2 (*b*) that as θ approaches 90°, the distance 1-2 increases by a relatively small amount for a given change in the angle that when θ is just increasing from its zero value. Hence *gravity-controlled instruments have scales which are not uniform but are cramped or crowded at their lower ends.*

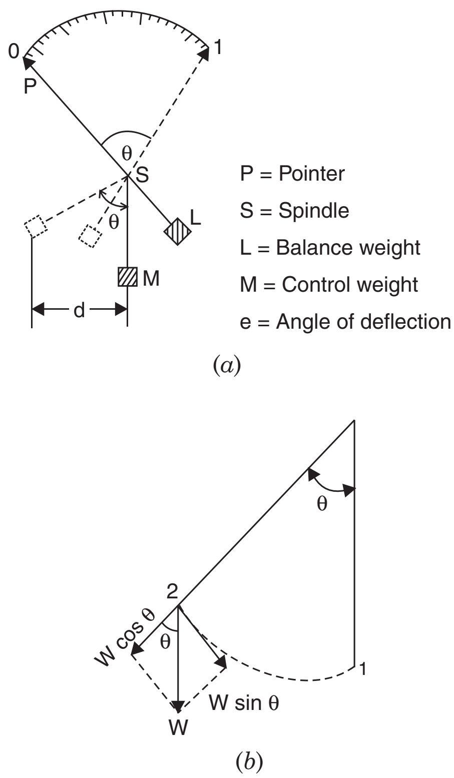


Fig. 2. Gravity control.

Advantages :

1. The gravity controlled instrument is cheaper than corresponding spring-controlled instrument.
2. It is not subjected to fatigue.
3. It is unaffected by temperature.

Disadvantages :

1. Gravity control gives a cramped scale.
2. The instrument must be levelled before use.

3. Damping Devices. Owing to the inertia of the moving system, when subjected to the deflecting and restoring torques, a number of vibration will be produced before coming finally to rest. To-avoid this, a *damping torque* is required which opposes the motion and ceases when the

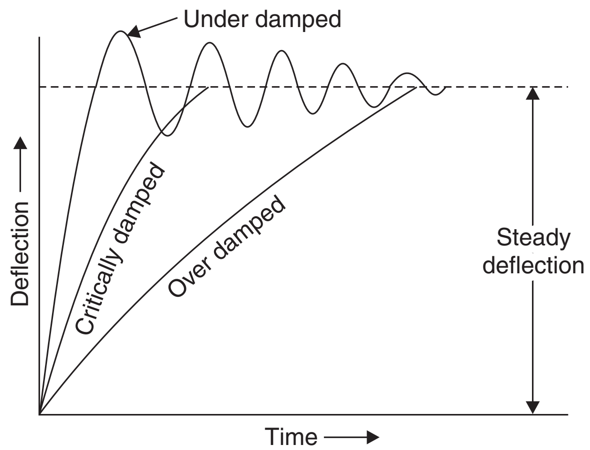


Fig. 3. Damping curves.

pointer comes to rest. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be *dead-beat.* If the instrument is over-damped the movement is very slow (and the instrument becomes lithargic) as shown in Fig. 3.

Damping can be provided by the following methods :

(*i*) Air damping.

(*ii*) Eddy current damping.

(*iii*) Fluid friction damping.

(*i*) **Air damping.** Fig. 4 shows an arrangement for obtaining air damping. It consists of a thin metal vane *MV* attached to the spindle *S* ; the vane moves in a sector-shaped box *B. Any tendency of the moving system to oscillate is damped by the action of the air on the vane.*

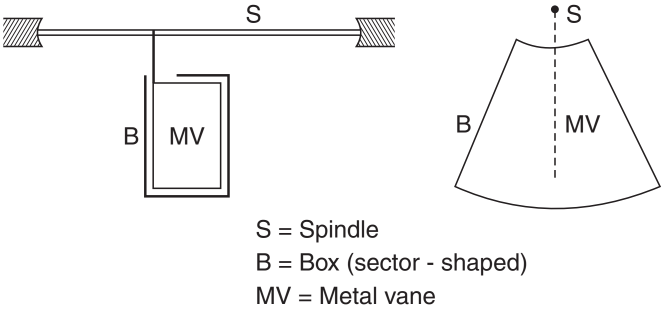


Fig. 4. Air damping.

(*ii*) **Eddy current damping.** This method of damping is based on the *principle* that when a conducting non-magnetic material is moved in a magnetic field an e.m.f. is induced in it which

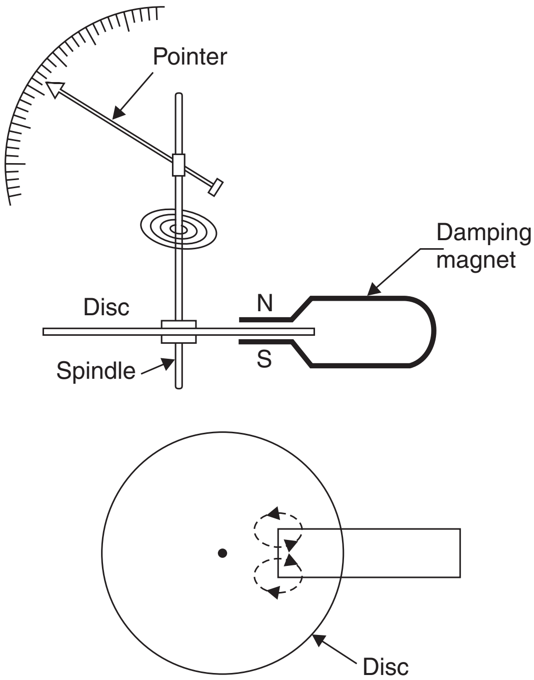


Fig. 5. Eddy current damping.

causes currents called the *eddy currents*. Due to these eddy currents a force exists between them and the field. Due to Lenz's law this force is always in opposition to the force causing rotation of the conducting material, thus, it provides the necessary damping.

- One form of eddy-current damping is shown in Fig. 5. Here a copper or aluminium disc, curried by a spindle, can move between the poles of a permanent magnet. If the disc moves clockwise, the e.m.f.'s induced in the disc circulate eddy currents as shown dotted. It follows from Lenz's law that these currents exert a force *opposing* the motion producing them, namely the clockwise movement of the disc.
- Another form of providing damping is used in the *moving coil instruments* using permanent magnet. The moving coil is mounted over a metallic former. When the coil is deflected eddy e.m.f.'s are induced in the two sides of the former, causing eddy forces as shown in Fig. 6.

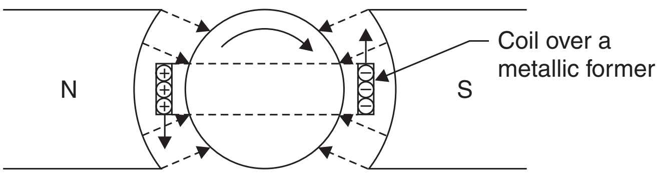


Fig. 6. Eddy current damping with permanent magnet.

(*iii*) **Fluid friction damping.** Fig. 7 shows the method of fluid friction damping. Here light vanes are attached to the spindle of the moving system. The vanes are dipped into a pot of damping oil and are completely submerged by the oil. The motion of the moving system is always opposed by the friction of the damping oil on the vanes. The damping force thus created always increases with the increase in velocity of vanes. There is no damping force when the vanes are stationary.

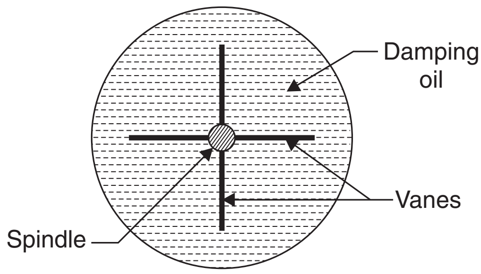


Fig. 7. Fluid friction damping.

The damping oil used must have the following properties :

(*i*) Must be a good insulator.

(*ii*) Should be non-evaporating.

(*iii*) Should not have corrosive action upon the metal of the vane.

(*iv*) The viscosity of the oil should not change with the temperature.

Though in this method of damping, no case is required as in the air friction damping but it is not much used due to the following **disadvantages :**

(*i*) Objectionable creeping of oil.

(*ii*) Using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The principle types of electrical indicating instruments, together with the methods of control and damping, are summarized below :

S. No.	*Type of instrument*	*Suitable for measuring*	*Method of control*	*Method of damping*
1.	Moving-iron	Current and voltage, D.C. and A.C.	Hair springs	Air
2.	Permanent magnet moving coil	Current and voltage, D.C. only	Hair springs	Eddy current
3.	Thermocouple	Current and voltage, D.C. and A.C.	As for moving coil	As for moving coil
4.	Electro-dynamic or dynamometer	Current, voltage and power, D.C. and A.C.	Hair springs	Air
5.	Electrostatic	Voltage only, D.C. and A.C.	Hair springs	Air or eddy current
6.	Rectifier	Current and voltage, A.C. only	As for moving coil	As for moving coil

Note. Apart from the electrostatic type of voltmeter, all voltmeters are in effect milliammeters connected in series with non-reactive resistor having a high resistance.

Difference between an ammeter and a voltmeter

An ammeter and a voltmeter work on the same principle. The ammeter has a *low resistance* so that when it is connected in **series** with any circuit, it does not change the current. The voltmeter has a *high resistance* and it is so designed that when connected in **parallel** to the circuit for measuring voltages it does not take appreciable current.

An ammeter of low range can be used as a voltmeter by connecting an external resistance in series with it.

AMMETERS AND VOLTMETERS

4. MOVING-IRON INSTRUMENTS (AMMETERS AND VOLTMETERS)

Moving-iron instruments are commonly used in laboratories and switchboards at commercial frequencies because they are *very cheap and can be manufactured with required accuracy.*

Moving-iron instruments can be divided into two types :

1. Attraction type ...... in which a sheet of soft iron is *attracted towards a solenoid.*

2. Repulsion type ...... in which two parallel rods or strips of soft iron, magnetised inside a solenoid, are regarded as *repelling each other.*

4.1. Attraction Type

Fig. 8 shows the sectional front and an end elevation of the attracted-iron type instrument. It consists of a solenoid (or coil) C and oval shaped soft-iron disc D in such a way that it can move in or out of the solenoid. To this iron a pointer P is attached so that it may deflect along with the moving iron over a graduated scale. The soft-iron disc is made of sheet metal specially shaped to give a scale as nearly uniform as possible.

When the current to be measured (or a definite fraction of the current to be measured or proportional to the voltage to be measured) is passed through the solenoid, a magnetic field is set up inside the solenoid, which in turn magnetises the iron. Thus the soft-iron disc is attracted into the solenoid/coil, causing the spindle and the pointer to rotate. *Damping* is provided by vane *V* attached to the spindle and moving in an *air chamber,* and *control* is by *hair spring.*

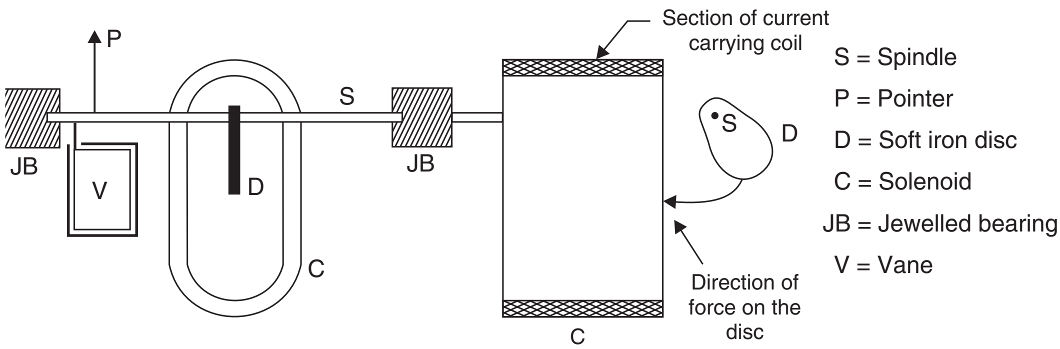


Fig. 8. Attraction-type moving-iron instrument.

4.2. Repulsion Type

Repulsion-type moving-iron instrument is shown in Fig. 9. Here there are two irons, one fixed (*A*) and the other mounted on a short arm fixed (*B*) to the instrument spindle. The two irons lie in the magnetic field due to a solenoid/coil *C*. When there is no current in

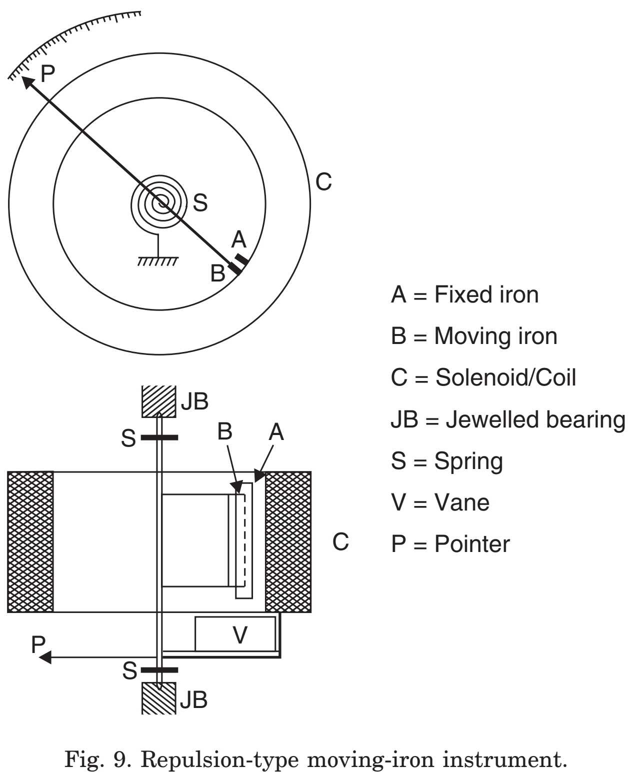


Fig. 9. Repulsion-type moving-iron instrument.

the coil the two iron pieces (moving one and fixed one) are almost touching each other and the pointer rests on zero position. When the current to be measured (or a definite fraction of it or proportional to the voltage to be measured) is passed through the solenoid, a magnetic field is set up inside the solenoid and the two iron pieces are **magentised in the same direction.** This sets up a **repulsive force** so moving iron piece, is repelled by fixed iron piece, thereby results in the motion of the moving iron piece, carrying the pointer. The pointer comes to rest in a deflected position when equilibrium is attained between the repulsive forces of the working elements and the controlling force.

Such instruments are commonly provided with *spring control* and *air friction damping.*

In commercial instruments, it is usual for the moving-iron B to be in the form of a *thin curved plate* and for the fixed iron A to be a tapered curved sheet. This construction can be arranged to give *a longer and more uniform scale* than is possible with the rods shown in Fig. 9.

Deflecting torque in moving-iron instruments :

In both the attraction and repulsion type moving-iron instruments it is found that for a given position of the moving system, the value of the *deflecting torque is proportional to the square of the current,* so long as the *iron is working below saturation.* Hence, if the current waveform is as shown in Fig. 10, the variation of the deflecting torque is represented by the dotted wave. If the

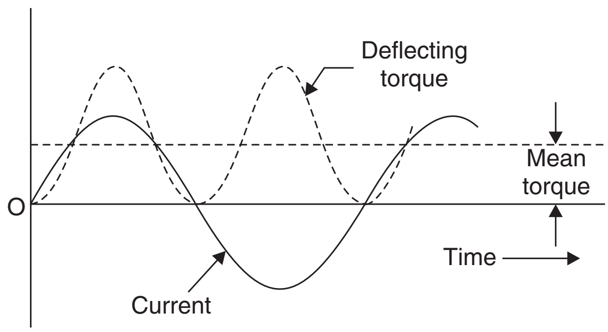


Fig. 10. Deflection torque in a moving-iron instrument.

supply frequency is 50 Hz, the torque varies between zero and a maximum 100 times a second, so that the moving system (due to its inertia) takes up a position corresponding to the mean torque, where

$$\text{mean torque} \propto \text{mean value of the square of the current} = kI^2$$

where k = a constant for a given instrument

and I = r.m.s. value of the current.

Thus the moving-iron instrument can be used to *measure both direct current and alternating current,* and in the latter case the instrument gives the r.m.s. value of the current.

Owing to the deflecting torque being proportional to the square of the current, the scale divisions are not uniform, being cramped at the beginning and open at the upper end of the scale.

Note. For both types of instruments (attraction-type and repulsion type) the necessary magnetic field is produced by the *ampere-turns of a current-carrying coil.*

— In case the instrument is to be used as an *ammeter,* the coil has *comparatively few turns of thick wire so* that the ammeter has *low resistance* because it is connected in *series* with the circuit.

— In case it is to be used as a *voltmeter,* the coil has *high impedance* so as to draw as small a current as possible since it is connected in *parallel with the circuit.* As current through the coil is small it has large *number of turns* in order to produce sufficient ampere-turns.

4.3. Advantages and Disadvantages of Moving-Iron Instruments

Advantages :

1. Can be used both in D.C. as well as in A.C. circuits.
2. Robust and simple in construction.
3. Possess high operating torque.
4. Can withstand overload momentarily.
5. Since the stationary parts and the moving parts of the instrument are simple so they are cheapest.
6. Suitable for low frequency and high power circuits.
7. Capable of giving an accuracy within limits of both precision and industrial grades.

Disadvantages :

1. Scales not uniform.
2. For low voltage range the power consumption is higher.
3. The errors are caused due to hysteresis in the iron of the operating system and due to stray magnetic field.
4. In case of A.C. measurements, change in frequency causes serious error.
5. With the increase in temperature the stiffness of the spring decreases.

4.4. Sources of Errors

A. Errors with both D.C. and A.C.

(*i*) **Errors due to hysteresis.** This source of error is due to *hysteresis* in the soft iron moving part, due to which too high values are recorded by the instrument, when the current is increasing and too low readings are liable to be indicated when the current is decreasing.

(*ii*) **Errors due to stray fields.** External stray magnetic fields are liable to affect adversely the accurate functioning of the instrument. Magnetic shielding of the working parts is obtained by using a covering case of cast iron.

B. Errors with A.C. only

Errors may be caused due to change in frequency because change in frequency produces (*i*) *change in impedance of the coil* and (*ii*) *change in magnitude of eddy currents.* The error due to the former is negligible in ammeters, as the coil current is determined by the external circuit and the error due to the latter can normally be made small.

5. MOVING-COIL INSTRUMENTS

The moving-coil instruments are of the following two types :

1. **Permanent-magnet type**......can be used for D.C. only.
2. **Dynamometer type**......can be used both for A.C. and D.C.

5.1. Permanent-magnet Moving-Coil Type (PMMC) Instruments

A permanent-magnet moving coil-type instrument works on the principle that *"when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field".*

Construction

- The instrument consists of a permanent magnet *M* and a rectangular coil *C* which consists of insulated copper wire wound on light aluminium frame fitted with polished steel pivots resting in jewel bearings. The magnet is made of Alnico and has soft-iron pole-pieces *PP* which are bored out cylindrically.
- The rectangular coil *C* is free to move in air gaps between the soft-iron pole pieces and a soft-iron cylinder *A* (central core), supported by a brass plate (not shown).

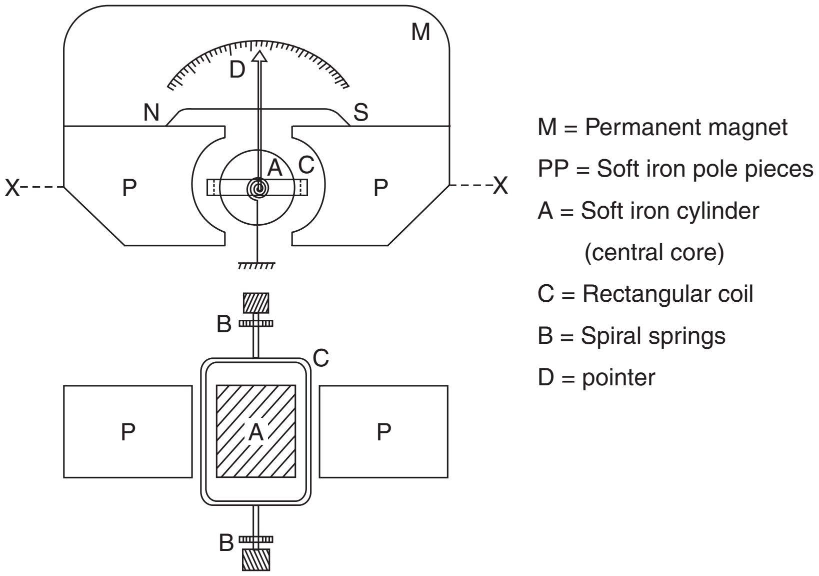


Fig. 11. Permanent-magnet moving-coil instrument.

The *functions of the central core A* are :

(*i*) To intensify the magnetic field by reducing the length of air gap across which the magnetic flux has to pass.

(*ii*) To give a radial magnetic flux of uniform density, thereby enabling the scale to be uniformly divided.

— The movement of the coil is controlled by two phosphor bronze hair springs *BB* (one above and one below), which additionally serve the purpose of leading the current in and out of the coil. The two springs are spiralled in *opposite directions* for neutralizing the effects of changes in temperature.

— The aluminium frame not only provides support for the coil but also *provides damping by eddy currents induced in it.*

Deflecting torque. Refer to Fig. 12. When current is passed through the coil, forces are set up on its both sides which produce *deflection torque.* If *I* amperes is the current passing through the coil, the magnitude of the force (*F*) experienced by each of its sides is given by

$$F = BIl \text{ newton}$$

where B = flux density in WB/m^2, and

l = length or depth of coil in metres.

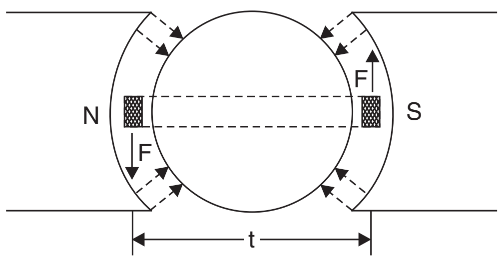


Fig. 12

For N turns, the force on each side of the coil is

$$= NBIl \text{ newton}$$

∴ Deflecting torque (T_d)

$$= \text{force} \times \text{perpendicular distance}$$
$$= NBIl \times b = NBI\,(l \times b) = NBIA \text{ Nm}$$

where b = breadth of the coil in metres, and

A = face area of the coil.

If B is constant, then

$$T_d \propto I \qquad (\text{i.e., current passing through the coil})$$
$$= kI \text{ where } k \text{ is a constant for a given instrument.}$$

Since such instruments are invariably spring controlled, the controlling torque (T_c) of the spiral springs ∝ angular deflection

i.e., $$T_c \propto \theta$$

or $$T_c = c\,\theta$$

where c = a constant for given springs, and

θ = angular deflection.

For a steady deflection,

Controlling torque (T_c) = deflecting torque (T_d)

Hence $$c\theta = kI$$

∴ $$\theta = \frac{k}{c}\,I$$

i.e., the *deflection is proportional to the current* and the *scale is therefore uniformly divided.*

Advantages and Disadvantages. The moving-coil permanent-magnet type instruments have the following advantages and disadvantages :

Advantages :

(*i*) Low power consumption.

(*ii*) Their scales are uniform.

(*iii*) No hysteresis loss.

(*iv*) High torque/weight ratio.

(*v*) They have very effective and efficient eddy-current damping.

(*vi*) Range can be extended with shunts or multipliers.

(*vii*) No effect of stray magnetic field as intense polarised or unidirectional field is employed.

Disadvantages :

(*i*) Somewhat costlier as compared to moving-iron instruments.

(*ii*) Cannot be used for A.C. measurements.

(*iii*) Friction and temperature might introduce errors as in case of other instruments.

(*iv*) Some errors are set in due to the ageing of control springs and the permanent magnets.

Ranges :

D.C. Ammeters

(*i*) *Without shunt*......0/5 micro-amperes up to 0/30 micro-amperes.

(*ii*) *With internal shunts*......upto 0/2000 amperes.

(*iii*) *With external shunts*......upto 0/5000 amperes.

D.C. Voltmeters

(*i*) *Without series resistance*......0/100 milli-volts.

(*ii*) *With series resistance*......upto 20000 or 30000 volts.

— Moving-coil permanent-magnet instruments can be used as :

(*i*) Ammeters......by using a low resistance shunt.

(*ii*) Voltmeters......by using a high series resistance.

(*iii*) Flux-meters......by eliminating the control springs.

(*iv*) Basllistic galvanometers......by making control springs of large moment of inertia.

Extension of Range. The following devices may be used for extending the range of instruments : (*i*) Shunts (*ii*) Multipliers (*iii*) Current transformers and (*iv*) Potential transformers. Use of ammeter shunts and voltage multipliers is discussed below.

1. Ammeter shunts. An ammeter shunt is merely a *low resistance* that is placed in parallel with the coil circuit of the instrument in order to *measure fairly large currents.* The greater part of the current in the main circuit is then diverted around the coil through the shunt. The connection diagram for a shunt and milliammeter for measuring large currents is shown in Fig. 13.

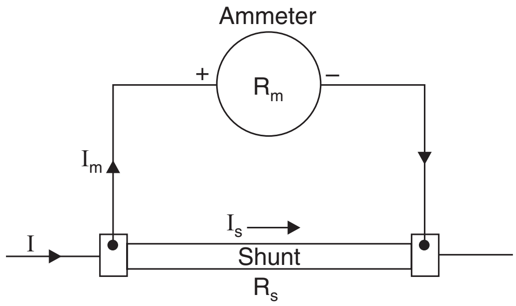


Fig. 13. Use of a shunt for measuring large currents.

The shunt is provided with four terminals, the milliammeter being connected across the potential terminals. If the instrument were connected across the current terminals, there might be considerable error due to the contact resistance at these terminals being appreciable compared with the resistance of the shunt.

Let I = current of the circuit to be measured,

I_m = current passing through the ammeter,

I_s = current passing through the shunt,

R_s = resistance of the shunt, and

R_m = resistance of the ammeter (plus its leads to the potential terminals).

Thus $$I = I_m + I_s \qquad ...(i)$$

Also, since the voltage drop across the shunt and the instrument in same,

$$\therefore \quad I_s R_s = I_m R_m$$

or $$R_s = \frac{I_m}{I_s} R_m$$

Substituting $I_s = I - I_m$ from equation (*i*), we get

$$R_s = \frac{I_m}{(I - I_m)} . R_m$$

or $$R_s = \frac{R_m}{\left(\frac{I}{I_m} - 1\right)} \qquad ...(ii)$$

The ratio of $\frac{I}{I_m}$ is known as '*multiplying power*' of the shunt. Denoting, the ratio by N, we have shunt resistance,

$$R_s = \frac{R_m}{N - 1} \qquad ...(iii)$$

The shunts are made of a material such as *manganin* (copper, manganese and nickel), which has a *negligible temperature coefficient of resistance.* The material, is employed in the form of thin strips, the ends of which are soldered to two large copper blocks. Each copper block carries two terminals-one current terminal and other potential terminal. The strips which form the shunt are spaced from each other to promote a good circulation of air and thus efficient cooling.

Note. A '*swamping*' resistor *r*, of material having negligible temperature co-efficient of resistance, is connected in series with the instrument (moving coil). The latter is wound with copper wire and the function of *r* is to *reduce* the error due to variation of resistance of the instrument with variation of temperature.

2. Voltmeter multipliers. The range of the instrument, when used as a voltmeter can be extended or multiplied by using a high non-inductive series resistance *R* connected in series with it as shown in Fig. 14.

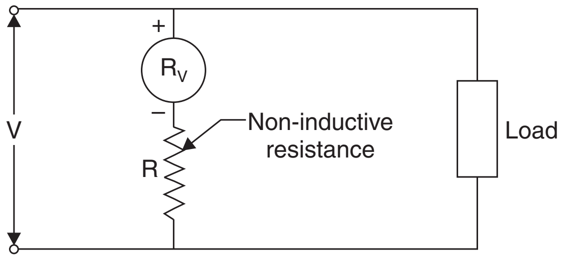


Fig. 14

Let I = full scale deflection current of voltmeter,

V = voltage of the circuit to be measured,

R_v = resistance of the voltmeter, and

R = external series resistance.

Now, voltage across supply leads

= voltage drop across the voltmeter + voltage drop across external resistance

$$\therefore \quad V = IR_v + IR$$

or

$$R = \frac{V - IR_v}{I} = \frac{V}{I} - R_v$$

The voltage multipliers to be used for D.C. measurements should satisfy the following requirements :

(*i*) The resistance should not change with time of usage.

(*ii*) The temperature co-efficient of resistance must be very low.

Note. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting *R* by a capacitor.

Example 1. *A milliammeter of 2.5 ohms resistance reads upto 100 milli amperes. What resistance is necessary to enable it to be used as :*

(*i*) *A voltmeter reading upto 10 V.*

(*ii*) *An ammeter reading upto 10 A.*

Draw the connection diagram in each case.

Solution. Resistance of the milliammeter, R_m = 2.5 Ω

Maximum current of the milliammeter, I_m = 100 mA = 0.1 A.

(*i*) Voltage to be measured, V = 10 volts

Resistance to be connected in series.

$$\mathbf{R} = \frac{V}{I_m} - R_m = \frac{10}{0.1} - 2.5 = \mathbf{97.5\ \Omega}. \quad \textbf{(Ans.)}$$

Connection diagram is shown in Fig. 15.

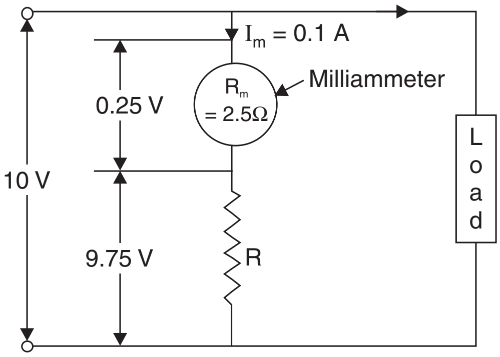


Fig. 15

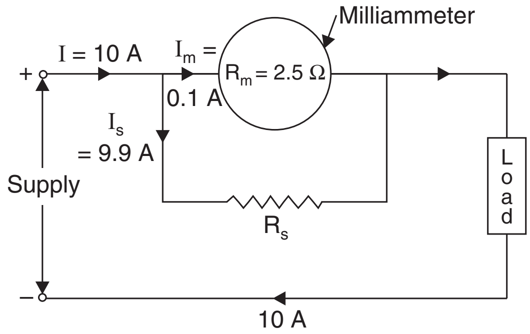


Fig. 16

(*ii*) Current to be measured, I = 10 A

Multiplying power of the shunt, $N = \frac{I}{I_m} = \frac{10}{0.1} = 100$

∴ Resistance to be connected in parallel,

$$R_s = \frac{R_m}{N-1} = \frac{2.5}{100-1} = \mathbf{0.02525\ \Omega.\ (Ans.)}$$

Connection diagram is shown in Fig. 16.

Example 2. *A moving-coil milliammeter having a resistance of 10 ohms gives full scale deflection when a current of 5 mA is passed through it. Explain how this instrument can be used for measurement of :*

(*i*) *Current upto 1 A.* (*ii*) *Voltage upto 5 V.*

Solution. Resistance of the milliammeter, R_m = 10 Ω

Full scale deflection current, I_m = 5 mA = 0.005 A.

(*i*) **To measure current upto 1 A :**

Resistance of the shunt, R_s : (Refer to Fig. 17)

Since voltage drop across the milliammeter and the shunt are equal

∴ $$I_m R_m = I_s R_s = (I - I_m) R_s$$

∴ $$\mathbf{R}_s = \frac{I_m R_m}{(I - I_m)} = \frac{0.005 \times 10}{(1 - 0.005)} = \mathbf{0.05025\ \Omega.\ (Ans.)}$$

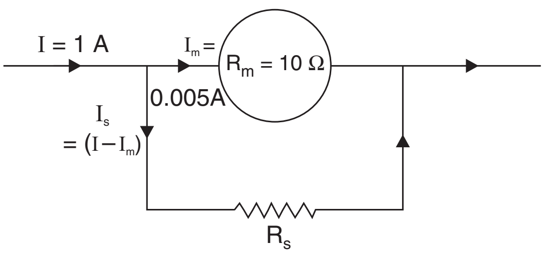


Fig. 17

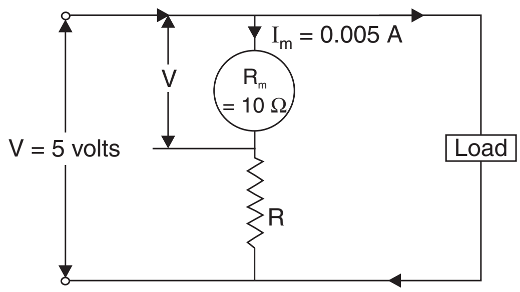


Fig. 18

(*ii*) **To measure voltage upto 5 V :**

The value of external series resistance, R : (Refer to Fig. 18)

Now, voltage across supply loads

= voltage drop across the milli-ammeter

+ voltage drop across external series resistance R

$\therefore \quad V = I_m R_m + I_m R$

or $\quad 5 = 0.005 \times 10 + 0.005\, R$

or $\quad \mathbf{R} = \dfrac{5 - 0.005 \times 10}{0.005} = \mathbf{990\ \Omega}.$ **(Ans.)**

Example 3. *If the moving coil of a voltmeter consists of 100 turns wound on a square former which has a length of 30 mm and the flux density in the air gap is 0.09 Wb/m^2, calculate the turning moment on the coil when it is carrying a current of 10 mA.*

Solution. Number of turns, $N = 100$

Length of each side, $l = 30$ mm $= 0.03$ m

Flux density, $B = 0.09$ Wb/m^2

Current through the coil, $I = 10$ mA $= 0.01$ A

We know that the force on each side of the coil,

$F = NBIl$ newton

$\therefore$ Turning moment (*i.e.*, deflecting torque),

$T = F \times \text{breadth} = F \times l = NBIl^2$ N-m

$= 100 \times 0.09 \times 0.01 \times (0.03)^2$ N-m

$= \mathbf{8.1 \times 10^{-5}}$ **Nm. (Ans.)**

Example 4. *A moving-coil instrument has a resistance of 5 Ω between terminals and full-scale deflection is obtained with a current of 0.015 A. This instrument is to be used with a manganin shunt to measure 100 A full scale. Calculate the error caused by a 20°C rise in temperature.*

(*i*) *When the internal resistance of 5 Ω is due to copper only.*

(*ii*) *When a 4 Ω manganin swamping resistor is used in series with a copper resistor of l Ω.*

The temperature-resistance co-efficients are :

Copper : $\alpha_c = 0.4\%$ *per °C, Manganin* : $\alpha_m = 0.015\%$ *per °C.*

Solution. Resistance of the instrument, $R_m = 5\ \Omega$

Current through the instrument, $I_m = 0.015$ A

Current to be measured, $I = 100$ A

Current through the shunt, $I_s = I - I_m = 100 - 0.015 = 99.985$ A

Voltage across the shunt $I_m R_m = 5 \times 0.015 = 0.075$ V

$\therefore$ Shunt resistance, $R_s = \dfrac{0.075}{99.985} = 0.00075\ \Omega \qquad [\because\ I_m R_m = (I - I_m) R_s]$

Shunt resistance after a rise of 20°C

$= 0.00075\,(1 + 20 \times 0.00015) = 0.000752\ \Omega.$

(*i*) The instrument resistance (which is wholly copper) after a rise of 20°C

$= 5(1 + 20 \times 0.004) = 5.4\ \Omega$

Hence, current through the instrument corresponding to 100 A in the line

$= \dfrac{0.000752}{(5.4 + 0.000752)} \times 100 = 0.01392$ A

Reading of the instrument $= 0.01392 \times 100/0.015 = 92.8$ A

$\therefore$ **Percentage error** $= 100 - 92.8 =$ **7.2. (Ans.)**

(*ii*) The instrument resistance after a rise of 20°C

$$= 1(1 + 20 \times 0.004) + 4(1 + 20 \times 0.00015)$$
$$= 1.08 + 4.012 + 5.092\ \Omega$$

Instrument current with a line current of 100 A

$$= \frac{0.000752}{(5.092 + 0.000752)} \times 100 = 0.01476\ \text{A}$$

$\therefore$ Instrument reading $= 0.01476 \times \frac{100}{0.015} = 98.4$ A

Percentage error = 100 – 98.4 = **1.6. (Ans.)**

Example 5. *The coil of a 250 V moving iron voltmeter has a resistance of 500 Ω and inductance of 1 H. The current taken by the instrument when placed on 250 V, D.C. supply is 0.05 A. Determine the percentage error when the instrument is placed on 250 V, A.C. supply at 100 Hz.*

Solution. Total ohmic resistance, $R = \frac{250}{0.05} = 500\ \Omega$

(Original calibration of the instrument is with direct current)

Reactance of the coil, $X_L = 2\pi fL = 2\pi \times 100 \times 1 = 628\ \Omega$

Coil impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(5000)^2 + (628)^2} = 5039\ \Omega$

$\therefore$ Voltage reading on A.C. $= \frac{250 \times 5000}{5039} = 248$ V

$\therefore$ Error = 248 – 250 = – 2 V

Percentage error $= \frac{2}{250} \times 100 =$ **0.8. (Ans.)**

Example 6. *A 15 volt moving iron voltmeter has a resistance of 300 Ω and an inductance of 0.12 H. Assuming that this instrument reads correctly on D.C., what will be its readings on A.C. at 15 volts when frequency is (i) 25 Hz and (ii) 100 Hz ?*

Solution. On *D.C., only ohmic resistance* is involved and the voltmeter reads correctly. But on *A.C.*, it is the *impedance* of the instrument which has to be taken into account.

(*i*) **When frequency is 25 Hz :**

Impedance at 25 Hz, $Z = \sqrt{R^2 + X_L^2}\sqrt{R^2 + (2\pi fL)^2}$

$$= \sqrt{(300)^2 + (2\pi \times 25 \times 0.12)^2} = 300.6\ \Omega$$

$\therefore$ **Voltmeter reading** $= 15 \times \frac{300}{300.6} =$ **14.97 V. (Ans.)**

(*ii*) **When frequency is 100 Hz :**

Impedance at 100 Hz, $Z = \sqrt{(300)^2 + (2\pi \times 100 \times 0.12)^2} = 309.3\ \Omega$

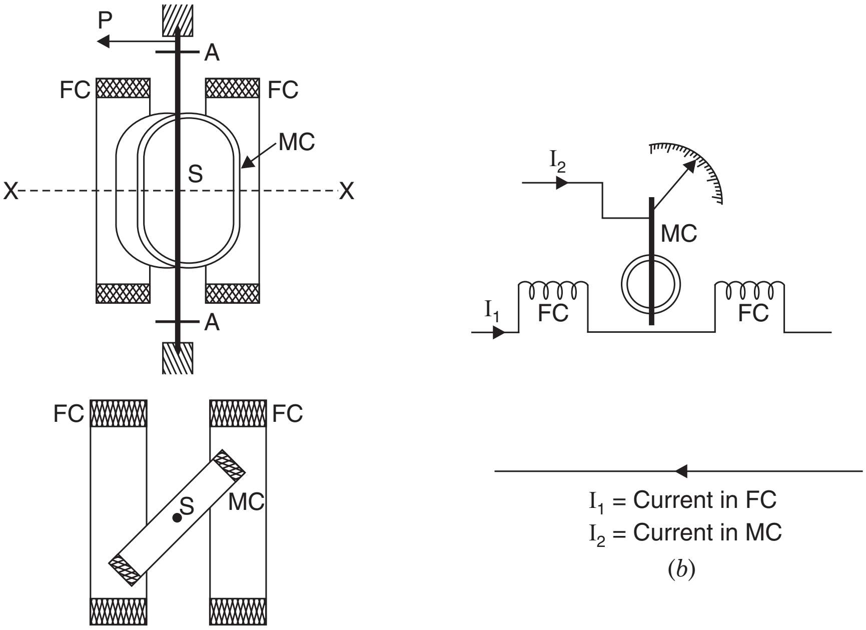
$\therefore$ **Voltmeter reading** $= 15 \times \frac{300}{309.3} =$ **14.55 V. (Ans.)**

Incidently, it may be noted that as the frequency is increased, the impedance of the voltmeter is also increased. Hence, the current is decreased and, therefore, the voltmeter readings are lower.

5.2. Electrodynamic or Dynamometer Instruments

In an electrodynamic instrument the *operating field is produced by another fixed coil* and *not* by *permanent magnet*. This instrument can be used as an ammeter or as voltmeter but is *generally used as a wattmeter*.

Refer to Figs. 19 (*a*) and (*b*).



I_1 = Current in FC
I_2 = Current in MC

(*b*)

FC = Field coils (divided into two halves)
MC = Moving coils
S = Spindle
A = Spiral hair springs

(*a*)

Fig. 19. Electrodynamic or dynamometer instrument.

These instruments essentially consist of fine wire *moving coil* placed in the magnetic field produced by *another fixed coil* when carrying currents. The coils are usually *air cored* to *avoid hysteresis, eddy currents and other errors* when the instrument is used on *A.C*. The *fixed coil FC* is divided into *two halves* placed close together and *parallel* to each other in order to provide a *fairly uniform field* within the range of the movement of the moving coil.

The upper diagram in Fig. 19 (*a*) shows a sectional elevation through fixed coil *FC* and the lower diagram represents a sectional plan on *XX*. The moving coil *MC* is carried by a spindle *S* and the controlling torque is exerted by spiral hair springs *A*, which may also serve to lead the current into and out of *MC*.

Deflecting torque. The deflecting torque is due to *interaction of the magnetic fields produced by currents in the fixed and moving coils.*

- Fig. 20 (*a*) shows the magnetic field due to current flowing through *FC* (I_1) in the direction indicated by the dots and cross.
- Fig. 20 (*b*) shows the magnetic field due to current (I_2) in *MC*.
- Fig. 20 (*c*) shows the combined effect of the above magnetic fields. By combining these magnetic fields it will be seen that when currents (I_1 and I_2) flow simultaneously through *FC* and *MC*, the resultant magnetic field is *distorted* and effect is to exert a clockwise torque on *MC*.

Since *MC* is carrying current (I_2) at right angles to the magnetic field produced by *FC*,

deflecting torque, $T_d \propto I_1 \times I_2$

or $T_d = KI_1I_2$, where K is a constant.

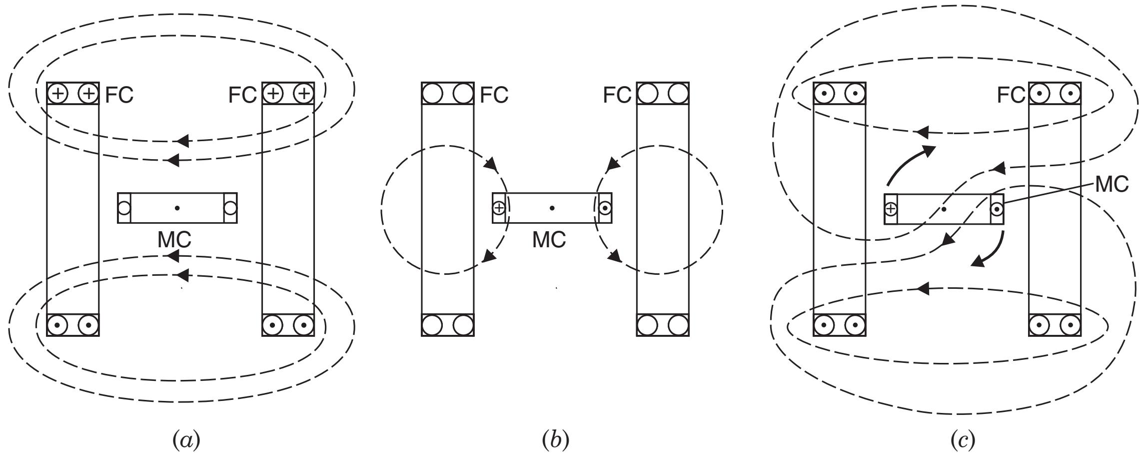


Fig. 20. Magnetic fields due to fixed and moving coils.

Since the instrument is spring-controlled, the restoring of control torque (T_c) is proportional to the angular deflection θ.

$\therefore$ $T_c \propto \theta$ or $T_c = K'\,\theta$

The two torques (T_d and T_c) are equal and opposite in the final deflected position.

$\therefore$ $T_d = T_c$

or $KI_1I_2 = K'\,\theta$

or $\theta \propto I_1I_2$

Use of the instrument as an ammeter. When the instrument is used as an ammeter then same current passes through both moving coil (*MC*) and fixed coils (*FC*) as shown in Fig. 21. In this case, $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 21 are used when *small currents* are to be measured.

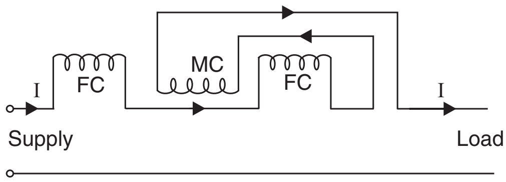


Fig. 21. Measurement of small currents.

In the case of *heavy currents,* a shunt is used to limit current through the moving coil as shown in Fig. 22.

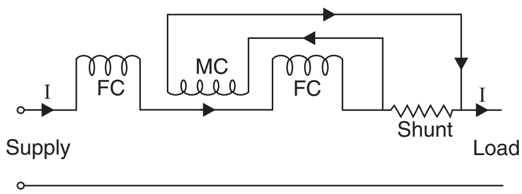


Fig. 22. Measurement of heavy currents.

Use of the instrument as voltmeter. When the instrument is used as a voltmeter, the fixed and moving coils are used in series along with a high resistance as shown in Fig. 23.

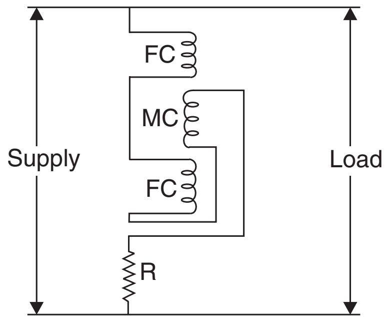


Fig. 23. Use of the instrument as a voltmeter.

Here again $I_1 = I_2 = I,$

where $I = \frac{V}{R}$ ......in D.C. circuits

and $I = \frac{V}{Z}$ ......in A.C. circuits

$\therefore$ $\theta \propto V \times V$ or $\theta \times V^2$

or $V = \sqrt{\theta}$

Thus whether the instrument is used as an ammeter or voltmeter its scale is *uneven* through the whole of its range and is cramped or crowded near the zero in particular.

Note. When the dynamometer instrument is used to measure an alternating current or voltage, the moving coil-due to its inertia-takes up a position where the average deflecting torque over one cycle is balanced by the restoring torque of the spiral springs. For that position, the deflecting torque is proportional to the mean value of the square of current or voltage, and the instrument scale can therefore be *calibrated to read the r.m.s. value.*

- In these instruments the damping is pneumatic (*i.e.*, air damping). Eddy current damping is admissible owing to weak operating field.

Ranges :

Ammeters. (*i*) With fixed and moving coils in series......0/0.01 A–0/0.05 A

(*ii*) With moving coil shunted or parallel connections......upto 0/30 A.

Voltmeters. Upto .0–750 volts.

Advantages and Disadvantages

Advantages :

(*i*) Can be used on both D.C. as well as A.C. systems.

(*ii*) They are free from hysteresis and eddy current errors.

(*iii*) It is possible to construct ammeters upto 10 A and volt-meters upto 600 V with precision grade accuracy.

Disadvantages :

(*i*) Since torque/weight ratio is small, such instruments have low sensitivity.

(*ii*) The scale is not uniform because $\theta \propto \sqrt{I}$.

(*iii*) Cost of these instruments is higher in comparison to those of moving iron instruments. So, these are only used as voltmeters and ammeters for precision measurements.

(*iv*) Higher friction losses.

6. WATTMETERS

A wattmeter is a combination of an ammeter and a voltmeter and, therefore consists of two coils known as *current coil* and *pressure coil*. The operating torque is produced due to interaction of fluxes on account of currents in current and pressure coils.

There are following three types of wattmeters :

1. Dynamometer wattmeter
2. Induction wattmeter
3. Electrostatic wattmeter.

We shall discuss here only the first and second type.

6.1. Dynamometer Wattmeter

In Fig. 24, the dynamometer is connected as a wattmeter. This is one of the advantages of this type of meter. If the coils are connected so that a value of current *proportional to the load voltage* flows in one, and a value of current *proportional to the load current* flows in the other,

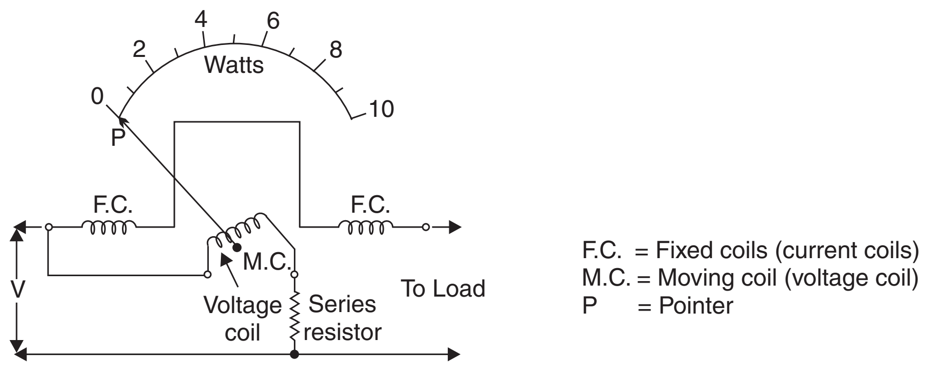


Fig. 24. Connection of dynamometer for measuring power.

the meter may be calibrated directly in watts. This is true because the indication depends upon the product of the two magnetic fields. The *strength of the magnetic fields depends upon the values of currents flowing* through the coils. If one current is proportional to load voltage and other current is the load current, then the meter can be calibrated in terms of watts or true power consumed by the load.

Let v = supply voltage,

i = load current, and

R = resistance of the moving coil circuit.

Current through fixed coils, $i_f = i$.

Current through the moving coil, $i_m = \dfrac{v}{R}$

Deflecting torque, $T_d \propto i_f \times i_m \propto \dfrac{iv}{R}$.

— For a *D.C. circuit the deflecting torque is thus proportional to the power.*

— For any circuit with *fluctuating torque, the instantaneous torque is proportional instantaneous power.* In this case due to inertia of moving parts the deflection will be proportional to the average torque *i.e.*, the deflection will be proportional to the *average power.* For sinusoidal alternating quantities the average power is $VI \cos \phi$, where

V = r.m.s. value of voltage,

I = r.m.s. value of current, and

ϕ = phase angle between V and I.

Hence an electrodynamic instrument, when connected as shown in Fig. 24, indicates the power, irrespective of the fact it is connected in an A.C. or D.C. circuit.

- Scales of such wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection, hence $\theta \propto$ power. Damping is *pneumatic*.

Errors :

- The error may creep in due to the inductance of the moving or voltage coil. However, the high non-inductive resistance connected in series with coil swamps, to a great extent, the phasing effect of the voltage coil inductance.
- There may be error in the indicated power due to the following :

(*i*) Some voltage drop in the current circuit.

(*ii*) The current taken by the voltage coil.

This error, however, in standard wattmeters may be overcome by having an additional compensating winding connected in series with the voltage coil but is so placed that it produces a field in opposite direction to that of the fixed or current coils.

Ranges :

(*i*) **Current circuit.** 0.25 to 100 A without employing current transformers.

(*ii*) **Potential circuit.** 5 to 750 V without employing potential transformers.

Advantages :

(*i*) The scale of the instrument is uniform (because deflecting torque is proportional to true power in both the cases *i.e.*, D.C. and A.C. and the instrument is spring controlled.)

(*ii*) High degree of accuracy can be obtained by careful design, hence these are used for calibration purposes.

Disadvantages :

(*i*) The error due to the inductance of pressure coil at low power factor is very serious (unless special features are incorporated to reduce its effect).

(*ii*) Stray field may effect the reading of the instrument. To reduce it, magnetic shielding is provided by enclosing the instrument in an iron case.

6.2. Induction Wattmeter

Induction wattmeters can be used on A.C. circuit only (in contrast with dynamometer wattmeters can be used both on D.C. and A.C. circuits) and are useful only when the frequency and supply voltage are constant.

The *operation* of all induction instruments depends on the *production of torque due to reaction between a flux* ϕ_1 (whose magnitude depends on the current or voltage to be measured) *and eddy currents induced in a metal disc or drum by another flux* ϕ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depends on the flux producing them, the *instantaneous value of the deflecting torque is proportional to the square of the current or voltage under measurement and the value of mean deflecting torque is proportional to the mean square of the current or voltage.*

Fig. 25 shows an induction wattmeter. It has two laminated electromagnets one is excited by the current in the main circuit—its exciting winding being joined in series with the circuit, hence it is also called *series magnet.*

The other electromagnet is excited by current which is *proportional to the voltage of the circuit*. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as *shunt magnet*.

A thin aluminium disc is mounted in such a way that it cuts the fluxes of both the magnets. Hence two eddy currents are produced in the disc. *The deflection torque is produced due to the interaction of these eddy currents and the inducing fluxes*. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as *to make the resultant flux in the shunt magnet lag behind the applied voltage by a suitable angle.*

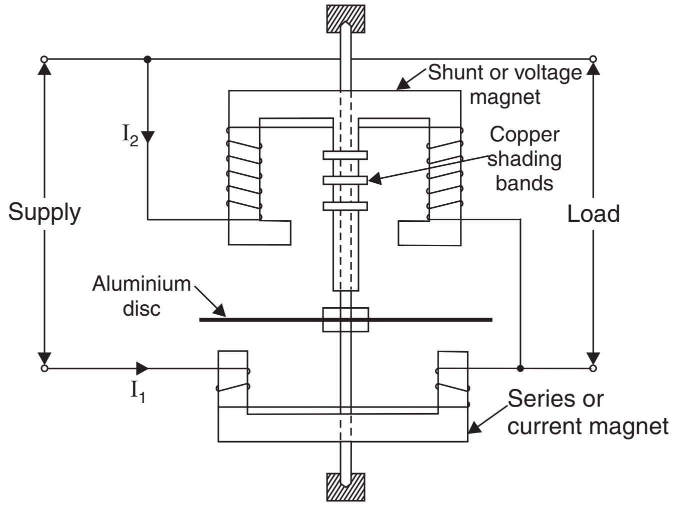


Fig. 25. Induction wattmeter.

This instrument is spring controlled, the spring being fitted to the spindle of the moving system which also carries a pointer. The scale is uniformly even and extends over 300°.

Such wattmeters can handle current upto 100 A. For handling greater currents they are used in conjunction with *current transformers.*

Advantages :

(*i*) Fairly long scale (extending over 300°).

(*ii*) Free from the effects of stray fields.

(*iii*) Good damping.

(*iv*) Practically free from frequency errors.

Disadvantages :

Sometimes subjected to serious temperature errors.

7. INTEGRATING METERS (ENERGY METERS)

Integrating or energy meters are *used to measure the quantity of electric energy supplied to a circuit in a given time.* They give *no direct indication of power i.e.*, as to the rate at which energy is being supplied because their registrations are independent of the rate at which at given quantity of electric energy is being consumed.

The main difference between an *energy meter* and *a wattmeter* is that the former is fitted with some type of *registration mechanism* whereby all the instantaneous readings of power are summed over a definite period of time whereas the latter indicates the value at a *particular instant* where it is read.

7.1. Essential Characteristics of Energy Meters

The essential characteristics of energy meters are given below :

1. They must be simple in design and must not contain any parts which may rapidly deteriorate.

2. The readings may be given directly by the dials and must avoid any multiplying factors.

3. The casing of the meter should be dust, water and insect proof.

4. Permanency of calibration is a prime requisite and to attain it, the friction at the pivots etc., and retarding torque of the magnetic brakes must remain constant. The magnets should be so placed that they are not affected in their strength by the magnetic field of the current coil.

5. The friction losses should be minimum (being unavoidable) and must remain practically constant over long periods of time. This requires that the moving parts should be light, and the jewels and pivots should be of best quality and kept in good order.

6. There should be minimum possible friction in the counter device. The torque of the meter should be high so that the unavoidable irregularities in friction may not cause inaccuracies.

7. The energy meter should maintain its accuracy under reasonably varying conditions of voltage and load.

8. The energy loss in the meter itself must be small.

7.2. Types of Energy Meters

Energy meters are generally of the following three types :

1. Electrolytic meters
2. Motor meters
3. Clock meters.

Here only motor meters will be discussed.

7.3. Motor Meters

The motor meters can be used in *D.C. as well as in A.C. circuits.* In principle the motor meter is a small motor of D.C. or A.C. type whose *instantaneous speed of rotation is proportional to the circuit current in case of an ampere-hour meter and to the power of the circuit in case of a watt-hour meter.*

The following are the essential parts of the motor meters :

1. **An operating torque system.** It produces a torque and causes the moving system to rotate continuously.

2. **A braking device.** It is usually a permanent magnet, known as *brake magnet.* This brake magnet induces current in some part of the moving system which in turn produces the braking torque. Thus the braking torque is *proportional to the induced currents* whereas the induced currents are *proportional to the speed of the moving system* (and hence the *braking torque is proportional to the speed of the moving system* (*disc*). When the braking torque is equal to the driving torque the moving system attains a steady speed.

3. **Revolution registering device.** This device is obtained by having a worm cut on the spindle of the instrument. The worm engages with a pinion and thus drives the train of wheels and registers ampere-hours and watt-hours directly.

Types of motor meters. The various types of motor meters are :

(*i*) Mercury motor meters
(*ii*) Commutator motor meters
(*iii*) Induction motor/energy meters.

— *Mercury motor meters* and *commutator meters* are used on *D.C. circuits*

— *Induction meters* are used on *A.C. circuit*.

7.4. Motor-driven Meter—Watt-hour Meter

The motor-driven meter shown in Fig. 26 can be used on direct or alternating current. It contains a small motor and an aluminium retarding disc. *The field winding is connected in series with the load, and the field strength is proportional to the load current*.

The armature is connected *across* the source, and the *current in the armature is proportional to the source or line voltage*. The torque produced in the armature is *proportional to the power* consumed by the load.

The armature shaft drives a series of counters that are calibrated in watt-hours. The power that is used can be read directly from the dials.

The aluminium disc attached to the armature is used to control the armature speed. The disc turns in a magnetic field produced by the permanent magnets, and the retarding force increases as the rotation increases and stops when the disc stops. The retarding force is produced by the aluminium conductor cutting through the lines of force of the permanent magnets. This is a form of magnetic damping.

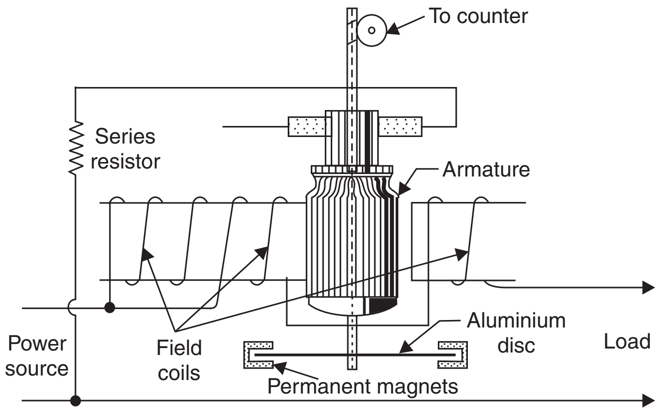


Fig. 26. Motor-driven meter (watt-hour meter) designed to operate on direct or alternating current.

The meter must overcome the friction of the bearings and indicators at *very light loads*. A portion of the field is produced by the armature current (coil in series with the armature winding). This coil is (called as compensating coil) wound to aid the field and is adjusted to the point where it just overcomes the meter friction.

7.5. Induction Type Watt-hour Meter

This is the most commonly used meter on A.C. circuits for measurement of energy.

Advantages :

(*i*) Simple in operation

(*ii*) High torque/weight ratio

(*iii*) Cheap in cost

(*iv*) Correct registration even at very low power factor

(*v*) Unaffected by temperature variations

(*vi*) More accurate than commutator type energy meter on light loads (owing to absence of a commutator with its accompanying friction).

Induction Type Single Phase Energy Meters. These are, by far, the most common form of A.C. meters met with in everyday *domestic* and *industrial installations.* These meters measure electric energy in kWh.

The principle of these meters is practically the same as that of induction watt meters. Instead of the control spring and the pointer of the watt-meter, the watt-hour meter, (energy meter) employs a *brake magnet and a counter attached to the spindle*. Just like other watt-hour meters, the eddy currents induced in the aluminium disc by the brake magnet due to the revolution of the disc, are utilised to control the continuously rotating disc.

Construction. The construction of a typical meter of this type is shown in Fig. 27. The brake-magnet and recording wheel-train being omitted for clearances. It consists of the following :

(*i*) Series magnet M_1

(*ii*) Shunt magnet M_2

(*iii*) Brake magnet

(*iv*) A rotating disc.

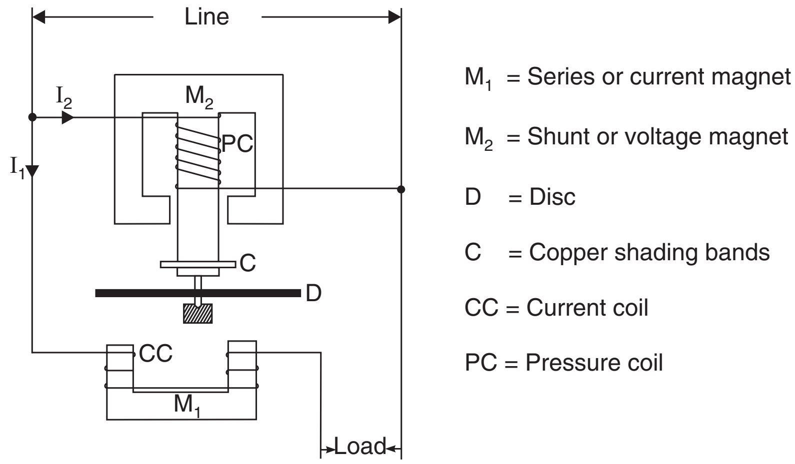


Fig. 27. Induction type single phase energy meter.

The *series electro-magnet* M_1 consists of a number of U-shaped iron laminations assembled together to form a core, wound with a *few turns of a heavy gauge wire*. This wound coil is known as *current coil* and is connected in one of the lines and in series with the load to be metered. The series electromagnet is energized and sets up a magnetic field cutting through the rotating disc, when load current flows through the current coil C.C. The rotating disc is an aluminum disc mounted on a vertical spindle and supported on a sapphire cup contained in a bottom screw. The bottom pivot, which is usually removable, is of hardened steel, and the end, which is hemispherical in shape, rests in the sapphire cup. The top pivot (not shown) merely serves to maintain the spindle in a vertical position under working condition and does not support any weight or exert any appreciable thrust in any direction.

The *shunt magnet* M_2 consists of a number of *M* shaped iron laminations assembled together to form a core. A core having *large number of turns of fine wire* is fitted on the middle limb of the shunt magnet, this coil is known as *pressure coil P.C.* and is connected across the supply mains.

The *brake magnet* consists of *C* shaped piece of alloy steel bent round to form a complete magnetic circuit, with the exception of a narrow gap between the poles. This magnet is mounted so that the disc revolves in the air gap between the polar extremities. The movement of the rotating disc through the magnetic field crossing the air gap *sets up eddy currents in the disc which react with the field and exerts a braking effect*. The speed of the rotating disc may be adjusted by changing the position of the brake magnet or by diverting some of the flux there from.

Working. The shunt electromagnet produces a magnetic field which is of pulsating character ; it cuts through the rotating disc and induces eddy currents there in, but normally *does not in itself produce any driving force*. Similarly series electromagnet induces eddy currents in the rotating disc, but *does not in itself produce any driving force*. In order *to obtain driving force in this type of meter, phase displacement of 90° between the magnetic field set up by shunt electromagnet and applied voltage V is achieved by adjustment of copper shading band C* (also known as power factor compensator or compensating loop). *The reaction between these magnetic fields and eddy currents set up a driving torque in the disc*.

Sources of Errors. The various sources of errors in an induction-type energy meter are given below :

(*i*) *Incorrect magnitude of the fluxes*. These may arise from abnormal voltages and load currents.

(*ii*) *Incorrect phase relation of fluxes*. These may arise from defective lagging, abnormal frequencies, changes in the iron losses etc.

(*iii*) *Unsymmetrical magnetic structure*. The disc may go on rotating while no current is being drawn but pressure coils alone are excited.

(*iv*) *Changes in the resistance of the disc*. It may occur due to changes in temperature.

(*v*) *Changes in the strength of the drag magnets*. It may be due to temperature or ageing.

(*vi*) Phase-angle errors due to lowering of power factor.

(*vii*) Abnormal friction of moving parts.

(*viii*) Badly distorted waveform.

(*ix*) Changes in the retarding torque due to the disc moving through the field of the current coils.

Example 7. *A 5 A, 230 V meter on full load unity power factor test makes 60 revolutions in 360 seconds. If the normal disc speed is 520 revolutions per kWh, what is the percentage error ?*

Solution. Energy consumed in 360 seconds

$$= \frac{VI \cos \phi \times t}{3600 \times 1000} \text{ kWh}$$

$$= \frac{230 \times 5 \times 1 \times 360}{3600 \times 1000} = 0.115 \text{ kWh}$$

where, t is in seconds.

Energy recorded by the meter

$$= \frac{60}{520} = 0.11538 \text{ kWh}$$

$\therefore$ **% age error** $= \dfrac{0.11538 - 0.115}{0.115} \times 100 =$ **0.33% (fast). (Ans.)**

Example 8. *The constant of a 230 V, 50 Hz, single phase energy meter is 185 revolutions per kWh. The meter takes 190 seconds for 10 revolutions while supplying a non-inductive load of 4.5 A at normal voltage. What is the percentage error of the instrument ?*

Solution. Energy consumed in 190 seconds

$$= \frac{VI \cos \phi}{1000} \times t = \frac{230 \times 4.5 \times 1}{1000} \times \frac{190}{3600} = 0.0546 \text{ kWh}$$

[cos ϕ = 1, since load supplied is non-inductive]

Energy registered by the meter $= \dfrac{10}{185} = 0.054$ kWh

$\therefore$ % age error $= \dfrac{0.054 - 0.546}{(0.054)} =$ **0.06% (slow). (Ans.)**

Example 9. *The name plate of a meter reads "1 kWh = 15000 revoltions". In a check up, the meter completed 150 revolutions during 45 seconds. Calculate the power in the circuit.*

Solution. Power metered in 150 revolutions

$$= 1 \times 150/15000 = 0.01 \text{ kWh}$$

If P kilowatt is the power in the circuit, then energy consumed in 45 seconds

$$= \frac{P \times 45}{3600} \text{ kWh} = 0.0125\, P \text{ kWh}$$

Equating the two amounts of energy, we have

$$0.0125\, P = 0.01$$

$$\therefore \quad P = 0.8 \text{ kW} = \mathbf{800\ W.\ (Ans.)}$$

Example 10. *A 230 V ampere-hour type meter is connected to a 230 V D.C. supply. If the meter completes 225 revolutions in 10 minutes when carrying 14 A, calculate : (i) The kWh registered by the meter, and (ii) The percentage error of the meter above or below the original calibration.*

The timing constant of the meter is 40 A-s/revolution.

Solution. During 225 revolutions the meter would register 40 × 225 A-*s* or coulombs. Since time taken is 10 minutes or 600 seconds it corresponds to a current of $\frac{40 \times 225}{600} = 15$ A.

(*i*) Energy recorded by the meter $= \frac{VI\,t}{1000}$ kWh

where t is in hour $= \frac{230 \times 15}{1000} \times \frac{10}{60} = 0.575$ kWh.

(*ii*) Actual energy consumed $= \frac{230 \times 14}{1000} \times \frac{10}{60} = 0.5367$ kWh

Percentage error $= \frac{0.575 - 0.5367}{0.575} \times 100 =$ **6.66%. (Ans.)**

8. MEGGER

Meggers (or megohmmeters) are instruments which measure the *insulation resistance* of *electric circuits relative to earth and one another.*

A megger consists of an *e.m.f. source* and a *voltmeter*. The scale of the voltmeter is calibrated in ohms (kilo-ohms or megohms, as the case may be). In measurements the e.m.f. of the self-contained source must be equal to that of the source used in calibration.

Fig. 28 shows diagrammatically a megger whose readings are independent of the speed of the self-contained generator. The moving system incorporates two coils 1 (current coil) and 2 (pressure coil) mounted on the same shaft and *placed in the field of a permanent magnet* (not shown) 90° apart. The generator energizes the two coils over separate wires. Connected in series with one coil is a fixed resistance R_1 (or several different resistances in order to extend the range of the instrument). The unknown resistance R_x is connected in series with the other coil. *The currents in the coils interact with the magnetic field and produce opposing torques.*

The deflection of the moving system depends on the ratio of the currents in the coils and is independent of the applied voltage. The unknown resistance is read *directly* from the scale of the instrument. (The accuracy of measurement is unaffected by variations in the speed of the generator between 60 and 180 r.p.m.).

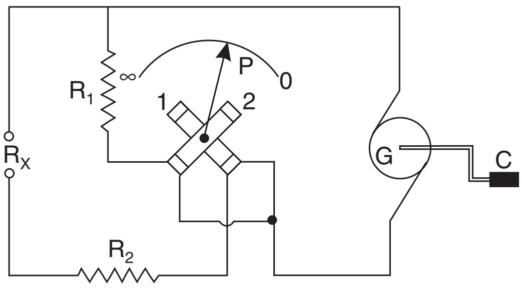


Fig. 28. Circuit diagram of megger.

HIGHLIGHTS

1. **Absolute instruments** are those instruments which indicate the quantity to be measured in terms of the *constants of the instrument* and in order to find out the quantity in the practical units it is necessary to multiply such deflections with an instrument constant.

2. **Secondary instruments** are those instruments in which the value of electrical quantity to be measured can be determined from the deflection of the instrument only when they have been precalibrated by comparison with an absolute instrument.

These are classified as :

(*i*) Indicating instruments
(*ii*) Recording instruments
(*iii*) Integrating instruments.

3. Essential features of indicating instruments are :

(*i*) Deflecting device
(*ii*) Controlling device
(*iii*) Damping device.

4. Moving iron instruments are of the following two types :

(*i*) Attraction type
(*ii*) Repulsion type.

5. Moving-coil instruments are of the following two types :

(*i*) Permanent magnet type
(*ii*) Dynamometer type.

6. Rectifier instruments can operate well into the R.F. range with the proper use of silicon or germanium rectifiers. They are more sensitive than any other type of A.C. meter.

7. A wattmeter consists of two coils known as current coil and pressure coil.

8. Integrating (or energy) meters are used to measure the quantity of electric energy supplied to a circuit in a given time. These are generally of the following three types :

(*i*) Electrolytic meters
(*ii*) Motor meters
(*iii*) Clock meters.

9. The resistance can be measured by the following methods :

(*i*) Voltmeter-ammeter method
(*ii*) Substitution method
(*iii*) By using wheatstone bridge
(*iv*) By using a potentiometer.

10. Meggers are instruments which measure the insulation resistance of electric circuits relative to earth and one another.

THEORETICAL QUESTIONS

1. Differentiate between an absolute instrument and a secondary instrument.
2. How are secondary instruments classified ?
3. What are the essential features of an indicating instrument ?
4. What are the advantages and disadvantages of gravity controlled instruments ?
5. Explain briefly the following :
(*i*) Air damping
(*ii*) Fluid friction damping
(*iii*) Eddy current damping.
6. Explain briefly, with the help of sketches, the construction and working of the following moving iron instruments :
(*i*) Attraction type
(*ii*) Repulsion type.
7. What are the advantages and disadvantages of moving-iron instruments ?
8. How are moving coil instruments classified ?
9. Give the construction and working of a permanent magnet moving-coil type instrument. Also enumerate its advantages and disadvantages.
10. Explain the construction and working of a dynamometer type instrument. How it can be used as an ammeter and a voltmeter ?
11. Give the characteristics of rectifier instruments.
12. Explain briefly the following :
(*i*) Rectifier ammeters
(*ii*) Rectifier voltmeters.
13. What is a wattmeter ?
14. Enumerate types of wattmeter.
15. Discuss the construction working of a dynamometer wattmeter with the help of a neat diagram.
16. Draw a neat sketch of an induction wattmeter and explain its working. Also state its advantages and disadvantages.
17. What is an integrating or energy meter ?
18. What are the essential characteristics of energy meters ?
19. Give the construction and working of a motor-driven meter-watt hour meter.
20. With the help of a neat sketch explain the construction and working of an induction type single phase energy meter. Also discuss the sources of errors prevalent in this type of energy meter.
21. What are the various methods by which a resistance can be measured ?
22. Explain any two of the following methods of measuring resistance :
(*i*) Voltmeter-ammeter method
(*ii*) Substitution method
(*iii*) Measurement of resistance by the Wheatstone bridge.
23. Explain briefly the following :
(*i*) The potentiometer
(*ii*) Megger.
24. What are instrument transformers ?
25. How are instrument transformers classified ?
26. Explain briefly, with near sketches the following :
(*i*) Potential transformers
(*ii*) Current transformers.

EXERCISE

1. A moving-coil instrument gives full-scale deflection with 15 mA and has a resistance of 5 Ω. Calculate the resistance to be connected :

(*i*) in parallel to enable the instrument to read upto 1 A.

(*ii*) in series to enable it to read upto 100 V. [**Ans.** (*i*) 0.076 Ω, (*ii*) 6661.7 Ω]

2. A moving-coil instrument has a resistance of 10 ohms and gives a full-scale deflection when carrying 50 mA. Show how it can be adopted to measure voltage upto 750 volts and current upto 100 amperes. [**Ans.** R = 14990 Ω, R_s = 0.005 Ω]

3. A 20 V moving-iron voltmeter reads correctly when put on D.C. and the instrument has a resistance of 600 ohms and inductance of 0.15 H. Find out the reading on 20 V A.C. mains (*i*) at 250 Hz (*ii*) 50 Hz. [**Ans** (*i*) 18.6 V, (*ii*) 19.87 V]

4. A 15 V moving-iron voltmeter has a resistance of 500 Ω and inductance of 0.12 H. Assuming that this instrument reads correctly on D.C. what will be its reading on A.C. at 15 V when the frequency is (*i*) 25 Hz and (*ii*) 100 Hz ? [**Ans.** (*i*) 14.99 V, (*ii*) 14.83 V]

5. The total resistance of a moving-iron voltmeter is 1000 Ω and coil has an inductance of 0.765 H. The instrument is calibrated with a full-scale deflection of 50 V D.C. Calculate the percentage error when the instrument is used on (*i*) 25 Hz supply, (*ii*) 50 Hz supply, the applied voltage being 50 V in each case. [**Ans.** (*i*) 0.72%, (*ii*) 36%]

6. In a moving-coil instrument, the moving coil has 40 turns and is of square shape with a mean length of 40 mm along each side. The coil hangs in a uniform radial field of 0.08 Wb/m^2. Find the turning moment of the coil when it is carrying a current of 10 mA. [**Ans.** 512×10^{-7} Nm]

7. A moving-coil instrument has a resistance of 5 Ω between terminals and full scale deflection is obtained with a current of 0.015 A. The instrument is to be used with a manganin shunt to measure 100 A full scale. Calculate the error caused by a 10°C rise in temperature. (*i*) when the internal resistance of 5 Ω is due to copper only, (*ii*) when a 4 Ω manganin swamping resistor is used in series with a copper resistor of 1 Ω.

Take : α_{copper} = 0.004/°C and $\alpha_{manganin}$ = 0.00015/°C. [**Ans.** (*i*) 3.7%, (*ii*) 0.8%]

8. A 230 V, 50 Hz, single phase energy meter has a constant of 200 revolutions per kWh. While supplying a non-inductive load of 4.4 A at normal voltage the meter takes 3 minutes for 10 revolutions. Calculate the percentage error of the instrument. [**Ans.** 1.186% (slow)]

9. The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 W load is connected, the disc rotates at 10.2 r.p.m. If the load is on for 12 hours, how many units are recorded as error ? [**Ans.** 0.24 kWh more]

10. The name plate of a meter reads "1 kWh = 15000 revolutions". In a check up, the meter completed 150 revolutions during 50 seconds. Calculate the power in the circuit. [**Ans.** 720 W]

UNIT-II : *ELECTRICAL MACHINES*

Chapters :

5

Direct Current (D.C.) Machines

1. CONSTRUCTION OF D.C. MACHINES

A D.C. machine consists of *two* main parts :

(*i*) **Stationary part.** It is designed mainly for *producing a magnetic flux*.

(*ii*) **Rotating part.** It is called the *armature*, where mechanical energy is converted into electrical (electrical generator), or conversely, electrical energy into mechanical (electric motor).

The stationary and rotating parts are separated from each other by an *air gap*.

- The *stationary part* of a D.C. machine consists of *main poles,* designed to create the magnetic flux, *commutating poles* interposed between the main poles and designed *to*

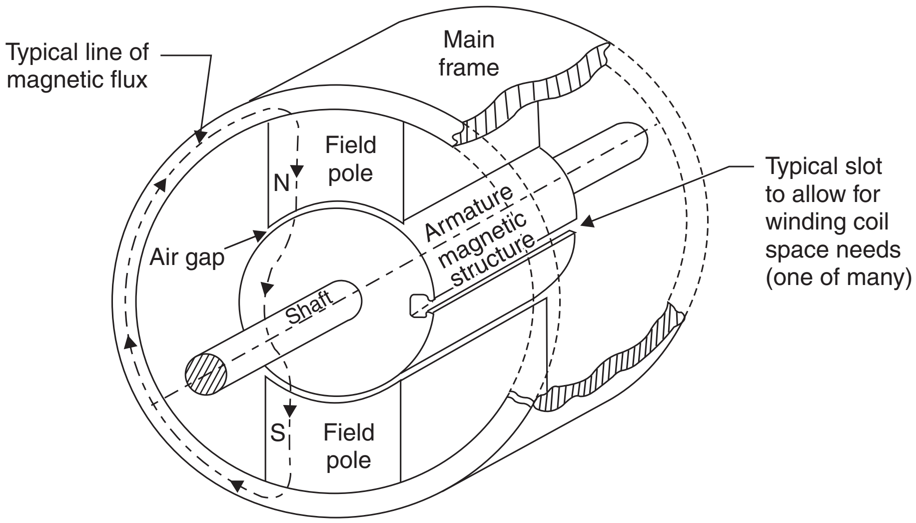


Fig. 1. Generator or motor magnetic structure.

ensure sparkless operation of the brushes at the commutator (in very small machines with a lack of space commutating poles are not used) ; and a *frame*/yoke.

- The *armature* is a cylindrical body rotating in the space between the poles and comprising a *slotted armature core,* a *winding* inserted in the armature core slots, a *commutator,* and *brush gear*.

Fig. 1 shows generator or motor magnetic structure.

Description of Parts of D.C. Machines :

1.1. Frame

Fig. 2 shows the sectional view of four pole D.C. machine.

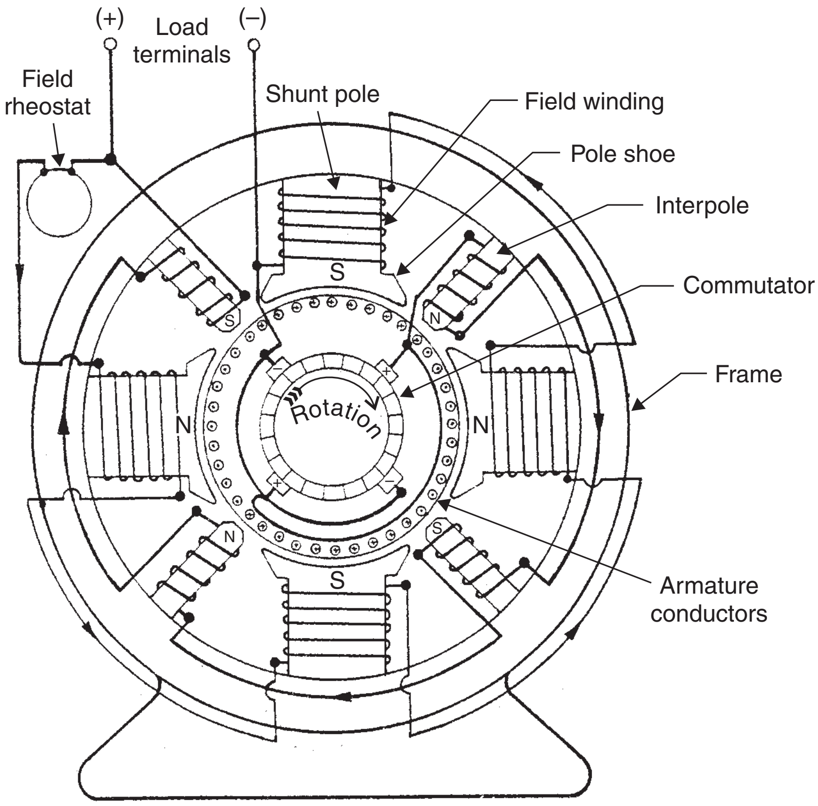


Fig. 2. Sectional view of a four pole D.C. machine.

- *The frame is the stationary part of a machine to which are fixed the main and commutating poles and by means of which the machine is bolted to its bed plate.*
- *The ring-shaped portion which serves as the path for the main and commutating pole fluxes is called the* '**yoke**'.

Cast iron used to be the material for the frame/yoke in *early machines* but now it has been *replaced by cast steel.* This is because cast iron is saturated by a flux density of about 0.8 Wb/m^2 while saturation with cast steel is at about 1.5 Wb/m^2. Thus, the cross-section of a cast iron frame is about twice that of a cast steel frame for the same value of magnetic flux. Hence, if it is necessary to reduce the weight of machine, cast steel is used. Another disadvantage with the use of cast iron is that its mechanical and magnetic properties are uncertain due to the presence of blow holes in the casting. Lately, rolled steel yokes have been developed with the improvements in the welding techniques. The *advantages of fabricated yokes are that there are no pattern charges and the magnetic and mechanical properties of the frame are absolutely consistent*.

It may be advantageous to use cast iron for frames but for medium and large sizes usually rolled steel is used.

- If the armature diameter does not exceed 35 to 45 cm, then, in addition to the poles, end shields or frame-heads which carry the bearings are also attached to the frame. When the armature diameter exceeds 1 m, it is common practice to use pedestal-type bearings, mounted separately, on the machine bed plate outside the frame.
- The end shield bearings, and sometimes the pedestal bearings, are of ball or roller type. However, more frequently plain pedestal bearings are used.
- In machines with large diameter armatures a brush-holder yoke is frequently fixed to the frame.

1.2. Field Poles

- Formerly the poles were cast integral with the yoke. This practice is still being followed for small machines. But in present day machines *it is usual to use either a completely laminated pole, or solid steel poles with laminated pole shoes*.
- Laminated construction is necessary because of the pulsations of field strength that result when the notched armature rotor magnetic structure passes the pole shoe. Variations in field strength result in internal eddy currents being generated in a magnetic structure. These eddy currents cause losses ; they may be largely prevented by having laminated magnetic structures. Laminated structures allow magnetic flux to pass along the length of the laminations, but do not allow electric eddy currents to pass across the structure from one lamination to another. The assembled stack of laminations is held together as a unit by appropriately placed rivets. *The outer end of the laminated pole is curved to fit very closely into the inner surface of the main frame*.
- Fig. 3 shows the constructional details of a field pole. *The pole shoe acts as a support to the field coils and spreads out the flux in the air gap and also being of larger cross-section reduces the reluctance of the magnetic path*.

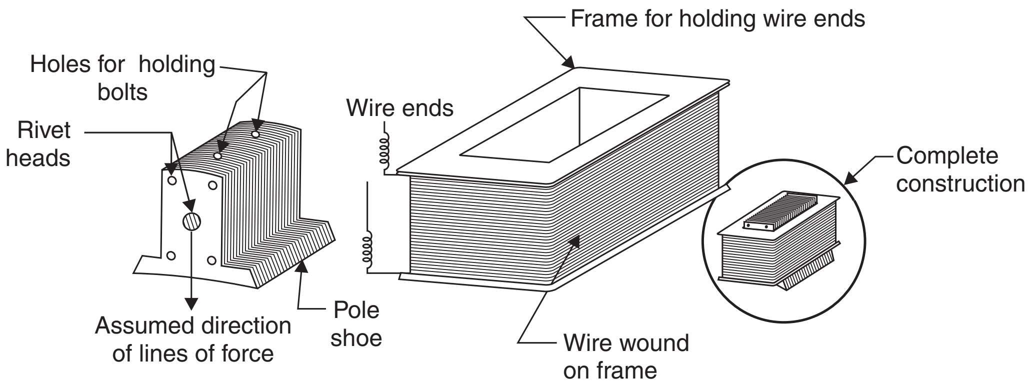


Fig. 3. Constructional details of a field pole.

- Different methods are used for attaching poles to the yoke. In case of *smaller sizes,* the back of the pole is drilled and tapped to receive pole bolts (*see* Fig. 4). In *larger sizes*, a circular or a rectangular pole bar is fitted to the pole. This pole bar is drilled and tapped and the pole bolts passing through laminations screw into the tapped bar (*see* Fig. 5).

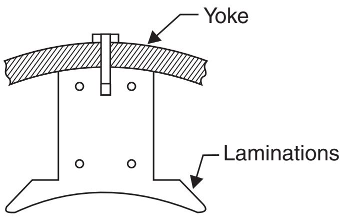


Fig. 4. Fixing pole to the yoke.

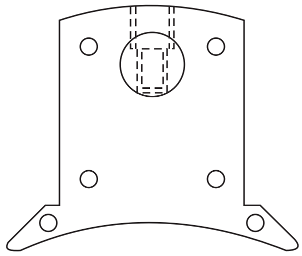


Fig. 5

1.3. Commutating Poles

- A commutating pole (also called *interpole*) is similar to a main pole and consists of core terminating in a pole shoe, which may have various shapes, and coil mounted on the core.
- The commutating poles *are arranged strictly midway between the main poles* and are bolted to the yoke.
- *Commutating poles are usually made of solid steel, but for machines operating on sharply varying loads they are made of sheet steel*.

1.4. Armature

- The armature consists of core and winding. *Iron being the magnetic material is used for armature core.* However, iron is also a good conductor of electricity. The rotation of solid iron core in the magnetic field results in eddy currents. The flow of eddy currents in the core leads to wastage of energy and creates the problem of heat dissipation. *To reduce the eddy currents the core is made of thin laminations.*
- *The armature of D.C. machines (see Fig. 6) is built up of thin laminations of low loss silicon steel.* The laminations are usually 0.4 to 0.5 mm thick and are insulated with varnish.
- The armature **laminations**, in *small machines,* are fitted directly on to the shaft and are clamped tightly between the flanges which also act as supports for the armature winding. One end flange rests against a shoulder on the shaft, the laminations are fitted and other end is pressed on the shaft and retained by a key.

The core (except in small size) is divided into number of packets by radial ventilation spacers. The spacers are usually I sections welded to thick steel laminations and arranged to pass centrally down each tooth.

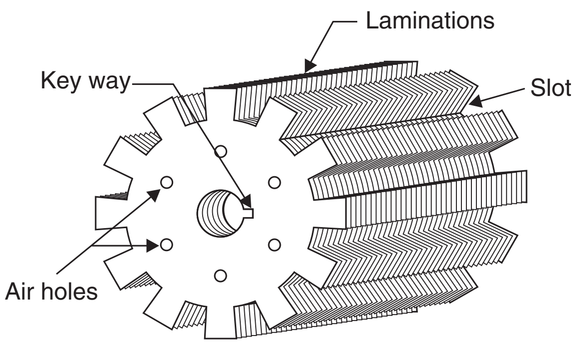


Fig. 6. Armature of a D.C. machine.

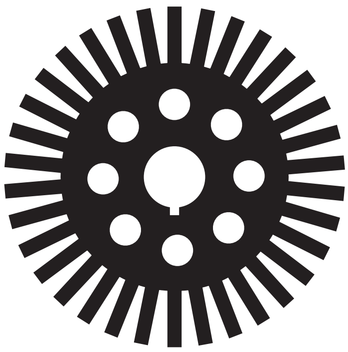


Fig. 7. Drum armature stamping with axial flow ventilation system.

— For *small machines* the laminations are punched in one piece (*see* Fig. 7). These laminations are built up directly on the shaft. With such an arrangement, it is necessary to provide *axial ventilation holes* so that air can pass into ventilating ducts.

— The armature laminations of *medium size machines* (having more than four poles) are built on a spider. The spider may be fabricated. Laminations up to a diameter of about 100 cm are punched in one piece and are directly keyed on the spider (*see* Fig. 8).

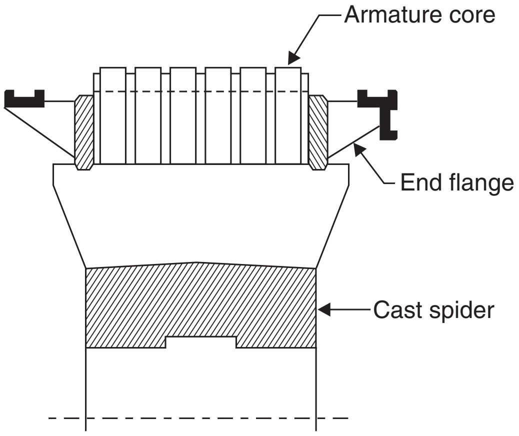


Fig. 8. Clamping of an armature core.

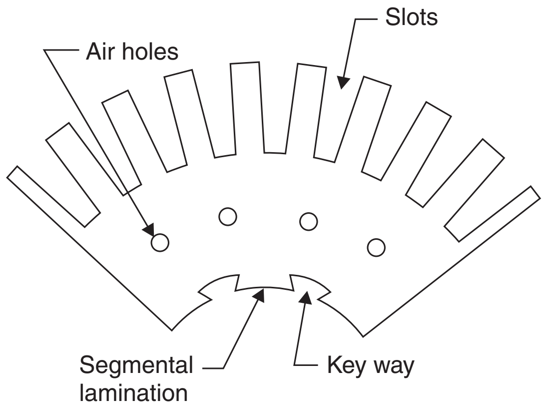


Fig. 9. Segmental stampings.

— In case of *large machines,* the laminations of such thin sections are difficult to handle because they tend to distort and become wavy when assembled together. Hence, circular laminations instead of being cut in one piece are cut in a number of suitable sections or *segments* which form part of a complete ring (*see* Fig. 9). A complete circular lamination is made up of four or six or even eight segmental laminations. Usually two keyways are notched in each segment and are dove-tailed or wedge shaped to make the laminations self-locking in position.

- The armature winding is housed in slots on the surface of the armature. The conductors of each coil are so spaced that when one side of the coil is under a north pole, the opposite is under a south pole.

Fig. 10 shows the arrangement of conductors and insulation in a slot.

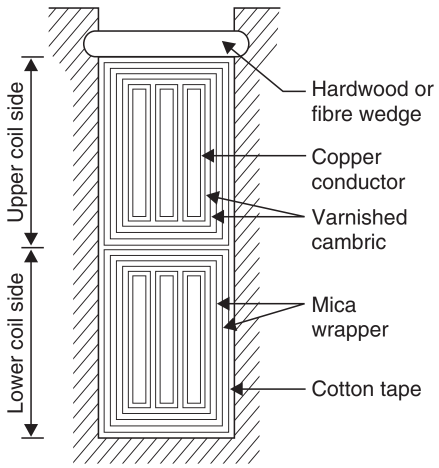


Fig. 10. Cross-section of an armature slot.

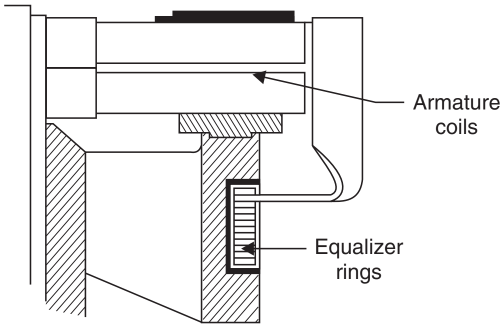


Fig. 11. Ring type equilizers.

- *In D.C. machines two layer winding with diamond shaped coils is used*. The coils are usually former wound. In *small machines,* the coils are held in position by band of steel wire, wound under tension along the core length. In *large machines*, it is useful to employ wedges of fibre or wood to hold coils in place in the slots. Wire bands are employed for holding the overhang. The *equilizer connections* are located under the overhang on the side of the commutator. Fig. 11 shows a typical arrangement for equilizers. The equilizers can be accommodated on the other end of the armature also.

1.5. Commutator

- *A commutator converts alternating voltage to a direct voltage.*
- A commutator is a cylindrical structure built up of segments made of hard drawn copper. These segments are separated from one another and from the frame of the machine by *mica strips.* The segments are connected to the winding through risers. The risers have air spaces between one another so that air is drawn across the commutator thereby keeping the commutator cool.

Fig. 12 shows the components of a commutator. The general appearance of a commutator when completed is as shown in Fig. 13 (*a*). The commutator and armature assembly is shown in Fig. 13 (*b*).

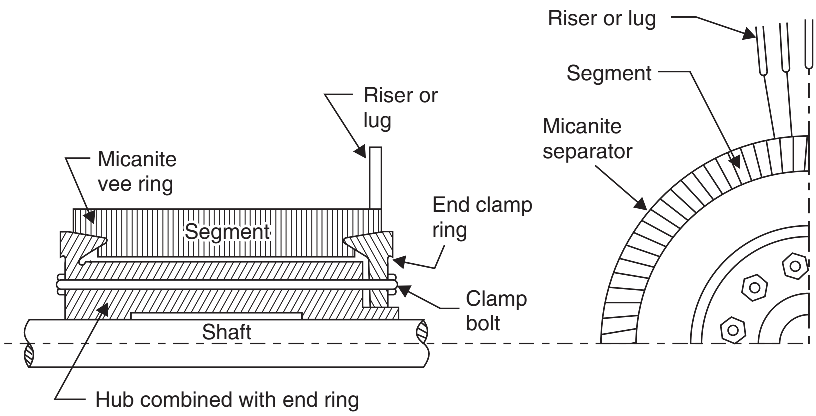


Fig. 12. Commutator components.

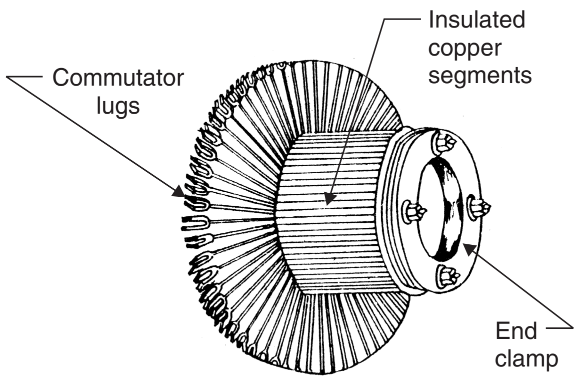


Fig. 13. (*a*) General appearance of a commutator after assembly.

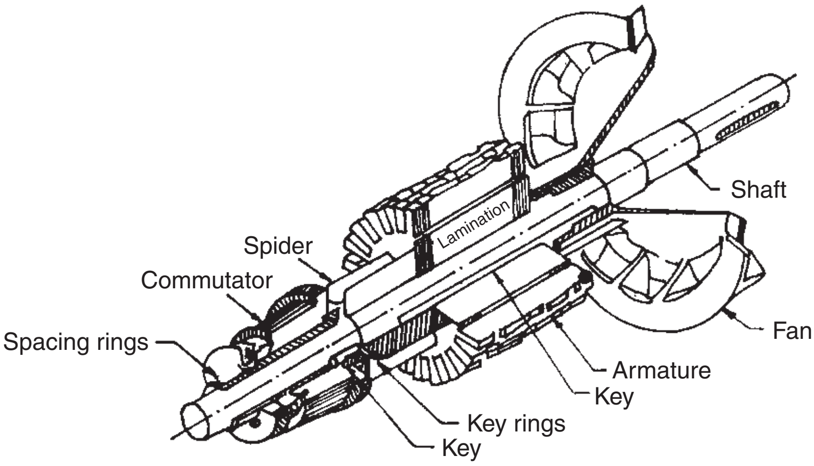


Fig. 13. (*b*) Commutator and armature assembly.

1.6. Brush Gear

To collect current from a rotating commutator or to feed current to it use is made of *brush-gear* which consists of :

(*i*) Brushes (*ii*) Brush holders

(*iii*) Brush studs or brush-holder arms (*iv*) Brush rocker

(*v*) Current-collecting busbars.

Brushes. The brushes used for D.C. machines are divided into five classes :

(*i*) Metal graphite (*ii*) Carbon graphite

(*iii*) Graphite (*iv*) Electro-graphite

(*v*) Copper.

- The allowable *current density* at the brush contact varies from 5 A/cm^2 in case of carbon to 23 A/cm^2 in case of copper.
- The use of *copper brushes* is made for machines designed for *large currents at low voltages.* Unless, very carefully lubricated, they cut the commutator very quickly and in any case, the wear is rapid. *Graphite and carbon graphite brushes are self-lubricating and, are, therefore, widely used*. Even with the softest brushes, however, there is a gradual wearing away of the commutator, and if mica between the commutator segments does not wear down so rapidly as the segments do, the high mica will cause the brushes to make poor contact with segments, and sparking will result and consequent damage to commutator. So, to prevent this, the mica is frequently '*undercut*' to a level below the commutator surface by means of a narrow milling cutter.

Brush holders. *Box type brush holders* are used in all ordinary D.C. machines. A box type brush holder is shown in Fig. 14. At the outer end of the arm, a brush box, open at top and bottom is attached. The brush is pressed on to the commutator by a *clock spring*. The *pressure can be adjusted by a lever arrangement provided with the spring*. The brush is connected to a flexible conductor called *pig tail*. The flexible conductor may be attached to the brush by a screw or may be soldered.

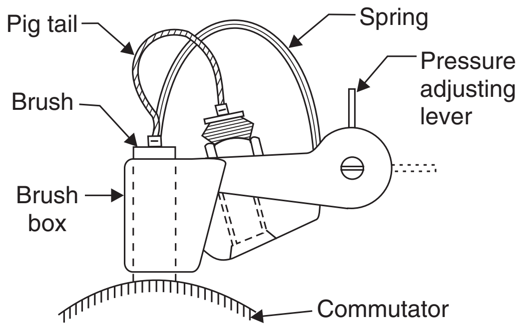


Fig. 14. Box type brush holder.

- The bush boxes are usually made of *bronze casting* or *sheet brass*. In low voltage D.C. machines where the commutation conditions are easy galvanised steel box may be used.
- Some manufacturers use individual brush holders while others use multiple holders, *i.e.*, a number of single boxes built up into one long assembly.

Brush rockers. Brush holders are fixed to brush rockers with bolts. The brush rocker is arranged concentrically round the commutator. *Cast iron is usually, used for brush rockers.*

1.7. Armature Shaft Bearings

- *With small machines roller bearings are used at both ends.*
- For *larger machines roller bearings are used for driving end and ball bearings are used for non-driving* (commutator) *end*.
- The bearings are housed in the end shields.
- For large machines pedestal bearings are used.

1.8. Armature Windings

The *armature winding* is very important element of a machine, as it directly takes part in the conversion of energy from one form into another. The requirements which a winding must meet are diverse and often of a conflicting nature. Among these requirements the following are of major importance.

- The winding must be designed with the *most advantageous utilisation of the material in respect to weight and efficiency*.
- The winding should *provide the necessary mechanical, thermal and electrical strength of the machine* to ensure the usual service life of 16–20 years.
- For D.C. machines proper current collection at the commutator (*i.e.*, absence of detrimental sparking) must be ensured.
- According to the degree of closure produced by winding, armature windings are of the following two types :

1. Open coil winding   2. Closed coil winding.

The closed armature windings are of two types :

(*i*) Ring winding   (*ii*) Drum winding

In general there are two types of drum armature windings :

(*i*) Lap winding   (*ii*) Wave winding.

"Lap winding" is *suitable for comparatively low voltage* but high current generators whereas **"wave of winding"** is used for high voltage, low current machines.

— In '*lap winding*' the finish of each coil is connected to the start of the next coil so that winding or commutator pitch is unity.

— In '*wave winding*' the finish of coil is connected to the start of another coil well away from the fixed coil.

2. E.M.F. EQUATION OF A GENERATOR

An **electrical generator** *is a machine which converts mechanical energy (or power) into electrical energy (or power). This energy conversion is based on the principle of the production of dynamically induced e.m.f. "Whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic induction." This e.m.f. causes current to flow if the conductor circuit is closed.* Therefore, the basic essential parts of an electrical generator are: (*i*) A magnetic field. and (*ii*) A conductor/conductors which can so move to cut the flux.

Let p = number of poles,

ϕ = flux/pole, webers (Wb),

Z = total number of armature conductors,

= number of slots × number of conductors/slot,

N = rotational speed of armature, r.p.m.,

a = number of parallel paths in armature, and

E_g = generated e.m.f. *per parallel path in armature*.

Average e.m.f. generated per conductor = $\frac{d\phi}{dt}$ volt.

Now, flux cut per conductor in one revolution, $d\phi = p\phi$ Wb.

Number of revolutions/second = $\frac{N}{60}$

$\therefore$ Time for one revolution, $dt = \frac{60}{N}$ seconds

Hence, according to Faraday's laws of electromagnetic induction,

E.m.f. generated per conductor = $\frac{p\phi N}{60}$ volts.

For a lap wound generator :

Number of parallel paths, $a = p$

Number of conductor (*in series*) in one path = $\frac{Z}{p}$

$\therefore$ E.m.f. generated *per path* = $\frac{\phi pN}{60} \times \frac{Z}{p} = \frac{\phi ZN}{60}$ volt.

For a wave wound generator :

Number of parallel paths, $a = p$

Number of conductor (in series) in one path = $\frac{Z}{2}$

$\therefore$ E.m.f. generator per path = $\frac{p\phi N}{60} \times \frac{Z}{2} = \frac{p\phi ZN}{120}$ volt.

In general, generated e.m.f.

$$E_g = \frac{\phi ZN}{60} \times \left(\frac{p}{a}\right) \text{ volt} = \frac{p\phi ZN}{60a} \quad ...(1)$$

where $a = p$ ...... for lap winding

$= 2$ ...... for wave winding.

Example 1. *A six-pole lap wound D.C. generator has 720 conductors, a flux of 40 m Wb per pole is driven at 400 r.p.m. Find the generated e.m.f.*

Solution. Number of poles, $p = 6$

Total number of conductors, $Z = 720$

Flux per pole, $\phi = 40 \text{ m Wb} = 40 \times 10^{-3}$ Wb

Speed of rotation, $N = 400$ r.p.m.

Number of parallel paths, $a = p = 6$ [Since the generator is *lap wound.*]

Generated e.m.f. E_g :

Using the relation, $E_g = \frac{p\phi ZN}{60a} = \frac{6 \times 40 \times 10^{-3} \times 720 \times 400}{60 \times 6} = 192$ V.

Hence, **generated e.m.f. E_g = 192 V. (Ans.)**

Example 2. *A six-pole lap connected generator has a useful flux/pole of 0.045 Wb. If the no load voltage at 400 r.p.m. is 300 V, find the conductors on the armature periphery.*

Solution. Number of poles, $p = 6$

Useful flux/pole, $\phi = 0.045$ Wb

No load voltage, $E_g = 300$ V

Number of conductors, Z :

Number of parallel paths, $a = p = 6$ [Since the generator is *lap wound*.]

We know that, $$E_g = \frac{p\phi ZN}{60a}$$

$$300 = \frac{6 \times 0.045 \times Z \times 400}{60 \times 6}$$

$$\therefore \quad Z = \frac{300 \times 60 \times 6}{6 \times 0.045 \times 400} \quad i.e., Z = 1000.$$

Hence, **total number of armature conductors = 1000. (Ans.)**

Example 3. *An 8-pole wave connected D.C. generator has 1000 armature conductors and flux/pole 0.035 Wb. At what speed must it be driven to generate 500 V ?*

Solution. Number of poles, $p = 8$

Total number of armature conductor, $Z = 1000$

Flux/pole, $\phi = 0.035$ Wb

Generated voltage, $E_g = 500$ V

Number of parallel paths, $a = 2$ [Since the generator is wave wound.]

Speed of rotation, N :

Using the relation, $$E_g = \frac{p\phi ZN}{60a}$$

$$500 = \frac{8 \times 0.035 \times 1000 \times N}{60 \times 2}$$

$$\therefore \quad N = \frac{500 \times 60 \times 2}{8 \times 0.035 \times 1000} = 214.3 \text{ r.p.m.}$$

Hence, **speed of generator = 214.3 r.p.m. (Ans.)**

Example 4. *The armature of a 6-pole D.C. generator has a wave winding containing 650 conductors. Calculate the generated e.m.f. when the flux per pole is 0.055 Wb and the speed is 300 r.p.m.*

Calculate speed at which the armature must be driven to generate an e.m.f. of 550 V if the flux per pole is reduced to 0.05 Wb.

Solution. Number of poles, $p = 6$

Total number of conductors, $Z = 650$

Flux per pole, $\phi = 0.055$ Wb

Speed of rotation, $N = 300$ r.p.m.

E.m.f. generated, $E_g = ?$

Generated e.m.f. (2nd case) = 550 V

Flux per pole (2nd case) = 0.05 Wb

Speed of rotation, $N = ?$

Case I. E.m.f. generated, E_g :

Using the relation, $$E_g = \frac{p\phi ZN}{60a}$$

$$= \frac{6 \times 0.055 \times 650 \times 300}{60 \times 2} \quad [\because \; a = 2, \text{ as the generator is wave wound}]$$

$$= 536.25 \text{ V.}$$

Hence, **e.m.f. generated = 536.25 V. (Ans.)**

Case II. Speed of rotation, N :

$$E_g = \frac{p\phi ZN}{60a}$$

$$550 = \frac{6 \times 0.05 \times 650 \times N}{60 \times 2}$$

$$N = \frac{550 \times 60 \times 2}{6 \times 0.05 \times 650} = 338.46 \text{ r.p.m.}$$

Hence, **speed of rotation = 338.46 r.p.m. (Ans.)**

Example 5. *A six pole lap wound D.C. armature has 70 slots with 20 conductors/slot. The ratio of pole arc to pole pitch is 0.68. The diameter of bore of the pole shoe is 0.46 m. The length of pole shoe is 0.3 m. If the air gap flux density is 0.3 Wb/m² and the e.m.f. induced in the armature is 500 V, find the speed at which it runs.*

Solution. Number of poles $p = 6$

Number of slots $= 70$

Conductors/slot $= 20$

∴ Total number of conductors, $Z = 70 \times 20 = 1400$

Ratio of pole arc to pole pitch $= 0.68$

Diameter of bore of the pole shoe $= 0.46$ m

Length of the pole shoe $= 0.3$ m

Air gap flux density, $B = 0.3$ Wb/m^2

E.m.f. induced, $E_g = 500$ V

Speed of rotation, N :

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.68$$

∴ Pole arc $= 0.68 \times \text{pole pitch} = 0.68 \times \frac{\pi D}{p} = \frac{0.68 \times \pi \times 0.46}{6} = 0.1638$ m

Area of pole shoe, $A = \text{pole arc} \times \text{length of pole shoe}$

$= 0.1638 \times 0.3 = 0.04914$ m^2

Now, $\phi = B \times A = 0.3 \times 0.04914 = 0.01474$ Wb

Using the relation, $E_g = \frac{p\phi ZN}{60a}$

$$500 = \frac{6 \times 0.01474 \times 1400 \times N}{60 \times 6}$$

[∵ $a = p = 6$, generator being lap wound]

∴ $$N = \frac{500 \times 60 \times 6}{6 \times 0.01474 \times 1400} = 1453.7 \text{ r.p.m.}$$

Hence, **speed of rotation = 1453.7 r.p.m. (Ans.)**

3. TYPES OF D.C. GENERATORS

- The power stations of modern design generate practically only three-phase alternating current. A large part of this power is used in the form of alternating current in industry, for *lighting* and *domestic needs*. When industrial needs make it necessary too or when it is of greater advantage to *use direct current* (for *chemical* and *metallurgical plants, electric traction*, etc.) *it is generally obtained by converting A.C. to D.C. with the help of converters of ionic or machine types.* In the latter case wide use is made of such installations as motor generator sets in which A.C. motor is coupled to a D.C. generator on a common shaft.
- As primary sources of power, D.C. generators are *mainly used in self-contained plants such as automobiles and air planes, for electric arc welding, train car lighting, in sub-marines, etc.*

3.1. Classification

According to *method of excitation* D.C. generators are classified as follows :

1. Separately excited generators, 2. Self-excited generators.

3.1.1. Separately excited generators

These are those generators whose field magnets are *energised* from an independent *external source* of D.C. current. Such a generator is shown in Fig. 15.

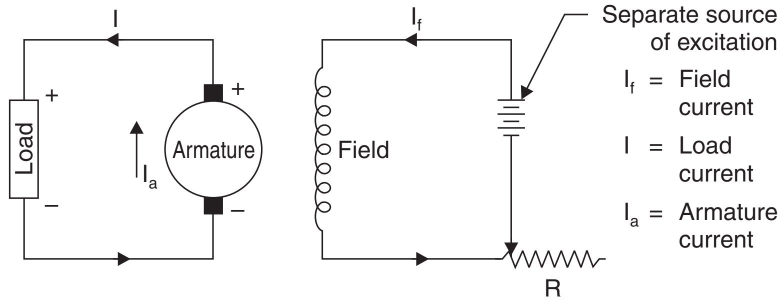


Fig. 15. Separately excited generator.

3.1.2. Self-excited generators

These are those generators whose field magnets are *energised by the current produced by the generators themselves.* Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current produced which is partly or fully passed through the field coils thereby *strengthening the pole flux.*

Self excited generators can be divided, in accordance with how the *field winding is connected into generators*, as follows :

(*i*) Shunt wound generators

(*ii*) Series wound generators

(*iii*) Compound wound generators :

(*a*) Short shunt

(*b*) Long shunt

(*i*) **Shunt wound generators :** Refer to Fig. 16. In these generators the field windings are connected *across or in parallel with the armature conductors,* and have the full voltage of the generator across them.

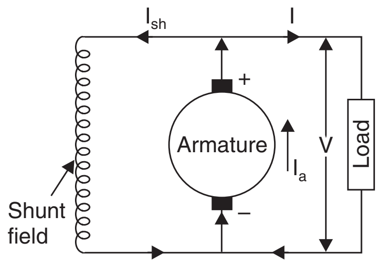


Fig. 16. Shunt wound generator.

Important relations : Refer to Fig. 16.

(*i*) $I_{sh} = \dfrac{V}{R_{sh}}$

(*ii*) $I_a = I_{sh} + I$

(*iii*) $V = E_g - I_a R_a$

(*iv*) Power developed $= E_g I_a$

(*v*) Power delivered $= VI$

where I_{sh} = shunt field current, I_a = armature current,

I (or I_l) = load current, R_a = armature resistance,

R_{sh} = shunt field resistance, E_g = generated e.m.f., and

V = terminal voltage.

(*ii*) **Series wound generators :** Refer to Fig. 17. In this case, the field windings are joined in series with armature conductors. *As they carry full load current, they consist of relatively few turns of thick wire or strip*. The use of such generators is limited to special purposes (as *boosters* etc.).

Important Relations. (*see* Fig. 17) :

(*a*) $I_a = I_{se} = I$ (I_{se} = series field current)

(*b*) $V = E_g - I(R_a + R_{se})$ (R_{se} = series field resistance)

(*c*) Power developed = $E_g I$

(*d*) Power delivered = VI.

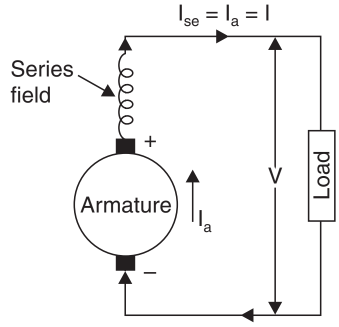


Fig. 17. Series wound generator.

(*iii*) **Compound wound generators.** It is a combination of a few series and a few shunt windings and be either short shunt or long shunt as shown in Figs. 18 and 19 respectively.

Important Relations :

(*a*) **Short shunt compound wound.** (*See* Fig. 18) :

(*i*) $I_{se} = I$

(*ii*) $I_{sh} = \dfrac{V + I_{se}R_{se}}{R_{sh}}$

(*iii*) $I_a = I + I_{sh}$

(*iv*) $V = E_g - I_aR_a - I_{se}R_{se}$

(*v*) Power developed = $E_g I_a$

(*vi*) Power delivered = VI.

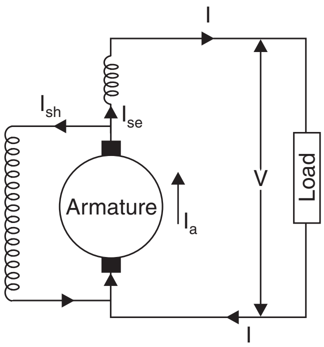


Fig. 18. Short shunt compound wound generator.

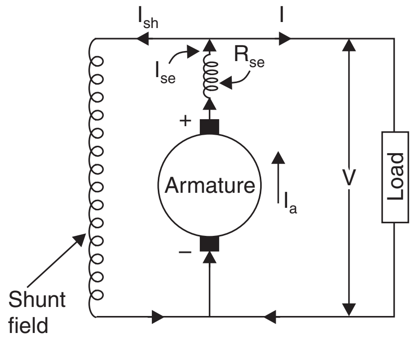


Fig. 19. Long shunt compound wound generator.

(*b*) **Long shunt compound wound.** (*See* Fig. 19) :

(*i*) $I_{sh} = \dfrac{V}{R_{sh}}$

(*ii*) $I_a = I_{se} = I + I_{sh}$

(*iii*) $V = E_g - I_aR_a - I_{se}R_{se} = E_g - I_a(R_a + R_{se})$

(*iv*) Power developed = $E_g I_a$

(*v*) Power delivered = VI.

Example 6. *The terminal voltage of a separately excited D.C. generator with constant excitation is constant and is equal to 250 V. Determine the percentage reduction in speed when the load changes from 250 kW to 150 kW. The armature resistance is 0.012 Ω and total contact drop at brushes = 2 V. Neglect armature reaction.*

Solution. Terminal voltage, $V = 250$ volts

Armature resistance, $R_a = 0.012\ \Omega$

Total contact drop at brushes $= 2$ V

Load change : From 250 kW to 150 kW

Percentage reduction in speed :

We know that generated e.m.f.,

$$E_g = \frac{p\phi ZN}{60a} \text{ i.e., } E_g \propto \phi N$$

Since ϕ is constant, $E_g \propto N$

— When the load is 250 kW, the armature current

$$I_{a1} = \frac{250 \times 1000}{250} = 1000 \text{ A}$$

and $E_{g1} = V + Ia_1\, Ra_1$ + drop in brushes

$= 250 + 1000 \times 0.012 + 2 = 250 + 12 + 2 = 264$ V

— When the load is 150 kW, the armature current

$$I_{a2} = \frac{150 \times 1000}{250} = 600 \text{ A}$$

and $E_{g2} = V + I_{a2}\, R_a$ + drop in brushes

$= 250 + 600 \times 0.012 + 2 = 250 + 7.2 + 2 = 259.2$ V

Now since $E_g \propto N$

$$\therefore \quad \frac{E_{g2}}{E_{g1}} = \frac{N_2}{N_1}$$

i.e., $$\frac{N_2}{N_1} = \frac{259.2}{264} \quad \text{or} \quad \frac{N_1}{N_2} = \frac{264}{259.2}$$

or $$\frac{N_1 - N_2}{N_1} = \frac{264 - 259.2}{264} = 0.01818$$

$\therefore$ **%age reduction in speed = 0.01818 × 100 = 1.818%. (Ans.)**

☞ **Example 7.** *A separately excited generator with constant excitation is connected to a constant resistance circuit. When the speed is 1200 r.p.m. it delivers 120 A at 500 V. At what speed will the current be reduced to 60 A ? Armature resistance = 0.1 Ω. Contact drop/brush = 1 V. Armature reaction may be ignored.*

Solution. Initial speed, $N_1 = 1200$ r.p.m.

Initial load current, $I_{a1} = 120$ A

Initial terminal voltage, $V_1 = 500$ V

Final load current, $I_{a2} = 60$ A

Armature resistance, $R_a = 0.1\ \Omega$

Contact drop/brush $= 1$ V

Speed N_2 :

Refer to Fig. 20.

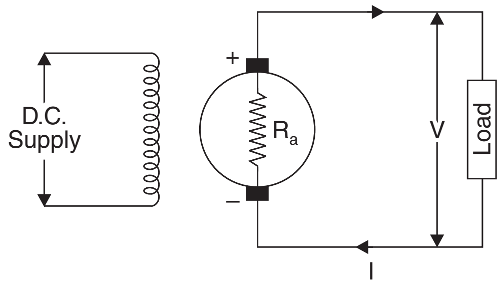


Fig. 20

We know that e.m.f. generated (at load current of 120 A)

$E_{g1} = V_1 + I_{a1}R_a$ + drop in brushes

$= 500 + 120 \times 0.1 + 2 \times 1 = 514$ V

Load resistance (constant),

$$R = \frac{500}{120} = 4.167\ \Omega$$

Let E_{g2} be the generated e.m.f. when current delivered is 60 A.

$$V_2 = I_{a2}R = 60 \times 4.167 = 250\ \text{V}$$

$$E_{g2} = V_2 + I_{a2}R_a + 2 \times 1$$

$$= 250 + 60 \times 0.1 + 2 \times 1 = 258\ \text{V}$$

Now using the relation, $\dfrac{N_2}{N_1} = \dfrac{E_{g2}}{E_{g1}}$

$$\frac{N_2}{1200} = \frac{258}{514}$$

$$N_2 = \frac{1200 \times 258}{514} = 602.33\ \text{r.p.m.}$$

Hence, **speed at a load of 60 A = 602.33 r.p.m. (Ans.)**

Example 8. *A shunt generator supplied 500 A at 500 V. Calculate its generated e.m.f. if its armature and shunt field resistances are 0.02 Ω and 125 Ω respectively.*

Solution. Load current, $I = 500$ A

Terminal voltage, $V = 500$ volts

Armature resistance, $R_a = 0.2\ \Omega$

Shunt field resistance, $R_{sh} = 125\ \Omega$

Refer to Fig. 21.

Shunt field current,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{125} = 4\ \text{A}$$

Armature current,

$$I_a = I + I_{sh} = 500 + 4 = 504\ \text{A}$$

Generated e.m.f.,

$$E_g = V + I_aR_a$$

$$= 500 + 504 \times 0.02 = 510.08\ \text{V}$$

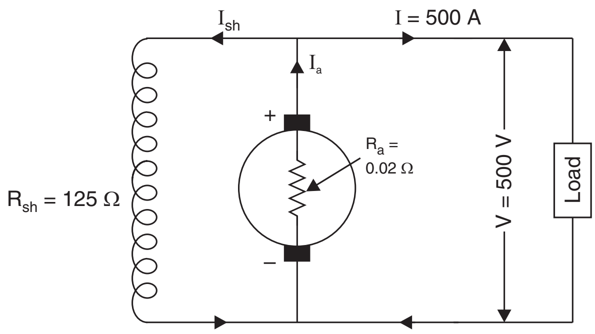


Fig. 21

Hence, **generated e.m.f. = 510.08 V. (Ans.)**

Example 9. *A 4-pole, D.C. shunt generator, with a shunt field resistance of 100 ohms and an armature resistance of 1 ohm, has 378 wave-connected conductors in its armature. The flux per pole is 0.02 Wb. If a load resistance of 10 ohms is connected across the armature terminals and the generator is driven at 1000 r.p.m., calculate power absorbed by load.*

Solution. *Given* : $p = 4$; $R_{sh} = 100\ \Omega$; $R_a = 1\ \Omega$, $Z = 378$; $a = 2$;
$\phi = 0.02$ Wb ; $R_L = 10\ \Omega$; $N = 1000$ r.p.m.

Power absorbed by the load :

The generator arrangement is shown in the Fig. 22.

For the generator,

$$E_g = \frac{p\phi ZN}{60\,a}$$

$$= \frac{4 \times 0.02 \times 378 \times 1000}{60 \times 2} = 252\ \text{V}$$

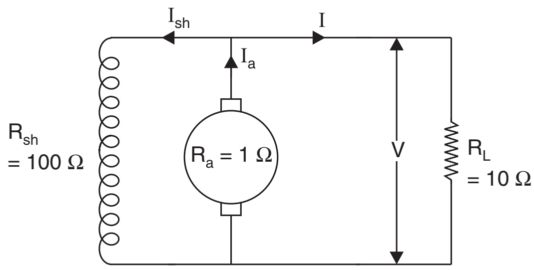


Fig. 22

$$V = E_g - I_a R_a$$

$$\therefore \quad I_a = \frac{E_g - V}{R_a} = \frac{252 - V}{1} = 252 - V$$

Now $$I = \frac{V}{R_L} = I_a - I_{sh} = (252 - V) - \frac{V}{100}$$

$$\therefore \quad \frac{V}{10} = (252 - V) - \frac{V}{100} \quad \text{or} \quad 252 - V = \frac{11V}{100}$$

or $$25200 - 100\,V = 11\,V \quad \text{or} \quad V = \frac{25200}{111} = 227 \text{ V}$$

$$I = \frac{227}{10} = 22.7 \text{ A}$$

∴ **Power absorbed by the load** $= I^2 R_L$

$= 22.7^2 \times 10 = 5153$ W or **5.153 kW. (Ans.)**

Example 10. *A 4-pole lap wound shunt generator supplies to 50 lamps of 100 watts, 200 V each. The field and armature resistances are 50 Ω and 0.2 Ω respectively. Allowing a brush drop of 1 V each brush, calculate the following :*

(*i*) *Armature current.* (*ii*) *Current per path.*

(*iii*) *Generated e.m.f.* (*iv*) *Power output of D.C. armature.*

Solution. Number of poles, $p = 4$

Total lamp load, $P = 50 \times 100 = 5000$ W

Terminal voltage, $V = 200$ volts

Field resistance, $R_{sh} = 50\ \Omega$

Armature resistance, $R_a = 0.2\ \Omega$

Voltage drop/brush $= 1$ V

Refer to Fig. 23.

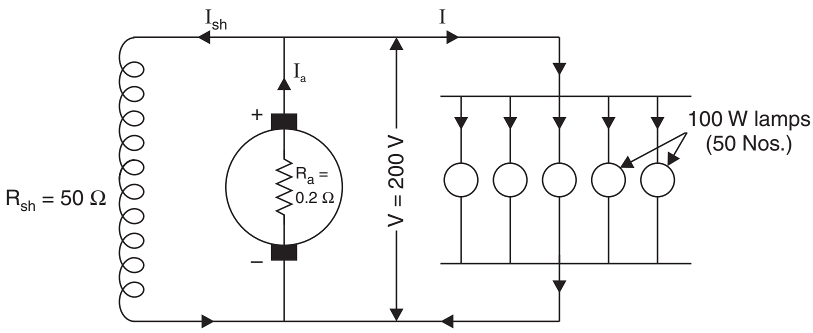


Fig. 23

(*i*) **Armature current, I_a :**

Load current, $$I = \frac{\text{Power consumed}}{\text{Terminal voltage}} = \frac{P}{V} = \frac{5000}{200} = 25 \text{ A}$$

Shunt field current, $$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{50} = 4 \text{ A}$$

∴ Armature current, $I_a = I + I_{sh} =$ **25 + 4 = 29 A. (Ans.)**

(*ii*) **Current per path :**

Current per path $= \dfrac{I_a}{a} = \dfrac{29}{4}$ [$\because$ $a = p = 4$, generator being lap wound]

$=$ **7.25 A. (Ans.)**

(*iii*) **Generated e.m.f., E_g :**

$$E_g = V + I_aR_a + \text{brush drop} = 200 + 29 \times 0.2 + 2 \times 1 = 207.8 \text{ V}$$

Hence, **generated e.m.f. = 207.8 V. (Ans.)**

(*iv*) **Power output of D.C. armature :**

Power output of D.C. armature $= \dfrac{E_gI_a}{1000} = \dfrac{207.8 \times 29}{1000}$ kW **= 6.026 kW. (Ans.)**

☞ **Example 11.** *A 4-pole, 500 V wave-wound shunt generator delivers a load current of 140 A. It has 65 slots with 12 conductors/slot and runs at 800 r.p.m. The shunt field and armature resistances are 250 Ω and 0.2 Ω respectively. The diameter of the bore of the pole shoe is 45 cm, the angle subtended by the pole shoe is 70° and it is 25 cm in length. Assuming contact drop/brush as 1 V, calculate the flux density in the air gap.*

Solution. Refer to Fig. 24.

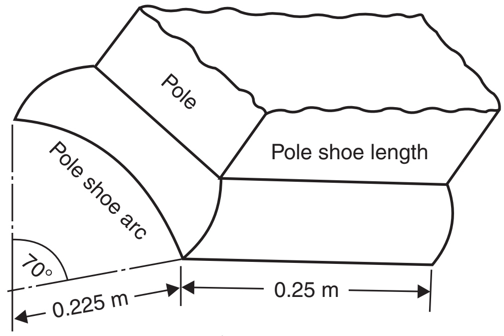


Fig. 24

Number of poles, $p = 4$

Number of parallel paths, $a = 2$

[generator being wave wound]

Terminal voltage, $V = 500$ volts

Load current, $I = 140$ A

Speed of rotation, $N = 800$ r.p.m.

Number of slots $= 65$

Number of conductors/slot $= 12$

$\therefore$ Total number of conductors,

$Z = 12 \times 65 = 780$

Shunt field resistance, $R_{sh} = 250\ \Omega$

Armature resistance, $R_a = 0.2\ \Omega$

Diameter of the bore of pole shoe, $D = 45$ cm ($= 0.45$ m)

Angle subtended by the pole shoe, $\theta = 70°$

Length of pole shoe $= 25$ cm ($= 0.25$ m)

Contact drop/brush $= 1$ V

Flux density in the air gap, B :

Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{500}{250} = 2$ A

Armature current, $I_a = I + I_{sh} = 140 + 2 = 142$ A

Generated e.m.f., $E_g = V + I_aR_a + \text{brush drop}$

$= 500 + 142 \times 0.2 + 2 \times 1 = 530.4$ V

Also, $E_g = \dfrac{p\phi ZN}{60\,a}$

$$530.4 = \frac{4 \times \phi \times 780 \times 800}{60 \times 2}$$

or $$\phi = \frac{530.4 \times 60 \times 2}{4 \times 780 \times 800} = 0.0255 \text{ Wb}$$

Arc length of pole shoe $= \frac{\pi D\theta}{360} = \frac{\pi \times 0.45 \times 70}{360} = 0.275$ m

Area of pole shoe, A = Arc length × length of pole shoe

$= 0.275 \times 0.25 = 0.06875$ m^2

Flux density in the air gap, $B = \frac{\phi}{A} = \frac{0.0255}{0.06875} = 0.371$ T (or Wb/m^2)

Hence, **flux density in the air gap = 0.371 T. (Ans.)**

Example 12. *A series generator delivers a current of 100 A at 250 V. Its armature and series field resistances are 0.1 Ω and 0.055 Ω respectively. Find* :

(*i*) *Armature current* (*ii*) *Generated e.m.f.*

Solution. *See* Fig. 25.

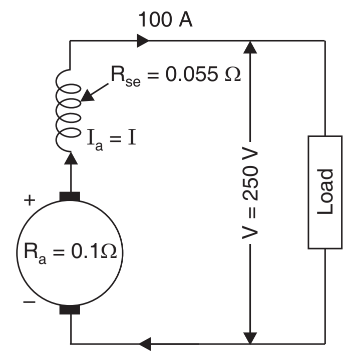


Fig. 25

Load current, $I = 100$ A

Terminal voltage, $V = 250$ volts

Armature resistance, $R_a = 0.1\ \Omega$

Series field resistance, $R_{se} = 0.055\ \Omega$

(*i*) **Armature current, I_a** :

Armature current (I_a) = load current (I)

$\therefore$ I_a = **100 A. (Ans.)**

(*ii*) **Generated e.m.f.** :

Generated e.m.f. $E_g = V + I(R_a + R_{se})$

$= 250 + 100(0.1 + 0.055) = 265.5$ V

Hence, **generated e.m.f. = 265.5 V. (Ans.)**

☞ **Example 13.** *A short shunt compound generator has armature, series field and shunt field resistances of 0.06 Ω, 0.03 Ω and 110 Ω respectively. It supplies 100 lamps rated at 250 V, 40 W. Find the generated e.m.f. Assume that contact drop/brush = 1 V.*

Solution. *See* Fig. 26.

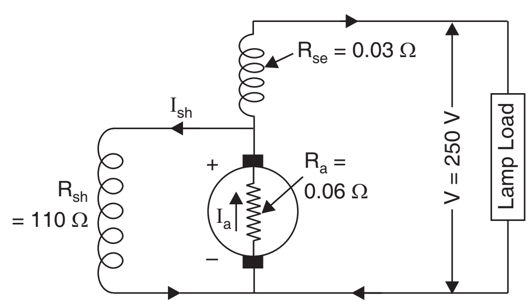


Fig. 26

Armature resistance, $R_a = 0.06\ \Omega$

Series field resistance, $R_{se} = 0.03\ \Omega$

Shunt field resistance, $R_{sh} = 110\ \Omega$

Terminal voltage, $V = 250$ Volts

Lamp load, $P = 100 \times 40 = 4000$ W

Contact drop/brush = 1 V

Generated e.m.f., E_g :

Load current, $I = \frac{P}{V} = \frac{4000}{250} = 16$ A

Voltage drop in series winding

$= IR_{se} = 16 \times 0.03 = 0.48$ V

Voltage across shunt field winding

$= V + IR_{se} = 250 + 0.48 = 250.48$ V

Shunt field current, $I_{sh} = \frac{250.48}{110} = 2.277$ A

Armature current, $I_a = I + I_{sh} = 16 + 2.277 = 18.277$ A

Generated e.m.f., E_g $= V + IR_{se} + I_aR_a$ + brush drop

$= 250 + 0.48 + 18.277 \times 0.06 + 2 \times 1 =$ **253.58 V. (Ans.)**

Example 14. *A long shunt compound generator has an armature, series field and shunt field resistances of 0.04 Ω, 0.03 Ω and 200 Ω respectively. It supplies a load current of 180 A at 400 V. Calculate the generated e.m.f. Assume contact drop/brush = 1 V.*

Solution. Refer to Fig. 27.

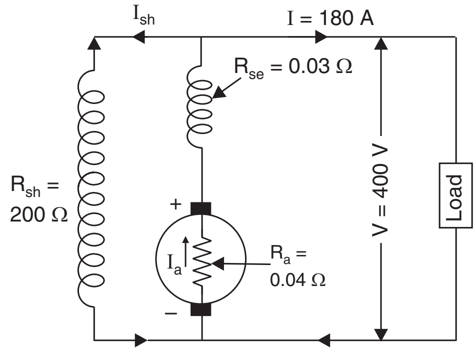


Fig. 27

Armature resistance, $R_a = 0.04\ \Omega$

Series field resistance, $R_{se} = 0.03\ \Omega$

Shunt field resistance, $R_{sh} = 200\ \Omega$

Load current, $I = 180$ A

Terminal voltage, $V = 400$ volts

Contact drop/brush $= 1$ V

Generated e.m.f., E_g :

Shunt current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{400}{200} = 2$ A

Armature current, $I_a = I + I_{sh}$

$= 180 + 2 = 182$ A

Generated e.m.f., $E_g = V + I_aR_a + I_aR_{se}$ + drop at brushes

$= 400 + 182 \times 0.04 + 182 \times 0.03 + 2 \times 1$

$= 400 + 7.28 + 5.46 + 2 = 414.74$ V

Hence, **generated e.m.f. = 414.74 V. (Ans.)**

4. POWER DIVISION IN A D.C. GENERATOR

Fig. 28 presents in graphical form power balances for D.C. generators with both shunt and series fields.

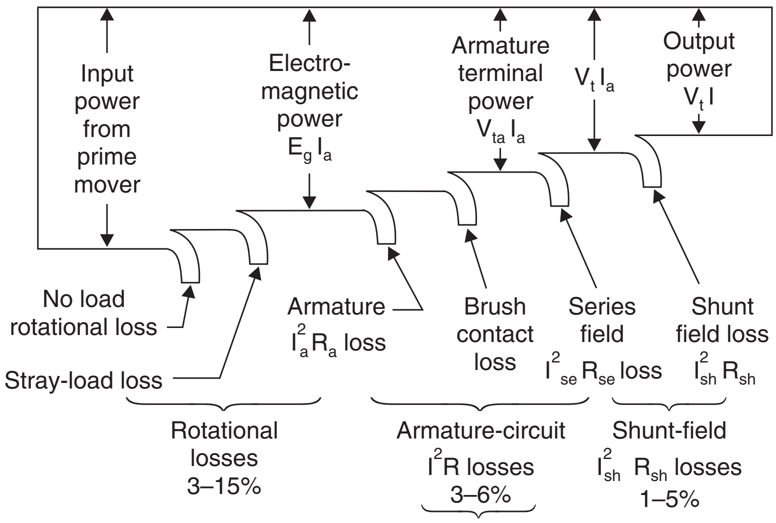


Fig. 28. Power division in a D.C. generator.

5. CHARACTERISTICS OF D.C. GENERATORS

The properties of generators are analysed with the aid of characteristics which give the relations between fundamental quantities determining the operation of a generator. These include the voltage across the generator terminals V, the field or exciting current I_f, the armature current I_a, and the speed of rotation N.

The three most important characteristics of D.C. generators are given below :

1. No load saturation characteristics $\left(\frac{E_0}{I_f}\right)$

2. Internal or total characteristics $\left(\frac{E}{I_a}\right)$

3. External characteristics $\left(\frac{V}{I}\right)$.

1. **No load saturation characteristic** $\left(\frac{\mathbf{E_0}}{\mathbf{I_f}}\right)$. It is also a known as *magnetic or open circuit characteristic* (O.C.C.). It shows the relationship between the no-load generated e.m.f. in armature, E_0 and field or exciting current I_f at a given fixed speed. The shape of the curve is practically the same for all types of generators whether they are separately excited or self-excited. It is just the magnetisation curve for the material of the electromagnets.

2. **Internal or total characteristic** $\left(\frac{\mathbf{E}}{\mathbf{I_a}}\right)$. It gives the relationship between the *e.m.f. E actually induced* in the armature after allowing for the demagnetising effect of armature reaction and the *armature current* I_a. This characteristic is of interest mainly to the designer.

3. **External characteristic** $\left(\frac{\mathbf{V}}{\mathbf{I}}\right)$

- This characteristic is also referred to as *performance characteristic* or sometimes *voltage-regulating curve*.
- It gives relation between the *terminal voltage V* and *load current I*.
- The curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The *values of V are obtained by subtracting* I_aR_a *from corresponding values of E*.
- The characteristic is of great importance in judging the *suitability of a generator for a particular purpose.*

The external characteristic can be obtained by the following two ways :

(*i*) By making simultaneous measurements with a suitable voltmeter and an ammeter on a loaded generator.

(*ii*) Graphically from the O.C.C. provided the armature and field resistances are known and also if the demagnetising effect of the armature reaction is known.

5.1. Separately Excited Generator

- Fig. 29 shows the connections of a separately excited generator, a battery being indicated as the source of exciting current, although any other constant voltage source could be used.

The field circuit is provided with a variable resistance and would normally contain a field switch and an ammeter, these being omitted from the diagram for simplicity. The armature is connected through 2-pole main switch to the bus bars, between which the load is connected.

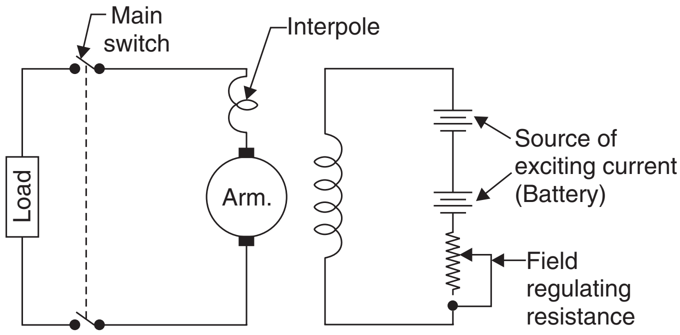


Fig. 29. Connection for a separately excited generator.

5.1.1. No-load saturation characteristic (or O.C.C.)

- If the generator is run at constant speed with the main switch open, and the terminal voltage is noted at various values of exciting or field current then the O.C.C. shown in Fig. 30 can be plotted. This is also referred to as the '*magnetisation curve*' since the same graph shows, to a suitably chosen scale, the amount of magnetic flux, there being a constant relationship (depending upon speed of rotation) between flux and induced voltage.
- It will be noticed that a small voltage is produced when the field current is zero, this being due to a small amount of permanent magnetism in the field poles. This is called *residual magnetism* and is usually sufficient to produce 2 or 3 per cent of normal terminal voltage, although in some special cases it is purposely increased to 10 per cent or more.
- The *first part of the curve is approximately straight* and *shows that the flux produced is proportional to the exciting current ;* but *after a certain point, saturation of the iron becomes perceptible as the curve departs from straight line form.*

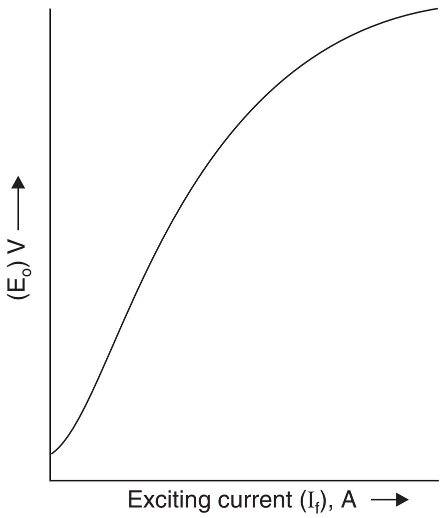


Fig. 30. Open-circuit characteristic of a separately excited generator.

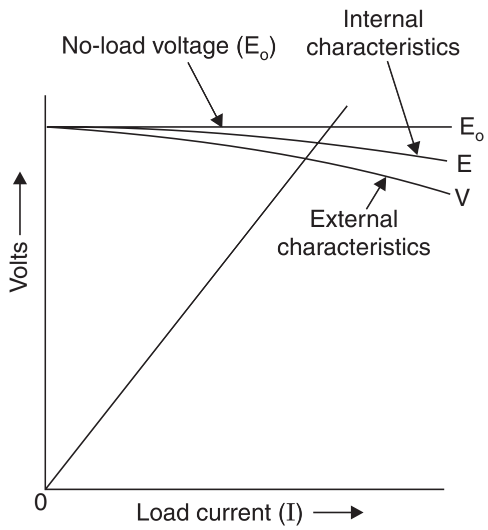


Fig. 31. Load characteristics of a separately excited generator.

5.1.2. Internal and external characteristics (or load characteristics)

- Load characteristics for a separately excited generator are shown in Fig. 31. The most important is the '*external characteristic*' (or total characteristic), which indicates the way in which the terminal voltage (V) varies as the load current is increased from zero to its full load value, the speed of rotation and exciting current being constant.

The voltage drop (drop of volts) at any particular load current, *indicated by the vertical distance between the external characteristic and the no-load voltage* is brought about by two causes :

(*i*) *Armature reaction which has a demagnetising effect upon the field.*

(*ii*) *Resistance drop*, this being the product of the armature current and the total armature-circuit resistance, consisting of the armature resistance, interpole resistance and brush contact resistance.

- The '*internal characteristic*' is obtained by *calculating the resistance drop for a few values of current and adding this to the voltage shown by the external characteristic. The vertical distance between the internal characteristic and no load voltage then represents the effect of armature reaction alone*.
- When the resistance of load is R, then voltage across its terminals is $V = IR$, where I represents the current, so that if the values of V corresponding to various values of I are calculated, the values will all lie upon a straight line such as OL in Fig. 31. *The load current and terminal voltage corresponding to this resistance are given by the inter-section of the line OL with the external characteristics.*

Note. The great *advantage of separate excitation* over all other forms of excitation is that the *current is entirely independent of the load current in the armature.* It is however, rather inconvenient to have to depend upon a separate source of supply and, therefore, the *method is used only in special cases, where the generator has to operate over a wide range of terminal voltage.*

5.2. Building up the Voltage of Self-excited Shunt Generator

One of the simplest forms of 'self-excited' generator is the shunt-wound machine, the connection diagram (without load) of which is shown in Fig. 32. The manner in which a self-excited generator manages to excite its own field and build a D.C. voltage across its armature is described with reference to Fig. 33 in the following **steps** :

(*i*) Assume that the generator starts from rest, *i.e.*, prime-mover speed is zero. Despite a residual magnetism, the generated e.m.f. E, is zero.

(*ii*) As the prime-mover rotates the generator armature and the speed approaches rated speed, the voltage due to residual magnetism and speed increases.

(*iii*) At rated speed, the voltage across the armature due to residual magnetism is small, E_1, as shown in the figure. But this voltage is also across the field circuit whose resistance is R_f. Thus, the current which flows in the field circuit I_1, is also small.

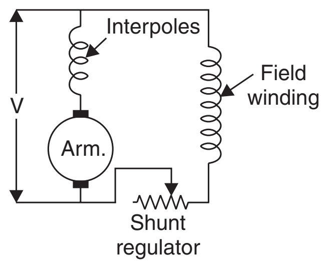


Fig. 32. Self-excited shunt generator.

(*iv*) When I_1 flows in the field circuit of the generator of Fig. 32, an increase in m.m.f. results (due to I_fT_f, T_f being field turns which aids the residual magnetism in increasing the induced voltage to E_2 as shown in Fig. 33.

(*v*) Voltage E_2 is now impressed across the field, causing a large current I_2 to flow in the field circuit. I_2T_f is an increased m.m.f., which produces generated voltage E_3.

(*vi*) E_3 yields I_3 in the field circuit, producing E_4. But E_4 causes I_4 to flow in the field producing E_5 ; and so on, up to E_8, the maximum value.

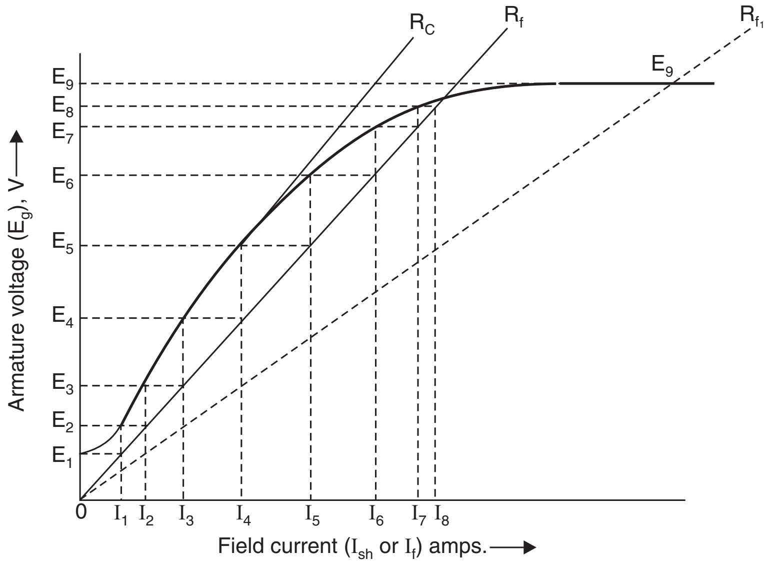


Fig. 33. Building up the voltage of a shunt generator.

(*vii*) The *process continues until that point where the field resistance line crosses the magnetisation curve* in Fig. 33. Here, the process stops. The induced voltage produced, when impressed across the field circuit, produces a current flow that in turn produces an induced voltage of the same magnitude, E_8, as shown in the figure.

Critical Resistance :

- In the above description a particular value of field resistance R_f was used for building up of self-excited shunt generator. If the field resistance were reduced by means of adjusting the field rheostat of Fig. 32 to a lower value say R_{f_1}. shown in Fig. 33, the build-up process would take place along field resistance line R_{f_1}, and build-up a somewhat higher value than E_8, *i.e.,* the point where R_{f1} intersects the magnetisation curve, E_9. Since the curve is extremely saturated in the vicinity of E_9, reducing the field resistance (to its limiting field winding resistance) will not increase the voltage appreciably. Conversely, increasing the field rheostat resistance and the field circuit resistance (to a value having a higher slope than R_f in the figure) will cause a reduction of the maximum value to which build-up can possibly occur.
- The field resistance may be increased until the field circuit reaches a *critical field resistance. Field circuit resistance above the critical field resistance will fail to produce build-up.*

This critical field circuit resistance, R_c, is shown as tangent to the saturation curve passing through the origin, O, of the axes of the curve of Fig. 33. Thus, a field circuit resistance higher than R_c will produce an armature voltage of E_1 approximately (and no more).

☞ **Reasons for Failure of Self-excited Shunt Generator to Build-up Voltage.** The reasons why a self-excited generator may fail to build-up voltage are given below :

1. **No residual magnetism.** The start of the build-up process requires some residual magnetism in the magnetic circuit of the generator. If there is little or no residual magnetism, because of inactivity or jarring in shipment, no voltage will be generated that can produce field current. To overcome this difficulty, a separate source of direct current is applied to the field for a short period of time and then removed. The magnetic field should now be sufficient to allow the voltage to build-up. *The application of a separate source of direct current to the field is called 'flashing the field'.*

2. **Field connection reversed.** The voltage generated due to residual magnetism is applied to the field. Current should flow in the field coils in such a direction as to produce lines of flux in the same direction as the residual flux. If the field connections are reversed, the *lines of flux produced by the current flow will oppose the residual flux* so that the generated voltage will *decrease* rather than increase when the field circuit is closed. In this instance it is necessary to *reverse the field connections with respect to the armature.*

3. **Field circuit resistance too high.** A field circuit resistance greater than critical value will prevent an appreciable build-up. At no load, resistance greater than the critical may be caused by the following :

- **Open field circuit connection.** The effects of an open circuit are apparent. The field circuit resistance is *much greater than the critical value* ; hence *generator will not build-up*.
- **Dirty commutator.** A dirty commutator does not permit good contact between the brushes and the commutator. This poor contact shows up as a high resistance to the flow of current in the field circuit and produces the same effect as a high field circuit resistance.

5.3. Shunt Generator Characteristics

In a shunt generator the field circuit is connected directly across the armature. Appliances, motors, light bulbs, and other electrical devices connected in *parallel* across the generator terminals represent a *load* on the generator. As more devices are connected in parallel, the load on the generator increases ; that is, the generator current increases. Because the generator current increases, the terminal voltage of the generator decreases. There are three factors that cause this decrease in voltage :

(*i*) Armature-circuit resistance (R_a),

(*ii*) Armature reaction, and

(*iii*) Reduction in field current.

(*i*) **Armature-circuit resistance.** The armature circuit of a generator, like every electrical circuit, contains resistance. This resistance includes the resistance of (*i*) *the copper conductors of the armature winding, (ii) the commutator, (iii) contact resistance between brushes and commutator,* and (*iv*) *the brushes themselves.* When no current flows through the armature, there is no *IR* drop in the armature and the voltage at the terminals is the same as the generated voltage. However, when there is current in the armature circuit, a voltage drop exists due to the armature resistance, and the terminal voltage is less than the generated voltage. The terminal voltage may be calculated from the following reaction :

$$V = E_g - I_a R_a$$

where V = voltage at terminals of generator,

E_g = generated or induced voltage,

I_a = total armature current, and

R_a = armature-circuit resistance.

(*ii*) **Armature reaction.** When current flows in the armature conductors a flux surrounds these conductors. The direction of this armature flux is such that it reduces the flux from the field poles, resulting in both a reduced generated voltage and terminal voltage.

(*iii*) **Reduction in the field current.** The field circuit is connected across the terminals of the generators. When the terminal voltage of the generator becomes smaller because of the armature-resistance volt drop and armature reaction, the voltage across the field circuit also becomes smaller and therefore field current will be less. A reduction in the magnitude of field current also reduces the flux from the field poles, which in turn reduces the generated voltage and also the terminal voltage.

External Characteristic

- See Figs. 34 and 35. The effect of the preceding three factors is shown in Fig. 35, which shows external (load-voltage) characteristic of a shunt generator.
- As shown in the circuit of Fig. 34, the readings of the voltage across the armature (and load), V are plotted as a function of load current, I. The voltage, V, is the same as E_g at no load (neglecting the I_aR_a and armature reaction drop produced by the field current). The effects of armature reaction, armature circuit voltage drop, and decrease in field current are all shown with progressive increase in load. Note that both the armature reaction and the I_aR_a drops are shown as dashed straight lines, representing theoretically linear voltage directly proportional to the increase in load current. The drop owing to decreased field current is a curved line, since it depends on the degree of saturation existing in the field at the value of load.

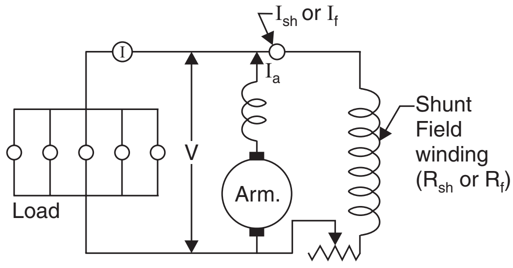


Fig. 34. Shunt generator under load.

- Generally, the external load-voltage characteristic decreases with application of load only to *a small extent up to its rated load* (current) *value*. Thus, the shunt generator is considered as having a fairly constant output voltage with application of load, and in practice, is *rarely operated beyond the rated load current value continuously for any appreciable time.*

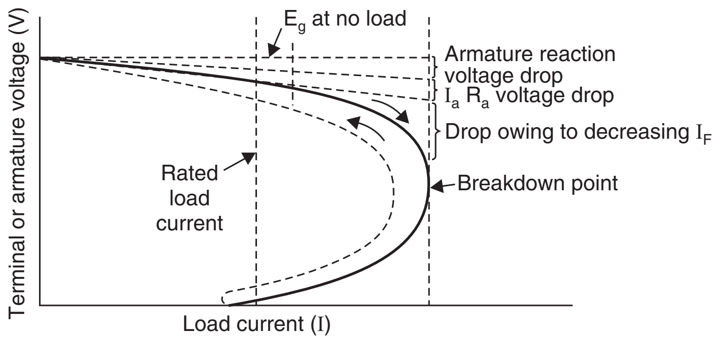


Fig. 35. Shunt generator load characteristics.

- As shown in Fig. 35 *further application of load causes the generator to reach a breakdown point beyond which further load causes it to 'unbuild' as it operates on the unsaturated portion of its magnetisation curve. This unbuilding process continues until the terminal voltage is zero, at which point the load current is of such magnitude that the internal armature circuit voltage drop equals the e.m.f. generated on the unsaturated or linear portion of its magnetisation curve.*
- It may be noted that if the *external load is decreased* (an increase of external load resistances), the *generator will tend to build-up gradually along the dashed line* shown in Fig. 35. Note that for any value of load current, the terminal or armature voltage is less (as the voltage increases) compared to the solid lines which yield a higher voltage (as the voltage decreases). This *difference is due to hysteresis*.

Effect of Varying Excitation. Fig. 36 shows the effect of varying excitation upon the external characteristic of a shunt generator. With normal excitation the initial slope is small and heavy load current can be varied. With reduced excitation, the fall of voltage is more rapid and the maximum load current is reduced.

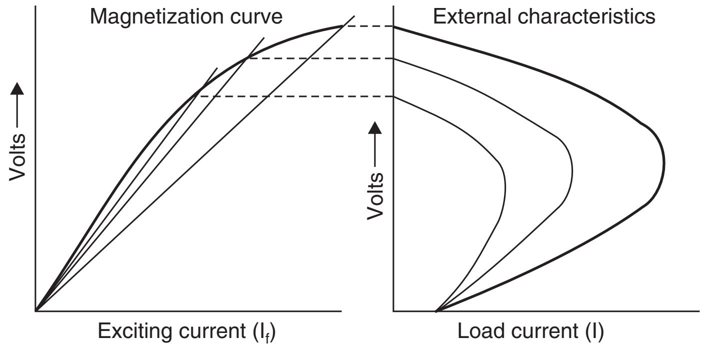


Fig. 36. Effect of varying excitation.

Voltage Regulation. The term '*voltage regulation*' is used to indicate the degree of change in armature voltage produced by application of load. If there is little change from no-load to full load, the generator or voltage-supplying device is said to possess good voltage regulation. If the *voltage changes appreciably with load, it is considered to have poor voltage regulation.*

'Voltage regulation' is defined as the *change in voltage from no-load to full load, expressed as a percentage of the rated terminal voltage* (armature voltage at full load).

i.e., Per cent voltage regulation

$$= \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100 \quad ...(2)$$

where V_{nl} = no load terminal voltage,

V_{fl} = full load (rated) terminal voltage.

Internal or Total Characteristic. To determine internal characteristic from external characteristic the following procedure is adopted [*see* Fig. 37].

Steps 1. From the given data, draw the external characteristic (*I*).

2. Draw the shunt field resistance line *OL* and armature resistance line *OM*.

3. On the external characteristic take any point say *F*.

4. From point *F* draw vertical and horizontal lines intersecting *X* and *Y* axes respectively. Let these lines be *FC* and *FA* respectively.

5. Take point *D* on *X*-axis so that *CD* = *AB* representing the shunt field current, I_{sh}.

6. From point *D* draw vertical *DE* and produce it intersecting line *AF* produced at *H*.

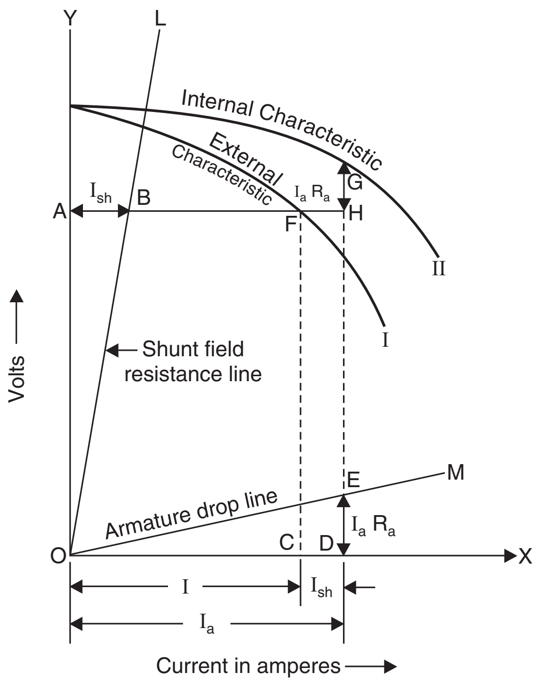


Fig. 37. Determination of internal characteristic from external characteristic.

7. Take point *G* on line *DH* produced so that $HG = DE$ ($= I_a R_a$) representing the armature drop.

8. Following the above procedure take a number of points on external characteristic and find corresponding points lying on internal characteristic.

9. Draw a curve passing through these points which is the required internal characteristic (*II*).

External Characteristic and No-load Saturation Curve. The external characteristic of a shunt generator can be obtained directly from its no-load saturation curve as explained below. Following two cases will be considered :

(*A*) When armature reaction is so small as to be *negligible*. This is more or less true for generators fitted with compoles.

(*B*) When armature reaction is *not negligible.*

(A) Armature Reaction Negligible :

Steps 1. From given data draw O.C.C. [*see* Fig. 38].

2. Draw shunt field resistance line (say *OS*) meeting O.C.C. at any point (say *A*).

3. From point *A* draw horizontal line intersection *Y*-axis, say at point *B*.

Hence, *OB* is the maximum no-load or open circuit voltage.

4. Take any point (say *L*) on the O.C.C. and draw an ordinate, say, *LMN* intersecting field resistance line at *M* and *X*-axis at *N*. Now, *LN represents the generated e.m.f., MN represents the terminal voltage and LM represents voltage drop in armature*.

5. From points *L* and *M* draw horizontal lines cutting vertical axis say at point *D* and *E* respectively.

6. Draw armature resistance line *OC*.

7. From point *E*, draw line *EF* parallel to line *OC* cutting line *LD* produced at *F*. Hence, *F* is lying on the internal characteristic.

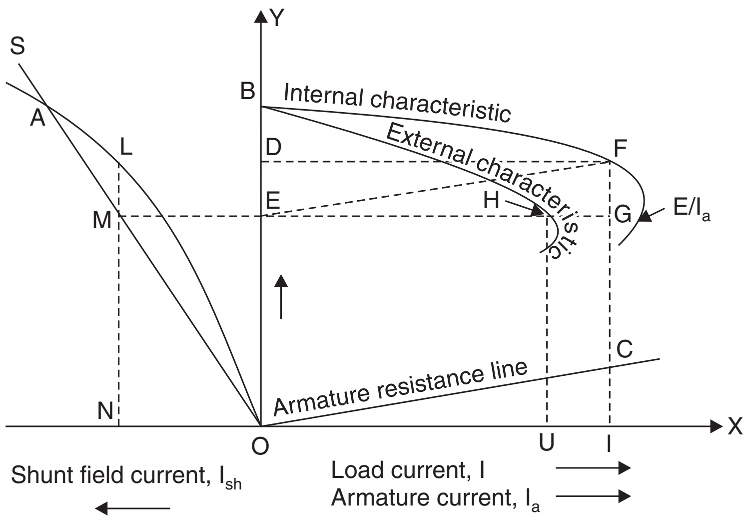


Fig. 38. Determination of external and internal characteristic from O.C.C.

Similarly other points can be obtained and internal characteristic may be drawn through these points :

8. From point F draw vertical line intersecting produced line ME at any point G and X-axis say, at point T. Since $FG = LM = CT$, hence point G lies on the *curve representing relation between armature current and terminal voltage*.

9. Take TU = shunt field current (I_{sh}) ON (scale being different). OU represents the load current corresponding to armature current represented by OT and terminal voltage OE.

10. From point U draw a vertical line intersecting line EG at H. Point H *lies on the external characteristic.*

Similarly other points may be obtained and curve may be drawn, which is the required *external characteristic.*

(B) Taking Armature Reaction into Account :

Here, in addition to considering the voltage drop in armature, voltage drop due to armature reaction is also taken into account.

Let $I_a R_a$ = voltage drop in armature

I_{sh} = increase in shunt field current to counteract the demagnetising effect.

Now if a right-angled triangle say lmn is drawn as that ln = voltage drop in armature, and mn = shunt field current. The triangle lmn is called as the *drop reaction triangle*.

In order to draw external and internal characteristic repeat the process as in A with following modifications in steps 4 and 9 respectively.

4. Take any point L on the O.C.C. and draw line LM parallel to the line lm of triangle lmn and complete the triangle LMN. Now from the points L and M draw vertical lines cutting X-axis at points N' and M'. Now LN' *represents the generated e.m.f.* MM' represents the terminal voltage, LN *represents the voltage drop in armature due to armature resistance,* ON' *is the shunt field current to induce an e.m.f. represented by* LN' and $N'M'$ is the *increase in shunt field current to counteract the demagnetising effect.*

9. Take TU = shunt field current ON' (scale being different). OU represents the load current corresponding to the armature current represented by OT and terminal voltage MM'.

Voltage Control of Shunt Generators

- The terminal voltage of a shunt generator may be kept constant at all loads with the use of adjustable resistance, called a *field rheostat, connected in series with the shunt-field circuit*. By adjusting the resistance of the rheostat to suit the load on the machine, changes in terminal voltage with load may be prevented. When the load changes gradually, hand control of the rheostat may be used, although *automatic control* employing a *voltage regulator* is far more satisfactory.
- The terminal voltage may also be controlled *automatically* by the addition of a series-field winding. This method has the advantage of being *automatic cheap*, and *generally satisfactory.*

5.4. Series Generator

The field winding of a series generator is connected in series with the armature winding as shown in Fig. 39. It consists of a *few series of heavy wire, capable of carrying the output current of the machine without overheating.* The characteristic curves of a D.C. series generator are drawn as given in next page :

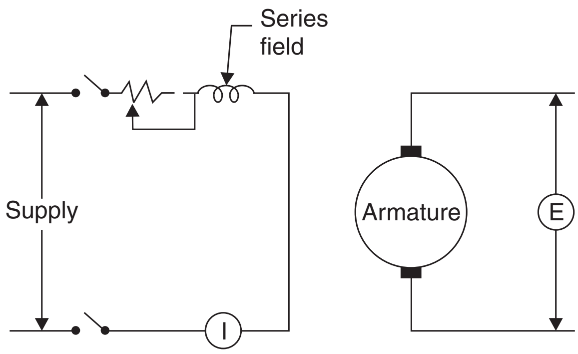
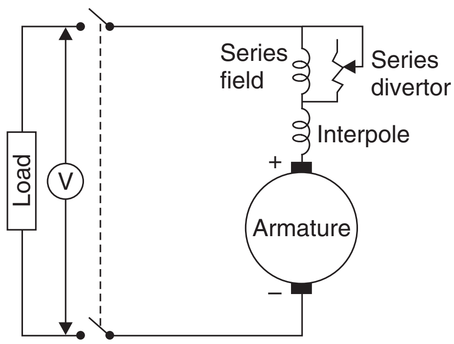


Fig. 39. Connection diagram for obtaining the saturation curve of a series generator.

- The *saturation* curve (1) may be obtained in a manner exactly similar to that already described for the shunt machine. The series-connected generator is illustrated in Fig. 39. The series connected generator illustrated in Fig. 40 must be capable of safely carrying the maximum current to be used, or about 125 per cent of rated load current. A plot of simultaneous readings of generated voltage and field current, taken at a rated speed, yields the magnetisation curve 1 of Fig. 41.
- **External Characteristic (curve 2).** To obtain the data for this curve, the machine is connected to the load as shown in Fig. 40, ammeter and voltmeter being inserted to read the load current I (= I_a) and the terminal voltage V respectively. The machine is run at constant (rated) speed, a series of simultaneous readings of voltage and current is taken while the load is varied from a minimum value to perhaps 125 per cent of rated load. When these readings are plotted, using V as co-ordinate and I_a (= I) as abscissa curve 2 (*see* Fig. 41) is obtained.



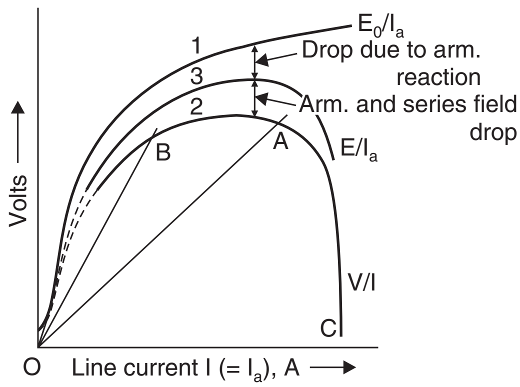


Fig. 40. Circuit for loading a series generator. Fig. 41. Characteristic curves of a D.C. series generator.

It may be noted that the *readings cannot begin at zero load* as with the shunt generator, for if *the resistance of the circuit including armature, series field and load is increased beyond a certain critical value, the generator unbuilds and loses its load entirely.* Thus, if *OA* is the resistance line for the circuit the terminal voltage is the ordinate to the curve at *A*. When the *resistance of the circuit* is gradually *increased*, the *load falls off* along the curve, and *A* approaches *B*. When the resistance line finally becomes tangent to the curve, however, *operation becomes unstable*, and any slight further increase in the resistance causes the machine to unbuild its voltage and lose its load. The resistance that brings about this condition is called the *critical resistance*. Therefore, to *begin with*, *the resistance of the circuit must be reduced below the critical value before the generator delivers any load.*

- **Internal Characteristic (curve 3).** This curve is obtained by adding the resistance drop $I_a (R_a + R_{se})$ to the external characteristic curve ; R_a and R_{se} being armature resistance and series field resistance respectively.

  The difference between curves 1 and 3 is the *reduction in voltage caused by armature reaction.*
- It is worth noting that between A and C a considerable change in resistance brings about only a slight change in load current. Over this range the *voltage decreases rapidly,* owing to increasing armature reaction (particularly when the brushes are shifted forward), while the *current remains nearly constant*. Thus, between A and C the machine may be used to supply power to a *constant current variable-voltage circuit, such as series arc circuit.*
- Owing to initially *rising characteristic,* the series generator is often used as a *voltage booster* to give an increase of voltage practically proportional to the current.
- A series generator also finds application in electric traction where '*dynamic braking*' is employed. The connections of the series traction motors are changed by means of a controller so that they act as generators ; the power absorbed in braking the vehicle being dissipated in resistances, which are also used for starting purposes when the machines are reconnected as motors.

5.5. Compound Wound Generator

In case of a series generator the voltage regulation is very poor but the ability of the series field to produce additional useful magnetisation in response to increased load cannot be denied. This useful characteristic of the series field, combined with the relatively constant voltage characteristic of the shunt generator, led to the compound generator. Fig. 42 shows connection diagrams for a compound generator.

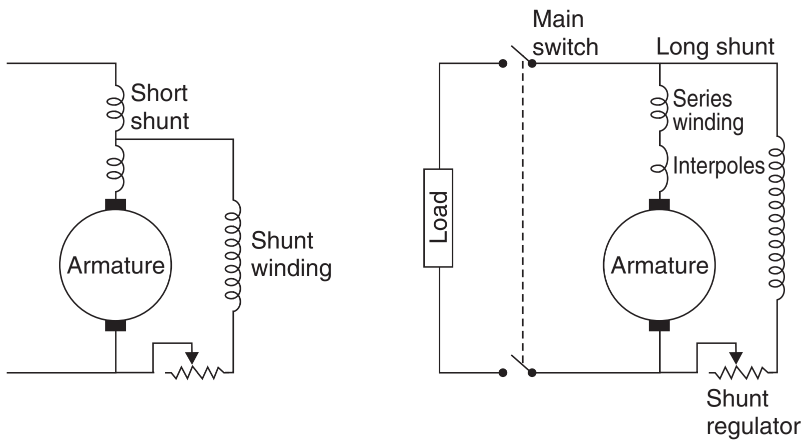


Fig. 42. Connection diagrams for a compound generator.

Regardless of the method of connection, the terminal voltage V of the short shunt or long shunt compound generator in same,

$$V = E_g - (I_{se}.\ R_{se} + I_a R_a).$$

The generated voltage, E_g, of a compound generator is the result of the combination of m.m.f.s produced by the series $(I_{se}T_{se})$ and shunt $(I_{sh}T_{sh})$ ampere-turns due to currents which flow in their field windings. In a compound generator, the *shunt field predominates and is much stronger of the two. When the series field m.m.f. aids the shunt field m.m.f.,* the generator is said to be '*cumulatively* *compounded* ' [*see* Fig. 43 (*a*)]. When the *series field m.m.f. opposes the shunt field m.m.f.,* the generator is said to be '*differentially compounded* ' [*see* Fig. 43 (*b*)].

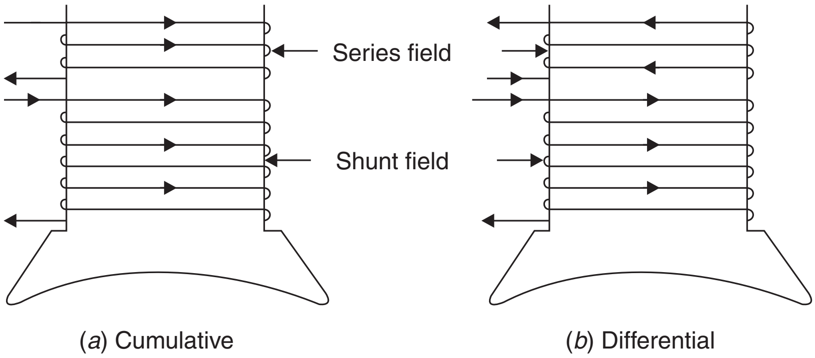


Fig. 43. Current directions in series and shunt-field coils of cumulative and differential-compound generators.

Characteristics of Cumulative Compound Generator

See Fig. 44. Depending on the relative additional aiding m.m.f., produced by the series field there are three types of load characteristics possible for the *cumulative* compound generator.

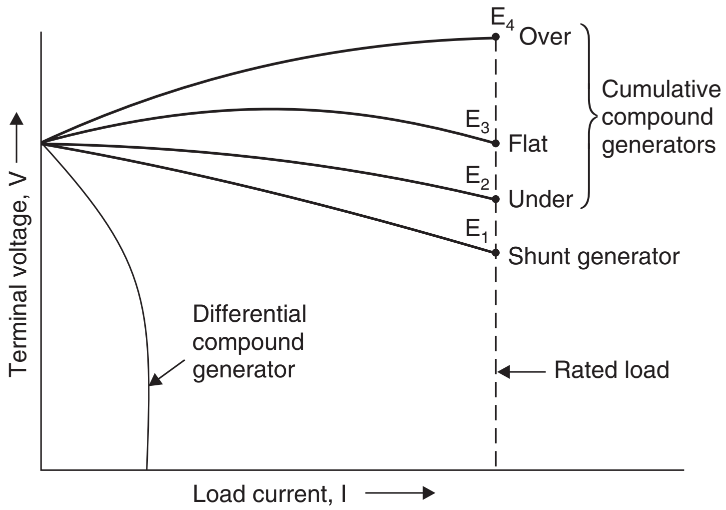


Fig. 44. External load voltage characteristics of cumulative and differential compound generators.

These types are called :

(*i*) Over-compound (*ii*) Flat-compound

(*iii*) Under-compound.

(*i*) **Over compound generator.** An over-compound generator is one whose terminal voltage *rises with the application of load* so that its full-load voltage *exceeds* its no-load voltage (*negative regulation*).

(*ii*) **Flat-compound generator.** A flat compound generator has a load-voltage characteristic in which the no-load and full-load voltages are equal (*zero per cent regulation*).

(*iii*) **Under-compound generator.** An under-compound generator has a load characteristic in which the full load voltage is somewhat *less* than no-load voltage, but whose aiding series-field ampere-turns cause its characteristic to have *better regulation than an equivalent shunt generator*.

- Most commercial compound D.C. dynamos, whether used as generators or motors, are normally supplied by the manufacturer as *over-compound machines.* The *degree of compounding* (over, flat or under) may be adjusted by means of *divertor* which shunts the series field.

Characteristic of Differential Compound Generator

- The *differential compound generator is defined as that compounding produced when the series field m.m.f. opposes the shunt field m.m.f.* The difference in current direction of the two windings is shown in Fig. 43 (*b*), where for the sake of clarity, the series field winding is shown above (rather than directly around) the shunt field winding.
- Fig. 45 shows the load characteristic of differential compound generator.

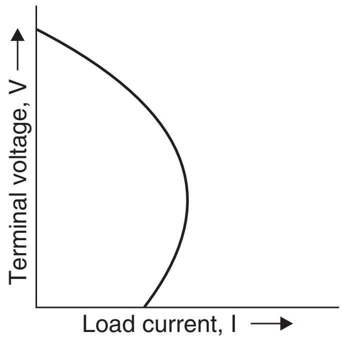


Fig. 45. Differential compound generator—load characteristic.

When the differential compound generator is *without load* it builds up and self-excites its shunt field in much the same manner as the shunt generator. However, *when a load is applied, the generated voltage E_g is now reduced by the* reduction in the main field flux created by the *opposing m.m.f. of the series field.* This reduction in E_g occurs *in addition to the armature and series circuit voltage drop, the armature reaction, and the reduction in field current produced by reduction of the armature voltage.* The result is a sharp drop in the terminal voltage with load as shown in Fig. 45, and the field is below saturation and rapidly unbuilds.

- *The differential compound generator is used as a constant-current generator for the same constant-current applications as the series generator.*

6. APPLICATIONS OF D.C. GENERATORS

6.1. Separately Excited Generators

(*i*) The separately excited generators are usually *more expensive than self-excited generators* as they require *a separate source of supply.* Consequently they are generally used where self-excited are relatively unsatisfactory. These *are used in Ward Leonard systems of speed control, because self-excitation would be unsuitable at lower voltages.*

(*ii*) These generators are also used where quick and requisite response to control is important (since separate excitation gives a quicker and more precise response to the changes in the resistance of the field circuit).

6.2. Shunt Generators

(*i*) These generators are used to advantage, in conjunction with automatic regulators, *as exciters for supplying the current required to excite the fields of A.C. generators.* The regulator controls the voltage of the exciter by cutting in and out some of the resistance of the shunt-field rheostat, thereby holding the voltage at whatever value is demanded by operating conditions. This is one of the most important applications of shunt generators.

(*ii*) Shunt generators are used *to charge batteries.* In this application the voltage should drop off slightly as the load increases, because the voltage of a lead battery is lower when battery is discharged than when battery is charged. When it is discharged, however, the battery can stand a large charging current than when it is charged. *Because of its drooping characteristic the shunt generator is admirably suited to battery charging service,* for, in a general way, the voltage curve of the generator has the same shape as the voltage curve of the battery itself. In both cases, as the load falls the voltage rises.

- Shunt generators can be operated in parallel without difficulty, and the wiring of parallel-operated shunt machines is quite a bit simpler than the corresponding wiring for compound machines. When a slight drop in voltage is not objectionable, as when a motor load is fed directly from the generator terminals, shunt machines may be used to advantage.

6.3. Series Generators

The field of application of series generator is limited. These are used for the following purposes :

(*i*) Series arc lighting.

(*ii*) Series incandescent lighting.

(*iii*) As a series booster for increasing the voltage across the feeder carrying current furnished by some other sources.

(*iv*) Special purposes such as supplying the field current for regenerative braking of D.C. locomotives.

6.4. Compound Generators

The compound generator is used for more than any other type.

(*i*) It may be built and adjusted automatically to supply an approximately *constant voltage at the point of use.,* throughout the entire range of load. This is *very great advantage.* It is possible to provide a constant supply voltage at the end of a long feeder by the simple expedient of *overcompounding* the generator, because the resistance drop in the line is compensated for by the rising characteristic of the generator.

When the point of utilisation is near the generator, a flat-compounded machine may be used.

(*ii*) *Differentially compounded generator* finds an useful application *as an arc welding generator* where the generator is practically short circuited every time the electrode touches the metal plates to be welded.

(*iii*) Compound generators are used to supply power to :

- Railway circuits,
- Motors of electrified steam rail-roads,
- Industrial motors in many fields of industry,
- Incandescent lamps, and
- Elevator motors etc.

Example 15. *Draw armature drop line if armature resistance is 0.2* Ω.

Solution. The *procedure* of drawing an armature drop line is as follows (*see* Fig. 46) :

(*i*) Take any value of armature current and find corresponding voltage in the armature resistance. For example, take armature current of 200 A, voltage drop in the armature for this value of armature current is 200 × 0.2 = 40 V.

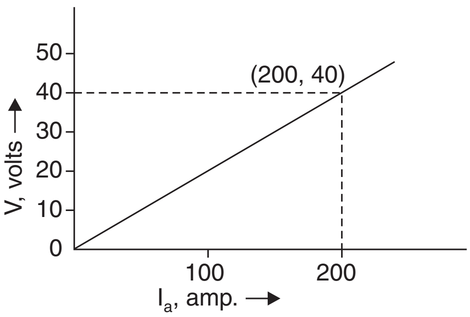


Fig. 46

So, a point (200, 40) lying on this line is obtained.

(*ii*) Join the above point with origin *O*, the armature drop or ohmic drop line is obtained.

Example 16. *The open circuit characteristic (O.C.C.) of a separately excited generator, at 600 r.p.m., is as under* :

Field Current, A	:	*1.6*	*3.2*	*4.8*	*6.4*	*8.0*	*9.6*	*11.2*
E.m.f., V	:	*148*	*285*	*390*	*460*	*520*	*560*	*590*

Find :

(*i*) *The voltage to which the machine will excite as a shunt generator with a field circuit resistance of 60 ohm.*

(*ii*) *The critical resistance, at this speed.*

Solution.

- Plot O.C.C. (E_o/I_f) as shown in Fig. 47.

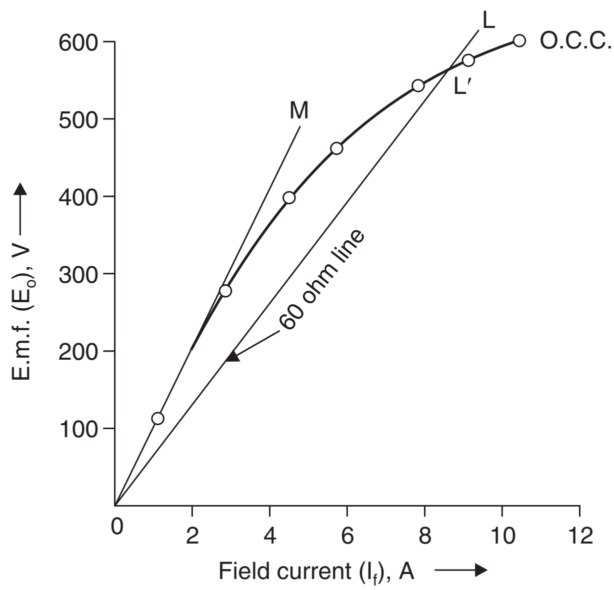


Fig. 47

- Line *OL* represents 60 Ω line.
- The voltage to which the machine will excite a shunt generator is given by point *L′ i.e.,* the intersection of O.C.C. and 60 Ω line. The machine will *excite* at 550 V.
- Draw line *OM* tangent to O.C.C. The *slope of this line represents critical resistance.* This value of critical resistance is 91 Ω.

Example 17. *The open circuit characteristic of a 4-pole, 250 V shunt generator having 610 lap-connected armature conductors running at 750 r.p.m. is as follows* :

Field current, A	:	*0*	*0.5*	*1.0*	*2.0*	*3.0*	*4.0*	*5.0*
E.m.f., V	:	*10*	*50*	*100*	*175*	*220*	*245*	*262*

Calculate :

(*i*) *Field circuit critical resistance.*

(*ii*) *Critical speed for field circuit resistance of 80 Ω.*

(*iii*) *Residual flux per pole.*

Solution. Number of poles of the generator, $p = 4$

Number of parallel paths, $a = p = 4$ [Generator being lap connected]

Number of armature conductors, $Z = 610$

- Plot O.C.C. from the given data as shown in Fig. 48.

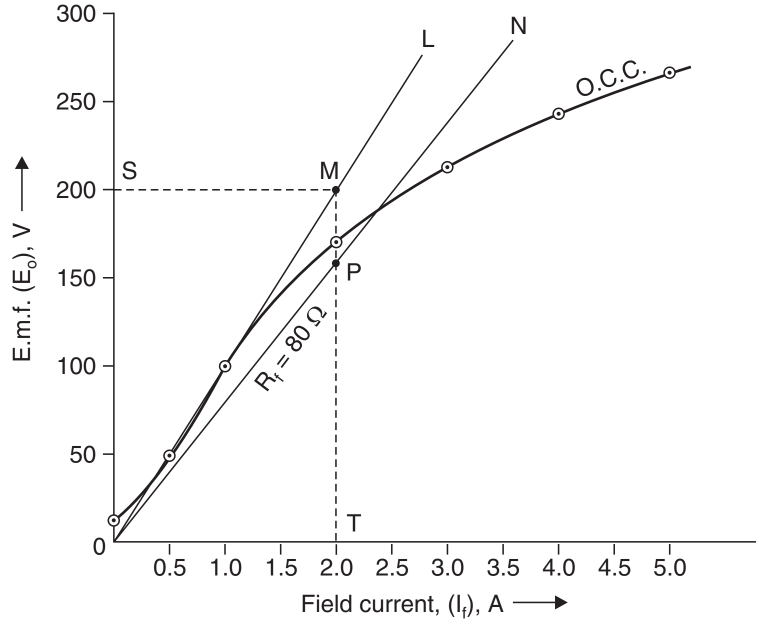


Fig. 48

- Draw line *OL* tangent to O.C.C. at the origin. To determine the slope of the line *OL*, take any point *M* on this line and from *M* draw horizontal and vertical lines cutting O.C. voltage axis and field current axis at points *S* and *T* respectively.

(*i*) Critical resistance of field circuit = slope of line *OL*

$$= \frac{OS \text{ in volts}}{OT \text{ in amperes}} = \frac{200}{2} = \textbf{100 ohms.} \quad \textbf{(Ans.)}$$

(*ii*) Draw line *ON* representing resistance of 80 ohms. Let ordinate drawn from point *M* intersect line *ON* at *P*.

Critical speed, $N_c = N \times \frac{TP}{TM} = 750 \times \frac{160}{200} = \textbf{600 r.p.m.}$ **(Ans.)**

(*iii*) **Residual flux per pole,** ϕ :

With no exciting current, the e.m.f. induced due to residual flux is 10 V at 750 r.p.m. (given).

Using the relation, $E_g = \frac{p\phi ZN}{60a}$

$$10 = \frac{4 \times \phi \times 610 \times 750}{60 \times 4}$$

$$\therefore \quad \phi = \frac{10 \times 60 \times 4}{4 \times 610 \times 750} = 0.00131 \text{ Wb.}$$

Hence, **Residual flux/pole = 0.00131 Wb. (Ans.)**

7. DIRECT CURRENT MOTOR

7.1. General Aspects

The electric motor is a machine which converts electric energy into mechanical energy. It depends for its operation on the force which is known to exist on a conductor carrying current while situated in a magnetic field.

Construction. A D.C. motor is similar in construction to a D.C. generator. As a matter of fact any D.C. generator will run as a motor when its field and armature windings are connected to a source of direct current. The field winding produces the necessary magnetic field. *The flow of current through the armature conductors produces a force which rotates the armature.*

Though the essential construction of D.C. motor is identical to that of a generator, the *external appearance of a motor may be somewhat different from that of a generator.* This is mainly due to the fact that the *frame of a generator may be partially open* because it is located in relatively clean environment and only skilled operators are present in its vicinity. A motor, on the other hand, may be operating in a rather dusty environment and only unskilled operators may be working in its vicinity. Therefore, frames of motors are to a large extent closed.

The body of *D.C. mill motors* is made in two halves bolted together for easy access to the field windings and inter-poles.

Applications. Because of their inherent characteristics D.C. motors find extensive application in :

(*i*) Steel plants (*ii*) Paper mills

(*iii*) Textile mills (*iv*) Printing presses

(*v*) Cranes (*vi*) Winches

(*vii*) Excavators etc., *where precise and accurate speed control over a wide range is required*.

Advantages. The D.C. motors possess the following *advantages* :

(*i*) High starting torque.

(*ii*) Speed control over a wide range, both below and above the normal speed.

(*iii*) Accurate stepless speed control with constant torque.

(*iv*) Quick starting, stopping, reversing and accelerating.

Disadvantages. The *disadvantages* of D.C. motors are :

(*i*) High initial cost.

(*ii*) Increased operating and maintenance costs because of the commutators and brushgear.

7.2. Principle of Operation of D.C. Motor

The principle of motor action can be stated as follows :

"*Whenever a current carrying conductor is placed in a magnetic field, it experiences a force whose direction is given by Fleming's left hand rule*".

- Fig. 49 illustrates this principle.

  Fig. 49 (*a*) shows the field set up by the poles.

  Fig. 49 (*b*) shows the conductor field due to flow of current in the conductor.

Fig. 49 (*c*) shows the resultant field produced when the current carrying conductor wire of Fig. 49 (*b*) is inserted in the air gap of Fig. 49 (*a*) with the *axis of the conductor at right angles to the direction of the flux*.

On the *upper side* of the conductor in Fig. 49 (c) the magnetizing forces of the field and of the current in the conductor are *additive* while on the *lower side* these are *subtractive*. This explains why the resultant field is strengthened above and weakened below the conductor (wire).

The above experiment shows that the wire in Fig. 49 (c) has a force on it which tends to move it downward. Thus, the *force acts in the direction of the weaker field*. When the current in the wire is reversed, the direction of the force is also reversed, as in Fig. 49 (d).

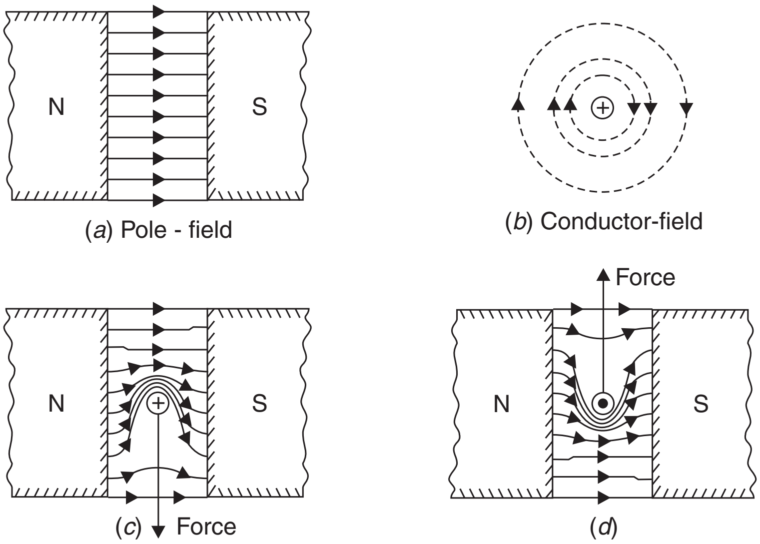


Fig. 49. The principle of motor action.

The force (F) developed in the conductor is given by the relation,

$$F = BIl \text{ newtons}$$

where B = flux density, T (Wb/m^2),

I = current in conductor, A, and

l = exposed length of conductor, m.

Now consider the magnetic field of a D.C. motor in which there is no current in the armature conductors ; the lines of force will be distributed as shown in Fig. 50.

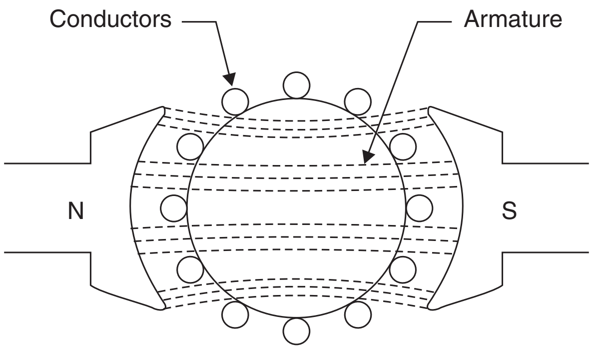


Fig. 50. Distribution of lines of force in a motor due to magnetic field only.

If, now, the armature carries current, each of its conductors will produce a magnetic field which, when super-imposed on the main field, causes a distribution of magnetic lines as shown in Fig. 51.

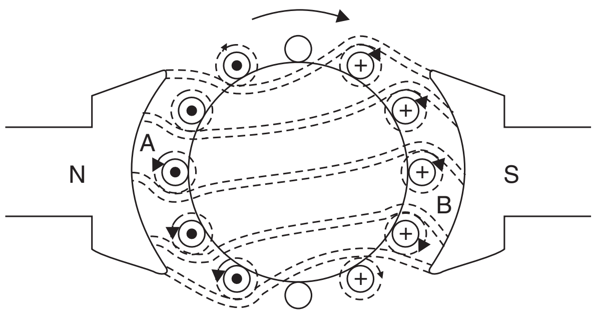


Fig. 51. Distribution of lines of force in a motor, on load, due to the armature and magnetic field.

The magnetic field is said to be distorted, since the lines of force no longer follow approximately straight paths.

These lines of force have the *property of tending to shorten* themselves, so that they may be regarded as *being in tension*. *Each conductor* in Fig. 51 *will experience a force like that exerted on a stone in a catapult*. Since these conductors are embedded in slots in the armature, the latter is caused to rotate in a clockwise direction.

7.3. Back or Counter E.M.F.

Refer to Fig. 52. In a D.C. motor when the armature rotates, the conductors on it cut the lines of force of magnetic field in which they revolve, so that an e.m.f. is induced in the armature as in a generator. The *induced e.m.f. acts in opposition to the current in the machine* and, therefore, *to the applied voltage*, so that *it is customary to refer to this voltage as the 'back e.m.f.'* That this is so can be deduced by Lenz's law, which states that the *direction of an induced e.m.f. is such as to oppose the change causing it, which is, of course, the applied voltage.*

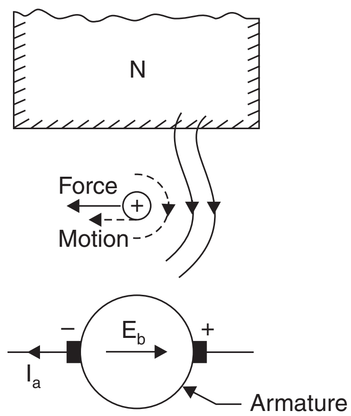


Fig. 52. Motoring operation.

The magnitude of the back or counter e.m.f. can be calculated by using formula for the induced e.m.f. in a generator, and it is important in the case of the motor, to appreciate that this is *proportional to the product of the flux and the speed*. Thus, if E_b denotes the back e.m.f., ϕ the flux and N the speed, we may write,

$$E_b = k\,\phi\,N$$

where k is a number depending on *nature of armature winding.*

The value of back e.m.f. (E_b) is always less than the applied voltage, although difference is small when the machine is running under normal conditions. It is the difference between these two quantities which actually drives current through the resistance of the armature circuit. If this resistance is represented by R_a, the back e.m.f. by E_b and the applied voltage by V, then we have

$$V = E_b + I_aR_a$$

where I_a is the current in the armature circuit.

7.4. Comparison between Motor and Generator Action

— Fig. 53 (*b*) shows a generator action, where a mechanical force moves a conductor in upward direction inducing an e.m.f. in the direction shown. When a current flows as a result of this e.m.f., there is a current-carrying conductor existing in a magnetic field ; hence motor action occurs. Shown as a dotted line in Fig. 53 (*b*), the *force developed as a result of motor action opposes the motion which produced it*.

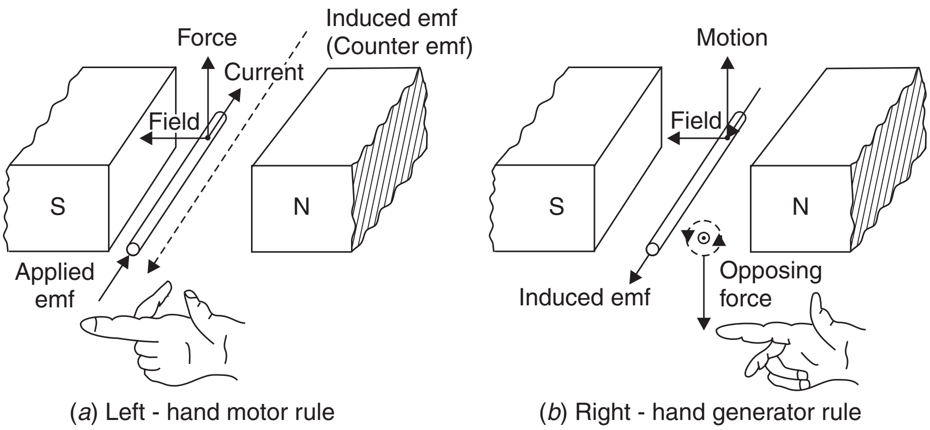


Fig. 53. Comparison of motor and generator action.

Thus, it can be stated categorically that in rotating electric machines *generator action and motor action occur simultaneously*. Hence, the same dynamo may be operated either as a motor or a generator or both (as in dynamo motor or synchronous converter).

- Fig. 54 presents a more graphic representation in terms of rotational elements, which compares the elementary motor and generator for the same direction of rotation and shows the electric circuits of each which is self-explanatory.

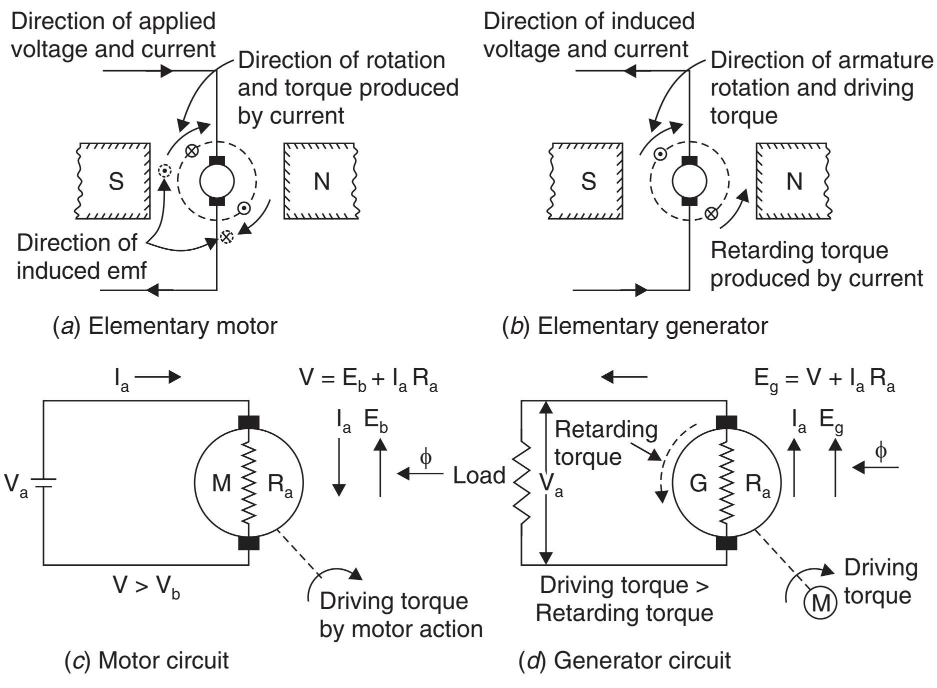


Fig. 54. Elementary motor action *versus* generator action.

It may be noted that when a dynamo is operating as a *motor*, the generated e.m.f., is *always less than the terminal voltage* (that produces motor action) and it *opposes the armature current.* On the other hand when a dynamo is operating as a *generator*, the *armature current is in the same direction as the generated e.m.f.*, and the *generated e.m.f.* E_g *exceeds the terminal voltage V applied across the load*. This distinction between generator and motor, in which the armature-generator voltage aids or opposes the armature current, respectively give rise to the following basic armature circuit equations :

For a motor, $V = E_b + I_aR_a$ ...(3)

For a generator, $E_g = V + I_aR_a$ ...(4)

where V = applied voltage (measurable terminal voltage) across the armature,

E_b = back or counter e.m.f. developed in the armature of the motor,

E_g = generated e.m.f. developed in the generator armature, and

I_aR_a = armature voltage drop due to a flow of armature current through an armature of a given resistance, R_a.

7.5. Torque Developed in a Motor

When the field of a machine (of the type described as generator) is excited and a potential difference is impressed upon the machine terminals, the current in the armature winding reacts with the air-gap flux to produce a turning moment or *torque* which tends to cause the armature to revolve. Fig. 55 illustrates production of torque in a motor.

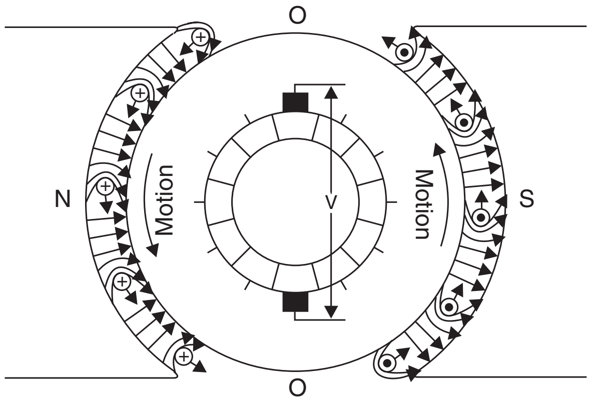


Fig. 55. Production of torque in a D.C. motor.

When the brushes are on the neutral axis, all the armature conductors lying under the north pole carry currents in a *given direction*, while those lying under south pole carry currents in the *reverse direction*. The commutator (just as in a generator) serves to reverse the current in each armature coil at the instant it passes through the neutral axis, so the above relation is always maintained as the armature rotates.

All *conductors under the north pole* carry inward-flowing currents which react with the air gap flux to *produce down-ward acting forces* and a *counter clockwise torque*. Similarly the *conductors under the south pole* carry outward-flowing currents which *produce upward-acting forces*. These *forces also give rise to counter clockwise torques*. If the air-gap flux is assumed to be radially directed at all points, each of the force acts tangentially and produces a turning moment equal to the force multiplied by its lever arm—the radial distance from the centre of the conductor to the centre of the shaft.

Magnitude of torque developed by each conductor

$= BIlr$ Nm

If the motor contains Z conductors, the total torque developed by the armature

$$T_a = BIlrZ \text{ Nm} \quad \text{...(5)}$$

where B = gap density, T (Wb/m^2)

I = armature current in a conductor, A

l = active length of each conductor, m

r = average lever arm of a conductor or the average radius at which conductors are placed, m

Z = total number of armature conductors.

It is more convenient to express T_a in terms of armature current I_a, total flux per pole ϕ and number of poles p.

$$I = \frac{I_a}{a}$$

and

$$B = \frac{\phi}{A}$$

where a = number of parallel paths,

and A = the cross-sectional area of flux path at radius r.

$$A = \frac{2\pi rl}{p}$$

Then

$$T_a = \frac{Z\phi lI_a r}{2\pi rl} \times \frac{p}{a} = \frac{Z\phi I_a p}{2\pi a} \text{ Nm}$$

i.e.,

$$T_a = 0.159 Z\phi p \times \frac{I_a}{a} \quad \text{...(6)}$$

or

$$T_a = k\phi I_a \text{ Nm} \quad \text{...(7)}$$

where $k = \frac{Zp}{2\pi a}$ is a *constant for any machine.*

Alternative proof :

The expression for the torque developed by the motor armature may also be deduced as follows :

Let T_a be the torque developed in Nm by the motor armature running at N r.p.m.

Power developed = work done per second

$$= T_a \times 2\pi N \text{ watts} \quad \text{...}(i)$$

Electrical equivalent of mechanical power developed by the armature also

$$= E_b I_a \text{ watts} \quad \text{...}(ii)$$

Equating (*i*) and (*ii*), we get

$$T_a \times \frac{2\pi N}{60} = E_b I_a$$

or

$$T_a = \frac{E_b I_a}{2\pi \left(\frac{N}{60}\right)}$$

Also since

$$E_b = \frac{p\phi ZN}{60a}$$

$$\therefore \quad T_a \times 2\pi \frac{N}{60} = \frac{p\phi ZN}{60a} \,.\, I_a \text{ or } T_a = \frac{Z\phi p}{2\pi} \,.\, \frac{I_a}{a} \text{ Nm}$$

i.e.,

$$T_a = 0.159\, Z\phi p \,.\, \frac{I_a}{a} \,.$$

Note. From the above equation for torque, we find that

$$T \propto \phi I_a$$

Then

(*i*) In the case of shunt motors, ϕ is practically constant,

hence $$T \propto I_a$$

(*ii*) In the case of series motors, ϕ is proportional to I_a before saturation (because field windings carry full armature current)

$\therefore$ $$T \propto I_a^2 .$$

Shaft torque (T_{sh}). The torque developed by the armature is the *gross torque*. Whole of this torque is not available at the pulley, since certain percentage of torque developed by the armature is lost to overcome the iron and friction losses. *The torque which is available for useful work is known as* **shaft torque** T_{sh}. It is so called because it is *available at the shaft*. The horse power obtained by using shaft torque is called brake horse power (B.H.P.).

$$\text{B.H.P. (metric)} = \frac{T_{sh} \times 2\pi N}{735.5}$$

$\therefore$ $$T_{sh} = \frac{\text{B.H.P. (metric)} \times 735.5}{\frac{2\pi N}{60}} \quad ...(8)$$

where N = speed of armature in r.p.m.

The difference $T_a - T_{sh}$ is known as **lost torque** (*i.e.,* torque lost in iron and friction losses)

$$= 0.159 \times \frac{\text{iron and friction losses}}{\frac{N}{60}} \text{ Nm.}$$

7.6. Mechanical Power Developed by Motor Armature

Refer to Fig. 56. The voltage V applied across the motor armature has to (*i*) overcome back e.m.f. E_b and (*ii*) supply the armature ohmic drop I_aR_a.

$\therefore$ $$V = E_b + I_aR_a$$

This is known as *voltage equation of a motor*.

Multiplying both sides by I_a, we get

$$VI_a = E_bI_a + I_a^2R_a$$

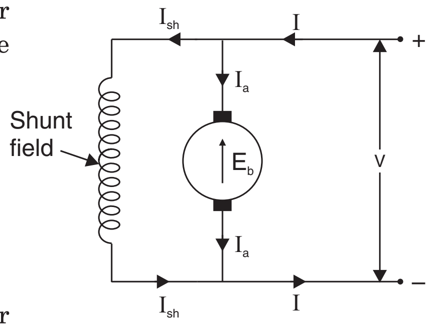


Fig. 56

Here VI_a = electrical input to the armature,

E_bI_a = electrical equivalent of mechanical power P_m developed in the armature, and

$I_a^2R_a$ = copper loss in the armature.

The power available at the pulley for doing useful work is *somewhat less than the mechanical power developed by the armature*.

This is evident, since there are certain *mechanical losses* (such as bearing and windage friction and iron losses) that must be supplied by the driving power of the motor.

Condition for maximum power. We know that, the mechanical power developed by the motor,

$$P_m = VI_a - I_a^2R_a$$

Differentiating both sides with respect to I_a

$$\frac{d(P_m)}{dI_a} = V - 2I_aR_a = 0$$

$\therefore$ $$I_aR_a = \frac{V}{2}$$

As $$V = E_b + I_aR_a$$

$\therefore$ $$V = E_b + \frac{V}{2}$$

i.e., $$E_b = \frac{V}{2} \quad ...(9)$$

Hence, *mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage.* In practice, however, *this condition is not realized* because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses, such as mechanical and magnetic, into consideration, the efficiency of the motor will be below 50 per cent.

7.7. Types of D.C. Motors

There are three main types of motors characterised by the connection of the field winding in relation to the armature. These are :

1. **Shunt wound motor** or the shunt motor, in which the *field winding is connected in parallel with the armature.*
2. **Series motor,** in which the *armature and field windings are connected in series.*
3. **Compound motor,** which has two field windings, *one of which is connected in parallel with the armature and the other in series with it.*

Shunt wound motor. Fig. 57 shows the connections of a shunt motor. From these connections it may be observed at once that the *field current is constant*, since it is connected directly to the supply which is assumed to be at constant voltage. Hence, the flux is approximately constant and, since also the *back e.m.f. is almost constant under normal conditions the speed is approximately constant*. This is not strictly true, but nevertheless, *it is usual for all practical purposes to regard the shunt motor as a constant speed machine*. It is, therefore, employed in practice for drives, the speeds of which are required to be *independent of the loads*. The speed can, of course, be varied when necessary and this is done by the inclusion of a variable resistor in series with the field winding, as shown in Fig. 57.

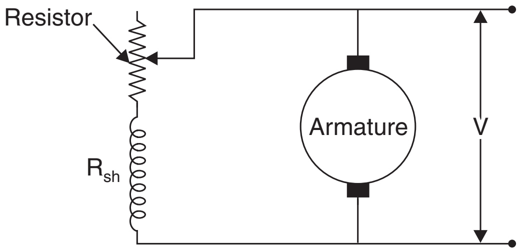


Fig. 57. Connections of a shunt motor.

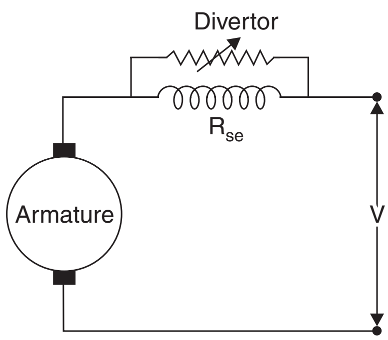


Fig. 58. Connections of a series motor.

Series motor. Fig. 58 shows the connections for the series motor. The current passing through the field winding is the same as that in the armature, since the armature current increases as the mechanical load on the shaft increases, so also does the field current. The resultant increase in magnetic flux causes a reduction in the speed, as can be observed from a consideration of the formula :

$$E_b = \frac{p\phi ZN}{60a} \text{ or } E_b = k\phi N$$

where $k = \dfrac{pZ}{60a}$ being constant

or $$N = \frac{E_b}{k\phi}.$$

This is a useful property for many drives in which it is desirable that a heavy increase in the load should automatically bring about a compensating reduction in speed. As with the shunt motor, the speed may also be varied independently of the load by the inclusion of a variable resistor in the field circuit. In this case, however, it is connected in parallel with the series winding as shown in Fig. 58, and is called a *divertor compound motor*.

Refer to Fig. 59. The compound motor has a shunt field winding in addition to the series winding so that the number of magnetic lines of force produced by each of its poles is the resultant of the flux produced by the shunt coil and that due to the series coil. The *flux so produced depends not only on the current and number of turns of each coil, but also on the winding direction of the shunt coil in relation to that of the series coil*. When the two fluxes *assist each other* the machine is a **cumulative compound motor**, while if they *oppose each other*, it is said to be a **differential compound motor.**

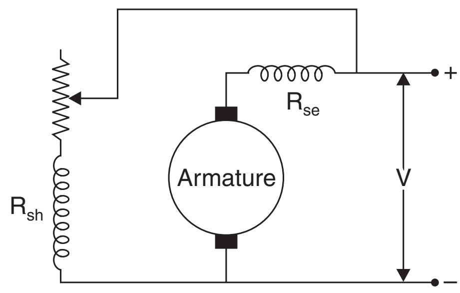


Fig. 59. Connections of a compound motor.

Fig. 60 shows the field windings and interpole connections of a *differential compound wound motor*. The shunt coil is made up of *many turns of fine wire,* whilst the series coil comprises relatively *few turns of thickwire*.

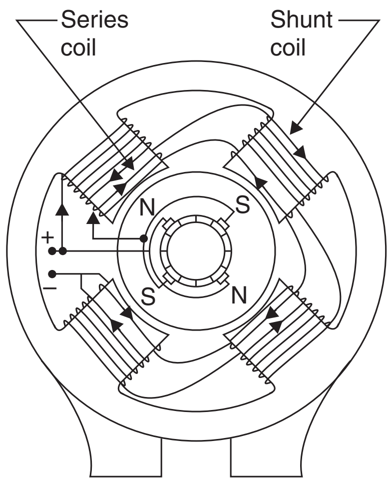


Fig. 60. Field windings of a differential compound motor.

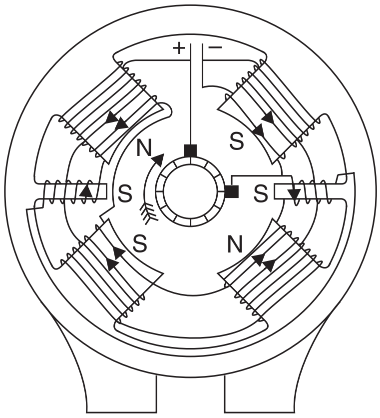


Fig. 61. Field windings and interpole connections of a cumulative compound wound motor.

Fig. 61 shows the field windings of a *cumulative compound motor*.

The flow of currents in the shunt and series coils is worth noting in Figs. 60 and 61.

7.8. Speed of a D.C. Motor

We know that the voltage equation of a motor is given by

$$V = E_b + I_a R_a$$

or $$E_b = V - I_a R_a \quad \text{or} \quad \frac{p\phi ZN}{60a} = V - I_a R_a$$

$\therefore$ $$N = \frac{(V - I_a R_a)}{\phi} \times \frac{60a}{Zp}$$

But $$V - I_a R_a = E_b$$

$\therefore$ $$N = \frac{E_b}{\phi} \times \frac{60a}{Zp}$$

or $$N = k' \frac{E_b}{\phi} \quad \text{...(10)}$$

where $k' = \dfrac{60a}{Zp}$ = constant.

The equation (10) shows that speed is *directly proportional to back e.m.f.* and *inversely to the flux*

or $$N \propto \frac{E_b}{\phi}.$$

For series motor :

Let N_1 = speed in the first case,

I_{a1} = armature current in the first case, and

ϕ_1 = flux in the first case.

N_2, I_{a2} and ϕ_2 = corresponding quantities in the second case.

Using the above relation $\left(i.e.\, N \propto \dfrac{E_b}{\phi}\right)$, we get

$$N_1 \propto \frac{E_{b1}}{\phi_1} \quad \text{and} \quad N_2 \propto \frac{E_{b2}}{\phi_2}$$

where $E_{b1} = V - I_{a1} R_a$

and $E_{b2} = V - I_{a2} R_a$

$\therefore$ $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \quad \text{...(11)}$$

Prior to saturation of poles :

$$\phi \propto I_a$$

$\therefore$ $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad \text{...[11(a)]}$$

For shunt motor :

Applying the same equation in this case also, we get

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

If $\phi_1 = \phi_2$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{...(12)}$$

7.9. Speed Regulation

The speed regulation of a D.C. motor is defined as follows :

"The change in speed when the load on the motor is reduced from rated value to zero, expressed as per cent of the rated load speed."

$\therefore$ Per cent speed regulation $= \dfrac{\text{no load speed} - \text{full load speed}}{\text{full load speed}}$.

SHUNT MOTORS

Example 18. *Determine the torque developed when a current of a 30 A passes through the armature of a motor with the following particulars : lap winding, 310 conductors, 4-pole, pole-shoes 16.2 cm long subtending an angle of 60° at the centre, bore radius 16.2 cm, flux density in air gap 0.7 tesla.*

Solution. Number of poles, $p = 4$

Number of parallel paths, $a = p = 4$ [Motor being lap-wound]

Number of armature conductors, $Z = 310$

Length of pole shoe = 16.2 cm

Bore radius = 16.2 cm

Flux density in air gap, $B = 0.7$ tesla

Armature current, $I_a = 30$ A

Torque developed, T :

We know that,

Pole-shoe arc $= \pi D \times \dfrac{60}{360} = \pi(2 \times 16.2) \times \dfrac{60}{360} = 16.967$ cm

$\therefore$ Pole area = pole-shoe length × pole-shoe arc

$= (16.2 \times 10^{-2}) \times (16.967 \times 10^{-2})$

$= 16.2 \times 16.967 \times 10^{-4}\ \text{m}^2 = 0.02748\ \text{m}^2$

Also flux, $\phi = B \times$ pole area

$= 0.7 \times 0.02748 = 0.01924$ Wb

Using the relation, $T = 0.159 Z\phi p \left(\dfrac{I_a}{a}\right) = 0.159 \times 310 \times 0.01924 \times 4 \times \dfrac{30}{4}$

$= 28.45$ Nm.

Hence, **torque developed = 28.45 Nm. (Ans.)**

Example 19. *A 230 V D.C. shunt motor takes 32 A at full load. Find the back e.m.f. on full load if the resistances of motor armature and shunt field windings are 0.2 ohm and 115 ohms respectively.*

Solution. Supply voltage, $V = 230$ Volts

Full load current, $I = 32$ A

Armature resistance, $R_a = 0.2$ ohm

Shunt field windings' resistance, $R_{sh} = 115$ ohms

Back e.m.f., E_b :

Shunt field current, $I_{sh} = \dfrac{230}{115} = 2$ A

∴ Armature current, $I_a = I - I_{sh} = 32 - 2 = 30$ A

Back e.m.f. on full load, $E_b = V - I_aR_a$

$= 230 - 30 \times 0.2 = 224$ V

Hence, **back e.m.f. on full load = 224 V. (Ans.)**

Example 20. *The power input to a 230 volts D.C. shunt motor is 8.477 kW. The field resistance is 230 Ω and armature resistance is 0.28 Ω. Find the input current, armature current and back e.m.f.*

Solution. *Given* : P_{in} = 8.477 kW ; R_{sh} = 230 Ω ; R_a = 0.28 Ω

$I : I_a : E_b$:

Input current, $I = \frac{P_{in}}{V} = \frac{8.477 \times 1000}{230}$

$=$ **36.86 A. (Ans.)**

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1$ A

∴ *Armature current*, $I_a = I - I_{sh}$

$= 36.86 - 1$

$=$ **35.86 A. (Ans.)**

Back e.m.f. $E_b = V - I_aR_a$

$= 230 - 35.86 \times 0.28 =$ **219.96 V. (Ans.)**

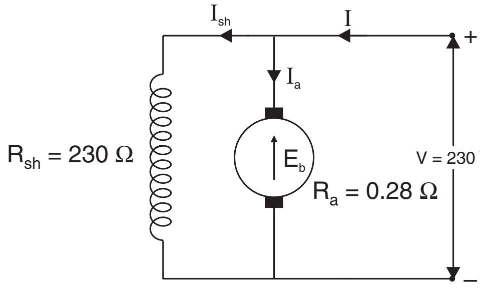


Fig. 62

Example 21. *A six-pole lap-connected 230 V shunt motor has 410 armature conductors. It takes 41 A on full load. The flux per pole is 0.05 weber. The armature and field resistances are 0.1 ohm and 230 ohms respectively. Contact drop per brush = 1 V.*

Determine the speed of motor at full load.

Solution. Number of poles, $p = 6$

Number of parallel paths, $a = 6$ [Motor being lap-connected]

Number of armature conductors, $Z = 410$

Full load current, $I = 41$ A

Flux per pole, $\phi = 0.05$ weber

Armature resistance, $R_a = 0.1$ ohm

Shunt field resistance, $R_{sh} = 230$ ohms

Contact drop/brush = 1 V

Speed of motor on full load, N :

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1$ A

Armature current, $I_a = I - I_{sh} = 41 - 1 = 40$ A

Back e.m.f. on full load, $E_b = V - I_aR_a$ – drop at brushes

$= 230 - 40 \times 0.1 - 2 \times 1 = 224$ V

We know that, $E_b = \frac{p\phi ZN}{60a}$

$$224 = \frac{6 \times 0.05 \times 410 \times N}{60 \times 6}$$

∴ $$N = \frac{224 \times 60 \times 6}{6 \times 0.05 \times 410} = 655.6 \text{ r.p.m.}$$

Hence, **speed of motor on full load = 655.6 r.p.m. (Ans.)**

Example 22. *A 250 volt d.c. shunt motor, on no load, runs at 1000 rpm and takes 5 A. The field and armature resistances are 250 ohms and 0.25 ohm respectively. Calculate the speed when the motor is loaded such that it takes 41 A if the armature reaction weakens the field by 3%.*

Solution. *Given* : $V = 250$ V ; $N_0 = 1000$ r.p.m. ; $I_0 = 5$ A ; $R_{sh} = 250\ \Omega$;

$$R_a = 0.25\ \Omega,\ I = 41\text{ A},\ \phi = \left(1 - \frac{3}{100}\right)\phi_0 = 0.97\ \phi_0$$

Speed of the motor, N :

Shunt current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{250} = 1$ A

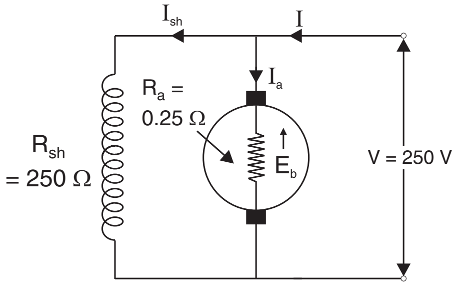


Fig. 63

At no-load :

$$I_0 = 5\text{ A} ;\ I_{as} = I_0 - I_{sh} = 5 - 1 = 4\text{ A}$$
$$E_{b0} = V - I_{a0}R_a = 250 - 4 \times 0.25 = 249\text{ V}$$

At full-load :

$$I = 41\text{ A} ;\ I_a = I - I_{sh} = 41 - 1 = 40\text{ A}$$
$$E_b = V - I_aR_a = 250 - 40 \times 0.25 = 240\text{ V}$$

Using the relation, $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}} \times \dfrac{\phi_0}{\phi}$

or $\dfrac{N}{1000} = \dfrac{240}{249} \times \dfrac{\phi_0}{0.97\phi_0}$

∴ $\mathbf{N} = 1000 \times \dfrac{240}{249} \times \dfrac{1}{0.97} =$ **993.67 r.p.m. (Ans.)**

Example 23. *A 120 volt d.c. shunt motor has an armature resistance of 0.2 ohms and a field resistance of 60 ohms. The full-load line current is 60 A and full-load speed is 1800 r.p.m. If the brush contact drop is 3 V, find the speed of the motor at half-load.*

Solution. *Given* : $V = 120$ V ; $R_a = 0.2\ \Omega$; $R_{sh} = 60\ \Omega$

$I_1 = 60$ A, $N_1 = 1800$ r.p.m. ;

Brush contact drop = 3 V

Speed of the motor at half-load, N_2 :

$$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2\text{ A}$$

Full load armature current,

$$I_{a1} = I_1 - I_{sh} = 60 - 2 = 58\text{ A}$$

Back e.m.f. $E_{b1} = V - I_{a1}R_a$ – brush contact drop
$= 120 - 58 \times 0.2 - 3 = 105.4$ V

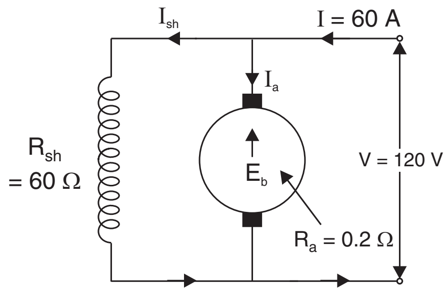


Fig. 64

Armature current at half-load, $I_{a2} = \dfrac{I_{a1}}{2} = \dfrac{58}{2} = 29$ A

Back e.m.f. at half-load, $E_{b2} = V - I_{a2}R_a$ – brush contact drop

$$= 120 - 29 \times 0.2 - 3 = 111.2\text{ V}$$

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2} = \dfrac{E_{b2}}{E_{b1}}$ [∵ $\phi_1 = \phi_2$ ...being shunt motor]

∴ $\dfrac{N_2}{1800} = \dfrac{111.2}{105.4}$

or $\mathbf{N_2} = \dfrac{1800 \times 111.2}{105.4} =$ **1899 r.p.m. (Ans.)**

☞ **Example 24.** *A 4-pole 500 V shunt motor takes 7 A on no-load, the no-load speed being 750 r.p.m. It has a shunt field current of 2 A. Calculate the full-load speed of the motor if it takes 122 A at full-load. Armature resistance = 0.2 ohm. Contact drop/brush = 1 V. Armature reaction weakens the field by 4% on full-load.*

Solution. Supply voltage, $V = 500$ Volts

No-load current, $I_0 = 7$ A

No-load speed, $N_0 = 750$ r.p.m.

Shunt field current, $I_{sh} = 2$ A

Full-load current, $I = 122$ A

Armature resistance, $R_a = 0.2$ ohm

Contact drop/ brush $= 1$ V

Flux at full-load, $\phi = \left(\frac{100 - 4}{100}\right)\phi_0 = 0.96\phi_0$

Full-load speed, N :

At no-load :

$$I_{a0} = I_0 - I_{sh} = 7 - 2 = 5 \text{ A}$$

$$E_{b0} = V - I_{a0}R_a - \text{contact drop at brushes}$$

$$= 500 - 5 \times 0.2 - 2 \times 1 = 497 \text{ V.}$$

At full-load :

$$I_a = I - I_{sh} = 122 - 2 = 120 \text{ A}$$

$$E_b = V - I_aR_a - \text{contact drop at brushes}$$

$$= 500 - 120 \times 0.2 - 2 \times 1 = 500 - 24 - 2 = 474 \text{ V}$$

We know that, $E_{b0} \propto \phi_0 N_0$

and $E_b \propto \phi N$

$$\therefore \quad \frac{E_b}{E_{b0}} = \frac{\phi}{\phi_0} \times \frac{N}{N_0}$$

$$\frac{474}{497} = \frac{0.96\phi_0}{\phi_0} \times \frac{N}{750}$$

$$\therefore \quad N = \frac{474 \times 750}{497 \times 0.96} = 745 \text{ r.p.m. (app.)}$$

Hence, **full-load speed of the motor = 745 r.p.m. (app.) (Ans.)**

☞ **Example 25.** *A 440 V, 4-pole, lap connected shunt motor has a no-load input current of 15 A and a shunt field current of 10 A. At full-load it takes a current of 150 A. If armature resistance = 0.1 ohm, flux per pole on no-load = 0.05 weber, number of armature conductors = 750 and contact drop per brush = 1 V, calculate :*

(*i*) *No-load speed* (*ii*) *Full-load speed*

(*iii*) *Speed regulation.*

Armature reaction weakens the field by 1.5% on full-load.

Solution. Supply voltage, $V = 440$ Volts

Number of poles, $p = 4$

Number of parallel paths, $a = p = 4$ [Motor being lap-connected]

Armature resistance, $R_a = 0.1$ ohm

Flux per pole on no-load, $\phi_0 = 0.05$ weber

Number of armature conductors, $Z = 750$

Contact drop per brush, $= 1$ V

No-load input current, $I_0 = 15$ A
Shunt field current, $I_{sh} = 10$ A
Full-load current, $I = 150$ A

No-load speed, N_0 :
Full-load speed, N :
Speed regulation :

(*i*) **No-load speed :**

Armature current, $I_{a0} = I_0 - I_{sh} = 15 - 10 = 5$ A

$$E_{b0} = V - I_{a0}R_a - \text{brush contact drop}$$
$$= 440 - 5 \times 0.1 - 2 \times 1 = 437.5 \text{ V}$$

Using the relation : $E_{b0} = \dfrac{p\phi_0 Z N_0}{60a}$

or $$437.5 = \frac{4 \times 0.05 \times 750 \times N_0}{60 \times 4}$$

∴ $$N_0 = \frac{437.5 \times 60 \times 4}{4 \times 0.05 \times 750} = 700 \text{ r.p.m.}$$

Hence, **no-load speed = 700 r.p.m. (Ans.)**

(*ii*) **Full-load speed :**

Armature current, $I_a = I - I_{sh} = 150 - 10 = 140$ A

$$E_b = V - I_aR_a - \text{brush contact drop}$$
$$= 440 - 140 \times 0.1 - 2 \times 1 = 424 \text{ V}$$
$$\phi = (1 - 0.015)\ \phi_0$$ [Since armature reaction weakens the field by 1.5 per cent]
$$= 0.985\ \phi_0$$

Using the relation : $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}} \times \dfrac{\phi_0}{\phi}$

or $$\frac{N}{700} = \frac{424}{437.5} \times \frac{\phi_0}{0.985\phi_0}$$

∴ $$N = 700 \times \frac{424}{437.5} \times \frac{\phi_0}{0.985\phi_0} = 688.73 \text{ r.p.m.}$$

Hence, **full-load speed = 688.73 r.p.m. (Ans.)**

(*iii*) **Percentage speed regulation**

$$= \frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}} \times 100$$
$$= \frac{700 - 688.73}{688.73} \times 100 = 1.637$$

Hence, **percentage speed regulation = 1.637%. (Ans.)**

Example 26. *A 250 V shunt motor takes a line current of 60 A and runs at 800 r.p.m. Its armature and field resistances are 0.2 ohm and 125 ohms respectively. Contact drop/brush = 1 V. Calculate* :

(*i*) *No-load speed if the no-load current is 6 A.*

(*ii*) *The percentage reduction in the flux per pole in order that the speed may be 1000 r.p.m. when the armature current is 40 A.*

Neglect the effects of armature reaction.

Solution. Supply voltage, $V = 250$ volts

Load current, $I_1 = 60$ A

Load speed, $N_1 = 800$ r.p.m.

Armature resistance, $R_a = 0.2$ ohm

Shunt field resistance, $R_{sh} = 125$ ohms

Contact drop $= 2$ V

Armature current, $I_{a2} = 40$ A

Load speed, $N_2 = 1000$ r.p.m.

No-load current, $I_0 = 6$ A

No-load speed, N_0 :

Percentage reduction in flux/pole :

Shunt field current, $I_{sh} = \frac{250}{125} = 2$ A

At no-load :

$$I_{a0} = I_0 - I_{sh} = 6 - 2 = 4 \text{ A}$$

$$E_{b0} = V - I_{a0}R_a - \text{brush drop}$$

$$= 250 - 4 \times 0.2 - 2 \times 1 = 247.2 \text{ V}$$

At load (I_1 = 60 A) :

$$I_{a1} = I_1 - I_{sh} = 60 - 2 = 58 \text{ A}$$

$$E_{b1} = V - I_{a1}R_a - \text{brush drop}$$

$$= 250 - 58 \times 0.2 - 2 \times 1 = 236.4 \text{ V}$$

Using the relation, $\frac{N_1}{N_0} = \frac{E_{b1}}{E_{b0}} \times \frac{\phi_0}{\phi_1}$

Since the motor is shunt wound and armature reaction effect is negligible so *flux may be assumed to be constant* and therefore speed is directly proportional to the back e.m.f. developed.

$$\therefore \quad \frac{N_1}{N_0} = \frac{E_{b1}}{E_{b0}}$$

$$\frac{800}{N_0} = \frac{236.4}{247.2}$$

$$N_0 = \frac{800 \times 247.2}{236.4} = 836.5 \text{ r.p.m.}$$

Hence, **no-load speed = 836.5 r.p.m. (Ans.)**

At armature current, I_{a2} = 40 A

Let the flux be reduced to ϕ_2

$$E_{b2} = V - I_{a2}R_a - \text{brush drop}$$

$$= 250 - 40 \times 0.2 - 2 \times 1 = 240 \text{ V}$$

Using the relation, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$

or $$\frac{1000}{800} = \frac{240}{236.4} \times \frac{\phi_1}{\phi_2}$$

$$\therefore \quad \frac{\phi_2}{\phi_1} = \frac{240 \times 800}{1000 \times 236.4} = 0.812$$

∴ Percentage reduction in flux

$$= \frac{\phi_1 - \phi_2}{\phi_1} \times 100 = \left(1 - \frac{\phi_2}{\phi_1}\right) \times 100 = (1 - 0.812) \times 100 = 18.8\%$$

Hence, **%age reduction in flux = 18.8%. (Ans.)**

Example 27. *A 220 V shunt motor takes 60 A when running at 800 r.p.m. It has an armature resistance of 0.1 ohm. Determine the speed and armature current if the magnetic flux is weakened by 20%. Contact drop per brush = 1 V.*

Total torque developed remains constant.

Solution. Supply voltage = 220 V

Load current, $= I_1 = I_{a1} = 60$ A [Neglecting shunt field current]

Speed at 60 *A*, $N_1 = 800$ r.p.m.

Armature resistance, $R_a = 0.1$ ohm

Contact drop/brush = 1 V

$\mathbf{N_2} = ?$, $\mathbf{I_{a2}} = ?$

$$\phi_2 = \left(1 - \frac{20}{100}\right) \phi_1 \quad \text{[Since magnetic flux is weakened by 20\%]}$$

$$= (1 - 0.20)\phi_1 = 0.8\phi_1$$

Since $T \propto \phi I_a$

and torque remains constant [Given]

$$\therefore \quad \phi_1 I_{a1} = \phi_2 I_{a2}$$

$$\phi_1 \times 60 = 0.8\phi_1 I_{a2}$$

$$\therefore \quad I_{a2} = \mathbf{75\ A.}$$

Back e.m.f., $E_{b1} = V - I_{a1}R_a$ – brush drop

$= 200 - 60 \times 0.1 - 2 \times 1 = 212$ V

Back e.m.f., $E_{b2} = V - I_{a2}R_a$ – brush drop

$= 220 - 75 \times 0.1 - 2 \times 1 = 210.5$ V

Using the relation,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{800} = \frac{210.5}{212} \times \frac{\phi_1}{0.8\phi_1}$$

$$\therefore \quad N_2 = \frac{800 \times 210.5}{212 \times 0.8} = \mathbf{992.9\ r.p.m.}$$

Hence, **Speed = 992.9 r.p.m. (Ans.)**

Armature current = 75 A. (Ans.)

☞ **Example 28.** *A 440 V shunt motor takes 105 A (armature current) from the supply and runs at 1000 r.p.m. Its armature resistance is 0.15 ohm. If total torque developed is unchanged, calculate the speed and armature current if the magnetic field is reduced to 70% of the initial value.*

Solution. Supply voltage, $V = 440$ Volts

Armature current, $I_{a1} = 105$ A

Speed, $N_1 = 1000$ r.p.m.

Armature resistance, $R_a = 0.15$ ohm

$\phi_2 = 0.7\ \phi_1$.

(*i*) **Armature current, I_{a2} :**

As the torque developed is same

$$\therefore \quad \phi_1 I_{a1} = \phi_2 I_{a2}$$

$$I_{a2} = \frac{\phi_1}{\phi_2} \times I_{a1} = \frac{\phi_1}{0.7\phi_1} \times 105 = 150 \text{ A}$$

Hence, **armature current = 150 A. (Ans.)**

Now, $E_{b1} = V - I_{a1}R_a = 440 - 105 \times 0.15 = 424.25$

and $E_{b2} = V - I_{a2}R_a = 440 - 150 \times 0.15 = 417.5$ V.

(*ii*) **Speed, N_2 :**

Using the relation, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{1000} = \frac{417.5}{424.25} \times \frac{\phi_1}{0.7\phi_1}$$

$$\therefore \quad N_2 = 1000 \times \frac{417.5}{424.25} \times \frac{1}{0.7} = 1405 \text{ r.p.m.}$$

Hence, **speed of the motor when magnetic field is reduced to 70% of initial value = 1405 r.p.m. (Ans.)**

SERIES MOTORS

Example 29. *A 230 V series motor develops torque of 310 Nm at 800 r.p.m. The torque lost due to iron and friction loss is 10 N-m. If the efficiency is 85% determine the current taken by the motor at 800 r.p.m.*

Solution. Supply voltage = 230 V

Torque developed, T_a = 310 N-m

Torque lost due to iron and friction, T_{lost} = 10 N-m

Efficiency = 85%

Speed of the motor, N = 800 r.p.m.

Current taken by motor at 800 r.p.m. :

$$T_{useful} = T_a - T_{lost} = 310 - 10 = 300 \text{ N-m}$$

Watts corresponding to useful torque

$$= \frac{T_{useful} \times 2\pi N}{60} = \frac{300 \times 2\pi \times 800}{60} = 25136 \text{ W}$$

Power in watts taken from mains $= \dfrac{25136}{0.85} = 29571.7$ W

Current taken by motor $= \dfrac{29571.7}{230} =$ **128.57 A. (Ans.)**

Example 30. *A 8-pole 240 V lap-wound, series motor has armature and series field resistances of 0.2 ohm and 0.02 ohm respectively. There are 660 armature conductors. If the flux/pole is 0.3 Wb and the total torque developed in the armature is 320 Nm, find the current taken by the motor and its speed.*

Solution. Number of poles, $p = 4$

Number of parallel paths, $a = p = 4$ [Motor being lap-wound.]

Armature resistance, R_a = 0.2 ohm

Series field resistance, R_{se} = 0.02 ohm

Number of armature conductors, $Z = 660$
Flux/pole, $\phi = 0.03$ Wb
Total torque developed, $I_a = 320$ Nm

(*i*) **Current taken by motor, I_a :**

$$E_b = \frac{p\phi ZN}{60a}$$

$$\therefore \quad \frac{E_b}{N} = \frac{p\phi Z}{60a} = \frac{4 \times 0.03 \times 660}{60 \times 4} = 0.33$$

Also, $$T_a \times \left(\frac{2\pi N}{60}\right) = E_b I_a$$

or $$T_a = \frac{E_b}{N} \cdot \frac{60 I_a}{2\pi} = 0.33 \times \frac{60 \times I_a}{2\pi}$$

$$\therefore \quad 320 = \frac{0.33 \times 60 \times I_a}{2\pi}$$

or $$I_a = \frac{320 \times 2\pi}{0.33 \times 60} = 101.56 \text{ A}$$

Hence, current taken by motor = **101.56 A. (Ans.)**

(*ii*) **Speed of motor, N :**

$$E_b = V - I_a(R_a + R_{se})$$
$$= 240 - 101.56(0.2 + 0.02) = 217.66 \text{ V}$$

But $$\frac{E_b}{N} = 0.33$$ [Already calculated above]

$$\therefore \quad N = \frac{E_b}{0.33} = \frac{217.66}{0.33} = \mathbf{659.6 \text{ r.p.m.}} \quad \textbf{(Ans.)}$$

☞ **Example 31.** *A six-pole, lap-wound 400 V series motor has the following data : Number of armature conductors = 920, flux/pole = 0.045 Wb, total motor resistance = 0.6 ohm, iron and friction losses = 2 kW. If the current taken by the motor is 90 A, find :*

(*i*) *Total torque ;* (*ii*) *Useful torque at the shaft ;*
(*iii*) *Power output ;*
(*iv*) *Pull at the rim of a pulley of 40 cm diameter connected to the shaft.*

Solution. Number of poles, $p = 6$
Supply voltage, $V = 400$ Volts
Number of parallel paths, $a = p = 6$ [Motor being lap-wound]
Number of armature conductors, $Z = 920$
Flux/pole, $\phi = 0.045$ Wb
Motor resistance, $R_m = 0.6$ ohm
Iron and friction losses = 2 kW or 2000 W
Current taken by the motor, $I_a = 90$ A
Radius of the pulley, = 40/2 = 20 cm or 0.2 m
Using the relation, $E_b = V - I_a R_m = 400 - 90 \times 0.6 = 346$ V

Also, $$E_b = \frac{p\phi ZN}{60a}$$

$$346 = \frac{6 \times 0.045 \times 920 \times N}{60 \times 6}$$

$\therefore \quad N = \dfrac{346 \times 60 \times 6}{6 \times 0.045 \times 920} = 501$ r.p.m.

(*i*) **Total torque, T_a :**

We know that, $\quad T_a = 0.159 \times Z\phi p \times \left(\dfrac{I_a}{a}\right)$

$= 0.159 \times 920 \times 0.045 \times 6 \times \dfrac{90}{6} = 592.4$ Nm

Hence, **Total torque = 592.04 Nm. (Ans.)**

(*ii*) **Useful torque, T_{useful} :**

$$T_{lost} \times \left(\frac{2\pi N}{60}\right) = \text{Iron and friction loss} = 2000 \text{ W}$$

$\therefore \quad T_{lost} = \dfrac{2000 \times 60}{2\pi N} = \dfrac{2000 \times 60}{2\pi \times 501} = 38.11$ Nm

$\therefore \quad T_{useful} = T_a - T_{lost} = 592.4 - 38.11$

$= 554.29$ Nm

Hence, **useful torque = 554.29 Nm. (Ans.)**

(*iii*) **Power output :**

Power output $= T_{useful}\left(\dfrac{2\pi N}{60}\right) = \dfrac{554.29 \times 2\pi \times 501}{60}$

$= 29084.4$ or 29.08 kW

Hence, **power output = 29.08 kW. (Ans.)**

(*iv*) If F is the pull at the rim of the pulley and r is the radius,

Torque at the shaft, $\quad T_{useful} = F \times r$

i.e., $\quad F \times 0.2 = 554.29$

$\therefore \quad F = 2771.45$ N

Hence, **pull at the rim of the pulley = 2771.45 N. (Ans.)**

Example 32. *A 240 V series motor takes 40 A when giving its rated output at 1500 r.p.m. Its resistance is 0.3 Ω. Find what resistance must be added to obtain rated torque* (*i*) *at starting and* (*ii*) *at 1000 r.p.m.*

Solution. *Given* : $V = 240$ volts ; $I (= I_a) = 40$ A, $N = 1500$ r.p.m., $R = 0.3\ \Omega$.

(*i*) **Resistance to be added to obtain rated torque at starting, R_{add} :**

Since the torque remains the same in both the cases, it is obvious that the current drawn by the motor remains constant at 40 A.

$\therefore \quad 40 = \dfrac{240}{R_{add} + 0.3}$

or $\quad R_{add} = \dfrac{240}{40} - 0.3 =$ **5.7 Ω. (Ans.)**

(*ii*) **Resistance to be added to obtain rated torque at 1000 r.p.m., R_{add} :**

We know that, $\quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$

$E_{b1} = 240 - 40 \times 0.3 = 228$ V

$\therefore \quad \dfrac{1000}{1500} = \dfrac{E_{b2}}{228}$

or $$E_{b2} = \frac{1000 \times 228}{1500} = 152 \text{ V}$$

Now $E_{b2} = V - 40(R_{add} + 0.3)$ or $152 = 240 - 40(R_{add} + 0.3)$

$\therefore$ $$\mathbf{R_{add}} = \frac{240 - 152}{40} - 0.3 = \mathbf{1.9}\ \Omega. \quad \textbf{(Ans.)}$$

Example 33. *A 200 V D.C. series motor runs at 700 r.p.m. when operating at its full-load current of 20 A. The motor resistance is 0.5 Ω and the magnetic circuit can be assumed unsaturated. What will be the speed if (i) the load torque is increased by 44% (ii) the motor current is 10 A.*

Solution. *Given* : $V = 200$ volts ; $N_1 = 700$ r.p.m., $I_1 = 20$ A, $R_m = (R_a + R_{sh}) = 0.5\ \Omega$

Speeds, (N_2, N_3) :

(*i*) **When the load torque is increased by 44% :**

$$T_2 = 1.44T_1$$

$$\phi_2 I_2 = 1.44\ \phi_1 I_1 \qquad (\because\ T \propto \phi I_a)$$

$$I_2^2 = 1.44\ I_1^2 \qquad (\because\ \phi \propto I)$$

or $$I_2 = I_1\sqrt{1.44} = 20 \times 1.2 = 24 \text{ A.}$$

Back e.m.f. $E_{b1} = V - I_1(R_a + R_{se}) = 200 - 20 \times 0.5 = 190$ V

Back e.m.f. $E_{b2} = V - I_2(R_a + R_{se}) = 200 - 24 \times 0.5 = 188$ V

Also $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

or $$\frac{N_2}{700} = \frac{188}{190} \times \frac{20}{24}$$

or $$\mathbf{N_2 = 577.2\ r.p.m.} \quad \textbf{(Ans.)}$$

(*ii*) **When the motor current is 10 A :**

Back e.m.f., $$E_{b3} = V - I_3(R_a + R_{se})$$
$$= 200 - 10 \times 0.5 = 195 \text{ V}$$

Also, $$\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b1}} \times \frac{\phi_1}{\phi_3}$$

$$\frac{N_3}{700} = \frac{195}{190} \times \frac{20}{10}$$

or $$\mathbf{N_3 \simeq 1437\ r.p.m.} \quad \textbf{(Ans.)}$$

Example 34. *A 220 V D.C. series motor draws full-load line current of 38 A at the rated speed of 600 r.p.m. The motor has armature resistance of 0.4 Ω and the series field resistance is 0.2 Ω. The brush voltage drop irrespective of load is 3.0 volts, find :*

(*i*) *The speed of the motor when the load current drops to 19 A.*

(*ii*) *The speed on removal of load when the motor takes only 1 A from supply.*

(*iii*) *The internal horse power developed in each of the above cases. Neglect the effect of armature reaction and saturation.*

Solution. *Given* : $V = 220$ volts ; $I_1 = 38$ A, $N_1 = 600$ r.p.m., $R_a = 0.4\ \Omega$, $R_{se} = 0.2\ \Omega$, Brush drop = 3 V ; $I_2 = 19$ A, $I_0 = 1$ A.

(*i*) **Speed of the motor N_2 (at 19 A)**

On full-load :

Back e.m.f., $$E_{b1} = V - I_1(R_a + R_{se}) - \text{brush voltage drop}$$
$$= 220 - 38(0.4 + 0.2) - 3 = 194.2 \text{ V.}$$

On-load current of 19 A

Back e.m.f., $E_{b2} = 220 - 19(0.4 + 0.2) - 3 = 205.6$ V

No-load, back e.m.f., $E_{b0} = 220 - 1(0.4 + 0.2) - 3 = 216.4$ V

We know that, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{600} = \frac{205.6}{194.2} \times \frac{I_1}{I_2} \qquad \left[\text{Assuming unsaturated field } i.e., \frac{\phi_1}{\phi_2} = \frac{I_1}{I_2}\right]$$

∴ $$\mathbf{N_2} = 600 \times \frac{205.6}{194.2} \times \frac{38}{19} = \mathbf{1270\ r.p.m.} \quad \textbf{(Ans.)}$$

(*ii*) **N_0 (at 1 A) :**

Now, $$\frac{N_0}{N_1} = \frac{E_{b0}}{E_{b1}} \times \frac{\phi_1}{\phi_0}$$

$$= \frac{E_{b0}}{E_{b1}} \times \frac{I_1}{I_0}$$

∴ $$N_0 = N_1 \times \frac{E_{b0}}{E_{b1}} \times \frac{I_1}{I_0} = 600 \times \frac{216.4}{194.2} \times \frac{38}{1} = \mathbf{25406\ r.p.m.} \quad \textbf{(Ans.)}$$

(*iii*) **Internal H.P. developed (in the above cases) :**

Internal H.P. developed at a load current 38 A

$$= \frac{194.2 \times 38}{735.5} = \mathbf{10.} \quad \textbf{(Ans.)}$$

Internal H.P. developed at a load current of 19 A

$$= \frac{205.6 \times 19}{735.5} = \mathbf{5.31.} \quad \textbf{(Ans.)}$$

Internal H.P. developed at no load current of 1 A

$$= \frac{216.4 \times 1}{735.5} = \mathbf{0.294.} \quad \textbf{(Ans.)}$$

Example 35. *A D.C. series motor draws a line current of 100 A from the mains while running at 1000 r.p.m. Its armature resistance is 0.15 Ω and the field resistance is 0.1 Ω. Assuming that the flux corresponding to a current of 25 A is 40% of that corresponding to 100 A, find the speed of the motor when it is drawing 25 A from 230 V supply.*

Solution. *Given* : $V = 230$ volts ; $I_1 = 100$ A ; $N_1 = 1000$ r.p.m. ;
$R_a = 0.15\ \Omega$; $R_{se} = 0.1\ \Omega$; $I_2 = 25$ A ; $\phi_2 = 0.4\phi_1$

Speed, N_2 :

While drawing line current of 100 A,

$$E_{b1} = V - I_1(R_a + R_{se})$$
$$= 230 - 100(0.15 + 0.1) = 205 \text{ V}$$

While drawing line current of 25 A,

$$E_{b2} = V - I_2(R_a + R_{se})$$
$$= 230 - 25(0.15 + 0.1) = 223.75 \text{ V}$$

We know that, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \text{ or } \frac{N_2}{1000} = \frac{223.75}{205} \times \frac{\phi_1}{0.4\phi_1}$$

∴ $$\mathbf{N_2} = 1000 \times \frac{223.75}{205} \times \frac{1}{0.4} = \mathbf{2729\ r.p.m.} \quad \textbf{(Ans.)}$$

7.10. Motor Characteristics

The properties of all motors and, in particular, D.C. motors are defined as a totality of the following characteristics :

(*i*) *Starting* ; (*ii*) *Operating and mechanical* ;
(*iii*) *Braking ; and* (*iv*) *Regulation*.

Starting characteristics. The starting characteristics *determine the operation of starting from the moment the motor begins running to the moment when steady-state operation is established* and *include* :

- The starting current I_{start} generally determined by the ratio $\dfrac{I_{\text{start}}}{I_{\text{run}}}$;
- The starting torque T_{start}, determined by the ratio $\dfrac{T_{\text{start}}}{T_{\text{run}}}$;
- The duration of starting t_{start} ;
- The economy of operation determined by the amount of energy consumed in starting ; and
- The cost and reliability of the starting equipment.

Operating characteristics. The operating characteristics are those that give the relation between *speed, torque* and *efficiency* as *functions* of the *useful power* or the *armature* current for V = constant and constant resistances in the armature and field circuit.

Mechanical characteristics. Of major importance for industrial drive mechanisms are the mechanical characteristics, which are the relation $N = f(T)$ (where N and T stand for speed and torque respectively) for conditions of constant voltage and resistances in the armature and field circuits. These also include the *braking characteristics*.

Regulation characteristics. These characteristics determine the *properties of motors when their speed is controlled.* These include :

- The regulation range determined by the ratio $\dfrac{N_{\max}}{N_{\min}}$;
- The efficiency of regulation from the point of view of the initial cost of the equipment and maintenance ;
- The nature of regulation—continuous or stepped ; and
- The simplicity of the control apparatus and methods.

The *D.C. motors possess versatile* and diverse regulation characteristics, and for this reason are indispensable in installations where wide-range control of speed is necessary.

The characteristic curves of a motor are those curves which show relation between the following quantities :

1. *Torque and armature current i.e.,* T_a/I_a characteristic. This is also known as **electrical characteristic.**

2. *Speed* and *armature current i.e.,* N/I_a characteristic.

3. *Speed* and *torque i.e.,* N/T_a characteristic. This is also known as **mechanical characteristic.**

This can be obtained from (1) and (2) above.

Following relations are worth *keeping in mind* while discussing motor characteristics :

$$N \propto \frac{E_b}{\phi} \text{ and } T_a \propto \phi I_a.$$

7.10.1. Torque-current characteristics

Shunt motor :

- When running on *no-load,* a small armature current flows to supply the field and to *drive the machine against the friction and other losses in it.*
- As the load is applied to the motor, and is increased, the torque rises *almost proportionally to the increase in current.* This is not quite true, because the flux has been assumed to be constant, whereas it decreases slightly owing to armature reaction. The effect of this is to cause the top of the curve connecting torque and line current to *bend over* as shown in Fig. 65.
- The starting torque of a motor is determined by the starting resistance, which in turn, governs the initial current through the machine when the main switch is closed. At this moment the speed is zero, so that the back e.m.f. is zero and the starting current is given by $I = V/R$, where V is the supply voltage and R is the total resistance, which includes the armature and starting resistance.

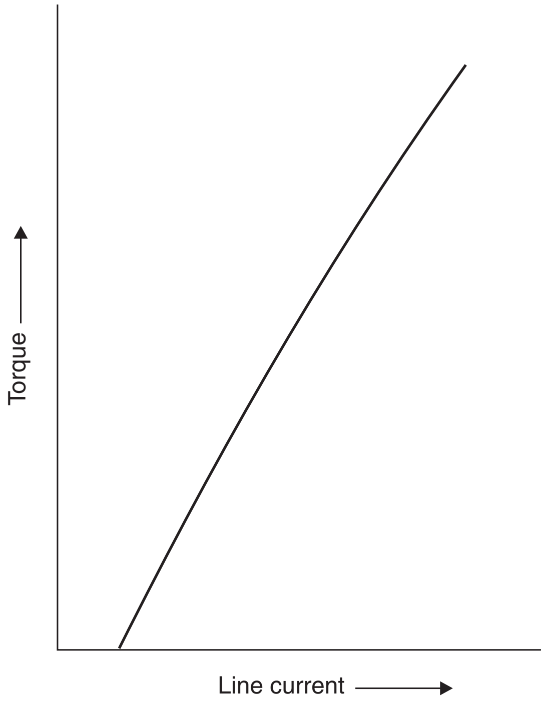


Fig. 65. Torque-current characteristic of a shunt motor.

If the starting current is *limited* by *heating considerations* to twice the full load current, then with normal supply voltage the starting torque of a shunt motor is twice the full load torque. If, however, the supply voltage is below normal, the flux is also less than twice full load torque. The importance of this will be appreciated when the starting torque of a series motor is compared with that of the shunt motor.

Series motor :

- In a series motor the torque ($T_a \propto \phi I_a$) increases much more than does the armature current. This is because the flux itself increases with the armature current, though, owing to the magnetic saturation, the two are not strictly proportional. Nevertheless, for all *but the heavy loads which tend to produce saturation of the field system,* it may be said that the *torque is approximately proportional to the square of the load current.*

Fig. 66 shows the relationship between torque and current. Here the current commences at the no-load value, rises *parabolically at first,* but *increases more slowly* as the *effects of armature reaction and magnetic saturation becomes appreciable.*

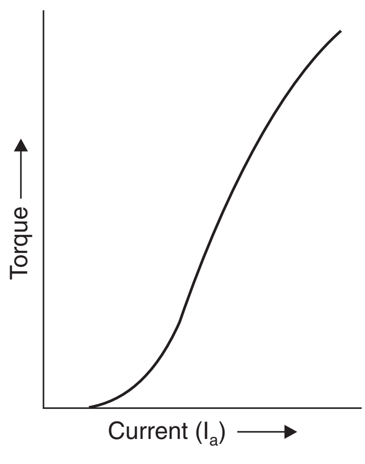


Fig. 66. Torque-current characteristic of a series motor.

- This property of a series motor, by virtue of which a *heavy current gives rise to a very high torque,* also influences its starting characteristics. In a case of a shunt motor, it has already been seen, that the current at the moment of starting may be as high as twice the full-load value ; if we allow for the armature of the magnetisation characteristic and for weakening effect of armature reaction and assume that the flux is increased to 1.5 times its full-load value, then it is obvious that the *starting torque of a series motor is three times the full-load torque.*

- Further more, if the supply voltage falls, the starting current may still be maintained at twice full-load value by cutting out some of the starting resistance, so that the high value of starting torque may still be maintained.

This type of motor (series motor) is *superior to shunt motor for drives in which machines have to be started and accelerated from rest when fully loaded,* as *is the case with traction equipment.*

Compound motors :

Differential compound motor. Refer Fig. 60. In this type of motor the two field windings (shunt and series) *oppose* each other. On *light loads,* such a machine *runs as a shunt motor*, since the series field winding, carrying only a small current, has relatively little effect.

On *heavy loads,* the series coils strengthen and since they are in opposition to the shunt winding, cause a *reduction in the flux* and a consequence *decrease in torque*.

On *heavy overloads* or *when starting upon load, the series winding may become as strong as shunt,* or *it may even predominate, in which the torque will be reduced to zero* or *may even be reversed.* In the latter case, the motor would tend to start-up in the *wrong direction*.

It is obvious that such characteristics may cause *dangerous results,* so that differential compound motors have only *very limited applications in practice*.

Fig. 67 shows the torque-current characteristic of a differential compound motor.

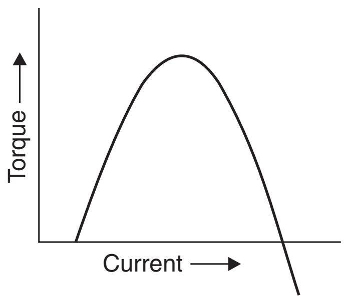


Fig. 67. Torque-current characteristic of a differential compound motor.

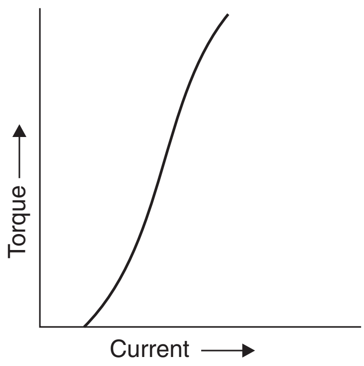


Fig. 68. Torque-current characteristic of a cumulative compound motor.

Cumulative compound motor. In this type of motor the two field windings *assist* each other as shown in Fig. 61. The *flux on no-load is that due to the shunt-winding, while on load the flux and torque rise with the load current.* The torque, therefore, increase more rapidly than in the case of the shunt machine, and on heavy loads it resembles the characteristic of the series motor.

Fig. 68 shows the torque-current characteristic of a cumulative compound motor.

The torque-current characteristics of shunt, series and compound (differential and cumulative) motors are showing in Fig. 69 from which their properties, may be compared, as far as torque is concerned.

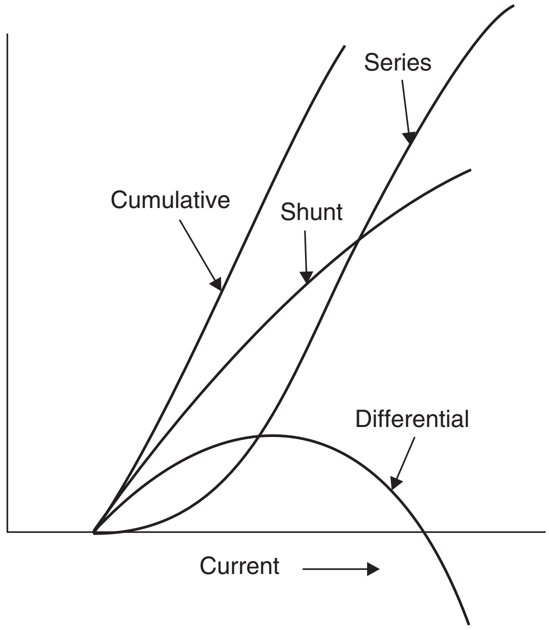


Fig. 69. Torque-current characteristics of shunt, series and compound motors.

7.10.2. Speed-current characteristics

The speed-current characteristics of various motors can be deduced from the following *motor equation,*

$$N \propto \frac{E_b}{\phi} \quad \text{or} \quad N \propto \frac{V - IR}{\phi} .$$

Shunt motor :

- In the shunt motor, the field circuit is connected to the supply terminals so that the exciting current remains constant as long the temperature of the machine is constant, and field regulator is not adjusted. Actually as the machine warms up, the field resistance increases and the exciting current decreases by about 4% for every 10°C rise in temperature. *Neglecting* this effect and also due to armature reaction, it is seen that the speed of a shunt machine falls slightly as the load increases. The *fall in speed is proportional to the volt drop IR in the armature circuit.* If, however, we *consider the effect of armature reaction, an increase of load causes a slight decrease in flux, unless the machine is fitted with compensating windings.* This weakening of the field *tends to raise the speed,* so that the actual fall in speed is less than that calculated by a consid-eration of the volt drop in the armature.

On the whole, the shunt motor may be regarded as one in which the *speed is approximately constant,* falling slightly as the load increases (*see* Fig. 70).

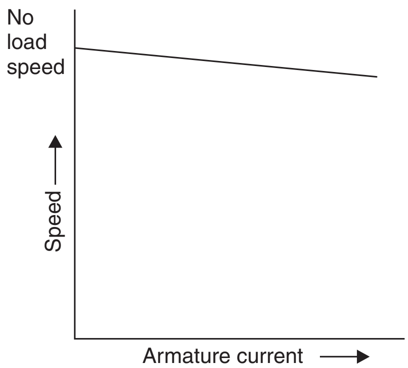


Fig. 70. Speed-current characteristic of a shunt motor.

- The *speed* of a shunt machine can be *increased by inserting resistance* in the field by means of a field regulator. This weakens the field and causes the motor to run faster in order to generate the necessary back e.m.f. of course, it is *impossible to reduce the speed by this method below that at which it runs with no field resistance in the circuit.*

Series motor :

- In case of a series motor the flux does not remain constant, or even approximately constant, because the field winding is in series with the load, so that as the load increases so also does the strength of the magnetic field. At first the flux increases approximately in proportion to the load, but as the field approaches saturation, owing to the heavier loads, the increase is not so rapid. The effects of temperature changes and of armature reaction may be neglected (in comparison with the above mentioned effect).
- It will be appreciated, while considering motor equation, that the back e.m.f. decreases as the armature current increases, as in shunt motor ; in the latter, however, the decrease is due to the volt drop in the armature, while in the series machine the loss in volts occurs in the field as well as in the armature, since they are in series. The back e.m.f. in a series motor, therefore, decreases more rapidly than in a corresponding shunt machine. *The speed,* however, is *proportional to the back e.m.f. divided by the flux ; the former decreases, while the latter increases with increasing load so that the speed decreases rapidly as the armature current increases.* This *property is a valuable feature in a drive of which the speed is required automatically to adjust to compensate for changes in load.*

The speed-current characteristic of a series motor are shown in Fig. 71 (*a*).

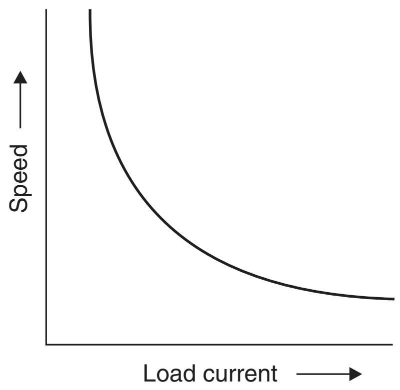


Fig. 71. (*a*) Speed-current characteristic of a series motor.

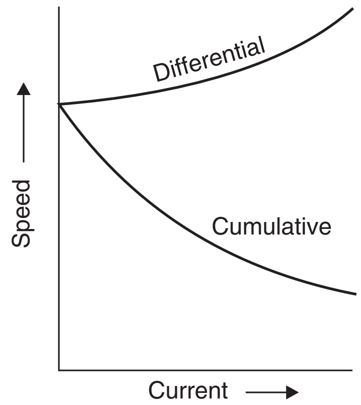


Fig. 71. (*b*) Speed-current characteristic of compound motors.

- On very *low current*, a *series motor runs at very high speeds,* or *tends to race*, as it is termed. This is *dangerous,* since the machine may be destroyed by the centrifugal forces set up in the rotating parts. *For this reason, when installing a series motor it must be positively connected to its load by gearing or by direct connection and never by belting.* Moreover, the minimum load should be great enough to keep the speed within safe limits, as is the case, for example, with *railway motors, hoists* and *rolling mills.*

Compound motor :

- A compound motor runs on '*no-load*' at a speed determined by its shunt winding since the series field contributes little to the total flux in this condition.
- In a *cumulative compound motor at* '*no-load*', the series field strengthens the shunt winding so that the speed falls as in a series machine. Since the flux at any load is equal to the shunt and series fluxes, the speed is less than it would be if running on either field alone.

The speed-current characteristic of such a machine is shown in Fig. 71 (*b*). Such a characteristic has two important **advantages.** These are :

(*i*) *The machine has the compensating action of reducing its speed on heavy loads, as is the case with series machine.*

(*ii*) The maximum speed on no-load is limited by the shunt winding, since this produces a maximum flux even on no-load.

This type of motor, therefore, is suitable for driving machines which operate *on a cycle consisting of a power or working stroke followed by a return or idle stroke.* The series winding produces a fall in speed on the working stroke, while the shunt winding permits the return stroke to be completed at a high, but safe, speed. *A fly-wheel is also provided to act as a load equilizer in such a drive.*

In case of a *differential compound motor,* since the series winding opposes the shunt, the resultant flux decreases as the load increases ; thus the machine runs at a higher speed than it would do as a shunt motor. If the series windings were relatively weak, this reduction in flux might be just sufficient for the fall in speed, brought about by the volt drop in the machine. *Such a motor would have a useful application in driving loads at a constant speed.* If the series field were strong, however, an increase in load would result in a decrease in the magnetic flux and a rise in speed would take place as shown in Fig. 49, the heavier the load, the faster would the motor tend to run. This is the property which may have *dangerous consequences,* since a heavy overload would result in such a high speed that the motor would destroy itself.

7.10.3. Speed-Torque (or Mechanical) characteristics

The speed-torque characteristics of the four types, *i.e.,* shunt, series, cumulative and differential of motors drawn on the same diagram are shown in Fig. 72 for the purpose of comparison.

The main properties of individual motors, from this diagram, may be summarised as under :

1. **Shunt motor.** As the *load torque increases* the *speed falls somewhat,* but the machine *may be regarded as an approximately constant speed motor*.

The shunt motor is *used* :

- When the speed is required to remain approximately constant from no-load to full-load.
- When the load has to be driven at a number of speeds, any one of which is required to remain approximately constant.

2. **Series motor.** As the *load torque increases the speed falls rapidly.* At low torque the speed becomes very high and machine tends to race.

The series motors are used :

- When *large starting torque is required* (as in *traction motors*).
- When the load is subject to heavy fluctuations, and a reduced speed is desired to compensate for the high torque, provided that there is no possibility of the machine '*losing*' its load.

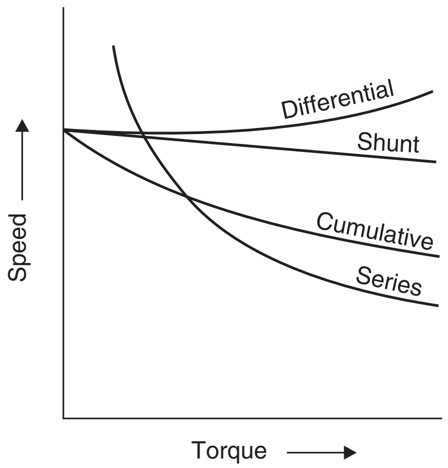


Fig. 72. Speed-torque characteristics of D.C. motors.

3. **Cumulative compound motor.** In this type of motor the *speed falls appreciably as the torque increases*, but on low torques the maximum speed is limited to a safe value. These motors are used :

- *When a large starting torque is required but when the load may fall so low that a series motor would race.*

- *When the load is of a fluctuating nature and a reduced speed is desirable on the heavy loads.*

In such a case a flywheel is usually fitted so that when speed is so reduced the kinetic energy stored in the flywheel at high speeds is given up to assist the motor in driving the heavy load.

- *When the supply voltage is subject to fluctuations* (as in traction systems).

4. **Differential compound motor.** The speed at low torque is limited by the shunt winding, as in the cumulative compound machine. At *high torques*, the speed may be arranged to remain constant or, with a stronger series field, the speed may rise with increasing load.

On *very heavy loads the machine may tend to race*.

Its use is usually restricted to applications *which require a constant speed*.

Industrial applications of D.C. Motors :

1. Shunt motors :

(*i*) Drills and milling machines
(*ii*) Line-shaft drives
(*iii*) Boring mills
(*iv*) Grinders and shapers
(*v*) Spinning and weaving machines
(*vi*) Wood working machines
(*vii*) Small printing presses
(*viii*) Light machine tools generally.

2. Series motors :

(*i*) Traction drives generally
(*ii*) Tram cars and railway cars
(*iii*) Cranes, derricks, hoists, elevators and winches
(*iv*) Fans and air compressors
(*v*) Vacuum cleaners, hair driers, sewing machines
(*vi*) Universal machines generally.

3. Cumulative compound motors :

(*i*) Punching, shearing and planing machines
(*ii*) Lifts, haulage gears and mine hoists
(*iii*) Pumps and power fans
(*iv*) Rolling mills, stamping presses and large printing presses
(*v*) Trolley buses.

4. Differential compound motors :

(*i*) Battery boosters
(*ii*) Experimental and research work.

7.11. Comparison of D.C. Motor Characteristics

Fig. 73 shows the comparison among shunt, series and compound operation for the *same motor* with its armature being acted upon by either or both fields. It is worth noting here that the outputs vary at the same armature current.

- Fig. 74 compares shunt, series and compound performance when the three are rated at the horse power (or kilowatts) and rpm (or radians/sec). In this type of plot all the characteristics cross at the design load point.
- Figs. 73 and 74 are drawn with the motors operating on their full rated voltage. *Different voltages will move the curves, but the relative shapes will remain similar.*

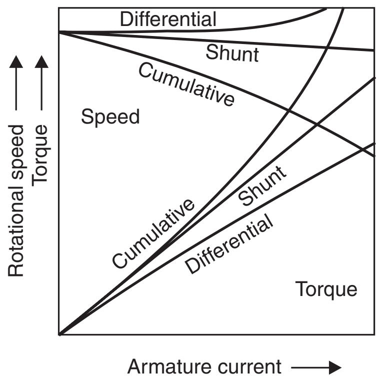


Fig. 73. Speed, torque relations for shunt and compound fields with the same armature current.

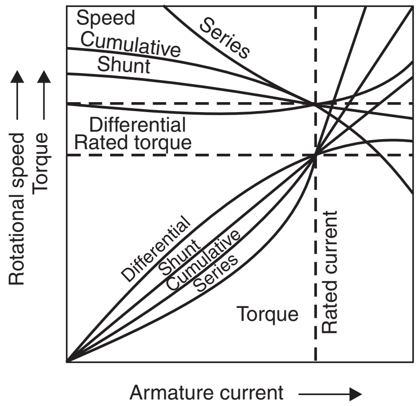


Fig. 74. Speed and torque relations for shunt, series and compound fields. Motors of same rated speed, torque and current.

The following may be deduced from Fig. 74.

(*i*) The curves diverge between no-load and full load. If the load torque were around mid-load, the shunt motor would draw the least current and the series the most.

(*ii*) At overloads the opposite is true. The series motor can develop an overload torque of around double the rated torque without drawing excessive current.

Fig. 73 shows that the speed characteristics for the same armature current are very much different. Therefore, the ability to take increasing torque loads that is the special contribution of a series field is accompanied by a much greater speed change.

7.12. Summary of Characteristics and Applications of D.C. Motors

The summary of characteristics and applications of D.C. motors is given in Table 1.

Table 1. Summary of Characteristics and Applications of D.C. motors

S. No.	*Type of motor*	*Characteristics*	*Applications*
1.	**Separately excited D.C. motors :**	● Possible to *obtain very accurate speeds.* ● Most suitable for applications requiring *speed variation from very low value to high value.*	● Paper machines. ● Steel rolling units. ● Diesel electric propultion of ships.
2.	**Shunt motors :** (*i*) *Constant speed :*	● Starting torque-medium, usually limited to 250% by a starting resistor but may be increased. ● Maximum operating torque usually limited to about 200% by commutation. ● Speed control : — increase upto 200% speed by field control. — decrease by armature voltage control.	● Employed for constant-speed applications ; may be used for adjustable speed not greater than 2 : 1 range. Field of applications includes : — Lathes ; — Centrifugal pumps ; — Fans and blowers — Machine tools ; — Wood working machines ; — Reciprocating pumps ; — Spinning and weaving machines ; — Printing presses, etc.

S No.	*Type of motor*	*Characteristics*	*Applications*
	(*ii*) *Adjustable speed :*	• Starting torque-medium, usually limited to 250% by a starting resistor but may be increased. • Maximum operating torque-usually limited to 200% by commutation. • Speed regulation – 10 to 15%. • Speed control : 6 : 1 range by field control.	• Same as above, For applications requiring adjustable speeds control, either constant torque or constant output.
3.	**Series motors :**	• Variable speed. • Adjustable varying speed. • *Starting torque* very high upto 500%. • *Maximum momentary operating* torque upto 400%. • *Speed regulation* : Widely variable, very high at no-lead. • *Speed control* : By series resistance.	• Suitable for drives requiring high starting torque and where adjustable, varying speed is satisfactory. Fields of application include : — Cranes ; — Hoists ; — Trolley cars ; — Conveyors ; — Electric locomotives etc. **Loads must be positively connected,, not belted.* *To prevent overspeed, lightest load should not be much less than 15 to 20% of full-load torque.
4.	**Compound motors :** (*i*) *Cumulative compound wound :*	• Variable speed. • Adjustable varying speed. • *Starting torque* high upto 450% depending upon the degree of compounding. • *Maximum momentary operating torque* higher than shunt, upto 350%. • *Speed regulation* : Varying, depending upon degree of compounding, upto 25–30%. • *Speed control* : Usually not used but may be upto 125% by field control.	• Suitable for drives requiring high starting torque and only fairly constant speed, pulsating loads with flywheel action. Fields of application include : — Shears ; — Punches ; — Elevators ; — Conveyors ; — Rolling mills ; — Heavy planes etc.
	(*ii*) *Differentially compound wound :*	• Torque and speed *almost constant.* • Tendency towards speed instability with a possibility of motor running away and strong possibility of motor starting in wrong direction.	• Employed for experimental and research work.

7.13. Starting of D.C. Motors

7.13.1. Need for starters

A motor at *rest* has no back or counter e.m.f. At staring therefore, the *armature current is limited only by the resistance of the armature circuit.* The *armature resistance is very low*, however, and if full voltage were impressed upon the motor terminals at stand still, the resulting armature current would be *many times full-load* value—*usually sufficient to damage the machine.* For this reason, *additional resistance* is introduced into the armature circuit at starting. As the motor gains speed, its back e.m.f. builds up and the starting resistance is cut-out.

Note. Very small D.C. motors, either shunt, series or compound wound, have sufficient armature resistance so that they may be started directly from the line without the use of a staring resistance and without injury to the motor.

Fig. 75 shows the connections of a starting resistance in three types of D.C. motors :

(*a*) A series motor; (*b*) A shunt motor; and

(*c*) A compound motor.

- In the case of *series motor* [Fig. 75 (*a*)], the armature, field and starting resistance are all in series.
- In the case of *shunt motor* [Fig. 75 (*b*)], it will be seen that the *top end of shunt field is connected to the first contact* on the starting resistance. This is to *ensure* that the *field winding receives the full supply at the moment of switching on.* If the fields were connected to the *last stud* of the starting resistance, then on starting, the field would receive only a *proportion of the supply voltage, the field current would be correspondingly weak and the torque might be too small to start the motor against the friction of the moving parts.*

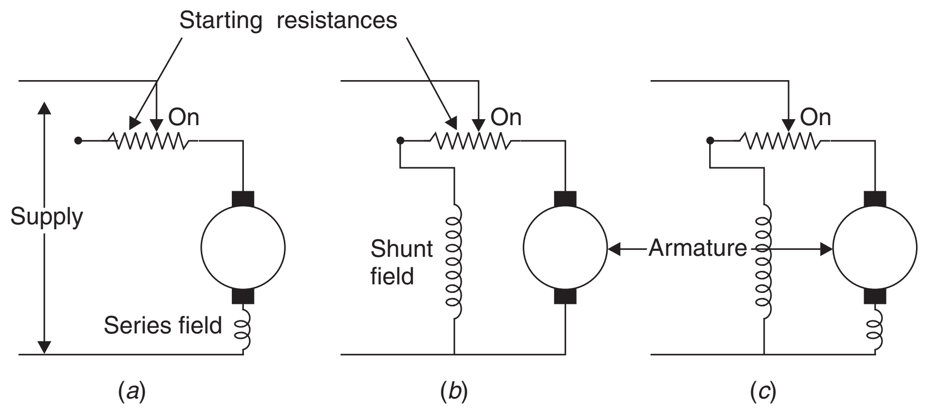


Fig. 75. Circuits incorporating starting resistances.

- The connections for the *compound motor* are seen from [Fig. 75 (*c*)] to be a combination of those of the series and the shunt connections.

7.13.2. Starters for shunt and compound motors

- The starters of D.C. motors are generally manufactured in convenient sizes and styles for use as auxiliaries with D.C. shunt and compound motors. Their *primary function is to limit the current in the armature circuit during the starting accelerating period.*
- The motor starters are always rated on the basis of output power and voltage of the motors with which they are to be used.

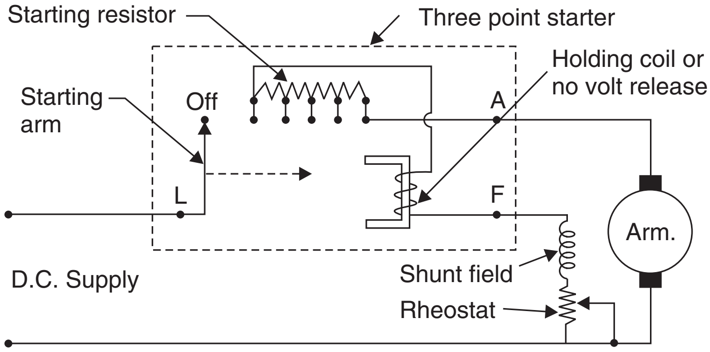
- There are two standard types of motor starters for shunt and compound motors. These are :

(*i*) Three-point type; and (*ii*) Four-point type.

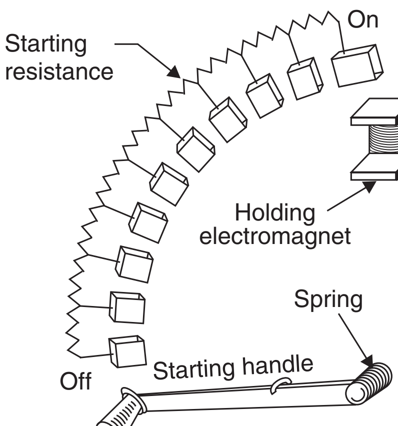
Three-point starters are not completely satisfactory when used with motors whose speeds must be controlled by inserting resistance in the shunt field circuit. However, when applications require *little or no speed control,* either may be employed.

Three-point starter. Refer to Fig. 76. The starter has three terminals *L, F* and *A*. The line terminal *L* must be connected to either side, positive or negative of the D.C. source on the main switch ; the field terminal *F* is connected to one field terminal on the motor ; the armature terminal *A* must be connected to either one of the motor armature terminals. The final connection must then be made from the second line terminal on the main switch to a junction of the remaining two armature and field terminals of the motor. If it is desired that the speed of the motor is controlled, a field rheostat is added as shown in Fig. 76 (*a*).

When the motor is at rest, the starter arm [represented by an arrow in Fig. 76 (*a*)] is held in the OFF position by a strong spiral spring.



(*a*)



(*b*)

Fig. 76. Three-point starter connected to a shunt motor.

Starting of motor :

- In order to *start the motor*, one hand is held on the handle of the open main switch while the starter arm is moved to the first stud [Figs. 76 (*a*) and (*b*)] with the other hand ; then the main switch is closed. If all the wiring is correct and the armature is free to turn, the motor will start.

- After the armature has accelerated sufficiently on the first stud, the starter arm is slowly moved to studs 2, 3, 4 etc., until the arm rests firmly against the iron poles of the *holding-coil* electromagnet. The entire starting process should take from 5 to 10 seconds. In the final position, the electromagnetic pull exerted by the holding coil will be greater than the force exerted by the spiral spring. *Should there be a power failure or should the field circuit be opened accidentally, the starter arm will fall back to its OFF position.* This function of starter is particularly important because :
  (*i*) if the power fails and starter arm is not restored to the OFF position, the motor might be damaged should the power, come on again ; and
  (*ii*) if the shunt field circuit were opened accidentally and the starter arm did not return to the OFF position, the motor speed might become dangerously high.
- Often the motors are protected against *overloads* by thermal *overload relays* in which *bimetallic is heated by motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contractor thereby isolating the motor from the supply.*

Drawback of a three-point starter. The use of a three-point starter presents a problem. The speed of the motor is controlled by means of the field rheostat. To increase the speed of motor necessitates the setting of the field rheostat to a higher resistance value. The current through the shunt field is reduced, and so is the current through the coil of the holding electromagnet. The *reduced current through the coil weakens the strength of the magnet and makes susceptible to line-voltage variations.* In the weakened condition a slight reduction in line voltage would further weaken the holding magnet, releasing the arm of the starter and thus *disconnecting the motor from the line*. Unscheduled stoppages of the motor make the three-point starter quite unpopular.

Four-point starter :

- Fig. 77 shows a simplified diagram of a *four-point starter*.
- In this starter the drawback/disadvantage of the three-point starter is eliminated. In addition to the same three-points that were used with the three-point starter, the other side of the line, L_2 is the *fourth point* brought to the starter. The coil of the holding magnet is connected across the line when the arm is moved from the 'off' position. The holding magnet and starting resistors function as in the three-point starter. *The possibility of accidentally opening the field circuit is quite remote ; hence the greater acceptance of the four-point starter over the three-point starter.*

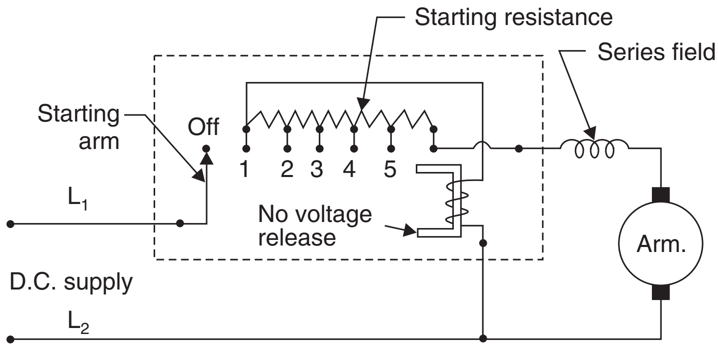


Fig. 77. Four-point starter.

— The four-point starter *provides the motor with no voltage protection.* Should the power fail, the motor must be disconnected from the line. If not, full line voltage will be applied to the armature without the benefit of starting resistors when power is restored. The holding magnet, being connected across the line, releases the arm *when the voltage drops below a specific value, thus protecting the motor when the power is restored.*

8. SPEED CONTROL OF D.C. MOTORS

8.1. Factors Controlling the Speed

D.C. machines are generally much more adaptable to adjustable speed service. *The ready availability of D.C. motors to adjustment of their operating speed over wide ranges and by a variety of methods is one of the important reasons for the strong competitive position of D.C. machinery in modern industrial applications.*

The speed of a D.C. motor can be expressed by the following relationship.

$$N \propto \frac{V - I_a R_a}{\phi} .$$

Therefore, the speed of D.C. motor can be regulated by changing ϕ, R or V, or in other words, by,

1. Field control
2. Rheostatic control
3. Voltage control.

Note. Thyristor control of D.C. motor is out of scope of this book.

8.2. Field Control Method

- **Field control** is the most common method and forms one of the *outstanding advantages of shunt motors*. The method is, of course, also applicable to compound motors. Adjustment of field current and hence the flux and speed by adjustment of the shunt field circuit resistance or with a solid-state control when the field is *separately excited* is accomplished simply, *inexpensively*, and *without much change in motor losses*.

The *speed is inversely proportional to the field current*

i.e.,
$$N \propto \frac{1}{I_f} \propto \frac{1}{\phi} .$$

- The *lowest speed obtainable is that corresponding to maximum field current* ; the *highest speed is limited electrically by the effects of armature reaction under weak- field conditions in causing motor instability and poor commutation*.
- *Since, voltage across the motor remains constant, it continues to deliver constant output.* This characteristic makes this method suitable for fixed output loads. The performance curve of a D.C. motor with voltage and field control is shown in Fig. 78.

Merits. The merits of this method are :

1. Good working efficiency.
2. Compact controlling equipment.
3. Capability of minute speed control.

4. The speed is not effected by load, and speed control can be performed effectively even at light loads.

5. Relatively inexpensive and simple to accomplish, both manually and automatically.

6. Within limits, field control does not affect speed regulation in the cases of shunt, compound, and series motors.

7. Provides relatively smooth and stepless control of speed.

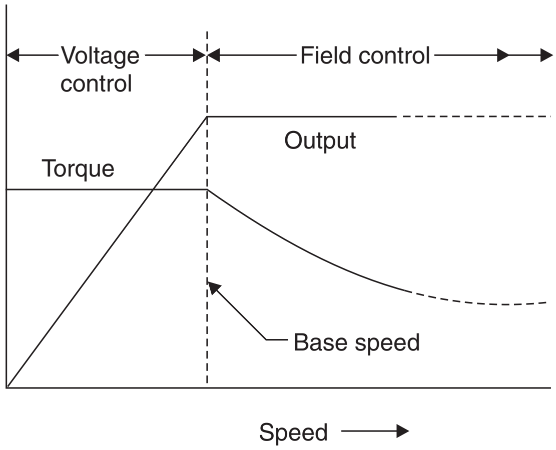


Fig. 78

Demerits. The demerits of field control as a method of speed control are :

1. Inability to obtain speeds below the basic speed.

2. Instability at high speeds because of armature reaction.

3. Commutation difficulties and possible commutator damage at high speeds.

Shunt Motors :

- The flux of a D.C. shunt motor can be changed by changing shunt field current (I_{sh}) with the help of a shunt field rheostat as shown in Fig. 79. Since the field current is very small, the power wasted in the controlling resistance is very small.

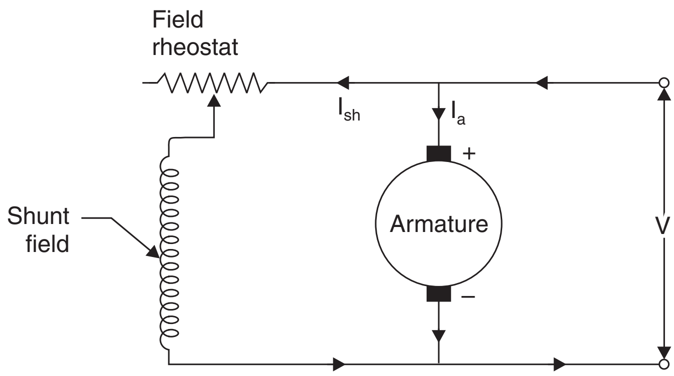


Fig. 79. Field rheostatic control for a D.C. shunt motor.

- In *non-interpolar machines* the speed can be increased by this method in the ratio **2 : 1**. In machines fitted *with interpoles* a ratio of maximum to minimum speeds of **6 : 1** is fairly common.

Series Motors. In a series motor, *variations of flux* can be brought about in anyone of the following ways :

(*i*) Field divertors (*ii*) Armature divertor

(*iii*) Tapped field control (*iv*) Paralleling field coils.

(*i*) **Field divertors.** A variable resistance, known as field divertor (Fig. 80) *shunts* the series windings. Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence, the *flux can be decreased and consequently the speed of the motor increased*.

(*ii*) **Armature divertor.** In order to *get speeds lower than the normal speed* a divertor across the armature can be used (Fig. 81). For a *given constant load torque,* if I_a is reduced due to armature divertor, then ϕ must increase ($\because \ T_a \propto I_a$). This results in an increase in current taken from the supply which increases the flux and a fall in speed $\left(\because \ N \propto \frac{1}{\phi}\right)$. The variations in speed can be controlled by varying the divertor resistance.

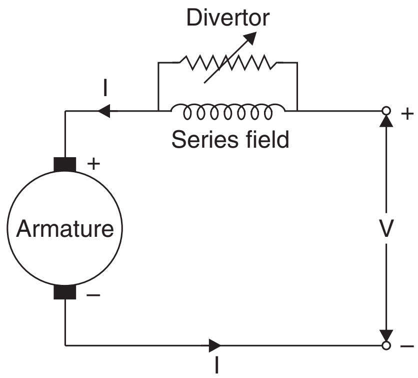


Fig. 80. Field divertor method of speed control for D.C. series motor.

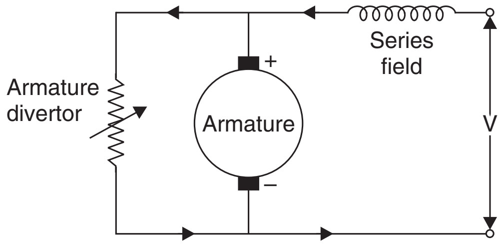


Fig. 81. Armature divertor method of speed control of D.C. series motor.

(*iii*) **Tapped field control.** In this method a number of tappings from the field winding are brought outside, as shown in Fig. 82. A number of series field turns can be short-circuited according to the requirement. *When all field turns are in circuit, the motor runs at lowest speed and speed increases with cutting out some of the series field turns.*

This method is often employed in *electric traction*.

(*iv*) **Paralleling field coils.** In this method of speed control several speeds can be obtained by regrouping the field coils as shown in Fig. 83 (*a*, *b*, *c*). This method is used for *fan motors*. It is seen that for a 4-pole motor, three fixed speeds can be obtained.

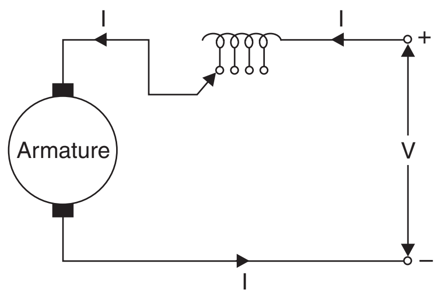
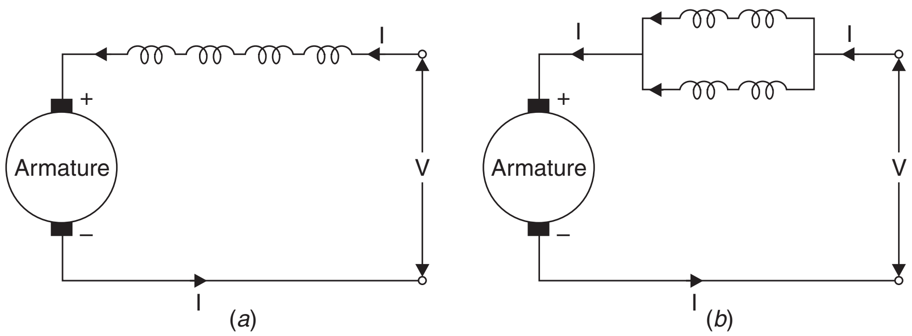
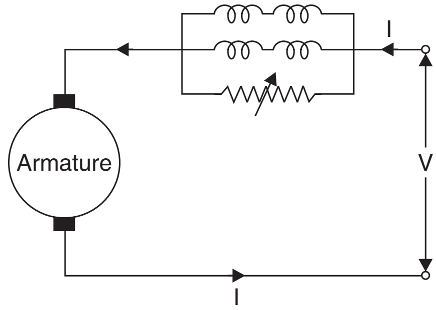


Fig. 82. Tapped field control for D.C. series motor.





(*c*)

Fig. 83. Paralleling field coils method for speed control of D.C. series motor.

Example 36. *A 220 V D.C. shunt motor draws a no-load armature current of 2.5 A when running at 1400 r.p.m. Determine its speed when taking an armature current of 60 A, if armature reaction weakens the flux by 3 per cent.*

Take armature resistance $= 0.2\ \Omega$.

Solution. Supply voltage, $V = 220$ Volts

No-load current, $I_{a0} = 2.5$ A

No-load speed, $N_0 = 1400$ r.p.m.

Armature resistance, $R_a = 0.2\ \Omega$

Armature current, $I_a = 60$ A

Full-load flux, $\phi = 0.97\ \phi_0$

Load speed N :

Back e.m.f. at no-load, $E_{b0} = V - I_{a0}R_a = 200 - 2.5 \times 0.2 = 219.5$ V

Back e.m.f. on load, $E_b = V - I_aR_a = 220 - 60 \times 0.2 = 208$ V.

Now using the relation, $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}} \times \dfrac{\phi_0}{\phi}$

$$\frac{N}{1400} = \frac{208}{219.5} \times \frac{\phi_0}{0.97\phi_0}$$

$$\therefore \qquad N = \frac{1400 \times 208}{219.5 \times 0.97} = 1367.7 \text{ r.p.m.}$$

Hence, **load speed = 1367.7 r.p.m. (Ans.)**

Example 37. *The armature and shunt field resistances of a 500 V shunt motor are 0.2 Ω and 100 Ω respectively. Find the resistance of the shunt field regulator to increase the speed from 800 r.p.m. to 1000 r.p.m., if the current taken by the motor is 450 A. The magnetisation characteristic may be assumed as a straight line.*

Solution. Supply voltage, $V = 500$ Volts

Armature resistance, $R_a = 0.2\ \Omega$

Shunt field resistance, $R_{sh} = 100\ \Omega$

Initial speed, $N_1 = 800$ r.p.m.

Final speed, $N_2 = 1000$ r.p.m.

Current drawn by the motor, $I = 450$ A

Resistance of shunt field regulator, R :

Since the magnetization characteristic is a straight line,

$$\phi \propto I_{sh}$$

Also, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$\therefore$ $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}.$$

At 800 r.p.m.

$$I_{sh1} = \frac{V}{R_{sh}} = \frac{500}{100} = 5 \text{ A}$$

Armature current, $I_{a1} = I - I_{sh1} = 450 - 5 = 445$ A

Back e.m.f. $E_{b1} = V - I_{a1}R_a = 500 - 445 \times 0.2 = 411$ V

At 1000 r.p.m.

Armature current, $I_{a2} = I - I_{sh2} = 450 - I_{sh2}$

Back e.m.f., $E_{b2} = V - I_{a2}R_a = 500 - (450 - I_{sh2}) \times 0.2 = 410 + 0.2I_{sh2}$

Now $$\frac{1000}{800} = \frac{(410 + 0.2I_{sh2})}{411} \times \frac{5}{I_{sh2}}$$

$$410 + 0.2I_{sh2} = \frac{1000}{800} \times \frac{411}{5}\ I_{sh2} = 102.75I_{sh2}$$

or $$102.55I_{sh2} = 410$$

$\therefore$ $$I_{sh2} = 4 \text{ A app.}$$

Total resistance in the field circuit

$$= \frac{V}{I_{sh2}} = \frac{500}{4} = 125\ \Omega$$

Resistance of the shunt field $= 100\ \Omega$

Resistance of the shunt field regulator,

$$R = 125 - 100 = 25\ \Omega$$

Hence, **resistance of the shunt field regulator = 25** Ω. **(Ans.)**

☞ **Example 38.** *A 500 V series motor has an armature resistance of 0.4 ohm and series field resistance of 0.3 ohm. It takes a current of 100 A at a speed of 600 r.p.m. Find the speed of the motor if a divertor of resistance 0.6 ohm is connected across the field, the load torque being kept constant.*

Neglect armature reaction and assume that flux is proportional to the current.

Solution. Supply voltage, $V = 500$ Volts

Armature resistance, $R_a = 0.4$ ohm

Series field resistance, $R_{se} = 0.3$ ohm

Divertor resistance, $R_{\text{div.}} = 0.6$ ohm

$I_{a1} = 100$ A, $N_1 = 600$ r.p.m.

Speed, N_2 :

Back e.m.f., $$E_{b1} = V - I_{a1}(R_a + R_{se})$$
$$= 500 - 100(0.4 + 0.3) = 430 \text{ V.}$$

Let I_{a2} be the current taken and ϕ_2 be the flux produced when a divertor is connected across the series field (Fig. 84)

Since the torque remains constant,

$\therefore$ $$\phi_1 I_{a1} = \phi_2 I_{a2} \quad ...(i)$$

But $\phi \propto$ current through series field

$\therefore$ $$\phi_1 \propto I_{a1}$$

Current through the series field when a divertor is connected

$$= I_{a2} \times \frac{R_{div.}}{R_{div.} + R_{se}}$$

$$= I_{a2} \times \frac{0.6}{0.6 + 0.3} = 0.667 I_{a2}$$

Flux in this case $\phi_2 \propto 0.667 I_{a2}$

Substituting in (*i*), we get

$$I_{a1}^2 = 0.667 I_{a2}^2$$

$$I_{a2}^2 = \frac{I_{a1}^2}{0.667}$$

$$\therefore \quad I_{a2} = \frac{I_{a1}}{\sqrt{0.667}} = \frac{100}{\sqrt{0.667}} = 122.44 \text{ A}$$

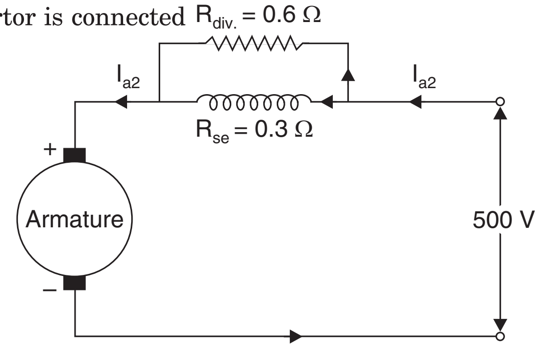


Fig. 84

Series field current, $I_{se} = 0.667 I_{a2}$

$= 0.667 \times 122.44 = 81.64$ A

Back e.m.f., $E_{b2} = V - I_{a2}R_a - I_{se}R_{se}$

$= 500 - 122.44 \times 0.4 - 81.64 \times 0.3$

$= 500 - 48.97 - 24.49 = 426.54$ V

Using the relation, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{600} = \frac{426.54}{430} \times \frac{100}{81.64}$$

$\therefore \quad N_2 = 729$ r.p.m.

Hence, **speed of the motor = 729 r.p.m. (Ans.)**

Example 39. *A D.C. series motor drives a load the torque of which varies are square of the speed. The motor takes a current of 20 A when the speed is 800 r.p.m. Calculate the speed and current when the motor field winding is shunted by a divertor of the same resistance as that of the field winding.*

Neglect all motor losses and assume that the magnetic circuit is unsaturated.

Solution. $I_{a1} = 20$ A, $N_1 = 800$ r.p.m.

$\mathbf{I_{a2} = ?, N_2 = ?}$

When the field winding is shunted by a divertor of equal resistance, then current through either is half the armature current. If I_{a2} is the new armature current, then $\frac{I_{a2}}{2}$ passes through the winding (Fig. 85).

$$\therefore \quad \phi_2 = \frac{I_{a2}}{2}$$

Now $T_1 \propto \phi_1 I_{a1} \propto N_1^2$ (Given)

and $T_2 \propto \phi_2 I_{a2} \propto N_2^2$ (Given)

From (*i*) and (*ii*), we get

$$\left(\frac{N_2}{N_1}\right)^2 = \frac{\phi_2 I_{a2}}{\phi_1 I_{a1}} \quad ...(i)$$

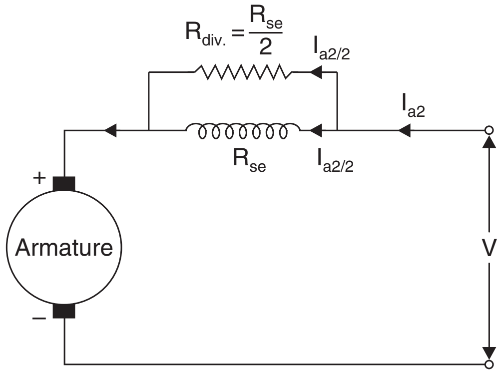


Fig. 85

Because all losses are negligible, hence the armature and series field resistances are negligible. This means that back e.m.f. in both cases is the same as the applied voltage.

∴ $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \text{ becomes}$$

$$\frac{N_2}{N_1} = \frac{\phi_1}{\phi_2} \qquad ...(ii)$$

[∵ $E_{b2} = E_{b1}$ = applied voltage]

Putting this value in (*i*) above, we get

$$\left(\frac{\phi_1}{\phi_2}\right)^2 = \frac{\phi_2 I_{a2}}{\phi_1 I_{a1}}$$

or $$\frac{I_{a2}}{I_{a1}} = \left(\frac{\phi_1}{\phi_2}\right)^3$$

$$\frac{I_{a2}}{20} = \left(\frac{20}{I_{a2}/2}\right)^3 \qquad [\because \ \phi_2 \propto I_a/2]$$

∴ $$I^4_{a2} = 20 \times 40^3$$

or $$I_{a2} = 33.63 \text{ A}$$

Hence, **current = 33.63 A. (Ans.)**

From (*ii*), we get $$\frac{N_2}{800} = \frac{20}{(33.63)/2}$$

$$N_2 = 800 \times \frac{40}{33.63} = 951.52 \text{ r.p.m.}$$

Hence, **speed of the motor = 951.53 r.p.m. (Ans.)**

8.3. Rheostatic Control

- This method consists of obtaining reduced speeds by the insertion of external series resistance in the armature circuit. It can be used with series, shunt and compound motors ;

for the last two types, the *series resistor must be connected between the shunt field and the armature, not between line and the motor*.

- It is *common method of speed control for series motors* and is generally analogous in action to wound-rotor induction-motor control by series rotor resistance.
- *This method is used when speeds below the no-load speed is required.*

Advantages :

1. The ability to achieve speeds below the basic speed.
2. Simplicity and ease of connection.
3. The possibility of combining the functions of motor starting with speed control.

Disadvantages :

1. The relatively high cost of large, continuously rated, variable resistors capable of dissipating large amounts of power (particularly at higher power ratings).
2. Poor speed regulation for any given no-load speed setting.
3. Low efficiency resulting in high operating cost.
4. Difficulty in obtaining stepless control of speed in higher power ratings.

Shunt motors :

- In armature or rheostatic control method of speed the voltage across the armature (which in normally constant) is varied by inserting a variable rheostat or resistance, called *controller resistance,* in *series* with the armature circuit. As the controller resistance is increased, the potential difference across the armature is decreased thereby decreasing the armature speed. For a load of constant torque, speed is approximately proportional to the potential difference across the armature.

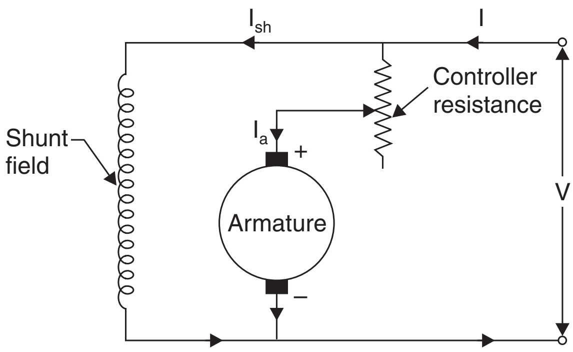


Fig. 86. Armature resistance control for D.C. shunt motor.

From the speed/armature current characteristic it is seen that *greater the resistance in armature, greater is the fall in speed.*

There is a particular load current for which the *speed would be zero. This is the maximum current and is known as 'stalling current'*.

- This method is *very wasteful, expensive and unsuitable for rapidly changing load*, because for a given value of R_t the speed will change with load. A more stable operation can be obtained by using a *divertor across the armature* (Fig. 88) in addition to armature control resistance. Now, the changes in armature current due to changes in the load torque *will not be so effective* in changing the potential difference across the armature and hence the speed of the armature.

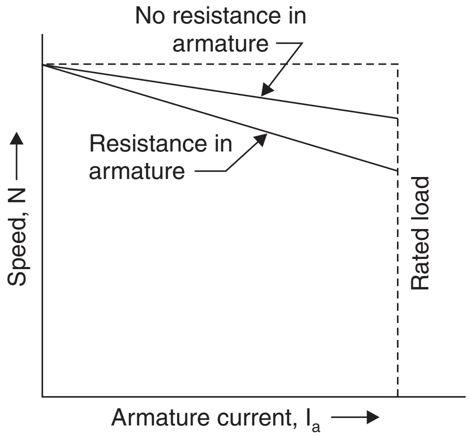


Fig. 87. Speed-current characteristic of D.C. shunt motor.

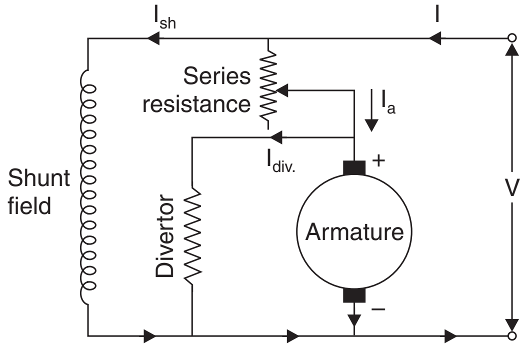


Fig. 88. Use of divertor across the armature for speed control of D.C. shunt motor.

Series motors. Armature resistance control is the most common method employed for D.C. series motors (Figs. 89 and 90).

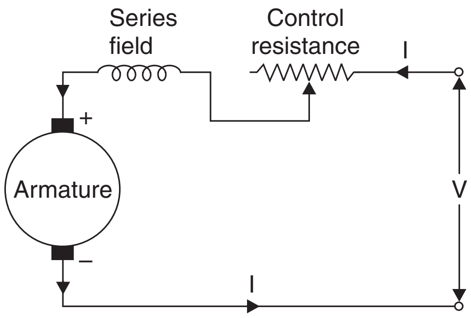


Fig. 89. Armature resistance control for D.C. series motor.

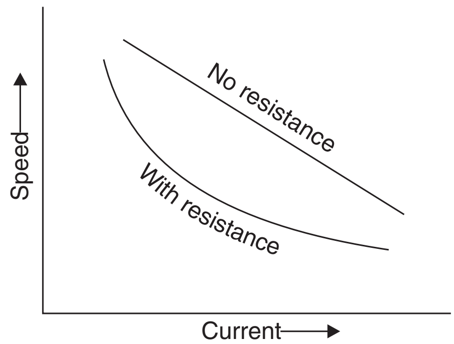


Fig. 90

By increasing the resistance in series with the armature the voltage applied across the armature terminals can be decreased. With the reduced voltage across the armature, the speed is reduced.

Since full motor current passes through the resistance, the loss of power is considerable.

Although terminal-voltage control by means of a variable voltage supply would effectively control the speed of a D.C. series motor, the high cost of the control equipment is seldom warranted.

Series-parallel control :

- This system is widely used in *electric traction.* Here two or more similar mechanically-coupled motors are employed.
- **At low speeds** the *motors are joined in series* as shown in Fig. 91 (*a*). The additional resistance is gradually cut-out by controller as the motors attain the speed, and finally the resistance is totally removed, then *each motor has half of line voltage*. In this arrangement, for *any given value of armature current, each motor will run at half of its normal speed*. As there is no external resistance in the circuit, therefore *there is no waste of energy and so motors operate at an efficiency nearly equal to that obtainable with full line voltage across the terminals of each motor.*

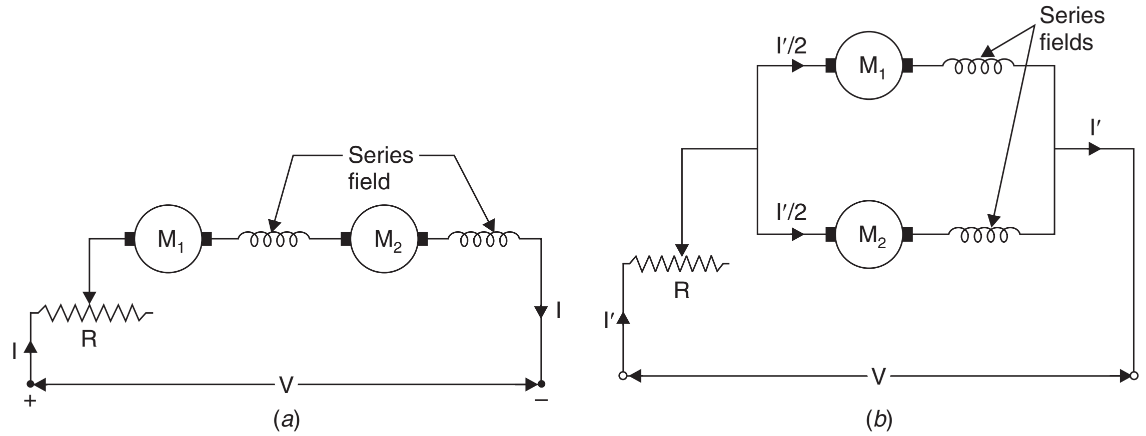


Fig. 91. Series-parallel control.

When the motors are connected in *series* and resistance R is *completely cut-out* :

Voltage across each motor $= \frac{V}{2}$

Current through each motor $= I$

$$\text{Speed} \propto \frac{\text{voltage}}{\text{current}} \propto \frac{V/2}{I} \propto \frac{V}{2I}$$

$$\text{Torque} \propto \phi I \propto I^2 \qquad [\text{Since } \phi \propto I, \text{ assuming unsaturated field}]$$

- **At high speeds the motors** *are joined in parallel* as shown in Fig. 91 (b). The variable resistance R is gradually cut out as motors attain the speed. After the resistance R is completely cut out each motor is connected across the full line voltage.

When the motors are connected in *parallel* and resistance R is completely cut-out :

Voltage across each motor $= V$

Current through each motor $= \frac{I}{2}$ $\left[= \frac{I'}{2} \text{ when resistance } R \text{ is } not \text{ completely cut-out}\right]$

$$\text{Speed} \propto \frac{\text{voltage}}{\text{current}} \propto \frac{V}{I/2} \propto \frac{2V}{I}$$

Also, $\text{torque} \propto \phi I \propto I^2$ $[\because \phi \propto I]$

$\because$ $$T \propto \left(\frac{I}{2}\right)^2 \propto \frac{I^2}{4}$$

The *torque is* $\frac{1}{4}$ *times that produced by motors when in series.*

Example 40. *The armature and shunt field resistances of a 230 V shunt motor are 0.1 ohm and 230 ohms respectively. It takes a current of 61 A at 1000 r.p.m. If the current taken remains unaltered find the resistance to be included in series with the armature circuit to reduce the speed to 750 r.p.m.*

Solution. Supply voltage, $V = 230$ Volts

Armature resistance, $R_a = 0.1$ ohm

Shunt field resistance, $R_{sh} = 230$ ohms

Load current, $I = 61$ A

Speed, $N_1 = 1000$ r.p.m.

Speed, $N_2 = 750$ r.p.m.

Additional resistance required, R :

Shunt field current,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1 \text{ A}$$

Armature current,

$$I_1 = I - I_{sh} = 61 - 1 = 60 \text{ A}$$

Using the relation,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \quad ...(i)$$

Since flux remains constant in shunt motor

$$\phi_1 = \phi_2$$

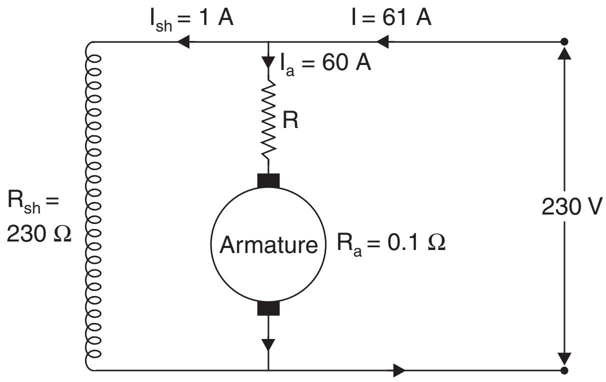


Fig. 92

Also it is given that the current taken by the motor remains constant

$\therefore$ $I = I_1 = I_2 = 61$ A

and $I_{a1} = I_{a2} = 60$ A

Thus, eqn. (*i*) reduces to

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad ...(ii)$$

$$E_{b1} = V - I_{a1}R_a = 230 - 60 \times 0.1 = 224 \text{ V}$$

$$E_{b2} = V - I_{a2}(R_a + R)$$

$$= 230 - 60(0.1 + R) = 230 - 6 - 60R$$

$$= 224 - 60\,R$$

Putting this value in (*ii*), we get

$$\frac{750}{1000} = \frac{224 - 60R}{224}$$

or

$$60R = 224 - \frac{750}{1000} \times 224$$

$\because$ $R = 0.933$ ohm

Hence, **additional resistance required = 0.933 ohm. (Ans.)**

Example 41. *The armature resistance of a 230 V D.C. shunt motor is 0.2 ohm. It takes 15 A at rated voltage and runs at 800 r.p.m. Calculate the value of additional resistance required in the armature circuit to reduce the speed to 600 r.p.m. when the load torque is independent of speed.*

Ignore the field current.

Solution. Supply voltage, $V = 230$ Volts

Armature resistance, $R_a = 0.2$ ohm

Armature current, $I_1 = I_{a1} = 15$ A

Speed, $N_1 = 800$ r.p.m.

Speed, $N_2 = 600$ r.p.m.

Additional resistance required, R :

Back e.m.f., $E_{b1} = V - I_{a1}R_a = 230 - 15 \times 0.2 = 227$ V

Since as per given data load torque is independent of speed and flux is constant

$$I_{a1} = I_{a2} = 15 \text{ A}$$

$$\phi_1 = \phi_2$$

Back e.m.f., $E_{b2} = V - I_{a2}(R_a + R) = 230 - 15(0.2 + R)$

$= 230 - 3 - 15R = 227 - 15R$

Using the relation, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2}$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \qquad [\because \quad \phi_1 = \phi_2]$$

$\therefore$ $$\frac{600}{800} = \frac{227 - 15R}{227}$$

$$15R = 227 - \frac{600}{800} \times 227$$

$\therefore$ $R = 3.783$ ohms

Hence, **additional resistance required = 3.783 ohms. (Ans.)**

8.4. Voltage Control

When the speed is controlled by regulating the motor terminal voltage while maintaining constant field current, it is called **voltage control.**

With voltage control, the change in speed is almost proportional to the change in voltage as shown in Fig. 78. The *output varies directly with speed and the torque remains constant.* Since the voltage has to be regulated without affecting the field, the application of voltage control is limited to *separately excited motors* (Fig. 93) *only.*

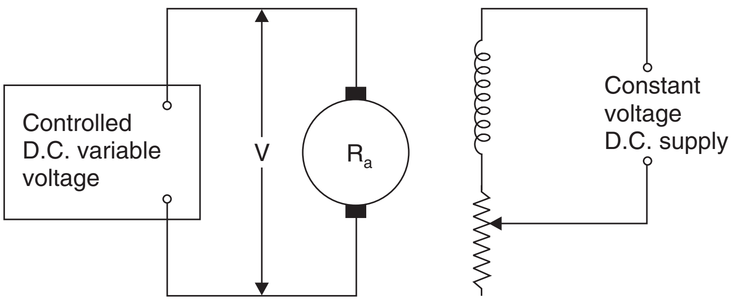


Fig. 93. Voltage control method.

- For D.C. motors of *fractional* and *relatively low power* rating, the variable D.C. voltage source may be a D.C. vacuum tube, a gas or thyraton tube, or a semi-conductor (silicon controlled rectifier) amplifier, operating from a three-phase or single phase A.C. supply.
- *Motors of moderate rating up to* 75 kW may be controlled by this method using Rototrol or Regulex or magnetic amplifiers as the adjustable D.C. voltage source.
- *Large D.C. motors* are controlled in this manner by means of rotary amplifiers such as the amplidyne or the Ward-Leonard control system.

Advantages :

1. Speed control over a wide range is possible.
2. This method eliminates the need for series armature starting resistance.
3. Uniform acceleration can be obtained.
4. Speed regulation is good.

Disadvantages :

1. Arrangement is costly as two extra machines are required.
2. The overall efficiency of the system is low, especially at light loads.

Applications :

Inspite of the high capital cost, this method finds wide applications in :

(*i*) Steel mills for reversing the rolling mills.

(*ii*) Seamless tube mills and shears.

(*iii*) High and medium speed elevators in tall buildings, mine hoists, paper machine drives and electric shovels.

Ward-Leonard System. This method of control not only gives a wide range of operating speeds, but reduces to the very minimum the wastage of energy that may take place at staring and stopping.

Fig. 94 shows the schematic arrangement of Ward-Leonard method.

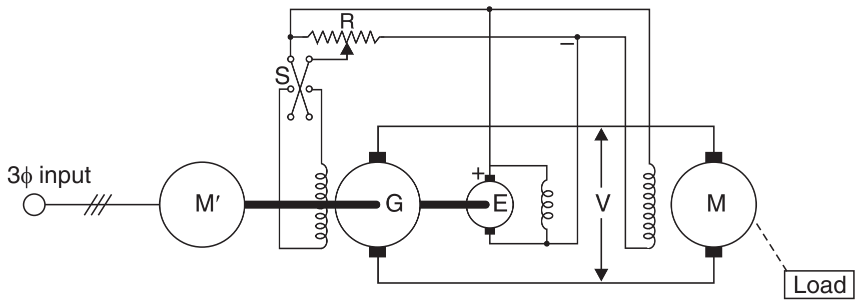


Fig. 94. Ward-Leonard method.

M = main motor whose speed is to be controlled

G = separately excited generator which feeds the armature of the motor *M*

E = an exciter (a small shunt generator) which provides field excitation to the generator *G* and motor *M*

M′ = driving motor—a constant speed motor which drives *G* and *E*.

[If the system is to work on A.C. supply, the driving motor *M′* is a 3-phase induction motor. If the system is to work on D.C. supply, the motor *M′* is a shunt motor. In the latter case the exciter *E* is not necessary because the excitation for the generator *G* and motor *M* can be obtained from D.C. mains. A diesel engine can also be used in place of motor *M′*].

R = a potentiometer rheostat

S = a double throw switch.

The working of this system is as follows :

- The motor *M′* drives the generator *G* and excitor *E* at constant speed. The voltage fed to motor *M* can be controlled by varying the setting of *R*. A change in voltage applied to motor *M* changes its speed. The speed can be adjusted to any value from zero to maximum in either direction by means of a rheostat *R* and switch *S*.
- When the sliding contact of *R* is at extreme right, the motor is running at full speed in one direction. To decrease the speed the sliding contact is moved to the left. When the sliding contact is at the extreme left position, the speed of motor *M* is zero. In order to reverse the speed of the motor, the sliding contact is shifted to the extreme left, the switch *S* is reversed and the sliding contact shifted to right again.
- A modification of the Ward-Leonard system is known as *Ward-Leonard Ilgner system,* which uses a small motor-generator set with the *addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit*. When the main motor *M* becomes suddenly overloaded, the driving motor *M′* slows down, thus allowing the

inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor M, then M' speeds up thereby again storing energy in the flywheel. When Ilgner system is driven by means of an A.C. motor (whether induction or synchronous) another refinement in the form of a 'slip regulator' can be usefully employed thus giving an additional control.

- One important feature of the Ward-leonard system is its *regenerative action.* When a locomotive, fitted with this system, is descending a slope, it speeds up due to the action of gravity. The speed of motor M increases until its back e.m.f. exceeds the applied voltage. Motor M then runs as generator and feeds the machine G which now works as a generator and feeds electrical energy back into the trolley wire. This results in salvaging of considerable amount of energy and a superior and smooth braking action. Such an action is known as *regenerative braking*.

Advantages of Ward-Leonard system :

1. A wide range of speed from standstill to high speeds in either direction.
2. Rapid and instant reversal without excessively high armature currents.
3. Starting without the necessity of series armature resistances.
4. Stepless control from standstill to maximum speed in either direction.
5. Larger units employing generator field reversal eliminate the need for heavy armature conductors for reversing, and at the same time prevent motor runway since the motor field is always excited.
6. The method lends itself to adaptation of intermediate electronic, semi-conductor, and magnetic amplifiers to provide stages of amplification for an extremely large motor. Thus, the power in the control circuit may be extremely small.
7. Extremely good speed regulation at any speed.

Disadvantages :

1. High initial cost.
2. Since the efficiency, neglecting the exciter efficiency, is essentially the product of the individual efficiencies of the two larger machines, the efficiency of this method is *not as high as rheostat speed control or the field control method.*

Example 42. *A series motor drives a fan for which the torque varies as square of the speed. Its resistance between terminals is 1.2 ohm. On 220 V, it runs at 350 r.p.m. and takes 30 A. The speed is to be raised to 450 r.p.m. by increasing the voltage. Find the voltage.*

Assume that flux varies directly as current.

Solution. Resistance between terminals = 1.2 ohm

$$I_{a1} = 30 \text{ A}, N_1 = 350 \text{ r.p.m.}, N_2 = 450 \text{ r.p.m.}$$

Since $\phi \propto I_a$

$\therefore$ $T \propto \phi I_a \propto I_a^2$ ...(*i*)

Also, $T \propto N^2$ ...[Given] ...(*ii*)

From (*i*) and (*ii*), we get $I_a^2 \propto N^2$ or $I_a \propto N$

or
$$\frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{450}{350}$$

$$\therefore \quad I_{a2} = I_{a1} \times \frac{450}{350} = 30 \times \frac{450}{350} = 38.57 \text{ A}$$

Back e.m.f., $E_{b1} = 220 - 30 \times 1.2 = 184$ V

Back e.m.f., $E_{b2} = V - 38.57 \times 1.2 = V - 46.28$

$$\frac{\phi_1}{\phi_2} = \frac{I_{a1}}{I_{a2}} = \frac{30}{38.57}$$

Now using the relation, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{450}{350} = \frac{V - 46.28}{184} \times \frac{30}{38.57}$$

$\therefore$ $$V - 46.28 = \frac{450}{350} \times 184 \times \frac{38.57}{30} = 304.15$$

$\therefore$ **V = 304.15 Volts. (Ans.)**

HIGHLIGHTS

1. **Basic type of D.C. machine** is that of commutator type. This is actually an alternating current (A.C.) machine, but furnished with a special device, a commutator, which under certain conditions converts alternating current into direct current.
2. An electrical **generator** is a machine which converts mechanical energy (or power) into electrical energy (or power). It works on the following principle :

   "Whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic induction".
3. A D.C. machine consists of two **main** parts :
   - **Stationary Part :** designed mainly for producing magnetic flux.
   - **Rotating Part :** called armature, where mechanical energy is converted into electrical (electric generator) or, conversely, electrical energy into mechanical (electric motor).

   The various parts of a D.C. machine are enumerated below :

   Frame, Field poles, Commutating poles, Armature, Commutator, Brush gear, Armature shaft bearings, Armature windings.
4. According to the degree of closure produced by winding, armature windings are of following two types :

   1. Open coil winding 2. Closed coil winding

   The closed armature windings are of two types :

   (*i*) Ring winding (*ii*) Drum winding

   In general there are two types of drum armature windings :

   (*i*) Lap winding (*ii*) Wave winding.
5. **'Lap winding'** is suitable for comparatively low voltage but high current generators whereas **'wave winding'** is used for high voltage, low current machines.
6. In the **'lap winding'** *t*he finish of each coil is connected to the start of the next coil so that winding or commulator pitch is unity.

   In **'wave winding'** *t*he finish of coil is connected to the start of another coil well away from the first coil.
7. The **rated output** of an electrical machine is its duty in the conditions for which it has been designed by the manufacturer. The rated duty is characterised by values given on the machine's rating plate, and termed rated value, as, for instance : rated output, rated voltage, rated current, rated speed, etc.
8. E.m.f. equation of a generator is given as follows :

$$E_g = \frac{p\phi ZN}{60a}$$

where E_g = generated e.m.f. per parallel path in armature.

p = number of poles

ϕ = flux/pole, Wb

Z = total number of conductors

= number of slots × number of conductors/slot

N = rotational speed of armature, r.p.m.

a = number of parallel paths in armature

[For a **wave wound** generator : $a = 2$
For a **lap wound** generator : $a = p$]

9. According to **method of excitation** D.C. generators are classified as follows :

1. Separately excited generators
2. Self excited generators.

Self excited generators can be divided in accordance with how the field winding is connected into generators as follows :

(*i*) Shunt wound generators
(*ii*) Series wound generators
(*iii*) Compound wound generators :
(*a*) Short shunt
(*b*) Long shunt.

10. The electric motor is a machine which converts electrical energy into mechanical energy.

11. Voltage equation of a motor is $V = E_b + I_a R_a$.

12. Condition for maximum power developed in armature is $E_b = \frac{V}{2}$.

13. Armature torque of a motor is given by $T_a = 0.159\, Z\phi p \times \frac{I_a}{a}$ Nm.

14. Shaft or useful torque is given by $T_{sh} = \frac{\text{B.H.P. (metric)} \times 735.5}{2\pi N / 60}$ Nm

Torque lost in iron and friction losses = $0.159 \times \frac{\text{iron and friction losses}}{N/60}$ Nm.

15. Speed of a D.C. motor is given by $N = \frac{E_b}{\phi}\left(\frac{60a}{Zp}\right)$ *r.p.m.*

16. Speed regulation of a motor is given by

Percentage speed regulation = $\frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}}$.

17. In a D.C. motor reversal is accomplished by changing the polarity of either the armature or the field, but not by changing both.

OBJECTIVE TYPE QUESTIONS

D.C. GENERATOR

Choose the Correct Answer :

1. Laminations of core are generally made of
(*a*) cast iron (*b*) carbon
(*c*) silicon steel (*d*) stainless steel.

2. Which of the following could be approximately the thickness of laminations of a D.C. machine ?
(*a*) 0.005 mm (*b*) 0.05 mm
(*c*) 0.5 mm (*d*) 5 mm.

3. The armature of D.C. generator is laminated to
(*a*) reduce the bulk (*b*) provide passage for cooling air
(*c*) insulate the core (*d*) reduce eddy current loss.

4. The resistance of armature winding depends on
(*a*) length of conductor (*b*) cross-sectional area of the conductor
(*c*) number of conductors (*d*) all of the above.

5. The field coils of D.C. generator are usually made of
(*a*) mica (*b*) copper
(*c*) cast iron (*d*) carbon.

6. The commutator segments are connected to the armature conductors by means of
(*a*) copper lugs (*b*) resistance wires
(*c*) insulation pads (*d*) brazing.

7. In a commutator
(*a*) copper is harder than mica (*b*) mica and copper are equally hard
(*c*) mica is harder than copper (*d*) none of the above.

8. In D.C. generators the pole shoes are fastened to the pole core by
(*a*) rivets (*b*) counter sunk screws
(*c*) brazing (*d*) welding.

9. According to Fleming's right-hand rule for finding the direction of induced e.m.f., when middle finger points in the direction of induced e.m.f., forefinger will point in the direction of
(*a*) motion of conductor (*b*) lines of force
(*c*) either of the above (*d*) none of the above.

10. Fleming's right-hand rule regarding direction of induced e.m.f., correlates
(*a*) magnetic flux, direction of current flow and resultant force
(*b*) magnetic flux, direction of motion and the direction of e.m.f. induced
(*c*) magnetic field strength, induced voltage and current
(*d*) magnetic flux, direction of force and direction of motion of conductor.

11. While applying Fleming's right-hand rule to find the direction of induced e.m.f., the thumb points towards
(*a*) direction of induced e.m.f.
(*b*) direction of flux
(*c*) direction of motion of the conductor if forefinger points in the direction of generated e.m.f.
(*d*) direction of motion of conductor, if forefinger points along the lines of flux.

12. The bearings used to support the rotor shafts are generally
(*a*) ball bearings (*b*) bush bearings
(*c*) magnetic bearings (*d*) needle bearings.

13. In D.C. generators, the cause of rapid brush wear may be
(*a*) severe sparking (*b*) rough commutator surface
(*c*) imperfect contact (*d*) any of the above.

14. In lap winding, the number of brushes is always
(*a*) double the number of poles (*b*) same as the number of poles
(*c*) half the number of poles (*d*) two.

15. For a D.C. generator when the number of poles and the number of armature conductors are fixed, then which winding will give the higher e.m.f. ?
(*a*) Lap winding (*b*) Wave winding
(*c*) Either of (*a*) and (*b*) above (*d*) Depends on other features of design.

16. A 4 pole D.C. generator rotates at 1500 r.p.m. How many times in a second will the e.m.f. induced in each armature conductor change its direction ?
(*a*) 100 (*b*) 50
(*c*) 400 (*d*) 200.

17. If B is the flux density, l the length of conductor and v the velocity of conductor, then induced e.m.f. is given by
(*a*) Blv (*b*) Blv^2
(*c*) Bl^2v (*d*) Bl^2v^2.

18. In case of a wave wound generator, the average pitch
(*a*) must be even (*b*) must be odd
(*c*) may be odd or even (*d*) is generally fractional.

19. In case of a 4 pole D.C. generator provided with a two layer lap winding with sixteen coils, the pole pitch will be
(*a*) 4 (*b*) 8
(*c*) 16 (*d*) 32.

20. The material for commutator brushes is generally
(*a*) mica (*b*) copper
(*c*) cast iron (*d*) carbon.

21. The insulating material used between the commutator segments is normally
(*a*) graphite (*b*) paper
(*c*) mica (*d*) insulating varnish.

22. In a D.C. generator, if p be the number of poles and N be the r.p.m. of rotor, then the frequency of magnetic reversals will be
(*a*) $\frac{Np}{2}$ (*b*) $\frac{Np}{60}$
(*c*) $\frac{Np}{120}$ (*d*) $\frac{Np}{3000}$.

23. For generating large currents on D.C. generators which winding is generally preferred ?
(*a*) Progressive wave winding (*b*) Lap winding
(*c*) Retrogressive wave winding (*d*) Current depends on design.

24. The purpose of providing dummy coils in a generator is
(*a*) to enhance flux density (*b*) to amplify voltage
(*c*) to provide mechanical balance for the rotor (*d*) to reduce eddy currents.

D.C. MOTOR

25. No-load speed of which of the following motor will be highest ?
(*a*) Shunt motor (*b*) Series motor
(*c*) Cumulative compound motor (*d*) Differentially compound motor.

26. The direction of rotation of a D.C. series motor can be changed by
(*a*) interchanging supply terminals (*b*) interchanging field terminals
(*c*) either of (*a*) and (*b*) above (*d*) none of the above.

27. Which of the following applications requires high starting torque ?
(*a*) Lathe machine (*b*) Centrifugal pump
(*c*) Locomotive (*d*) Air blower.

28. In case a D.C. motor is connected across A.C. supplies
(*a*) the motor will burn (*b*) the motor will run at rated speed
(*c*) the motor will run at corresponding synchronous speed
(*d*) the motor will run at slow speed.

29. If a D.C. motor is to be selected for conveyors, which motor would be preferred ?
(*a*) Series motor (*b*) Shunt motor
(*c*) Differentially compound motor (*d*) Cumulative compound motor.

30. Which D.C. motor will be preferred for machine tools ?
(*a*) Series motor (*b*) Shunt motor
(*c*) Cumulative compound motor (*d*) Differentially compound motor.

31. Which D.C. motor will be preferred for constant speed line shafting ?
(*a*) Cumulatively compound motor (*b*) Differentially compound motor
(*c*) Shunt motor (*d*) Series motor.

32. Differentially compound D.C. motors can find applications requiring
(*a*) high starting torque
(*b*) low starting torque
(*c*) variable speed
(*d*) frequent on-off cycles.

33. Which D.C. motor is preferred for elevators ?
(*a*) Shunt motor
(*b*) Series motor
(*c*) Differentially compound motor
(*d*) Cumulatively compound motor.

34. According to Fleming's left-hand rule, when the forefinger points in the direction of the field or flux, the middle finger will point in the direction of
(*a*) current in the conductor
(*b*) movement of conductor
(*c*) resultant force on conductor
(*d*) none of the above.

35. If the field of a D.C. shunt motor gets opened while motor is running
(*a*) the speed of motor will be reduced
(*b*) the armature current will reduce
(*c*) the motor will attain dangerously high speed
(*d*) the motor will continue to run at constant speed.

36. Starters are used with D.C. motors because
(*a*) these motors have high starting torque
(*b*) these motors are not self-starting
(*c*) back e.m.f. of these motors is zero initially
(*d*) to restrict armature current as there is no back e.m.f. while staring.

37. Which of the following D.C. motors has self-load properties ?
(*a*) Series motor
(*b*) Shunt motor
(*c*) Cumulatively compound motor
(*d*) Differentially compound motor.

38. In D.C. shunt motors as load is reduced
(*a*) the speed will increase abruptly
(*b*) the speed will increase in proportion to reduction in load
(*c*) the speed will remain almost constant
(*d*) the speed will reduce.

39. What will happen if the back e.m.f. of a D.C. motor vanishes
(*a*) the motor will stop
(*b*) the motor will continue to run
(*c*) the armature may burn
(*d*) the motor will run noisy.

40. In case of D.C. shunt motors the speed is dependent on back e.m.f. only because
(*a*) back e.m.f. is equal to armature drop
(*b*) armature drop is negligible
(*c*) flux is proportional to armature current
(*d*) flux is practically constant in D.C. shunt motors.

41. In a D.C. shunt motor, under the conditions of maximum power, the current in the armature will be
(*a*) almost negligible
(*b*) rated full-load current
(*c*) less than full-load current
(*d*) more than full-load current.

42. Which D.C. motor will have least percentage increase of input current, for the same percentage increase in the torque ?
(*a*) Shunt motor
(*b*) Series motor
(*c*) Cumulatively compound motor
(*d*) Separately excited motor.

43. These days D.C. motors are widely used in
(*a*) pumping sets
(*b*) air compressors
(*c*) electric traction
(*d*) machine shops.

44. By looking at which part of the motor, it can be easily confirmed that a particular motor is D.C. motor ?
(*a*) frame
(*b*) shaft
(*c*) commutator
(*d*) stator.

45. In which of the following applications D.C. series motor is invariably used ?
(*a*) Starter for a car
(*b*) Drive for a water pump
(*c*) Fan motor
(*d*) Motor operation in A.C. or D.C.

46. In D.C. machines fractional pitch winding is used
(*a*) to improve cooling (*b*) to reduce copper losses
(*c*) to increase the generated e.m.f. (*d*) to reduce the sparking.

47. A three point starter is considered as suitable for
(*a*) shunt motors (*b*) shunt as well as compound motors
(*c*) shunt, compound and series motors (*d*) all D.C. motors.

48. Small D.C. motors up to 5 H.P. usually have
(*a*) 2 poles (*b*) 4 poles
(*c*) 6 poles (*d*) 8 poles.

49. In case the conditions for maximum power for a D.C. motor are established, the efficiency of the motor will be
(*a*) 100% (*b*) around 90%
(*c*) anywhere between 75% and 90% (*d*) less than 50%.

50. A shearing machine has cyclic load consisting of intermittent light and heavy loads. Which of the following D.C. motors will be suitable for this purpose ?
(*a*) Series motor (*b*) Shunt motor
(*c*) Cumulative compound motor (*d*) Differentially compound motor.

51. The ratio of starting torque to full-load torque is least in case of
(*a*) series motor (*b*) shunt motor
(*c*) compound motors (*d*) none of the above.

52. In D.C. motor which of the following can sustain the maximum temperature rise ?
(*a*) Slip rings (*b*) Commutator
(*c*) Field winding (*d*) Armature winding.

53. Which of the following law/rule can be used to determine the direction of rotation of D.C. motor ?
(*a*) Lenz's law (*b*) Faraday's law
(*c*) Coulomb's law (*d*) Fleming's left-hand rule.

54. Which of the following loads normally needs staring torque more than the rated torque ?
(*a*) Blowers (*b*) Conveyors
(*c*) Air compressors (*d*) Centrifugal pumps.

55. The starting resistance of a D.C. motor is generally
(*a*) low (*b*) around 500 Ω
(*c*) 1000 Ω (*d*) infinitely large.

56. In case of conductively compensated D.C. series motors, the compensating winding is provided
(*a*) as separately wound unit (*b*) in parallel with armature winding
(*c*) in series with armature winding (*d*) in parallel with field winding.

57. Sparking at the commutator of a D.C. motor may result in
(*a*) damage to commutator segments (*b*) damage to commutator insulation
(*c*) increased power consumption (*d*) all of the above.

58. Which of the following motors is preferred for operation in highly explosive atmosphere ?
(*a*) Series motor (*b*) Shunt motor
(*c*) Air motor (*d*) Battery operated motor.

59. If the supply voltage for a D.C. motor is increased, which of the following will decrease ?
(*a*) Staring torque (*b*) Operating speed
(*c*) Full-load current (*d*) All of the above.

60. When the speed of a D.C. motor increases
(*a*) back e.m.f. increases and current drawn decreases
(*b*) back e.m.f. as well as current drawn both increase
(*c*) back e.m.f. as well as current drawn both decrease
(*d*) back e.m.f. decreases and current drawn increases.

61. As compared to an induction motor, the air gap in a D.C. motor is
(*a*) less than 50% (*b*) between 50% and 90%
(*c*) same (*d*) more.

62. Field winding of a D.C. series motor is usually provided with thick wire
(*a*) to provide large flux (*b*) to reduce the use of insulating materials
(*c*) as it carries large load current (*d*) in order to reduce eddy current.

63. Which one of the following is not the function of pole shoes in a D.C. machine ?
(*a*) To reduce eddy current loss (*b*) To support the field coils
(*c*) To spread out flux for better uniformity (*d*) To reduce the reluctance of the magnetic path.

64. The mechanical power developed by a shunt motor will be maximum when the ratio of back e.m.f. to applied voltage is
(*a*) 4.0 (*b*) 2.0
(*c*) 1.0 (*d*) 0.5.

65. The condition for maximum power in case of D.C. motor is
(*a*) back e.m.f. = 2 × supply voltage (*b*) back e.m.f. = $\frac{1}{2}$ × supply voltage
(*c*) supply voltage = $\frac{1}{2}$ × back e.m.f. (*d*) supply voltage = back e.m.f.

66. For which of the following applications a D.C. motor is preferred over an A.C. motor ?
(*a*) Low speed operation (*b*) High speed operation
(*c*) Variable speed operation (*d*) Fixed speed operation.

67. What will happen in case 220 V D.C. series motor is connected to 220 V A.C. supply ?
(*a*) The armature winding of motor will burn
(*b*) The motor will vibrate violently
(*c*) The motor will run with less efficiency and more sparking
(*d*) The motor will not run.

68. In D.C. machines the residual magnetism is of the order of
(*a*) 2 to 3 per cent (*b*) 10 to 15 per cent
(*c*) 20 to 25 per cent (*d*) 50 to 75 per cent.

69. If T_a be the torque and I_a the armature current for a D.C. motor, then which of the following relation is valid before saturation ?
(*a*) $T_a \propto I_a$ (*b*) $T_a \propto \frac{I}{I_a}$
(*c*) $T_a \propto I_a^2$ (*d*) $T_a \propto \frac{I}{(I_a)^2}$.

70. Which D.C. motor is generally preferred for cranes and hoists ?
(*a*) Series motor (*b*) Shunt motor
(*c*) Cumulatively compound motor (*d*) Differentially compound motor.

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*d*)	**4.** (*d*)	**5.** (*b*)	**6.** (*a*)	**7.** (*c*)
8. (*b*)	**9.** (*b*)	**10.** (*b*)	**11.** (*d*)	**12.** (*a*)	**13.** (*d*)	**14.** (*b*)
15. (*b*)	**16.** (*a*)	**17.** (*a*)	**18.** (*c*)	**19.** (*b*)	**20.** (*d*)	**21.** (*c*)
22. (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*b*)	**27.** (*c*)	**28.** (*a*)
29. (*a*)	**30.** (*b*)	**31.** (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*a*)	**35.** (*c*)
36. (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*c*)	**40.** (*d*)	**41.** (*d*)	**42.** (*b*)
43. (*c*)	**44.** (*c*)	**45.** (*a*)	**46.** (*d*)	**47.** (*b*)	**48.** (*a*)	**49.** (*d*)
50. (*c*)	**51.** (*b*)	**52.** (*c*)	**53.** (*d*)	**54.** (*b*)	**55.** (*a*)	**56.** (*c*)
57. (*d*)	**58.** (*c*)	**59.** (*c*)	**60.** (*a*)	**61.** (*d*)	**62.** (*c*)	**63.** (*a*)
64. (*d*)	**65.** (*b*)	**66.** (*c*)	**67.** (*c*)	**68.** (*a*)	**69.** (*c*)	**70.** (*a*).

THEORETICAL QUESTIONS

D.C. GENERATOR

1. What is a basic type of D.C. machine ?
2. Explain the principle on which a generator works.
3. Explain the construction and working of an elementary generator.
4. Describe briefly various parts of a D.C. machine.
5. Why is a commutator and brush arrangement necessary for operation of a D.C. machine ?
6. Name the main parts of a D.C. machine and state the materials of which each part is made.
7. Enumerate all the parts of a D.C. machine and indicate their functions.
8. Give the materials and functions of the following parts of a D.C. machine :
   (*i*) Field poles (*ii*) Yoke
   (*iii*) Commutator (*iv*) Commutating poles
   (*v*) Armature.
9. Give the constructional features and working principle of a D.C. generator. Draw a cross-sectional view of a 4 pole D.C. generator and label there on all parts. Also sketch the shapes of :
   (*i*) Commutator segment with riser, and
   (*ii*) Armature stamping.
10. Give the advantages and uses of lap and wave windings.
11. Explain the following terms as applied to a D.C. armature winding.
   (*i*) Front pitch (*ii*) Back pitch
   (*iii*) Pole pitch (*iv*) Commutator pitch.
12. Draw up the winding table for a 4 pole, wave connected armature having 30 coil sides and give a developed diagram of the winding showing the polarity and position of the brushes, the main poles and the direction of motion of the armature for a D.C. motor.
13. Draw the developed winding diagram of lap winding for 6 poles, 18 slots with two coil sides/slot, double layer showing therein position of poles, direction of motion, direction of generated e.m.f. and position of brushes.
14. Derive the e.m.f. equation of a D.C. generator.
15. Deduce an expression for the voltage induced in a D.C. generator.
16. Write a short note on equiliser rings.
17. How are ratings of electrical machines expressed ?
18. Draw the magnetic circuit of a 4 pole D.C. machine.
19. Explain different methods of excitation of D.C. generators with suitable diagrams.
20. How are D.C. generators classified ?
21. What is the difference between a separately excited and a self-excited generator ?
22. Sketch the following types of D.C. generators :
   (*i*) Shunt (*ii*) Series
   (*iii*) Compound.
   State with reason(s) where each is used ?
23. What is the difference between the short-shunt and long shunt compound generators ?
24. With the help of a neat diagram show power division in a D.C. generator.
25. Answer the following questions briefly :
   (*i*) State the principle on which generators operate.
   (*ii*) Name the materials of which the following parts of a D.C. machine are made and why ?
   (*a*) Frame (*b*) Armature
   (*c*) Bushes.
   (*iii*) Why are field coils provided in a D.C. generator ?
   (*iv*) What is the function of an armature in a D.C. generator ?
   (*v*) Of what material are the laminations of an armature made up of ?
   (*vi*) What is the function of a commutator ?
   (*vii*) What purpose is served by brushes in a D.C. machine ?

D.C. MOTOR

26. What is the working principle of a D.C. motor ?

27. "In every D.C. generator motor action occurs and in every D.C. motor a generator action occurs". Explain.

28. What is torque ? What is the source of the torque force in a D.C. motor ?

29. What is back e.m.f. or counter voltage ?

30. What is the effective voltage across a D.C. motor armature ?

31. What is the meaning of armature power ?

32. Is electrical power related to mechanical power ?

33. Explain the function of commutator in a D.C. motor. In what respects is the commutation process in D.C. motors different from that in generators ?

34. What is mean by speed regulation ?

35. What is meant by the term 'shunt motor' ?

36. What is meant by the term 'series motor' ?

37. What is the dominant speed characteristic of a shunt motor ?

38. Explain speed-current and torque-current characteristics of a series motor.

39. What is meant by a compound motor ?

40. What conditions require the use of a compound motor ?

41. How is D.C. motor reversed ?

42. Why is a starter necessary for a motor ? Give the diagram and explain the working of a three-point starter for a shunt motor.

43. What are the functions of 'no-volt release' and 'overload release' in a starter ? Discuss the operation of these two features in a shunt motor starter.

44. Give the reasons for the following :

(*i*) A series motor should not be connected to a load through a belt.

(*ii*) A series motor develops a high starting torque.

(*iii*) A differential compound motor is very rarely used.

(*iv*) A shunt motor runs at almost constant speed irrespective of load current.

45. Derive an expression for the torque of a D.C. motor.

46. Differentiate between the generator action and motor action of a D.C. machine.

47. How may the direction of rotation of a D.C. shunt motor be reversed ? What is the effect of reversing the line terminals ?

48. Develop the general expression for the speed of a motor in terms of supply voltage, armature resistance and flux per pole.

49. Explain the 'armature reaction', in D.C. shunt motors, indicating also few remedies to its adverse effects.

50. Explain in detail how the shunt motor behaves as a constant speed motor and the series motor as a variable speed motor.

51. Draw the characteristic curves of D.C. shunt and series motors. Use these curves to explain the applications for which these motors are used.

52. Discuss the applications of shunt, series and compound motors.

53. State the types of loads for which shunt, series and compound D.C. motors are suitable. Give reasons in support of your answer.

54. Why is a starter necessary for D.C. motor ? Explain the working of a three-point starter with the help of a neat diagram.

55. What is the difference between three-point and four-point starters ? What are the additional features incorporated in a four-point starter ?

56. Explain what happens, giving reasons thereof, in the following cases :

(*i*) In a D.C. shunt motor armature terminals are reversed.

(*ii*) In a D.C. shunt motor field terminals are reversed.

(*iii*) The line terminals to a D.C. motor interchanged.

(*iv*) A D.C. shunt motor is directly connected to the supply mains.

(*v*) A D.C. motor is stopped by throwing back the starting arm.

57. Explain with neat diagrams construction and working of a series motor starter.

58. How does D.C. motor automatically adjust input to match the mechanical load on the motor ?

EXERCISE

D.C. GENERATOR

1. An eight-pole lap connected armature runs at 750 r.p.m. It has 800 conductors and flux/pole of 60 mWb. Calculate the e.m.f. generated. [**Ans.** 600 volts]

2. An 8-pole lap connected generator has a useful flux/pole of 0.05 Wb. If the no load voltage at 400 r.p.m. is 300 V find the conductors on the armature periphery. [**Ans.** 900]

3. An eight-pole wave connected armature has 300 conductors and runs at 400 r.p.m. Determine the useful flux/pole if the electromotive force generated on open circuit is 500 V. [**Ans.** 6.25×10^{-2} Wb]

4. A 6-pole wave connected D.C. generator has 52 slots and each slot has 20 conductors. Find the speed of the generator. The generated e.m.f. is 240 V and the flux per pole is 5 mWb. [**Ans.** 923 r.p.m.]

5. An 8-pole wave connected D.C. generator has 900 armature conductors and a flux/pole of 0.04 Wb. At what speed it must be driven to generate 500 V ? [**Ans.** 208.3 r.p.m.]

6. The lap wound armature of a 4-pole generator has 51 slots. Each slot contains 20 conductors. What will be the e.m.f. generated in machine when driven at 1500 r.p.m. if useful flux per pole is 0.01 Wb ? [**Ans.** 255 V]

7. An 8-pole lap wound generator armature has 960 conductors, a flux of 40 mWb and a speed of 400 r.p.m. Calculate the e.m.f. generated on open circuit. If the same armature is wave wound, at what speed must it be driven to generate 400 volts ? [**Ans.** 256 V, 156.25 r.p.m.]

8. An 8-pole D.C. shunt generator with 788 wave connected conductors and running at 500 r.p.m. supplies a load of 12.5 Ω resistance at a terminal voltage of 250 V. The armature resistance is 0.24 Ω and the field resistance is 250 Ω. Find :

(*i*) Armature current (*ii*) Induced e.m.f.

(*iii*) Flux/pole. [**Ans.** (*i*) 21 A, (*ii*) 255.04 V, (*iii*) 9.83 mWb]

9. A 4-pole lap wound D.C. shunt generator has a useful flux/pole 0.07 Wb. The armature winding consists of 220 turns, each turn having a resistance of 0.004 Ω. Calculate the terminal voltage when running at 900 r.p.m. if the armature current is 50 A. [**Ans.** 459.25 V]

10. A shunt generator has a generated voltage on open circuit of 127 V. When the machine is on load, terminal voltage is 120 V. Find the load current if the field resistance be 15 Ω and the armature resistance 0.02 Ω. Neglect armature reaction. [**Ans.** 342 A]

11. A short shunt compound generator has armature, series field and shunt field resistances of 0.08 Ω, 0.04 Ω and 100 Ω respectively. It supplies 125 lamps each rated at 250 V, 40 W. Find the generated e.m.f. and the armature current. Drop under each brush may be taken as 1 V. [**Ans.** 22.508 A, 254.6 V]

D.C. MOTOR

12. A 250 V D.C. shunt motor takes 41 A at full-load. Find the back e.m.f. on full-load if the resistances of motor armature and shunt field windings are 0.1 ohm and 250 ohms respectively. [**Ans.** 246 V]

13. A 230 V motor has an armature circuit resistance of 0.6 ohm. If the full-load armature current is 30 A and the no-load armature current is 4 A, find the change in back e.m.f. from no-load to full-load. [**Ans.** 15.6 V]

14. A six-pole lap-connected 250 V shunt motor has 396 armature conductors. It takes 30 A on full-load. The flux per pole is 0.04 weber. The armature and field resistances are 0.1 ohm and 200 ohms respectively. Contact drop per brush = 1 V.

Determine the speed on full-load. [**Ans.** 928.5 r.p.m.]

15. A four-pole, 440 V shunt motor takes 8 A on no-load, the no-load speed being 750 r.p.m. It has a shunt field current of 1.5 A. Calculate the full-load speed of the motor if it takes 100 A at full-load. Armature resistance = 0.25 ohm. Contact drop per brush = 1 V. Armature reaction weakens the field by 5 per cent on full-load. [**Ans.** 748 r.p.m.]

16. A four-pole D.C. shunt motor working on 250 V takes a current of 2 A when running at 1,000 r.p.m. What will be its back e.m.f., speed and the percentage speed drop if the motor takes 51 A at a certain load ? Armature resistance = 0.2 ohm and shunt field resistance = 250 ohms.

[**Ans.** 240 V, 963 r.p.m., 3.84%]

17. A four-pole lap wound 500 V shunt motor takes a full-load armature current of 50 A, the full-load speed being 800 r.p.m. There are 440 armature conductors and armature resistance is 0.3 ohm, contact drop per brush = 1 V.

Find the useful flux per pole. [**Ans.** 0.08233 weber]

18. A 500 V, 6-pole lap connected shunt motor has a no-load input current of 16 A and a shunt field current of 10 A. At full-load it takes a current of 160 A. If armature resistance = 0.1 ohm, flux/pole at no-load = 0.06 weber, number of armature conductors = 744 and contact drop/brush = 1 V, calculate :

(*i*) No-load speed (*ii*) Full-load speed

(*iii*) Speed regulation.

Armature reaction weakens the field by 2 per cent. [**Ans.** (*i*) 669 r.p.m., (*ii*) 662.7 r.p.m., (*iii*) 0.9504%]

19. 250 V shunt motor takes a line current of 50 A and runs at 600 r.p.m. Its armature and field resistances are 0.4 ohm and 125 ohms respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate :

(*i*) No-load speed if the no-load line current is 5 A

(*ii*) Percentage reduction in the flux per pole in order that the speed may be 800 r.p.m. when the armature current is 40 A. [**Ans.** (*i*) 647 r.p.m., (*ii*) 23.7%]

20. A 250 V shunt motor on no-load runs at 1000 r.p.m. and takes 5 A. The total armature and shunt field resistances are respectively 0.2 ohm and 250 ohms. Calculate the speed when loaded and taking a current of 50 A, if armature reaction weakens the field by 3%. [**Ans.** 994 r.p.m.]

21. A D.C. series motor of resistance 1 ohm between terminals runs at 800 r.p.m. at 200 V with a current of 15 A. Find the speed at which it will run when connected in series with a 5 ohms resistance and taking the same current. [**Ans.** 476 r.p.m.]

22. A 250 V shunt motor takes 70 A when running at 1000 r.p.m. It has an armature resistance of 0.1 ohm. Determine the speed and armature current if the magnetic flux is weakened by 30 per cent. Contact drop per brush = 1 V.

Total torque developed remains constant. [**Ans.** 1411 r.p.m., 100 A]

23. A 500 V shunt motor takes a no-load current of 10 A at 500 V. It takes 100 A at 500 V on full-load. Armature and shunt field resistances are 0.2 ohm and 200 ohms respectively. Contact drop/brush = 1 V. If the no-load speed is 800 r.p.m. find the full-load speed.

Armature reaction weakens the field by 3 per cent on full-load. [**Ans.** 795 r.p.m.]

24. A 250 V shunt motor on no-load runs at 1000 r.p.m. and takes 5 A. The total armature and shunt field resistances are respectively 0.2 ohm and 250 ohms. Calculate the speed when loaded and taking a current of 50 A, if armature reaction weakens the field by 3 per cent. [**Ans.** 994 r.p.m.]

25. A 250 V shunt motor takes a current of 52 A and runs at 1000 r.p.m. at full-load. Armature and shunt field resistances are 0.3 ohm and 125 ohms respectively. Determine :

(*i*) Speed at full-load torque (*ii*) Speed at double full-load torque

(*iii*) Stalling torque in terms of the full-load torque.

Assume that the flux remains constant. [**Ans.** 681 r.p.m., 298 r.p.m., 2.78 times]

26. A 4-pole, 250 V lap connected shunt motor has armature and field resistances of 0.05 ohm and 100 ohms respectively. There are 492 armature conductors and the useful flux per pole is 0.03 weber. It takes a current of 7.5 A on no-load and a current of 120 A on full-load. Determine :

(*i*) No-load speed (*ii*) Full-load speed

(*iii*) Speed regulation.

Neglect drop/brush = 1 V. Armature reaction may be neglected.

[**Ans.** (*i*) 1007 r.p.m., (*ii*) 984 r.p.m., (*iii*) 2.337%]

27. A 500 V shunt motor has an armature resistance of 0.2 ohm. It takes an armature current of 60 A at 750 r.p.m. Find the speed and armature current if the flux is increased to 150% of its initial value. Assume the torque to be constant throughout. Contact drop/brush = 1 V. [**Ans.** 504.2 r.p.m., 40 A]

28. Determine the value of torque in Nm established by the armature of a 4-pole motor having 774 conductors, two paths in parallel , 24 m Wb flux per pole when the total armature current is 50 amperes. [**Ans.** 295.3 Nm]

29. Calculate torque on an armature carrying a total of 25000 ampere-conductors, the diameter of the core being 0.9144 m, the length 30.5 cm, the polar arc 70% and the flux density in the air gap 0.6 tesla. [**Ans.** 1464 Nm]

30. A 220 V series motor in which the total armature and field resistance is 0.1 ohm is working with unsaturated field, taking 100 A and running at 800 r.p.m. Calculate at what speed the motor will run when developing half the torque ? [**Ans.** 1147 r.p.m.]

31. A 4-pole wave wound shunt motor, running at 1000 r.p.m, takes 100 A at 250 V. There are 252 armature conductors. Useful flux per pole = 0.04 Wb. The shunt field current of motor is 5 A. Determine :

(*i*) Total torque developed

(*ii*) Actual torque if the friction and iron loss is 2 kW. [**Ans.** (*i*) 304.7 Nm, (*ii*) 285.61 Nm]

32. A 4-pole 250 V D.C. shunt motor has lap connected 960 conductors. The flux per pole is 20 mWb. Determine the torque developed by the armature and the useful torque in Nm when current drawn by the motor is 32 A. The armature resistance is 0.1 ohm and shunt field resistance is 125 ohms. The rotational losses of the machine amount to 825 watts. [**Ans.** 91.6 Nm ; 81.4 Nm]

33. A shunt motor operating on 230 V takes an armature current of 6 A at no-load and runs at 1200 r.p.m. The armature resistance is 0.25 ohm. Determine the speed and electromagnetic torque when the armature takes 36 A with the same flux. [**Ans.** 1161 r.p.m, 65.44 Nm]

34. A 44.76 kW, 250 V, 4-pole lap-connected D.C. shunt motor has 30 slots with 12 conductors/slot. The armature and shunt field resistances are 0.06 ohm and 100 ohms respectively. The flux/pole is 0.03 Wb. If the full-load efficiency is 88%, find at full-load :

(*i*) The speed ;

(*ii*) Useful torque at shaft. [**Ans.** (*i*) 1323 r.p.m., (*ii*) 323.3 Nm]

35. A 4-pole, lap-wound, 250 V shunt motor has the following particulars : No. of armature conductors = 720 ; armature resistance = 0.311 ohm, useful flux/pole = 0.04 Wb. If the total torque developed by the motor is 147.15 Nm, find the armature current taken and the speed. [**Ans.** 32.12 A ; 500 r.p.m.]

36. The input to a 220 V D.C. shunt motor is 11 kW. The other particulars of the motor are : no-load current = 5 A ; no-load speed = 1150 r.p.m. ; armature resistance = 0.5 ohm ; shunt field resistance = 110 ohms. Calculate :

(*i*) The torque developed (*ii*) The efficiency

(*iii*) The speed at this load. [**Ans.** 87.1 Nm ; 79.6% ; 1031 r.p.m.]

6

Single-Phase Transformer

1. General aspects. 2. Basic definitions. 3. Working principle of a transformer. 4. Transformer ratings. 5. Kinds of transformers. 6. Transformer construction 7. Transformer windings, terminals, tappings and bushings 8. Transformer cooling 9. Single phase transformer : Elementary theory of an ideal transformer—E.m.f. equation of a transformer—Voltage transformation ratio—Transformer with losses but no magnetic leakage—Resistance and magnetic leakage—Transformer with resistance and leakage reactance—Equivalent resistance and reactance—Voltage drop in a transformer—Equivalent circuit—Transformer tests—Regulation of a transformer—Percentage resistance and reactance—Transformer losses—Transformer efficiency—All-day efficiency—Transformer noise—Auto-transformer—Polarity of transformers. —*Highlights—Objective Type Questions—Theoretical Questions—Exercise.*

1. GENERAL ASPECTS

Although the transformer is not classified as an electric machine, the principles of its operation are fundamental for the induction motor and synchronous machines. Since A.C. electric machines are normally built for low frequencies only the low-frequency power transformer will be considered in this text.

Function. *The function of a transformer, as the name implies, is to transform alternating current energy from one voltage into another voltage*. The transformer has *no rotating parts*, hence it is often called a *static transformer.*

When energy is transformed into a higher voltage the transformer is called a *step-up transformer* but when the case is otherwise it is called a *step-down transformer*. Most power transformers operate at constant voltage, *i.e.,* if the power varies the current varies while the voltage remains fairly constant.

Applications. A transformer performs many important functions in prominent areas of electrical engineering.

- In *electrical power engineering* the transformer makes it possible to convert electric power from a generated voltage of about 11 kV (as determined by generator design limitations) to higher values of 132 kV, 220 kV, 400 kV, 500 kV and 765 kV thus permitting transmission of huge amounts of power along long distances to appropriate distribution points at tremendous savings in the cost of transmission lines as well as in power losses.
- At *distribution points* transformers are used to reduce these high voltages to a safe level of 400/230 volts for use in homes, offices etc.
- In *electric communication circuits* transformers are used for a variety of purposes *e.g.*, as an impedance transformation device to allow maximum transfer of power from the input circuit to the output device.

- In *radio and television circuits* input transformers, interstage transformers and output transformers are widely used.
- Transformers are also used in *telephone circuits, instrumentation circuits* and *control circuits*.

2. BASIC DEFINITIONS

- A *transformer* is a *static electromagnetic device designed for the transformation of the (primary) alternating current system into another (secondary) one of the same frequency with other characteristics, in particulars, other voltage and current*.
- As a rule a transformer consists of a core assembled of sheet transformer steel and two or several windings coupled *electromagnetically*, and in the case of *autotransformer, also electrically*.
- A transformer with two windings is called *double-wound transformer* ; a transformer with three or more windings is termed a *triple wound* or *multi-winding* one.
- According to the kind of current, transformers are distinguished as single-phase, three-phase and poly-phase ones. A *poly-phase transformer winding is a group of all phase windings of the same voltage*, connected to each other in a definite way.
- **Primary and secondary windings.** The transformer winding to which the energy of the alternating current is delivered is called the *primary winding* ; the other winding from which energy is received is called the *secondary winding*.
- In accordance with the names of the windings, all quantities pertaining to the primary winding as, for example, power, current, resistance etc., are also primary, and those pertaining to the secondary winding secondary.
- **h.v. and l.v. windings.** The winding connected to the circuit with the higher voltage is called the *high-voltage winding* (h.v.), the winding connected to the circuit with the lower voltage is called the *low-voltage winding*. (l.v.). If the secondary voltage is *less* than the primary one, the transformer is called a *step-down transformer* and if *more-a step-up transformer*.
- *A tapped transformer* is one whose windings are fitted with special taps for changing its voltage or current ratio.
- **Oil and dry transformers.** To avoid the deterimental effect of the air on the winding insulation and improve the cooling conditions of the transformer its core together with the windings assembled on it is immersed in a tank filled with transformer oil. Such transformers are called **oil transformers**. Transformers not immersed in oil are called **dry transformers.**

3. WORKING PRINCIPLE OF A TRANSFORMER

A transformer operates on the principle of *mutual inductance,* between two (and sometimes more) inductively coupled coils. It consists of two windings in close proximity as shown in Fig. 1. *The two windings are coupled by magnetic induction.* (There is no conductive connection between the windings). One of the windings called *primary* is energised by a sinusoidal voltage. The second winding, called *secondary* feeds the load. The alternating current in the primary winding sets up an alternating flux (ϕ) in the core. The secondary winding is linked by most of this flux and e.m.fs are induced in the two windings. The e.m.f. induced in the secondary winding drives a current through the load connected to the winding. Energy is transferred from the primary circuit to the secondary circuit through the medium of the magnetic field.

In brief, a transformer is a *device* that :

(*i*) *transfers electric power from one circuit to another* ;

(*ii*) *it does so without change of frequency ; and*

(*iii*) *it accomplishes this by electromagnetic induction (or mutual inductance).*

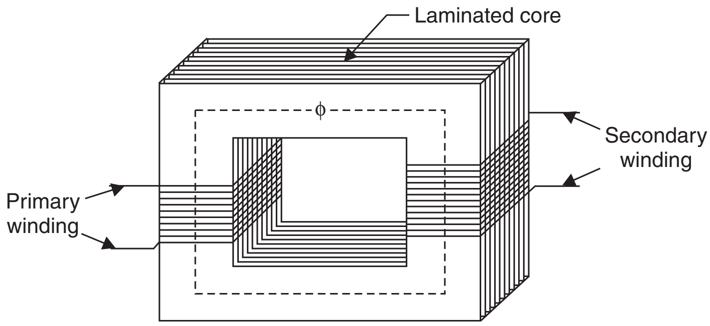


Fig. 1. Two winding transformer.

4. TRANSFORMER RATINGS

The rated quantities of a transformer, its power, voltage, frequency, etc., are given in Manufacturer's name plate, which should always be arranged so as to be accessible. But the term 'rated' can also be applied to quantities not indicated on the name plate, but relating to the rated duty, as for example, the rated efficiency, rated temperature conditions of the cooling medium, etc. :

- *The rated duty* of a transformer is determined by the quantities given in the name plate.
- The *rated power* of the transformer is the power at the secondary terminals, indicated in the name plate and expressed in kVA.
- *The rated primary voltage* is the voltage indicated in the transformer name plate ; if the primary is provided with taps, the rated tapped voltage is specially noted.
- *The rated secondary voltage* is the voltage across the transformer *secondary terminals at no-load* and with the rated voltage across the primary terminals ; if the secondary winding has taps, then their rated voltage is specially indicated.
- *The rated currents of the transformer*, primary and secondary, are the currents indicated in the name plate of the transformer and calculated by using the corresponding rated values of power and voltage.

5. KINDS OF TRANSFORMERS

The following kinds of transformers are the most important ones :

1. **Power transformers.** *For the transmission and distribution of electric power.*

2. **Auto-transformers.** *For converting voltages within relatively small limits to connect power systems of different voltages, to start A.C. motors etc.*

3. **Transformer for feed installations with static convertors.** (Mercury arc rectifiers, ignitrons, semi-conductor valves, etc.) *When converting A.C. into D.C.* (rectifying) *and convering D.C. into A.C. (inverting).*

4. **Testing transformers.** *For conducting tests at high and ultra-high voltages.*

5. **Power transformers for special applications.** Furnace, welding etc.

6. **Radio-transformers.** *It is used in radio engineering etc.*

Note. *Distribution transformers* should be designed to have maximum efficiency at a load much *lower than full-load* (*about 50 per cent*).

Power transformers should be designed to have maximum efficiency *at or near full-load.*

6. TRANSFORMER CONSTRUCTION

All transformers have the following essential elements :

1. Two or more **electrical windings** insulated from each other and from the core (except in auto-transformers).

2. A **core,** which in case of a single-phase distribution transformers usually comprises *cold-rolled silicon-steel strip* instead of an assembly of punched silicon-steel laminations such as are used in the larger power-transformer cores. The *flux path in the assembled core is parallel to the directions of steel's grain or 'orientation'.* This results in a *reduction in core losses* for a given flux density and frequency, or *it permits the use of higher core densities* and *reduced size of transformers for given core losses.*

Other necessary parts are :

- A *suitable container* for the assembled core and windings.
- A *suitable medium* for insulating the core and its windings from each other and from the container.
- Suitable *bushings* for insulating and bringing the terminals of the windings out of the case.

The two basic types of transformer construction are :

1. *The core type.*
2. *The shell type.*

The above two types differ in their relative arrangements of copper conductors and the iron cores. In the '*core type*', the *copper virtually surrounds the iron core*, while *in the 'shell type'*, *the iron surrounds the copper winding*.

6.1. Core Type Transformer

The completed magnetic circuit of the core-type transformer is in the shape of the hollow rectangle, exactly as shown in Fig. 2 in which I_0 is the no-load current and ϕ is the flux produced by it. N_1 and N_2 are the number of turns on the primary and secondary sides respectively.

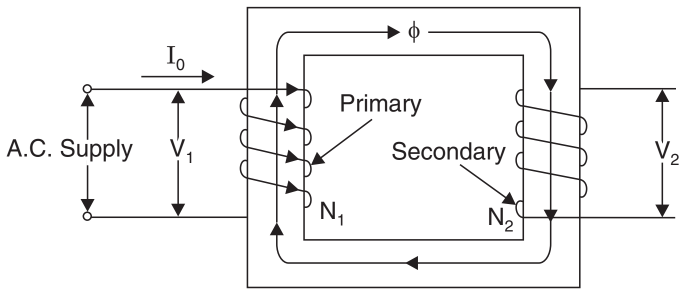


Fig. 2. Magnetic circuit of a core-type transformer.

The core is made up of *silicon-steel laminations* which are, either rectangular or L-shaped. With the coils wound on two legs the appearance is that of Fig. 3. If the two coils shown were the respective high and low-side coils as in Fig. 3, the *leakage reactance would be much too great.* In order to provide maximum *linkage* between windings, the group on *each leg is made up of both high-tension and low-tension coils.* This may be seen in Fig. 4, where a cross-sectional cut is taken across the legs of the core. By placing the high-voltage winding around the low-voltage winding, only one layer of high-voltage insulation is required, that between the two coils. *If the high-voltage coils were adjacent to the core, an additional high-voltage insulation layer would be necessary between the coils and the iron core.*

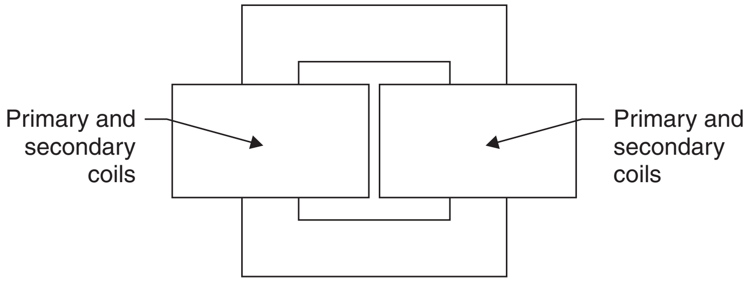


Fig. 3. Core-type transformer.

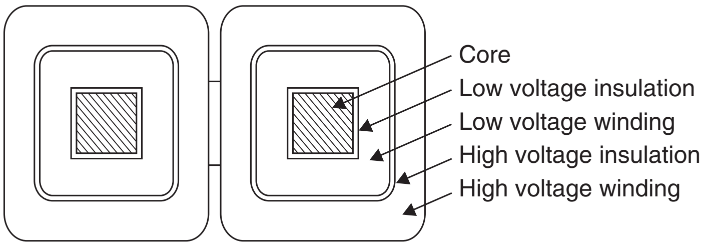


Fig. 4. Cross-section of a core-type transformer.

Fig. 5 shows the coils and laminations of a core-type transformer with a cruciform core and circular coils.

- Fig. 6 shows *the different types of cores used in core type transformers.*

Rectangular cores [Fig. 6 (*a*)] with rectangular cylindrical coils can be used for small size core-type transformers. For large size transformers it becomes wasteful to use rectangular cylindrical coils and so circular cylindrical coils are preferred. For such purposes, '*square cores*' may be used as shown in Fig. 6 (*b*) where circles represent the tubular former carrying the coils. Evidently a considerable amount of useful space is still wasted. A common improvement on the square core is to employ a '*cruciform core*' [Fig. 6 (*c*)] which demands, atleast, two sizes of core strips. For *very large transformers*, further core stepping is done as in Fig. 6 (*d*) where atleast three sizes of core plates are necessary. *Core stepping not only gives high space factor but also results in reduced length of the mean turn and the consequent I^2R loss. Three stepped core is the most commonly* used although more steps may be used for very large transformers as shown in Fig. [6 (*e*)].

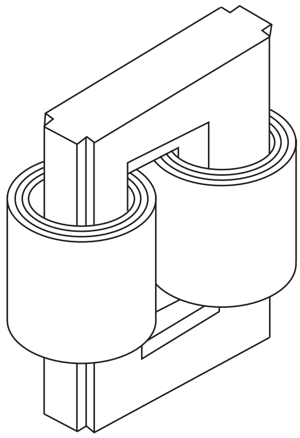


Fig. 5. Coils and laminations of a core-type transformer.

6.2. Shell Type Transformer

In the shell-type construction the iron almost entirely surrounds the copper (Fig. 7). The core is made up of E-shaped or F-shaped laminations which are stacked to give a rectangular figure eight. All the windings are placed on the centre leg, and in order to reduce leakage, each high-side coil is adjacent to a low-side coil. The coils actually occupy the entire space of both windows, are flat or pencake in shape, and are usually constructed of strip copper. Again, *to reduce the amount the high-voltage insulation required, the low-voltage coils are placed adjacent to the iron core.*

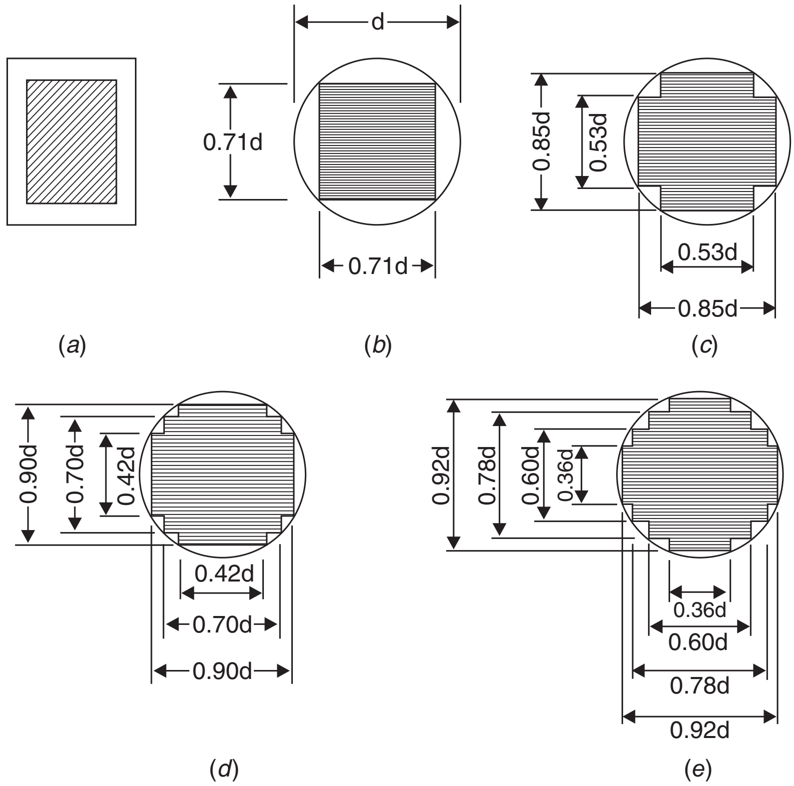


Fig. 6. Various types of cores.

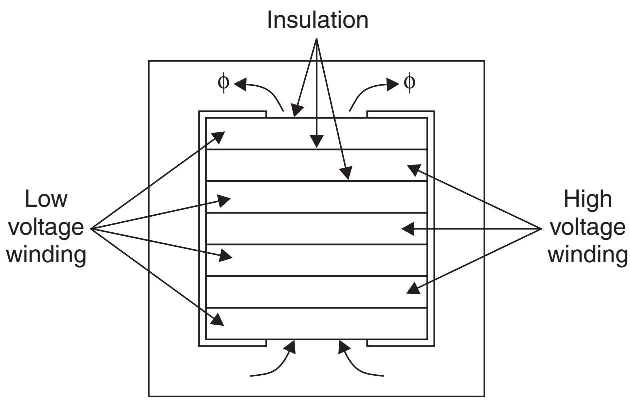


Fig. 7. Shell-type transformer.

Fig. 8 shows the coils and laminations of a typical shell-type transformer.

Choice of Core- or Shell Type Construction. In general, the core-type has a longer mean length of core and a shorter mean length of coil turn. The core type also has a smaller cross-section of iron and so will need a greater number of turns of wire, since, in general, not as high a flux may be reached in the core. However, *core type is better adopted for some high-voltage service since there is more room for insulation*. The *shell type has better provision for mechanically supporting and bracing the coils*. This allows better resistance to the very high mechanical forces that develop during a high-current short circuit.

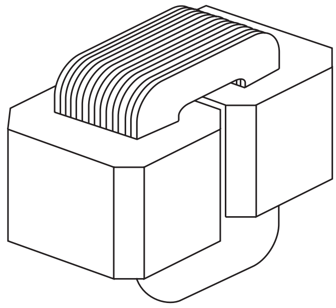


Fig. 8. Coils and laminations of a shell-type transformer.

The choice of core- or shell type construction is usually one of *cost*, for similar characteristics can be obtained with both types.

Both core and shell forms are used, and selection is based upon many factors such as voltage rating, kVA rating, weight, insulation stress, mechanical stress, and heat distribution.

6.3. Spiral Core Transformer

The typical spiral core is shown in Fig. 9. The core is assembled either of a *continuous* strip of transformer steel wound in the form of a circular or elliptical cylinder or of a group of short strips assembled to produce the same elliptical-shaped core. By using this construction the core flux always follows along the grain of the iron. Cold-rolled steel of high silicon content enables the designer to use higher operating flux densities with lower loss per kg. *The higher flux density reduces the weight per kVA.*

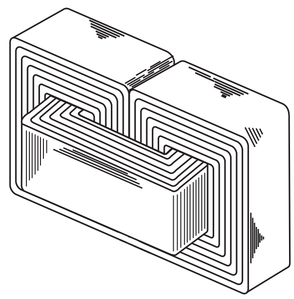


Fig. 9. Spiral-core transformer.

7. TRANSFORMER WINDINGS, TERMINALS, TAPPINGS AND BUSHINGS

7.1. Transformer Windings

The most important requirements of transformer windings are :

1. The winding should be *economical both as regards initial cost*, with a view to the market availability of copper, and the efficiency of the transformer in service.

2. The heating conditions of the windings should *meet standard requirements*, since departure from these requirements towards allowing higher temperature will drastically shorten the service life of the transformer.

3. The winding should be *mechanically stable* in respect to the forces appearing when sudden short circuit of the transformer occur.

4. The winding should have the *necessary electrical strength in respect to over voltages.*

The different *types of windings* are *classified* and briefly discussed below :

1. Concentric windings :

(*i*) Cross-over (*ii*) Helical

(*iii*) Disc.

2. Sandwich windings

1. Concentric windings. Refer Fig. 10. These windings are *used for core type transformers.* Each limb is wound with a group of coils consisting of both primary and secondary turns which may be concentric cylinders. The l.v. winding is placed next to the core and h.v. winding on the outside. But the two windings can be sub-divided, *and interlaced with high tension and low tension section alternately to reduce leakage reactance.* These windings can be further divided as follows :

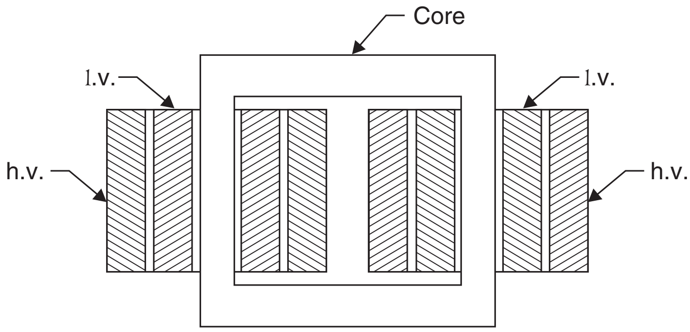


Fig. 10. Concentric coils.

(*i*) **Cross-over windings.** Cross-over windings are used for currents up to 20 A and so they are suitable for h.v. winding of small transformers. The conductors are either cotton covered round wires or strips insulated with paper. Cross-over coils are wound over formers and each coil consists of a number of layers with a number of turns per layer. The complete winding consists of a number of coils connected in series. Two ends of each coil are brought out, one from inside and one from outside. The inside end of a coil is connected to the outside end of the adjacent coil.

(*ii*) **Helical winding.** A helical winding consists of rectangular strips wound in the form of a helix. The strips are wound in parallel radially and each turn occupies the total radial depth of winding.

Helical coils are well suited for *l.v. windings of large transformers*. They can also be used for h.v. windings by putting extra insulation between layers in addition to insulation of conductors.

(*iii*) **Continuous disc winding.** This type of winding consists of a number of flat strips wound spirally from inside (radially) outwards. The conductor is used in such lengths as are sufficient for complete winding or section of winding between tappings. The conductor can either be a single strip or a number of strips in parallel, wound on the flat. This gives a robust construction for each disc. The discs are wound on insulating cylinders spaced from it by strips along the length of cylinder. The discs are separated from each other with press board sectors attached to the vertical strips. The vertical and horizontal spacers provide ducts for free circulation of oil which is in contact with every turn.

2. Sandwich coils. Sandwich coils (Fig. 11) are employed in transformers of *shell type.* Both high and low voltage windings are split into a number of sections. Each high voltage section lies between the low voltage sections.

The *advantage of sandwich coils is that their leakage can be easily controlled and so any desired value of leakage reactance can be had by the division of windings*.

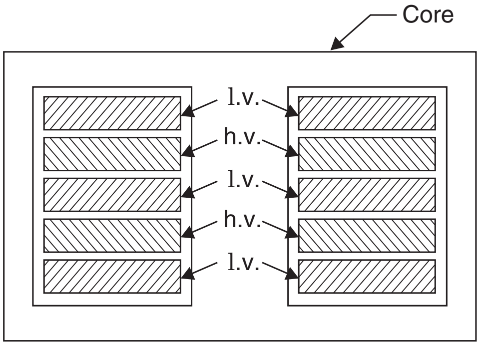


Fig. 11. Sandwich coils.

7.2. Terminals and Leads

The connection to the windings are of insulated copper rods or bars. The shape and size of leads is important in high voltage transformers owing to dielectric stress and corona which are caused at bends and corners. Connections from windings are directly taken to the busbars in the case of air-cooled transformers while they are taken to insulated bushings in the case of oil-cooled transformers.

7.3. Tappings

In a supply network the voltage can be controlled by changing the transformation ratio. This can be done by tapping the winding in order to alter the number of turns. The change in number of turns may be effected when the transformer is out of circuit (known as off load tap changing) or when on load (known as on load tap changing). *The tappings are provided on the high voltage winding because a fine voltage variation is obtained owing to large number of turns. It is difficult to obtain voltage variation within close percentage limits in low voltage winding as there are few turns and voltage per turn is a large percentage of the total voltage.*

In transformers, the tappings can be provided at :

(*i*) phase ends ; and

(*ii*) neutral point or in the middle of the windings.

- The advantage of providing tappings at *phase ends* is that the number of bushing insulators is reduced, this is important where the cover space is limited. Some transformers have reinforced insulation at the phase ends. It is essential that in such cases either the tapping should not be provided at end turns or the reinforcement should be carried beyond the lower tap.
- When the tappings are made at the *neutral point* the insulation between various parts is small. *This arrangement is economical especially in the case of high voltage transformers*.

7.4. Bushings

The bushings are *employed for insulating and bringing out terminals of the winding from the container to the external circuit*. For low-voltage transformers this is accomplished by employing *bushings of porcelain around the conductor at the point of entry.* For high voltages it is necessary to employ bushings of larger sizes. In modern transformers the problem is met by using *large porcelain or composition bushings for voltages as high as 33 kV, above that oil filled* or *condenser type bushings are used.*

8. TRANSFORMER COOLING

8.1. Cooling Methods

The transformers get heated due to iron and copper losses occurring in them. It is necessary to dissipate this heat so that the temperature of the windings is kept below the value at which the insulation begins to deteriorate. The *cooling of transformers is more difficult than that of rotating machines because the rotating machines create a turbulent air flow which assists in removing the heat generated due to losses*. Luckily the losses in transformers are comparatively small. Nevertheless the elaborate cooling arrangements have been devised to deal with the whole range of sizes.

As far as cooling methods are concerned, the transformers are of following two types :

1. Dry type. 2. Oil immersed type.

Dry Type Transformers. Small transformers upto *25 kVA size* are of the *dry type* and have the following cooling arrangements :

(*i*) **Air natural.** In this method the natural circulation of surrounding air is utilized to carry away the heat generated by losses. A sheet metal enclosure protects the winding from mechanical injury.

(*ii*) **Air blast.** Here the transformer is cooled by a continuous blast of cool air forced through the core and windings (Fig. 12). The blast is produced by a fan. The air supply must be filtered to prevent accumulation of dust in ventilating ducts.

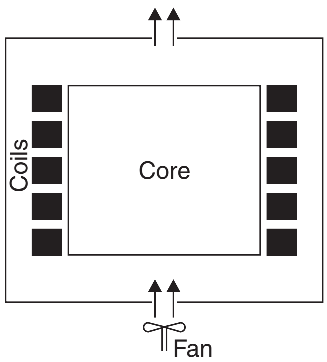


Fig. 12

Oil Immersed Transformers. In general most transformers are of oil immersed types. The *oil provides better insulation than air and it is a better conductor of heat than air*. Mineral oil is used for this purpose.

Oil immersed transformers are classified as follows :

(*i*) **Oil immersed self-cooled transformers.** The transformer is immersed in oil and heat generated in cores and windings is passed to oil by conduction. Oil in contact with heated parts rises and its place is taken by cool oil from the bottom. The natural oil transfers its heat to the tank walls from where heat is taken away by the ambient air. The oil gets cooler and falls to the bottom from where it is dissiptated into the surroundings. The tank surface is the best dissipator of heat but a plain tank will have to be excessively large, if used without any auxiliary means for high rating transformers. As both space and oil are costly, these auxiliary means should not increase the cubic capacity of the tank. The heat dissipating capacity can be increased by providing (*i*) corrugations, (*ii*) fins, (*iii*) tubes (Fig. 13) and (*iv*) radiator tanks.

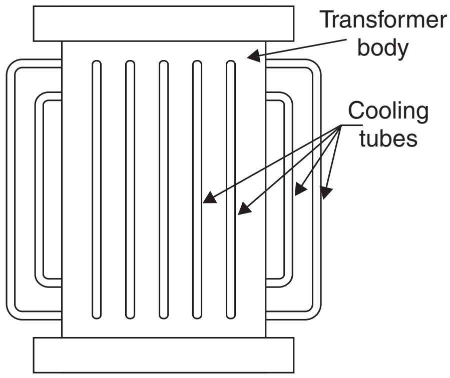


Fig. 13. Transformer with cooling tubes.

The *advantages of 'oil natural' cooling is that it does not clog the ducts and the windings are free from effects of moisture.*

(*ii*) **Oil immersed forced air-cooled transformers.** In this type of cooling, air is directed over the outer surfaces of the tank of the transformer immersed in oil.

(*iii*) **Oil immersed water-cooled transformers.** Heat is extracted from the oil by means of a stream of water pumped through a metallic coil immersed in the oil just below the top of the tank. The heated water is in turn cooled in a spray pond or a cooling tower.

(*iv*) **Oil immersed forced oil cooled transformers.** In such transformers heat is extracted from the oil by pumping the oil itself upward through the winding and then back by way of external radiators which may themselves be cooled by fans. The *extra cost of oil pumping equipment must of course be economically justified but it has incidentally the advantage of reducing the temperature difference between the top and bottom of enclosing tank*.

Fig. 14 shows the cooling of transformers having capacities from 10000 kVA and higher. In such cases air blast cooling of radiator is used.

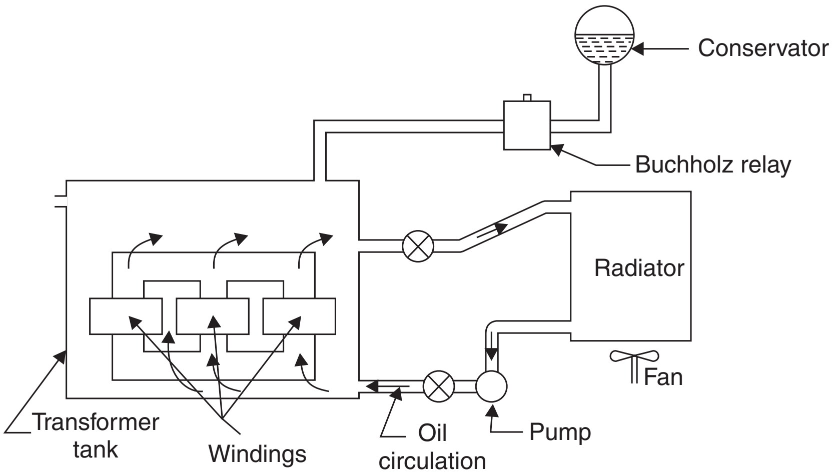


Fig. 14. Air blast cooling of radiator.

8.2. Transformer Oil

It is a *mineral oil obtained by refining crude petroleum*. It serves the following *purposes* :

(*i*) Provides additional insulation.

(*ii*) Carries away the heat generated in the core and coils.

(*iii*) Protects the paper from dirt and moisture.

The transformer oil should possess the following *properties* :

1. High dielectric strength.
2. Low viscosity to provide good heat transfer.
3. Good resistance to emulsion.
4. Free from inorganic acid, alkali and corrosive sulphur.
5. Free from sludging under normal operating conditions.
6. High flash/fire point.

8.3. Conservator and Breather

Conservator. The oil should not be allowed to come in contact with atmospheric air as it may take up moisture which may spoil its insulating properties. Also air may cause acidity and sludging of oil. To prevent this, many transformers are provided with conservators. The *function of a conservator (Fig. 14) is to take up contraction and expansion of oil without allowing it to come in contact with outside air*. The conservator consists of an air tight metal-drum fixed above the level of the top of the tank and connected with it by a pipe. The main tank is completely filled with oil when cold. The *conservator is partially filled with oil*. So the oil surface in contact with air is greatly reduced. The sludge thus formed remains in the conservator itself and does not go to the main tank.

Breather. When the temperature changes, the oil expands or contracts and there is a displacement of air. When the transformer cools, the oil level goes down, and air is drawn in. This is known as *breathing*. The air, coming in, is passed through an apparatus called *breather* for the purpose of extracting moisture. The *breather consists of a small vessel which contains a drying agent like silica get crystal impregnated with cobalt crystal*.

Note. *Sludging* means the slow formation of solid hydrocarbons due to heating and oxidation. The sludge deposit itself on the windings and cooling ducts producing overheating. This makes transformer still hotter producing more sludge. This process may continue till the transformer becomes unusable due to overheating. So the contact of oil with air should be avoided as the air contains oxygen.

9. SINGLE PHASE TRANSFORMER

9.1. Elementary Theory of an Ideal Transformer

The basic theory of a transformer is not difficult to understand. To simplify matters as much as possible, let us first consider *an ideal transformer*, that is, *one in which the resistance of the windings is negligible and the core has no losses*.

Let the secondary be open (Fig. 15), and let a sine wave of potential difference v_1 (Fig. 16) be impressed upon the primary. The impressed potential difference causes an alternating current to

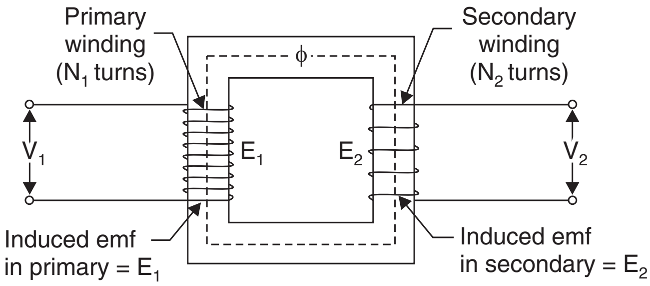
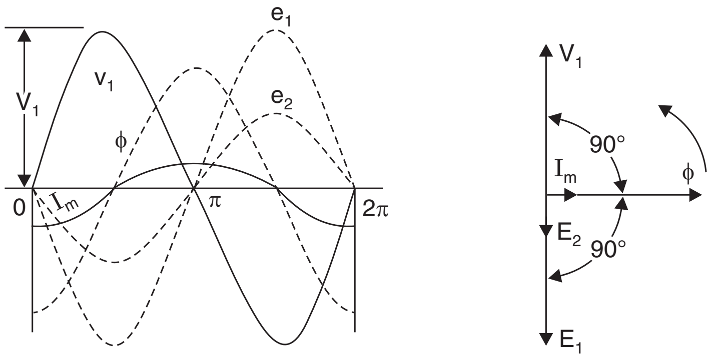


Fig. 15. Elementary diagram of an ideal transformer with an open secondary winding.

flow in the primary winding. Since the primary resistance is negligible and there are no losses in the core, the effective resistance is zero and the circuit is purely reactive. Hence the current wave i_m lags the impressed voltage wave v_1 by 90 time degrees, as shown in Fig. 17. The reactance of circuit is very high and the magnetizing current is very small. This current in the N_1 turns of the primary magnetizes the core and produces a flux ϕ that is at all times proportional to the current (if the permeability of the circuit is assumed to be constant), and therefore in time phase with the current. The flux, by its rate of change, induces in the primary winding E_1 which at every instant of time is



Figs. 16 and 17. Current, voltage and flux curves of an ideal (no loss) transformer.

equal in value and opposite in direction to V_1. It is called *counter e.m.f.* of the primary. The value which the primary current attains must be such that the flux which it produces in the core is of sufficient value to induce in the primary the required counter e.m.f.

Since the flux also threads (or links) the secondary winding a voltage e_2 is induced in the secondary. This voltage is likewise proportional to the rate of change of flux and so is in time phase with e_1, but it may have any value depending upon the number of turns N_2 in the secondary.

9.2. E.M.F. Equation of a Transformer

Let N_1 = number of turns in primary,

N_2 = number of turns in secondary,

ϕ_m = maximum flux in the core, Wb.

$= B_m \times A$, [where B_m is the maximum flux density in the core and A is the core area], and

f = frequency of a.c. input, Hz.

Refer to Fig. 18. Since the flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle *i.e.,* in $\frac{T}{4}$ or $\frac{1}{4f}$ second (T being time-period of the cycle),

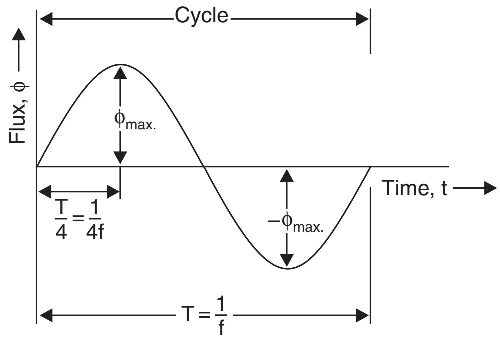


Fig. 18

∴ Average rate of change of flux

$$= \frac{\phi_m}{\frac{1}{4f}} = 4f\phi_{max} \text{ Wb/s or volt}$$

If flux ϕ varies *sinusoidally,* then r.m.s. (root mean square) value of inducted e.m.f. is obtained by multiplying the average value with form factor.

But, form factor $= \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$

∴ r.m.s. value of e.m.f./turn

$$= 1.11 \times 4f\phi_{max} = 4.44\phi_{max} \text{ volt}$$

Now, r.m.s. value of induced e.m.f. in the whole of primary winding,

$$E_1 = 4.44f\phi_{max} N_1 \quad ...(1)$$

Similarly r.m.s. value of induced e.m.f. in secondary is,

$$E_2 = 4.44f\phi_{max} N_2 \quad ...(2)$$

In an ideal transformer on no-load $V_1 = E_1$ and $V_2 = E_2$ (Fig. 15).

9.3. Voltage Transformation Ratio (K)

The *transformation ratio is defined as the ratio of the secondary voltage to primary voltage*. It is denoted by the letter K.

From eqns. (1) and (2), $\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$ ...(3)

- If $N_2 > N_1$ *i.e.,* $K > 1$, then transformer is called *step-up transformer*.
- If $N_2 < N_1$ *i.e.,* $K < 1$, then transformer is called *step-down transformer.*

Again for an *ideal transformer*

$$\text{Input } (VA) = \text{Output } (VA)$$

$$V_1I_1 = V_2I_2 \text{ or } E_1I_1 = E_2I_2$$

or
$$\frac{I_2}{I_1} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K} \quad ...(4)$$

i.e., Primary and secondary currents are inversely proportional to their respective turns.

Example 1. *A 40 kVA, single phase transformer has 400 turns on the primary and 100 turns on the secondary. The primary is connected to 2000 V, 50 Hz supply. Determine* :

(*i*) *The secondary voltage on open circuit.*

(*ii*) *The current flowing through the two windings on full-load.*

(*iii*) *The maximum value of flux.*

Solution. Rating = 40 kVA

Primary turns, $N_1 = 400$

Secondary turns, $N_2 = 100$

Primary induced voltage, $E_1 = V_1 = 2000$ V

(*i*) **Secondary voltage on open, circuit $\mathbf{V_2}$:**

Using the relation, $$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = E_1 \times \frac{N_2}{N_1}$$

$\therefore$ $$E_2 = V_2 = 2000 \times \frac{100}{400} = 500 \text{ V}$$

Hence, $\mathbf{V_2 = 500\ V.}$ **(Ans.)**

(*ii*) **Primary current, $\mathbf{I_1}$:**

Secondary current, $\mathbf{I_2}$:

Primary full-load current, $$I_1 = \frac{\text{kVA} \times 100}{V_1} = \frac{40 \times 1000}{2000} = \mathbf{20\ A.} \quad \textbf{(Ans.)}$$

Secondary full-current, $$I_2 = \frac{\text{kVA} \times 1000}{V_2} = \frac{40 \times 1000}{500} = \mathbf{80\ A.} \quad \textbf{(Ans.)}$$

(*iii*) **Maximum value of flux, $\boldsymbol{\phi_{max}}$:**

Using e.m.f. equation, $E_1 = 4.44 f \phi_{max} N_1$

$\therefore$ $2000 = 4.44 \times 50 \times \phi_{max} \times 400$

$\therefore$ $$\phi_{max} = \frac{2000}{4.44 \times 50 \times 400} = 0.0225 \text{ Wb.}$$

Hence, $\phi_{max} = \mathbf{0.0225\ Wb.}$ **(Ans.)**

Example 2. *The no-load ratio required in a single-phase 50 Hz transformer is 6600/600 V. If the maximum value of flux in the core is to be about 0.08 Wb, find the number of turns in each winding.*

Solution. Primary, $E_1 = V_1 = 6600$ V

Secondary, $E_2 = V_2 = 600$ V

Maximum value of flux, $\phi_{max} = 0.08$ Wb.

Primary turns, $\mathbf{N_1}$:

Secondary turns, $\mathbf{N_2}$:

Using the relation, $E_1 = 4.44 f \phi_{max} N_1$

$$6600 = 4.44 \times 50 \times 0.08 \times N_1$$

$\therefore$ $$N_1 = \frac{6600}{4.44 \times 50 \times 0.08} \simeq 372$$

Hence, $\mathbf{N_1 = 372.}$ **(Ans.)**

Also $$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$\therefore$ $$N_2 = \frac{E_2 N_1}{E_1} = \frac{600 \times 372}{6600} \simeq 34$$

Hence, $\mathbf{N_2 = 34.}$ **(Ans.)**

Example 3. *A single-phase transformer is connected to a 230 V, 50 Hz supply. The net cross-sectional area of the core is 60 cm². The number of turns in the primary is 500 and in the secondary 100. Determine* :

(*i*) *Transformation ratio.* (*ii*) *Maximum value of flux density in the core.*
(*iii*) *E.m.f. induced in secondary winding.*

Solution. Primary turns, $N_1 = 500$

Secondary turns, $N_2 = 100$

Primary, $E_1 = V_1 = 230$ V

Core area, $a = 60 \text{ cm}^2 = 60 \times 10^{-4} \text{ m}^2$

(*i*) **Transformation ratio, K :**

$$K = \frac{N_2}{N_1} = \frac{100}{500} = 0.2$$

Hence, **K = 0.2. (Ans.)**

(*ii*) **Maximum value of flux density, $\mathbf{B_{max}}$:**

Using the e.m.f. equation, $E_1 = 4.44 f \phi_{max} N_1$

$\therefore$ $230 = 4.44 \times 50 \times \phi_{max} \times 500$

or $$\phi_{max} = \frac{230}{4.44 \times 50 \times 500} = 0.00207 \text{ Wb}$$

Now, $$B_{max} = \frac{\phi_{max}}{A} = \frac{0.00207}{60 \times 10^{-4}} = 0.345 \text{ T}$$

[where T stands for tesla (Wb/m²)]

Hence, $\mathbf{B_{max} = 0.345\ T.}$ **(Ans.)**

(*iii*) **E.m.f. induced in the secondary winding, $\mathbf{E_2}$:**

Using the relation, $$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{E_2}{230} = \frac{100}{500}$$

$\therefore$ $\mathbf{E_2 = 46\ V.}$ **(Ans.)**

Example 4. *3300/300 V single-phase 300 kVA transformer has 1100 primary turns. Find* :

(*i*) *Transformation ratio.* (*ii*) *Secondary turns.*
(*iii*) *Voltage/turn.*
(*iv*) *Secondary current when it supplies a load of 200 kW at 0.8 power factor lagging.*

Solution. Primary, $E_1 = 3300$ V

$N_1 = 1100$

Secondary, $E_2 = 300$ V

Rating of the transformer = 300 kVA

Output = 200 kW

(*i*) **Transformation ratio, K = ?**

$$K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{300}{3300} = \frac{1}{11}$$

Hence, $\mathbf{K = \frac{1}{11}}$. **(Ans.)**

(*ii*) **Secondary turns, N_2 = ?**

Using the relation, $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$\therefore$ $N_2 = \frac{E_2 N_1}{E_1} = \frac{300 \times 1100}{3300} = 100$

Hence, $\mathbf{N_2 = 100}$. **(Ans.)**

(*iii*) **Voltage/turn = ?**

Voltage/turn $\frac{E_1}{N_1} = \frac{3300}{1100} = \mathbf{3\ V}$. **(Ans.)**

(*iv*) **Secondary current,** $I_2 = \frac{\text{Output}}{V_2 \cos\phi} = \frac{200 \times 1000}{300 \times 0.8} = 833.33$ A

Hence, $\mathbf{I_2 = 833.33\ A}$. **(Ans.)**

Example 5. *The voltage per turn of a single-phase transformer is 1.1 V. When the primary winding is connected to a 220 V, 50 Hz A.C. supply, the secondary voltage is found to be 550 V. Find :*

(*i*) *Primary and secondary turns.*

(*ii*) *Core area if the maximum flux density is 1.1 T.*

Solution. Voltage per turn = 1.1 V

Primary, E_1 = 220 V

Secondary, E_2 = 550 V

Max. flux density, B_{max} = 1.1 T

(*i*) **Primary turns,** $\mathbf{N_1} = \frac{E_1}{1.1} = \frac{200}{1.1} = \mathbf{200}$. **(Ans.)**

Secondary turns, $\mathbf{N_2} = \frac{E_2}{1.1} = \frac{550}{1.1} = \mathbf{500}$. **(Ans.)**

(*ii*) **Core area A :**

Using the relation, $E_1 = 4.44 f \phi_{max} N_1$

$220 = 4.44 \times 50 \times \phi_{max} \times 200$

$\therefore$ $\phi_{max} = \frac{220}{4.44 \times 50 \times 200} = 0.004955$ Wb

Core area, $A = \frac{\phi_{max}}{B_{max}} = \frac{0.004955}{1.1} = 0.004504\ m^2 = \mathbf{45.04\ cm^2}$. **(Ans.)**

Example 6. *The core of 1000 kVA, 11000/550 V, 50 Hz, single-phase transformer has a cross-section of 20 cm × 20 cm. If the maximum core density is not to exceed 1.3 tesla, calculate :*

(*i*) *The number of h.v. and l.v. turns per phase.*

(*ii*) *The e.m.f. per turns.*

Assume a stacking factor of 0.9.

Solution. *Given* : A = 20 cm × 20 cm = 400 × 10^{-4} m^2 ; B_{max} = 1.3 Wb/m^2, f = 50 Hz ;
E_1 = 11000 V ; E_2 = 550 V ; Stacking factor = 0.9

(*i*) **Number of turns N_1 (h.v.) ; N_2 (l.v.) :**

We know that, Flux = flux density × area of cross-section × stacking factor

$$\therefore \quad \phi_{max} = 1.3 \times 400 \times 10^{-4} \times 0.9 = 0.0468 \text{ Wb}$$

$$E_1 = 4.44 f\phi_{max} N_1$$

or
$$11000 = 4.44 \times 50 \times 0.0468 \times N_1$$

$$\therefore \quad \mathbf{N_1} = \frac{11000}{4.44 \times 50 \times 0.0468} = 1058.75 \simeq \mathbf{1059.} \quad \textbf{(Ans.)}$$

and
$$\mathbf{N_2} = \frac{E_2}{E_1} \times N_1 = \frac{550}{11000} \times 1059 = 52.95 \simeq \mathbf{53.} \quad \textbf{(Ans.)}$$

(*ii*) **The e.m.f. per turn :**

$$\text{E.M.F. per turn} = \frac{E_1}{N_1} = \frac{11000}{1059} = \mathbf{10.387\ V.} \quad \textbf{(Ans.)}$$

9.4. Transformer with Losses but no Magnetic Leakage

We shall consider the following two cases :

1. When transformer is on no-load.
2. When transformer is on load.

9.4.1. Transformer on no-load. A transformer is said to be on *no-load* if its secondary side is open and primary is connected to a sinusoidal alternating voltage V_1. The alternating applied voltage will cause *flow of alternating current in the primary winding which will create alternating flux*. This primary input current (I_0) under no-load conditions supply :

(*i*) *Iron losses in the core* (*i.e.,* hysteresis loss and eddy current loss).

(*ii*) A very *small amount of copper losses in primary* (there being no copper loss in the secondary as it is open).

Thus I_0 is not at 90° behind V_1, but lags it by an angle $\phi_0 < 90°$.

No-load power input, $\quad P_0 = V_1 I_0 \cos \phi_0 \quad$ ...(5)

where $\cos \phi_0$ = primary power factor under no-load conditions.

As is evident from Fig. 19, primary current I_0 has the following two components :

(*i*) *Active* or *working or iron loss component* I_w. This component is in *phase with* V_1 and mainly supplies the iron loss plus small quantity of primary copper loss.

$$I_w = I_0 \cos \phi_0 \quad ...(i)$$

(*ii*) *Magnetising component* I_m. This component is *in quadrature with* V_1 and its function is to *sustain the alternating flux in the core.* It is *wattless.*

$$I_m = I_0 \sin \phi_0 \quad ...(ii)$$

Also
$$I_0 = \sqrt{I_w^2 + I_m^2} \quad ...(iii)$$

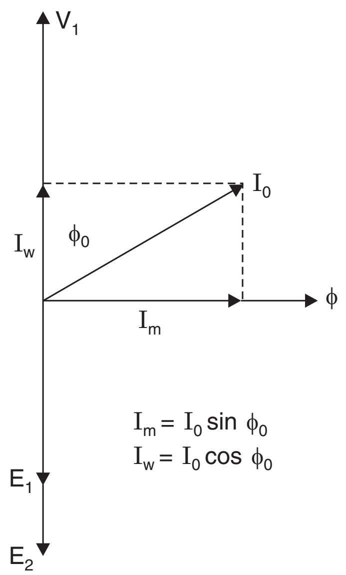


Fig. 19. No-load vector diagram.

The following points are *worthnoting* :

- The no-load primary current I_w is *very small as compared to the full-load primary current.*
- As I_0 is very small, the no-load primary copper loss is negligibly small which means that *no-load primary input is practically equal to the iron loss in the transformer.*
- Since, it is primarily the core loss which is responsible for shift in the current vector, angle ϕ_0 is known as **hysteresis angle of advance.**

Example 7. *A 3300/300 V single-phase transformer gives 0.6 A and 60 W as ammeter and wattmeter readings when supply is given to the low voltage winding and high voltage winding is kept open, find* :

(*i*) *Power factor of no-load current.* (*ii*) *Magnetising component.*

(*iii*) *Iron loss component.*

Solution. No-load losses, $P_0 = 60$ W

No-load current, $I_0 = 0.6$ A

(*i*) $P_0 = VI_0 \cos \phi_0$

$\therefore$ $60 = 300 \times 0.6 \times \cos \phi_0$

or $$\cos \phi_0 = \frac{60}{300 \times 0.6} = 0.33 \text{ (lagging)}$$

Hence, **power factor of no-load current = 0.33. (Ans.)**

(*ii*) Magnetising component, $I_m = I_0 \sin \phi_0 = 0.6 \sqrt{1 - \cos^2 \phi_0} = 0.6 \sqrt{1 - (0.33)^2} = 0.566$ A

$\therefore$ $I_m =$ **0.566 A. (Ans.)**

(*iii*) Iron loss component, $I_w = I_0 \cos \phi_0 = 0.6 \times 0.33 =$ **0.198 A. (Ans.)**

[or $I_w = \sqrt{I_0^2 - I_m^2} = \sqrt{(0.6)^2 - (0.566)^2} = 0.198$ A].

Example 8. *Find (i) active and reactive components of no-load current ; and (ii) no-load current of a 440/220 V single-phase transformer if the power input on no-load to the high voltage winding is 80 W and power factor of no-load current is 0.3 lagging.*

Solution. Primary, $E_1 = 440$ V

Secondary, $E_2 = 220$ V

Power factor, $\cos \phi_0 = 0.3$ (lagging)

No-load losses, $P_0 = 80$ W

(*i*) **Active component** (or *wattful component*),

$$I_w = (I_0 \cos \phi_0) = \frac{P_0}{V_1} = \frac{80}{440} = \mathbf{0.182\ A.} \quad \textbf{(Ans.)}$$

$\cos \phi_0 = 0.3$; $\phi_0 = \cos^{-1}(0.3) = 72.54°$

$\therefore$ $\tan \phi_0 = 3.18$

Reactive component (or *magnetising components*)

$I_m = I_w \tan \phi_0 = 0.182 \times 3.18 =$ **0.578 A. (Ans.)**

(*ii*) $I_w = I_0 \cos \phi_0$

$\therefore$ $$I_0 = \frac{I_w}{\cos \phi_0} = \frac{0.182}{0.3} = \mathbf{0.606.} \quad \textbf{(Ans.)}$$

[or $I_0 = \sqrt{I_w^2 + I_m^2} = \sqrt{(0.182)^2 + (0.578)^2} = 0.606$ A].

Example 9. *A 3300/220 V, 30 kVA, single-phase transformer takes a no-load current of 1.5 A when the low voltage winding is kept open. The iron loss component is equal to 0.4 A find* :

(*i*) *No-load input power.*

(*ii*) *Magnetising component and power factor of no-load current.*

Solution. Rating of transformer = 30 kVA

Primary, $E_1 = 3300$ V

Secondary, $E_2 = 220$ V

No-load current, $I_0 = 1.5$ A

Iron loss component, $I_m = 0.4$ A

(*i*) **No-load input power,**

$$P_0 = V_1 I_0 \cos \phi_0 = V_1 I_w \qquad (\because \; I_w = I_0 \cos \phi_0)$$

$$= 3300 \times 0.4 = \mathbf{1320\ W.} \quad \textbf{(Ans.)}$$

(*ii*) **Magnetising component,**

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{1.5^2 - 0.4^2} = \mathbf{1.44\ A.} \quad \textbf{(Ans.)}$$

No-load power factor,

$$\cos \phi_0 = \frac{I_w}{I_0} = \frac{0.4}{1.5} = \mathbf{0.267.} \quad \textbf{(Ans.)}$$

9.4.2. Transformer on load. *The transformer is said to be loaded when the secondary circuit of a transformer is completed through an impedance or load.* The magnitude and phase of secondary current I_2 with respect to secondary terminal voltage will depend upon the characteristic of load, *i.e.,* current I_2 will be in phase, lag behind and lead the terminal voltage V_2 respectively when the load is purely resistive, inductive and capacitive.

The secondary current I_2 sets up its own ampere-turns (= $N_2 I_2$) and creates its own flux ϕ_2 opposing the main flux ϕ_0 created by no-load current I_0. The opposing secondary flux ϕ_2 weakens the primary flux ϕ_0 momentarily hence primary counter or back e.m.f. E_1 tends to be reduced. V_1 gains the upper hand over E_1 momentarily and hence causes more current to flow in primary. Let this additional primary current be I_2'. It is known as *load component of primary current*. The additional primary m.m.f. $N_1 I_2'$ sets up its own flux ϕ_2' which is in opposition to ϕ_2 (but is in the same direction as ϕ_0) and is equal to it in magnitude. Hence they *cancel* each other. Thus we find that the magnetic effects of secondary current I_2 are immediately neutralised by the additional primary current I_2' which is brought into existence exactly at the same instant as I_2.

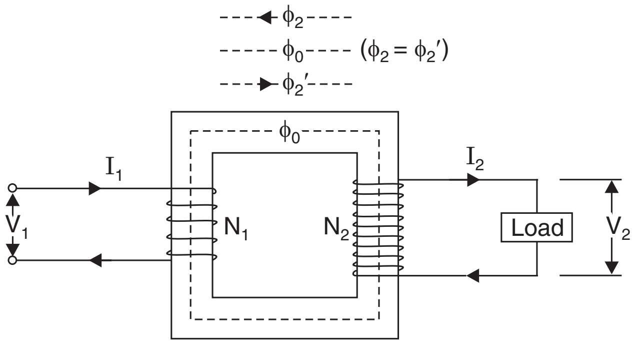


Fig. 20. An ideal transformer on load.

From above discussion it can be concluded that :

- Whatever be the load conditions, the *net flux passing through the core is approximately the same as at no-load*.
- Since the core flux remains constant at all loads, the core *loss almost remains constant under different loading conditions*.

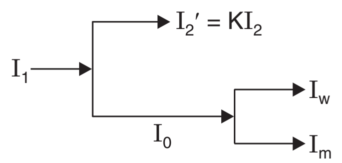
Since $\phi_2 = \phi_2'$

$\therefore$ $N_2 I_2 = N_1 I_2'$

i.e., $$I_2' = \frac{N_2}{N_1} \times I_2 = KI_2 \qquad \left(\because \frac{N_2}{N_1} = K\right)$$

The total primary current is the vector sum of I_0 and I_2' ; the current I_2' is in *antiphase* with I_2 and K times in magnitude.

The components of primary current can be shown as below :



The vector diagrams for transformer on non-inductive, inductive and capacitive loads are shown in Fig. 21 (*a, b, c*).

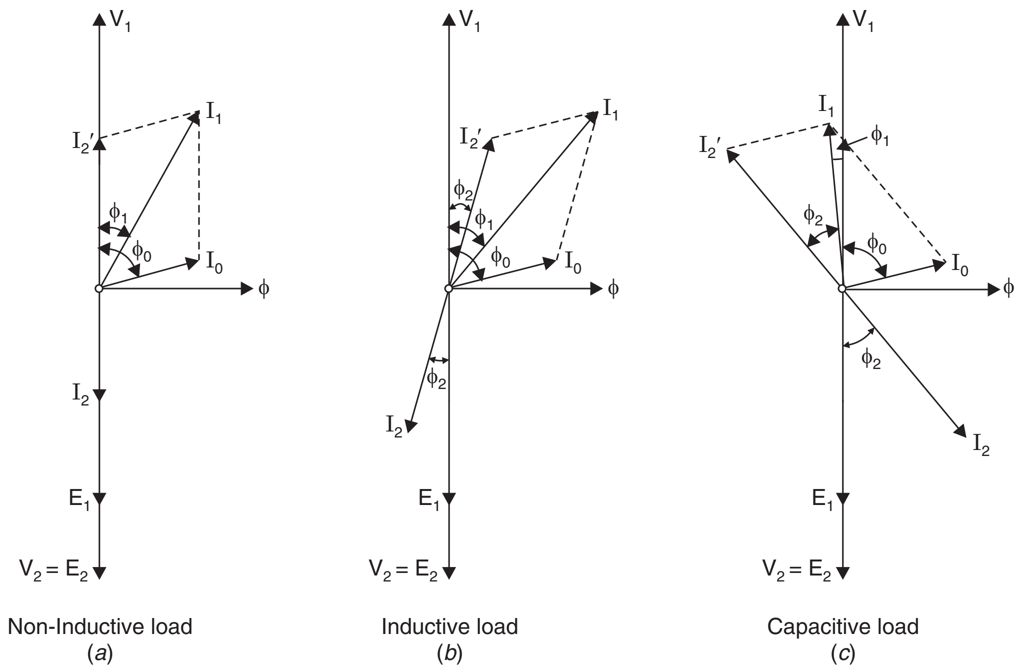


Fig. 21

9.5. Resistance and Magnetic Leakage

In an ideal transformer it is assumed that windings have got no resistance and there is no leakage flux. But in actual practice it is not possible to have an ideal transformer.

Resistance. In an actual transformer the primary as well secondary winding possess some resistance due to which some voltage drop takes place in them.

Magnetic leakage. Magnetic flux cannot be confined into a desired path. The greater portion of the flux (*i.e.*, the mutual flux) remains confined to the core and links both the windings but a small portion, called the *leakage flux,* completes its path through the air surrounding the coils. As shown in Fig. 22 each of the winding is associated with a leakage flux. Since the path of the leakage flux is largely in air the leakage flux and the voltage induced by it vary *linearly with the current. The primary leakage flux varies linearly with the primary current and the secondary leakage flux varies linearly with the secondary current.* The effect of primary leakage may be simulated by assigning to the primary a leakage inductance (equal to primary leakage flux per ampere of primary current). The reactance corresponding to this primary leakage inductance (*i.e.,* $2\pi f \times$ primary leakage inductance) is known as primary leakage reactance X_1. Similarly the effect of secondary leakage flux may be simulated by a secondary leakage inductance and the corresponding leakage reactance X_2.

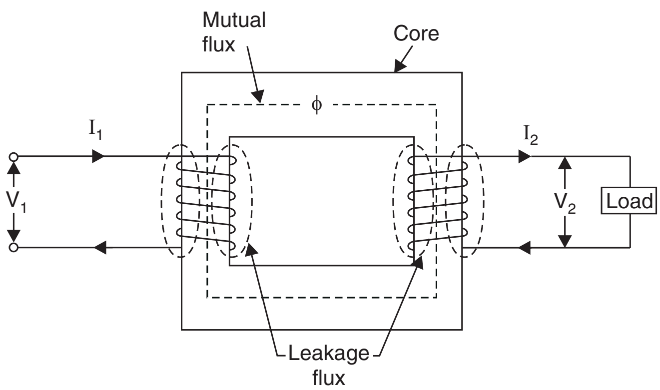


Fig. 22. Magnetic flux in a transformer.

The terminal voltage V_1 applied to the primary must have a component $jI_1 X_1$ to balance the primary leakage e.m.f. Similarly V_2 *i.e.,* the output voltage from secondary will be less than the induced e.m.f. E_2 by a component $jI_2 X_2$ to account for the secondary leakage flux.

A transformer with winding resistance and magnetic leakage is equivalent to an ideal transformer (having no resistance and leakage resistance) having resistive and inductive coils connected in series with each winding as shown in Fig. 23.

9.6. Transformer with Resistance and Leakage Reactance

Let us consider transformer (Fig. 23) having primary and secondary windings of resistances R_1 and R_2 and reactances X_1 and X_2 respectively.

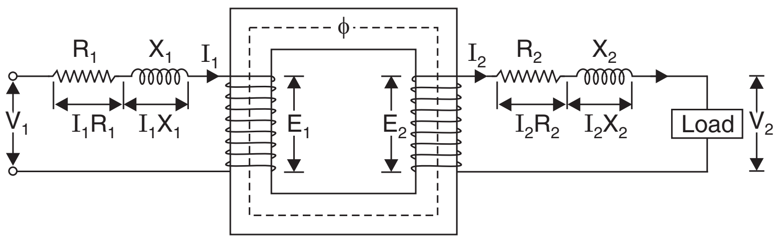
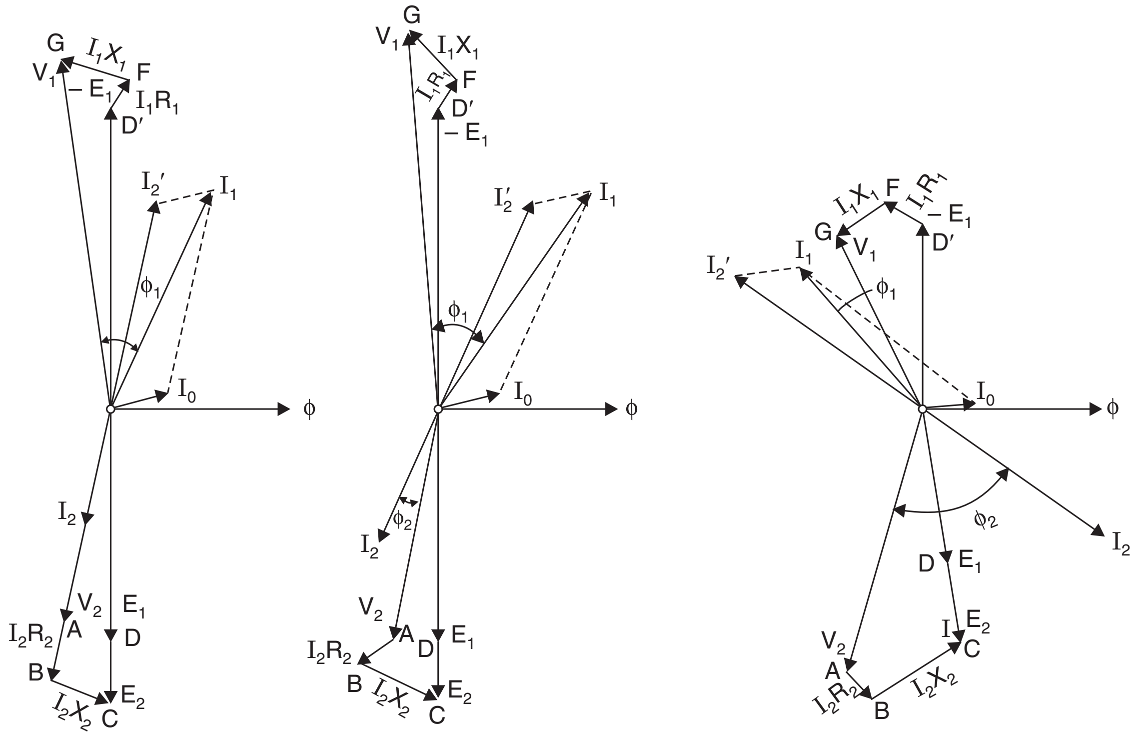


Fig. 23. An equivalent diagram of an actual transformer.

Impedance of primary winding, $Z_1 = R_1 + jX_1$

Impedance of secondary winding, $Z_2 = R_2 + jX_2$

The *vector diagrams* for different kinds of loads for such a transformer are shown in Fig. 24 (*a, b, c*). *In these diagrams, vectors for resistive drops are drawn parallel to current vectors* whereas *reactive drops are drawn perpendicular to the current vectors*. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.



(*a*) (*b*) (*c*)

Fig. 24. (*a*) Unity p.f., (*b*) lagging p.f. and (*c*) leading p.f.

9.7. Equivalent Resistance and Reactance

Equivalent Resistance. In Fig. 25 is shown a transformer whose primary and secondary windings have resistances of R_1 and R_2 respectively (shown external to the windings). The resistances of the two windings can be transferred to any one of the two windings. If both the resistances are concentrated in one winding the calculations become simple since then we can work in one winding only.

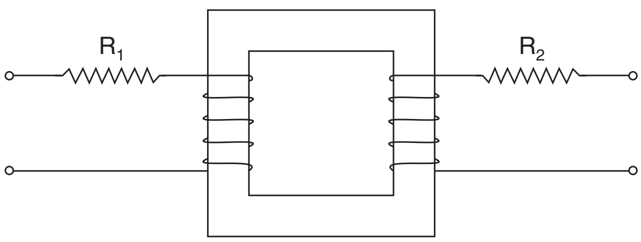


Fig. 25

We shall now prove that a resistance of R_2 in secondary is equivalent to $\dfrac{R_2}{K^2}$ in primary and its value will be denoted by R_2', — *the equivalent secondary resistance referred to primary.*

In the secondary, copper loss is $I_2{}^2R^2$. This loss is supplied by primary which takes a current of I_1. Hence, if R_2' is the equivalent resistance referred to primary which would have caused the same loss as R_2 in secondary, then

$$I_1{}^2R_2' = I_2{}^2R_2 \text{ or } R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

If no-load current I_0 is neglected,

then,
$$\frac{I_2}{I_1} = \frac{1}{K}$$

Hence,
$$R_2' = \frac{R_2}{K^2}$$

Similarly, equivalent primary resistance as referred to secondary is

$$R_1' = K^2R_1$$

In Fig. 26, secondary resistance has been transferred to primary side (leaving secondary circuit resistanceless).

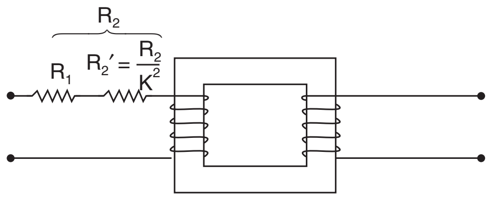


Fig. 26

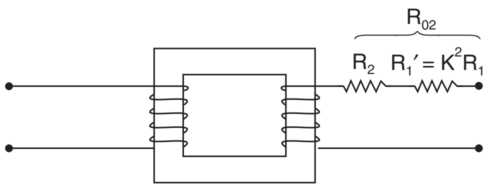


Fig. 27

The resistance $R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$ is known as the *equivalent or effective resistance of the transformer as referred to the primary* and may be designated as R_{01}.

Thus
$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}. \qquad ...(6)$$

Similarly, the equivalent resistance of the transformer as referred to the secondary (Fig. 27) is

$$R_{02} = R_2 + R_1' = R_2 + K^2R_1 \qquad ...(7)$$

The following points are worth remembering :

(*i*) When shifting resistance *to the secondary, multiply* it by K^2.

(*ii*) When shifting resistance *to the primary, divide* by K^2.

Leakage reactance. Leakage reactance can also be transferred from one winding to the other in the same way as resistance (Fig. 28 and Fig. 29).

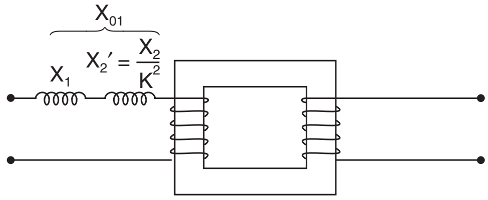


Fig. 28

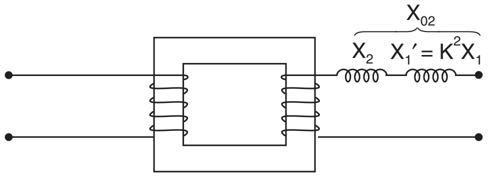


Fig. 29

$$X_2' = \frac{X_2}{K^2} \text{ and } X_1' = K^2X_1$$

and $$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} \quad ...(8)$$

$$X_{02} = X_2 + X_1' = X_2 + K^2X_1 \quad ...(9)$$

Total impedance. It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \quad \text{(Fig. 30) ...(10)}$$

and $$Z_{02} = \sqrt{R_0^2 + X_{02}^2} \quad \text{(Fig. 31) ...(11)}$$

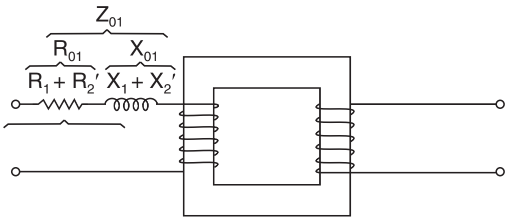


Fig. 30

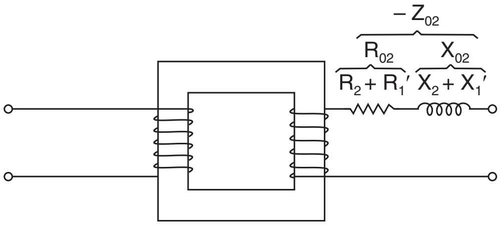


Fig. 31

9.8. Total Voltage Drop in a Transformer

(*i*) **Approximate voltage drop.** When there is no-load on the transformer, then,

$$V_1 = E_1 \text{ (approximately).}$$

and $$E_2 = KE_1 = KV_1$$

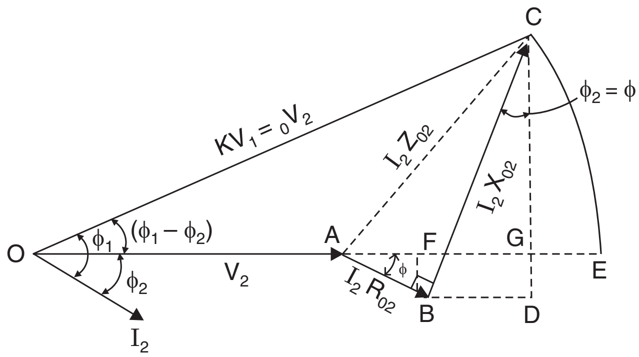


Fig. 32. Lagging power factor.

Also $E_2 = {}_0V_2$, where ${}_0V_2$ is secondary terminal voltage on no-load

or $E_2 = {}_0V_2 = KV_1$

V_2 = secondary voltage on load.

Refer to Fig. 32. The procedure of finding the approximate voltage drop of the transformer as *referred to secondary* is given below :

- Taking O as centre, radius OC draw an arc cutting OA produced at E.

  The total voltage drop $I_1Z_{02} = AC = AE$ which is approximately equal to AG.

- From B draw BF perpendicular on OA produced. Draw CG perpendicular to OE and draw BD parallel to OE.

Approximate voltage drop $= AG = AF + FG = AF + BD$ [$\because$ $FG = BD$]

$= I_2R_{02} \cos \phi + I_0X_{02} \sin \phi$

This is the value of approximate voltage drop for a *lagging power factor*.

Figs. 33 and 34 refer to *unity* and *leading power factor* respectively.

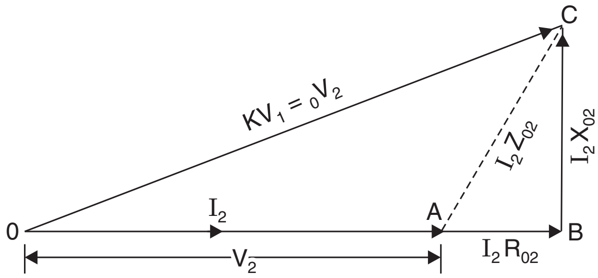


Fig. 33. Unity power factor.

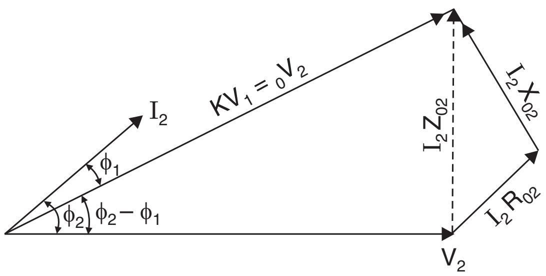


Fig. 34. Leading power factor.

- The approximate voltage drop for a *leading power factor* becomes :

$$(I_2R_{02} \cos \phi - I_2X_{02} \sin \phi)$$

- The approximate voltage drop for a transformer *in general* is given by :

$$(I_2R_{02} \cos \phi \pm I_1X_{02} \sin \phi) \quad ...(12)$$

- The voltage drop as *referred to primary* is given by :

$$(I_1R_{01} \cos \phi \pm I_1X_{02} \sin \phi)$$

- *Percentage* voltage drop in *secondary*

$$= \left(\frac{I_2R_{02} \cos \phi \pm I_2X_{02} \sin \phi}{{}_0V_2} \right) \times 100 \quad ...(13)$$

(*ii*) **Exact voltage drop.** When we refer to Fig. 32 we find that exact drop is AE and not AG. If quantity GE is added to AG the exact value of voltage drop will be obtained.

From right angled triangle OCG, we get

$$CG^2 = OC^2 - OG^2 = (OC + OG)\,(OC - OG)$$

$$= (OC + OG)\,(OE - OG) = GE \times 2OC$$

$$\therefore \quad GE = \frac{CG^2}{2OC}$$

Now $CG = I_2X_{02} \cos \phi - I_2R_{02} \sin \phi$

$\therefore$ $$GE = \frac{(I_2X_{02} \cos \phi - I_2R_{02} \sin \phi)^2}{2\,{}_0V_2}$$

$\therefore$ The exact voltage drop for a *lagging* power factor is

$$= AG + GE$$

$$= (I_2R_{02} \cos \phi + I_2X_{02} \sin \phi) + \frac{(I_2X_{02} \cos \phi - I_2R_{02} \sin \phi)^2}{2\,{}_0V_2} \quad ...(14)$$

For a *leading* power, the expression becomes

$$= (I_2R_{02} \cos \phi - I_2X_{02} \sin \phi) + \frac{(I_2X_{02} \cos \phi + I_2R_{02} \sin \phi)^2}{2\,{}_0V_2} \quad ...(15)$$

In *general*, the exact voltage drop is

$$= (I_2R_{02} \cos \phi \pm I_2X_{02} \sin \phi) + \frac{(I_2X_{02} \cos \phi \mp I_2R_{02} \sin \phi)^2}{2\,{}_0V_2}$$

Percentage drop is $$= \frac{(I_2R_{02} \cos \phi \pm I_2X_{02} \sin \phi) \times 100}{{}_0V_2} + \frac{(I_2X_{02} \cos \phi \mp I_2R_{02} \sin \phi)^2 \times 100}{2\,{}_0V_2^{\,2}} \quad ...(16)$$

[Upper signs ... for *lagging* power factor
Lower signs ... for *leading* power factor]

9.9. Equivalent Circuit

In transformers, the problems concerning voltages and currents can be solved by the use of phasor diagrams. However, it is more convenient to represent the transformer by an equivalent circuit. If an equivalent circuit is available the computations can be done by the direct application of circuit theory. An *equivalent circuit is merely a circuit interpretation of the equations which describe the behaviour of the device*.

The transformer windings, in the equivalent circuit, are shown as ideal. The resistance and leakage reactance of the primary and secondary are shown separately in the primary and secondary circuits. The effect of magnetising current is represented by X_0 connected in parallel across the winding. The effect of core loss is represented by a non-inductive resistance R_0 as shown in Fig. 35.

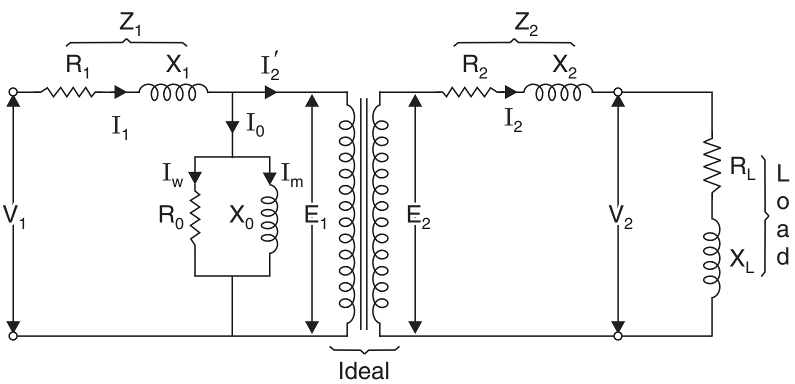


Fig. 35. Equivalent circuit of a transformer.

The equivalent circuit can be simplified by transferring the secondary resistances and reactances to the primary side in such a way that the ratio of E_2 to E_1 is not affected in magnitude or phase. Let R_2' be the resistance which must be placed in the primary circuit to produce the same drop as produced by R_2 in the secondary. Then R_2' causes a voltage drop in primary equal to $I_2'R_2'$. The ratio of $I_2'R_2'$ and I_2R_2 must be the same as the turn ratio $\frac{N_1}{N_2}$.

Thus, $$\frac{I_2' R_2'}{I_2R_2} = \frac{N_1}{N_2} = \frac{1}{K} \text{ or } R_2' = R_2 \times \frac{I_2}{I_2'} \times \frac{1}{K}$$

But $$\frac{I_2}{I_2'} = \frac{N_1}{N_2} = \frac{1}{K}$$

∴ $$R_2' = \frac{R_2}{K^2}$$

Similarly, $$X_2' = \frac{X_2}{K^2}.$$

The load resistance and reactance can be transferred to the primary side in the same way. When all the secondary impedances have been transferred to the primary side, the winding need not be shown in the equivalent circuit. The exact equivalent circuit of the transformer with all impedances transferred to the primary side is shown in Fig. 36.

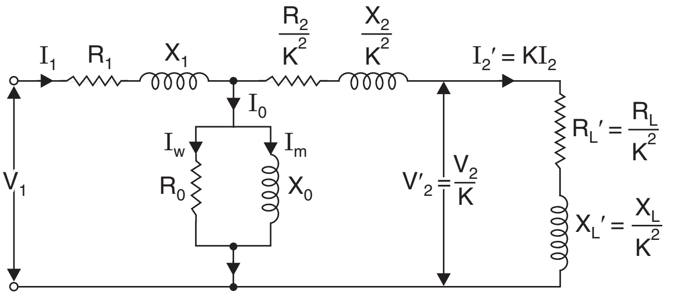


Fig. 36. Equivalent circuit with secondary impedances transferred to primary.

Just as the secondary impedances have been transferred to the primary it is possible to transfer the primary winding resistance and reactance to the secondary side. The primary resistance R_1 and leakage reactance X_1 when transferred to secondary are denoted by R_1' and X_1' and are given by

$$R_1' = K^2R_1$$
$$X_1' = K^2X_1.$$

Approximate equivalent circuit. It is seen that E_1 differs from V_1 by a very small amount. Moreover, the no-load current I_0 is only a small fraction of full-load primary current so that I_2' is practically equal to I_1. Consequently the equivalent circuit can be simplified by transferring the parallel branch consisting of R_0 and X_0 to the extreme left position of the circuit as shown in Fig. 37. This circuit is known as *approximate equivalent circuit*. Analysis with the approximate equivalent circuit gives almost the same results as the analysis with the exact equivalent circuit. *However, the analysis with the approximate equivalent circuit is simple because the resistances R_1 and R_2' and leakage reactances X_1 and X_2' can be combined.*

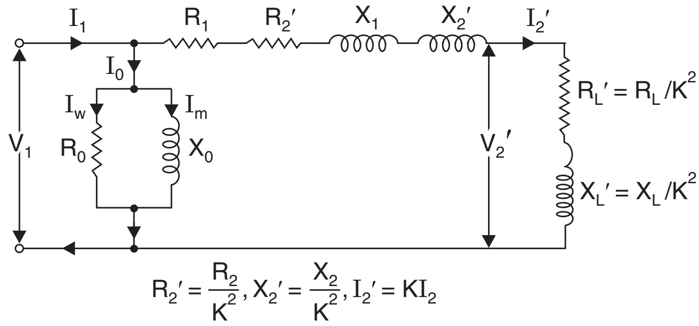


Fig. 37. Approximate equivalent circuit of transformer.

9.10. Transformer Tests

The performance of a transformer can be calculated on the basis of its equivalent circuit which contains the following four main parameters :

(*i*) Equivalent resistance R_{01} as referred to primary (or secondary R_{02}).

(*ii*) Equivalent leakage resistance X_{01} as referred to primary (or secondary X_{02}).

(*iii*) Core loss conductance G_0 (or resistance R_0).

(*iv*) Magnetising susceptance B_0 (or reactance X_0).

These parameters or constants can be determined by the following *two tests* :

1. Open-circuit or no-load test.
2. Short-circuit or impedance test.

The above two tests are *convenient to perform and very economical because they furnish the required information without actually loading the transformer*.

9.10.1. Open-circuit or no-load test (O.C. Test)

An open-circuit or no-load test is conducted to find :

(*i*) No-load loss or core loss.

(*ii*) No-load current I_0 which is helpful in finding R_0 and X_0.

The connections for this test are made as shown in Fig. 38. One winding of the transformer (*usually high voltage winding*) is *left open* and the other is connected to its supply of *normal voltage and frequency*. Ammeter *A* and wattmeter *W* are connected to measure *no-load current* (I_0) *and no-load input power* (P_0) respectively.

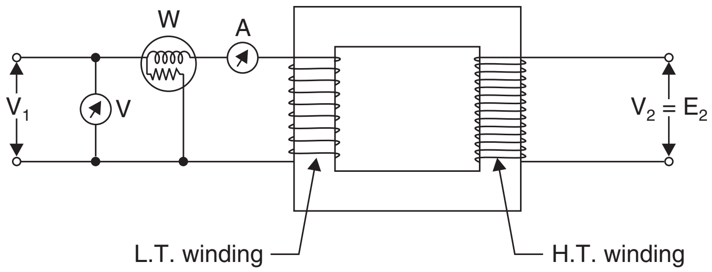


Fig. 38. Circuit diagram for open-circuit test.

As the primary no-load current I_0 (as measured by ammeter) is small (usually 3 to 10% of rated load current) copper loss is negligibly small in primary (L.T. winding) and nil in secondary winding (it being open). Hence the *wattmeter reading represents practically the core-loss under no-load conditions* (and *this loss is same for all loads*).

From the data available from this test R_0, X_0, $\cos \phi_0$ (no-load power factor), I_w and I_m can be calculated as follows :

Now, Iron loss $= P_i$ = input power on no-load

$= P_0$ watts (say)

No-load current $= I_0$

Applied primary voltage $= V_1$

Also $P_0 = V_1 I_0 \cos \phi_0$ (where $\cos \phi_0$ = no-load power factor)

$$\therefore \quad \cos \phi_0 = \frac{P_0}{V_1 I_0} \quad ...(17)$$

or

$$\phi_0 = \cos^{-1} \frac{P_0}{V_1 I_0}$$

No-load current *wattful component*,

$$I_w = I_0 \cos \phi_0 = \frac{P_0}{V_1} \quad ...(18)$$

No-load current *magnetising component*,

$$I_m = \sqrt{I_0^2 - I_w^2} \quad ...(19)$$

The no-load resistance, $$R_0 = \frac{V_1}{I_w} = \frac{V_1^2}{P_0} \quad ...(20)$$

The no-load reactance, $$X_0 = \frac{V_1}{I_m} = \frac{V_1}{\sqrt{I_0^2 - I_w^2}} \quad ...(21)$$

The no-load vector diagram is shown in Fig. 19.

Since the current is practically all-exciting current when a transformer is on no-load (*i.e.,* $I_0 = I_m$) and the voltage drop in primary leakage impedance is small, hence the exciting admittance Y_0 of the transformer is given by

$$I_0 = V_1 Y_0 \quad \text{or} \quad Y_0 = \frac{V_1}{I_0} \quad ...(22)$$

The exciting conductance $$G_0 = \frac{P_0}{V_1^2} \quad ...(23)$$

The exciting susceptance $$B_0 = \sqrt{Y_0^2 - G_0^2} \quad ...(24)$$

Separation of core losses. The core loss is made up of the following two parts :

(*i*) Eddy current loss.

(*ii*) Hysteresis loss.

Eddy current loss, $P_e = AB^2_{max} f^2$, where A is constant.

Hysteresis loss, $P_h = BB^{1.6}_{max} f$, where B is constant.

Total loss $= P_e + P_h = AB^2_{max} f^2 + BB^{1.6}_{max} f$.

The values of constants A and B can be found out by conducting two experiments using two different frequencies but the same maximum flux density ; thereafter eddy current and hysteresis loss can be found separately.

9.10.2. Short-circuit or impedance test (S.C. Test)

This test is conducted to determine the following :

(*i*) Full-load copper loss.

(*ii*) Equivalent resistance and reactance *referred to metering side*.

In this test (Fig. 39) the terminals of the secondary winding (*usually low voltage winding*) are short-circuited by a thick conductor or through an ammeter which may serve the additional purpose of indicating rated load current. A low voltage, usually 5 to 10% of normal primary voltage, at correct frequency is applied to the primary and *is continuously increased till full-load currents flow in the primary as well as secondary windings* (as indicated by the respective ammeters).

Since applied voltage is very low so flux linking with the core is very small and therefore, iron losses are so small that these can be neglected, the reading of the wattmeter gives total copper losses at full-load.

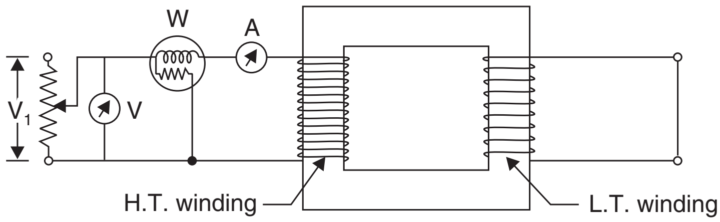


Fig. 39. Short-circuit test.

The equivalent circuit of the transformer under short-circuit condition is shown in Fig. 40.

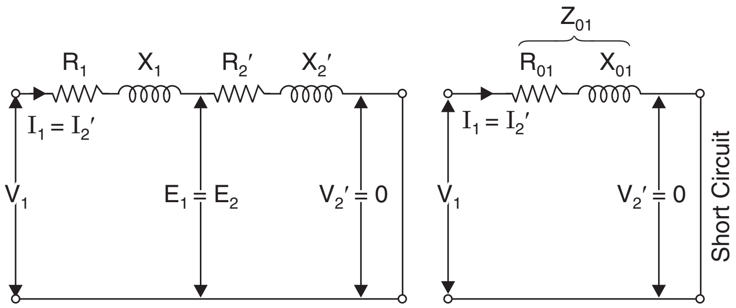


Fig. 40. Equivalent circuit of transformer under short-circuit condition.

Let V_{SC} = voltage required to circulate rated load currents
I_1 = reading of the ammeter on the primary side
Z_{01} = total impedance as referred to primary side
R_{01} = total resistance as referred to primary side
X_{01} = total reactance as referred to primary side.

Then, equivalent impedance as referred to primary side,

$$Z_{01} = \frac{V_{SC}}{I_1} \quad ...(25)$$

Also $$P = I_1^2 R_{01}$$

$\therefore$ $$R_{01} = \frac{P}{I_1^2} \quad ...(26)$$

and $$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} \quad ...(27)$$

9.11. Regulation of a Transformer

- Due to the resistances of the windings and leakage reactances voltage drop takes place in a transformer. Accordingly the output voltage under load conditions is different from the output voltage under no-load conditions. **Voltage regulation** is defined as :

"*The change in secondary voltage when rated load at a specified power is removed*".

It is specified as a percentage of the rated secondary voltage.

Thus, if $_0V_2$ = secondary terminal voltage at *no-load*

$= E_2 = KE_1 = KV_1$ because at no-load the impedance drop is negligible

V_2 = secondary terminal voltage on *full-load*

Then, % regulation (down) $= \dfrac{_0V_2 - V_2}{_0V_2} \times 100$ ...(28)

% regulation (up) $= \dfrac{_0V_2 - V_2}{V_2} \times 100$ ...(29)

For calculating the regulation it is convenient to refer the total resistance and reactance to the secondary side.

- Refer to Fig. 32.

% regulation $= \left(\dfrac{I_2R_{02} \cos \phi \pm I_2X_{02} \sin \phi}{_0V_2}\right) \times 100$ (app.) ...(30)

[+ sign for *lagging* power factor
– sign for *leading* power factor]

The less this value, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load.

The regulation may also be expressed in terms of primary values as follows :

% regulation $= \dfrac{V_1 - V_2'}{V_1} \times 100$ (neglecting angle between V_1 and V_2')

$= \left(\dfrac{I_1R_{01} \cos \phi \pm I_1X_0 \sin \phi}{V_1}\right) \times 100$ ...(31)

- In the above definitions of regulation, *primary voltage was supposed to be kept constant* and changes in secondary terminal were considered. But as the transformer is loaded, the secondary terminal voltage falls (for lagging power factor) ; hence to keep the circuit voltage constant, the primary voltage must be increased. The rise *in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer*.

If the primary voltage has to be raised from its rated value V_1 to V_1', then

% regulation $= \dfrac{V_1' - V}{V_1} \times 100$ ...[31 (*a*)]

9.12. Percentage Resistance and Reactance

- Invariably the equivalent resistance and reactance of a transformer are expressed in percent.

Percentage resistance is the *resistance drop in volts at rated current and frequency as a percentage of the rated voltage i.e.*,

$$\text{Percent } R = \frac{IR}{V} \times 100 \quad ...(i)$$

Percentage reactance is the *reactance drop in volts at rated current and frequency expressed as a percentage of the rated voltage i.e.*,

$$\text{Percent } X = \frac{IX}{V} \times 100 \quad ...(ii)$$

and

$$\text{Percent } Z = \frac{IZ}{V} \times 100 \quad ...(iii)$$

The important **advantage** of expressing resistance and reactance of a transformer in percentage is that the *percentage resistance and percentage reactance have same values whether determined referred to primary or secondary whereas when expressed in ohms they have different values when referred to primary and secondary.*

Further, *percent reactances and impedances are convenient for computing the current that will flow in a transformer when the secondary is accidentally shorted.* When the secondary is shorted, $I_1 = \frac{E_1}{Z_0}$ or $E_1 = I_1Z_0$ *i.e.*, when secondary is shorted, I_1Z_0 is 100 percent of E_1. Therefore, the short-circuit current is given by the equation,

$$\text{Short circuit current ; } I_{SC} = \frac{100}{\text{Percent impedance}} \times \text{full load rated current} \quad ...(iv)$$

Unless the transformer is quite small, equivalent resistance is negligible and percent reactance may be substituted for percent impedance in the above equation (*iv*).

- **Another method of designating the resistances and reactances** is by the **per unit values**. The per unit values are equal to the percentage values divided by 100.

$$\therefore \quad \text{Per unit } R = \frac{IR}{V} \quad ...(v)$$

and

$$\text{Per unit } X = \frac{IX}{V} \quad ...(vi)$$

9.13. Transformer Losses

The losses in a transformer are classified as follows :

1. Iron losses (or core losses).
2. Copper losses.

1. **Iron or core losses.** It includes *hysteresis loss* and *eddy current loss*.

(*i*) **Hysteresis loss.** Since the flux in a transformer core is alternating, power is required for the continuous reversals of the elementary magnets of which the iron is composed. This loss is known as *hysteresis loss.*

$$\text{Hysteresis loss} = K_h f B_{max}^{1.6} \quad ...(32)$$

where f is the frequency in Hz, B_{max} is the maximum flux density in core and K_h is a constant.

(*ii*) **Eddy current loss.** This is due to the flow of eddy currents in the core. Thin laminations, insulated from each other, reduce the eddy current loss to small proportion.

$$\text{Eddy current loss} = K_e f^2 B_{max}^2 \quad ...(33)$$

where K_e is a constant.

Iron or core loss is found from open circuit test. The input of the transformer when on no-load measures the core loss.

2. **Copper losses.** These losses are due to the ohmic resistance of the transformer windings.

$$\text{Total copper loss} = I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$$

These losses, as is evident, are proportional to square of the current (or kVA)^2.

The value of copper losses is found from the *short-circuit test*.

9.14. Transformer Efficiency

The efficiency of a transformer at a particular load and power factor is defined as *the ratio of power output to power input*.

$$\therefore \quad \text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$= \frac{\text{output}}{\text{output + losses}} = \frac{\text{output}}{\text{output + cu loss + iron loss}} \quad ...(34)$$

or
$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}} \quad ...(35)$$

It may be noted that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to volt-amperes. Hence at any volt-ampere load, the efficiency depends on power factor, being *maximum at unity power factor*.

Efficiency can be calculated by determining core losses from open-circuit test and copper losses from short-circuit test.

Condition for maximum efficiency :

Iron losses, P_i = hysteresis loss + eddy current loss

$= P_h + P_e$

Copper losses, $P_c = I_1^2 R_{01}$ or $I_2^2 R_{02}$

Considering primary side :

Input to primary $= V_1 I_1 \cos \phi_1$

Efficiency,
$$\eta = \frac{V_1 I_1 \cos \phi_1 - \text{losses}}{V_1 I_1 \cos \phi_1} = \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - P_i}{V_1 I_1 \cos \phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{P_i}{V_1 I_1 \cos \phi_1}$$

Differentiating both sides w.r.t. I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi_1} + \frac{P_i}{V_1 I_1^2 \cos \phi_1}$$

For η to be maximum,

$\frac{d\eta}{dI_1} = 0$. Hence the above equation reduces to

$$\frac{R_{01}}{V_1 \cos \phi_1} = \frac{P_i}{V_1 I_1^2 \cos \phi_1}$$

or
$$P_i = I_1^2 R_{01} \text{ or } I_2^2 R_{02} \quad ...(36)$$

or **Copper losses = Iron losses**

The output current corresponding to maximum efficiency is

$$I_2 = \sqrt{\frac{P_i}{R_{02}}} \quad ...(37)$$

By proper design it is possible to make the maximum efficiency occur at any desired load.

Variation of efficiency with power factor. We know that transformers efficiency,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{(V_2 I_2 \cos \phi + \text{losses})}$$

Let,
$$\frac{\text{losses}}{V_2 I_2} = \beta$$

$\therefore$
$$\eta = 1 - \left(\frac{\text{losses}/V_2 I_2}{\cos \phi + \text{losses}/V_2 I_2} \right) \text{ or } = 1 - \frac{\beta}{(\cos \phi + \beta)}$$

or $$\eta = 1 - \frac{(\beta/\cos\phi)}{1+(\beta/\cos\phi)} \qquad ...(38)$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Fig. 41.

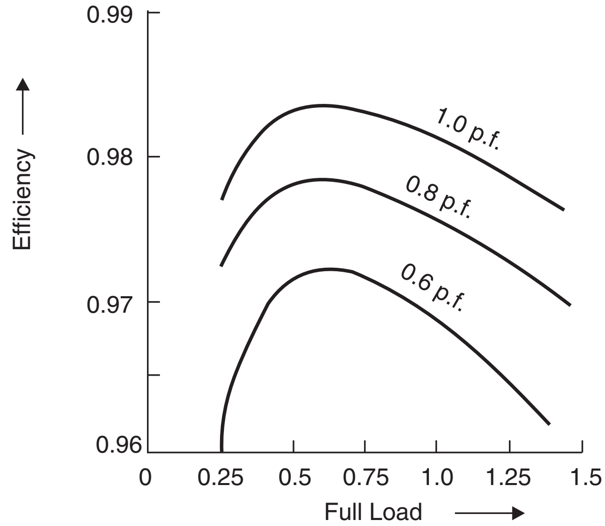


Fig. 41. Variations of efficiency with power factor at different loadings.

Transformer on Load

☞**Example 10.** *A 230 V/115 V single-phase transformer takes a no-load current of 2 A at a power factor of 0.2 lagging with low voltage winding kept open. If the low voltage winding is now loaded to take a current of 15 A at 0.8 power factor lagging find the current taken by high voltage winding.*

Solution. Primary, $E_1 = V_1 = 230$ V

Secondary, $E_2 = V_2 = 115$ V

No-load current $I_0 = 2$ A

No-load power factor, $\cos\phi_0 = 0.2$ or $\phi_0 = 78.46°$ or $78° 29'$

Load power factor, $\cos\phi_2 = 0.8$ or $\phi_2 = 36.9°$ or $36° 54'$

Current taken by h.v. winding, I_1 :

Now, transformation ratio,

$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{115}{230} = \frac{1}{2}$$

∴ Secondary current referred to primary,

$$I_2' = KI_2 = \frac{1}{2} \times 15 = 7.5 \text{ A}$$

Phase angle, between I_0 and I_2'

$= 78.46° - 36.9° = 41.56°$ or $41° 34'$

Using parallelogram law of forces (Fig. 42), we get,

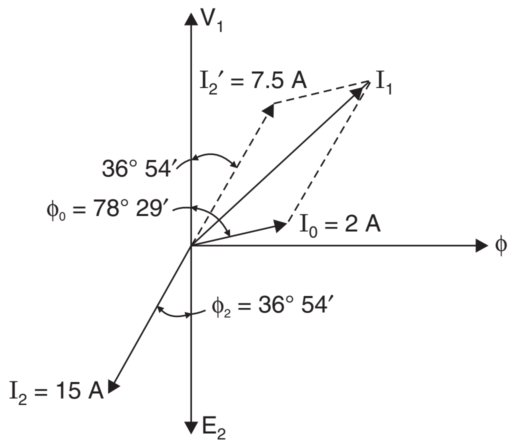


Fig. 42

$$I_1 = \sqrt{2^2 + 7.5^2 + 2 \times 2 \times 7.5 \times \cos 41.56°}$$
$$= 9.09 \text{ A}$$

Hence, **current taken by h.v. winding,**

$$\mathbf{I_1 = 9.09\ A.} \quad \textbf{(Ans.)}$$

Example 11. *The number of turns on the primary and secondary windings of a transformer are 1000 and 200 respectively. When the load current on the secondary is 100 A at 0.8 power factor lagging, the primary current is 30 A at 0.707 power factor lagging. Determine the no-load current of the transformer and its phase with respect to the voltage.*

Solution. Primary turns, $N_1 = 1000$

Secondary turns, $N_2 = 200$

Secondary current, $I_2 = 100$ A

Power factor, $\cos \phi_2 = 0.8$ or $\phi_2 = 36.9°$ or $36° \, 54'$

Primary current, $I_1 = 30$ A

Power factor, $\cos \phi_1 = 0.707$ or $\phi_1 = 45°$

No-load current, I_0, ϕ_0 :

Transformation ratio, $K = \dfrac{200}{1000} = \dfrac{1}{5}$

Secondary current referred to primary,

$$I_2' = KI_2 = \frac{1}{5} \times 100 = 20 \text{ A}$$

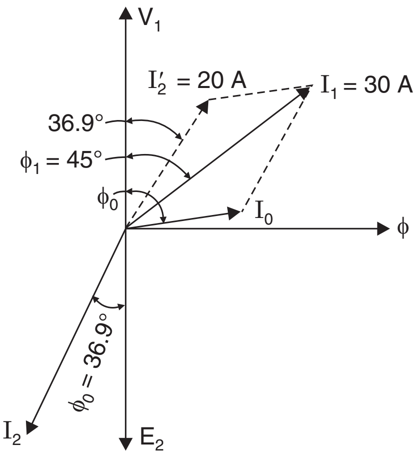


Fig. 43

Refer Fig. 43. I_1 is the vector sum of I_0 and I_2'. Let I_0 lag behind V_1 by an angle ϕ_0.

Resolving currents into their X-and Y-components, we get

$$I_0 \cos \phi_0 + 20 \cos 36.9° = 30 \cos 45°$$

$\therefore$ $$I_0 \cos \phi_0 = 30 \cos 45° - 20 \cos 36.9°$$
$$= 21.21 - 16 = 5.21 \text{ A} \quad ...(i)$$

$$I_0 \sin \phi_0 + 20 \sin 36.9° = 30 \sin 45°$$

$\therefore$ $$I_0 \sin \phi_0 = 30 \sin 45° - 20 \sin 36.9°$$
$$= 21.21 - 12 = 9.21 \text{ A} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\tan \phi_0 = \frac{9.21}{5.21} = 1.767$$

$\therefore$ $$\phi_0 = 60.5°$$

Putting the value of ϕ_0 in (*i*), we get

$$I_0 \cos 60.5° = 5.21$$

$\therefore$ $$I_0 = \frac{5.21}{\cos 60.5°} = 10.58 \text{ A}$$

Hence, **no-load current = 10.58 A**

and $\boldsymbol{\phi_0}$ **= 60.5°. (Ans.)**

Example 12. *A 30 kVA, 2000/200 V, single-phase, 50 Hz transformer has a primary resistance of 3.5 Ω and reactance of 4.5 Ω. The secondary resistance and reactance are 0.015 Ω and 0.02 Ω respectively. Find :*

(*i*) *Equivalent resistance, reactance and impedance referred to primary.*

(*ii*) *Equivalent resistance, reactance and impedance referred to secondary.*

(*iii*) *Total copper loss of the transformer.*

Solution. Primary resistance, $R_1 = 3.5\ \Omega$

Primary reactance, $X_1 = 4.5\ \Omega$

Secondary resistance, $R_2 = 0.015\ \Omega$

Secondary reactance, $X_2 = 0.02\ \Omega$

Transformation ratio, $K = \dfrac{200}{2000} = \dfrac{1}{10}$

(*i*) **Equivalent resistance *referred to primary*,**

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 3.5 + \frac{0.015}{\left(\frac{1}{10}\right)^2} = \mathbf{5.0\ \Omega.} \quad \textbf{(Ans.)}$$

Equivalent reactance *referred to primary,*

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 4.5 + \frac{0.02}{\left(\frac{1}{10}\right)^2} = \mathbf{6.5\ \Omega.} \quad \textbf{(Ans.)}$$

Equivalent impedance *referred to primary*,

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2} = \sqrt{(5)^2 + (6.5)^2} = \mathbf{8.2\ \Omega.} \quad \textbf{(Ans.)}$$

(*ii*) **Equivalent resistance *referred to secondary*,**

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

$$= 0.015 + \left(\frac{1}{10}\right)^2 \times 3.5 = \mathbf{0.05\ \Omega.} \quad \textbf{(Ans.)}$$

Equivalent reactance *referred to secondary*,

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.02 + \left(\frac{1}{10}\right)^2 \times 4.5 = \mathbf{0.065\ \Omega.} \quad \textbf{(Ans.)}$$

Equivalent impedance *referred to secondary*,

$$Z_{02} = \sqrt{(R_{02})^2 + (X_{02})^2} = \sqrt{(0.05)^2 + (0.065)^2} = \mathbf{0.082\ \Omega.} \quad \textbf{(Ans.)}$$

(*iii*) **Total copper loss :**

Secondary current $= \dfrac{30 \times 1000}{200} = 150$ A

Total copper loss $= I_2^2 R_{02} = 150^2 \times 0.05 =$ **1125 W. (Ans.)**

☞**Example 13.** *A single-phase transformer has the following data :*

Turn ratio 20 : 1 ; $R_1 = 20\ \Omega$, $X_1 = 80\ \Omega$; $R_2 = 0.04\ \Omega$; $X_2 = 0.2\ \Omega$. *No-load current = 1.2 A leading the flux by 30°.*

The secondary delivers 180 A at a terminal voltage of 400 V and at a power factor of 0.8 lagging. Determine by the aid of a vector diagram :

(*i*) *The primary applied voltage.* (*ii*) *The primary power factor.*

(*iii*) *The efficiency.*

Solution. Refer to Fig. 44 :

(*i*) **Primary applied voltage, V_1 :**

Taking V_2 as the reference vector

$\therefore \quad V_2 = 400 \angle 0° = 400 + j0$

$I_2 = 180(0.8 - j0.6) = 144 - j108$

$Z_2 = (0.04 + j0.2)$

$E_2 = V_2 + I_2Z_2 = (400 + j0) + (144 - j108) \times (0.04 + j0.2)$

$= 400 + (5.76 + j28.8 - j4.32 + 21.6)$

$= (427.36 + j24.48) = 428.1 \angle 3.28°.$

Obviously,

$\beta = 3.28°$

$$E_1 = \frac{E_2}{K} = 20E_2 = 20(427.36 + j24.48) = 8547 + j490$$

$\therefore \quad -E_1 = -8547 - j490 = 8561 \angle 183.28°$

Secondary current referred to primary,

$$I_2' = -KI_2 = \frac{(-144 + j108)}{20} = -7.2 + j5.4$$

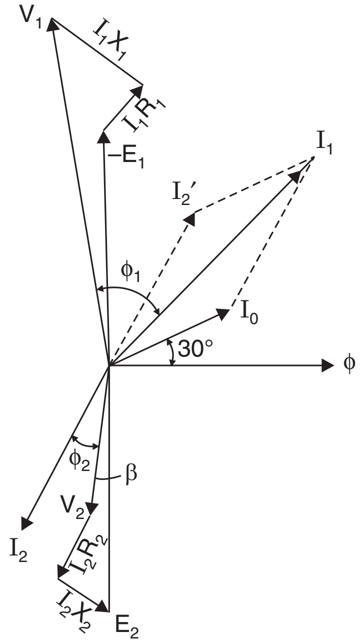


Fig. 44

As seen from Fig. 44, I_0 leads V_2 by an angle

$3.28° + 90° + 30° = 123.28°$

$\therefore \quad I_0 = 1.2 \angle 123.28° = 1.2(\cos 123.28° + j \sin 123.28°)$

$= 1.2(-0.548 + j0.836) = -0.657 + j1.003.$

Primary current,

$I_1 = -I_2' + I_0 = (-7.2 + j5.4) + (-0.657 + j1.003)$

$= -7.857 + j6.403 = 10.14 \angle 140.8°$

$V_1 = -E_1 + I_1Z_1 = (-8547 - j490) + (-7.857 + j6.403)(20 + j80)$

$= -8547 - j490 + (-157.14 - j628.56 + j128.06 - 512.24)$

$= -9216.38 - j990.5 =$ **$9269 \angle 186.13°$. (Ans.)**

(*ii*) **Primary power factor, $\cos \phi_1$:**

Phase angle between V_1 and I_1,

$\phi_1 = 186.13° - 140.8° = 45.33°$

$\therefore$ Primary power factor $= \cos 45.33° =$ **0.703 (lag). (Ans.)**

(*iii*) **Efficiency :**

No-load primary input power

$= V_1I_1 \cos \phi_0 = 9269 \times 1.2 \times \cos 60° = 5561.4$ W

$$R_{02} = R_2 + K^2R_1 = 0.04 + \left(\frac{1}{20}\right)^2 \times 20 = 0.09\ \Omega$$

Total copper losses as referred to secondary

$= I_2^2R_{02} = (180)^2 \times 0.09 = 2916$ W

Output $= V_2I_2 \cos \phi_2 = 400 \times 180 \times 0.8 = 57600$ W

Total losses $= 5561.4 + 2916 = 8477.4$ W

Input $=$ Output $+$ Losses $= 57600 + 8477.4 = 66077.4$ W

$$\therefore \quad \text{Efficiency, } \eta = \frac{\text{output}}{\text{input}} = \frac{57600}{66077.4} = \mathbf{0.872} \text{ or } \mathbf{87.2\%}. \quad \textbf{(Ans.)}$$

Example 14. *The high voltage and low voltage windings of a 2200/220 V single-phase 50 Hz transformer has resistances of 4.8 Ω and 0.04 Ω and reactances 2 Ω and 0.018 Ω respectively. The low voltage winding is connected to a load having an impedance of (6 + j4) Ω. Determine* :

(*i*) *Current in l.v. winding,* (*ii*) *Current in h.v. winding,*

(*iii*) *Load voltage, and* (*iv*) *Power consumed by the load.*

Solution. Primary resistance, $R_1 = 4.8\ \Omega$

Primary reactance, $X_1 = 2\ \Omega$

Secondary resistance, $R_2 = 0.04\ \Omega$.

Secondary reactance, $X_2 = 0.018\ \Omega$

Impedance of load, $Z_L = (6 + j4)$

Transformation ratio, $K = \frac{E_2}{E_1} = \frac{220}{2200} = \frac{1}{10}$

Equivalent resistance referred to l.v. side,

$$R_{02} = R_2 + K^2R_1 = 0.04 + \left(\frac{1}{10}\right)^2 \times 4.8 = 0.088\ \Omega$$

Equivalent reactance referred to l.v. side,

$$X_{02} = X_2 + K^2X_1 = 0.018 + \left(\frac{1}{10}\right)^2 \times 2 = 0.038\ \Omega$$

Equivalent impedance referred to l.v. side,

$$Z_{02} = R_{02} + jX_{02} = (0.088 + j0.038)\ \Omega$$

(*i*) **Current in l.v. winding,**

$$I_2 = \frac{220}{Z_{02} + Z_L} = \frac{220}{(0.088 + j0.038) + (6 + j4)}$$

$$= \frac{220}{6.088 + j4.038} = \frac{220}{\sqrt{(6.088)^2 + (4.038)^2}}$$

$$= \mathbf{30.11\ A.}\quad \textbf{(Ans.)}$$

(*ii*) **Current in h.v. winding,**

$$I_1 \simeq I_2' = KI_2 = \frac{1}{10} \times 30.11 = \mathbf{3.011\ A.}\quad \textbf{(Ans.)}$$

(*iii*) **Load voltage,** $= I_2Z_L = 30.11(6 + j4) = 30.11 \times \sqrt{(6)^2 + (4)^2}$

$$= \mathbf{217.12\ V.}\quad \textbf{(Ans.)}$$

(*iv*) **Power consumed by the load,**

$$= I^2R_L = 30.11^2 \times 6 = \mathbf{5439.6\ W.}\quad \textbf{(Ans.)}$$

☞**Example 15.** *A single-phase transformer has $Z_1 = 1.4 + j5.2\ \Omega$ and $Z_2 = 0.0117 + j0.0465\ \Omega$. The input voltage is 6600 V and the turn ratio is 10.6 : 1. The secondary feeds a load which draws 300 A at 0.8 power factor lagging. Find the secondary terminal voltage and the kW output. Neglect no-load current I_0.*

Solution. Impedance $Z_1 = 1.4 + j5.2\ \Omega$

Impedance $Z_2 = 0.0117 + j0.0465\ \Omega$

Input voltage $V_1 = 6600$ V

Turn ratio, $K = 10.6 : 1$

Secondary load current, $I_2 = 300$ A

Power factor, $\cos \phi_2 = 0.8$

Secondary terminal voltage, V_2 :

kW output :

Let the secondary terminal voltage = $V_2 \angle 0°$

$$I_2 = 300 \angle -36.87° \text{ A}$$
$$E_2 = V_2 \angle 0° + I_2 Z_2$$
$$= V_2 \angle 0° + (300 \angle -36.87°)(0.0117 + j0.0465) = V_2 \angle 0° + 14.4 \angle 39°$$
$$E_1 = 10.6E_2 = 10.6V_2 \angle 0° + 152.64 \angle 39°$$
$$I_1 = \frac{300}{10.6} \angle -36.87° = 28.3 \angle -36.87°$$
$$V_1 = E_1 + I_1 Z_1$$
$$= 10.6V_2 \angle 0° + 152.64 \angle 39° + (28.3 \angle -36.87°)(1.4 + j5.2)$$
$$= 10.6V_2 + 152.64 \angle 39° + 152.4 \angle 38°$$
$$= 10.6V_2 + 118.62 + j96.06 + 120 + j93.83$$
$$= 10.6V_2 + 238.62 + j189.89$$

Since $|V_1| = 6600$ V

$\therefore \quad (10.6V_2 + 238.62)^2 + (189.89)^2 = 6600^2$

or $V_2 \simeq 600$ V

Hence, **secondary terminal voltage = 600 V. (Ans.)**

Output :

Output $= V_2 I_2 \cos \phi_2 = 600 \times 300 \times 0.8 =$ **144000 W** or **144 kW. (Ans.)**

Example 16. *The full-load copper loss on h.v. side of 100 kVA, 11000/317 V 1-phase transformer is 0.62 kW and on the l.v. side is 0.48 kW.*

(*i*) *Calculate R_1, R_2 and R_2' in ohms :*

(*ii*) *The total reactance is 4 percent, find X_1, X_2 and X_2' in ohms if the reactance is divided in the same proportion as resistance.*

Solution. *Given* : Rated kVA = 100 ; V_1 = 11000 V ; V_2 = 317 V ; loss on *h.v.* side = 0.62 kW ; loss of *l.v.* side = 0.48 kW ; total reactance = 4%.

(*i*) **R_1, R_2 and R_2' :**

h.v. side full-load current, $I_1 = \dfrac{\text{rated kVA} \times 1000}{V_1} = \dfrac{100 \times 1000}{11000} = 9.09$ A

l.v. side full-load current, $I_2 = \dfrac{\text{rated kVA} \times 1000}{V_2} = \dfrac{100 \times 1000}{317} = 315.46$ A

Primary winding resistance, $\mathbf{R_1} = \dfrac{\text{full-load copper loss in primary winding}}{I_1^2}$

$$= \frac{0.62 \times 1000}{(9.09)^2} = \mathbf{7.5\ \Omega}. \quad \textbf{(Ans.)}$$

Secondary winding resistance, $\mathbf{R_2} = \dfrac{\text{full-load copper loss in secondary winding}}{I_2^2}$

$$= \frac{0.48 \times 1000}{(315.46)^2} = \mathbf{0.00482\ \Omega}. \quad \textbf{(Ans.)}$$

Transformation ratio, $K = \dfrac{V_2}{V_1} = \dfrac{317}{11000} = 0.02882$

Secondary resistance referred to primary,

$$R_2' = \frac{R_2}{K^2} = \frac{0.00482}{(0.02882)^2} = \mathbf{5.8\ \Omega.}\quad \textbf{(Ans.)}$$

(*ii*) $\mathbf{X_1, X_2}$ **and** $\mathbf{X_2'}$ **:**

Percentage reactance referred to *h.v.* side $= \dfrac{I_1 X_{01}}{V_1} \times 100 = 4$

or transformer total reactance referred to primary,

$$X_{01} = \frac{4 \times V_1}{100 I_1} = \frac{4 \times 11000}{100 \times 9.09} = 48.4\ \Omega.$$

Now, $$\frac{R_1}{R_2'} = \frac{X_1}{X_2'} \qquad \text{...(Given)}$$

or $$\frac{R_1 + R_2'}{R_2'} = \frac{X_1 + X_2'}{X_2'}$$

or $$\frac{7.5 + 5.8}{5.8} = \frac{48.4}{X_2'} \qquad (\because\ X_1 + X_2' = X_{01})$$

or $$\mathbf{X_2'} = \frac{48.4 \times 5.8}{(7.5 + 5.8)} = \mathbf{21.1\ \Omega.}\quad \textbf{(Ans.)}$$

Primary winding reactance, $\mathbf{X_1} = X_{01} - X_2' = 48.4 - 21.1 = \mathbf{27.3\ \Omega.}$ **(Ans.)**

Secondary winding reactance, $\mathbf{X_2} = K^2 X_2' = \left(\dfrac{377}{11000}\right)^2 \times 21.1 = \mathbf{0.0175\ \Omega.}$ **(Ans.)**

Hysteresis and Eddy Current Losses

Example 17. *A 230 V, 2.5 kVA single-phase transformer has an iron loss of 100 W at 40 Hz and 70 W at 30 Hz. Find the hysteresis and eddy current losses at 50 Hz.*

Solution. Iron loss at 40 Hz = 110 W

Iron loss at 30 Hz = 75 W

Hysteresis and eddy current loss at 50 Hz :

We know that hysteresis loss,

$$P_h \propto f = Af$$

Eddy current loss, $P_e \propto f^2 = Bf^2$

Iron loss, $P_i = P_h + P_e = Af + Bf^2$

At 40 Hz :

$$100 = 40A + (40)^2 B$$

i.e., $40A + 1600B = 100$

or $A + 40B = 2.5$ ...(*i*)

At 30 Hz :

$$70 = 30A + (30)^2 B$$

i.e., $30A + 900B = 70$

or $3A + 90B = 7$ ...(*ii*)

Solving (*i*) and (*ii*), we get $A = \dfrac{11}{6}$, $B = \dfrac{1}{60}$.

Hysteresis loss at 50 Hz

$$= Af = \frac{11}{6} \times 50 = \mathbf{91.67\ W.} \quad \textbf{(Ans.)}$$

Eddy current loss at 50 Hz

$$= Bf^2 = \frac{1}{60} \times 50^2 = \mathbf{41.67\ W.} \quad \textbf{(Ans.)}$$

Example 18. *When a transformer is supplied at 400 V, 50 Hz the hysteresis loss is found to be 310 W and eddy current loss is found to be 260 W. Determine the hysteresis loss and eddy current loss when the transformer is supplied at 800 V, 100 Hz.*

Solution. Hysteresis loss at 400 V, 50 Hz = 310 W

Eddy current loss at 400 V, 50 Hz = 260 W

We know that,

Induced e.m.f., $E = 4.44 f \phi_{max} N = 4.44 (B_{max} A) f N$

i.e. $E \propto B_{max} f$

or $B_{max} \propto \frac{E}{f} = a \frac{E}{f}$ [a = constant]

In the **Ist case,** $B_{max_1} = a \cdot \frac{E_1}{f_1} = \frac{400}{50} a = 8a.$

In the **IInd case,** $B_{max_2} = a \frac{E_2}{f_2} = \frac{800}{100} a = 8a$

Hence, $B_{max_1} = B_{max_2}$

[$\because$ Core area A, and number of turns N remain the same]

Hence, $P_h = Af$ and $P_e = Bf^2$

$P_{h1} = 310$ W at $f_1 = 50$ Hz

$\therefore$ $A = \frac{310}{50} = 6.2$

Hysteresis loss at 100 Hz

$$= Af_2 = 6.2 \times 100 = \mathbf{620\ W.} \quad \textbf{(Ans.)}$$

$P_{e1} = 260$ W at $f_1 = 50$ Hz

$\therefore$ $B = \frac{260}{50^2} = 0.104$

Eddy current loss at 100 Hz

$$= Bf_2^2 = 0.104 \times 100^2 = \mathbf{1040\ W.} \quad \textbf{(Ans.)}$$

Transformer Tests (O.C. and S.C.) and Equivalent Circuit

☞**Example 19.** *A 50 Hz, single-phase transformer has a turn ratio of 5. The resistances are 0.8 Ω, 0.02 Ω and reactances are 4 Ω and 0.12 Ω for high-voltage and low-voltage windings respectively. Find :*

(*i*) *The voltage to be applied to the h.v. side to obtain full-load current of 180 A in the l.v. winding on short-circuit.*

(*ii*) *The power factor on short-circuit.*

Draw the equivalent circuit and vector diagram.

Solution. Turn ratio, $\frac{N_1}{N_2} = 5, K = \frac{1}{5}$

Primary resistance, $R_1 = 0.8\ \Omega$

Primary reactance, $X_1 = 4\ \Omega$

Secondary resistance, $R_2 = 0.02\ \Omega$

Secondary reactance, $X_2 = 0.12\ \Omega$

Total resistance referred to primary,

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.8 + 0.02 \times (5)^2 = 0.8 + 0.5 = 1.3\ \Omega$$

Total reactance referred to secondary,

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 4 + 0.12(5)^2 = 4 + 3 = 7\ \Omega$$

Total impedance referred to primary,

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(1.3)^2 + (7)^2} = 7.12 \text{ ohm}$$

Also $$I_1 = KI_2 = \frac{1}{5} \times 180 = 36 \text{ A}$$

(*i*) **Voltage to be applied** on h.v. side to obtain full-load current I_1,

$$V_{SC} = I_1 Z_{01} = 36 \times 7.12 = \mathbf{256.3\ V.} \quad \textbf{(Ans.)}$$

(*ii*) The **power factor** on short-circuit,

$$\cos \phi_0 = \frac{R_{01}}{Z_{01}} = \frac{1.3}{7.12} = \mathbf{0.1825.} \quad \textbf{(Ans.)}$$

Equivalent circuit and vector diagram are shown in Figs. 45, 46 and 47.

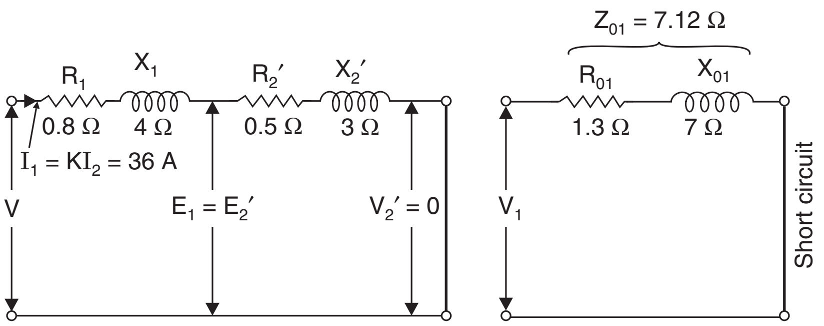


Fig. 45

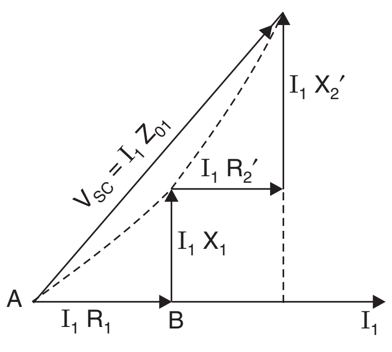


Fig. 46

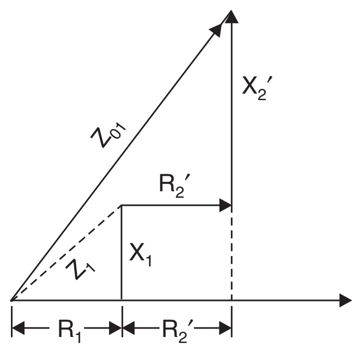


Fig. 47

Example 20. *A 12 kVA 4000/400 V transformer has primary and secondary winding resistances of 13 Ω and 0.15 Ω and leakage reactances of 20 Ω and 0.25 Ω respectively. The magnetising reactance is 6000 Ω and the resistance representing core loss is 12000 Ω. Determine* :

(*i*) *Equivalent resistance and reactance as referred to primary.*

(*ii*) *Input current with secondary terminals open circuited.*

(*iii*) *Input current when the secondary load current is 30 A at 0.8 power factor lagging.*

Solution. Transformation ratio, $K = \frac{400}{4000} = \frac{1}{10}$

Primary resistance, $R_1 = 13\ \Omega$

Primary reactance, $X_1 = 20\ \Omega$

Secondary resistance, $R_2 = 0.15\ \Omega$

Secondary reactance, $X_2 = 0.25\ \Omega$

Magnetising reactance, $X_0 = 6000\ \Omega$

Resistance representing the core loss, $R_0 = 12000\ \Omega$

(*i*) **Equivalent resistance referred to primary,**

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$= 13 + \frac{0.15}{\left(\frac{1}{10}\right)^2} = 13 + 15 = \mathbf{28\ \Omega.} \quad \textbf{(Ans.)}$$

Equivalent reactance referred to primary,

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$= 20 + \frac{0.25}{\left(\frac{1}{10}\right)^2} = 20 + 25 = \mathbf{45\ \Omega.} \quad \textbf{(Ans.)}$$

(*ii*) **Input current with secondary terminals open-circuited (or input current at no-load) :**

Magnetising component, $I_m = \frac{4000}{6000} = 0.667$ A

Wattful component, $I_w = \frac{4000}{12000} = 0.333$ A

Hence input current at no-load,

$$I_0 = 0.333 - j0.667 = \mathbf{0.745 \angle -63.5° \ A.} \quad \textbf{(Ans.)}$$

(*iii*) **Input current $\mathbf{I_1}$:**

Secondary output current, $I_2 = 30$ A (*given*)

Power factor, $\cos \phi_2 = 0.8$ (lagging)

i.e., $I_2 = 30 \angle -36.9°$

Secondary current referred to primary,

$$I_2' = KI_2 = \frac{30}{10} \angle -36.9° = 3 \angle -36.9° = 2.4 - j1.8$$

∴ Input current, $I_1 = I_2' + I_0 = (2.4 - j1.8) + (0.333 - j0.667)$

$$= 2.733 - j2.467 = \mathbf{3.68 \angle -42.07° \ A.} \quad \textbf{(Ans.)}$$

☞**Example 21.** *Obtain the approximate equivalent circuit of a given 200/2000 V single-phase 30 kVA transformer having the following test results :*

O.C. test : 200 V, 6.2 A, 360 W on l.v. side

S.C. test : 75 V, 18 A, 600 W on h.v. side.

Solution. O.C. test (l.v. side)

Primary voltage, $V_1 = 200$ V

No-load current, $I_0 = 6.2$ A

No-load loss, $P_0 = 360$ W

Now, $P_0 = V_1 I_0 \cos \phi_0$

$360 = 200 \times 6.2 \times \cos \phi_0$

$$\therefore \quad \cos \phi_0 = \frac{360}{200 \times 6.2} = 0.29$$

$\sin \phi_0 = 0.957$

Wattful component of no-load current,

$$I_w = I_0 \cos \phi_0 = 6.2 \times 0.29 = 1.8 \text{ A.}$$

Resistance representing the core loss,

$$R_0 = \frac{V_1}{I_w} = \frac{200}{1.8} = \mathbf{111.11\ \Omega.} \quad \textbf{(Ans.)}$$

Magnetising component of no-load current,

$$I_m = I_0 \sin \phi_0 = 6.2 \times 0.95 = 5.93 \text{ A}$$

Magnetising reactance, $X_0 = \frac{V_1}{I_m} = \frac{200}{5.93} = \mathbf{33.7\ \Omega.}$ **(Ans.)**

S.C. test (h.v. side) :

Short-circuit voltage, $V_{SC} = 75$ V

Short-circuit current, $I_{SC} = 18$ A

Losses, $P_{SC} = 600$ W

Impedance of transformer referred to h.v. side,

$$Z_{02} = \frac{V_{SC}}{I_{SC}} = \frac{75}{18} = 4.167\ \Omega$$

$P_{SC} = I_{SC}^2 \times R_{02}$

$600 = 18^2 \times R_{02}$

$$\therefore \quad R_{02} = \frac{600}{18^2} = 1.85\ \Omega$$

Transformation ratio, $K = \frac{2000}{200} = 10$

Referred to 200 V side :

$$Z_{01} = \frac{Z_{02}}{K^2} = \frac{4.167}{(10)^2} = 0.04167\ \Omega$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{1.85}{(10)^2} = 0.0185\ \Omega$$

$\therefore \qquad X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.04167)^2 - (0.0185)^2}$

$= \mathbf{0.0373\ \Omega.}$ **(Ans.)**

Approximate equivalent circuit is shown in Fig. 48.

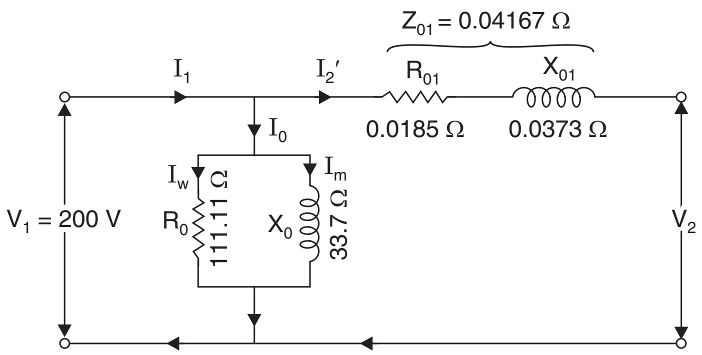


Fig. 48

Example 22. *The following readings were obtained on O.C. and S.C. tests on a 200/400 V, 50 Hz, single-phase transformer.*

O.C. test (l.v. side) : 200 V, 0.6 A, 60 W

S.C. test (h.v. side) : 15 V, 9 A, 80 W

Calculate the secondary voltage when delivering 4 kW at 0.8 power factor lagging, the primary voltage being 200 V.

Solution. O.C. test—l.v. side (*Instruments in the primary side* and secondary open)

Primary voltage, $V_1 = 200$ V

No-load current, $I_0 = 0.6$ A

No-load loss, $P_0 = 60$ W

Now, $P_0 = V_1 I_0 \cos \phi_0$

$60 = 200 \times 0.6 \times \cos \phi_0$

$\therefore \qquad \cos \phi_0 = \dfrac{60}{200 \times 0.6} = 0.5$

$\sin \phi_0 = 0.866$

Wattful component of no-load current

$I_w = I_0 \cos \phi_0 = 0.6 \times 0.5 = 0.3$ A

Magnetising component of no-load current,

$I_m = I_0 \sin \phi_0 = 0.6 \times 0.866 = 0.52$ A

$\therefore$ Resistance representing the core loss,

$R_0 = \dfrac{V_1}{I_w} = \dfrac{200}{0.3} = \mathbf{666.67\ \Omega.}$ **(Ans.)**

Magnetising reactance, $X_0 = \dfrac{V_1}{I_m} = \dfrac{200}{0.52} = \mathbf{384.6\ \Omega.}$ **(Ans.)**

S.C. test—h.v. side (*Instruments in the secondary side* and primary short-circuited)

Short circuit voltage, $V_{SC} = 15$ V

Short circuit current, $I_{SC} = 9$ A

Losses, $P_{SC} = 80$ W

Impedance of transformer referred to secondary,

$$Z_{02} = \frac{V_{SC}}{I_{SC}} = \frac{15}{9} = 1.667\ \Omega$$

Also, $P_{SC} = I_{SC}^2 \times R_{02}$

$$80 = 9^2 \times R_{02}$$

$\therefore$ $R_{02} = \frac{80}{81} = \mathbf{0.987\ \Omega.}$ **(Ans.)**

Referred to 200 V side :

Transformation ratio, $K = \frac{400}{200} = 2$

$$Z_{01} = \frac{Z_{02}}{K^2} = \frac{1.667}{2^2} = 0.417\ \Omega$$

$$R_{01} = \frac{R_{02}}{2^2} = \frac{0.987}{4} = 0.247\ \Omega$$

$\therefore$ $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.417)^2 - (0.247)^2} = 0.336\ \Omega$

Output, $\text{kVA} = \frac{4}{0.8} = 5$ kVA

Output current, $I_2 = \frac{5 \times 1000}{400} = \mathbf{12.5\ A.}$ **(Ans.)**

This value of I_2 is *approximate* because V_2 (which is to be calculated as yet) has been taken equal to 400 V (which in fact is equal to E_2 or V_2)

Now, $Z_{02} = 1.667\ \Omega$

$R_{02} = 0.987\ \Omega$

$\therefore$ $X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.667^2 - 0.987^2} = 1.343\ \Omega$

Total drop as referred to secondary

$$= I_2 (R_{02} \cos \phi_2 + X_{02} \sin \phi_2)$$

$$= 12.5(0.987 \times 0.8 + 1.343 \times 0.6) = 19.94 \text{ V}$$

$\therefore$ $V_2 = 400 - 19.94 \simeq 380$ V

Hence, **secondary voltage = 380 V. (Ans.)**

Approximate equivalent circuit is shown in Fig. 49.

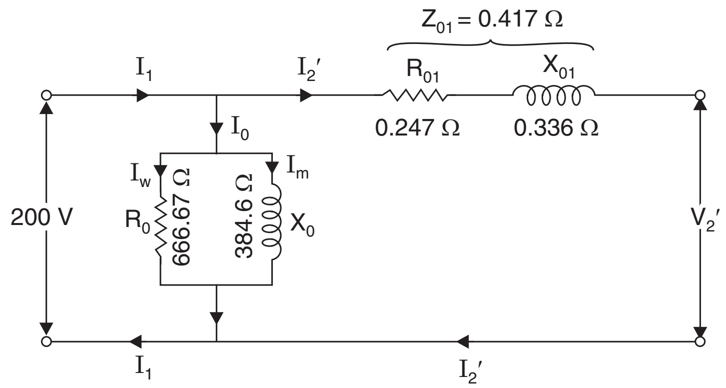


Fig. 49

Example 23. *A 4 kVA, 400/200 V, 50 Hz single-phase transformer has the following test data :*

O.C. test (l.v. side) 200 V, 1 A, 64 W

S.C. test (h.v. side) 15 V, 10 A, 80 W

Determine :

(*i*) *Equivalent circuit referred to l.v. side, and*

(*ii*) *Secondary load voltage on full-load at 0.8 power factor lagging.*

Solution. (*i*) **O.C. test—**l.v. side :

Voltage, $V_0 = 200$ V

No-load current, $I_0 = 1$ A

No-load loss, $P_0 = 64$ W

Now, $P_0 = V_0 I_0 \cos \phi_0$

$64 = 200 \times 1 \times \cos \phi_0$

$\therefore$ $\cos \phi_0 = 0.32$

$\sin \phi_0 = 0.9474$

Wattful component of no-load current,

$$I_w = I_0 \cos \phi_0 = 1 \times 0.32 = 0.32 \text{ A}$$

Magnetising component of no-load current,

$$I_m = I_0 \sin \phi_0 = 1 \times 0.9474 = 0.9474 \text{ A}$$

$\therefore$ Resistance representing the core loss,

$$R_0 = \frac{V_0}{I_w} = \frac{200}{0.32} = \mathbf{625\ \Omega}. \quad \textbf{(Ans.)}$$

Magnetising reactance, $X_0 = \dfrac{V_0}{I_m} = \dfrac{200}{0.9474} = \mathbf{211.1\ \Omega}.$ **(Ans.)**

S.C. test—h.v. side :

Short-circuit voltage, $V_{SC} = 15$ V

Short-circuit current, $I_{SC} = 10$ A

Losses, $P_{SC} = 80$ W

Impedance of the circuit referred to h.v. side,

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{15}{10} = \mathbf{1.5\ \Omega}. \quad \textbf{(Ans.)}$$

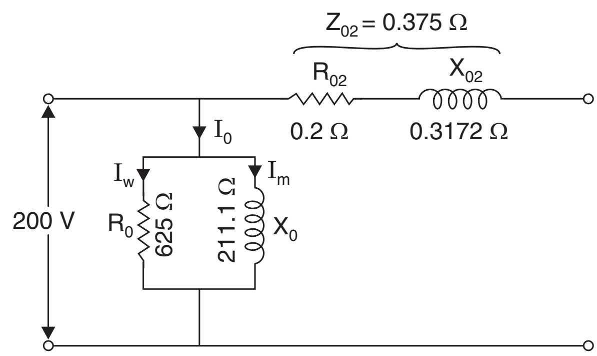


Fig. 50

Also $P_{SC} = I_{SC}^2 \times R_{01}$

$\therefore$ $80 = 10^2 \times R_{01}$

$\therefore$ $R_{01} = \dfrac{80}{100} = \mathbf{0.8\ \Omega}$. **(Ans.)**

Referred to l.v. side :

Transformation ratio, $K = \dfrac{200}{400} = \dfrac{1}{2}$

$$Z_{02} = K^2 Z_{01} = \left(\tfrac{1}{2}\right)^2 \times 1.5 = \mathbf{0.375\ \Omega}. \quad \textbf{(Ans.)}$$

$$R_{02} = K^2 R_{01} = \left(\tfrac{1}{2}\right)^2 \times 0.8 = \mathbf{0.2\ \Omega}. \quad \textbf{(Ans.)}$$

$\therefore$ $X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{(0.375)^2 - (0.2)^2} = \mathbf{0.3172\ \Omega}$. **(Ans.)**

The approximate equivalent circuit is shown in Fig. 52.

(*ii*) **Secondary load voltage, $\mathbf{V_2}$:**

Secondary full-load current, $I_2 = \dfrac{4 \times 1000}{200} = 20$ A

$\cos \phi_2 = 0.8$

$\sin \phi_2 = 0.6$

$\therefore$ $I_2 = 20(0.8 - j0.6) = (16 - j12)$

$Z_{02} = R_{02} + jX_{02} = 0.2 + j0.3172$

Secondary load voltage, $V_2 = 200 - I_2 Z_{02} = 200 - (16 - j12)(0.2 + j0.3712)$

$= 200 - (3.2 + j5.075 - j2.4 + 3.806)$

$= 200 - (7 + j2.675) = 193 - j2.675 \simeq$ **193 V. (Ans.)**

Example 24. *A single-phase step-down transformer has a turn ratio of 3. The resistance and reactance of the primary winding are 1.2 Ω and 6 Ω and those of the secondary winding are 0.05 Ω and 0.03 Ω respectively. If the h.v. winding is supplied at 230 V, 50 Hz with l.v. winding short circuited, find* :

(*i*) *Current in the l.v. winding,* (*ii*) *Copper loss in the transformer, and*

(*iii*) *Power factor.*

Solution. Turn ratio $= 3$, *i.e.*, $K = \dfrac{1}{3}$

Resistance of primary, $R_1 = 1.2\ \Omega$

Reactance of primary, $X_1 = 6\ \Omega$

Resistance of secondary, $R_2 = 0.05\ \Omega$

Reactance of secondary, $X_2 = 0.3\ \Omega$

Referred to *h.v. side*

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 1.2 + 0.05 \times (3)^2 = 1.65\ \Omega$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 6 + 0.3 \times (3)^2 = 8.7\ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(1.65)^2 + (8.7)^2} = 8.855\ \Omega$$

Current in h.v. winding when l.v. winding is short-circuited,

$$I_{SC} = \frac{V_{SC}}{Z_{01}} = \frac{230}{8.855} = 25.97 \text{ A}$$

Neglecting I_0, $I_1 = I_2' = 25.97$ A

(*i*) **Current in the l.v. winding,**

$$I_2 = \frac{I_2'}{K} = \frac{25.97}{\left(\frac{1}{3}\right)} = \mathbf{77.91\ A.} \quad \textbf{(Ans.)}$$

(*ii*) **Total copper loss** $= I_1^2 R_{01} = (25.97)^2 \times 1.65 =$ **1112.8 W. (Ans.)**

(*iii*) **Primary power factor :**

Power input on short-circuit,

$$P_{SC} = V_{SC} I_{SC} \cdot \cos \phi_{SC}$$

$$1112.8 = 230 \times 25.97 \times \cos \phi_{SC}$$

$$\therefore \quad \cos \phi_{SC} = \frac{1112.8}{230 \times 25.97} = \mathbf{0.1863.} \quad \textbf{(Ans.)}$$

☞**Example 25.** *A single-phase, 3 kVA, 230/115 V, 50 Hz transformer has the following constants :*

Resistance : Primary 0.3 Ω, secondary 0.09 Ω.

Reactance : Primary 0.4 Ω, secondary 0.1 Ω.

Resistance of equivalent exciting circuit referred to primary, $R_0 = 600$ *Ω.*

Reactance of equivalent exciting circuit referred to primary, $X_0 = 200$ *Ω.*

What would be the readings of the instruments when the transformer is connected for (i) O.C. test, (ii) S.C. test. In both tests supply is given to h.v. side.

Solution. Primary resistance, $R_1 = 0.3$ Ω

Primary reactance, $X_1 = 0.4$ Ω

Secondary resistance, $R_2 = 0.09$ Ω

Secondary reactance, $X_2 = 0.1$ Ω

$R_0 = 600$ Ω

$X_0 = 200$ Ω

(*i*) **O.C. test :**

Wattful component of no-load current,

$$I_w = \frac{V_1}{R_0} = \frac{230}{600} = 0.383 \text{ A}$$

Magnetising component of no-load current,

$$I_w = \frac{V_1}{x_0} = \frac{230}{200} = 1.15 \text{ A}$$

$\therefore$ No-load current, $I_0 = \sqrt{I_w^2 + I_m^2} = \sqrt{(0.383)^2 + (1.15)^2} = 1.212$ A

Input on no-load to h.v. winding

$$= V_1 I_w = 230 \times 0.383 = 88.09 \text{ W}$$

Hence, the readings of the instruments are :

Voltmeter reading = 230 V. (Ans.)

Ammeter reading = 1.212 A. (Ans.)

Wattmeter reading = 88.09 W. (Ans.)

(*ii*) **S.C. test :**

Transformation ratio, $K = \frac{115}{230} = \frac{1}{2}$

Referred to *h.v. side*, $R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.3 + \frac{0.09}{\left(\frac{1}{2}\right)^2} = 0.66\ \Omega$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 0.4 + \frac{0.1}{\left(\frac{1}{2}\right)^2} = 0.8\ \Omega$$

$\therefore$ $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(0.66)^2 + (0.8)^2} = 1.037\ \Omega$

Full-load current in the h.v. winding

$$I_1 = \frac{3 \times 1000}{230} = 13.04 \text{ A}$$

Voltage to be applied to h.v. winding on short-circuiting l.v. winding to pass full-load current of 13.04 A through h.v. winding.

$$V_{SC} = I_1 Z_{01} = 13.04 \times 1.037 = 13.52 \text{ V}$$

Power supplied to h.v. winding on short-circuit

$$= I_1^2 \cdot R_{01} = (13.04)^2 \times 0.66 = 112.2 \text{ W}$$

Hence, the readings of the instruments are :

Voltmeter reading = 13.52 V. (Ans.)

Ammeter reading = 13.04 A. (Ans.)

Wattmeter reading = 112.2 W. (Ans.)

Regulation and Efficiency of a Transformer

Example 26. *A 4 kVA 220/440 V, 50 Hz, single-phase transformer gave the following test figures. No load test performed on 220 V side keeping 440 V side open : 220 V, 0.7 A, 60 W. Short circuit test performed short circuiting the 440 V side through an ammeter : 9 V, 6 A, 21.6 W calculate.*

(*a*) *The magnetising current and the component corresponding to iron loss at normal voltage and frequency.*

(*b*) *The efficiency at full-load at unity power factor and the corresponding secondary terminal voltage.*

(*c*) *Draw the phasor diagram corresponding to the full load operation.*

Solution. *Open circuit test* on 220 V (l.v.) side : $V_1 = 220$ V ; $P_0 = 60$ W ; $I_0 = 0.7$ A

Short circuit test on 440 V (h.v.) side : $V_{SC} = 9$ V ; $I_{SC} = 6$ A ; $P_{SC} = 21.6$ W

(*a*) $P_0 = V_1 I_0 \cos \phi_0$ or $60 = 220 \times 0.7 \times \cos \phi_0$

or $\cos \phi_0 = 0.3896$, and $\sin \phi_0 = 0.921$

$\therefore$ Iron loss component of current, $I_w = I_0 \cos \phi_0 = 0.7 \times 0.3896 =$ **0.273 A. (Ans.)**

Magnetising component of current, $I_m = I_0 \sin \phi_0 = 0.7 \times 0.921 =$ **0.6447 A. (Ans.)**

(*b*) Full-load secondary current, $I_2 = \frac{4 \times 1000}{440} = 9.1$ A

Full-load primary current, $I_1 = \frac{4 \times 1000}{220} = 18.2$ A

Full-load copper losses, $P_c = \left(\frac{I_1}{I_{SC}}\right)^2 \times P_{SC} = \left(\frac{18.2}{6}\right)^2 \times 21.6 = 198.7$ W

∴ Full-load efficiency at unity power factor,

$$\eta = \frac{4 \times 1000 \times 1}{4 \times 1000 \times 1 + 60 + 198.7} \times 100 = \mathbf{93.92\%.} \quad \textbf{(Ans.)}$$

From short circuit test, we have

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{9}{6} = 1.5\ \Omega$$

$$R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{21.6}{6^2} = 0.6\ \Omega$$

∴ $$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{1.5^2 - 0.6^2} = 1.375\ \Omega$$

Now, $$R_{02} = K^2 R_{01} = (2)^2 \times 0.6 = 2.4\ \Omega \qquad \left(\because\ K = \frac{440}{220} = 2\right)$$

$$X_{02} = K^2 X_{01} = 2^2 \times 1.375 = 5.5\ \Omega$$

Since $\cos\phi_2 = 1$ ∴ $\sin\phi_2 = \sin^{-1}(\cos^{-1} 1) = 0$

$${}_0V_2^2 = (V_2 + I_2R_{02})^2 + (I_2X_{02})^2$$

$$(440)^2 = (V_2 + 9.1 \times 2.4)^2 + (9.1 \times 5.5)^2$$

or $$(V_2 + 21.84)^2 = (440)^2 - (9.1 \times 5.5)^2 = 191095$$

or $$\mathbf{V_2} = (\sqrt{191095} - 21.84) = \mathbf{415.3\ V.} \quad \textbf{(Ans.)}$$

(*c*) The phasor diagram of transformer corresponding to full-load operation is shown in Fig. 51.

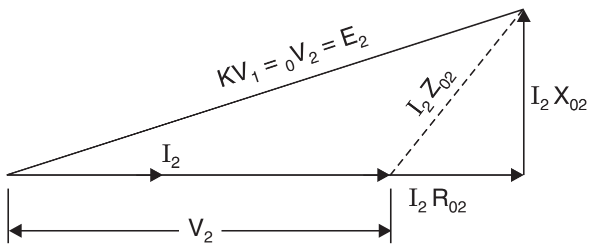


Fig. 51

Example 27. *The parameters of the equivalent circuit of a 100 kVA, 2000/200 volt single-phase transformer are as follows :*

Primary resistance	*= 0.2 ohms ;*
Secondary resistance	*= 2 milli ohms.*
Primary leakage reactance	*= 0.45 ohms ;*
Secondary leakage reactance	*= 4.5 milli ohms.*
Core loss resistance	*= 10 kilo ohms ;*
Magnetizing reactance	*= 1.55 kilo ohms.*

Using the circuit referred to primary, determine the :

(*i*) *Voltage regulation.*

(*ii*) *Efficiency of the transformer operating at rated load with 0.8 lagging power factor.*

Solution. *Given* : Rating = 100 kVA ; $R_1 = 0.2\ \Omega$; $R_2 = 0.002\ \Omega$;

$X_1 = 0.45\ \Omega$; $X_2 = 0.0045\ \Omega$; $R_0 = 10000\ \Omega$; $X_0 = 1550\ \Omega$.

(*i*) **Voltage regulation :**

Transformation ratio, $K = \frac{200}{2000} = 0.1$

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2} = 0.2 + \frac{0.002}{0.1^2} = 0.4\ \Omega$$

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2} = 0.45 + \frac{0.0045}{0.1^2} = 0.9\ \Omega$$

Primary full-load current, $I_1 = \frac{100 \times 1000}{2000} = 50$ A,

$$\cos\phi = 0.8,\ \sin\phi = \sin[\cos^{-1}(0.8)] = 0.6$$

Refer to Fig. 52.

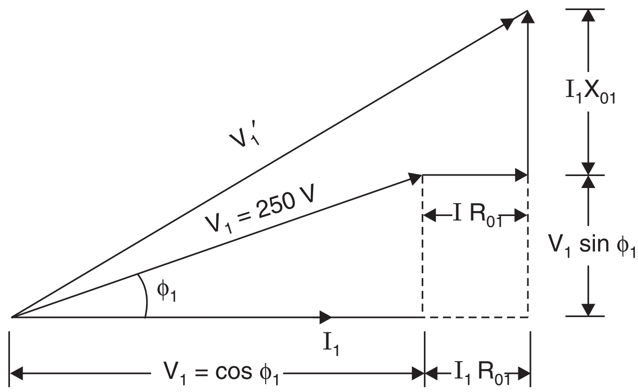


Fig. 52

Applied voltage, $V'_1 = \sqrt{(V_1 \cos\phi + I_1R_{01})^2 + (V_1 \sin\phi + I_1X_{01})^2}$

$$= \sqrt{(2000 \times 0.8 + 50 \times 0.4)^2 + (2000 \times 0.6 + 50 \times 0.9)^2} = 2043 \text{ V}$$

$\therefore$ % voltage regulation $= \frac{V'_1 - V_1}{V_1} \times 100 = \frac{2043 - 2000}{2000} \times 100 =$ **2.15%. (Ans.)**

(*ii*) **Efficiency of the transformer, η :**

Working component of no-load current, $I_w = \frac{V}{R_0} = \frac{2000}{10000} = 0.2$ A

Iron loss $= V_1I_0 \cos\phi_0 = V_1I_w = 2000 \times 0.2 = 400$ W

Full-load copper losses, $P_c = I_1^2 R_{01} = (50)^2 \times 0.4 = 1000$ W

Efficiency of the transformer at 0.8 p.f.,

$$\% \eta = \frac{\text{kVA} \times 1000 \times \cos\phi}{(\text{kVA} \times 1000 \times \cos\phi) + P_i + P_c} \times 100$$

$$= \frac{(100 \times 1000 \times 0.8)}{(100 \times 1000 \times 0.8 + 400 + 1000)} \times 100$$

= **98.28%. (Ans.)**

Example 28. *In a 25 kVA, 2000/200 V transformer, the constant and variable losses are 350 W and 400 W respectively. Calculate the efficiency on u.p.f. at*

(*i*) *Full load, and* (*ii*) *Half full load.*

Solution. Rating of transformer = 25 kVA

Constant or iron losses, P_i = 350 W or 0.35 kW

Variable or copper losses, P_c = 400 W or 0.4 kW

Power factor, $\cos \phi = 1$

Efficiency, η :

(*i*) **At full load :**

$$\eta = \frac{\text{output}}{\text{output + losses}} = \frac{\text{kVA} \cos \phi}{\text{kVA} \cos \phi + P_i + P_c}$$

$$= \frac{25 \times 1}{25 \times 1 + 0.35 + 0.4} = \mathbf{0.9708} \text{ or } \mathbf{97.08\%}. \quad \textbf{(Ans.)}$$

(*ii*) **At half full load :**

$$\eta = \frac{(0.5 \times 25) \times 1}{(0.5 \times 25 \times 1) + 0.35 + \left[\left(\frac{1}{2}\right)^2 \times 0.4\right]}$$

$$= \mathbf{0.9653} \text{ or } \mathbf{96.53\%}. \quad \textbf{(Ans.)}$$

☞**Example 29.** *The following readings were obtained from O.C. and S.C. tests on 8 kVA 400/120 V, 50 Hz transformer.*

O.C. test (l.v. side) : 120 V ; 4 A ; 75 W.

S.C. test (h.v. side) : 9.5 V ; 20 A ; 110 W.

Calculate :

(*i*) *The equivalent circuit (approximate) constants,*

(*ii*) *Voltage regulation and efficiency for 0.8 lagging power factor load, and*

(*iii*) *The efficiency at half full-load and 0.8 power factor load.*

Solution. (*i*) Transformation ratio, $K = \frac{120}{400} = \frac{3}{10}$

O.C. test :

It is seen from the O.C. test, that *primary open*, the secondary draws a no-load current of 4 A.

Since $K = \frac{3}{10}$, the corresponding no-load current $I_0 = 4 \times \frac{3}{10} = 1.2$ A.

Also $P_0 = V_1 I_0 \cos \phi_0$

$\therefore$ $75 = 400 \times 1.2 \times \cos \phi_0$

i.e., $\cos \phi_0 = \frac{75}{400 \times 1.2} = 0.156$

and $\sin \phi_0 = 0.987$

Now, wattful component of no-load current, I_0,

$$I_w = I_0 \cos \phi_0 = 1.2 \times 0.156 = 0.187 \text{ A}$$

and magnetising component of no-load current I_0,

$$I_m = I_0 \sin \phi_0 = 1.2 \times 0.987 = 1.184 \text{ A}$$

$\therefore$ Resistance representing the core loss,

$$R_0 = \frac{V_1}{I_w} = \frac{400}{0.187} = \mathbf{2139\ \Omega}. \quad \textbf{(Ans.)}$$

Magnetising reactance, $X_0 = \frac{V_1}{I_m} = \frac{400}{1.184} = \mathbf{337.8\ \Omega}. \quad \textbf{(Ans.)}$

S.C. test :

During S.C. test the *instruments have been placed in primary*.

Here, $V_{SC} = 9.5$ V

$I_{SC} = 20$ A

$P_{SC} = 110$ W

Now, $Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{9.5}{20} = \mathbf{0.475\ \Omega.}$ **(Ans.)**

$R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{110}{(20)^2} = \mathbf{0.25\ \Omega.}$ **(Ans.)**

$\therefore$ $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.475)^2 - (0.25)^2} = \mathbf{0.404\ \Omega.}$ **(Ans.)**

The equivalent circuit is shown in Fig. 53.

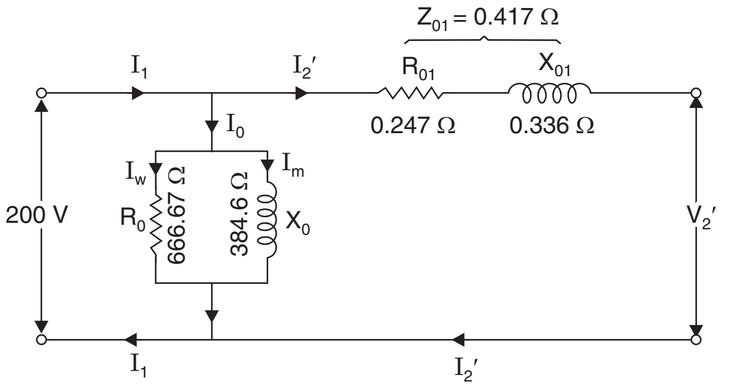


Fig. 53

(*ii*) Total *approximate voltage drop* as referred to primary is

$= I_1 (R_{01} \cos \phi + X_{01} \sin \phi)$

Now, full-load, $I_1 = \frac{8 \times 1000}{400} = 20$ A

$\therefore$ Voltage drop $= 20 (0.25 \times 0.8 + 0.404 \times 0.6) = 8.85$ V

% **Voltage regulation** $= \frac{8.85}{400} \times 100 =$ **2.21%. (Ans.)**

Iron loss, $P_i = 75$ W

Copper loss, $P_c = 110$ W

Full-load losses $= P_i + P_c = 75 + 110 = 185$ W

$\therefore$ Full-load output $= 8 \times 1000 \times 0.8 = 6400$ W

$\therefore$ **Full-load efficiency,** $\eta = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{6400}{6400 + 185} =$ **0.972** or **97.2%. (Ans.)**

(*iii*) **Half full-load :**

Iron loss, $P_i = 75$ W

Copper loss, $P_c = (\frac{1}{2})^2 \times 110 = 27.5$ W

Total losses $= P_i + P_c = 75 + 27.5 = 102.5$ W

Output at half full-load $= \frac{1}{2} \times 8 \times 1000 \times 0.8 = 3200$ W

$\therefore$ **Efficiency** $= \dfrac{\text{output}}{\text{output + losses}} = \dfrac{3200}{3200 + 102.5} =$ **0.969** or **96.9%. (Ans.)**

Example 30. *Consider a 20 kVA, 2200/220 V, 50 Hz transformer. The OC/SC test results are as follows :*

O.C. Test : 220 V, 4.2 A, 148 W (l.v. side)

S.C. Test : 86 V, 10.5 A, 360 W (h.v. side)

(*i*) *Determine the regulation at 0.8 p.f. lagging at full load.*

(*ii*) *What is the p.f. on short-circuit ?*

Solution. *Given* :

S.C. Test :

$$V_{SC} = 86 \text{ V}, I_{SC} = 10.5 \text{ A}, P_C = 360 \text{ W}$$

(*i*) **Regulation :** Equivalent impedance referred to primary, $Z_{01} = \dfrac{V_{SC}}{I_{SC}} = \dfrac{86}{10.5} = 8.19\ \Omega$

[$\because$ Short-circuit test has been conducted on the h.v. (primary) side]

Equivalent resistance referred to primary, $R_{01} = \dfrac{P_{SC}}{I_{SC}^2} = \dfrac{360}{(10.5)^2} = 3.27\ \Omega$

Equivalent reactance referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{8.19^2 - 3.27^2} = 7.51\ \Omega$

Full load primary current, $I_1 = \dfrac{20 \times 1000}{2200} = 9.09$ A

Percentage regulation $= \dfrac{I_1 (R_{01} \cos \phi + X_{01} \sin \phi)}{V_1} \times 100$

$= \dfrac{9.09 (3.27 \times 0.8 + 7.51 \times 0.6)}{2200} \times 100 =$ **2.94%. (Ans.)**

(*ii*) **P.f. on short-circuit, cos ϕ_{SC}** :

$$\cos \phi_{SC} = \frac{R_{01}}{Z_{01}} = \frac{3.27}{8.19} = \mathbf{0.399 \text{ (lag)}}. \quad \textbf{(Ans.)}$$

Example 31. *High voltage side short circuit test data for 20 kVA, 2300/230 V transformer are :*

Power = 250 watts ; *Current = 8.7 A* ; *Voltage = 50 V.*

Calculate equivalent impedance, resistance, reactance referred to h.v. side. Find the transformer regulation at 0.7 lagging power factor.

Solution. *Given* : **h.v. side :** $P_{SC} = 250$ W ; $I_{SC} = 8.7$ A ; $V_{SC} = 50$ V ; $\cos \phi = 0.7$.

From S.C. test with measurements performed on h.v. side the various parameters referred to *h.v.* side are :

$$R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{250}{8.7^2} = 3.303\ \Omega$$

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{50}{8.7} = 5.747\ \Omega$$

$$\therefore \quad X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{5.747^2 - 3.303^2} = 4.703\ \Omega$$

We know that, $\% \text{ Regulation} = \dfrac{I_1 R_{01} \cos \phi + I_1 X_{01} \sin \phi}{V_1} \times 100$

where $I_1 = \dfrac{20 \times 10^3}{2300} = 8.696$ A (rated current) ; and

$\cos \phi = 0.7$ (given) and hence $\sin \phi = \sin (\cos^{-1} 0.7) = 0.714$.

$\therefore$ **% Regulation** $= \dfrac{8.696 \times 3.303 \times 0.7 + 8.696 \times 4.703 \times 0.714}{2300} \times 100 =$ **2.14%. (Ans.)**

Example 32. *A 10 kVA, 2300/230 V, 50 Hz, distribution transformer has h.v. winding resistance 3.96 Ω and leakage resistance of 15.8 Ω. The l.v. winding has corresponding value of 0.0396 Ω and 0.158 Ω respectively. The transformer has a core loss of 58 W under normal operating conditions. Find :*

(*i*) *Load terminal voltage when transformer delivers rated current at 0.8 p.f. lagging to a load on l.v. side, with h.v. side voltage held at rated value and compute efficiency at this load.*

(*ii*) *The h.v. side voltage necessary to maintain rated voltage at load terminals when the transformer is loaded as above. Is efficiency same as in (i) ?*

Solution. *Given* : $R_1 = 3.96\ \Omega$; $X_1 = 15.8\ \Omega$; $R_2 = 0.0396\ \Omega$; $X_2 = 0.158\ \Omega$;
$P_i = 58$ W ; *p.f.* = 0.8 lagging

(*i*) **Load terminal voltage :**

Rated secondary current, $I_2 = \dfrac{10 \times 10^3}{230} = 43.48$ A

$$R_{02} = R_2 + K^2 R_1 = 0.0396 + \left(\frac{230}{2300}\right)^2 \times 3.96 = 0.0792\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 0.158 + \left(\frac{230}{2300}\right)^2 \times 15.8 = 0.316\ \Omega$$

$\therefore$ Voltage drop on load side $= I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$
$= 43.48 (0.0792 \times 0.8 + 0.316 \times 0.6) = 11$ V

$\therefore$ **Load terminal voltage** = 230 – 11 = **219 V. (Ans.)**

Copper loss (P_C) $= I_2^2 R_{02} = 43.48^2 \times 0.0792 = 149.73$ W

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Output} + \text{Core loss } (P_i) + \text{Copper loss } (P_C)}$$

$$= \frac{10 \times 10^3 \times 0.8}{(10 \times 10^3 \times 0.8) + 58 + 149.73} = \mathbf{0.9747} \text{ or } \mathbf{97.47\%}. \quad \textbf{(Ans.)}$$

(*ii*) To maintain rated voltage at load terminals of 230 V, the O.C. voltage required at secondary terminals would be 230 + 11 = 241 V. Hence primary induced voltage required

$$= 241 \times \frac{2300}{230} = \mathbf{2410\ V.} \quad \textbf{(Ans.)}$$

Example 33. *The following test results were obtained for a 1000/100 V, 100 kVA single-phase transformer* :

O.C. test :

Primary volts = 1000, secondary volts = 100, watts in primary = 950.

S.C. test :

Primary volts for full-load current = 20, watts in primary = 1000.

Determine the regulation and efficiency of the transformer at full-load and at 0.8 power factor lagging.

Solution. Transformation ratio, $K = \frac{100}{1000} = \frac{1}{10}$

O.C. test :

Primary volts, $V_1 = 1000$ V

Iron loss, $P_i = 950$ W = 0.95 kW

S.C. test :

Full-load copper loss, $P_c = 1000$ W = 1 kW

In S.C. test 20 V in primary are equivalent to $20 \times \frac{1}{10} = 2$ V in secondary.

Now, full-load secondary current,

$$I_2 = \frac{100 \times 1000}{100} = 1000 \text{ A}$$

Also $$Z_{02} = \frac{2}{1000} = 0.002\ \Omega$$

and $$R_{02} = \frac{1000}{(1000)^2} = 0.001\ \Omega$$

$$\therefore \quad X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{0.002^2 - 0.001^2} = 0.00173\ \Omega$$

Approximate voltage drop at 0.8 power factor lagging

$$= I_2(R_{02} \cos \phi + X_{02} \sin \phi)$$
$$= 1000(0.001 \times 0.8 + 0.00173 \times 0.6) = 1.838 \text{ V}$$

$\therefore$ **% regulation down** $= \frac{1.838}{100} \times 100 =$ **1.838%. (Ans.)**

Full-load output $= 100 \times 0.8 = 80$ kW

Total losses $= P_i + P_c = 0.95 + 1 = 1.95$ kW

$\therefore$ **Efficiency,** $\eta = \frac{\text{output}}{\text{output + losses}} = \frac{80}{80 + 1.95} =$ **0.976** or **97.6%. (Ans.)**

Example 34. *The primary and secondary winding resistances of a 30 kVA, 6600/250 V single-phase transformer are 8 Ω and 0.015 Ω respectively. The equivalent leakage reactance as referred to the primary winding is 30 Ω. Find the full-load regulation for load power factors of :*

(*i*) *unity* (*ii*) *0.8 lagging* (*iii*) *0.8 leading.*

Solution. Transformer ratio, $K = \frac{250}{6600} = \frac{1}{26.4}$

Primary resistance, $R_1 = 8\ \Omega$

Secondary resistance, $R_2 = 0.015\ \Omega$

Equivalent leakage reactance as referred to primary,

$$X_{01} = 30\ \Omega$$

Now, $$R_{02} = R_2 + K^2R_1 = 0.015 + \left(\frac{1}{26.4}\right)^2 \times 8 = 0.02648\ \Omega$$

$$X_{02} = K^2X_{01} = \left(\frac{1}{26.4}\right)^2 \times 30 = 0.04304\ \Omega$$

Secondary full-load current, $I_2 = \frac{30 \times 1000}{250} = 120$ A.

(*i*) **Unity power factor,** $\cos \phi_2 = 1$:

Voltage drop $= I_2R_{02} \cos \phi_2 + I_2X_{02} \sin \phi_2 = 120 \times 0.02648 \times 1 + 0 = 3.177$ V

% **regulation** $= \frac{3.177}{250} \times 100 =$ **1.27%. (Ans.)**

(*ii*) **0.8 power factor (lagging)** :

Voltage drop $= I_2R_{02} \cos \phi_2 + I_2X_{02} \sin \phi_2$

$= 120 \times 0.02648 \times 0.8 + 120 \times 0.04304 \times 0.6 = 5.64$ V

% **regulation** $= \frac{564}{250} \times 100 =$ **2.256%. (Ans.)**

(*iii*) **0.8 power factor (leading) :**

Voltage drop $= I_2R_{02} \cos \phi_2 - I_2X_{02} \sin \phi_2$

$= 120 \times 0.02648 \times 0.8 - 120 \times 0.04304 \times 0.6 = -0.557$ V

% **regulation** $= -\frac{0.557}{250} \times 100 =$ **– 0.223%. (Ans.)**

Example 35. *Short-circuit test is conducted on a 5 kVA, 400 V/100 V single phase transformer with 100 V winding shorted. The input voltage at full load current is 40 V. The wattmeter, on the input reads 250 W. Find the power factor for which regulation at full load is zero.*

Solution. *Given* : Rating of transformer : 5 kVA, 400 V/100 V

Input power, on short-circuit, $P_{SC} = 250$ W

The input voltage at full load current, $V_{SC} = 40$ V

Power factor for which regulation at full load is zero :

Input current I_{SC} = full load current, $I_1 = \frac{5 \times 1000}{400} = 12.5$ A

Equivalent resistance as referred to primary, $R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{250}{(12.5)^2} = 1.6\ \Omega$

Equivalent impedance referred to primary, $Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{40}{12.5} = 3.2\ \Omega$

Equivalent reactance referred to primary, $X_{01} = \sqrt{{Z_{01}}^2 - {R_{01}}^2} = \sqrt{3.2^2 - 1.6^2} = 2.77\ \Omega$

Regulation will be zero when phase angle,

$$\phi = \tan^{-1} \frac{R_{02}}{X_{02}} = \tan^{-1} \frac{R_{01}}{X_{01}} = \tan^{-1} \left(\frac{1.6}{2.77} \right) = \mathbf{30^\circ}$$

or **power factor** $= \cos \phi = \cos (30^\circ) =$ **0.866. (Ans.)**

Example 36. *A 50 MVA, 76.2 V/33 kV, 1-phase, 50 Hz, two-winding transformer with tap changer has percentage impedance of 0.5 + j7.0. What tapping must be used to maintain rated voltage at the secondary on*

(*i*) *full load at 0.8 lagging power factor, and*

(*ii*) *40 MVA load at 0.6 lagging power factor.*

Assume that the tap changer is provided on the h.v. side.

Solution. *Given* : Rated MVA = 50 ; $f = 50$ Hz ; $E_1 = 76.2$ V ; $E_2 = 33$ kV ;

percentage impedance $= 0.5 + j7.0$.

(*i*) Percentage regulation $= \dfrac{L_2R_{02}\cos\phi + I_2X_{02}\sin\phi}{E_2} \times 100$

$= \dfrac{I_2R_{02}}{E_2} \times 100\cos\phi + \dfrac{I_2X_0}{E_2} \times 100\sin\phi$

$= \%R\cos\phi + \%X\sin\phi$

$= 0.5 \times 0.8 + 7 \times 0.6 = 4.6\%$

i.e., voltage to be raised for maintaining rated voltage = 4.6%

So tap setting required on h.v. side = – 4.6% or **4.6% down. (Ans.)**

(*ii*) At load of 40 MVA *i.e.*, 80% of full load or 0.8 times full load,

voltage regulation $= 0.8\,(\%R\cos\phi + \%X\sin\phi)$

$= 0.8(0.5 \times 0.6 + 7 \times 0.8) = 4.72\%$

Tap setting required on h.v. side = **4.72% down. (Ans.)**

Note. *If the tap changer is on the primary side, tap setting will be down and if it is to be provided on secondary side, tap setting will be up for raising the secondary voltage on load.*

Example 37. *The percentage resistance and reactance of a transformer are 2% and 4% respectively. Find the approximate regulation on full-load at :*

(*i*) *Unity power factor,* (*ii*) *0.8 power factor lagging, and*

(*iii*) *0.8 power factor leading.*

Solution. Percentage resistance = 2%

Percentage reactance = 4%

Approximate % regulation = % resistance × cos ϕ ± % reactance × sin ϕ

(*i*) **Unity power factor :**

Approximate % regulation = % resistance cos ϕ ($\because$ $\sin\phi = 0$)

$= 2 \times 1 =$ **2%. (Ans.)**

(*ii*) **0.8 power factor lagging :**

Approximate % regulation = % resistance × cos ϕ + % reactance × sin ϕ

$= 2 \times 0.8 + 4 \times 0.6 = 1.6 + 2.4 =$ **4%. (Ans.)**

(*iii*) **0.8 power factor leading :**

Approximate % regulation = % resistance × cos ϕ – % reactance × sin ϕ

$= 2 \times 0.8 - 4 \times 0.6 = 1.6 - 2.4 =$ **– 0.8%. (Ans.)**

Example 38. *A single-phase 80 kVA, 2000/200 V, 50 Hz transformer has impedance drop of 8% and resistance drop of 4% :*

(*i*) *Find the regulation at full-load 0.8 power factor lagging.*

(*ii*) *At what power factor is the regulation zero.*

Solution. Impedance drop = 8%

Resistance drop = 4%

i.e., $$\frac{I_2Z_{02}}{V_2} = \frac{8}{100} \quad \text{or} \quad I_2Z_{02} = \frac{200 \times 8}{100} = 16 \text{ V}$$

and $$\frac{I_2R_{02}}{V_2} = \frac{4}{100} \quad \text{or} \quad I_2R_{02} = \frac{200 \times 4}{100} = 8 \text{ V}$$

$$\therefore \quad I_2X_{02} = \sqrt{(I_2Z_{02})^2 - (I_2R_{02})^2} = \sqrt{16^2 - 8^2} = 13.86 \text{ V.}$$

(i) % regulation $= \dfrac{I_2R_{02} \cos \phi_2 + I_2X_{02} \sin \phi_2}{V_2}$

$= \dfrac{8 \times 0.8 + 13.86 \times 0.6}{200} = \dfrac{6.4 \times 8.316}{200}$ = **7.36%. (Ans.)**

(ii) *Regulation can be zero only when the power factor is leading.* Then

$$\frac{I_2R_{02} \cos \phi_2 - I_2X_{02} \sin \phi_2}{V_2} = 0$$

or $$I_2R_{02} \cos \phi_2 - I_2X_{02} \sin \phi_2 = 0$$

or $$I_2R_{02} \cos \phi_2 = I_2X_{02} \sin \phi_2$$

or $$\tan \phi_2 = \frac{I_2R_{02}}{I_2X_{02}} = \frac{8}{13.86} = 0.577 \text{ or } \phi_2 = 30°$$

∴ **Power factor** = $\cos \phi_2$ = **0.866 leading. (Ans.)**

Example 39. *The high voltage of a single-phase 200 kVA 4400/220 V transformer takes a current of 35 A and power of 1250 W at 80 V when the low voltage winding is short-circuited. Determine :*

(i) *The voltage to be applied to the high voltage winding on full-load at 0.8 power factor lagging if the full-load secondary terminal voltage is to be kept at 220 V.*

(ii) *% regulation.*

Solution. (i) Transformation ratio, $K = \dfrac{220}{4400} = \dfrac{1}{20}$

Voltage applied to h.v. winding in S.C. test

$$V_{SC} = 80 \text{ V}$$

and $$I_{SC} = 35 \text{ A}$$

$$P_{SC} = 1250 \text{ W}$$

Referred to *h.v. side,*

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{80}{35} = 2.28\ \Omega$$

$$R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{1250}{35^2} = 1.02\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{2.28^2 - 1.02^2} = 2.04\ \Omega$$

Full-load current in h.v. winding,

$$I_1 = \frac{200 \times 1000}{4400} = 45.45 \text{ A}$$

Voltage drop in resistance $= I_1R_{01} = 45.45 \times 1.02 = 46.36$ V

Voltage drop in reactance $= I_1X_{01} = 45.45 \times 2.04 = 92.72$ V

Full-load secondary voltage referred to primary,

$$V_2' = \frac{V_2}{K} = \frac{220}{\frac{1}{20}} = 4400 \text{ V}$$

To get full-load secondary voltage equal to 220 V, supply voltage V_1 must be equal to the vector sum of 4400 V, I_1R_{01} and IX_{01}.

From to Fig. 54,

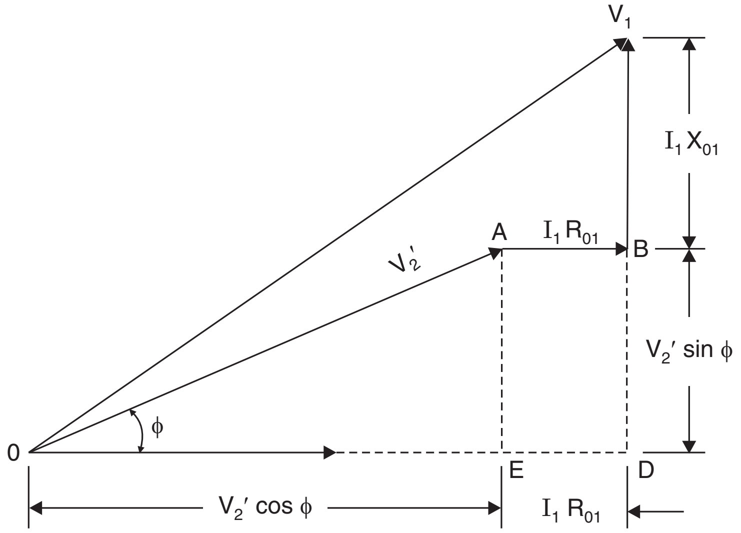


Fig. 54

$$OC^2 = OD^2 + CD^2 = (OE + ED)^2 + (BD + BC)^2$$

$$V_1^2 = (V_2' \cos \phi + I_1R_{01})^2 + (V_2' \sin \phi + I_1X_{01})^2$$

$$= (4400 \times 0.8 + 46.36)^2 + (4400 \times 0.6 + 92.72)^2$$

or $$V_1 = 4493 \text{ V}$$

Voltage to be applied to h.v. winding to get a full-load secondary voltage of 220 V = 4493 V. (Ans.)

(*ii*) % **regulation** $= \dfrac{V_1 - V_2'}{V_1} \times 100 = \dfrac{4493 - 4400}{4493} \times 100 =$ **2.07%. (Ans.)**

Example 40. *The high voltage and low voltage windings of a 50 kVA, 4400/220 V, 50 Hz transformer have resistances of 2.2 Ω and 0.005 Ω respectively. The full-load current is obtained when 160 V at 50 Hz is applied to h.v. winding with l.v. winding short-circuited. Find :*

(*i*) *The equivalent resistance and reactance of the transformer referred to h.v. side, and*

(*ii*) *Reactance of each winding.*

Assume that the ratio of resistance to reactance is the same for each winding and the full-load efficiency of transformer is 0.98.

Solution. (*i*) Transformation ratio, $K = \dfrac{220}{4400} = \dfrac{1}{20}$

Primary resistance, $R_1 = 22\ \Omega$

Secondary resistance, $R_2 = 0.005\ \Omega$

Full-load output $= 50 \text{ kVA} = 50 \times 10^3 \text{ VA}$

Full-load input $= \dfrac{\text{output}}{\text{efficiency}} = \dfrac{50 \times 10^3}{0.98} \text{ VA}$

Full-load input current, $I_1 = \dfrac{\text{input in VA}}{\text{supply voltage}} = \dfrac{50 \times 10^3}{0.98 \times 4400} = 11.6 \text{ A}$

Impedance referred to h.v. side

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{160}{11.6} = 13.8\ \Omega$$

Total resistance referred to h.v. side,

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 2.2 + \frac{0.005}{\left(\frac{1}{20}\right)^2} = 2.2 + 2 = \mathbf{4.2\ \Omega.}\quad \textbf{(Ans.)}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(13.8)^2 - (4.2)^2} = \mathbf{13.15\ \Omega.}\quad \textbf{(Ans.)}$$

(*ii*) Since the ratio of resistance to reactance remains same (given),

$$\therefore \qquad \frac{R_1}{X_1} = \frac{R_2}{X_2} = \frac{R_2'}{X_2'}$$

i.e.,

$$\frac{R_1}{R_2'} = \frac{X_1}{X_2'}$$

or

$$\frac{R_1 + R_2'}{R_2'} = \frac{X_1 + X_2'}{X_2'} \quad \text{or} \quad \frac{R_{01}}{R_2'} = \frac{X_{01}}{X_2'}$$

or

$$X_2' = \frac{R_2'\, X_{01}}{R_{01}} = \frac{2 \times 13.15}{4.2} = 6.26\ \Omega$$

Reactance of primary winding,

$$X_1 = X_{02} - X_2' = 13.15 - 6.26 = \mathbf{6.85\ \Omega.}\quad \textbf{(Ans.)}$$

Reactance of secondary winding,

$$X_2 = K^2 X_2' = \left(\frac{1}{20}\right)^2 \times 6.26 = \mathbf{0.01565\ \Omega.}\quad \textbf{(Ans.)}$$

☞**Example 41.** *The following test results were obtained in a 250/500 V transformer :*

O.C. test (l.v. side) : 250 V, 1 A, 80 W.

S.C. test (l.v. winding short-circuited) : 20 V, 12 A, 100 W.

Determine :

(*i*) *The circuit constants.*

(*ii*) *The applied voltage and efficiency when the output is 10 A at 500 V and 0.8 power factor lagging.*

Solution. Transformation ratio, $K = \frac{500}{250} = 2$

(*i*) **Circuit constants :**

O.C. test *(l.v. side)*

Voltage, $V_1 = 250$ V

No-load current, $I_0 = 1$ A

No-load loss, $P_0 = 80$ W

Also, $P_0 = V_1 I_0 \cos \phi_0$

$80 = 250 \times 1 \times \cos \phi_0$

$$\cos \phi_0 = \frac{80}{250} = 0.32$$

Wattful component of no-load current I_0,

$$I_w = I_0 \cos \phi_0 = 1 \times 0.32 = 0.32 \text{ A}$$

Magnetising component of no-load current, I_0,

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{1^2 - 0.32^2} = 0.95 \text{ A}$$

Now, resistance representing the core loss,

$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.32} = \mathbf{781.25\ \Omega.} \quad \textbf{(Ans.)}$$

Magnetising reactance, $X_0 = \frac{V_1}{I_m} = \frac{250}{0.95} = \mathbf{263.16\ \Omega.} \quad \textbf{(Ans.)}$

S.C. test *(l.v. winding short-circuited)*

Short-circuit voltage, $V_{SC} = 20$ V

Short-circuit current, $I_{SC} = 12$ A

Losses, $P_{SC} = 100$ W

As the primary is short-circuited all *values refer to secondary winding.*

$$\therefore \quad R_{02} = \frac{P_{SC}}{I_{SC}^2} = \frac{100}{(12)^2} = 0.694\ \Omega$$

$$Z_{02} = \frac{V_{SC}}{I_{SC}} = \frac{20}{12} = 1.677\ \Omega$$

and $$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{(1.667)^2 - (0.694)^2} = 1.516\ \Omega$$

As R_0 and X_0 refer to primary, let us transfer these values to primary as follows :

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.694}{(2)^2} = \mathbf{0.174\ \Omega}$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.516}{(2)^2} = \mathbf{0.38\ \Omega}$$

$$Z_{01} = \frac{Z_{02}}{K^2} = \frac{1.669}{(2)^2} = \mathbf{0.417\ \Omega}$$

The equivalent circuit is shown in Fig. 55.

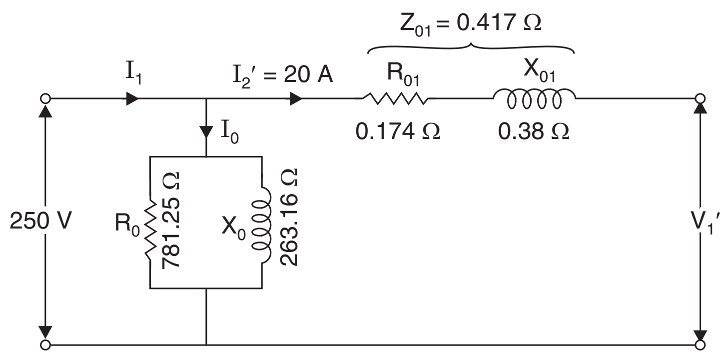


Fig. 55

(*ii*) **Applied voltage, V_1' :**

The applied voltage V_1' is the vector sum of V_1 and I_1Z_{01} (Fig. 56).

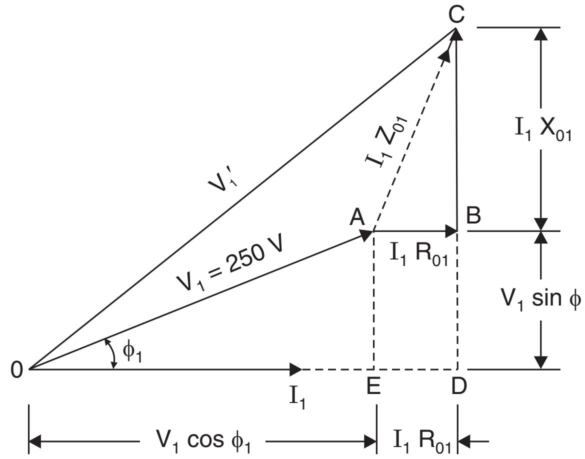


Fig. 56

Output current, $I_2 = 10$ A

$\therefore$ $I_1 = KI_2 = 2 \times 10 = 20$ A

Now, $I_1R_{01} = 20 \times 0.174 = 3.48$ V

$I_1X_{01} = 20 \times 0.38 = 7.6$ V

Refer Fig. 56. Neglecting the angle between V_1 and V_1', we have

$$OC^2 = OD^2 + DC^2$$

$$OC = V_1' = \sqrt{(OE + ED)^2 + (DB + BC)^2}$$

$$= \sqrt{(V_1 \cos \phi_1 + I_1R_{01})^2 + (V_1 \sin \phi_1 + I_1X_{01})^2}$$

$$= \sqrt{(250 \times 0.8 + 3.48)^2 + (250 \times 0.6 + 7.6)^2} = 257.4 \text{ V}$$

Hence, **applied voltage = 257.4 V. (Ans.)**

Efficiency :

Iron loss, $P_i = 80$ W

Total copper loss, $P_c = I_2^2 R_{02} = 10^2 \times 0.694 = 69.4$ W

Total loss, $= P_i + P_c = 80 + 69.4 = 149.4$ W

Output $= 500 \times 10 \times 0.8 = 4000$ W

$\therefore$ **Efficiency,** $\eta = \dfrac{\text{Output}}{\text{output + losses}} = \dfrac{4000}{4000 + 149.4}$ = **0.964** or **96.4%. (Ans.)**

☞**Example 42.** *A 25-kVA, 2200/220 V, 50-Hz distribution transformer is tested for efficiency and regulation as follows :*

O.C. test (l.v. side) : 220 V, 4 A, 150 W.

S.C. test (h.v. side) : 90 V, 10 A, 350 W.

Determine :

(*i*) *Core loss,*

(*ii*) *Equivalent resistance referred to primary,* (*iii*) *Equivalent reactance referred to secondary,*

(*iv*) *Equivalent reactance referred to primary,*

(*v*) *Equivalent reactance referred to secondary,*

(*vi*) *Regulation of transformer at 0.8 power factor lagging current, and*

(*vii*) *Efficiency at full-load and half-load at 0.8 power factor lagging current.*

Solution. Transformation ratio, $K = \dfrac{220}{2200} = \dfrac{1}{10}$

(*i*) **Core loss :**

Since no-load primary input is practically equal to the core loss, hence, **core loss as found from no-load test, is 150 W. (Ans.)**

(*ii*) **From S.C. test :**

$$V_{SC} = 90 \text{ V (short-circuit voltage)}$$
$$I_{SC} = 10 \text{ A (short-circuit current)}$$
$$P_{SC} = 350 \text{ W (copper loss)}$$

∴ **Equivalent resistance referred to primary**,

$$R_{01} = \frac{P_{SC}}{I_{SC}^2} = \frac{350}{(10)^2} = \mathbf{3.5\ \Omega.} \quad \textbf{(Ans.)}$$

(*iii*) **Equivalent resistance referred to secondary,**

$$R_{02} = K^2 R_{01} = \left(\frac{1}{10}\right)^2 \times 3.5 = \mathbf{0.035\ \Omega.} \quad \textbf{(Ans.)}$$

(*iv*) Also $\quad Z_{01} = \dfrac{V_{SC}}{I_{SC}} = \dfrac{90}{10} = 9\ \Omega$

∴ **Equivalent reactance referred to primary,**

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{9^2 - 3.5^2} = \mathbf{8.29\ \Omega.} \quad \textbf{(Ans.)}$$

(*v*) **Equivalent reactance referred to secondary**,

$$X_{02} = K^2 X_{01} = \left(\frac{1}{10}\right)^2 \times 8.29 = \mathbf{0.0829\ \Omega.} \quad \textbf{(Ans.)}$$

(*vi*) % **regulation :**

Let us find the rise in voltage necessary to maintain the output terminal voltage constant from no-load to full-load.

Rated primary current $= \dfrac{25 \times 1000}{2200} = 11.36$ A

Now, using the relation,

$$V_1' = \sqrt{(V_1 \cos\phi + I_1 R_0)^2 + (V_1 \sin\phi + I_1 X_{01})^2}$$
$$= \sqrt{(2200 \times 0.8 + 11.36 \times 3.5)^2 + (2200 \times 0.6 + 11.36 \times 8.29)^2}$$
$$= 2289 \text{ V (app.)}$$

∴ % **regulation** $= \dfrac{V_1' - V_1}{V_1} \times 100$ (Art. 9.11)

$$= \frac{2289 - 2200}{2200} \times 100 = \mathbf{4.045\%.} \quad \textbf{(Ans.)}$$

We can get the same result by working in the secondary :

Rated secondary current,

$$I_2 = \frac{I_1}{K} = \frac{11.36}{1/10} = 113.6 \text{ A}$$

$$_0V_2 = \sqrt{(V_2 \cos\phi + I_2R_{02})^2 + (V_2 \sin\phi + I_2X_{02})^2}$$

$$= \sqrt{(220 \times 0.8 + 113.6 \times 0.035)^2 + (220 \times 0.6 + 113.6 \times 0.0829)^2}$$

$$= 228.9 \text{ V}$$

$\therefore$ % regulation $= \dfrac{_0V_2 - V_2}{V_2} \times 100 = \dfrac{228.9 - 220}{220} \times 100 =$ **4.045%. (Ans.)**

(*vii*) **Efficiency :**

Core loss, $P_i = 150$ W

Copper loss at full-load,

$$P_c = I_1^2 R_{01} = 11.36^2 \times 3.5 = 451.7 \text{ W}$$

Copper loss at half-load,

$$P_c = \left(\frac{11.36}{2}\right)^2 \times 3.5 = 112.9 \text{ W}$$

$\therefore$ Efficiency at *full-load*

$$= \frac{\text{output}}{\text{output} + P_i + P_c} = \frac{25 \times 1000 \times 0.8}{(25 \times 1000 \times 0.8) + 150 + 451.7}$$

= **0.9707** or **97.07%. (Ans.)**

Efficiency at half *full-load*

$$= \frac{(25 \times 1000/2) \times 0.8}{(25 \times 1000/2) \times 0.8 + 150 + 112.9}$$ = **0.9744** or **97.44%. (Ans.)**

☞**Example 43.** *Two similar 100 kVA, single-phase transformers gave the following test readings when tested by Sumpner's test.*

Supply power = 2.4 kW.

Power supplied to secondary circuit in passing full-load current through it = 3.2 kW.

Find the efficiency and regulation of each transformer at unity power factor.

Solution. Core loss (or iron loss) of the transformers

= 2.4 kW

Iron loss of each transformer,

$$P_i = \frac{2.4}{2} = 1.2 \text{ kW}$$

Full-load copper loss of both transformers

= 3.2 kW

Full-load copper loss of each transformer

$$P_c = \frac{3.2}{2} = 1.6 \text{ kW}$$

Total full-load losses of each transformer

$$= P_i + P_c = 1.2 + 1.6 = 2.8 \text{ kW}$$

Output at full-load at unity power factor

$$= 100 \times 1 = 100 \text{ kW}$$

$\therefore$ **Full-load efficiency,** $\eta = \dfrac{\text{output}}{\text{output} + \text{losses}} = \dfrac{100}{100 + 2.8}$ = **0.9727** or **97.27%. (Ans.)**

Example 44. *A 10 kVA, 2500/250 V, single phase transformer gave the following test results :*

Open circuit test : 250 V, 0.8 A, 50 W

Short circuit test : 60 V, 3 A, 45 W

(*i*) *Calculate the efficiency of half full-load at 0.8 p.f.*

(*ii*) *Calculate the load kVA at which maximum efficiency occurs and also the maximum efficiency at 0.8 p.f.*

(*iii*) *Compute the voltage regulation at 0.8 p.f. leading.*

Solution. Full rated current $= \dfrac{10 \times 1000}{2500} = 4 \text{ A}$

Hence reading of wattmeter corresponding to full-load current of 4 A

$$= 45 \times \left(\frac{4}{3}\right)^2 = 80 \text{ W}$$

$\therefore$ Full-load copper losses = 80 W, and

Full-load iron losses = 50 W

(*i*) **Efficiency at half-load at 0.8 p.f. :**

$$\eta = \frac{\text{half-load output}}{\text{half-load output + losses}}$$

$$= \frac{(10 \times 1000 \times 0.5) \times 0.8}{[(10 \times 1000 \times 0.5) \times 0.8] + 50 + 80 \times \left(\frac{1}{2}\right)^2}$$

$$= \frac{4000}{4000 + 50 + 20} = \mathbf{0.983} \text{ or } \mathbf{98.3\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Load kVA for maximum efficiency, and its value :**

For maximum efficiency :

Copper loss = Iron loss = 50 W

$\therefore$ Current at which maximum efficiency occurs $= \dfrac{50 \times 4}{80} = 2.5 \text{ A}$

$\therefore$ **Load kVA** $= 10 \times \dfrac{2.5}{4}$ = **6.25 kVA. (Ans.)**

η_{max} (at 0.8 p.f.) $= \dfrac{(6.25 \times 1000 \times 0.8)}{(6.25 \times 1000 \times 0.8) + 50 + 50}$ = **0.98** or **98%. (Ans.)**

(*iii*) **Voltage regulation at 0.8 p.f. leading :**

From short circuit test :

$$V_{SC} = 60 \text{ V} ; I_{SC} = 3 \text{ A} ; P_{SC} = 45 \text{ W}$$

Now, $R_{01} = \dfrac{P_{SC}}{I_{SC}^2} = \dfrac{45}{3^2} = 5 \ \Omega$

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{60}{3} = 20 \ \Omega$$

$\therefore$ $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{20^2 - 5^2} = 19.36 \ \Omega$

∴ Voltage drop at leading p.f. of 0.8

$= I_1 R_{01} \cos\phi - I_1 X_{01} \sin\phi$

$= 4 \times 5 \times 0.8 - 4 \times 19.36 \times 0.6 = -30.46$ V

Hence % **voltage regulation** $= \frac{-30.46}{2500} \times 100 =$ **$-$ 1.218%. (Ans.)**

Example 45. *A 50 kVA single-phase transformer has a full-load primary current of 250 A and total resistance referred to primary is 0.006 ohm. If the iron loss amounts to 200 W, find the efficiency on full-load and half-load at (i) unity power factor and (ii) 0.8 power factor.*

Solution. Full-load primary current,

$I_1 = 250$ A

Total resistance referred to primary,

$R_{01} = 0.006\ \Omega$

Iron loss, $P_i = 200$ W

Full-load copper loss, $P_c = I_1^2 R_{01} = (250)^2 \times 0.006 = 375$ W

Total full-load loss $= P_i + P_c = 200 + 375 = 575$ W $= 0.575$ kW

Half full-load copper loss $= \left(\frac{1}{2}\right)^2 P_c = \left(\frac{1}{2}\right)^2 \times 375 = 93.75$ W

Total loss on half-load $= 200 + 93.75 = 293.75$ W $= 0.294$ kW (say)

(*i*) **At unity power factor :**

Full-load output, $= 50 \times 1 = 50$ kW

Efficiency at full-load $= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{50}{50 + 0.575} =$ **0.9886** or **98.86%. (Ans.)**

Half-full-load output $= 50 \times \frac{1}{2} \times 1 = 25$ kW

Efficiency at half full-load at unity power factor

$= \frac{25}{25 + 0.294} =$ **0.9984** or **99.84%. (Ans.)**

(*ii*) **At 0.8 power factor :**

Full-load output $= 50 \times 0.8 = 40$ kW

Efficiency on full-load at 0.8 power factor

$= \frac{40}{40 + 0.575} =$ **0.9858** or **98.58%. (Ans.)**

Half full-load output $= 50 \times \frac{1}{2} \times 0.8 = 20$ kW

Efficiency on half full-load at 0.8 power factor

$= \frac{20}{20 + 0.294} =$ **0.9855** or **98.55%. (Ans.)**

☞**Example 46.** *A 40 kVA, single-phase transformer has an iron loss of 300 W and full-load copper loss of 600 W,*

(*i*) *Find the load at which maximum efficiency occurs and the value of maximum efficiency at unity power factor.*

(*ii*) *If the maximum efficiency occurs at 80% of full-load, find the new core loss and full-load copper loss assuming that total full-load loss is a constant.*

Solution. Rating of transformer = 40 kVA

Iron loss, $P_i = 300$ W

Full-load copper loss, $P_c = 600$ W

(*i*) Let the maximum efficiency occurs at x times full-load, then

$$x^2 P_c = P_i$$

or $$x = \sqrt{\frac{P_i}{P_c}} = \sqrt{\frac{300}{600}} = 0.707 = 70.7\%$$

Hence, **efficiency occurs at 70.7% of full-load. (Ans.)**

Maximum efficiency :

Output at unity power factor

$$= 0.707 \times 40 \times 1 = 28.28 \text{ kW}$$

Total losses $= P_i + P_c = 2P_i$

$= 2 \times 300 = 600$ W $\left[\begin{array}{c}\text{Because when efficiency is maximum,} \\ P_i = P_c\end{array}\right]$

$= 0.6$ kW

$\therefore$ **Maximum efficiency** $= \dfrac{\text{output}}{\text{output + losses}} = \dfrac{28.28}{28.28 + 0.6} =$ **0.9792** or **97.92%. (Ans.)**

(*ii*) **New core loss, P_i' :**

New copper loss, P_c' :

Maximum efficiency occurs at 80% of full-load

Now, $P_i' + P_c' = P_i + P_c$ (*given*)

$= 300 + 600 = 900$ W

Also $$0.8 = \sqrt{\frac{P_i'}{P_c'}} \quad \text{or} \quad \frac{P_i'}{P_c'} = [0.8]^2 = 0.64$$

or $$\frac{P_i'}{P_i' + P_c'} + \frac{0.64}{1 + 0.64} = 0.3902 \text{ or } \frac{P_i'}{900} = 0.3902$$

$\therefore$ $P_i' = 351.2$ W

and $P_c' = 900 - P_i' = 900 - 351.2 = 548.8$ W

Hence, **new iron loss = 351.2 W. (Ans.)**

new copper loss = 548.8 W. (Ans.)

Example 47. *The primary and secondary resistances of a 1100/220 V transformer are 0.3 Ω and 0.02 Ω respectively. If iron loss amounts to 260 W determine the secondary current at which maximum efficiency occurs and find the maximum efficiency at 0.8 power factor.*

Solution. Transformation ratio, $K = \dfrac{220}{1100} = \dfrac{1}{5}$

Primary resistance, $R_1 = 0.3\ \Omega$

Secondary resistance, $R_2 = 0.02\ \Omega$

Iron loss, $P_i = 260$ W

Total resistance referred to secondary,

$$R_{02} = R_2 + K^2 R_1 = 0.02 + \left(\frac{1}{5}\right)^2 \times 0.3 = \mathbf{0.032\ \Omega.} \quad \textbf{(Ans).}$$

Let I_2 be the secondary current at maximum efficiency.

At maximum efficiency,

Copper loss = Iron loss

$$I_2^2 R_{02} = 260$$

$$\therefore \qquad I_2^2 = \frac{260}{R_{02}} = \frac{260}{0.032} = 8125 \text{ or } I_2 = 90.14 \text{ A}$$

Hence, **secondary current at maximum efficiency = 90.14 A. (Ans.)**

Maximum efficiency at 0.8 power factor :

Output at maximum efficiency at 0.8 power factor

$$= V_2 I_2 \cos\phi_2 = 220 \times 90.14 \times 0.8 = 15865 \text{ W}$$

Losses $= P_i + P_c = 2P_i$ $\qquad (\because \; P_i = P_c)$

$$= 2 \times 260 = 520 \text{ W}$$

$$\therefore \quad \textbf{Maximum efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{15865}{15865 + 520} = \textbf{0.9683} \text{ or } \textbf{96.83\%}. \quad \textbf{(Ans.)}$$

9.15. All-day Efficiency

All-day efficiency is the *ratio of energy (kWh) delivered in a 24 hour period divided by the energy (kWh) input in the same length of time*.

$$\therefore \qquad \eta_{\text{all-day}} = \frac{\text{output in kWh}}{\text{input in kWh}} \text{ (for 24 hours)} \qquad ...(39)$$

Transformers used on residence-lighting circuits (and distribution circuits generally) are either idle or only lightly loaded during much of 24-hour period. However, they must at all times be connected to the line and ready to serve, so that the core losses are being supplied continually. *It is therefore very important that such transformers be designed for minimum core loss*. The copper losses are relatively less important, since they depend on the load. Because they are lightly loaded much of the time, *distribution transformers are designed for relatively large full-load copper loss and have their maximum power efficiencies at light loads*. This design results in improved all-day efficiency for these transformers. *Power transformers*, on the other hand are loaded more or less continuously and *are designed for full-load copper losses equal to about twice the no-load losses*.

To calculate all-day efficiency, it is necessary to know how the load on the transformer varies from hour to hour. The quotient obtained by dividing the energy output by the energy output plus energy losses over a 24-hour period yields the efficiency expressed as a decimal fraction.

The use of a *load factor* facilitates practical calculations.

Example 48. *A 15 kVA, 2000/200 V transformer has an iron loss of 250 W and full-load copper loss 350 W. During the day it is loaded as follows* :

No. of hours	*Load*	*Power factor*
9	$\frac{1}{4}$ *load*	*0.6*
7	*full-load*	*0.8*
6	$\frac{3}{4}$ *load*	*1.0*
2	*no-load*	—

Calculate the all-day efficiency.

Solution. Rating of transformer = 15 kVA

Iron loss, $P_i = 250 \text{ W} = 0.25 \text{ kW}$

Full-load copper loss, $P_c = 350 \text{ W} = 0.35 \text{ kW}$

Iron loss/day $= 0.25 \times 24 = 6 \text{ kWh}$

Copper loss at $\frac{1}{4}$ *load* $= \left(\frac{1}{4}\right)^2 \times P_c = \frac{1}{16} \times 0.35 = 0.0218 \text{ kW}$

Copper loss for 9 hours at $\frac{1}{4}$ load

$= 9 \times 0.0218 = 0.196$ kWh

Copper loss at *full-load* $= P_c = 0.35$ kW

Copper loss for 7 hours on full-load

$= 7 \times 0.35 = 2.45$ kWh

Copper loss at $\frac{3}{4}$ *load* $= \left(\frac{3}{4}\right)^2 \times P_c = \frac{9}{16} \times 0.35 = 0.197$ kW

Copper loss for 6 hours at $\frac{3}{4}$ load $= 0.197 \times 6 = 1.18$ kWh

Copper loss/day $= 0.196 + 2.45 + 1.18 = 3.826$ kWh

Total loss/day = iron loss/day + copper loss/day = 6 + 3.826 = 9.826 kWh

Total output/day $= \frac{1}{4} \times 15 \times 0.6 \times 9 + 15 \times 0.8 \times 7 + \frac{3}{4} \times 15 \times 1.0 \times 6$

$= 20.25 + 84 + 67.5 = 171.75$ kWh

All-day efficiency $= \frac{\text{output}}{\text{output + losses}} = \frac{171.75}{171.75 + 9.826}$

= **0.9459** or **94.59%.** **(Ans.)**

9.16. Transformer Noise

The "**hum**" caused by energized power transformer, under *no-load conditions*, originates in the *core where the laminations tend to vibrate by magnetic forces*. The noise is transmitted through the oil to the tank side and thence to the surroundings.

The following are the *main factors which produce noise in transformers* :

1. *Magnetostriction* (occurrence of dimensional changes both parallel to, and perpendicular to the direction of magnetisation).
2. The mechanical vibrations caused by the laminations, depending upon the tightness of clamping, size, gauge, associated structural parts, etc.
3. The mechanical vibration of trank walls.
4. The damping.

The noise emission may be reduced by the following methods/means :

1. Prevention of vibration of core-plate by the use of a lower flux density and giving attention to constructional feature (such as clamping bolts, proportions and dimensions of the '*steps*' in plate width, tightness of clamping and uniformity of plates).
2. Using cushions, padding, or oil barriers to sound insulate the transformer from tank.
3. Designing suitably the tank and stiffeners to check tank wall vibration.
4. Sound insulating the tank from the ground or surrounding air.

However, the *noise problem cannot be solved completely.*

9.17. Auto-transformer

A transformer in which part of the winding is common to both the primary and secondary circuits is known as an **auto-transformer.** *The primary is electrically connected to the secondary, as well as magnetically coupled to it*.

Refer to Fig. 57. *LM* is primary winding having N_1 turns and *MS* is secondary winding having N_2 turns. If no-load current and iron losses are neglected.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = K.$$

The current in the section MS is vector difference of I_2 and I_1 ; but since the two currents are practically in phase opposition, the resultant is $(I_2 - I_1)$ where $I_2 > I_1$.

Saving of copper (in comparison to conventional two winding transformer) :

The volume and hence weight of copper is proportional to the length and area of cross-section of the conductors. But the *length of conductor is proportional to the number of turns and cross-section depends on current*. Hence the weight of copper is proportional to the *product of number of turns and current to be carried*.

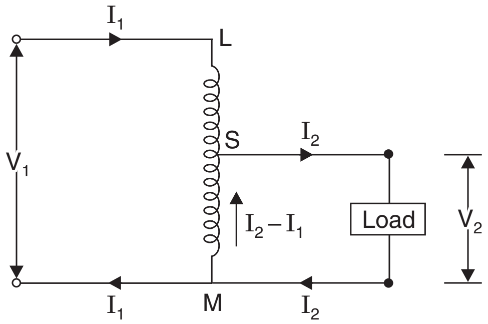


Fig. 56A. Auto-transformer.

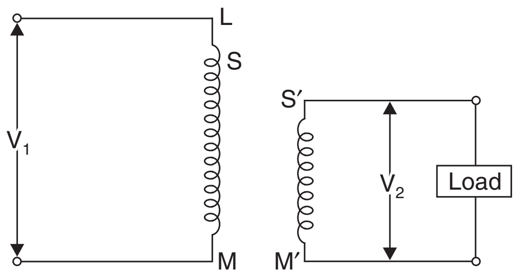


Fig. 57. Conventional two winding transformer.

Weight of copper in conventional two winding transformer

$$\propto (N_1 I_1 + N_2 I_2).$$

Weight of copper in auto-transformer

= weight of copper in section LS + weight of copper is section MS.

But weight of copper in section $LS \propto (N_1 - N_2) I_1$

and weight of copper in section $MS \propto N_2 (I_2 - I_1)$

$\therefore$ Weight of copper in auto-transformer

$$\propto (N_1 - N_2)\, I_1 + N_2\, (I_2 - I_1)$$

$$\therefore \quad \frac{\text{Weight of copper in auto-transformer } (W_a)}{\text{Weight of copper in ordinary transformer } (W_0)}$$

$$= \frac{(N_1 - N_2)\, I_1 + N_2\, (I_2 - I_1)}{N_1 I_1 + N_2 I_2} = \frac{(N_1 - 2N_2)\, I_1 + N_2 I_2}{N_1 I_1 + N_2 I_2}$$

$$= \frac{\dfrac{N_1}{N_2} - 2 + \dfrac{I_2}{I_1}}{\dfrac{N_1}{N_2} + \dfrac{I_2}{I_1}} = \frac{\dfrac{1}{K} - 2 + \dfrac{1}{K}}{\dfrac{1}{K} + \dfrac{1}{K}}$$

$= 1 - K$ ...(40) $\left[\because \frac{N_2}{N_1} = K, \frac{I_2}{I_1} = \frac{1}{K}\right]$

∴ Saving in copper $= W_0 - W_a = W_0 - (1 - K)\, W_0 = KW_0$

∴ Saving in copper $= K \times$ weight of copper in ordinary transformer

It can be proved that power transformed = input $(1 - K)$

The rest of the power is conducted *directly from the source to the load*.

Advantages of auto-transformer. An auto-transformer entails the following *advantages* :

(*i*) Higher efficiency

(*ii*) Small size

(*iii*) Lower cost

(*iv*) Better voltage regulation when compared with a conventional two-winding transformer of the same rating.

Disadvantages. Following are the *disadvantages/limitations* of auto-transformers :

(*i*) The primary and secondary are conductively connected, rather than isolated as in the conventional (ordinary) transformer. Because of this, *both sides are subject to any stresses set up by disturbances on either side*. The low-voltage side is subject to high-voltage stress and should be insulated for the higher voltage. In the case of a step down transformer, the high voltage may still be impressed upon equipment connected to the low-voltage side. This is shown in Fig. 58 where the low-voltage coil has accidentally opened. The voltage on the load is nearly 2300 V, being less than that by the impedance volt drop in the primary coil.

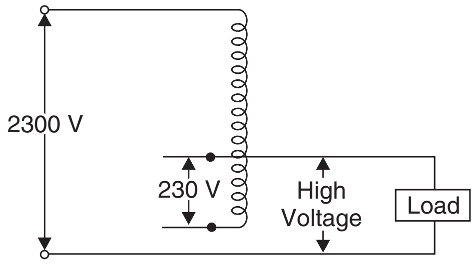


Fig. 58. Open circuit on auto-transformer.

(*ii*) As the voltage ratio of an auto-transformer increases, the common coil is much smaller compared with the entire winding. This means that the economy gained is only a small part of the transformer and therefore this advantage is minimized.

Thus, because of the above disadvantages, lack of isolation and decreased economy, *auto-transformers are rarely used in ratios greater than 4 : 1, except for low-power devices on low- voltage systems*.

Uses. The auto-transformers find the following *applications* :

1. To tie together transmission or distribution circuits of slightly different voltages (*e.g.*, 11000 V system with a 13200 V system).

2. To obtain partial line voltages for starting induction and synchronous motors with squirrel-cage windings.

3. To give a small boost to a distribution cable to correct for the voltage drop.

4. As furnace transformers for getting a convenient supply to suit the furnace winding from a 230 V supply.

5. As regulating transformers.

6. To obtain a neutral in a 3-wire A.C distribution system in the same way as a balancer set is used in a 3-wire D.C. distribution system.

7. A continuously variable auto-transformer finds useful applications in electrical testing laboratory.

9.18. Polarity of Transformers

In a transformer each of the primary terminals becomes alternately positive and negative with respect to the other and the same is true about the secondary terminals. If the transformer is to be used alone, the polarity is not important but if the transformer is to be used in parallel with another transformer the *instantaneous polarity is important* because the terminals having identical instantaneous polarity have to be connected together.

Refer to Figs. 59 and 60. The terminals of the high voltage (h.v.) winding (*i.e.*, H_1 and H_2) and the terminals of the low voltage (l.v.) winding (*i.e.*, X_1 and X_2) are so marked that when the instantaneous voltage is directed from H_1 to H_2 in the high voltage winding, it is directed from X_1 to X_2 in the low voltage winding. In other words if the high voltage terminal H_1 is positive with respect to H_2 at any instant when the low voltage terminal X_1 will be positive with respect to X_2 at the same instant. It follows that when the terminals are arranged as in Fig. 59 the polarity is subtractive whereas the additive polarity is represented as in Fig. 60.

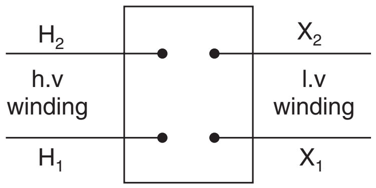


Fig. 59. Subtractive polarity.

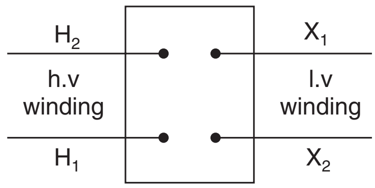


Fig. 60. Additive polarity.

Polarity Test. A polarity test is carried out to *find out the terminals having the same instantaneous polarity* assuming that the terminals are not marked. The connections are shown in Fig. 61.

- One h.t. and one l.t. terminals are joined together. A voltmeter is placed between the remaining two terminals.
- A convenient moderate voltage is impressed on the h.t. winding.

If the voltage V′, is 'greater' than the applied voltage V, then the transformer has 'additive' polarity.

If V′ is 'less' than V, the transformer has 'subtractive' polarity.

The terminals are then marked accordingly.

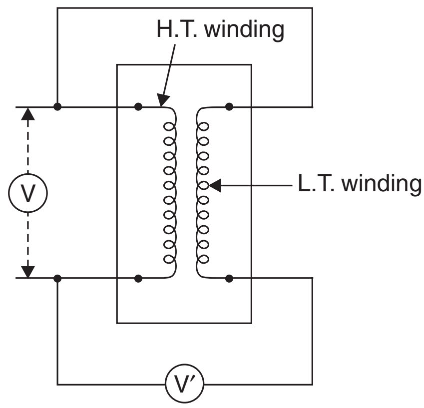


Fig. 61. Polarity test.

Example 49. *A load of 6 kW is supplied by an auto-transformer at 120 V and at unity power factor. If the primary voltage is 240 V, determine* :

(*i*) *Transformation ratio,* (*ii*) *Secondary current,*

(*iii*) *Primary current,*

(*iv*) *Number of turns across secondary if the total number of turns is 280.*

(*v*) *Power transformed, and*

(*vi*) *Power conducted directly from supply mains to load.*

Solution. Load supplied = 6 kW

Output voltage, $V_2 = 120$ V

Primary voltage, $V_1 = 240$ V

Total number of turns, $N_1 = 280$

(*i*) **Transformation ratio,** $K = \frac{V_2}{V_1} = \frac{120}{240} = \mathbf{\frac{1}{2}}$. **(Ans.)**

(*ii*) **Secondary current,** $I_2 = \frac{6 \times 1000}{V_2 \cos \phi} = \frac{6000}{120 \times 1}$ = **50 A. (Ans.)**

(*iii*) **Primary current,** $I_1 = KI_2 = \frac{1}{2} \times 50 =$ **25 A. (Ans.)**

(*iv*) **Turns across secondary,** $N_2 = kN_1 = \frac{1}{2} \times 280 =$ **140. (Ans.)**

(*v*) **Power transformed** $= \text{Load} \times (1 - K) = 6 \times (1 - \frac{1}{2}) =$ **3 kW. (Ans.)**

(*vi*) **Power conducted directly from supply mains**

$= 6 - 3 =$ **3 kW. (Ans.)**

Example 50. *The primary and secondary voltages of an auto-transformer are 600 V and 500 V respectively. Show with the aid of a diagram the current distribution in the windings when the secondary current is 210 A. Calculate the economy in copper.*

Solution. Primary voltage, $V_1 = 600$ V

Secondary voltage, $V_2 = 500$ V

Secondary current, $I_2 = 210$ A

Economy in copper :

Transformation ratio, $\frac{V_2}{V_1} = \frac{500}{600} = \frac{5}{6}$

Primary current, $I_1 = KI_2 = \frac{5}{6} \times 210 = 175$ A

The current distribution is shown in Fig. 62.

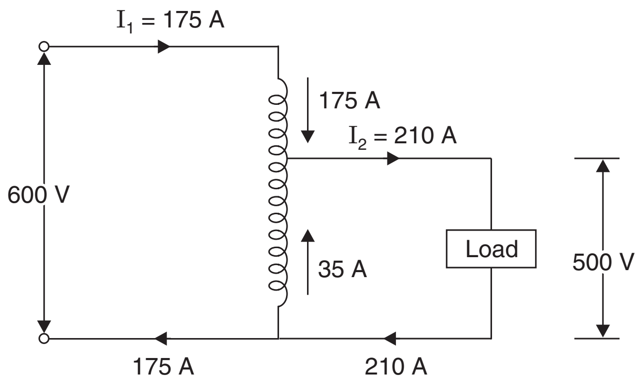


Fig. 62

We know that,

$$\frac{\text{Weight of copper on auto-transformer}}{\text{Weight of copper on ordinary transformer}} = 1 - K$$

Economy in copper

$$= \frac{\text{wt. of Cu on ordinary transformer} - \text{wt. of Cu on auto-transformer}}{\text{wt. of Cu on ordinary transformer}}$$

$$= 1 - \frac{\text{wt. of Cu on auto-transformer}}{\text{wt. of Cu on ordinary transformer}}$$

$= 1 - (1 - K) = K = \mathbf{\frac{5}{6}}$ or **83.33%. (Ans.)**

Example 51. *A 4 kVA single-phase 50 Hz transformer has a full-load efficiency of 95.5% and iron loss of 45 W. The transformer is now connected as an auto-transformer to 220 V supply. If it delivers 4 kW load at unity power factor to a 110 V circuit, calculate the efficiency of the operation and the current drawn by the high-voltage side.*

Solution. Fig. 63 shows the connections for a 2-winding transformer. In Fig. 64 the same unit has been connected as an auto-transformer to a 220 V supply. Since the two-windings are connected in series, hence voltage across each is 110 V.

In both connections the iron loss would remain the same. Since the auto-transformer windings will each carry but *half the current* as the conventional two-winding transformer, the copper losses will be $\frac{1}{4}$*th of previous value.*

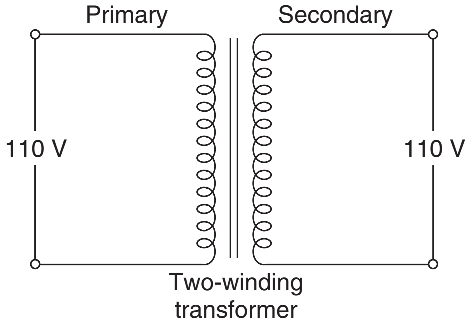


Fig. 63. Two-winding transformer.

Two-winding transformer :

Efficiency = 95.5% or 0.955

$$\therefore \quad 0.955 = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{4000}{4000 + 45 + \text{copper loss}}$$

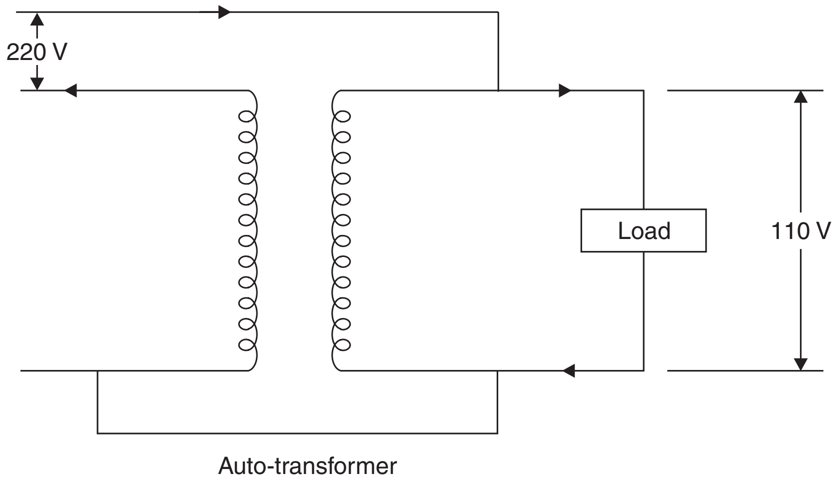


Fig. 64. Auto-transformer

$\therefore$ Copper loss $= \dfrac{4000}{0.955} - (4000 + 45) = 143.48$ W

Auto-transformer :

Copper loss $= \dfrac{143.48}{4} = 35.87$ W

Iron loss = 45 W

$\therefore$ **Efficiency** $= \dfrac{4000}{4000 + 35.87 + 45} =$ **0.9802** or **98.02%. (Ans.)**

HIGHLIGHTS

1. The function of a transformer is to transform alternating current energy from one voltage into another voltage. It operates on the principle of mutual inductance (between two or more inductively coupled coils).

2. **Distribution transformers** should be designed to have maximum efficiency at a load much lower than full-load (about 50%).

Power transformers should be designed to have maximum efficiency at or near full-load.

3. The transformation ratio (K) is defined as the ratio of the secondary voltage to primary voltage.

4. **Approximate** voltage drop $= I_2R_{02} \cos \phi \pm I_2 X_{02} \sin \phi$

Exact voltage drop $= (I_2R_{02} \cos \phi \pm I_2X_{02}) + \dfrac{(I_2X_{02} \cos \phi \mp I_2R_{02} \sin \phi)^2}{2\ {}_0V_2}$

5. Transfer of resistance or reactance from
Primary to secondary ...... $\times K^2$

Secondary to primary ...... $\times \dfrac{1}{K^2}$

6. The change in secondary voltage when rated load at a specified power is removed.

$$\% \text{ regulation} = \frac{I_2R_{02} \cos \phi \pm I_2X_{02} \sin \phi}{{}_0V_2} \times 100$$

$$\eta_{\text{all-day}} = \frac{\text{output in kWh}}{\text{input in kWh}} \text{ (for 24 hours).}$$

OBJECTIVE TYPE QUESTIONS

(A) Choose the Correct Answer :

1. Which of the following does not change in a transformer ?
(*a*) Current (*b*) Voltage
(*c*) Frequency (*d*) All of the above.

2. In a transformer the energy is conveyed from primary to secondary
(*a*) through cooling coil (*b*) through air
(*c*) by the flux (*d*) none of the above.

3. A transformer core is laminated to
(*a*) reduce hysteresis loss (*b*) reduce eddy current losses
(*c*) reduce copper losses (*d*) reduce all above losses.

4. Which loss is not common between a transformer and rotating machines ?
(*a*) Eddy current loss (*b*) Copper loss
(*c*) Windage loss (*d*) Hysteresis loss.

5. The degree of mechanical vibrations produced by the laminations of a transformer depends on
(*a*) tightness of clamping (*b*) gauge of laminations
(*c*) size of laminations (*d*) all of the above.

6. The no-load current drawn by transformer is usually what percent of the full-load current ?
(*a*) 0.2 to 0.5 per cent (*b*) 2 to 5 per cent
(*c*) 12 to 15 per cent (*d*) 20 to 30 per cent.

7. In case there are burrs on the edges of the laminations of the transformer, it is likely to result in
(*a*) vibrations (*b*) noise
(*c*) higher eddy current loss (*d*) higher hysteresis loss.

8. The path of a magnetic flux in a transformer should have
(*a*) high resistance (*b*) high reluctance
(*c*) low resistance (*d*) low reluctance.

9. No-load test on a transformer is carried out to determine
(*a*) copper loss (*b*) magnetising current
(*c*) magnetising current and loss (*d*) efficiency of the transformer.

10. The dielectric strength of transformer oil is expected to be
(*a*) 1 kV (*b*) 33 kV
(*c*) 100 kV (*d*) 330 kV.

11. Sumpner's test is conducted on transformers to determine
(*a*) temperature (*b*) stray losses
(*c*) all-day efficiency (*d*) none of the above.

12. The permissible flux density in case of cold rolled grain oriented steel is around
(*a*) 1.7 Wb/m^2 (*b*) 2.7 Wb/m^2
(*c*) 3.7 Wb/m^2 (*d*) 4.7 Wb/m^2.

13. During the short-circuit test on a small transformer the frequency is increased from 50 Hz to 200 Hz. The copper losses will increase by a factor of
(*a*) 16 (*b*) 4
(*c*) 1 (*d*) $\frac{1}{4}$.

14. The efficiency of a transformer will be maximum when
(*a*) copper losses = hysteresis losses (*b*) hysteresis losses = eddy current losses
(*c*) eddy current losses = copper losses (*d*) copper losses = iron losses.

15. No-load current in a transformer
(*a*) lags behind the voltage by about 75° (*b*) leads the voltage by about 75°
(*c*) lags behind the voltage by about 15° (*d*) leads the voltage by about 15°.

16. The purpose of providing an iron core in a transformer is to
(*a*) provide support to windings (*b*) reduce hysteresis loss
(*c*) decrease the reluctance of the magnetic path (*d*) reduce eddy current losses.

17. Which of the following is not a part of transformer installation ?
(*a*) Conservator (*b*) Breather
(*c*) Buchholz relay (*d*) Exciter.

18. While conducting short-circuit test on a transformer the following side is short-circuited
(*a*) high voltage side (*b*) low voltage side
(*c*) primary side (*d*) secondary side.

19. In a transformer the toppings are generally provided on
(*a*) primary side (*b*) secondary side
(*c*) low voltage side (*d*) high voltage side.

20. The use of higher flux density in the transformer design
(*a*) reduces weight per kVA (*b*) reduces iron losses
(*c*) reduces copper losses (*d*) increases part load efficiency.

21. The chemical used in breather for transformer should have the quality of
(*a*) ionizing air (*b*) absorbing moisture
(*c*) cleansing the transformer oil (*d*) cooling the transformer oil.

22. The chemical used in breather is
(*a*) asbestos fibre (*b*) silica sand
(*c*) sodium chloride (*d*) silica gel.

23. If a pump motor is run on 2/3rd of its supply voltage, it will
(*a*) continue to deliver same power (*b*) burn
(*c*) stall (*d*) continue to run at lower speed.

24. An ideal transformer has infinite values of primary and secondary inductances. The statement is
(*a*) true (*b*) false.

25. The transformer ratings are usually expressed in terms of
(*a*) volts (*b*) amperes
(*c*) kW (*d*) kVA.

26. The noise resulting from vibrations of laminations set by magnetic force, is termed as
(*a*) magnetostriction (*b*) boo
(*c*) hum (*d*) zoom.

27. Hysteresis loss in a transformer varies as (B_{max} = maximum flux density)
(*a*) B_{max} (*b*) $B_{max}^{1.6}$
(*c*) B_{max}^{2} (*d*) $B_{max}^{2.4}$.

28. Material used for construction of transformer core is usually
(*a*) wood (*b*) copper
(*c*) aluminium (*d*) silicon steel.

29. The thickness of laminations used in a transformer is usually
(*a*) 0.4 mm to 0.5 mm (*b*) 4 mm to 5 mm
(*c*) 14 mm to 15 mm (*d*) 25 mm to 40 mm.

30. The function of conservator in a transformer is
(*a*) to protect against internal fault (*b*) to reduce copper as well as core losses
(*c*) to cool the transformer oil
(*d*) to take care of the expansion and contraction of transformer oil due to variation of temperature of surroundings.

31. The highest voltage for transmitting electrical power in India is
(*a*) 33 kV (*b*) 66 kV
(*c*) 132 kV (*d*) 400 kV.

32. In a transformer the resistance between its primary and secondary is
(*a*) zero (*b*) 1 Ω
(*c*) 1000 Ω (*d*) infinite.

33. A transformer oil must be free from
(*a*) sludge (*b*) odour
(*c*) gases (*d*) moisture.

34. A Buchholz relay can be installed on
(*a*) auto-transformers (*b*) air-cooled transformers
(*c*) welding transformers (*d*) oil cooled transformers.

35. Gas is usually not liberated due to dissociation of transformer oil unless the oil temperature exceeds
(*a*) 50°C (*b*) 80°C
(*c*) 100°C (*d*) 150°C.

36. The main reason for generation of harmonics in a transformer could be
(*a*) fluctuating load (*b*) poor insulation
(*c*) mechanical vibrations (*d*) saturation of core.

37. Distribution transformers are generally designed for maximum efficiency around
(*a*) 90% load (*b*) zero load
(*c*) 25% load (*d*) 50% load.

38. Which of the following properties is not necessarily desirable in the material for transformer core ?
(*a*) Mechanical strength (*b*) Low hysteresis loss
(*c*) High thermal conductivity (*d*) High permeability.

39. Helical coils can be used on
(*a*) low voltage side of high kVA transformers (*b*) high frequency transformers
(*c*) high voltage side of small capacity transformers
(*d*) high voltage side of high kVA rating transformers.

40. High frequency transformers sometimes make use of ferrite cores because it has
(*a*) high specific gravity (*b*) high resistance
(*c*) high hysteresis (*d*) low permeability.

41. Cross over windings are used on
(*a*) low voltage side of high kVA rating transformers
(*b*) current transformers
(*c*) high voltage side of high kVA rating transformers
(*d*) high voltage side of low kVA rating transformers.

42. While conducting short-circuit test on a transformer the following side is short-circuited.
(*a*) h.v. side (*b*) l.v. side
(*c*) primary side (*d*) secondary side.

43. A substance is placed in strong magnetic field. Which of the following will have maximum influence on the capacity of the substance to get magnetised ?
(*a*) Permeability (*b*) Susceptibility
(*c*) Permittivity (*d*) Resistivity.

44. Harmonics in transformer result in
(*a*) increased core losses (*b*) increased I^2R losses
(*c*) magnetic interference with communication circuits
(*d*) all of the above.

45. Tertiary winding is provided in transformers having
(*a*) mesh/star winding (*b*) mesh/mesh winding
(*c*) star/star winding (*d*) any of the above.

46. The core used in high frequency transformer is usually
(*a*) copper core (*b*) cost iron core
(*c*) air core (*d*) mild steel core.

47. The full-load copper loss of a transformer is 1600 W. At half-load the copper loss will be
(*a*) 6400 W (*b*) 1600 W
(*c*) 800 W (*d*) 400 W.

48. The value of flux involved in the e.m.f. equation of a transformer is
(*a*) average value (*b*) r.m.s. value
(*c*) maximum value (*d*) instantaneous value.

49. Silicon steel used in laminations mainly reduces
(*a*) hysteresis loss (*b*) eddy current losses
(*c*) copper losses (*d*) all of the above.

50. Which winding of the transformer has less cross-sectional area ?
(*a*) Primary winding (*b*) Secondary winding
(*c*) Low voltage winding (*d*) High voltage winding.

51. Power transformers are generally designed to have maximum efficiency around
(*a*) no-load (*b*) half-load
(*c*) near full-load (*d*) 10% overload.

52. Which of the following is the main advantage of an auto-transformer over a two winding transformer ?
(*a*) Hysteresis losses are reduced (*b*) Saving in winding material
(*c*) Copper losses are negligible (*d*) Eddy losses are totally eliminated.

53. During short-circuit test iron losses are negligible because
(*a*) the current on secondary side is negligible (*b*) the voltage on secondary side does not vary
(*c*) the voltage applied on primary side is low
(*d*) full-load current is not supplied to the transformer.

54. Two transformers are connected in parallel. These transformers do not have equal percentage impedance. This is likely to result in
(*a*) short-circuiting of the secondaries
(*b*) power factor of one of the transformers is leading while that of the other lagging
(*c*) transformers having higher copper losses will have negligible core losses
(*d*) loading of the transformers not in proportion to their kVA ratings.

55. The changes in volume of transformer cooling oil due to variation of atmospheric temperature during day and night is taken care of by which part of transformer ?
(*a*) Conservator (*b*) Breather
(*c*) Bushings (*d*) Buchholz relay.

56. The transformer laminations are insulated from each other by
(*a*) mica strip (*b*) thin coat of varnish
(*c*) paper (*d*) any of the above.

57. Which type of winding is used in 3-phase shell-type transformer ?
(*a*) Circular type (*b*) Sandwich type
(*c*) Cylindrical type (*d*) Rectangular type.

58. During open circuit test of a transformer
(*a*) primary is supplied rated voltage (*b*) primary is supplied full-load current
(*c*) primary is supplied current at reduced voltage (*d*) primary is supplied rated kVA.

59. Open circuit test on transformers is conducted to determine
(*a*) hysteresis losses (*b*) copper losses
(*c*) core losses (*d*) eddy current losses.

60. Short circuit test on transformers is conducted to determine
(*a*) hysteresis losses (*b*) copper losses
(*c*) core losses (*d*) eddy current losses.

61. For the parallel operation of single-phase transformers it is necessary that they should have
(*a*) same efficiency (*b*) same polarity
(*c*) same kVA rating (*d*) same number of turns on the secondary side.

62. The transformer oil should have ...... voltatility and ...... viscosity
(*a*) low ...... low (*b*) high ...... high
(*c*) low ...... high (*d*) high ...... low.

63. The function of breather in a transformer is
(*a*) to provide oxygen inside the tank (*b*) to cool the coils during reduced load
(*c*) to filter the transformer cooling oil
(*d*) to arrest flow of moisture when outside air enters the transformer.

64. The secondary winding of which of the following transformers is always kept closed ?
(*a*) Step-up transformer (*b*) Step-down transformer
(*c*) Potential transformer (*d*) Current transformer.

65. The size of a transformer core will depend on
(*a*) frequency (*b*) area of the core
(*c*) flux density of the core material (*d*) (*a*) and (*b*) both.

66. Natural air cooling is generally restricted for transformers up to
(*a*) 1.5 MVA (*b*) 5 MVA
(*c*) 15 MVA (*d*) 50 MVA.

67. A transformer can have regulation closer to zero
(*a*) on full-load (*b*) on overload
(*c*) on leading power factor (*d*) on zero power factor.

68. A transformer transforms
(*a*) voltage (*b*) current
(*c*) current and voltage (*d*) power.

69. Which of the following is not the standard voltage for power supply in India
(*a*) 11 kV (*b*) 33 kV
(*c*) 66 kV (*d*) 122 kV.

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*b*)	**4.** (*a*)	**5.** (*d*)	**6.** (*b*)	**7.** (*c*)
8. (*d*)	**9.** (*c*)	**10.** (*b*)	**11.** (*a*)	**12.** (*a*)	**13.** (*a*)	**14.** (*d*)
15. (*a*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)	**21.** (*b*)
22. (*d*)	**23.** (*c*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27** (*b*)	**28.** (*d*)
29. (*a*)	**30.** (*d*)	**31.** (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*d*)	**35.** (*d*)
36. (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*a*)	**40.** (*b*)	**41.** (*d*)	**42.** (*b*)
43. (*b*)	**44.** (*d*)	**45.** (*c*)	**46.** (*c*)	**47.** (*d*)	**48.** (*c*)	**49.** (*a*)
50. (*d*)	**51.** (*c*)	**52.** (*b*)	**53.** (*c*)	**54.** (*d*)	**55.** (*a*)	**56.** (*b*)
57. (*b*)	**58.** (*a*)	**59.** (*c*)	**60.** (*b*)	**61.** (*b*)	**62.** (*a*)	**63.** (*d*)
64. (*d*)	**65.** (*d*)	**66.** (*a*)	**67.** (*c*)	**68.** (*d*)	**69.** (*d*)	

(B) Say 'Yes' or 'No'

1. When a transformer raises the voltage it is called the step-up transformer.
2. A transformer must not be connected to D.C. source.
3. The ratio of primary voltage to secondary voltage is known as transformation ratio.
4. An ideal transformer is one in which the resistance of the windings is negligible and the core has no losses.
5. Primary and secondary currents are directly proportional to their respective turns.
6. The function of the magnetising component of no-load current is to sustain the alternating flux in the core.
7. The no-load primary input is practically equal to the iron loss in the transformer.
8. A transformer is said to be loaded when the secondary circuit of a transformer is completed through an impedance load.
9. Magnetic flux can be confined into a designed path.
10. When shifting resistance to the secondary, divide it by K^2.
11. An open-circuit test is conducted to find no-load or core loss.
12. Short-circuit test is conducted to find full-load copper loss.
13. The change in voltage when rated load at a specified power is removed is termed as voltage regulation.
14. Iron or core losses include copper loss and eddy current loss.
15. Iron or core losses are found from short-circuit test.
16. The efficiency of a transformer at a particular load and power factor is defined as the ratio of power output to power input.
17. Copper losses = iron losses is the condition for maximum efficiency of a transformer.
18. $\eta_{\text{all-day}} = \dfrac{\text{output in kWh}}{\text{input in kWh}}$ (for 24 hours).

ANSWERS

1. Yes	**2.** Yes	**3.** No	**4.** Yes	**5.** No	**6.** Yes	**7.** Yes
8. Yes	**9.** No	**10.** No	**11.** Yes	**12.** Yes	**13.** Yes	**14.** No
15. No	**16.** Yes	**17.** Yes	**18.** Yes.			

THEORETICAL QUESTIONS

1. What is a transformer ? How does it transfer electric energy from one circuit to another ?
2. Explain the principle of operation of a transformer.
3. What is meant by *transformer action* ? Under what conditions will it take place ?
4. If an alternating current is impressed on one coil, what will be the frequency of the induced voltage in another coil with which it is coupled ?
5. Enumerate the various kinds of transformers.
6. Explain the need for stepping up and stepping down voltages in a power system. How does a transformer accomplish ?
7. Why are transformer windings divided into several coils ?
8. What properties should a good transformer oil possess ?
9. What purposes are served by placing transformers in oil-filled tanks ?
10. Why are the tanks of some large transformers corrugated ?
11. Write a short note on 'transformer cooling'.
12. Enumerate and explain briefly different types of windings.
13. Derive an expression for induced e.m.f. in a transformer in terms of frequency, the maximum value of flux and the number of turns on the windings.
14. Derive an expression for the e.m.f. of an ideal transformer winding.
15. Explain the function of the active and reactive components of the no-load current of a static transformer.
16. Why are some transformers constructed with primary and secondary windings divided into two halves ?
17. Distinguish between *power* and *distribution* transformers.
18. Draw and explain the no-load phasor diagram for a single-phase transformer.
19. Draw the vector diagram of a power transformer under full-load condition.
20. What is meant by *equivalent resistance* of a transformer ? How may it be calculated in primary terms and secondary terms ?
21. What is meant by the *equivalent reactance* of a transformer ? How may it be calculated in primary terms and secondary terms ?
22. How can the equivalent impedance of a transformer be determined ?
23. Outline carefully the procedure for performing the short-circuit test.
24. What useful information is obtained from the short-circuit test ?
25. Outline carefully the procedure for performing the open-circuit test.
26. What useful information is obtained from open-circuit test ?
27. What are the two components of the core loss ?
28. How is the hysteresis loss affected by a change in flux density ?
29. Develop the equivalent circuit of a single phase transformer.
30. Draw the equivalent circuit of a transformer and show how the constants of the primary and secondary windings may be combined to give a simplified equivalent circuit with the values of constants given in terms of the secondary winding.
31. Describe the method of calculating the regulation and efficiency of a single-phase transformer by open circuit and short circuit tests.
32. Distinguish between the 'efficiency' and the 'regulation' of a transformer. Show how power factor affects both of them.
33. Explain with circuit diagrams, the open-circuit and short-circuit tests to be carried out in the laboratory for the determination of the parameters of a single-phase transformer. Derive the condition for maximum efficiency.
34. What is Sumpner's test ? Draw a circuit diagram to conduct this test and explain its principle.
35. Define all-day efficiency.
36. What is an auto-transformer ? What advantages are possessed by auto-transformers ?
37. Derive an expression for saving of copper when an auto-transformer is used.
38. State the applications of auto-transformers. Why is this transformer not used as a distribution transformer ? Prove that for the same capacity and voltage ratio, an auto-transformer requires less copper than an ordinary transformer.

39. What are the sources of heat in a transformer ? Describe briefly the various methods used for cooling of transformers.

40. To what does the '*polarity*' of a transformer refer ?

EXERCISE

E.m.f. Equation-Turn Ratio

1. The no-load ratio required in a single-phase 50 Hz transformer is 6600/300 V. If the maximum value of flux in the core is to be about 0.09 weber, find the number of turns in each winding.
[**Ans.** $N_1 = 330$; $N_2 = 15$]

2. A 20 kVA, single-phase transformer has 200 turns on the primary and 40 turns on the secondary. The primary is connected to 1000 V, 50 Hz supply. Determine :
(*i*) the secondary voltage on open circuit ;
(*ii*) the currents flowing through the two windings on full-load ; and
(*iii*) the maximum value of flux. [**Ans.** (*i*) 200 V ; (*ii*) 20 A, 100 A ; (*iii*) 0.0225 Wb]

3. A single-phase transformer is connected to a 230 V, 50 Hz supply. The net cross-sectional area of the core is 50 cm^2. The number of turns in the primary is 460 and in the secondary 80. Determine :
(*i*) Transformation ratio ; (*ii*) Peak value of flux density in the core ; and
(*iii*) E.m.f. in the secondary winding. [**Ans.** 0.1739 ; 0.4504 T ; 40 V]

4. A 6600/440 V single-phase 600 kVA transformer has 1200 primary turns. Find :
(*i*) Transformation ratio ; (*ii*) Secondary turns ;
(*iii*) Voltage/turn ; and
(*iv*) Secondary current when it supplies a load of 400 kW at 0.8 power factor lagging.

$$\left[\textbf{Ans.}\ \frac{1}{15}\ ;\ 80\ ;\ 5.5\ \text{V}\ ;\ 1136\ \text{A}\right]$$

5. Find the primary and secondary turns of a 3300/300 V, single-phase, 50 Hz, 30 kVA transformer if the flux in the core is to be about 0.06 Wb. Also determine the primary and secondary currents if the losses are to be neglected. [**Ans.** 264 ; 24 ; 100 A ; 9.09 A]

6. The voltage/turn of a single-phase transformer is 1.1 V, when the primary winding is connected to a 220 V, 50 Hz A.C. supply, the secondary voltage is found to be 660 V. Find :
(*i*) Primary and secondary turns ; and
(*ii*) Core area if the maximum flux density is 1.2 T. [**Ans.** 200 ; 600 ; 41.29 cm^2]

Transformer on No-load

7. A 2000/200 V single-phase transformer gives 0.5 A and 40 W as ammeter and wattmeter readings when supply is given to the low voltage winding and high voltage winding is kept open. Find :
(*i*) The magnetising component, (*ii*) The iron loss component, and
(*iii*) The power factor of no-load current. [**Ans.** 0.4582 A ; 0.2 A ; 0.4 lagging]

8. Find (*i*) active component and reactive components of no-load current and ; (*ii*) no-load current of a 230 V/115 V single-phase transformer if the power input on no-load to the high voltage winding is 70 W and power factor of no-load current is 0.25 lagging. [**Ans.** 0.3044 A ; 1.179 A ; 1.2176 A]

9. A single-phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the core is 150 cm and the joints are equivalent to an air-gap of 0.1 mm. When a potential difference of 3000 V is applied to the primary, maximum flux density is 1.2 T. Calculate :
(*i*) The cross-sectional area of the core, (*ii*) No-load secondary voltage,
(*iii*) The no-load current drawn by the primary, and
(*iv*) Power factor on no-load.
Given that AT/cm for a flux density of 1.2 T in the iron to be 5, the corresponding iron loss to be 2 W/kg at 50 Hz and density of iron as 7.8 g/cm^3. [**Ans.** 225 cm^2 ; 240 V ; 1.208 A ; 0.1457]

Transformer on Load

10. A 230 V/115 V single phase transformer takes a no-load current of 1.7 A at a power factor of 0.18 lagging with low voltage winding kept open. If the low voltage winding is now loaded to take a current of 13 A at 0.8 power factor lagging find the current taken by high voltage winding. [**Ans.** 7.834 A]

11. A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage. [**Ans.** I_0 = 5.93 A ; ϕ_0 = 73.3°]

12. A 8 : 1 step down, single-phase transformer takes a no-load current of 0.6 A at a power factor of 0.8 lagging with 1.v. winding kept open. If the secondary is connected to a load taking a current of 80 A at (*i*) 0.8 power factor lagging ; and (*ii*) 0.8 power factor leading find the primary current and power factor.

[**Ans.** 10.49 A, 0.7996 lagging ; 9.817 A, 0.8333 leading]

13. A 20 kVA, 2000/200 V, single-phase, 50 Hz transformer has a primary resistance of 2.5 Ω and reactance of 4.8 Ω. The secondary resistance and reactance are 0.01 Ω and 0.018 Ω respectively. Find :

(*i*) Equivalent resistance referred to primary,

(*ii*) Equivalent impedance referred to primary,

(*iii*) Equivalent resistance, reactance and impedance referred to secondary, and

(*iv*) Total copper loss of the transformer.

[**Ans.** (*i*) 3.5 Ω ; (*ii*) 7.47 Ω ; (*iii*) 0.035, 0.066 Ω, 0.0747 Ω ; (*iv*) 350 W]

14. A 50 kVA, 4400/220 V transformer has R_1 = 3.45 Ω, R_2 = 0.009 Ω. The values of reactances are X_1 = 5.2 Ω and X_2 = 0.015 Ω. Calculate for the transformer :

(*i*) Equivalent resistance as referred to primary,

(*ii*) Equivalent resistance as referred to secondary,

(*iii*) Equivalent reactance as referred to both primary and secondary,

(*iv*) Equivalent impedance as referred to both primary and secondary, and

(*v*) Total copper loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.

[**Ans.** (*i*) 7.05 Ω ; (*ii*) 0.0176 Ω ; (*iii*) 11.2 Ω ; (*iv*) 13.23 Ω ; 0.0331 Ω ; (*v*) 909 W]

15. A single-phase transformer has the following data :

Turn ratio = 19.5 : 1, R_1 = 25 Ω , X_1 = 100 Ω, R_2 = 0.06 Ω, X_2 = 0.25 Ω. No-load current = 1.25 A leading the flux by 30°.

The secondary delivers 200 A at a terminal voltage of 500 V and a power factor of 0.8 lagging. Determine by the aid of a vector diagram the primary applied voltage, the primary power factor and the efficiency.

[**Ans.** 12540 ∠186.7°, 0.698 (lag), 86.74%]

16. The high voltage and low voltage windings of a 1100/110 V single-phase 50 Hz transformer has resistances of 2.4 Ω and 0.02 Ω and reactances 1 Ω and 0.009 Ω respectively. The low voltage winding is connected to a load having an impedance of (3 + *j*2) Ω. Determine :

(*i*) Current in 1.v. winding,
(*ii*) Current in h.v. winding,
(*iii*) The load voltage, and
(*iv*) Power consumed by the load.

[**Ans.** (*i*) 30.11 A ; (*ii*) 3.011 A ; (*iii*) 108.5 V ; (*iv*) 2720.7 W]

Equivalent Circuit and O.C. and S.C. Tests

17. The parameters of a 2300/230 V, 50 Hz transformer are given below :

R_1 = 0.286 Ω R_2' = 0.319 Ω

X_1 = 0.73 Ω X_2' = 0.73 Ω

R_0 = 250 Ω X_0 = 1250 Ω

The secondary load impedance Z_L = 0.387 + *j*0.29. Solve the exact equivalent circuit with normal voltage across the primary. [**Ans.** η = 78.8%, % Regulation = 2.7%]

18. A 230 V, 3 kVA single-phase transformer has an iron loss of 100 W at 40 Hz and 70 W at 30 Hz. Find the hysteresis and eddy current losses at 50 Hz. [**Ans.** 91.67 W, 41.67 W]

19. When a transformer is supplied at 400 V, 50 Hz the hysteresis loss is found to be 300 W and eddy current loss is found to be 250 W. Determine the hysteresis loss and eddy current loss when the transformer is supplied at 800 V, 100 Hz. [**Ans.** 600 W ; 1000 W]

20. When the primary of a transformer rated at 2200 V, 50 Hz is supplied at 2200 V, 50 Hz the wattmeter gives a reading of 1200 W on no-load. When it is supplied at 1100 V, 25 Hz, the wattmeter gives a reading of 400 W on no-load. If the wattmeter is connected in the input circuit find the hysteresis loss and eddy current loss at normal voltage and frequency. [**Ans.** 400 V ; 800 W]

21. A 4400 V, 50 Hz transformer has a hysteresis loss of 1000 W, eddy current loss of 1500 W and full-load copper loss of 3500 W. If the transformer is supplied at 6600 V, 75 Hz, what will be the losses ? Assume that the full-load current remains the same. [**Ans.** P_h = 1500 W, P_e = 3375 W]

22. A 50 Hz, single-phase transformer has a turn ratio of 6. The resistances are 0.9 Ω, 0.03 Ω and reactances are 5 Ω and 0.13 Ω for high-voltage and low voltage windings respectively. Find :

(*i*) The voltage to be applied to the h.v. side to obtain full-load current of 200 A in the l.v. winding on short-circuit.

(*ii*) The power factor on short-circuit.

Draw the equivalent circuit and show therein all the values. [**Ans.** 329.3 V, 0.2]

23. A 200/2000 V transformer is fed from a 200 V supply. The total winding resistance and leakage reactance as referred to low voltage side are 0.16 Ω and 7.0 Ω respectively. The representing core loss is 400 Ω and the magnetising reactance is 231 Ω. A load of impedance 596 + j444 Ω is connected across the secondary terminals. Calculate :

(*i*) Input current ; (*ii*) The secondary terminal voltage ;

(*iii*) Primary power factor. [**Ans.** (*i*) 25.96 ∠– 40.78° A ; (*ii*) 1859 V ; (*iii*) 0.757 lagging]

24. Determine the approximate equivalent circuit of a given 200/2000 V single-phase 40 kVA transformer having the following test results :

O.C. test : 200 V, 6.4 A, 384 W .... on l.v. side.

S.C. test : 78 V, 20 A, 620 W ...... on h.v. side. [**Ans.** R_{01} = 0.0155 Ω, X_{01} = 0.0358 Ω, R_0 = 104.2 Ω, X_0 = 32.75 Ω]

25. Determine the equivalent circuit of a 200/400 V, 50 Hz, single-phase transformer from the following test data :

O.C. test (l.v. side) : 200 V, 0.7 A, 70 W

S.C. test (h.v. side) : 15 V, 10 A, 85 W

Calculate the secondary voltage when delivering 5 kW at 0.8 power factor, lagging, the primary voltage being 200 V. [**Ans.** R_0 = 571.4 Ω, X_0 = 330 Ω, R_{01} = 0.21 Ω, X_{01} = 0.31 Ω, V_2 = 377.8 V]

26. A single-phase, 10 kVA, 500/250 V, 50 Hz transformer has the following constants :

Resistance : Primary 0.2 Ω, Secondary 0.5 Ω

Reactance : Primary 0.4 Ω, Secondary 0.1 Ω

Resistance or equivalent exciting circuit referred to primary, R_0 = 1500 Ω, reactance of the equivalent exciting circuit referred to primary, X_0 = 750 Ω.

What would be the readings of the instruments when the transformer is connected for the open-circuit and short-circuit tests ? [**Ans.** O.C. test : 500 V, 0.745 A, 167 W ; S.C. test : 46.8 V, 20 A, 880 W]

27. Find the approximate equivalent circuit of a single-phase 400/4400 V transformer having the following test readings :

O.C. test (l.v. side) : 400 V, 5.2 A, 600 W

S.C. test (h.v. side) : 155 V. 50 A, 1850 W [**Ans.** R_0 = 266.7 Ω, X_0 = 80.34 Ω, Z_{01} = 0.02563 Ω, R_{01} = 0.006115 Ω, X_{01} = 0.02488 Ω]

28. The following readings were taken in open-circuit and short-circuit tests on a single-phase 20 kVA, 2000/200 V transformer :

O.C. test (l.v. side) : 200 V, 2 A, 120 W

S.C. test (h.v. side) : 30 V, 10 A, 140 W

Determine :

(*i*) Equivalent circuit referred to l.v. side

(*ii*) Secondary terminal voltage on full-load at 0.8 power factor leading.

[**Ans.** (*i*) R_0 = 333.3 Ω, X_0 = 104.8 Ω, R_{02} = 0.014 Ω, X_{02} = 0.02654 Ω ; (*ii*) 200.47 V]

Regulation and Efficiency of a Transformer

29. The corrected instrument readings obtained from open and short-circuit tests on 10 kVA, 450/120 V, 50 Hz transformer are :

O.C. Test :

V_1 = 120 V, I_1 = 4.2 A, W_1 = 80 W ; V_1, W_1 and I_1 were read on the low-voltage side.

S.C. Test :

V_1 = 9.65 V ; I_1 = 22.2 A ; W_1 = 120 W with low-voltage winding short-circuited.

Calculate :

(*i*) The equivalent circuit (approximate) constants.

(*ii*) Efficiency and voltage regulation for 0.8 lagging power.

(*iii*) The efficiency at half full-load and 0.8 lagging power factor load.

[**Ans.** (*i*) R_0 = 25.30 Ω, X_0 = 409 Ω ; (*ii*) 97.57%, 2.04% ; (*iii*) 97.34%]

30. The primary and secondary winding resistance of a 40 kVA, 6600/250 V single-phase transformer are 10 Ω and 0.02 Ω respectively. The equivalent leakage reactance as referred to the primary winding is 35 Ω. Find the full-load regulation for load power factors of (*i*) unity ; (*ii*) 0.8 lagging ; and (*iii*) 0.8 leading. [**Ans.** (*i*) 2.202% ; (*ii*) 3.69% ; (*iii*) – 0.166%]

31. The percentage resistance and reactance of a transformer are 2.5% and 4% respectively. Find the approximate regulation on full-load at

(*i*) unity power factor, (*ii*) 0.8 power factor lagging, and

(*iii*) 0.8 power factor leading. [**Ans.** (*i*) 2.5% ; (*ii*) 4.4% ; (*iii*) – 0.4%]

32. A single-phase 100 kVA, 2000/200 V, 50 Hz transformer has impedance drop of 10% and resistance drop of 5%.

(*i*) Find the regulation at full-load, 0.8 power factor lagging.

(*ii*) At what power factor is the regulation zero ? [**Ans.** (*i*) 9.196% ; (*ii*) 0.866 leading]

33. The high voltage of a single-phase 200 kVA 4400/220 V transformer takes a current of 30 A and power of 1200 W at 75 V when the low voltage winding is short-circuited. Determine :

(*i*) The voltage to be applied to the high voltage winding on full-load 0.8 power factor lagging if the secondary terminal voltage is to be kept at 220 V, and

(*ii*) % regulation. [**Ans.** (*i*) 4506 V ; (*ii*) 2.355%]

34. A 20 kVA, 2200/220 V 50 Hz distribution transformer is tested for efficiency and regulation as follows ;

O.C. test (l.v. side) : 220 V, 4.2 A, 148 W.

S.C. test (h.v. side) : 86 V, 10.5 A, 360 W.

Determine :

(*i*) Core loss

(*ii*) Equivalent resistance referred to primary.

(*iii*) Equivalent resistance referred to secondary.

(*iv*) Equivalent reactance referred to primary.

(*v*) Equivalent reactance referred to secondary.

(*vi*) Regulation of transformer at 0.8 power factor lagging current.

(*vii*) Efficiency at full-load and half full-load at 0.8 power factor lagging current.

[**Ans.** 148 W, 3.26 Ω, 0.0326 Ω, 7.51 Ω, 0.0751 Ω, 2.95%, 97.4%, 97.3%]

35. A 100 kVA single-phase transformer has a full-load primary current of 400 A and total resistance referred to primary is 0.006 Ω. If the iron loss amounts to 500 W, find the efficiency on full-load and half-load at

(*i*) unity power factor, and (*ii*) 0.8 power factor.

[**Ans.** 98.58%, 98.56%, 98.22%, 98.19%]

Maximum Efficiency

36. A 50 kVA, single phase transformer has an iron loss of 400 W and full-load copper loss of 800 W.

(*i*) Find the load at which maximum efficiency occurs and the value of maximum efficiency at unity power factor.

(*ii*) If the maximum efficiency occurs at 80% of full-load, find the new core loss and full-load copper loss assuming that total full-load loss is constant.

[**Ans.** (*i*) 70.7% of full-load, 97.79% ; (*ii*) P_i = 468.36 W, P_c = 731.64 W]

37. A 1100/220 V transformer has a primary resistance of 0.25 Ω and secondary resistance of 0.03 Ω. If iron loss amounts to 250 W, determine the secondary current at which maximum efficiency occurs and find the maximum efficiency at 0.8 power factor. [**Ans.** 111.8 A ; 97.52%]

38. A single-phase 250 kVA transformer has an efficiency of 96% on full-load at 0.8 power factor and on half full-load at 0.8 power factor. Find :

(*i*) Iron loss, and (*ii*) full-load copper loss.

[**Ans.** P_i = 2.767 kW, P_c = 5.533 kW]

39. The efficiency of a 300 kVA, single-phase transformer is 97.8% when delivering full-load at 0.8 power factor lagging and 98.4% when delivering half full-load at unity power factor. Determine the efficiency at 80% of full-load at 0.8 power factor lagging. [**Ans.** 97.95%]

40. A 12 kVA, 400/200 V single-phase 50 Hz transformer has maximum efficiency of 95% at 85% of full-load at unity power factor. Determine the efficiency at full-load at 0.8 power factor lagging. [**Ans.** 93.76%]

41. A 50 kVA, 2000/250 V single-phase transformer has resistances of 1.1 Ω and 0.015 Ω and reactances of 4.4 Ω and 0.06 Ω for the high voltage and low voltage windings respectively. It has a maximum efficiency of 96.05% at 80% of full-load at unity p.f. The magnetising current for the h.v. side at 2000 V is 1.25 A. Find the readings of suitable instruments for open circuit test and short-circuit tests, supply being given to h.v. side in both cases.

$$\begin{bmatrix} \textbf{Ans.}\ \text{O.C. test} : 2000\ \text{V}, 1.316\ \text{A}, 824\ \text{W} \\ \text{S.C. test} : 212.3\ \text{V}, 25\ \text{A}, 1287\ \text{W} \end{bmatrix}$$

42. A 100 kVA, 3300/300 V single-phase transformer has an efficiency of 97.5% both on full-load at unity p.f. and on half full-load at unity power factor. The power factor of the no-load current is 0.3 lagging and the regulation on full-load at 0.8 p.f. lagging is 3 per cent. Draw the equivalent circuit referred to the l.v. side.

$$\begin{bmatrix} \textbf{Ans.}\ R_{01} = 105.3\ \Omega, & X_0 = 33.11\ \Omega \\ R_{02} = 0.01538\ \Omega, & X_{02} = 0.02451\ \Omega \end{bmatrix}$$

43. The maximum efficiency of a single-phase 240 kVA, 2000/250 V transformer occurs at 70% of full-load and is equal to 98% at 0.8 p.f. lagging. Determine the efficiency and regulation on full-load at 0.8 p.f. lagging if the impedance of the transformer is 8 per cent. [**Ans.** 97.81%, 5.699%]

44. A 5 kVA, 230/115 V transformer takes 1.2 A and 75 W when 230 V is applied to h.v. winding and l.v. winding is kept open. It takes 21.75 A and 150 W when the l.v. winding is short-circuited and 17.4 V is applied to the h.v. winding. Find :

(*i*) The no-load current as a fraction of full-load input current at 0.8 p.f. lagging ;

(*ii*) Percentage regulation on full-load at 0.8 p.f. lagging ; and

(*iii*) The load at which maximum efficiency occurs and the maximum efficiency at unity p.f.

[**Ans.** 5.162% of full-load input current ; 6.566% ; 70.7% of full-load, 95.94%]

45. The efficiency of a 20 kVA, 2000/200 V transformer is 96.8% at full-load at unity p.f. and 96% at 60% of a load at 0.8 p.f. Find the regulation at full-load at :

(*i*) 0.8 p.f. lagging ; and (*ii*) 0.8 p.f. leading if the impedance is 7 per cent.

[**Ans.** (*i*) 5.469%, (*ii*) – 2.385%]

46. A 100 kVA, 2000/200 V single-phase transformer takes a current of 50 A and 2400 W at 100 V when the low voltage winding is short circuited. Determine the load-voltage and percentage regulation when delivering full-load current at 0.8 p.f. leading, the supply voltage being 2000 V.

[**Ans.** 200.142 V ; – 0.071%]

47. A single-phase, 25 kVA transformer has an iron loss of 240 W and full-load copper loss of 600 W.

(*i*) Find the load at which maximum efficiency occurs and maximum efficiency at 0.8 p.f.

(*ii*) If the maximum efficiency occurs at 70% of full-load, find the core loss and copper loss assuming the total loss to be the same as in the previous case.

[**Ans.** (*i*) 63.25% of full-load, 96.36% ; (*ii*) 276.3 W, 563.7 W]

All-Day Efficiency

48. A 20 kVA, 2000/200 V transformer has an iron loss of 300 W and full-load copper loss of 400 W. During the day it is loaded as follows :

No. of hours	*Load*	*Power factor*
8	$\frac{1}{4}$ load	0.5
6	Full-load	0.8
6	$\frac{3}{4}$ load	unity
4	No-load	—

Find the all-day efficiency. [**Ans.** 94.84%]

49. A 30 kVA transformer has got a maximum efficiency of 97% at 80% of load at unity p.f. During the day it is loaded as follows :

No. of hours	*Load*	*Power factor*
10	4 kW	0.6 lag
9	20 kW	0.8 lag
5	24 kW	0.85 lag

Find the all-day efficiency. [**Ans.** 95.68%]

50. The all-day efficiency of 200 kVA transformer is 96% when it is loaded as follows :

No. of hours	*Load*	*Power factor*
12	120 kW	0.8
8	150 kW	unity
4	No-load	—

If maximum efficiency of this transformer occurs at 80% of full-load, find iron loss and full-load copper loss. [**Ans.** P_i = 2.645 kW, P_c = 4.133 kW]

Auto Transformer

51. An auto transformer supplies a load of 5 kW at 125 V and at unity power factor. If the primary voltage is 250 V, determine :

(*i*) Transformer ratio, (*ii*) Secondary current,

(*iii*) Primary current,

(*iv*) Number of turns across secondary if the total number of turns is 250,

(*v*) Power transformed, and

(*vi*) Power conducted directly from the supply mains to load.

[**Ans.** (*i*) $\frac{1}{2}$, (*ii*) 40 A, (*iii*) 20 A, (*iv*) 125, (*v*) 9.5 kW, (*vi*) 2.5 kW]

52. The primary and secondary voltages of an auto-transformer are 500 V and 400 V respectively. Show with the aid of diagram the current distribution in the windings when the secondary current is 100 A and calculate the economy in copper. [**Ans.** 80 per cent]

53. A 200/250 V auto-transformer draws power from a 200 V line and supplies a 5 kW load with a power factor of 0.8 lagging. A second load of 2 kW is supplied at unity power factor from 100 V winding. Neglecting losses, calculate the current drawn by the transformer from the 200 V line and its power factor. [**Ans.** 42.64 A, 0.898]

54. A 5 kVA, single-phase 50 Hz transformer has full-load efficiency of 95 per cent and an iron loss of 50 W. The transformer is now connected as an auto-transformer to a 220 V supply. If it delivers a 5 kW load at unity power factor to a 110 V circuit, calculate the efficiency of the operation and the current drawn by the high-voltage side. [**Ans.** 79.27%, 23.2 A]

55. A 480/120 V, 5 kVA two winding transformer is to be used as an auto-transformer to supply power at 480 V from 600 V source. Draw the connection diagram and determine the kVA capacity as an auto-transformer. [**Ans.** 25 kVA]

7

Single-Phase Induction Motors

1. Gerneral aspects. 2. Types of single-phase motor. 3. Single-phase induction motors. 4. Split-phase motors. 5. Single-phase commulator motors—*Highlights—Objective Type Questions—Theoretical Questions*.

1. GENERAL ASPECTS

- The number of machines operating from single-phase supplies is greater than all other types taken in total. For the most part, however, they are only used in the smaller sizes, less than 5 kW and mostly in the fractional H.P. range. They operate at *lower power-factors* and are relatively *inefficient when compared with polyphase motors*. Though simplicity might be expected in view of the two-line supply, the *analysis is quite complicated*.
- *Single phase motors* perform a great variety of useful services in the home, the office, the factory, in business establishments, on the farm, and many other places where electricity is available. Since the requirements of the numerous applications differ so widely, the motor-manufacturing industry has developed several types of such machines, each type having operating characteristics that meet definite demands. For example, one type operates satisfactorily on direct current or any frequency up to 60 cycles ; another rotates at absolutely constant speed, regardless of load ; another develops considerable starting torque and still another, although not capable of developing much starting torque, is nevertheless extremely cheap to make and very rugged.

2. TYPES OF SINGLE-PHASE MOTOR

The single-phase motors may be of the following types :

1. **Single-phase Induction Motors :**

A. Split-phase motors

(*i*) Resistance-start motor (*ii*) Capacitor-start motor

(*iii*) Permanent-split (single-value) capacitor motor

(*iv*) Two-value capacitor motor.

B. Shaded-pole induction motor.

C. Reluctance-start induction motor.

D. Repulsion-start induction motor.

2. **Commutator-Type, Single-Phase Motors :**

A. Repulsion motor. B. Repulsion-induction motor.

C. A.C. series motor. D. Universal motor.

3. **Single-phase Synchronous Motors :**

A. Reluctance motor. B. Hysteresis motor.

C. Sub-synchronous motor.

3. SINGLE-PHASE INDUCTION MOTORS

3.1. Applications and Disadvantages

Applications :

- Single phase induction motors are in very wide use in industry especially in *fractional horse-power field*.

  They are extensively used for electric drive for *low power constant speed apparatus* such as *machine tools, domestic apparatus* and *agricultural machinery* in circumstances where a three-phase supply is *not* readily available.
- There is a large demand for single-phase induction motors in sizes ranging from a *fraction of horse-power upto about 5 H.P.*

Disadvantages :

Though these machines are useful for small outputs, they are not used for large powers as they suffer from many disadvantages and are never used in cases where three-phase machines can be adopted.

The main *disadvantages* of single-phase induction motors are :

1. Their output is only 50% of the three-phase motor, for a given frame size and temperature rise.
2. They have lower power factor.
3. Lower-efficiency.
4. These motors do not have inherent starting torque.
5. More expensive than three-phase motors of the same output.

3.2. Construction and Working

Construction :

- A single phase induction motor is similar to a 3-ϕ squirrel cage induction motor in physical appearance. Its rotor is essentially the same as that used in 3-ϕ induction motors. Except for shaded pole motors, the stator is also very similar. There is a uniform air-gap between the stator and rotor but no electrical connection between them. It can be wound for any even number of poles, two, four and six being most common. Adjacent poles have opposite magnetic property and synchronous speed equation, $N_s = \frac{120f}{p}$ also applies.
- The stator windings differ in the following two aspects :
  — **Firstly**, single phase motors are usually provided with concentric coils.
  — **Secondly**, these motors normally have two stator windings. In motors that operate with both windings energised, the winding with the *heaviest wire* is known as the **main winding** and the other is called the **auxiliary winding**. If the motor runs with auxiliary winding open, these windings are usually referred as *running* and *starting*.
  — In *most of motors the main winding is placed at the bottom of the slots and the starting winding on top but shifted 90° from the running winding*.

Working :

When the stator winding of a single phase induction motor is connected to single phase A.C. supply, a *magnetic field* is developed, *whose axis is always along the axis of stator coils*. The magnetic field produced by the stator coils is pulsating, varying sinusoidally with time. Currents are induced in the rotor conductors by transformer action, these currents being in such a direction as to oppose the stator m.m.f. Then the axis of the rotor m.m.f., wave coincides with that of the stator field, the *torque angle* is, therefore, *zero* and *no torque is developed on starting*. However, if the rotor is given a push by hand or by other means in either direction, it will pick-up the speed and continue to rotate in the same direction developing operating torque. Thus a single phase induction motor is not inherently self-starting and requires some special means for starting.

The above mentioned behaviour of this type of motor can be explained by anyone of the following theories :

1. Double revolving field theory 2. Cross-field theory.

The results given by both the theories are approximately same.

Double revolving field theory is described below :

The magnetic field produced by the stator coils is *pulsating*, varying sinusoidally with time. *Ferrari* pointed out that *such a field can be resolved into two equal fields but rotating in opposite directions with equal angular velocities. The maximum value of each component is equal to half the maximum of the pulsating field*.

If the initial time is such that the rotating vectors of the two component fields are along the *Y*-axis in the positive direction, the two component waves ϕ_1 and ϕ_2 coincide. The resultant of these two is ϕ_{max}. After a short interval of time the two vectors rotate, through an angle θ in their respective directions and the waves are shown to occupy the positions in Fig. 1. These waves intersect at *A* on the *Y*-axis and as the waves travel *A* moves along the *Y*-axis. Hence the resultant of these two component waves at any instant is equal to 2*OA*.

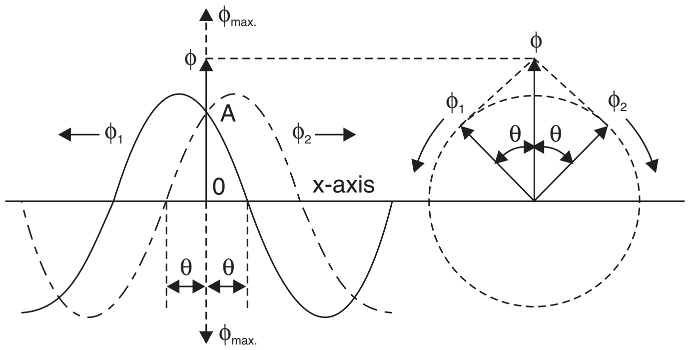


Fig. 1

$$\phi_1 = OA = \phi_{1(max)} \cos (\omega t - \theta) \quad ...(i)$$

$$\phi_2 = OA = \phi_{2(max)} \cos (\omega t + \theta) \quad ...(ii)$$

and

$$\phi_{1(max)} = \phi_{2(max)}$$

By expanding and adding (*i*) and (*ii*),

$$\phi_1 + \phi_2 = 2\phi_{1(max)} \cos \theta \cos \omega t$$

$$2OA = \phi_{(max)} \cos \theta \cos \omega t \quad ...(iii)$$

which is the equation of the pulsating field and **proves Ferrari's statement.** Thus a single-phase induction motor is *not inherently self-starting*.

The existence of these two fluxes (*forward and backward*) rotating in opposite directions can be verified by supplying a fractional horsepower single-phase induction motor with rated voltage. The motor does not start, but if the shaft is turned by hand, say in clockwise direction, the rotor picks up speed. This means that the rotor conductors are rotating in the direction of that field which rotates in clockwise direction. When the motor is braked and stopped without switching off the supply, the rotor remains at rest. If now the shaft is turned by hand in anticlockwise direction, the motor picks up speed in that direction. This means that the rotor conductors are now rotating in the direction of the other field.

This behaviour of the motor is *due to the presence of two opposing torques due to the two field.* When the rotor is at rest, (*i.e.,* slip = 1) the two torques are equal but opposite in direction. Hence the net torque is zero and therefore the rotor remains at rest. Fig. 2 shows the torque variations due to the two fields. If the rotor is made to speed up in one direction, say in that in which T_1 increases, T_1 exceeds the opposing torque T_2 and the motor beings to accelerate. T_2 goes on diminishing until at the working speed it is negligibly small. Hence the single-phase induction motor rotates in the direction in which it is made to run.

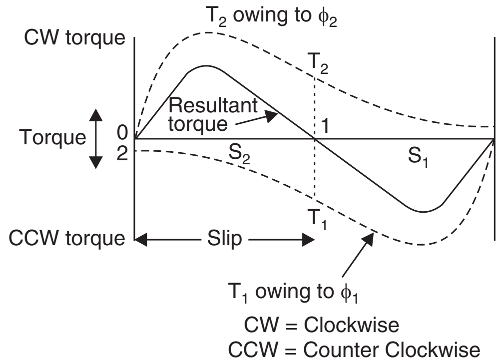


Fig. 2. Balanced torque at standstill in squirrel cage rotor excited by a single-phase winding.

Thus, if the rotor is made to run at speed N by some external means in any direction, say in the direction of forward field, the two slips are now s and $(2 - s)$, as shown below :

The slip of the rotor w.r.t. the forward rotating field F_f,

$$s_f = \frac{N_s - N}{N_s} = s \quad ...(1)$$

The slip of the rotor w.r.t. the backward rotating field F_b,

$$s_b = \frac{N_s - (-N)}{N_s} = \frac{2N_s - (N_s - N)}{N_s} = (2 - s) \quad ...(2)$$

- Under normal running condition $(2 - s) >> s$ and as a consequence the backward field rotor currents are much larger than at standstill and have a low power factor. The corresponding opposing rotor m.m.f., owing to stator impedance, causes the backward field to be greatly reduced in strength. On the other hand, the low-slip forward rotating field induces smaller currents of a high power factor in the rotor than at standstill. This leads to greatly strengthening of forward field. Thus weakening of backward field and strengthening of forward field depends upon the slip or speed of rotor and the difference increases with the decrease in slip w.r.t. the forward field or with the increase in rotor speed in forward direction.
- In a single phase induction motor, *the increase in rotor resistance increases the effectiveness of the backward field, which reduces the breakdown torque, lowers the efficiency and increases the slip corresponding to maximum torque*.
- The **performance characteristics** of a single phase induction motor are somewhat *inferior* to that of a 3-phase induction motor due to the presence of backward rotating field.

  — A single phase induction motor has a *lower breakdown torque at larger slip and increased power losses*.

— Greater power input.

— The *speed regulation* tends to be *poorer* than that for a polyphase motor.

— The *power factor tends to be lower* (since the normal slip of a single phase induction motor under load conditions is rather greater than that of the corresponding 3-phase motor).

- In view of the above factors, a single phase induction motor has a *larger frame size* than that of 3-phase motor.
- Single phase motors *tend to be somewhat noiser than 3-phase motors which have no such pulsating torque.*

4. SPLIT-PHASE MOTORS

Since the single-phase induction is not self-starting, means must be provided to create an initial torque. But the initial torque is only possible if a rotating flux is created in the stator. It is known that *a rotating flux is produced when there is a difference of 90° between the currents of two stationary coils. Or if the stator possesses two fluxes having a large phase difference, the result is a rotating flux*.

4.1. Split-phase Resistance—Start Induction Motor

- In a split-phase induction motor the stator is provided with two parallel windings displaced 90 electrical degrees in space and somewhat less than 90° in time. Fig. 3 (*a*) shows the winding diagram of the *two windings* of a split-phase induction motor.

  The *starting winding* has fewer turns and is wound of smaller diameter copper than the running winding. The starting winding, therefore, *has a high resistance and low reactance*.

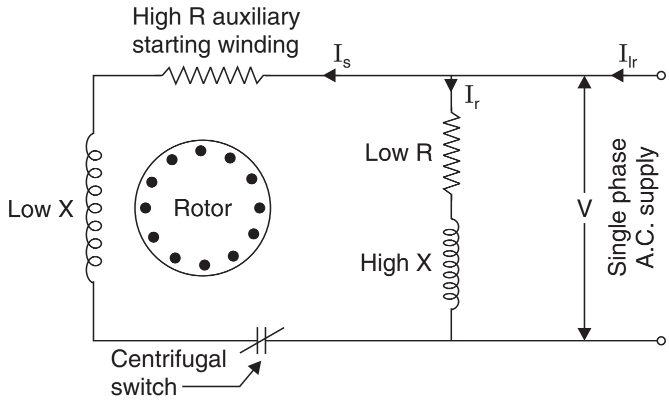
  The *running or main, winding* (heavier wire of more turns) has a *low resistance* and *high reactance.* Because of its lower impedance, the current in the running winding I_r is *higher* than the current in the starting winding, I_s.

  The phase relations of the lock-motor currents at the instant of starting are shown in Fig. 3 (*b*). The starting winding I_s lags the supply voltage by about 15°, while the greater running winding current lags the single-phase voltage by about 40°. Despite the fact that the current in the two space-quadrature windings are *not* equal, the quadrature components are practically equal.

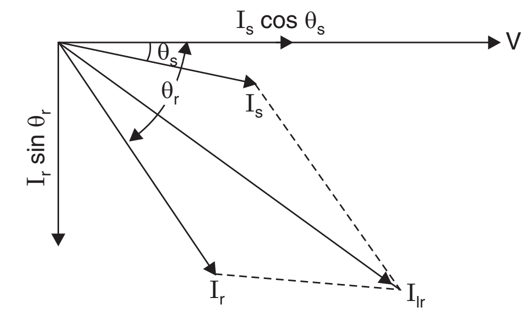
  If the windings are displaced by 90° in space, and if their quadrature current components, which are displaced by 90° in time, are practically equal, an equivalent two-phase rotating field is produced at starting which develops sufficient starting torque to accelerate the rotor, in the direction of the rotating field produced by the currents.

- As the *motor speeds up,* the *torque developed increases.* Above 85 per cent of synchronous speed, the torque developed by the running winding (main winding) alone is actually greater than that developed by both windings, and it might be advantageous to *open the auxiliary circuit at this cross over point.* To allow for individual variations among motors and switches, however, the contacts are usually designed to open at 75 per cent of synchronous speed. This does not seriously affect the operation, because the running (or main) winding alone usually develops approximately 200 per cent of full-load torque at this speed.
- The *starting winding is not designed for continuous operation, and care should be exercised that it does not remain connected to the supply after it should have been disconnected by the switch*. This series switch is usually *centrifugally operated, and is rather inexpensive*.

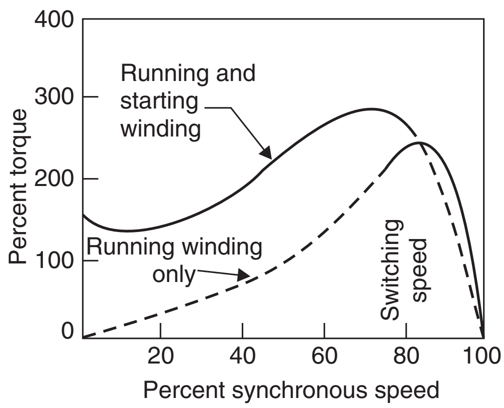
In case of a hermatically sealed motor, the switch is magnetically operated, and is opened in the de-energized condition.



(*a*) Connection diagram



(*b*) Phase relations



(*c*) Typical torque speed characteristic

Fig. 3. Split phase resistance-start induction motor.

- Split-phase induction motors may be *reversed by reversing the line connections of either the main or the auxiliary winding.* If however, reversal is attempted under normal running condition, nothing will happen.

  If it is necessary to *reverse the motor while it is rotating,* then some means must be incorporated to slow the motor down to the speed where the starting-switch contacts close, placing the starting winding across the supply lines. This may be done by *incorporating a timing device* which first disconnects the motor entirely from the line and then reverses one field at the proper time. A mechanical braking device which can be electrically operated may also be used.

- Speed control of split-phase windings is a *relatively difficult* matter since the synchronous speed of the rotating stator flux is determined by the frequency and number of poles developed in the running stator winding $\left(N_s = \dfrac{120f}{p} \right)$. By adding stator windings to change the number of poles, speed variation may be obtained. This, however, is a stepped speed change, as in polyphase induction motor, rather than a continuous variation. It must be pointed out, however, that *all speed changes must be accomplished in a range above that at which the centrifugal switch operates.*

Fig. 3 (*c*) shows the typical torque speed characteristics :

- The starting torque is 1.5 times to twice the full-load starting torque and starting current is 6 to 8 times full-load current.
- The speed regulation is very good.
- The percent slip is about 4-6 percent.
- Such a motor may operate with a power factor of 0.55–0.65 and efficiency of 60–65 percent.
- These motors are made in fractional kW $\left(\frac{1}{20} \text{ to } \frac{1}{4} \text{ kW}\right)$ ratings with speed ranging from 2875 to 700 r.p.m.

Shortcomings and uses. The major objections to the motor are (1) its *low starting torque* ; and (2) that, when heavily loaded, the slip exceeds 5 per cent, reducing the e.m.f. and producing an elliptical or *pulsating torque which makes* the motor somewhat *annoyingly noisy*. For this reason, the split-phase motor is used in appliances to drive loads which are themselves noisy : *oil burners, machine tools, grinders, dish washers, washing machines, air blowers and air compressors*.

- *Because of low starting torque these motors are seldom employed in sizes larger than* $\frac{1}{4}$ *kW.*

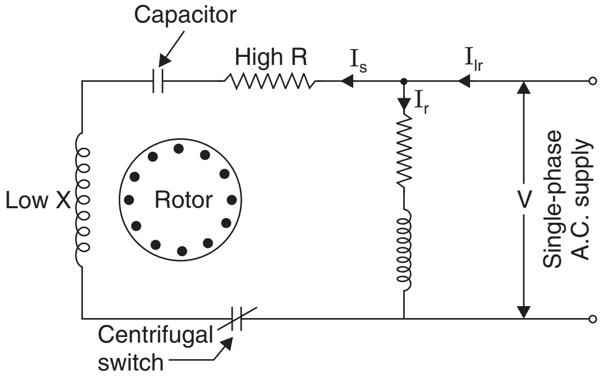
4.2. Split-phase Capacitor—Start Induction Motor

Another method of splitting the single-phase supply into two phases to be applied to the stator windings is *placing a capacitor in series with the starting auxiliary winding.* In this manner, the current in the starting winding may be made to lead the line voltage. Since the running winding current lags the line voltage, the phase displacement between the two currents be made to approximately 90° on starting. The circuit of capacitor-start motor is shown in Fig. 4 (*a*), while the vector diagram of the currents and voltage is shown in Fig. 4 (*b*). The values of the angles shown are fairly representative, and are rounded off for convenience. *One of the factors upon which the starting torque depends is the sine of the angle between the currents in the two windings.* The value of series capacitor may therefore be reduced, while maintaining a phase-shift angle of about 90°.

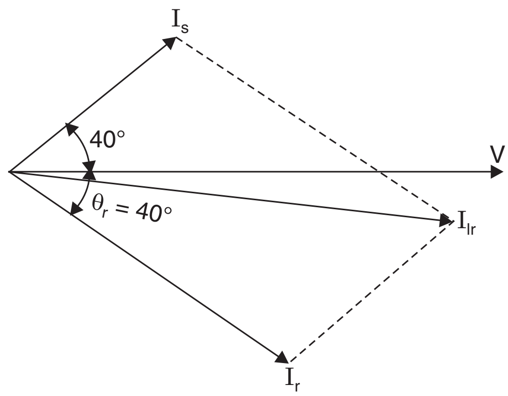
- The increase in phase angle between starting and running winding currents is not the only difference between the split-phase and capacitor-start motors. *The split phase motor must keep the number of starting-winding turns low, so that the current may be nearly in phase with the line voltage.* This, *however, is unnecessary in a capacitor-start motor, since the capacitor can overcome the inductance of the winding while still providing the proper phase shift.* There are thus *more auxiliary starting turns* in the capacitor-start motor than in the comparable split-phase motor. This provides a greater number of ampere-turns, hence a *larger rotating flux,* and therefore a *further increase in the starting torque*.

Also it is seen that for the same magnitudes of field currents, the *current* I_{lr} *is less in capacitor-start motor,* because of the greater angle between the two field currents. In addition, the *starting power factor is also better*. For a given line current, the starting torque is thus much higher for a capacitor-start motor than for a split-phase induction motor. *The starting torque of capacitor-start motor is from 3 to 4.5 times the full-load torque, while that of split-phase resistance-start induction motor rarely exceeds twice the full-load torque.*

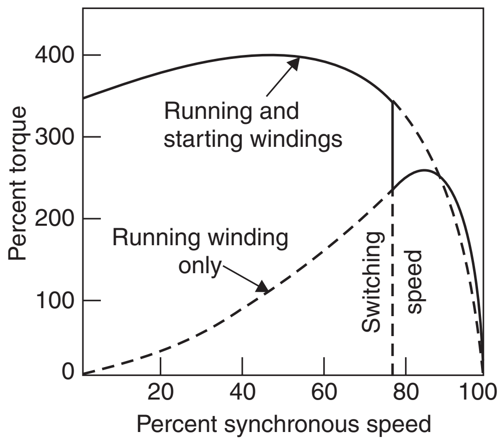
- These motors are manufactured in *ratings ranging from* $\frac{1}{10}$ *kW to* $\frac{3}{4}$ *kW, but larger sizes are also available.*



(*a*) Circuit diagram



(*b*) Vector diagram



(*c*) Typical torque-speed characteristic

Fig. 4. Capacitor-start induction motor.

- The capacitor-start motor may be reversed by changing the connections of one of the windings, but it is subject to the same limitations as the resistance-start induction motor.

Uses. By virtue of their higher starting torque, capacitor-start split-phase motors are used for *pumps, compressors, refrigeration units, air-conditioners,* and *large washing machines,* where a split-phase motor is required that will develop high starting torque under load.

4.3. Permanent-split Capacitor Induction Motor

- A permanent-split capacitor motor (also called single-value capacitor motor) is a single-phase induction motor which has the *same capacitor* in series with the starting (or auxiliary) winding for both starting and running. Because it runs continuously as a permanent split-phase motor, no centrifugal switch is required.
- The motor starts and runs by virtue of the quadrature phase-splitting produced by the two identical windings. As a result, *it does not possess the high running torque* produced by either the resistance-start or the capacitor-start motor.

Furthermore, the capacitor used is designed for continuous duty and is of an oil-filled type. The value of the capacitor is based on its optimum running rather than its starting characteristics. The result is that this motor *has a very poor starting torque,* about 50 to 100 percent of rated torque.

Fig. 5 shows the connection diagram and phase relations of a permanent split-phase motor.

- Because of the fairly uniform rotating magnetic field created by equal windings whose currents are displaced by almost 90°, the torque is fairly uniform and the motor *does not exhibit the characteristic pulsating hum* developed by most single-phase motors when loaded.

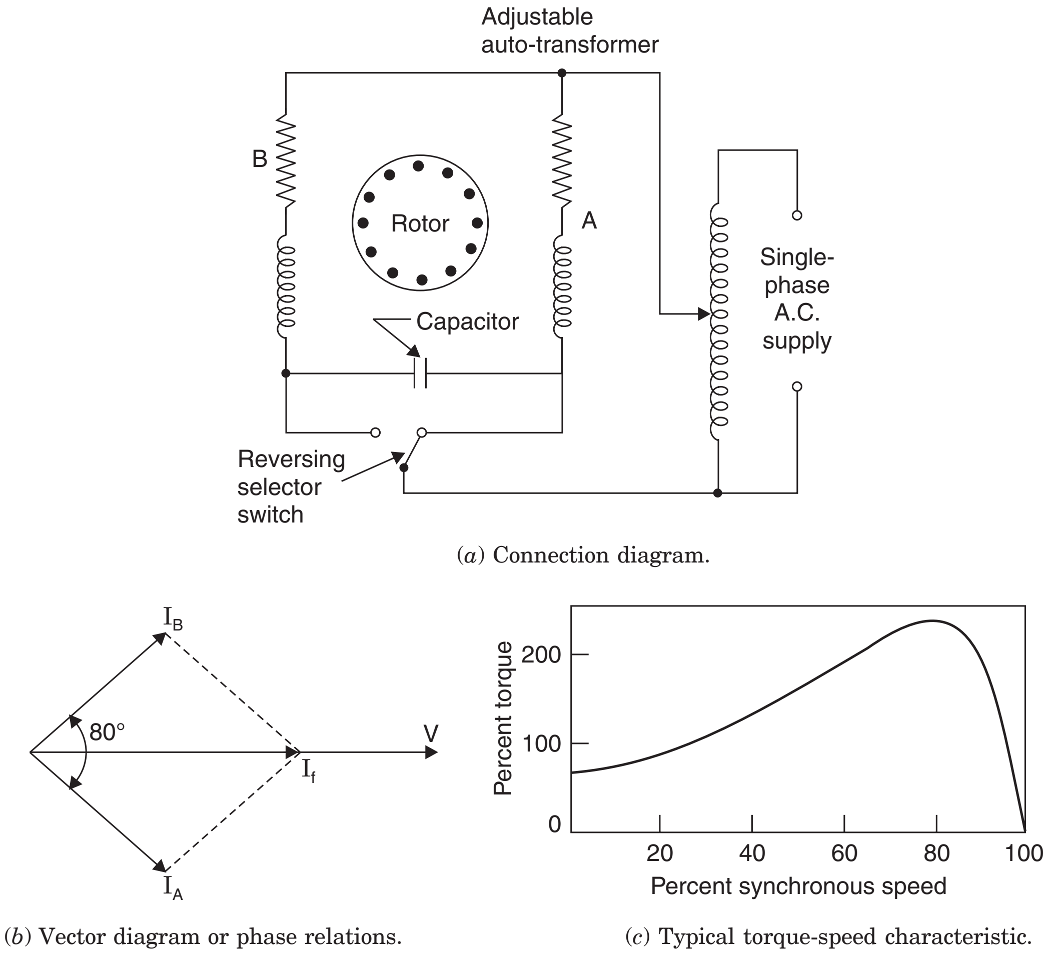


Fig. 5. Permanent split-phase motor.

This motor possesses the following *merits* :

(*i*) Higher power factor at full-load

(*ii*) Lower full-load line current

(*iii*) Higher full-load efficiency

(*iv*) Increased pull-out torque.

- The permanent-split capacitor motor is *more expensive* than the equivalent split-phase or capacitor-start induction motor. This is primarily due to the fact that the auxiliary (or starting) winding is now also a running winding. It must therefore have a continuous duty rating and as such is heavier than if it were short-time-rated.

Uses. Because of its *instant response as a reversing motor,* its *quieter operation,* and the *possibility of speed control*, the permanent-split capacitor motor is used for *exhaust and intake fans* and *blowers, unit heaters* and *office machines.*

4.4. Two-value Capacitor Induction Motor

- Refer Figs. 6 and 7. *The two-value capacitor motor combines the quiet operation and limited speed control advantages of a running permanent-split capacitor with high starting torque of the capacitor-start motor.*

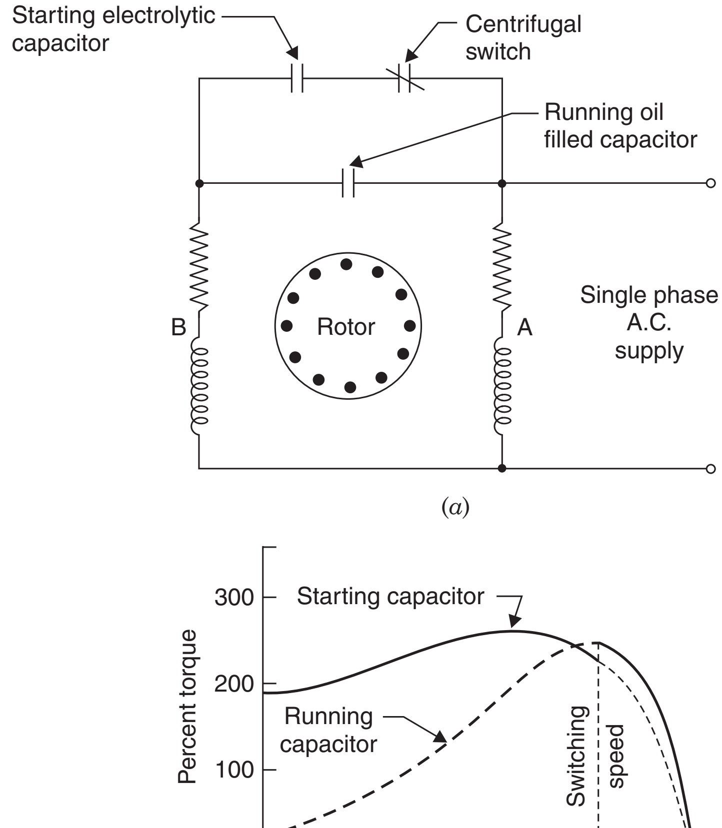


Fig. 6. Connection diagram for two-value capacitor motor and typical torque-speed characteristic.

- Two capacitors (Fig. 6) are employed during the starting period. One of these, an electrolytic starting capacitor, similar to that used for the intermittent duty of the capacitor-start split-phase motor, is of fairly high capacity (about 10 to 15 times the value of the running capacitor) and is cut out of the circuit by a centrifugal switch when slip reaches about 25 per cent.
- The *major advantage of the two value capacitor motor is its high starting torque, coupled with quiet operation and good running torque.* Still classed as a reversing motor, when the

line leads of one winding are reversed, it is reversed in the usual manner. When the speed drops below 25% slip during reversing, the centrifugal switch closes its starting contact, providing the maximum torque as the motor slows down and reverses. The contacts open again when the motor is up to 75% synchronous speed in the reverse direction. Frequent reversals, therefore will reduce the life of the centrifugal switch. For this reason, where frequent reversals are accomplished, permanent-split, single-value capacitor, using no centrifugal switch whatever, is preferable.

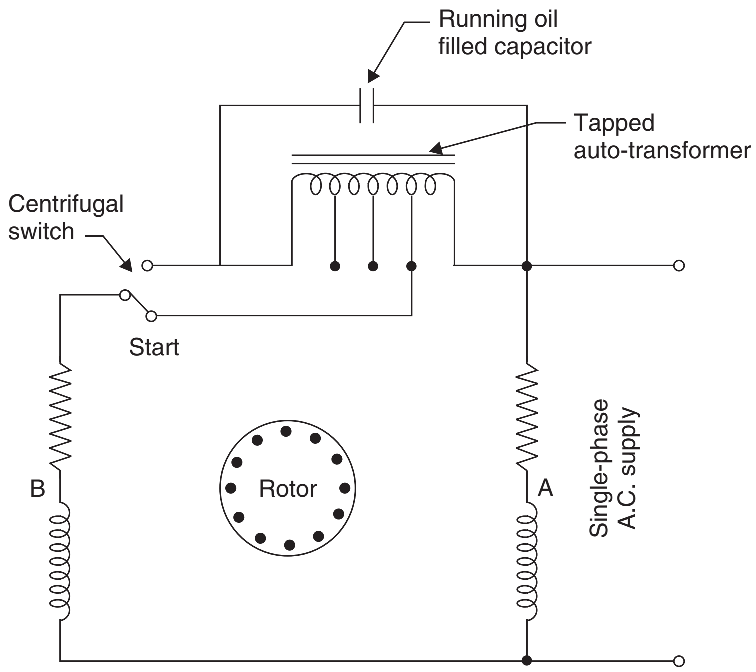


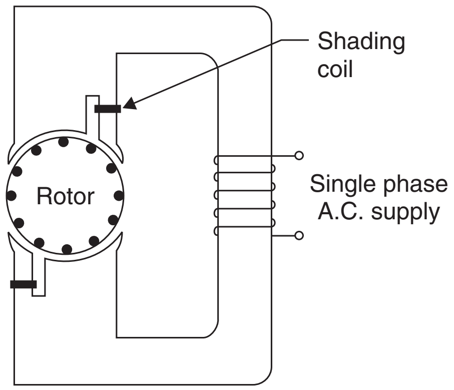
Fig. 7. Two-value capacitor motor with auto-transformer.

Uses. This two-value capacitor motor finds use in *smaller home air-conditioning units* which use this motor in its *compressor* and operates on a 15 amp. branch circuit. The lower starting current and lower running current (7.5 amp. maximum) at an improved power factor over the capacitor-start motor, are obtained through the *precise selection of starting and running capacitors* for the fixed compressor load.

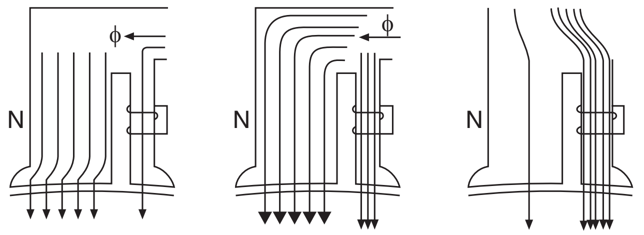
4.5. Shaded-pole Induction Motor

A shaded-pole motor is one of the *simplest* and *cheapest* of manufactured motors. It is essentially an induction machine, since its squirrel-cage rotor receives power in much the same way as does the rotor of the polyphase induction motor. There is however, one extremely important difference between the two. Whereas the poly-phase induction motor creates a true revolving field, in the sense that it is constant in magnitude and rotates at synchronous speed *completely round the entire core*, the field of the shaded-pole motor is not constant in magnitude but *merely shifts from one side of the pole to the other.* Because the shaded-pole motor does not create a true revolving field, the *torque is not uniform but varies from instant to instant.*

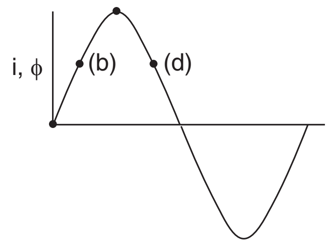
Fig. 8 shows the *general construction and principle of shaded pole motor.*



(*a*) General construction of 2 pole shaded pole motors



(*b*) Increasing ϕ (*c*) Constant ϕ (*d*) Decreasing flux



(*e*) Rate of change of current and flux in poles

Fig. 8. General construction and principle of shaded pole motor.

Construction. Each of the laminated poles of the stator has a slot cut *across* the laminations about one-third the distance from one edge. Around the smaller of the two areas formed by this slot is placed a heavy *copper short-circuited coil,* called a *shading coil* ; the iron around which the shading coil is placed is called the *shaded* part of the pole, while the free portion of the pole is the *unshaded* part. The exciting coil surrounds the entire coil.

Principle of operation. When the exciting winding is connected to an A.C. source of supply, the *magnetic axis will shift from the unshaded part of the pole to the shaded part of the pole*. This shift in the magnetic axis is, in effect *equivalent to an actual physical motion of the pole* ; the result is that the squirrel-cage rotor will rotate in a direction from the unshaded part to the shaded part. The shifting of flux is explained below.

- Refer Fig. 8 (*b*). When the flux in the field poles tend to increase, a short-circuit current is induced in the shading coil, which by Lenz's law opposes the force and the flux producing it. Thus, as the flux increases in each field pole, there is a *concentration of flux in the main segment of each pole*, while the shaded segment opposes the main field flux.

- At point (*c*) shown in Fig. 8 (*e*), the rate of change of flux and of current is zero, and there is no voltage induced in the shaded coil. Consequently, the *flux is uniformly distributed across the poles* [Fig. 8 (*c*)].
- When the flux decreases, the current reverses in the shaded coil to maintain the flux in the same direction. The result is that the flux crowds in the *shaded segment of the pole* [Fig. 8 (*d*)]

A typical torque-speed characteristic is shown in Fig. 9.

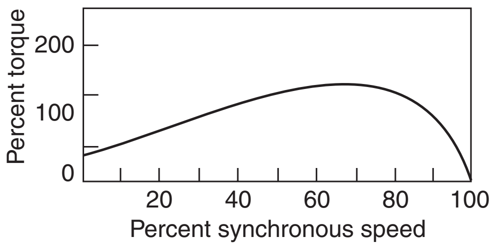


Fig. 9. Typical torque-speed characteristic of shaded pole motor.

- Shaded pole motors are built up to about 40 W.

Merits :

(*i*) Rugged construction (*ii*) Cheaper in cost

(*iii*) Small in size (*iv*) Requires little maintenance

(*v*) Its stalling locked-rotor current is only slightly higher than its normal rated current, so that it can remain stalled for short periods without harm.

Demerits :

(*i*) Very low starting torque (*ii*) Low efficiency

(*iii*) Low power factor.

Uses. Its low starting torque limits its application to *phono-motors* or *turntables, motion picture projectors, small fans and blowers, bending machines, rotating store-window display tables, and relatively light loads.*

Reversing shaded-pole motor. Fig. 10 shows a reversing shaded-pole motor with the switch in the clockwise rotation position. This places 1 and 3 actively in the circuit, being short-circuited by

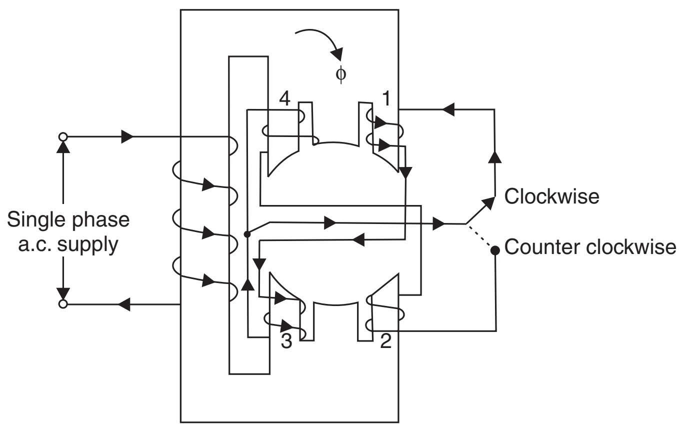


Fig. 10. Reversing shaded pole motor.

the reversing switch. Coils 2 and 4 remain in series with each other, but are on open circuit and so are inactive. The main-field flux is shown increasing vertically downward, and the arrows on the shading coils show the current in them. It is then seen that the *shading coil must be connected in such a manner as not to have their induced voltages in opposition, or there may not be any current in them, and hence no flux lag*.

4.6. Reluctance-start Induction Motor

A reluctance-start induction motor is shown in Fig. 11. Its characteristics are similar to that of shaded pole motor. In this motor too the magnetic field shifts across the pole, but the effect is obtained by the *non-uniform air gap of salient poles. Where there is a greater air gap, the flux in that portion is more nearly in phase with the current. There is a greater lag between flux and current where there is a lower reluctance or where the air gap is smaller.* Since both fluxes are produced by the same current, the flux across the larger air gap *leads* the flux across the smaller one. The two fluxes are obviously displaced in time, and so the *magnetic field shifts across the poles from larger air gap to the shorter gap*. Thus the *direction of rotation is firmly fixed by the construction, and the motor cannot be reversed at all*.

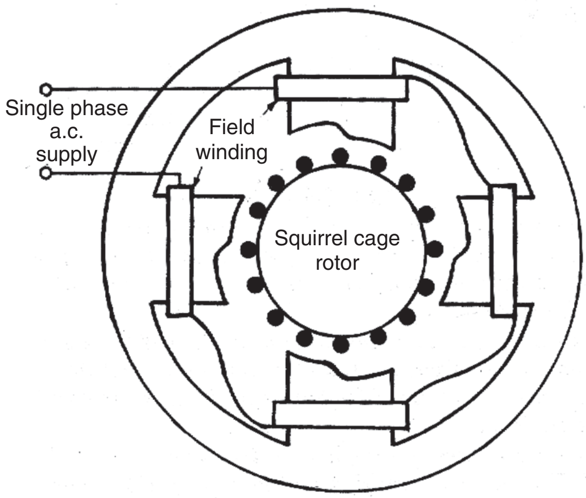


Fig. 11. Reluctance start motor.

Uses. For most small power applications, the shaded-pole motor is preferred, and the reluctance-start motor has limited use, usually only where *starting torque requirements are low*.

Note. This motor is an induction motor and should not be confused with *reluctance motor* which is actually a non-excited synchronous motor.

5. SINGLE-PHASE COMMUTATOR MOTORS

The commutator motors are so called because the wound rotor of this kind of motor is equipped with a *commutator and brushes.* This group consists of the following two classes :

1. Those operating on '*repulsion principle*' (repulsion motors) *in which energy is inductively transferred from the single-phase stator field winding to the rotor.*

2. Those operating on the *principle of the series motor* in which the energy is *conductively* carried both to the rotor armature and its series-connected single-phase stator field.

5.1. Repulsion Motor

A repulsion motor in its simplest form consists of a *field comprising a distributed winding,* housed in slots, in a smooth-cored stator and *an armature carrying a distributed winding connected to a commutator*. The stator winding, which produces the *main field,* is connected to the main supply. The armature or rotor winding is not connected electrically to the main circuit, but the brushes, which are set at an angle to the direction of the main flux, are short-circuited as shown in Fig. 12.

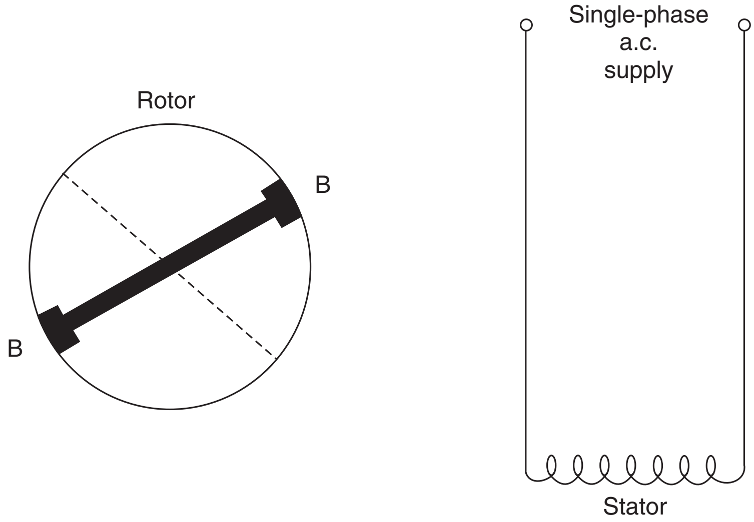


Fig. 12. Repulsion motor.

- If the brush axis *BB* is set *at right angles* to the direction of the stator flux, the e.m.f. induced in one-half of the rotor winding is *exactly balanced* by the e.m.f. induced in the other half, so that resultant e.m.f. is *zero* ; no current flows in the rotor winding and *no-torque is developed.*
- If the brush axis is placed *in line with the direction* of the stator flux, the e.m.f. in one half assists that in the other, so that *a maximum current flows.* Here again *no torque is developed,* since the torque of one half of the rotor conductors is exactly balanced by that due to the other half.
- If *the brush axis makes an angle with the stator flux as shown, a resultant torque is produced.* The torque is a *maximum theoretically, if this angle is 45°, but in practice the angle of inclination is about half this value*.

It is clear from above that the *speed of repulsion motor depends upon the brush position. Speed control of* such a machine can be provided by *mounting the brushes on a rocker* which can be rotated by a lever handle mounted on the motor end-shield. If remote control is required, the lever handle may be manipulated by a simple system of rods and cranks.

Alternatively, if the motor is to be totally enclosed, or remote control from a considerable distance is required, speed control may be obtained by the *use of an external series resistance with fixed brush gear.*

- The direction of rotation of a simple repulsion motor may be reversed by swinging the brushes into the position shown *dotted* in Fig. 12.

Atkinson repulsion motor. A modification of the simple repulsion motor is the Atkinson repulsion motor, in which the stator winding comprises two windings at right-angles to each other and connected in series, as shown in Fig. 13. One advantage obtained by this method is that the *direction of rotation can be reversed by reversing the connections to one of the stator windings.* Instead of moving the brush rocker, it is necessary only to throw the reversing switch, shown in Fig. 13.

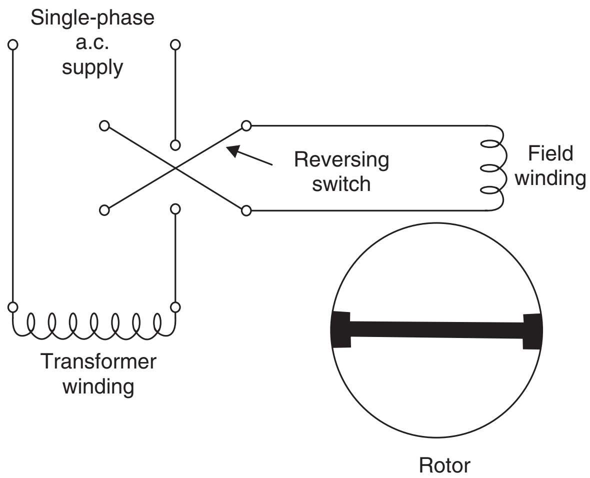


Fig. 13. Atkinson repulsion motor with reversing switch.

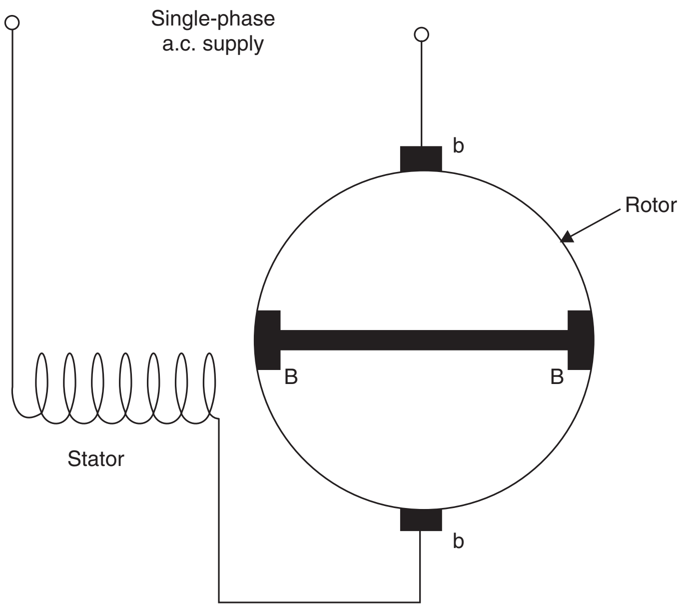


Fig. 14. Compensated repulsion motor.

It will be observed that, as the rotor is electrically connected to the stator, the compensated repulsion motor is not able to operate directly from a high-voltage supply, as was the case with the simple repulsion motor.

Fig. 15 shows the typical speed-torque characteristics of single-phase repulsion motor.

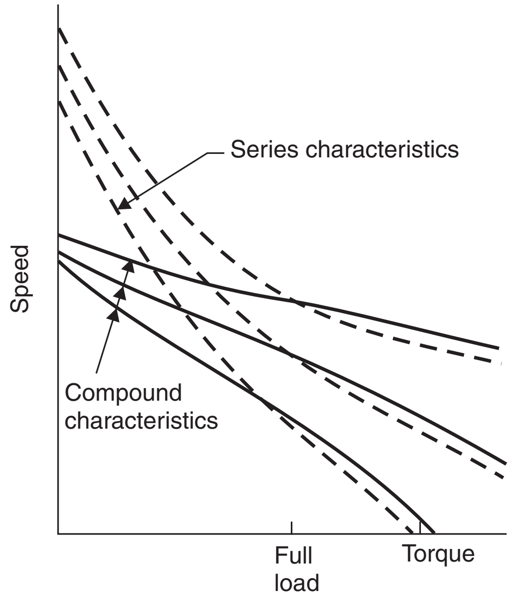


Fig. 15. Typical speed-torque characteristics of single-phase repulsion motors.

Compensated repulsion motor. The *power factor of the machine may be improved by compensation, a*nd this is done by providing *an additional set of brushes* as shown in Fig. 14. The axis of one pair, *BB*, of brushes coincides with the stator winding, these brushes are *short-circuited.* The other pair, *bb*, of brushes is set at right-angles to the former and is connected in *series with the stator winding*.

5.2. Repulsion-start Induction Motor

- As its name implies the repulsion-start induction motor *starts as a repulsion motor* with its brushes set to the maximum torque position. When the load has been accelerated to about 75 per cent of synchronous speed, a built in centrifugal device *places a shorting ring* in contact with the commutating bars, *converting the armature to squirrel-cage rotor*. The motor then runs as induction motor on its induction characteristic (Fig. 16).
- Although at one time this type of motor was used almost exclusively where high starting torque was required, it has been replaced in nearly all cases by the capacitor motors because of the following reasons :

(*i*) Requires more maintenance.

(*ii*) More expensive.

(*iii*) Makes quite a bit of noise on starting.

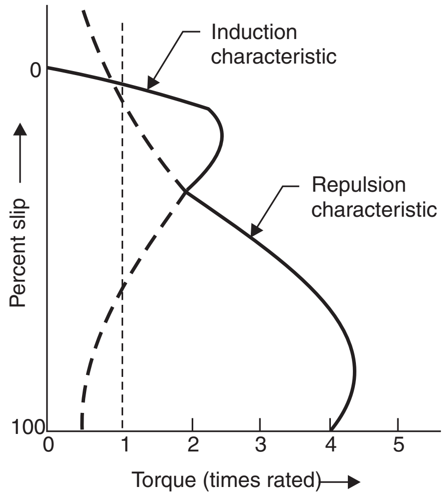


Fig. 16. Speed-torque characteristic of repulsion-start induction motor.

(*iv*) Causes radio interference when starting.

(*v*) Cannot be reversed easily.

Repulsion-start motors, despite these disadvantages, are still used in *integral-horsepower sizes* because of the following reasons :

(*i*) High starting torque. (*ii*) Low starting current.

(*iii*) Ability to accelerate a heavy load more rapidly than high-capacitance dual-capacitor motors.

5.3. Repulsion-induction Motor

- A single-phase repulsion-induction motor combines the constant-speed characteristics of the single-phase induction motor with the good starting characteristics of the repulsion motor.
- The *stator of this machine has a simple single-phase winding like that of single-phase induction motor.* The rotor, however, is built up of *laminations,* each of which has *two concentric sets of slots. These slots contain two distinct windings ; in the outer slots is wound a commutator winding similar to that of a D.C. armature, while in the inner slots is a cast aluminium squirrel-cage winding which clamps the laminations.*

Fig. 17 (*a*) illustrates an armature for a repulsion induction motor complete with a squirrel-cage winding.

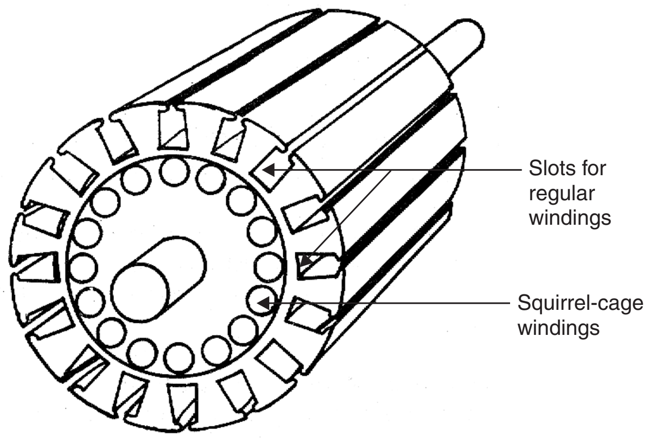


Fig. 17. (*a*) Repulsion induction motor.

At starting and during the acceleration period, the magnetic flux produced by the stator *embraces only the commutator winding in the outer slots owing to the high reactance of the squirrel-cage*. The *motor starts up virtually as a repulsion motor and develops a high starting torque. As the motor speeds up the reactance of the squirrel-cage decreases, so that this winding assists the commutator winding to supply the running torque.*

Fig. 17 (*b*) shows the speed-torque characteristic of repulsion-induction motor.

Merits. The repulsion-induction motor has the following merits :

(*i*) High starting torque. (*ii*) Fairly good speed regulation.

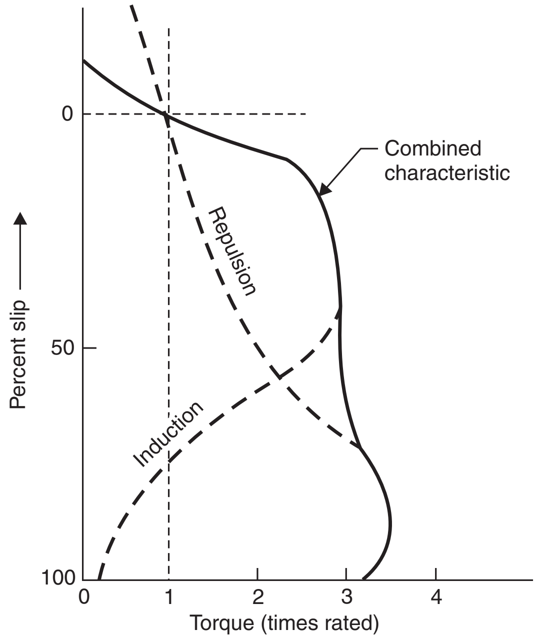


Fig. 17. (*b*) Speed-torque characteristic of repulsion-induction motor.

(*iii*) Major virtue is the ability to continue to develop torque under sudden, heavy applied loads without breaking down.

Uses. Such motors are suitable for all single-phase power applications *which require a high starting torque and constant speed when running* ; they also operate *at a very high power factor.* They are particularly well adapted to drive *machine tools,* lifts, hoists, *mixing machines, centrifugal pumps, fans and blowers.*

5.4. A.C. Series Motor

The series motor due to its desirable speed-torque characteristics *is almost exclusively used in railway service*. While the D.C. motor is entirely satisfactory for this class of work service and is generally used on street railway cars and trolley coaches, the fact that it is more convenient and more economical to transmit power and to transform voltages in A.C. systems than with direct currents has lead to the development of the A.C. series motor for use on some of the important steam-road electrifications.

Working principle. The working principle of an A.C. series motor is the same as that of the D.C. series motor. The armature and field are wound and interconnected in the same manner as the D.C. series motor.

When an alternating e.m.f. is applied to the terminals, since field and armature windings are connected in series, the field flux and armature current reverse simultaneously every half cycle, but the direction of the torque remains unchanged. The torque is pulsating, but its average value is equal to that which a D.C. motor will develop if it had the same r.m.s. value of flux and current. Motor connections, direction of torque, etc., for two successive half cycles are shown in Fig. 18. If the field and armature core are run at low saturation, the air-gap flux is approximately proportional to the current and the torque is approximately proportional to current squared.

Although it is theoretically possible to operate a D.C. series motor from an A.C. circuit, the following structural changes must be made in the motor to make it a practical and reasonable efficient machine :

- The entire magnetic circuit must be *laminated,* and materials with low iron-loss co-efficients should be used as in transformers.

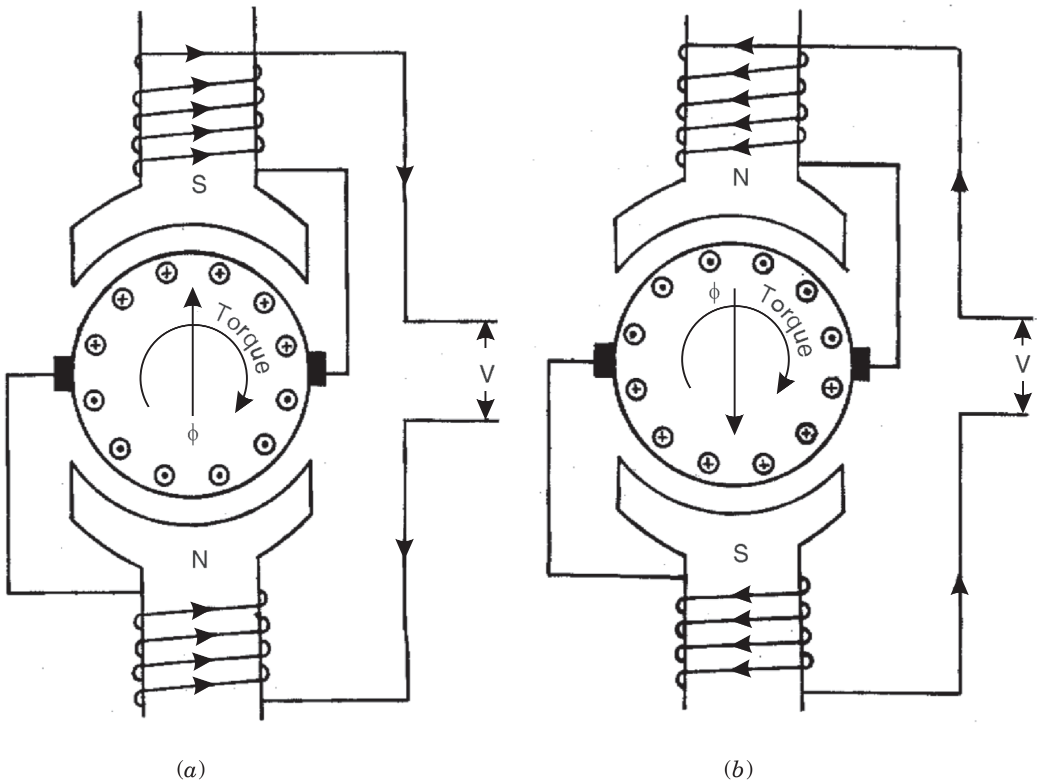


Fig. 18. Working principle of the A.C. series motor.

- The field circuit must be designed for a *much lower reactance* than the corresponding D.C. motor field in order to reduce the reactance voltage drop of the field to a minimum and to improve the power factor of the motor.
- A *distributed compensating winding* is required to reduce the reactance of the armature winding by reducing the leakage flux and to neutralize the cross-magnetising effect of the armature ampere turns.

The compensating winding may be connected in series with the series-field and armature windings, or it may be short-circuited upon itself and receive its excitation voltage by transformer action, since it is inductively coupled with the armature cross-field (Fig. 19). In the first case, the motor is said to be *conductively* compensated, while in the second it is *inductively* compensated. *Conductive compensation* is required on motors which are intended for operation in *D.C. as well as A.C. circuits*.

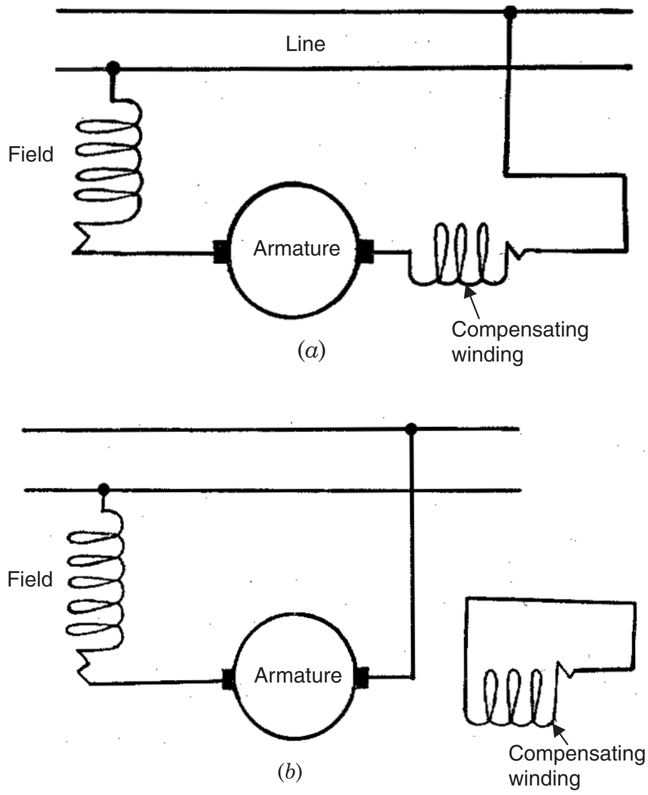


Fig. 19. Connections for (*a*) conductive compensation; (*b*) inductive compensation.

- Special provision must be made to secure *satisfactory commutation*.

Fig. 20 shows the A.C. series motor characteristic.

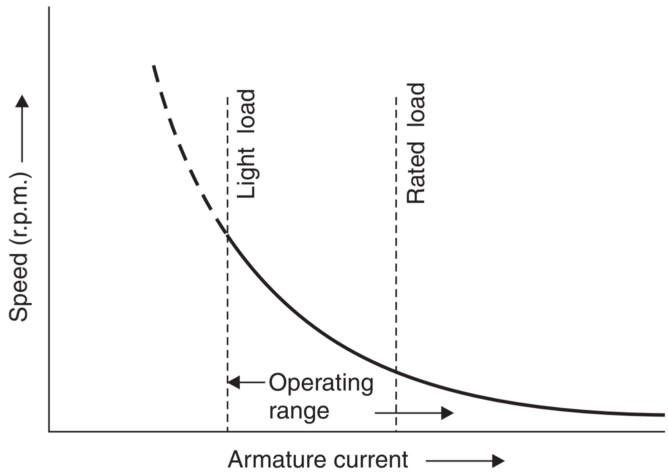


Fig. 20. A.C. series motor characteristics.

5.5. Universal Motor

- Fractional-horsepower series motors that are adapted for use on either D.C. or A.C. circuits of a given voltage are called *universal motors*.
- The universal motor is designed for commercial frequencies from 60 cycles down to D.C. (zero frequency), and for voltage from 250 V to 1.5 V. A commercial universal motor may have a somewhat weaker series field and more armature conductors than a D.C. series motor of equivalent horsepower. *It is manufactured in ratings up to 3/4 H.P., particularly for vacuum cleaners and industrial sewing machines.* In smaller sizes of $\frac{1}{4}$ H.P. or less, it is used in *electric hand drills*.

Like all series motors, the *no-load speed of the universal motor is universally high.* Quite frequently, *gears trains* are built into the motor housing of some universal motors to provide exceedingly high torque at low speeds.

When these motors are used in commercial appliances such as *electric shavers, sewing machines, office machines, and small hand hair dryers or vacuum cleaners,* they are always *directly loaded* with little danger of motor runaway.

Advantages of a universal motor :

1. High speed from above 3600 r.p.m. to around 25000 r.p.m.
2. High power output in small physical sizes for use in portable tools.
3. High torque at low and intermediate speeds to carry a particularly severe load.
4. Variable speed by adjustable governor, by line voltage or especially by modern pulse techniques.

Disadvantages :

1. Increased service requirement due to use of brushes and commutators. The life of these parts is limited in severe service.
2. Relatively high noise level at high speeds.
3. Moderate to severe radio and television interference due to brush sparking.
4. Requirement for careful balancing to avoid vibration.
5. Requirement for reduction gearing in most portable tools.

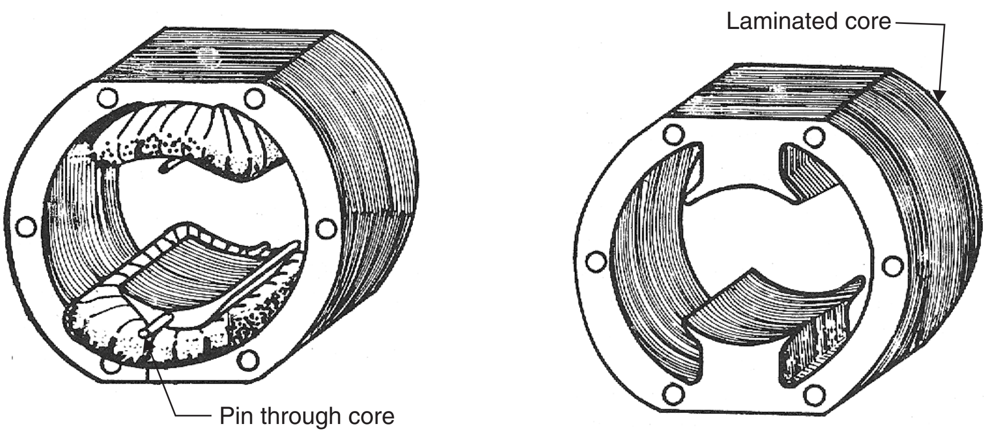


Fig. 21. Field core of a two pole universal motor.

Universal motors are manufactured in two types :

1. Concentrated-pole, non-compensated type (low H.P. rating).

2. Distributed field compensated type (high H.P. rating).

Fig. 21 shows the laminated field structure of a typical concentrated field universal motor.

Operation of a Universal Motor. As explained in Art. 5.4, such motors develop unidirectional torque regardless of whether they operate on D.C. or A.C. supply. The production of unidirectional torque when the motor runs on A.C. supply can be easily understood from Fig. 22. The motor works on the same principle as a D.C. motor *i.e.*, the force between the main pole flux and the current carrying armature conductors. This is true regardless of whether current is alternating or direct.

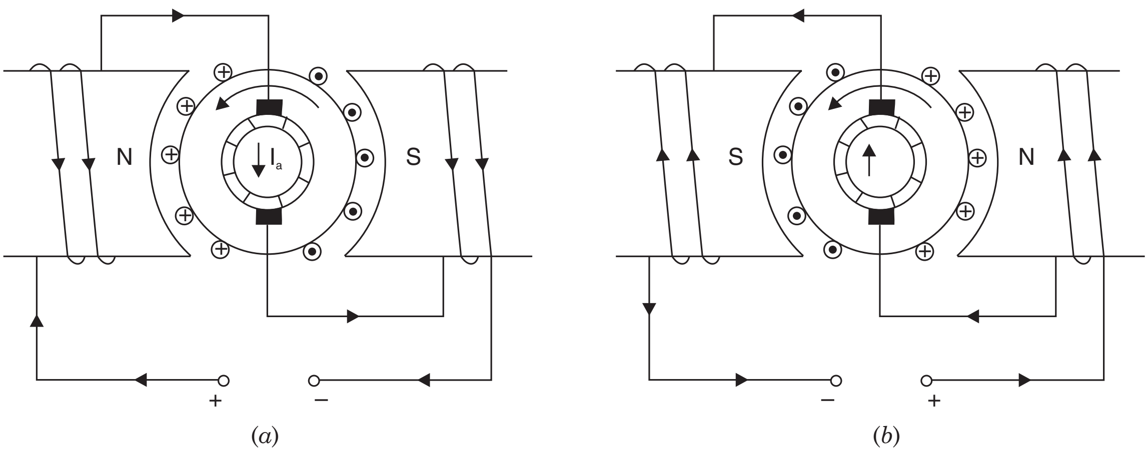


Fig. 22. Universal motor operation.

- Fig. 23 shows the typical torque characteristics of a universal motor both for D.C. and A.C. supply.

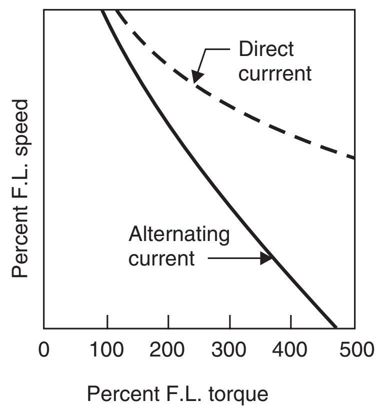


Fig. 23. Typical torque characteristics of a universal motor.

- The speed of a universal motor may be controlled by the following methods :

(*i*) Reactance method.

(*ii*) Tapped-field method.

(*iii*) Centrifugal mechanism.

HIGHLIGHTS

1. There are two basic types of single-phase induction motors which start on the split-phase principle, namely resistance-start induction-run motor and the capacitor-start induction-run motor.
2. The resistance-start induction-run motor has a relatively poor starting torque because the phase angle between the running winding current and the starting winding current is only from 30 to 50 electrical degrees.
3. The capacitor-start induction-run motor has a relatively good starting torque because the phase angle between the running winding current and the starting current is practically 90 electrical degrees.
4. The capacitor-start, capacitor-run induction motor has relatively good starting torque and operates at a comparatively high power factor. The main winding and auxiliary windings are both energized at all times the motor is in operation.
5. A resistance-start induction-run motor or a capacitor-start induction-run motor may be reversed by interchanging the leads of the starting winding circuit. The direction of rotation of either type of motor can also be reversed by interchanging the leads of the running winding circuit. However, **never attempt to change the rotation of either type of motor by reversing the line wires.**
6. The **repulsion motor** operates on the repulsion principles at all times. It has **excellent starting torque** and **a relatively wide range of speeds.**
7. The repulsion-start induction-run motor starts as a repulsion motor and after it reaches 75 per cent of rated speed, operates as an induction motor. It has excellent starting torque and very good speed regulation.
8. There are two types of brush mechanism used with repulsion-start induction-run motors : The **brush-lifting type** *and* **brush-riding** type.
9. To change the direction of rotation of either a repulsion motor or a repulsion-start induction-run motor, move the brushes to the other side of stator field poles so that there is an angle of 15 electrical degrees in the new position between the brushes and the stator field pole centres.
10. The A.C. series motor has excellent starting torque and can be used in those applications requiring different speeds. It has characteristics comparable to those of the D.C. series motor.
11. The direction of rotation of an A.C. series motor can be reversed by interchanging the connections to the armature.
12. The shaded-pole induction motor is usually made in fractional horsepower sizes in excess of 1/10 horsepower. This type of motor has no centrifugal starting device or mechanism which requires maintenance. However, the **starting torque** for this type of motor is **small.**
13. A *stepper motor* is an incremental motion machine. It does not rotate continuously as a conventional motor does.
14. The stepper motor is used in digitally controlled position control system in open loop mode.
15. Various types of stepper motors are :
    (*i*) Permanent-magnet stepper motor ;
    (*ii*) Variable-reluctance stepper motor ;
    (*iii*) Hybrid stepper motor.
16. D.C. servo-motors are preferred for very high power systems.
17. A.C. servo-motors are best suited for low power applications.

OBJECTIVE TYPE QUESTIONS

(A) Choose the Correct Answer :

1. In a split-phase motor, the running winding should have
   (*a*) high resistance and low inductance (*b*) low resistance and high inductance
   (*c*) high resistance as well as high inductance (*d*) low resistance as well as low inductance.

2. If the capacitor of a single-phase motor is short-circuited
(*a*) the motor will not start (*b*) the motor will burn
(*c*) the motor will run in reverse direction
(*d*) the motor will run in the same direction at reduced r.p.m.

3. In capacitor-start single-phase motors
(*a*) current in the starting winding leads the voltage
(*b*) current in the starting winding lags the voltage
(*c*) current in the starting winding is in phase with voltage in running winding.
(*d*) none of the above.

4. In a capacitor-start and run motors the function of the running capacitor in series with the auxiliary winding is to
(*a*) improve power factor (*b*) increase overload capacity
(*c*) reduce fluctuations in torque (*d*) to improve torque.

5. In a capacitor-start motor, the phase displacement between starting and running winding can be nearly
(*a*) 10° (*b*) 30°
(*c*) 60° (*d*) 90°.

6. In a split-phase motor
(*a*) the starting winding is connected through a centrifugal switch
(*b*) the running winding is connected through a centrifugal switch
(*c*) both starting and running windings are connected through a centrifugal switch
(*d*) centrifugal switch is used to control supply voltage.

7. The torque developed by a single-phase motor at starting is
(*a*) more than the rated torque (*b*) rated torque
(*c*) less than the rated torque (*d*) zero.

8. Which of the following motors will give relatively high starting torque ?
(*a*) Capacitor-start motor (*b*) Capacitor run motor
(*c*) Split-phase motor (*d*) Shaded-pole motor.

9. Which type of capacitor is preferred for capacitor-start and run motor ?
(*a*) Electrolyte capacitor (*b*) Paper capacitor (oil filled)
(*c*) Ceramic capacitor (*d*) Air capacitor.

10. Which of the following motors will have relatively higher power factor ?
(*a*) Capacitor-run motor (*b*) Shaded-pole motor
(*c*) Capacitor-start motor (*d*) Split-phase motor.

11. In a shaded-pole motor, the shading coil usually consist of
(*a*) a single turn of heavy wire which is in parallel with running winding
(*b*) a single turn of heavy copper wire which is short-circuited and carries only induced current
(*c*) a multilayer fine gauge copper wire in parallel with running winding
(*d*) none of the above.

12. In a shaded-pole single-phase motor, the revolving field is produced by the use of
(*a*) inductor (*b*) capacitor
(*c*) resistor (*d*) shading coils.

13. A centrifugal switch is used to disconnect starting winding when motor has
(*a*) run for about 1 minute (*b*) run for about 5 minutes
(*c*) picked up about 50 to 70 per cent of rated speed
(*d*) picked up about 10 to 25 per cent of rated speed.

14. If a particular application needs high speed and high starting torque, then which of the following motors will be preferred ?
(*a*) Universal motor (*b*) Shaded-pole type motor
(*c*) Capacitor-start motor (*d*) Capacitor-start and run motor.

15. Which motor has the highest power to weight ratio ?
(*a*) Capacitor-run motor (*b*) Capacitor-start motor
(*c*) Universal motor (*d*) None of the above.

16. In a universal motor the width of the brush is nearly
(*a*) half the width of commutator segment (*b*) equal to width of the commutator segment
(*c*) equal to width of two commutator segment (*d*) equal to width of four commutator segment.

17. The range of efficiency for shaded-pole motors is
(*a*) 95% to 99% (*b*) 80% or 90%
(*c*) 50% to 75% (*d*) 5% to 35%.

18. In a capacitor-start single-phase motor, when capacitor is replaced by a resistance
(*a*) torque will increase (*b*) the motor will consume less power
(*c*) motor will run in reverse direction (*d*) motor will continue to run in same direction.

19. The power factor of a single-phase induction motor is usually
(*a*) lagging (*b*) always leading
(*c*) unity (*d*) unity to 0.8 leading.

20. A shaded-pole motor can be used for
(*a*) toys (*b*) hair dryers
(*c*) circulators (*d*) any of the above.

21. A hysteresis motor works on the principle of
(*a*) hysteresis loss (*b*) magnetisation of rotor
(*c*) eddy current loss (*d*) electromagnetic induction.

22. Which of the following motors will give the highest starting torque ?
(*a*) D.C. shunt motor (*b*) Schrage motor
(*c*) Repulsion-start and induction-run motor (*d*) Universal motor.

23. In repulsion motors by shifting the position of brushes
(*a*) only speed can be controlled (*b*) speed and direction of rotation can be controlled
(*c*) speed, direction of rotation and torque can be controlled
(*d*) none of the above.

24. Which of the following single-phase motors is reversible ?
(*a*) Capacitor-start split-phase motor (*b*) Resistance-start split-phase motor
(*c*) Reluctance motor (*d*) None of the above.

25. For which of the applications a reluctance motor is preferred ?
(*a*) Electric shavers (*b*) Refrigerators
(*c*) Signalling and timing devices (*d*) Lifts and hoists.

26. In repulsion motors
(*a*) speed remains constant (*b*) speed variation is within 5%
(*c*) speed varies with load
(*d*) speed varies only with change in applied voltage.

27. In single-phase repulsion motors power factor is
(*a*) always leading (*b*) high at low speeds
(*c*) always unity (*d*) high at high speeds.

28. The motor used on small lathes is usually
(*a*) universal motor (*b*) d.c. shunt motor
(*c*) single-phase capacitor run motor (*d*) 3-phase synchronous motor.

29. Which of the following motors is preferred for tape recorders ?
(*a*) Shaded-pole motor (*b*) Hysteresis motor
(*c*) Two value capacitor motor (*d*) Universal motor.

30. A single-phase induction motor is
(*a*) inherently self-starting with high torque (*b*) inherently self-starting with low torque
(*c*) inherently non-self-starting with low torque (*d*) inherently non-self-starting with high torque.

31. A schrage motor can run on
(*a*) zero slip (*b*) negative slip
(*c*) positive slip (*d*) all of the above.

32. A universal motor can run on
(*a*) A.C. only (*b*) D.C. only
(*c*) either A.C. or D.C. (*d*) none of the above.

33. Which of the following single-phase motors is suitable for timing and control purposes ?
(*a*) Reluctance motor (*b*) Series motor
(*c*) Repulsion motor (*d*) Universal motor.

34. In case of a shaded-pole motor the direction of rotation of the motor is
(*a*) from main-pole to shaded-pole (*b*) from shaded-pole to main pole
(*c*) either of the above depending on voltage (*d*) either of the above depending on power factor.

35. In case of high speed universal motor which of the following needs more attention ?
(*a*) End play (*b*) Air gap
(*c*) Insulation in rotor (*d*) Balancing of rotor.

36. The wattage rating for a ceiling fan motor will be in the range
(*a*) 50 to 250 W (*b*) 250 to 500 W
(*c*) 50 to 150 W (*d*) 10 to 20 W.

37. The wattage of motor for driving domestic sewing machine will be around
(*a*) 100 to 150 W (*b*) 40 to 75 W
(*c*) 10 to 30 W (*d*) 5 to 10 W.

38. Which of the following single-phase motors has relatively poor starting torque ?
(*a*) Universal motor (*b*) Repulsion motor
(*c*) Capacitor motor
(*d*) All single phase motors have zero starting torque.

39. Which type of load is offered by cranes and hoists ?
(*a*) gradually varying load (*b*) non-reversing, no-load start
(*c*) reversing, light start (*d*) reversing, heavy start.

40. The speed of a universal motor is generally reduced by using
(*a*) gear trains (*b*) V-belts
(*c*) brakes (*d*) chains.

41. Which of the following motors can be used for unity power factor ?
(*a*) Capacitor run motor (*b*) Shaded-pole motor
(*c*) Hysteresis motor (*d*) Schrage motor.

42. When a D.C. series motor is connected to A.C. supply, the power factor will be low because of
(*a*) high inductance of field and armature circuits
(*b*) induced current in rotor due to variations of flux
(*c*) fine copper wire winding (*d*) none of the above.

43. The direction of rotation of universal motor can be reversed by reversing the flow of current through
(*a*) armature winding (*b*) field winding
(*c*) either armature winding or field winding (*d*) none of the above.

44. In which single-phase motor, the rotor has no teeth or winding ?
(*a*) Split-phase motor (*b*) Reluctance motor
(*c*) Hysteresis motor (*d*) Universal motor.

45. Which motor is normally free from mechanical and magnetic vibrations ?
(*a*) Split-phase motor (*b*) Universal motor
(*c*) Hysteresis motor (*d*) Shaded-pole motor.

46. As hysteresis motors are free from mechanical and magnetic vibrations therefore these are considered as suitable for
(*a*) fans (*b*) blowers
(*c*) sound equipment (*d*) mixer grinders.

47. A reluctance motor
(*a*) is self-starting (*b*) is constant speed motor
(*c*) needs no D.C. excitation (*d*) all of the above.

48. In a hysteresis motor, the rotor must have
(*a*) retentivity (*b*) resistivity
(*c*) susceptibility (*d*) none of the above.

49. The rotor of a hysteresis motor is made of
(*a*) aluminium (*b*) cast iron
(*c*) chrome steel (*d*) copper.

50. The electric motor used in portable drills is
(*a*) capacitor-run motor (*b*) hysteresis motor
(*c*) universal motor (*d*) repulsion motor.

51. Which of the following applications always has some load whenever switched on ?
(*a*) Vacuum cleaners (*b*) Fan motors
(*c*) Pistol drills (*d*) All of the above.

52. The speed control of universal motor used for sewing machines is by
(*a*) friction (*b*) varying the resistance
(*c*) tapping the field (*d*) centrifugal mechanism.

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*d*)	**6.** (*a*)
7. (*d*)	**8.** (*a*)	**9.** (*b*)	**10.** (*a*)	**11.** (*b*)	**12.** (*d*)
13. (*c*)	**14.** (*a*)	**15.** (*c*)	**16.** (*c*)	**17.** (*d*)	**18.** (*d*)
19. (*a*)	**20.** (*d*)	**21.** (*a*)	**22.** (*b*)	**23.** (*b*)	**24.** (*a*)
25. (*c*)	**26.** (*c*)	**27.** (*d*)	**28.** (*c*)	**29.** (*b*)	**30.** (*c*)
31. (*d*)	**32.** (*c*)	**33.** (*a*)	**34.** (*a*)	**35.** (*d*)	**36.** (*c*)
37. (*a*)	**38.** (*c*)	**39.** (*d*)	**40.** (*a*)	**41.** (*d*)	**42.** (*a*)
43. (*c*)	**44.** (*c*)	**45.** (*c*)	**46.** (*c*)	**47.** (*d*)	**48.** (*a*)
49. (*c*)	**50.** (*c*)	**51.** (*c*)	**52.** (*b*).		

(B) Say 'Yes' or 'No' :

1. In a capacitor-start motor, the phase displacement between starting and running winding can be nearly 90°.
2. In a split-phase motor the staring winding is connected through a centrifugal switch.
3. In a shaded-pole single-phase motor, the revolving field is produced by the use of capacitor.
4. A universal motor has the highest power to weight ratio.
5. The range of efficiency for shaded-pole motors is 80 to 90%.
6. The power factor of a single-phase induction motor is usually lagging.

7. A hysteresis motor works on the principle of eddy current loss.

8. Capacitor-start split-phase motor is reversible.

9. A reluctance motor is preferred for signalling and timing devices.

10. The wattage rating for a ceiling fan motor varies from 50 to 150 W.

11. A ...... motor is an incremental motion machine.

12. A stepper motor rotates continuously.

13. The stepper motor is used in digitally controlled position control system in open loop mode.

14. In stepper motors no sensors are needed for position and speed sensing.

15. A stepper motor cannot be readily interfaced with microprocessor.

16. In ...... stepper motor the rotor is made of ferite or rare earth material which is permanently magnetised.

17. In a ...... stepped motor the stator has only one set of winding-excited poles which interact with the two rotor stacks.

18. ...... servo-motors are preferred for very high power systems.

19. A.C. servo-motors are best suited for low power applications.

20. ...... construction is used for very low inertia applications.

ANSWERS

1. Yes	**2.** Yes	**3.** No	**4.** Yes	**5.** No	**6.** Yes
7. No	**8.** Yes	**9.** Yes	**10.** Yes	**11.** stepper	**12.** No
13. Yes	**14.** Yes	**15.** No	**16.** permanent-magnet		**17.** hybrid
18. D.C.	**19.** Yes	**20.** Drag-cup.			

THEORETICAL QUESTIONS

1. Explain briefly the split-phase method of motor starting.

2. Explain why the starting torque of a capacitor-start induction run motor is better than that of a resistance-start induction-run motor.

3. How is the direction of rotation reversed for each of the following ?

(*i*) Resistance-start induction-run motor, and

(*ii*) Capacitor-start induction-run motor.

4. (*a*) Compare operating characteristics of a resistance-start induction-run motor with those of a capacitor-start induction-run motor.

(*b*) List three applications for :

(*i*) A resistance-start induction-run motor

(*ii*) A capacitor-start induction-run motor.

5. Explain how a repulsion motor operates.

6. Explain how a repulsion-start induction-run motor operates.

7. How is the direction of rotation of each of the following motors reversed ?

(*i*) The repulsion motor, and (*ii*) The repulsion-start induction-run motor.

8. What is the difference between A.C. and D.C. series motor ?

9. Why are small fractional horsepower A.C. series motors called universal motors ?

10. What is the difference between a conductively compensated series motor and an inductively compensated series motor ?

11. Describe the construction and operation of a shaded-pole motor.

UNIT-III : *SEMICONDUCTOR DEVICES AND APPLICATIONS*

Chapter :

8. Semiconductor Devices and Applications

8

Semiconductor Devices and Applications

1. INTRODUCTION TO SEMICONDUCTORS

Semiconductors *are solid materials, either non-metallic elements or compounds, which allow electrons to pass through them so that they conduct electricity in much the same way as a metal.*

1.1. Characteristics of Semiconductors

Semiconductors possess the following *characteristics* :

1. The resistivity is usually high.

2. The temperature coefficient of resistance is *always negative.*

3. The contact between semiconductor and a metal forms a layer which has a higher resistance in one direction than the other.

4. When some suitable metallic impurity (*e.g.*, Arsenic, Gallium, etc.) is added to a semiconductor, its *conducting properties change appreciably.*

5. They exhibit a rise in conductivity in the increasing temperature, with the decreasing temperatures their conductivity falls off, and at low temperatures semiconductors become dielectrics.

6. They are usually metallic in appearance but (unlike metals) are generally hard and brittle.

Both the resistivity and the contact effect are as a rule very sensitive to small changes in physical conditions, and the *great importance* of semiconductors for a wide range of uses apart from rectification depend on the *sensitiveness*.

Examples of semiconducting materials

Of all the elements in the periodic table, *eleven* are *semiconductors* which are listed below :

S. No.	Element	Symbol	Group in the periodic table	Atomic No.
1.	Boron	B	III	15
2.	Carbon	C	IV	6
3.	Silicon	Si	IV	14
4.	Germanium	Ge	IV	32
5.	Phosphorus	P	V	15
6.	Arsenic	As	V	33
7.	Antimony	Sb	V	51
8.	Sulphur	S	VI	
9.	Sellinium	Se	VI	
10.	Tellurium	Te	VI	
11.	Iodine	I	VIII	

Examples of semiconducting *compounds* are given below :

(*i*) Alloys : Mg_3Sb_2, ZnSb, Mg_2Sn, CdSb, AlSb, InSb, GeSb.

(*ii*) Oxide : ZnO, Fe_3O_4, Fe_2O_3, Cu_2O, CuO, BaO, CaO, NiO, Al_2O_3, TiO_2, UO_2, Cr_2O_3, WO_2, MoO_3.

(*iii*) Sulphides : Cu_2S, Ag_2S, PbS, ZnS, CdS, HgS, MoS_2.

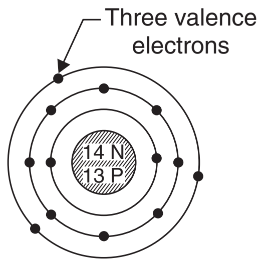
(*iv*) Halides : AgI, CuI.

(*v*) Selenides and Tellurides.

PbS is *used in photo-conductive devices, BaO in oxide coated cathodes, caesium antimonide in photomultipliers, etc.*

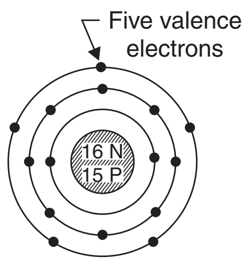
1.2. Atomic Structure

To understand how semiconductors work it is necessary to study briefly the structure of matter. All atoms are made of electrons, protons and neutrons. Most solid materials are classed, from the stand point of electrical conductivity, as conductors, semiconductors or insulators. *To be conductor, the substance must contain some mobile electrons—one that can move freely between the atoms.* These free electrons come only from the valence (outer) orbit of the atom. Physical force associated with the valence electrons bind adjacent atoms together. The inner electrons below the valence level, do not normally enter into the conduction process.



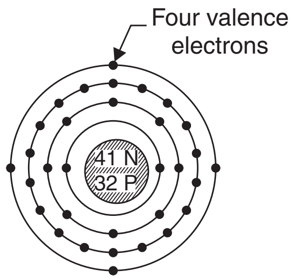
Aluminium

Fig. 1



Phosphorus

Fig. 2



Germanium

Fig. 3

Conductivity depends on the number of electrons in the valence orbit. Electron diagrams for three typical elements, aluminium, phosphorus and germanium are shown in Figs. 1, 2, 3.

These elements can all be used in semiconductor manufacture. The *degree of conductivity* is determined as follows :

1. Atoms with *fewer than four valence* electrons are *good conductors.*
2. Atoms with *more than four valence* electrons are *poor conductors.*
3. Atoms with *four valence electrons are semiconductors.*

Fig. 1 shows aluminium which has *three valence electrons*. When there are less than four valence electrons they are loosely held so that at least one electron per atom is normally free ; hence aluminium is a good conductor. This ready availability of free electrons is also true of copper and most other metals.

Fig. 2 shows Phosphorus with *five valence electrons*. When there are more than four valence electrons, they are lightly held in orbit so that normally *none are free.* Hence phosphorus and similar elements are poor conductors (insulators).

Germanium (Fig. 3) has four *valence electrons*. This makes it neither a good conductor nor a good insulator, hence its name "semiconductor". Silicon also has four valence electrons and is a semiconductor.

Note. The energy level of an electron increases as its distance from the nucleus increases. Thus an electron in the second orbit possess more energy than electron in the first orbit ; electrons in the third orbit have higher energy than in the second orbit and so on. It follows, therefore, that electrons in the last orbit will possess very high energy. These high energy electrons are less bound to the nucleus and hence they are more mobile. It is the mobility of last orbit electrons that they acquire the property of combining with other atoms. Further it is due to this combining power of last orbit electrons of an atom that they are called *valence electrons.*

1.3. Intrinsic Semiconductor

A *pure semiconductor is called "intrinsic semiconductor"*. Here no free electrons are available since all the co-valent bonds are complete. *A pure semiconductor, therefore behaves as an insulator.* It exhibits a peculiar behaviour even at room temperature or with rise in temperature. The *resistance of a semiconductor decreases with increase in temperature.*

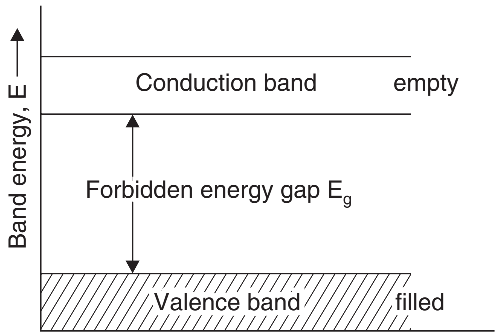


Fig. 4. Energy diagram for intrinsic (pure) semiconductor at absolute zero.

When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0°K, conduction electrons move to the anode and, the holes (when an electron is liberated into the conduction band a positively charged hole is created in valence band) move to cathode. Hence semiconductor current consists of movement of electrons in opposite direction.

1.4. Extrinsic Semiconductor

In a pure semiconductor, which behaves like an insulator *under ordinary conditions,* if small amount of certain *metallic impurity* is *added* it attains *current conducting properties.* The impure semiconductor is then called *"impurity semiconductor" or "extrinsic semiconductor". The process of adding impurity (extremely in small amounts, about 1 part in* 10^8*) to a semiconductor to make it extrinsic (impurity) semiconductor is called* **Doping.**

Generally following doping agents are used :

(*i*) *Pentavalent atom* having five valence electrons (arsenic, antimony, phosphorus) ... called *donor atoms.*

(*ii*) *Trivalent atoms* having three valence electrons (gallium, aluminium, boron) ... called *acceptor atoms.*

With the addition of suitable impurities to semiconductor, two type of semiconductors are :

(*i*) *N*-type semiconductor.

(*ii*) *P*-type semiconductor.

N-type semiconductor :

The presence of *even a minute quantity of impurity,* can produce *N*-type semiconductor. If the impurity atom has *one valence electron more* than the semiconductor atom which it has substituted, this *extra electron* will be loosely bound to the atom. For example, an atom of *Germanium* possesses *four valence electrons* ; when it is replaced in the crystal lattice of the substance by an impurity atom of antimony (Sb) which has *five valence electrons,* the fifth valence electron (free electron) produces extrinsic *N*-type conductivity *even at room temperature.* Such an impurity into a semiconductor is called *donor impurity* (or donor). The conducting properties of germanium will depend upon the *amount of antimony* (*i.e.*, impurity) *added.* This means that controlled conductivity can be obtained by proper addition of impurity. Fig. 5 (*a*) shows the loosely bound excess electron controlled by the donor atom.

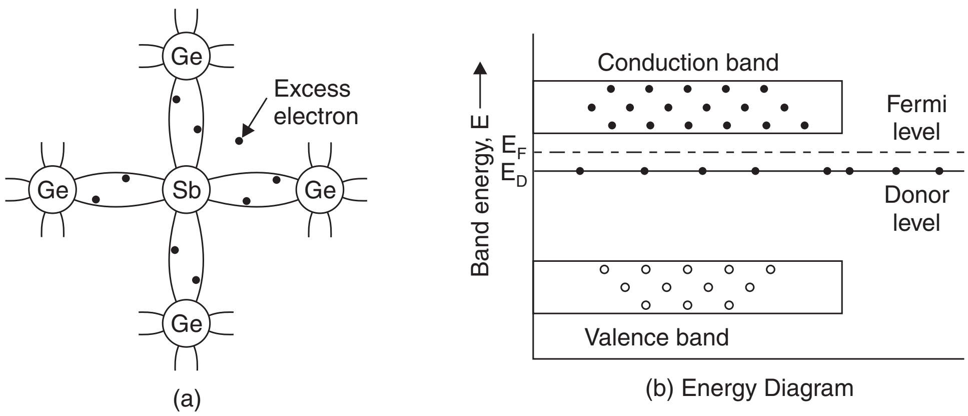


Fig. 5. *N*-type semiconductor.

- It may be noted that by giving away its one electron, the *donor atom* becomes a *positively-charged ion.* But it cannot take part in conduction because it is firmly fixed or tied into the crystal lattice. In addition to the electrons and holes *intrinsically available in germanium,* the addition of antimony greatly increases the number of conduction electrons. Hence, *concentration of electrons in the conduction band is increased and exceeds the concentration of holes in the valence band.* Consequently, *Fermi level shifts upwards towards the bottom of the conduction band* as shown in Fig. 5 (*b*). [Since the number of electrons as compared to the number of holes increases with temperature, the *position of Fermi level also changes considerably with temperature*].

- It is worth noting that even though *N*-type semiconductor has excess of electrons, still it is *electrically neutral.* It is so because by addition of donor impurity, number of electrons available for conduction purposes becomes more than the number of holes available intrinsically. But the total charge of the semiconductor does not change because the donor impurity brings in as much negative charge (by way of electrons) as positive charge (by way of protons).

Note. In terms of energy levels, the fifth antimony electron has as energy level (called donor level) just below the conduction band. Usually, the donor level is 0.01 eV below conduction band for germanium and 0.054 eV for silicon.

P-type semiconductor :

- *P*-type extrinsic semiconductor can be produced if the impurity atom has *one valence electron less* than the semiconductor atom that it has replaced in the crystal lattice. This impurity atom cannot fill all the *interatomic* bonds, and the free bond can accept an electron from the neighbouring bond ; leaving behind a vacancy of *hole.* Such an impurity is called an *acceptor impurity (or acceptor)*. Fig. 6 (*a*) shows structure of *P*-type semiconductor (Germanium and Boron).
- In this type of semiconductor, conduction is by means of holes in the valence band. Accordingly, *holes form the majority carriers whereas electrons constitute minority carriers.* The process of conduction is called *deficit conduction.*
- Since the concentration of holes in the valence band is more than the concentration of electrons in the conduction band, Fermi level shifts nearer to the valence band [Fig. 6 (*b*)]. The acceptor level lies immediately above the Fermi level. *Conduction is by means of hole movement at the top of valence band, the acceptor level readily accepting electrons from the valence band.*

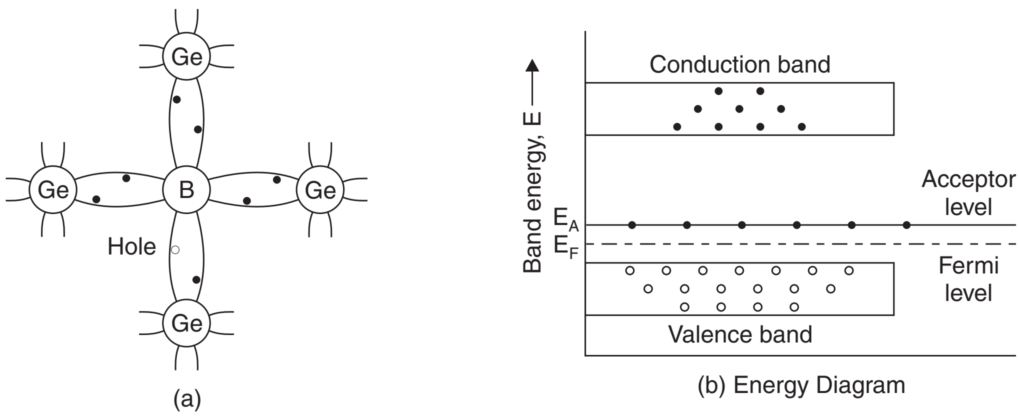


Fig. 6. *P*-type semiconductor.

It may be noted again that even though *P*-type semiconductor has excess of holes for conduction purposes, as a whole it is electrically neutral for the same reasons as discussed earlier.

2. P-N JUNCTION DIODE

In an **N-type material** (Fig. 7) *the electron is called the majority carrier and the hole is the minority carrier.*

In a **P-type material** (Fig. 8) *the hole is the majority carrier and the electron is the minority carrier.* The *N*- and *P*-type materials represent the *basic building blocks of semiconductor devices.*

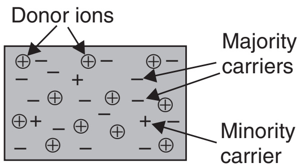


Fig. 7. *N*-type material.

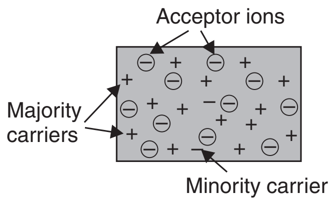


Fig. 8. *P*-type material.

The semiconductor diode is simply bringing these materials together (constructed from the same base-Ge or Si). At instant the two materials are "*joined*" the electrons and holes in the region of the junction will combine resulting in a lack of carriers in the region near the junction. *This region of uncovered positive and negative ions is called the depletion region due to the depletion of carriers in this region.*

2.1. Construction and Types of P-N Junction Diodes

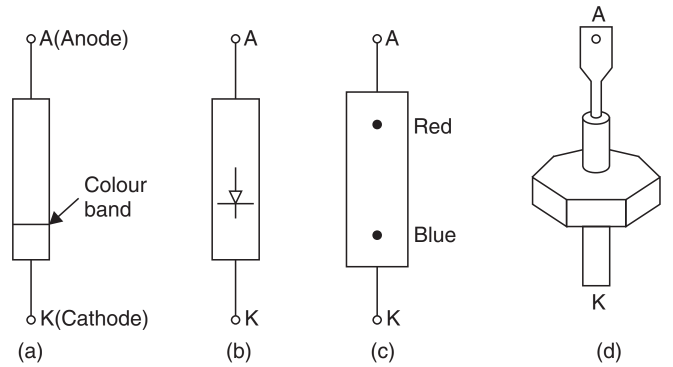
The most extensively used elements in the manufacture of junction diodes are *germanium and silicon* (although some other materials are also assuming importance in recent years).

A *P-N* junction diode (known as a semiconductor or crystal diode) consists of a *P-N* junction, formed either in germanium or silicon crystal. The diode has two terminals namely *anode and cathode.* The anode refers to the *P*-type region and cathode refers to the *N*-type region as shown in Fig. 9 (*a*). Its circuit symbol is as shown in Fig. 9 (*b*).

Fig. 9. *P-N* junction diode.

The arrow head, shown in the circuit symbol, points the direction of current flow, when it is "forward biased" (It is the same direction in which the movement of holes takes place).

The commercially available diodes, usually have some *notations* to identify the *P* and *N* terminals of leads. *The standard notation consists of type numbers preceded by IN,* such as *IN* 240 and *IN* 1250. Here 240 and 1250 *correspond* to colour bands. *In some diodes, the schematic symbol of a diode is painted or the colour dots are marked on the body.*



(*a*), (*b*) = Low current diodes ; (*c*) = Medium current diode ;
(*d*) = High current or power diode.

Fig. 10. Low, medium and high current diodes.

Fig. 10 shows low, medium and high current diodes.

— *Refer to Fig. 10 (a).* The diode shown has a colour band located near one of the ends. The end, which is near the colour band, is identified as cathode ; and the other end, obviously, is the anode (*A*).

— *Refer to Fig. 10 (b).* The diode has a schematic symbol actually painted at its cathode (*K*) and the other end as anode.

The diodes of Fig. 10 (*a*) and (*b*) *can pass a forward current of 100 mA and are known as* **low current diodes.**

— *Refer to Fig. 10 (c).* The diode has colour dots marked on its body. The end lying near the blue dot is a cathode, while the other end is anode. Sometimes this diode is *shown bigger in size* than that of diodes shown in Fig. 10 (*a*) and (*b*). The diodes of this size can *pass a forward current of 500 mA and are known as* **medium current diodes.**

— *Refer to Fig. 10 (d).* It shows a diode, which *can pass a forward current of several amperes.* Therefore it is known as a **power diode or a high current diode.**

- *The outstanding property of P-N junction/crystal diode to conduct current in one direction only permits it to be used as a* **rectifier.**

2.2. Potential Barrier and Biasing

A *P-N* junction diode which consists of *P-* and *N*-type semiconductors formed together to make a *P-N* junction is shown in the Fig. 11. The *place dividing the two zones* is known as a *"junction"*.

Potential barrier :

As a result of *diffusion* some electrons and holes migrate across the junction there by forming a *depletion layer* on either side of the junction by neutralisation of holes in the *P*-regional and of free electrons in the *N*-region. *This diffusion of holes and electrons across the junction continues till a* ***potential barrier*** *is developed in the depletion layer which then prevents further diffusion.* By the application of an external voltage this potential barrier is either *increased* or *decreased.*

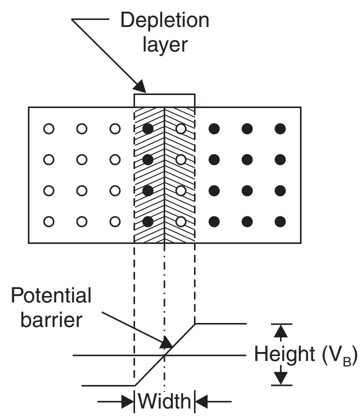
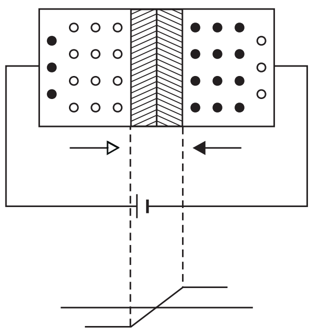


Fig. 11.

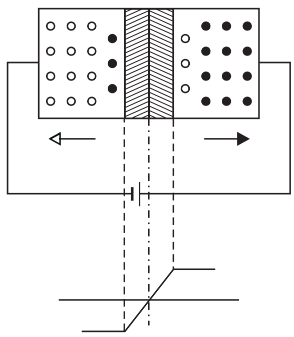
The barrier voltage of a *P-N* junction depends upon three factors namely *density*, *electronic charge* and *temperature*. For a given *P-N* junction, the first two factors are constant, thus making the value of V_B dependent only on temperature. It has been observed that for both germanium and silicon the value of V_B decrease by $2mV$/°C. Mathematically, the decrease in barrier voltage, $\Delta V_B = -0.002 \times \Delta t$, where Δt is the increase in temperature in °C.

Forward biasing :

The junction is said to be biased in the forward direction when then positive battery terminal is connected to *P*-type region and the negative battery terminal to the *N*-type (Fig. 12). This arrangement permits the flow of current across the *P-N* junction. The *holes are repelled by the positive battery terminal and electrons by the negative battery terminal with the result that both holes and electrons will be driven towards the junction where they will recombine.* Hence as long as the battery voltage is applied large current flows. In other words, the *forward bias lowers the potential barrier across the depletion layer* thereby *allowing more current to flow* across the junction.



Potential barrier decreased
Fig. 12. Forward biasing.



Potential barrier increased
Fig. 13. Reverse biasing.

Reverse biasing (Zener diode) :

The junction is said to be reversed biased when battery connections to the battery are reversed as shown in Fig. 13. In this arrangement holes are attracted by the negative battery terminal and electrons by the positive battery terminal so that both holes and electrons move *away* from the junction. Since there is no recombination of electron-hole pairs, diode current is negligible and the junction has high resistance. Reverse biasing *increases the potential barrier* at the junction, thereby *allowing very little current to flow through* the junction.

2.3. V-I Characteristics of a P-N Junction Diode

The *V-I* (volt-ampere) characteristic of a typical *P-N* junction diode with respect to breakdown voltage (V_{BR}) is shown in Fig. 14.

- For typical junction concentrations and current densities at a temperature of 300 K, forward voltage ranges between 0.2 and 0.3 V in germanium and between 0.5 and 0.75 V in silicon.
- The *reverse current is related to minority carrier concentration, which depends upon temperature and the energy gap of the material.*

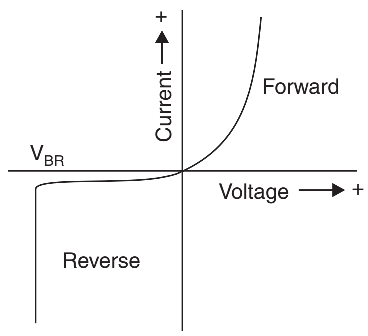
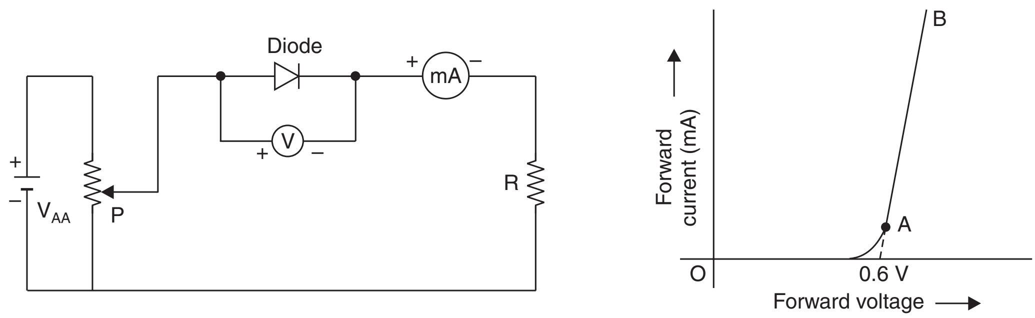


Fig. 14. *V-I* characteristic of a *P-N* junction diode.

Reverse current increases exponentially with temperature. It is a limiting factor in the high-temperature junction of semiconductor junction devices.

- The high-frequency response of a semiconductor diode may be seriously limited by charge stored in the depletion region. This charge gives a capacitive effect since it changes with voltage. The value of the stored charge is that of the ionized impurity atoms in the depletion regions on either side of the junction. *The width of the depletion region increases with higher reverse voltage and higher doping*. The result is lower capacitance, as in the case of a parallel-plate capacitor with wider spacing between plates.
- The *maximum reverse voltage of a P-N junction is limited by the field in the depletion region.* The field accelerates carriers, which may gain enough energy to create new hole-electron pairs by colliding with atoms of the lattice structure. Each of these carriers may also create a hole-electron pair. As reverse voltage is increased, as *avalanche breakdown point is reached at which this multiplicative action causes the current to increase abruptly. Avalanche breakdown voltage is higher in lightly doped regions, since depletion region is wider, making the internal electric field smaller for any given voltage.*
- *P-N* junction diodes *usually made of germanium or silicon, are commonly used as* **power rectifiers.**

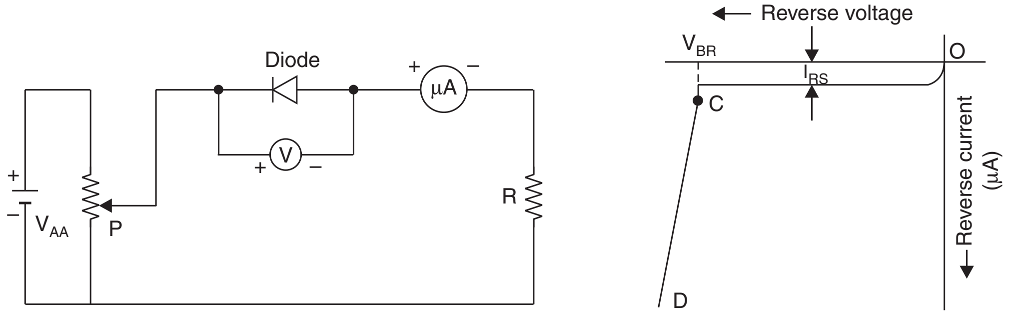
The circuit arrangements for obtaining *forward and reverse characteristics* of a *P-N* junction diode in a laboratory are shown in Figs. 15 (*a*), (*b*) and 16 (*a*), (*b*) respectively.



(a) Circuit arrangement. (b) Forward characteristic.

Fig. 15

- *Refer to Fig. 15.* It may noted that if excessive current is permitted to flow through the diode, it may get permanently damaged.
- *Refer to Fig. 16.* The circuit is similar to that shown in Fig. 15 (*a*), except two changes namely the *diode terminals are reversed and milliammeter is replaced by a microammeter.* It may be noted that negative terminal of the voltage source is connected to the anode of a diode and positive terminal to the cathode. Hence, the diode is *reversed.*



(a) Circuit arrangement. (b) Reverse characteristic.

Fig. 16

2.4. Diode Current Equation

The mathematical equation, which describes the forward and reverse characteristics of a semiconductor diode is called the **diode current equation.**

Let I = Forward (or reverse) diode current,

I_{RS} = Reverse saturation current,

V = External voltage (It is positive for forward bias and negative for reverse bias),

η = A constant

= 1 for germanium diodes, 2 for silicon diodes for relative *low value of diode current* (*i.e.,* at or below the knee of the curve)

= 1 for germanium and silicon for *higher levels of diode current* (*i.e.,* in the rapidly increasing section of the curve), and

V_T = Volt-equivalent of temperature. Its value is given by the relation, $\frac{T}{11600}$, where T is the absolute temperature

= 26 mV at room temperature (300 K).

For a *forward-biased diode*, the current equation is given by the relation,

$$I = I_{RS}\ [e^{V/(\eta \times V_T)} - 1] \qquad ...(i)$$

Substituting the value of V_T = 26 mV or 0.026 V (at room temperature) in eqn. (*i*), we get

$$I = I_{RS}\ (e^{40V/\eta})$$

∴ Diode current at or below the knee, for *germanium,*

$$I = I_{RS}\ (e^{40V} - 1) \qquad (\because \ \eta = 1)$$

and, for *silicon,*

$$I = I_{RS}\ (e^{20V} - 1) \qquad (\because \ \eta = 2)$$

When the value of applied voltage is greater than unity (*i.e.,* for the diode current in the rapidly increasing section of curve), the equation of diode current for germanium or and silicon,

$$I = I_{RS}\ .\ e^{20V} \qquad (\because \ \eta = 2)$$

The current equation for a reverse biased diode may be obtained from eqn. (*i*) by *changing the sign of the applied voltage (V).* Thus the *diode current for reverse bias,*

$$I = I_{RS}\ [e^{-V/(\eta \times V_T)} - 1]$$

When $V >> V_T$, then the term $e^{-V/(\eta \times V_T)} << 1$. Therefore $I = I_{RS}$. Thus the *diode current under reverse bias is equal to the reverse saturation current as long as the external voltage is below its breakdown value.*

Example 1. *The current flowing in a certain P-N junction diode at room temperature is 1.8 × 10⁻⁷ A, when large reverse voltage is applied. Calculate the current flowing, when 0.12 V forward bias is applied at room temperature.*

Solution. *Given :* $I_{RS} = 1.8 \times 10^{-7}$ A ; $V_F = 0.12$ V

The current flowing through the diode under forward bias is given by,

$$I = I_{RS}\,(e^{40V_F} - 1)$$

or $$\mathbf{I} = 1.8 \times 10^{-7}\,(e^{40 \times 0.12} - 1) = 21.69 \times 10^{-6}\text{ A} = \mathbf{21.69\ \mu A.} \quad \textbf{(Ans.)}$$

Example 2. *Determine the germanium P-N junction diode current for the forward bias voltage of 0.2 V at room temperature 24°C with reverse saturation current equal to 1.1 mA. Take $\eta = 1$.*

Solution. *Given :* $V_F = 0.2$ V ; $T = 24 + 273 = 297$ K ; $I_{RS} = 1.1$ mA $= 1.1 \times 10^{-3}$ A ; $\eta = 1$

We know that, $V_T = \dfrac{T}{11600} = \dfrac{297}{11600} = 0.0256$ V (*i.e.,* 25.6 mV)

∴ The diode current, $\mathbf{I} = I_{RS}\,[e^{V_F/(\eta \times V_T)} - 1]$

$$= 1.1 \times 10^{-3}\,[e^{0.2/(1 \times 0.0256)} - 1] = \mathbf{2.717\ A.} \quad \textbf{(Ans.)}$$

2.5. Static and Dynamic Resistance of a Diode

Static forward resistances (R_F). A diode has a definite value of resistance when forward biased. It is given by the *ratio of the D.C. voltage across the diode to D.C. current flowing through it.*

Mathematically, $R_F = \dfrac{V_F}{I_F}$

R_F may be obtained *graphically* from the diode forward characteristics as shown in Fig. 17. From the operating point P, the static forward resistance,

$$R_F = \frac{0.8}{16} = 0.05\ \Omega.$$

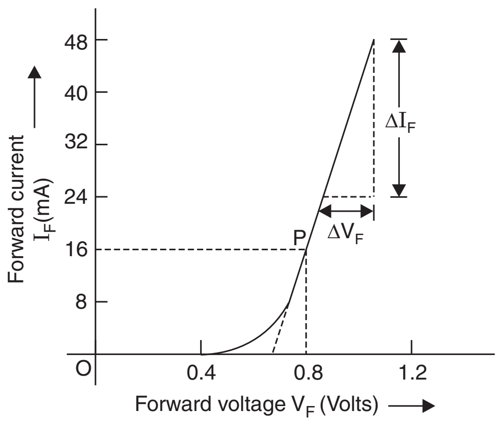


Fig. 17. Static and dynamic forward resistances of a diode from the characteristic curve.

Dynamic or A.C. resistance. In practice we don't use static forward resistance, instead, we use the dynamic or A.C. resistance. The A.C. resistance of a diode, at a particular D.C. voltage, is *equal to the reciprocal of the slope of the characteristic at that point ; i.e,* the A.C. resistance,

$$r_{\text{A.C.}} = \frac{1}{\Delta I_F/\Delta V_F} = \frac{\Delta V_F}{\Delta I_F} = \frac{\text{Change in voltage}}{\text{Resulting change in current}}.$$

Owing to the non-linear shape of the forward characteristic, the value of A.C. resistance of a diode is in the range of 1 to 25 Ω. Usually it is *smaller than D.C. resistance of a diode.*

Reverse resistance. When a diode is *reverse biased*, besides the forward resistance, it also possesses another resistance known as *reverse resistance.* It can be either D.C. or A.C. depending upon whether the reverse bias is direct or alternating voltage. Ideally, the reverse resistance of a diode is infinite. However, in actual practice, the reverse resistance is never infinite. It is *due to the existence of leakage current* in a reverse biased diode.

Its value for germanium and silicon diodes is of *several megaohms.*

The A.C. resistance of a diode may also be determined from the following *two resistances* :

1. Bulk resistance
2. Junction resistance.

1. **Bulk resistance r_B.** *The resistance of P- and N-semiconductor materials of which the diode is made of*, is known as "*bulk or body resistance*". It also includes the resistance introduced by the connection between the semiconductor material and external metallic conductor, contact resistance.

Mathematically $$r_B = r_P + r_N$$

where, r_P = Ohmic resistance of *P*-type semiconductor, and

r_N = Ohmic resistance of *N*-type semiconductor.

The typical values of bulk resistance may be :

For high power devices...0.1 Ω

Low-power general purpose diodes..........................2 Ω

The total voltage drop across the diode,

$$V_F = V_B + I_F \cdot r_B \qquad ...(1)$$

$$= 0.6 + I_F \cdot r_B \qquad \text{...For silicon diode} \quad ...[1\,(a)]$$

$$= 0.2 + I_F \cdot r_B \qquad \text{...for germanium diode} \quad ...[1\,(b)]$$

2. **Junction resistance r_J.** The value of junction resistance for a forward-biased *P-N* junction depends upon the value of forward D.C. current and is given by relation,

$$r_J = \frac{26}{I_F} \qquad ...(2)$$

where, I_F = Forward current in 'milliamperers'

Mathematically, the A.C. resistance, $$r_{A.C.} = r_J + r_B \qquad ...(3)$$

Example 3. *A silicon diode has a bulk resistance of 2.2 Ω and a forward current of 11 mA. What is the actual value of V_F for the device.*

Solution. *Given :* $r_B = 2.2\ \Omega$; $I_F = 11\text{ mA} = 11 \times 10^{-3} = 0.011\text{ A}$

Now, $$V_F = 0.6 + I_F \cdot r_B \qquad ...[\text{Eqn. } 1\,(a)]$$

$$= 0.6 + 0.011 \times 2.2 = \mathbf{0.6242\ \Omega}. \quad \textbf{(Ans.)}$$

2.6. Power and Current Ratings of a Diode

The power dissipation for a *forward biased diode is given by,*

$$P_{DF} = V_F \times I_F \qquad ...(4)$$

where, P_{DF} = Power dissipated by the diode,

V_F = Forward voltage drop, and

I_F = Forward current.

Similarly, power dissipation for a *reverse biased diode,*

$$P_{DR} = V_R \times I_R$$

where, V_R = Reverse voltage drop, and

I_R = Reverse current.

The maximum value of power, which a diode can dissipate without failure, is called its **rating**. Thus the power dissipation should not exceed power rating in any case, otherwise the diode will get destroyed.

The diode manufacturers more oftenly list the maximum current, which a device can handle, (called *current rating*), rather than power rating. It is because of the fact that it is *easy to measure current rating than power rating.*

2.7. Applications of a Diode

An important characteristic of the *P-N* junction diode that it conducts well in forward direction and poorly in reverse direction has made it useful in several applications listed below :

1. As zener diodes in voltage stabilizing circuits.
2. As rectifiers or power diodes in D.C. power supplies.
3. As a switch in logic circuits in computers.
4. As signal diodes in communication circuits.
5. As varactor diodes in radio and T.V. receivers.

3. ZENER DIODE

A properly doped P-N junction crystal diode which has a sharp breakdown voltage is known as ***Zener diode.***

The voltage-regulator diode is commonly called a **'Zener' diode.** It is a *voltage limiting diode* that has some applications in common with the older voltage-regulator gas tubes but serves a much wider field of application, because the devices cover a wide spectrum of voltages and power levels.

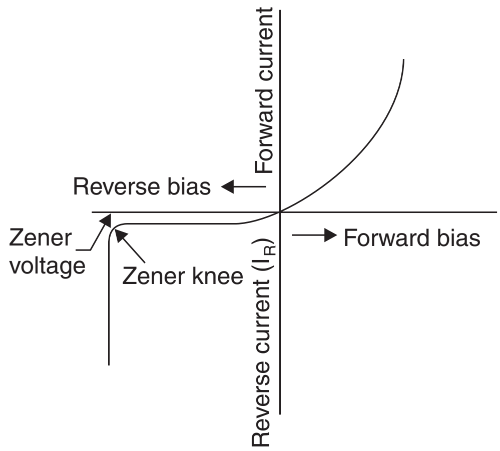


Fig. 18

3.1. Performance/Operation

The electrical performance of a zener diode is based on the *avalanche characteristics* of the *P-N* junction. When a source of voltage is applied to a diode in the *reverse direction* (negative to anode), a reverse current I_R is observed (see Fig. 18). As the reverse potential is increased beyond the "*Zener knee*" avalanche breakdown becomes well developed at zener voltage V_Z. At voltage V_Z, the high counter resistance drops to a low value and the junction current increases rapidly. The current must of necessity be limited by an external resistance, since the voltage V_Z developed across the zener diode remains essentially constant. *Avalanche breakdown of the operating zener diode is not destructive as long as the rated power dissipation of the junction is not exceeded.*

Externally, the zener diode looks much like other silicon rectifying devices, and electrically it is capable of rectifying alternating current.

The following points about the *Zener diode* are worth noting :

(*i*) It looks like an ordinary diode except that it is properly doped so as to have a sharp breakdown voltage.

(*ii*) It is always reverse connected *i.e.,* it is *always reverse biased.*

(*iii*) It has sharp breakdown voltage, called Zener voltage V_Z.

(*iv*) When forward biased, its characteristics are just those of ordinary diode.

(*v*) It is not immediately burnt just because it has entered the breakdown region (The current is limited only by both external resistance and power dissipation of Zener diode).

- The location of Zener region can be controlled by varying the doping levels. An *increase in doping, producing an increase in the number of added impurities, will decrease the Zener potential.*

- Zener diodes are available having Zener potentials of 1.8 to 200 V with power ratings from $\frac{1}{4}$ to 50 W. *Because of its higher temperature and current capability, silicon is usually preferred in the manufacture of Zener diodes.*

3.2. Equivalent Circuit of Zener Diode

The complete equivalent circuit of the Zener diode in the Zener region includes a small dynamic resistance and D.C. battery equal to the Zener potential, as shown in Fig. 19.

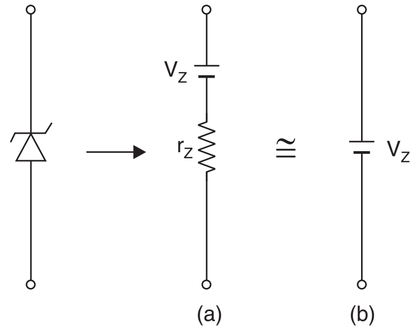


Fig. 19. Zener equivalent circuit : (a) Complete ; (b) Approximately.

"ON" state. When reverse voltage across a Zener diode is equal to or more than breakdown voltage V_Z, the current increases very sharply. In this region curve is almost vertical ; it means that voltage across Zener diode is constant at V_Z even though the current through it changes. Therefore, in the breakdown region, an ideal Zener diode (this assumption is fairly reasonable as the impedance of Zener diode is *quite small in the breakdown region*) can be represented by a battery of voltage V_Z as shown in Fig. 20 (*b*). Under such conditions, the Zener diode is said to be in the "ON" state.

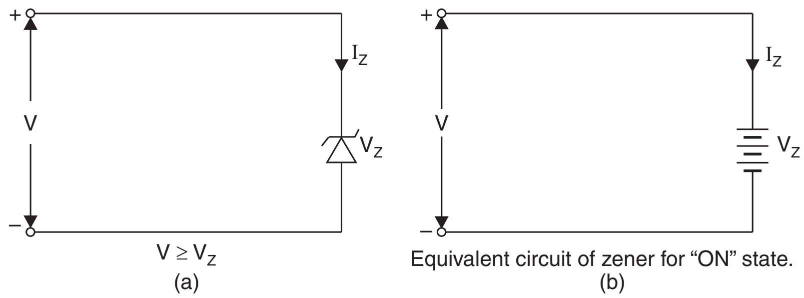


Fig. 20

"OFF" state. When the reverse voltage across the Zener diode is less than V_Z but greater than 0 V, the Zener diode is in the "OFF" stage. Under such conditions, the Zener diode can be represented by an open circuit as shown in Fig. 21 (*b*).

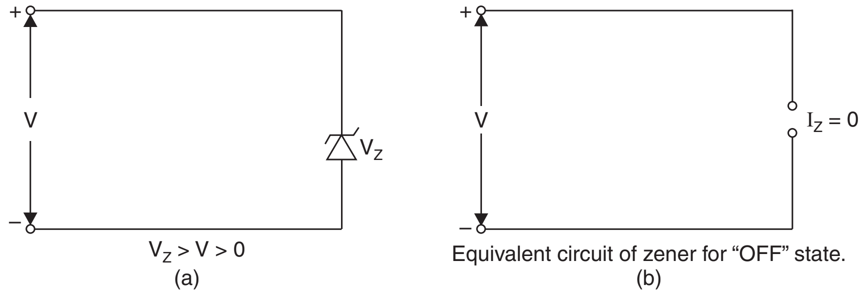


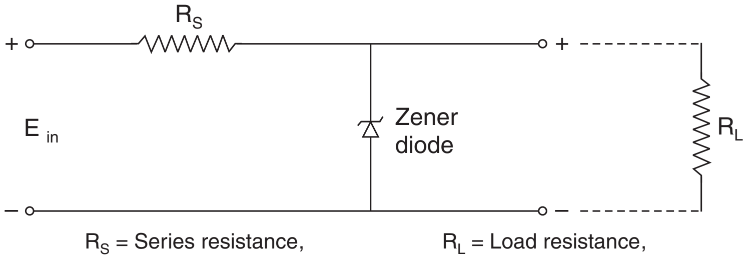
Fig. 21

3.3. Applications of Zener Diode

Zener diode serves in the following variety of *applications :*

1. **Voltage reference or regulator element :**

The primary use of a zener diode is as a *voltage reference or regulator element*. Fig. 22 shows the fundamental circuit for the Zener diode employed as a shunt regulator. In the circuit, diode element and load R_L draw current through the series resistance R_S. If E_{in} increases, the current through the Zener element will increase and thus maintain an essentially fixed voltage across R_L. This ability to maintain the desired voltage is determined by the temperature coefficient and the diode impedance of the zener device.



R_S = Series resistance, R_L = Load resistance

Fig. 22. Basic Zener-diode regulator circuit.

2. **Shunt transistor regulator :**

The Zener diode may also be used to control the reference voltage of a transistor regulated power supply. An example of this in a shunt transistor regulator is shown in Fig. 23, where Zener element is used to control the operating point of the transistor. The advantage of this circuit over that shown in Fig. 22 are *increased power handling capability and a regulating factor improved by utilizing the current gain of the transistor.*

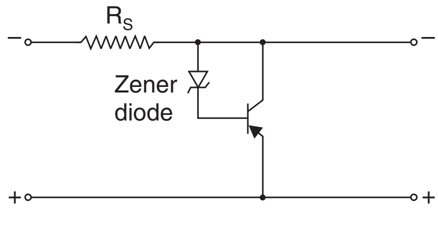


Fig. 23. Shunt transistor regulator.

3. **Audio or r-f applications :**

The Zener diode also finds use in audio or *r-f* (radio frequency) applications where a source of stable reference voltage is required, as in bias supplies. Frequently, *Zener diodes are connected in series package,* with, for example, one junction operating in the reverse within a single direction and possessing a positive temperature V_Z coefficient ; the remaining diodes are connected to operate in the forward direction and exhibit negative temperature V_Z coefficient characteristics. The net result is close neutralization of V_Z drift versus temperature change ; such reference units are frequently used to *replace standard voltage cells.*

4. **Computer circuits :**

Zener diodes also find use in *computer circuits designed for switching about the avalanche voltage of the diode.* Design of the Zener diode permits it to absorb *overload surges and thereby serves the function of protecting delicate circuitry form overvoltage.*

— The usual *voltage specifications* V_Z on Zener diodes are 3.3 to 200 V with ± 1, 2, 5, 10 or 20% tolerances.

— Typical *power dissipation ratings* are 500 mW, 1, 10 and 50 W.

— The temperature coefficient range on V_Z is as low as 0.001% °C.

Example 4. *Determine the current flowing through the Zener diode for the circuit shown in Fig. 24, if* $R_L = 4000\ \Omega$*, input voltage is 50 volts,* $R_S = 1800\ \Omega$ *and output voltage is 32 volts.*

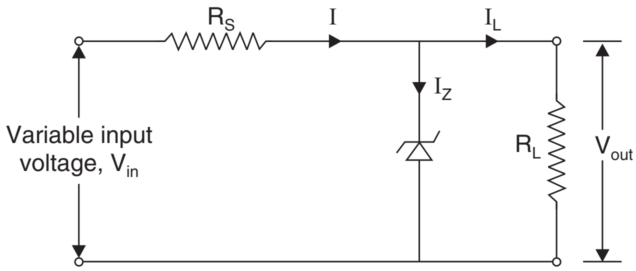


Fig. 24

Solution. Input voltage, $V_{in} = 50$ V

Output voltage, $V_{out} = 32$ V

Voltage drop in series resistor, $R_S = V_{in} - V_{out} = 50 - 32 = 18$ V

Current through series resistance, $I = \dfrac{V_{in} - V_{out}}{R} = \dfrac{18}{1800} = .01$ A or 10 mA

Load current, $I_L = \dfrac{V_{out}}{R_L} = \dfrac{32}{4000} = 0.008$ A or 8 mA

Current through Zener diode, $\mathbf{I_Z} = I - I_L = 10 - 8 =$ **2 mA. (Ans.)**

Example 5. *Determine the maximum and minimum values of Zener current if in the circuit shown in Fig. 24 the load resistance,* $R_L = 4000\ \Omega$*, series resistance = 8000 Ω, output voltage = 32 V and source voltage varies between 100 V and 128 V.*

Solution. Refer to Fig. 23. *Given :* $R_L = 4000\ \Omega$; $R = 8000\ \Omega$; $V_{out} = 32$ V ;

Load current, $I_L = \dfrac{V_{out}}{R_L} = \dfrac{32}{4000} = 0.008$ A or 8 mA

The Zener current will be maximum when input voltage is maximum *i.e.,* 128 V

Corresponding current through series resistance,

$$I = \frac{V_{in(max)} - V_{out}}{R_S} = \frac{128 - 32}{8000} = 0.012 \text{ A or } 12 \text{ mA}$$

Corresponding Zener current, $\mathbf{(I_Z)_{max.}} = I - I_L = 12 - 8 =$ **4 mA. (Ans.)**

The Zener current will be minimum when input voltage is minimum *i.e.,* 100 V.

Corresponding, current through series resistance,

$$I' = \frac{V_{in(min)} - V_{out}}{R_S} = \frac{100 - 32}{8000} = 0.0085 = 8.5 \text{ mA}$$

Corresponding Zener current, $(\mathbf{I_Z})_{\mathbf{min.}} = I' - I_L = 8.5 - 8 = \mathbf{0.5\ mA.}$ **(Ans.)**

Example 6. *In the simple Zener-diode based voltage regulator shown in Fig. 25, a 5.6 V, 0.25 W Zener diode is used. For reliable operation, the minimum* I_Z *should be 1 mA. The load* R_L *varies between 20 Ω and 50 Ω. Find the range of* R_S *for reliable and safe operation of the voltage regulator.*

Solution. (*i*) Let $R_S = 20\ \Omega$

$$I = \frac{5.6}{20} = 0.28 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.28 + 0.001} = 15.66\ \Omega \approx 16\ \Omega.$$

(*ii*) Let, $R_S = 50\ \Omega$

$$I = \frac{5.6}{50} = 0.112 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.112 + 0.001} = 38.93\ \Omega \approx 39\ \Omega.$$

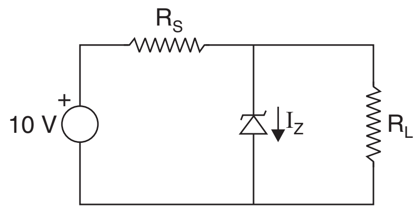


Fig. 25

∴ **R ranges from 16 Ω to 39 Ω. (Ans.)**

4. BIPOLAR JUNCTION TRANSISTOR (BJT)

4.1. Introduction

A transistor may be *defined* as follows :

— The word *transistor* was derived from the two word combination, *transfer-resistance* (Transfer + resistor → Transistor). A **transistor** *is a device to transfer a low resistance into a circuit having a high resistance.*

— A '*transistor*' is *a semiconductor device in which current flows in semiconductor materials.*

— When a thin layer of *P*-type or *N*-type semiconductor is between a pair of opposite types it constitutes a transistor.

- The **transistor** *is a solid state device, whose operation depends upon the flow of electric charge carriers within the solid.*

A **transistor** *is a semiconductor device having both rectifying and amplifying properties.*

"*The main difference between a vacuum triode and a transistor is that while a vacuum triode is a voltage controlled device, a transistor is a current controlled device*".

The transistor was invented by a team of three scientists at Bell Laboratories, USA in 1947. Although the first transistor was not a bipolar junction device, yet it was the beginning of a technological revolution that is still continuing in the twenty first century. All of the complex electronic devices and systems developed or in use today, are an outgrowth of early developments in semiconductor transistors.

The two basic types of transistors are :

1. Bipolar junction transistor (BJT)
2. Field-effect transistor (FET)

The *bipolar junction transistor is used in the following two broad area of electronics :*

(*i*) *As a* **linear amplifier** *to boost an electric signal.*

(*ii*) As an **electronic switch.**

4.2. Advantages and Disadvantages of Transistors Over Electron Tubes

Advantages :

1. A transistor works at quite a low voltage. It means no high tension is required.

2. Operating life of a transistor is practically unlimited provided it is not subjected to high temperatures, beyond permissible limits. While the life of an electron tube/thermionic valve is always limited to a few years.

3. They operate more *efficiently* than the electron tubes.

4. They can sustain mechanical shocks also, as they are solid crystals.

5. *Very compact electron devices* can be prepared by employing transistors instead of tubes because their size and weight are very small.

6. They can be made to oscillate with a very small power consumption of the order of a few microwatt only.

Disadvantages :

The transistors have the following *disadvantages* as compared to an electron tubes :

1. A transistor cannot withstand large temperature changes as its characteristics are very sensitive to temperature variations.

It cannot be used above 75°C. This limits the use of transistors to a great extent.

2. Its application is limited upto a few mega-cycles only.

3. Noise or *hum* is lower in case of transistors than electron tubes under similar conditions.

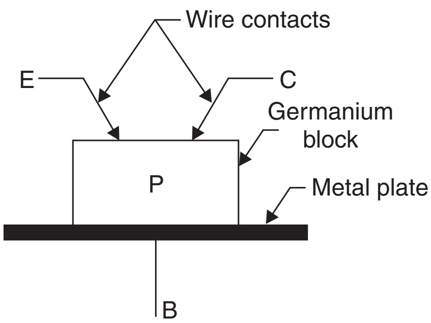
4.3. Transistor Types

Transistors are of the following two types :

1. Point-contact transistors
2. Junction transistors.

4.3.1. Point-contact transistor

Refer to Fig. 26. A point-contact transistor has a block of *P*-type germanium mounted on a metal plate with two wire contacts attached to the opposite side of the crystal. The two wire contacts are very close together ; one is referred to as the *emitter* (*E*) ; the other is *collector* (*C*) ; and *P*-type germanium block is termed the base (*B*). *The emitter and the base form a point-contact rectifier having a forward direction from the wire contact toward the plate contact.*



E = emitter, *C* = collector, *B* = base

Fig. 26. Basic construction of a point-contact transistor.

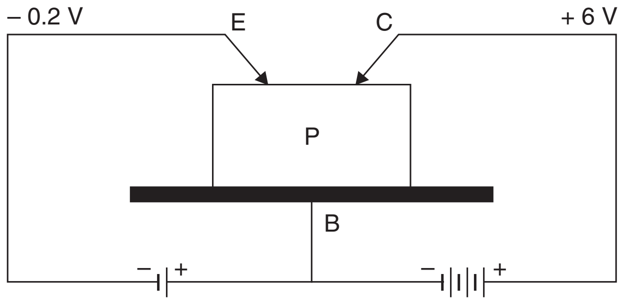


Fig. 27. A basic point-contact transistor.

Fig. 27 shows the emitter being acted on by a negative e.m.f. of – 0.2 V causing an electron flow of 0.008 A. In the same manner, the collector and the base combination are acted on by an e.m.f. in the reverse direction. Normally the positive 6 volts produce only 0.00024 A of electron flow to the collector due to the small number of free electrons within the *P*-type crystals. However, with the emitter injecting a great number of electrons into the crystal very near the collector, the flow of electrons to the collector becomes *at least double that to the emitter.*

It is important to know that *the emitter injects electrons* (*or holes*) *into the crystal to effect a reduction of the reverse resistance between the base and the collector. When the base of the point-contact transistor is of the N-type germanium material, the polarities of electromotive forces will be reversed and the emitter will literally inject holes.*

4.3.2. Junction transistor

A *trade transistor* consists of two *P-N* junction diodes placed back to back. The following are the two common types of *junction transistors :*

1. Grown junction type
2. Alloy-junction type.

1. Grown junction transistor :

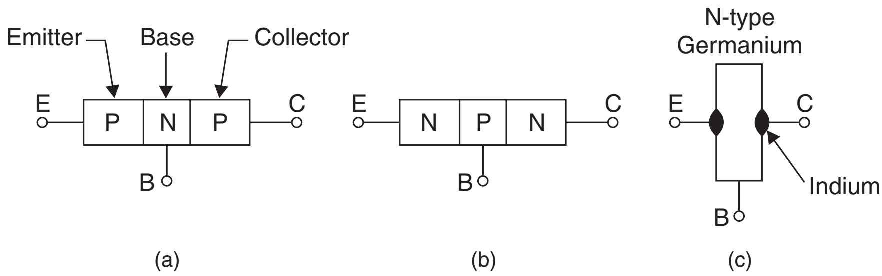


Fig. 28

Fig. 28 (*a*) shows a *grown P-N-P Junction triode transistor.*

Fig. 28 (*b*) shows the form of *N-P-N* junction transistor.

In the manufacture of *grown junction transistors the single crystal growing process is employed.*

— The *left* hand section or region is called the *emitter.*

— The *right* hand section or region is called the *collector.*

— The *middle* section is called the *base region or base.* It is *extremely thin* as compared to either the emitter or collector and is *lightly doped.*

Function of the emitter is to inject majority charge carriers into the base and that of the collector is to collect or attract these carriers through the base.

2. Alloy-junction transistor :

Fig. 28 (*c*) shows an alloy-junction transistor which consists of two leads of indium metal alloyed on the opposite sides of a thin slice of an *N*-type germanium. Collector is larger in size than the emitter. These transistors may be also of *P-N-P* type or *N-P-N* type.

Junction Transistors—Illustration of Emitter, Base and Collector :

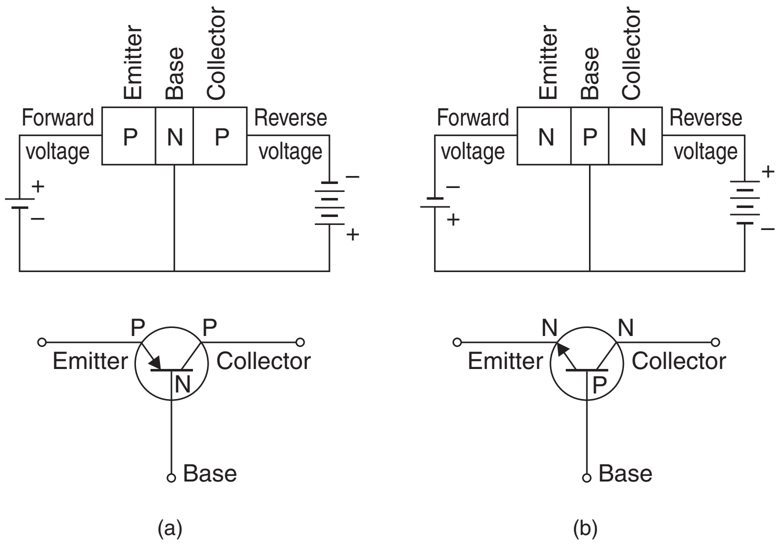


Fig. 29. Junction transistors illustrating emitter, base and collector.

As shown in Fig. 29, the transistor consists of a *P-N* junction and an *N-P* junction, by making either *P* or *N* semiconductor between opposite types. The purpose is to have the first section supply charges, either holes or electrons, to be collected by the third section, through the middle section. The electrode that supplies charges is *emitter* ; the electrode at the opposite end to collect the charges is the *collector*. The *base* in the middle forms two junctions between emitter and collector, *to control the collector current.*

Emitter :

- It is more heavily doped than any of other regions because its main function is to supply majority charge carriers (either electrons or holes) to the base.
- The emitter-base junction is biased with forward voltage. (Typical values : Ge = 0.2 V, Si = 0.6 V).

— As shown for the *P-N-P* transistor in Fig. 29 (*a*), the *P* emitter supplies *hole charges* to its junction with the base. This direction is indicated by the emitter arrow for forward hole current in the schematic symbol. The *arrow pointed into the base* shows a *P-N* junction between emitter and base, corresponding to the symbol for a *P-N* diode.

— For the *N-P-N* transistor in Fig. 29 (*b*), the emitter supplies electrons to the base.

Therefore, the symbol for the *N* emitter shows the *arrow out from the base*, opposite to the direction of electron flow.

In the schemic symbols, only the emitter has an arrow. The *arrow pointing into the base means a P-N-P transistor, the arrow out from the base means an N-P-N transistor.*

Practically all small transistors for audio and r.f. amplifiers are N-P-N, made of silicon, with a typical forward bias of 0.6 V between base and emitter collector.

The *function of a collector is to remove charges from the junction with the base.* In Fig. 29 (*a*), the *P-N-P* transistor has a *P* collector receiving *hole charges.* For the N-P-N transistor in Fig. 29 (*b*), the *N* collector receives *electrons.* The *collector base junction* always has *reverse voltage.* Typical values are 4 to 100 V. This polarity means *no majority charges can flow from collector to base.* However, in the opposite direction, from base to collector, the collector voltage attracts the charges in the base supplied by the emitter.

Base :

- The *base* in the middle *separates* the emitter and collector. *The base-emitter junction is forward based.* As a result, the resistance is *very low for the emitter circuit.* The *base-collector junction is reverse biased,* providing a much higher resistance in the collector circuit.
- It is *very lightly doped.*
- It is *very thin* (10^{-6} m) as compared to either emitter or collector.

Collector current :

The final requirement for transistor action is to have the *collector current controlled by the emitter-base circuit.* The *emitter has heavy doping* to *supply majority charges. However, the base has only light doping and is very thin, so that its charges can move to the collector junction.* The *collector voltage is relatively high.* Because of these factors, practically all the charges supplied by the emitter to the base are made to flow in the collector circuit. Typically, *98 to 99% or more of the emitter charges provide collector current* (I_C). The remaining *1 to 2% or less* becomes base current (I_B).

- *The key to the transistor action is the lightly doped thin base between the heavily doped emitter and moderately doped collector.*
- A transistor has two junctions (emitter-base and collector-base junctions), and each of these two junctions, may be forward biased or reverse biased, therefore, there are *four possible ways of biasing these two junctions*. Accordingly it may operate in different conditions as listed below :

Condition	Emitter-Base (EB) Junction	Collector-Base (CB) Junction	Region of Operation
1. Forward-Reverse (FR)	Forward-Biased	Reverse-Biased	Active
2. Forward-Forward (FF)	Forward-Biased	Forward-Biased	Saturation
3. Reverse-Reverse (RR)	Reverse-Biased	Reverse-Biased	Cut-off
4. Reverse-Forward (RF)	Reverse-Biased	Forward-Biased	Inverted

4.3.3. P-N-P and N-P-N transistors

To understand the basic mechanism of transistor operation the following facts need to be kept in mind :

1. Since *emitter is to provide charge carriers, it is always "forward biased".*

2. *First letter of transistor type indicates the polarity of the emitter voltage with respect to base.*

3. Collector's job is *to collect or attract those carriers through the base, hence it is always "reverse-biased".*

4. *Second letter* of transistor type *indicates the polarity of collector voltage with respect to the base.*

The above points apply both to *P-N-P* and *N-P-N* transistors.

4.3.4. Working of P-N-P transistor

Fig. 30 shows a **P-N-P transistor** connected in the common-base (or grounded-base) configuration (it is so called because both the emitter and collector are returned to the base terminals). The *emitter junction is forward-biased whereas the collector junction is reverse-biased.* The holes in the emitter are repelled by the positive battery terminal towards the *P-N* or emitter junction. The potential barrier at the junction is reduced due to the forward-bias, hence holes cross the junction and enter the *N*-type base. Because the base is thin and lightly-doped, majority of the holes (about 95%) are able to drift across the base without meeting electrons to combine with. The balance of 5% of holes are lost in the base region due to recombination with electrons. The holes which after crossing the *N-P* collector junction enter the collector region are swept up by the negative collector voltage V_C.

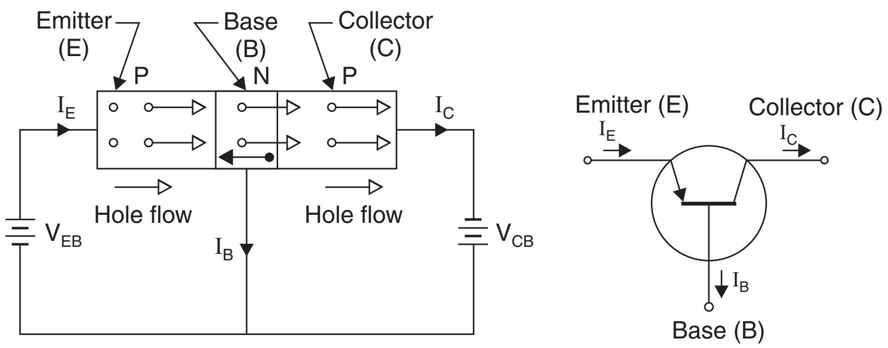


Fig. 30. *P-N-P* transistor.

The following points are worth noting :

1. In a *P-N-P* transistor *majority charge carriers are holes.*

2. *The collector current is always less than the emitter current because some recombination of holes and electrons take place.*

$$(I_C = I_E - I_B).$$

3. The *current amplification* (α) (or gain of *P-N-P* transistor) for steady conditions when connected in common base configuration is expressed as :

$$\alpha = \frac{I_C \text{ (collector current)}}{I_E \text{ (emitter current)}} < 1.$$

4. *Emitter arrow shows the direction of flow of conventional current.* Evidently, electron flow will be in the opposite direction.

4.3.5. Working of N-P-N transistor

Fig. 31 shows a **N-P-N junction transistor.** The emitter is forward-biased and the collector is reverse-biased. The electrons in the emitter region are repelled by the negative battery terminal towards the emitter or *N-P* junction. The *electrons cross over into the P-type base region because potential barrier is reduced due to forward bias.* Since the base is thin and lightly doped, most of the electrons (about 95%) cross over to the collector junction and enter the collector region where they are readily swept up by the positive collector voltage V_C. Only about 5% of the emitter electrons combine with the holes in the base and are lost as charge carriers.

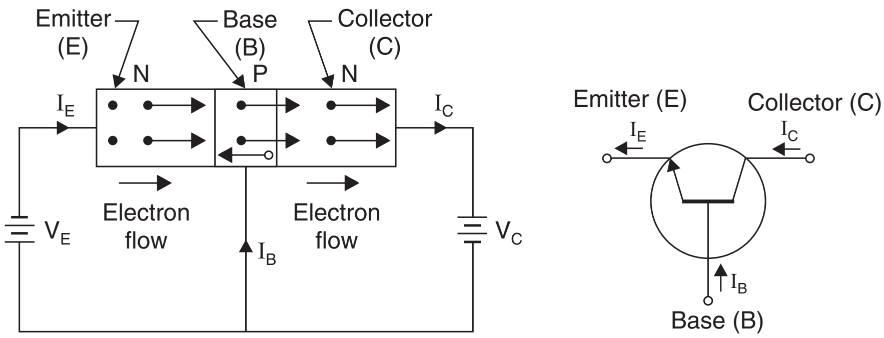


Fig. 31. *N-P-N* transistor.

The following points are worth noting :

1. In a *N-P-N* transistor, *majority charge carriers are electrons.*
2. I_C *(collector current) is less than* I_C *(emitter current) so that* $\alpha < I$.
3. *Emitter arrow shows the direction of flow of conventional current.*

- The choice of *N-P-N* transistor is *made more often because majority charge carriers are electrons whose mobility is much more than that of holes.*

Note. *The junction transistors have been made in power ranges from a few milliwatts to tens of watts. The tiny junction transistor is unparalleled in that it can be made to work at power levels as I microwatt.*

4.3.6. Transistor circuit configurations

A transistor is a three-terminal device (having three terminals namely *emitter, base* and *collector*) but we require four terminals-two for the input and two for the output for connecting it in a circuit. Hence one of the terminals of the transistor is made common to the input and output circuits. Thus there are three types of configurations for operation of a transistor. These configurations are :

(*i*) Common-base (CB) configuration.

(*ii*) Common-emitter (CE) configuration.

(*iii*) Common-collector (CC) configuration.

The term 'common' is used to denote the electrode that is common to the input and output circuits. Because the common electrode is generally grounded, these modes of operation are frequently referred to as ground-base, ground-emitter and grounded-collector configurations as shown in Fig. 32 for a *N-P-N* transistor.

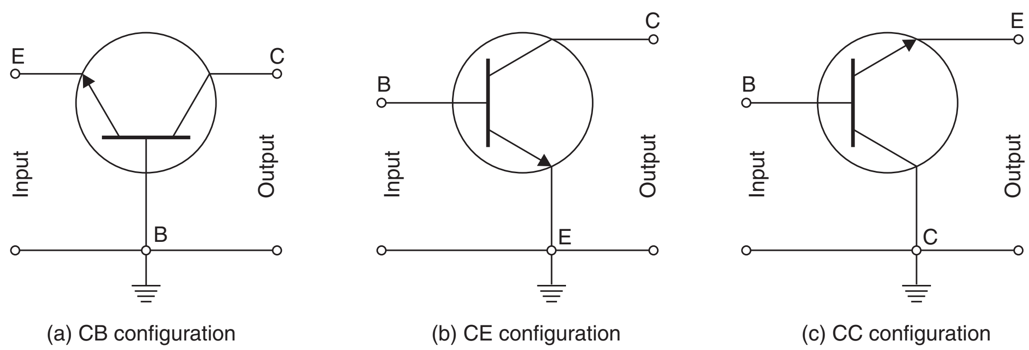


Fig. 32. Different circuit configurations for *N-P-N* transistor.

Each circuit configuration has specific advantages and disadvantages. It may be noted here that regardless of circuit connection, the *emitter is always biased in the forward direction, while the collector always has a reverse bias*.

4.3.7. Common-base (CB) configuration

In this circuit configuration, input is applied between emitter and base and output is taken from collector and base. Here, base of the transistor is common to both input and output circuits and hence the name common base configuration. A common-base configuration for *N-P-N* transistor is shown in Fig. 33.

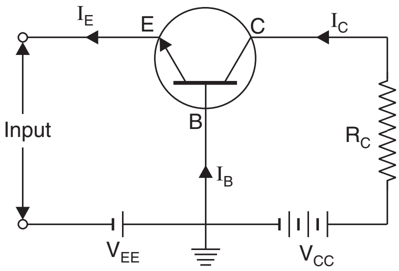


Fig. 33. Common-base *N-P-N* transistor.

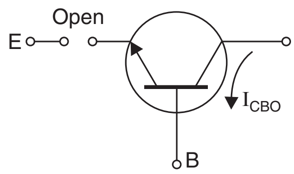


Fig. 34

Current amplification factor (α). *It is the ratio of output current to input current.* In CB configuration, the input current is the emitter current I_E and output current is the collector current I_C.

The ratio of change in collector current to the change in emitter current at constant collector-base voltage V_{CB} *is known as* ***current amplification factor*** *i.e.,*

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \text{ at constant } V_{CB} \quad ...(5)$$

If *only D.C. values are considered,* then $\alpha = \frac{I_C}{I_E}$ ...(6)

α is *less than unity.* This value can be increased (not more than unity) by decreasing the base current. This is accomplished by *making the base thin and doping it lightly.*

In commercial transistors, practical value of α varies from 0.9 to 0.99.

Collector current (I_C) :

Total collector current, $I_C = \alpha I_E + I_{leakage}$

(αI_E is the part of emitter current that reaches the collector terminal)

where, I_E = Emitter current, and

$I_{leakage}$ = Leakage current (This current is due to movement of minority carriers across base-collector junction on account of it being reversed ; it is *much smaller* than αI_E)

When emitter is open (Fig. 34) $I_E = 0$, but a small leakage current still flows in the collector circuit. This $I_{leakage}$ is abbreviated as I_{CBO}, meaning collector-base current with emitter open.

$$I_C = \alpha I_E + I_{CBO} \quad ...(7)$$

$$\therefore \quad I_C = \alpha(I_C + I_B) + I_{CBO} \quad (\because \; I_E = I_C + I_B)$$

or $$I_C(1 - \alpha) = \alpha I_B + I_{CBO}$$

or $$I_C = \left(\frac{\alpha}{1-\alpha}\right) I_B + \frac{I_{CBO}}{(1-\alpha)} \quad ...(8)$$

— In view of improved construction techniques, the magnitude of I_{CBO} for general-purpose and low-powered transistors (especially silicon transistors) is usually very small and may be neglected in calculations.

— For high power calculations, I_{CBO} appears in µA range.

— I_{CBO} is temperature dependent, therefore, at high temperature it must be considered in calculations.

Example 7. *In a common-base configuration, current amplification factor is 0.92. If the emitter current is 1.2 mA, determine the value of base current.*

Solution. *Given :* $\alpha = 0.94$; $I_E = 1.2$ mA

We know that, $\alpha = \frac{I_C}{I_E}$

or $I_C = \alpha I_E = 0.92 \times 1.2 = 1.1$ mA

Also, $I_E = I_C + I_B$

∴ $\mathbf{I_B} = I_E - I_C$

$= 1.2 - 1.1 =$ **0.1 mA. (Ans.)**

Example 8. *In a common-base configuration, the emitter current is 0.9 mA. If the emitter circuit is open, the collector current is 45 µA. Find the total collector current. Given that α = 0.9.*

Solution. *Given :* $I_E = 0.9$ mA ; $I_{CBO} = 45$ µA $= 45 \times 10^{-3}$ mA ; $\alpha = 0.9$.

Collector current, $I_C = \alpha I_E + I_{CBO}$ ...[Eqn. (7)]

$= 0.9 \times 0.9 + 45 \times 10^{-3} =$ **0.855 mA. (Ans.)**

Example 9. *In a CB configuration, α = 0.92. The voltage drop across 2.5 kΩ resistance which is connected in the collector is 2.5 V. Find the base current.*

Solution. *Given.* The common-base configuration of the transistor is shown in Fig. 35.

The voltage drop across R_C (= 2.5 kΩ)

= 2.5 V ...(Given)

∴ $I_C = \frac{2.5 \text{ V}}{2.5 \text{ k}\Omega} = 1$ mA

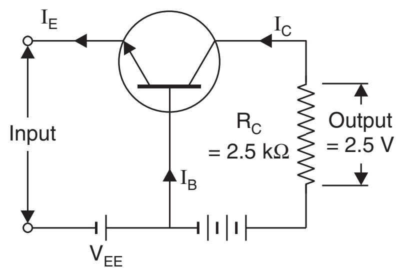


Fig. 35

Now, $\alpha = \frac{I_C}{I_E}$

∴ $I_E = \frac{I_C}{\alpha} = \frac{1}{0.92} = 1.087$ mA

Also, $I_E = I_C + I_B$

∴ $\mathbf{I_B} = I_E - I_C = 1.087 - 1 =$ **0.087 mA. (Ans.)**

Characteristics of Common-base transistor

Curves representing the variation of current with voltage in a transistor triode circuit are called **transistor characteristic curves.** There are the following two types of characteristic curves :

1. **Input characteristic curves** of I_E *versus emitter-base voltage* (V_{EB}).

2. **Output characteristic curves** of *collector current* (I_C) *versus collector-base voltage* (V_{CB}).

Fig. 36 shows the circuit of an *N-P-N* junction triode (common-base) studying characteristic curve.

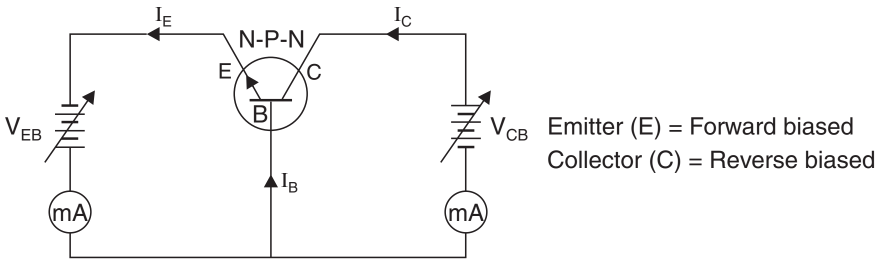


Fig. 36. Circuit of an *N-P-N* junction triode.

1. **Input characteristic curves :**

— To plot these curves the collector voltage is first put at zero potential (say), *i.e.*, $V_{CB} = 0$.

— The emitter-base voltage (V_{EB}) is now increased from zero onwards and emitter current (I_E) is recorded.

— A graph is plotted between I_E and V_{EB} as shown in Fig. 37.

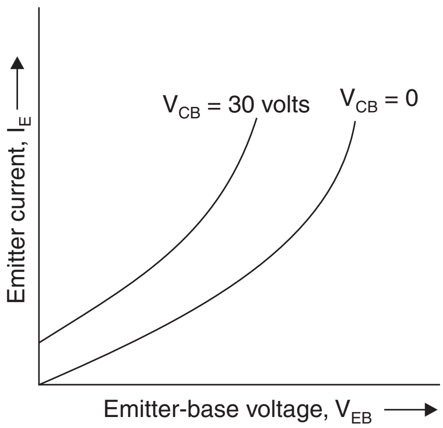


Fig. 37. Input characteristic curves.

— Another similar graph is plotted for V_{CB} = 30 volts (say).

From the graph we observe that :

(*i*) For a given collector voltage, the emitter current rises rapidly even with a very small increase in emitter potential. It means that *the input resistance* $R_i \left(= \dfrac{\Delta V_{EB}}{\Delta I_E} \text{ at constant } V_{CB} \right)$ *of the emitter-base circuit is very low.*

(*ii*) The emitter current is nearly *independent* of collector-base voltage.

2. **The output characteristic curves :**

These curves are obtained by plotting the variation of collector current (I_C) with collector-base voltage (V_{CB}) at different constant values of emitter current (I_E).

— These *curves* shown in Fig. 38, indicate that some collector current is present even when the collector voltage is zero. To make the collector current zero, we have to give a certain amount of *negative potential* to the collector.

— The *curves* also indicate that the collector current attains a high value even at a very low collector voltage and further increase in collector voltage does not produce any appreciable increase in collector current. It means that the *output resistance* $\left(R_o = \dfrac{\Delta V_{CB}}{\Delta I_C} \text{ at constant } I_E\right)$ *of the collector-base circuit is very large.*

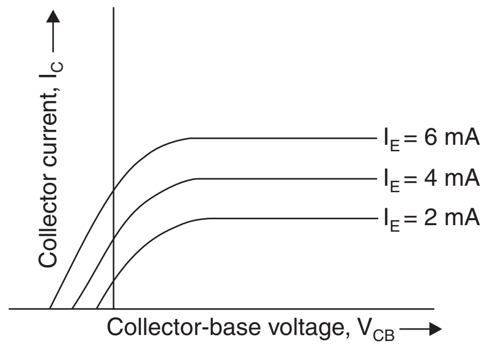


Fig. 38. Output characteristic curves.

The *collector current is always a little less than the emitter current* because of the neutralisation of a few holes and electrons within the base due to recombination.

3. **Feed back characteristic curves :**

These curves represent the variation of *collector current* (I_C) with *emitter-base voltage* (V_{EB}) for a constant-emitter current. A number of emitter current values are selected at which measurements are made. The nature of curves is shown in Fig. 39.

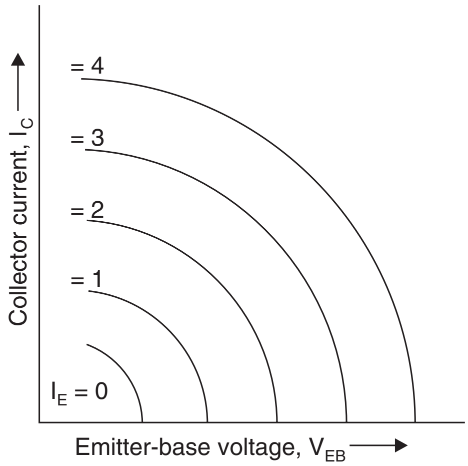


Fig. 39. Feed back characteristic.

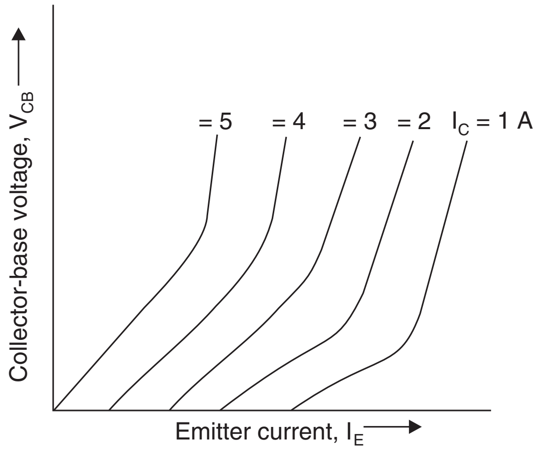


Fig. 40. Forward characteristic.

4. **Forward characteristic curves :**

Refer to Fig. 40. This type of curve is a graph between *emitter current* (I_E) and *collector-base voltage* at constant value of collector current.

4.3.8. Common-emitter (CE) configuration

In CE configuration, input is applied between base and emitter and output is taken from the collector and emitter. Here, emitter of the transistor is common to both input and output circuits and hence the name common-emitter configuration. Fig. 41 shows common-emitter *N-P-N* transistor circuit :

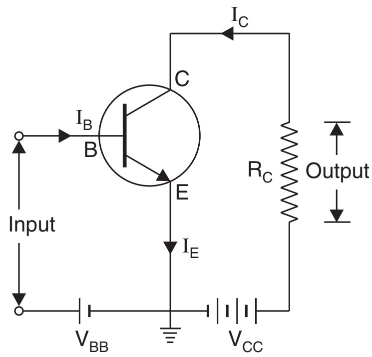


Fig. 41. Common emitter *N-P-N* transistor.

Base current amplification factor (β). In CE configuration, input current is I_B and output current is I_C. *The ratio of change in collector current (ΔI_C) to the change in base current (ΔI_B) is known as "base current amplification factor" i.e.,*

$$\beta = \frac{\Delta I_C}{\Delta I_B} \qquad ...(9)$$

If D.C. values are considered, $\beta = \dfrac{I_C}{I_B}$ ...[9 (*a*)]

In almost every transistor 5% of emitter current flows as the base current. Therefore the value of β is generally greater than 20, β usually varies from 20 to 500.

- *CE configuration is frequently used as it gives appreciable current gain as well as voltage gain.*

Relation between β and α. The relation between β and α is derived as follows :

$$\beta = \frac{\Delta I_C}{\Delta I_B} \qquad ...(i)$$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \qquad ...(ii)$$

Now, $I_E = I_B + I_C$

or $\Delta I_E = \Delta I_B + \Delta I_C$ or $\Delta I_B = \Delta I_E - \Delta I_C$

Inserting the value of ΔI_B in (*i*), we get

$$\beta = \frac{\Delta I_C}{\Delta I_E - \Delta I_C} \qquad ...(iii)$$

Dividing the numerator and denominator of R.H.S. by ΔI_E, we get

$$\beta = \frac{\Delta I_C/\Delta I_E}{(\Delta I_E/\Delta I_E) - (\Delta I_C/\Delta I_E)} = \frac{\alpha}{1-\alpha} \qquad \left(\because \ \alpha = \frac{\Delta I_C}{\Delta I_E}\right)$$

$$\therefore \quad \beta = \frac{\alpha}{1-\alpha} \qquad ...(10)$$

It is evident from the above expression that when α approaches unity, β approaches infinity. In other words the *current gain in CE configuration is very high.* It is due to *this reason that this circuit arrangement is used is about 90 to 95 percent of all transistor applications.*

Collector current. In *CE* configuration, I_B is the input current and I_C is the output current :

Now, $I_E = I_B + I_C$ ...(*i*)

and $I_C = \alpha I_E + I_{CBO}$ ...(*ii*)

or $I_C = \alpha(I_B + I_C) + I_{CBO}$

or $I_C(1 - \alpha) = \alpha I_B + I_{CBO}$

or $$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO} \qquad ...(iii)$$

It is evident from (*iii*) that if $I_B = 0$ (*i.e.,* base circuit is open), the collector current will be the current to the emitter. This is abbreviated as I_{CEO} meaning collector-emitter current with base open.

Inserting the value of $\frac{1}{1-\alpha} I_{CBO} = I_{CEO}$ in (*iii*), we get

$$I_C = \frac{\alpha}{1-\alpha} I_B + I_{CEO}$$

or $$I_C = \beta I_B + I_{CEO} \quad ...(11) \quad \left(\because \beta = \frac{\alpha}{1-\alpha}\right)$$

It may be noted that, $I_{CEO} = (\beta + 1) I_{CBO}$ ...(12)

Example 10. *Find the* α *rating of the transistor shown in Fig. 42. Hence determine the value of* I_C *using both* α *and* β.

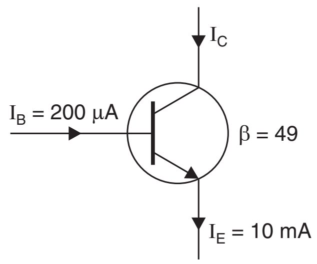


Fig. 42

Solution. Refer to Fig. 42.

$$\beta = \frac{\alpha}{1-\alpha} \quad ...[\text{Eqn. } (10)]$$

or $$\beta(1 - \alpha) = \alpha$$

or $$\beta - \alpha\beta = \alpha$$

or $$\beta = \alpha(1 + \beta)$$

$\therefore$ $$\alpha = \frac{\beta}{1+\beta} = \frac{49}{1+49} = 0.98$$

$\therefore$ $$I_C = \alpha I_E = 0.98 \times 10 \text{ mA} = \mathbf{9.8\ mA.} \quad \textbf{(Ans.)}$$

Also $$I_C = \beta I_B = 49 \times 200\ \mu\text{A} = 49 \times 0.2 \text{ mA} = \mathbf{9.8\ mA.} \quad \textbf{(Ans.)}$$

Example 11. *A transistor is connected in common-emitter (CE) configuration in which collector supply is 10 V and voltage drop across resistance* R_C *connected in the collector circuit is 0.6 V.*

The value of $R_C = 600\ \Omega$. *If* $\alpha = 0.95$, *determine :*

(*i*) *Collector-emitter voltage.*

(*ii*) *Base current.*

Solution. *Given :* $V_{CC} = 10$ V ; $R_C = 600\ \Omega$; $\alpha = 0.95$.

The required *CE* configuration with various values is shown in Fig. 43.

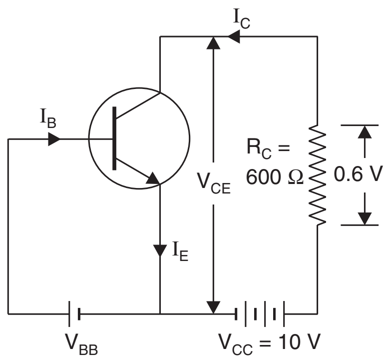


Fig. 43

(*i*) **Collector-emitter voltage V_{CE} :**

$$V_{CE} = V_{CC} - 0.6 = 10 - 0.6 = \mathbf{9.4\ V.} \quad \textbf{(Ans.)}$$

(*ii*) **Base current I_B :**

$$I_C = \frac{0.6 \text{ V}}{600\ \Omega} = 1 \text{ mA}$$

Now, $$\beta = \frac{\alpha}{1-\alpha} = \frac{0.95}{1-0.95} = 19$$

$\therefore$ Base current, $$I_B = \frac{I_C}{\beta} = \frac{1}{19} = \mathbf{0.0526\ mA.} \quad \textbf{(Ans.)} \quad \left(\because \beta = \frac{I_C}{I_B}\right)$$

Characteristics of common-emitter transistor :

Fig. 44 shows the circuit of a *N-P-N* common-emitter junction transistor for the study of characteristic curves.

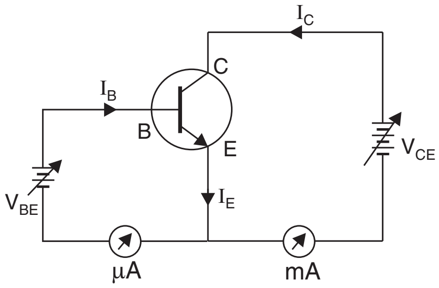


Fig. 44. Circuit of *N-P-N* common emitter junction transistor.

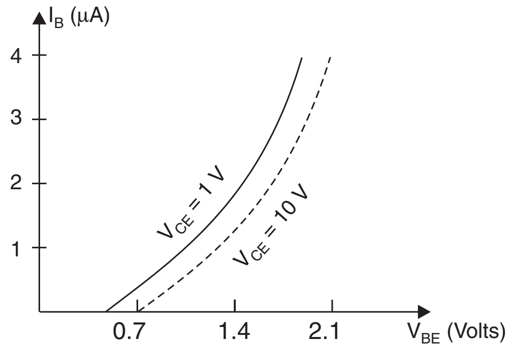


Fig. 45

1. **Input characteristic curves.** It is the curve between base current I_B and the base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE} (Refer to Fig. 45).

Input resistance, $R_i = \frac{\Delta V_{BE}}{\Delta I_B}$ at constant V_{CE}. Its value is of the order of a few hundred ohms.

- Fig. 46 shows the graph of collector current (I_C) with base current (I_B) at constant collector-emitter voltage. It may be noted from the curve that there is a collector current even when the basic current is zero. This is known as *collector leakage current* :

It increases with rise in temperature and also arises due to the reverse biasing between base and collector. The value of leakage current ranges from 100 μA to 500 μA.

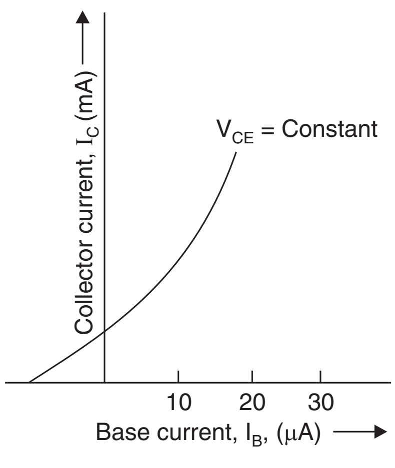


Fig. 46

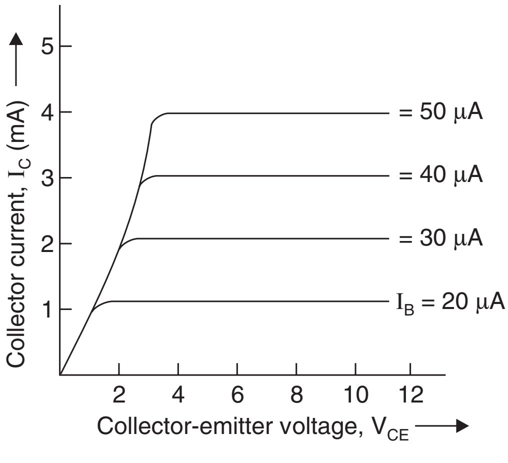


Fig. 47

2. **Output characteristic curves.** The collector-emitter voltage (V_{CE}) is varied and the corresponding collector current (I_C) is noted for various fixed values of base current (I_B).

The shape of the curves is shown in Fig. 47.

Such common-emitter characteristics are widely used for design purpose.

It may be noted $I_C >> I_B$

Output resistance $R_o = \frac{\Delta V_{CE}}{\Delta I_C}$ at constant I_B. Its value is of the order of 50 kΩ (less than that of *CB* circuit).

4.3.9. Common-collector (CC) configuration

In this type of configuration, input is applied between base and collector while output is taken between the emitter and collector. Here, collector of the transistor is common to both input and output circuits and hence the name common-collector connection. Fig. 48 shows the common-collector *N-P-N* transistor.

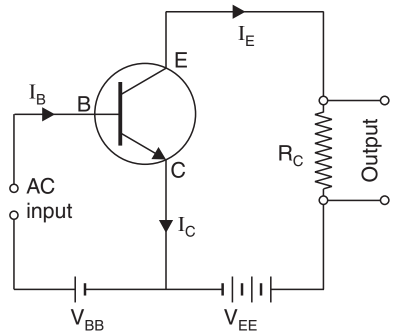


Fig. 48. Common-collector *N-P-N* transistor.

Current amplification factor γ. In *CC* configuration, the input current is the base current I_B and output current is the emitter current I_E. *The ratio of change in emitter current* (ΔI_E) *to the change in base current* (ΔI_B) *is known as "current amplification factor" i.e.,*

$$\gamma = \frac{\Delta I_E}{\Delta I_B}$$

This circuit provides the same gain as the common-emitter configuration as $\Delta I_E \simeq \Delta I_C$. However, *its voltage gain is always less than one.*

Relation between γ and α.

$$\gamma = \frac{\Delta I_E}{\Delta I_B} \quad ...(i)$$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad ...(ii)$$

Now, $I_E = I_B + I_C$

or $\Delta I_E = \Delta I_B + \Delta I_C$ or $\Delta I_B = \Delta I_E - \Delta I_C$

Inserting the value of ΔI_B in (*i*), we get

$$\gamma = \frac{\Delta I_E}{\Delta I_E - \Delta I_C}$$

Dividing the numerator and denominator of R.H.S. by ΔI_E, we get

$$\gamma = \frac{\Delta I_E/\Delta I_E}{(\Delta I_E/\Delta I_E) - (\Delta I_C/\Delta I_E)} = \frac{1}{1-\alpha} \quad \left(\because \ \alpha = \frac{\Delta I_C}{\Delta I_E}\right)$$

$$\therefore \quad \gamma = \frac{1}{1-\alpha} \quad ...(13)$$

Collector current :

We know that, $I_C = \alpha I_E + I_{CBO}$ ...[Eqn. (7)]

Also, $I_E = I_B + I_C = I_B + (\alpha I_E + I_{CBO})$

or $I_E(1 - \alpha) = I_B + I_{CBO}$

or $$I_E = \frac{I_B}{1-\alpha} + \frac{I_{CBO}}{1-\alpha}$$

or $$I_C, I_E = (\beta + 1)I_B + (\beta + 1)I_{CBO} \qquad ...(14)$$

$$\left[\beta = \frac{\alpha}{1-\alpha} \quad \therefore \quad \beta + 1 = \frac{\alpha}{1-\alpha} + 1 = \frac{1}{1-\alpha}\right]$$

Commonly used transistor connection :

Out of the three configuration, the **CE configuration** *is the most efficient.* It is used in about 90 to 95% of all transistor applications. This is due to following *reasons :*

1. *High current gain ;* it may range from 20 to 500.
2. *High voltage and power gain.*
3. *Moderate output to input impedance ratio* (this ratio is small, to the tune of 50). This *makes this configuration an ideal one for coupling between various transistor stages.*

4.3.10. Power rating of transistor

The maximum power that a transformer can handle without deterioration is known as **power rating** of the *transistor*.

When a transistor is in operation, almost all power is dissipated at *reverse biased *collector-base junction.*

The power rating or maximum power dissipation is given by,

$$P_D = \text{Collector current} \times \text{Collector-base voltage}$$
$$= I_C \times V_{CB}$$

Now, $V_{CE} = V_{CB} + V_{BE}$,

Since V_{BE} is very small, $V_{CB} \simeq V_{CE}$

$\therefore$ $$P_D = I_C \times V_{CE} \qquad ...(15)$$

- While connecting a transistor in the circuit it must be ensured that its power rating is not exceeded otherwise it may get destroyed due to overheating.

5. RECTIFIERS

A **rectifier** is *a circuit, which uses one or more diodes to convert A.C. voltage into pulsating D.C. voltage.*

A rectifier may be broadly categorized in the following *two types* :

1. Half-wave rectifier, and
2. Full-wave rectifier.

5.1. Half-wave Rectifier

Fig. 49 (*a*) shows a half-wave rectifier circuit. It consists of a single diode in series with a load resistor. A *P-N* junction diode can easily be used as a rectifier because it conducts current only when forward biased voltage is acting, and does not conduct when reverse bias voltage is acting.

The input to the half-wave rectifier is supplied from the 50 Hz A.C. supply, whose wave form is shown in Fig. 49 (*b*).

---

*Power dissipated at the base-emitter junction is *negligible* [The base-emitter junction conducts about the same current as the collection-base junction ($I_E \approx I_C$), but V_{BE} is very small (0.3 V and 0.7 V for Ge and Si transistors respectively.)]

Operation :

When and A.C. voltage source is connected across the junction diode as shown in Fig. 49 (*a*) the *positive half cycle of the input acts as a forward bias voltage* and the output across the load resistance varies correspondingly. The *negative half cycle* of the input acts as a *reverse bias* and practically no current flows in the circuit. The output is, therefore, *intermittent*, *pulsating* and *unidirectional*.

It is evident from the above discussion, that as the *circuit uses only one-half cycle of the A.C. input voltage, therefore, it is popularly known as a "half-wave rectifier"*.

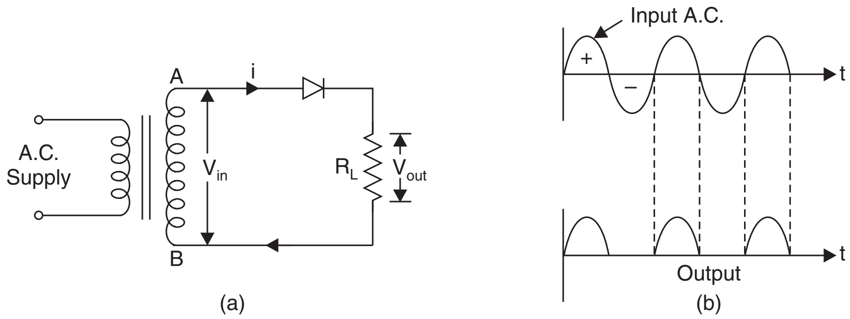


Fig. 49. Half-wave rectifier.

Disadvantages :

The main *disadvantages* of a half-wave rectifier are :

(*i*) The A.C. supply delivers power only half the time ; therefore, its *output is low*.

(*ii*) The pulsating current in the load contains alternating component whose basic frequency is equal to the supply frequency. Therefore, an elaborate filtering is required to produce steady direct current.

Efficiency of half-wave rectifier :

The ratio of D.C. power output to the applied A.C. input power is known as ***rectifier efficiency.***

i.e., Rectifier efficiency, $\eta = \dfrac{\text{D.C. power output}}{\text{A.C. input power}}$

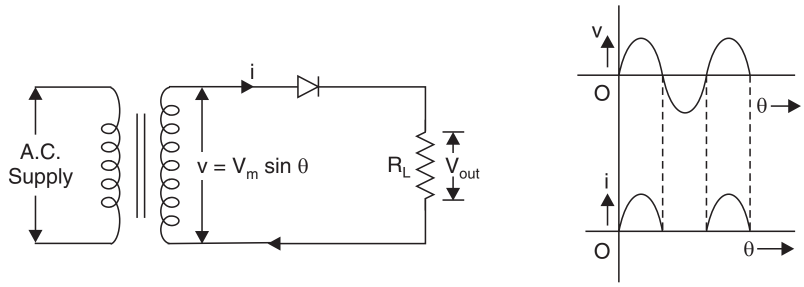


Fig. 50

Consider a half-wave rectifier shown in Fig. 50. Let $v = V_{max} \sin \theta$ be the alternating voltage that appears across the secondary winding.

Let, r_f = Diode forward resistance, and

R_L = Load resistance.

D.C. power output :

The output current is pulsating direct current. Therefore, in order to find D.C. power, average current has to be found out,

$$I_{av.} = I_{dc} = \frac{1}{2\pi}\int_0^{\pi} i\, d\theta = \frac{1}{2\pi}\int_0^{\pi} \frac{V_{max} \sin\theta}{(r_f + R_L)}\, d\theta$$

$$= \frac{V_{max}}{2\pi(r_f + R_L)}\int_0^{\pi} \sin\theta\, d\theta = \frac{V_{max}}{2\pi(r_f + R_L)}\Big[-\cos\theta\Big]_0^{\pi}$$

$$= \frac{V_{max}}{2\pi(r_f + R_L)} \times 2 = \frac{V_{max}}{\pi(r_f + R_L)} \times \frac{1}{\pi}$$

$$= \frac{I_{max}}{\pi} \qquad \left[\because \; I_{max} = \frac{V_{max}}{(r_f + R_L)}\right]$$

$\therefore$ D.C. power, $$P_{dc} = I_{dc}^2 \times R_L$$

$$= \left(\frac{I_{max}}{\pi}\right)^2 \times R_L \qquad ...(i)$$

A.C. power input : The A.C. power input is given by,

$$P_{ac} = I_{rms}^2\,(r_f + R_L)$$

For a half-wave rectified wave, $I_{r.m.s.} = \frac{I_{max}}{2}$

$$\therefore \qquad P_{ac} = \left(\frac{I_{max}}{2}\right)^2 \times (r_f + R_L) \qquad ...(ii)$$

$$\therefore \qquad \textit{Rectifier efficiency} = \frac{\text{D.C. output power}}{\text{A.C. input power}}$$

$$= \frac{(I_{max}/\pi)^2 \times R_L}{(I_{max}/2)^2\,(r_f + R_L)}$$

i.e., $$\eta_{\text{rectifier(half-wave)}} = \frac{0.406\,R_L}{r_f + R_L} = \frac{0.406}{1 + \frac{r_f}{R_L}} \qquad ...(16)$$

The efficiency will be maximum when r_f is negligible as compared to R_L.

$\therefore$ *Maximum rectifier (half-wave) efficiency* = **40.6%.**

This means in half-wave rectification, a maximum of 40.6% of A.C. power is converted into D.C. power.

Example 12. *A diode crystal having internal resistance r_f = 25 Ω is used for half-wave rectification. If the applied voltage v = 60 sin ωt and load resistance = 725 Ω, find :*

(*i*) I_{max}, I_{dc}, I_{rms}. (*ii*) *A.C. input power and D.C. output power.*

(*iii*) *D.C. output voltage.* (*iv*) *Efficiency of rectification.*

Solution. *Given :* r_f = 25 Ω ; R_L = 725 Ω ; $v = 60 \sin \omega t$.

(*i*) $\mathbf{I_{max}, I_{dc}, I_{rms}}$ **:**

Now, $$v = 60 \sin \omega t \qquad ...(Given)$$

i.e., $$V_{max} = 60 \text{ V}$$

$$\therefore \quad I_{max} = \frac{V_{max}}{r_f + R_L} = \frac{60}{25 + 725} = 0.08 \text{ A or } \mathbf{80\ mA.} \quad \textbf{(Ans.)}$$

$$I_{dc} = \frac{I_{max}}{\pi} = \frac{60}{\pi} = \mathbf{25.46\ mA.} \quad \textbf{(Ans.)}$$

$$I_{rms} = \frac{I_{max}}{2} = \frac{80}{2} = \mathbf{40\ mA.} \quad \textbf{(Ans.)}$$

(*ii*) **A.C. input power, D.C. output power :**

$$P_{ac} = (I_{rms})^2 \times (r_f + R_L) = \left(\frac{40}{1000}\right)^2 \times (25 + 725) = \mathbf{1.2\ W.} \quad \textbf{(Ans.)}$$

$$P_{dc} = (I_{dc})^2 R_L = \left(\frac{25.46}{1000}\right)^2 \times 725 = \mathbf{0.47\ W.} \quad \textbf{(Ans.)}$$

(*iii*) **D.C. output voltage, $\mathbf{V_{dc}}$:**

$$V_{dc} = I_{dc} \times R_L = \frac{25.46}{1000} \times 725 = \mathbf{18.46\ V.} \quad \textbf{(Ans.)}$$

(*iv*) **Efficiency of rectification :**

$$\text{Efficiency of rectification} = \frac{P_{dc}}{P_{ac}} = \frac{0.47}{1.2} = 0.3917 \text{ or } \mathbf{39.17\%.} \quad \textbf{(Ans.)}$$

Example 13. *A half-wave rectifier is used to supply 60 V D.C. to a resistive load of 600 Ω. If the diode resistance is 20 Ω, calculate A.C. voltage required.*

Solution. *Given :* V_{dc} = 60 V ; R_L = 600 Ω, r_f = 20 Ω

Let V_{max} be the maximum value of A.C. voltage required.

Now, $$V_{dc} = I_{dc} \times R_L$$

$$= \frac{I_{max}}{\pi} \times R_L = \frac{V_{max}}{\pi(r_f + R_L)} \times R_L \qquad \left[\because \ I_{max} = \frac{V_{max}}{r_f + R_L}\right]$$

or $$60 = \frac{V_{max}}{\pi(20 + 600)} \times 600$$

$$\therefore \quad \mathbf{V_{max}} = \frac{60 \times \pi(20 + 600)}{600} = \mathbf{194.78\ V.} \quad \textbf{(Ans.)}$$

5.2. Full-wave Rectifier

A ***full-wave rectifier*** *is a circuit, which allows a unidirectional current to flow through the load during the entire input cycle.* This can be achieved with *two diodes working alternately.* For the positive half-cycle of input voltage, one diode supplies current to the load and for the negative half-cycle, the other diode does so ; *current being always in the same direction through the load.*

For full-wave rectification the following *two circuits* are commonly used :

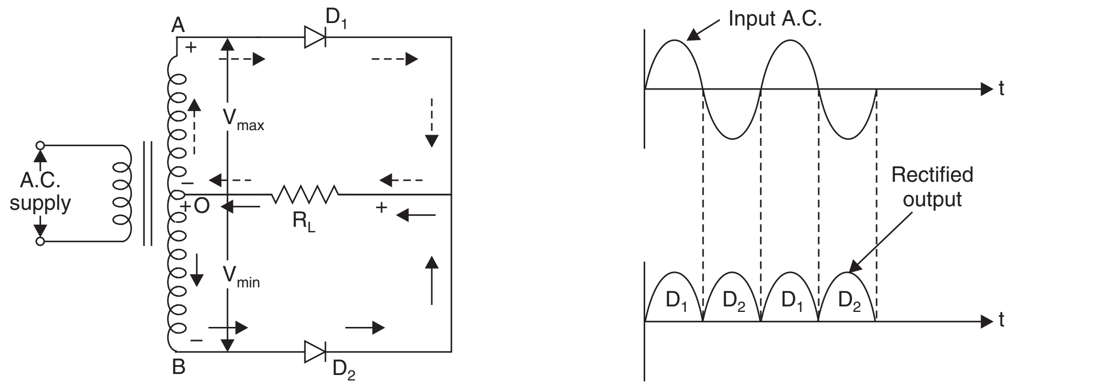
1. Centre-tapped full-wave rectifier
2. Full-wave bridge rectifier.

5.2.1. Centre-tapped full-wave rectifier

Fig. 51 shows the circuit of a centre-tapped full-wave rectifier. The circuit uses two diodes (D_1 and D_2) which are connected to the centre-tapped secondary winding *AB* of the transformer.

Operation :

- During the *positive half-cycle* of secondary voltage, the end A of the secondary winding is positive and end B negative. This makes the diode D_1 forward biased and diode D_2 reverse biased. Therefore, diode D_1 conducts while diode D_2 does not. The conventional current flows through diode D_1, load resistor R_L and the upper half of secondary winding as shown by the dotted arrows.
- During the *negative half-cycle,* the end A of the secondary becomes negative and end B positive. Therefore, D_2 conducts while diode D_1 does not. The conventional current flow is through D_2, R_L and lower half winding as shown by solid arrows.



(a) (b) Wave forms of full-wave rectifier.

Fig. 51. Centre-tapped full-wave rectifier.

It may be noted [Fig. 51 (*a*)] that the current in the load R_L is in the *same direction for both the cycles* of input *A.C.* voltage. Therefore, D.C. is obtained from the load R_L.

Also, Peak inverse voltage (*PIV*) = Twice the maximum voltage across the half-secondary winding

i.e., $PIV = 2\ V_{max}$.

Advantages :

1. The D.C. output voltage and load current values are twice than those of half-wave rectifiers.
2. The ripple factor is much less (0.482) than that of half-wave rectifier (1.21).
3. The efficiency is twice that of half-wave rectifier.

For a full-wave rectifier, the maximum possible value of efficiency is 81.2% while that of half-wave rectifier is 40.6%.

Disadvantages :

1. The diodes used must have high peak inverse voltage.
2. It is difficult to locate the centre tap on the secondary winding.
3. The D.C. output is small as each diode utilises only one-half of the transformer secondary voltage.

5.2.2. Full-wave bridge rectifier

It uses four diodes (D_1, D_2, D_3, D_4) across the main supply, as shown in Fig. 52 (*a*). The A.C. supply to be rectified is applied to the diagonally opposite ends of the bridge through the transformer. Between other two ends of the bridge, the load resistance R_L is connected.

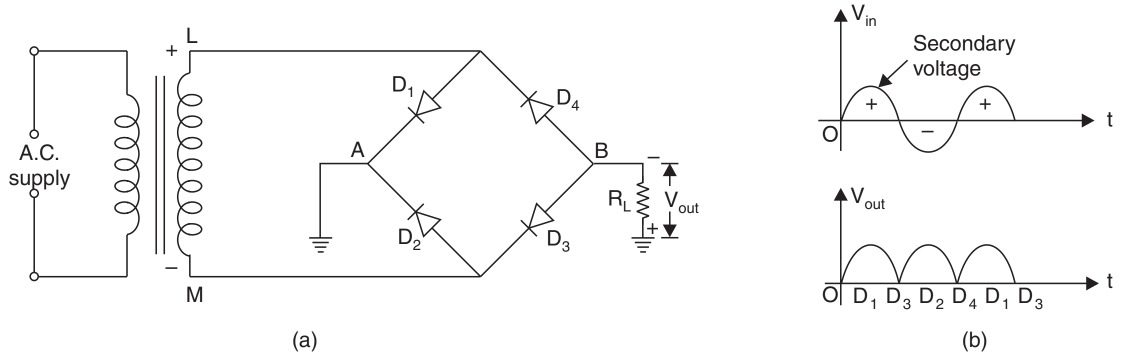


Fig. 52. Full-wave bridge rectifier.

Operation :

- During *positive half-cycle* of secondary voltage, the end L of the secondary winding becomes positive and end M negative. This makes D_1 and D_3 forward biased while diodes D_2 and D_4 are reverse biased. Therefore, only dioded D_1 and D_3 conduct. These two diodes will be in series through the load R_L as shown in Fig. 53 (*a*). The current flows (dotted arrows) from A to B through R_L.

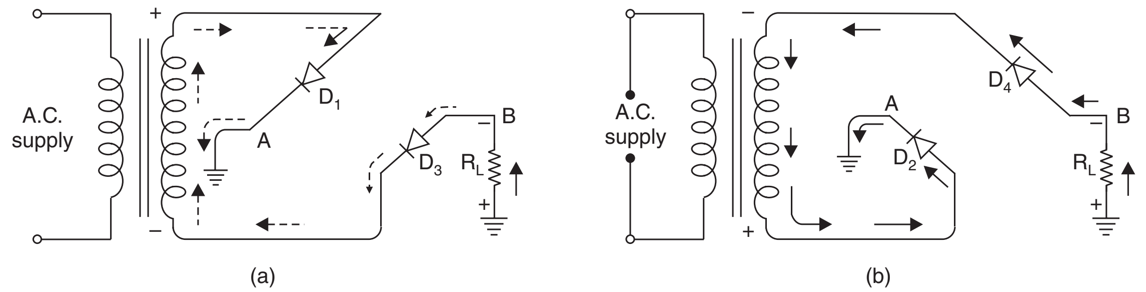


Fig. 53

- During the *negative half-cycle* of the secondary voltage, end L becomes negative and M positive. This makes D_2 and D_4 forward biased whereas diodes D_1 and D_3 are reverse biased. Therefore, only diodes D_2 and D_4 conduct. These two diodes will be in series with R_L as shown in Fig. 53 (*b*). The current flows (*solid arrows*) from A to B through R_L *i.e.,* in the same direction as for positive half-cycle. Therefore, D.C. output is obtained across R_L.

Further it may noted that *peak inverse voltage (PIV) of each diode is equal to the maximum secondary voltage of transformer.*

Advantages :

1. It can be used with advantage in applications allowing floating input terminals *i.e.,* no output terminal is grounded.

2. The transformer is less costly as it is required to provide only half the voltage of an equivalent centre-tapped transformer used in a full-wave rectifier circuit.

3. No centre-tap is required on the transformer.

4. The output is *twice* that of the centre-tapped circuit for the secondary voltage.

Disadvantages :

1. It uses four diodes as compared to two diodes for centre-tapped full wave rectifier.

2. Since during each half-cycle of A.C. input two diodes that conduct are in series, therefore, voltage drop in the internal resistance of the rectifying unit will be twice as great as in the centre-tapped circuit. This is objectionable when secondary voltage is small.

- These days, the *bridge rectifiers are so common that manufacturers* are packing them as a single unit with bakelite or some other plastic encapsulation with external connections brought out.

Efficiency of full-wave Rectifier :

Fig. 54 shows the *process of full rectification.*

The instantaneous current i is given by

$$i = \frac{v}{(r_f + R_L)} = \frac{V_{max} \sin \theta}{(r_f + R_L)}$$

where r_f and R_L are the diode forward resistance and load resistance respectively.

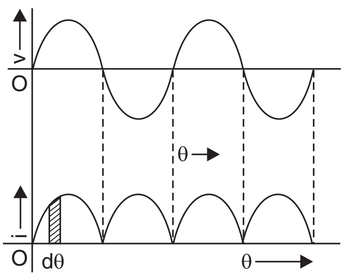


Fig. 54

D.C. output power (P_{dc}) :

$$I_{dc} = \frac{2I_{max}}{\pi}$$

∴ D.C. power output,

$$P_{dc} = I_{dc}^2 \times R_L = \left(\frac{2I_{max}}{\pi}\right)^2 \times R_L \qquad ...(17)$$

A.C. input power (P_{ac}) :

$$P_{ac} = (I_{rms})^2 (r_f + R_L)$$

For a full-wave rectified wave, we have

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

∴
$$P_{ac} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 (r_f + R_L) \qquad ...(18)$$

∴ Full-wave rectification efficiency,

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{(2I_{max}/\pi)^2 R_L}{(I_{max}/\sqrt{2})^2 (r_f + R_L)}$$

$$= \frac{8}{\pi^2} \times \frac{R_L}{(r_f + R_L)} = \frac{0.812 R_L}{(r_f + R_L)}$$

i.e.,
$$\eta = \frac{0.812}{1 + \frac{r_f}{R_L}} \qquad ...(19)$$

The efficiency will be maximum if r_f is negligible as compared to R_L.

∴ η_{max} = **81.2%**

This is double the efficiency than that of half-wave rectifier. Therefore, a full-wave rectifier is *twice as effective as a half-wave rectifier.*

Example 14. *A full-wave rectifier uses two diodes, the internal resistance of each diode may be assumed constant at 25 Ω. The transformer r.m.s. secondary voltage from centre tap to each end of secondary is 60 V and load resistance is 750 Ω. Find :*

(*i*) *The mean load current.*

(*ii*) *The r.m.s. value of load current.*

Solution. *Given :* $r_f = 25\ \Omega$; $R_L = 750\ \Omega$

(*i*) **The mean load current, I_{dc} :**

$$V_{max} = 60 \times \sqrt{2} = 84.85 \text{ V}$$

Maximum load current, $I_{max} = \dfrac{V_{max}}{(r_f + R_L)}$

$$= \frac{84.85}{25 + 750} = 0.109 \text{ A or } 109 \text{ mA}$$

$$\therefore \quad I_{dc} = \frac{2I_{max}}{\pi} = \frac{2 \times 109}{\pi} = \mathbf{69.39\ mA.} \quad \textbf{(Ans.)}$$

(*ii*) **The r.m.s. value of load, I_{rms} :**

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{109}{\sqrt{2}} = \mathbf{77.1\ mA.} \quad \textbf{(Ans.)}$$

5.3. Ripple Factor

The output voltage (or load current) of a rectifier consists of two components namely D.C. component and A.C. component. (Any wave which *varies in a regular manner has an A.C. component). The A.C. component present in the output is called a* **ripple**. As a matter of factor, the *ripple is undesirable and accounts for pulsations in the rectifier output :*

The *effectiveness of a rectifier depends upon the magnitude of ripple in the output. Smaller the ripple, more effective will be the rectifier.* Mathematically, the ripple factor,

$$\gamma = \frac{\text{The r.m.s. value of A.C. component of output voltage}}{\text{The D.C. component of output voltage}}$$

$$= \frac{V_{r(\text{r.m.s})}}{V_{dc}} = \frac{I_{r(\text{r.m.s.})}}{I_{dc}}$$

where, $V_{r(\text{r.m.s})}$ = The r.m.s. value of the A.C. component of the output voltage,

V_{dc} = The average or D.C. value of the output voltage,

$I_{r(\text{r.m.s})}$ = The r.m.s. value of the A.C. components of current, and

I_{dc} = The average or D.C. value of the load current.

We know that the r.m.s. value of the rectified load current,

$$I_{\text{r.m.s.}} = \sqrt{(I_{dc})^2 + (I_{r(\text{r.m.s.})})^2}$$

Dividing both sides by I_{dc}, we get

$$\frac{I_{\text{r.m.s.}}}{I_{dc}} = \frac{\sqrt{(I_{dc})^2 + (I_{r(\text{r.m.s.})})^2}}{I_{dc}} = \sqrt{1 + \left[\left\{\frac{I_{r(\text{r.m.s.})}}{I_{dc}}\right\}^2\right]}$$

Squaring and rearranging the above expression,

$$\frac{I_{r(\text{r.m.s.})}}{I_{dc}} = \sqrt{\left(\frac{I_{\text{r.m.s.}}}{I_{dc}}\right)^2 - 1}$$

or Ripple factor,

$$\gamma = \sqrt{\left(\frac{I_{\text{r.m.s.}}}{I_{dc}}\right)^2 - 1} \qquad ...(20)$$

(*i*) ***Ripple factor of a half-wave rectification.*** In half wave rectification,

$$I_{\text{r.m.s.}} = \frac{I_{\max}}{2}\ ;\ I_{dc} = \frac{I_{\max}}{\pi}$$

∴ Ripple factor,

$$\gamma = \sqrt{\left[\frac{(I_{\max}/2)}{(I_{\max}/\pi)}\right]^2 - 1} = 1.21$$

This indicates that A.C. component exceeds the D.C. component in the output of a half-wave rectifier. This results in greater pulsations in the output, hence half-wave rectifier is *not* very successful for conversion of A.C. into D.C.

(*ii*) ***Ripple factor of a full-wave rectifier.*** In full-wave rectification,

$$I_{\text{r.m.s.}} = \frac{I_{\max}}{\sqrt{2}}\ ;\ I_{dc} = \frac{2I_{\max}}{\pi}$$

∴ Ripple factor,

$$\gamma = \sqrt{\left(\frac{I_{\max}/\sqrt{2}}{2I_{\max}/\pi}\right)^2 - 1} = 0.48$$

6. REGULATED POWER SUPPLY

6.1. Ordinary Power Supply

Generally, electronic circuits require a source of D.C. power which is most conveniently obtained from commercial A.C. lines by using rectifier-filter system, called D.C. power supply (D.C. batteries are rarely employed because of their heavy cost). *Such a rectifier-filter combinations is known as ordinary D.C. power supply.*

Limitations. (*i*) The D.C. output voltage changes with a change in A.C. supply voltage.

(*ii*) The D.C. output voltage decreases considerably with the increase in load due to voltage drop in *transformer winding, rectifier and filter circuit.*

Thus ordinary power supply is not suitable for some of the electronic circuits, with such circuits we have to employ a regulated power supply which gives fixed output voltage.

6.2. Regulated Power Supply

A power supply that maintains the output voltage constant irrespective of A.C. mains fluctuations or load variations is known as a ***regulated power supply.***

Fig. 55 shows the block diagram of a regulated power supply. The functions of various components/elements are as follows :

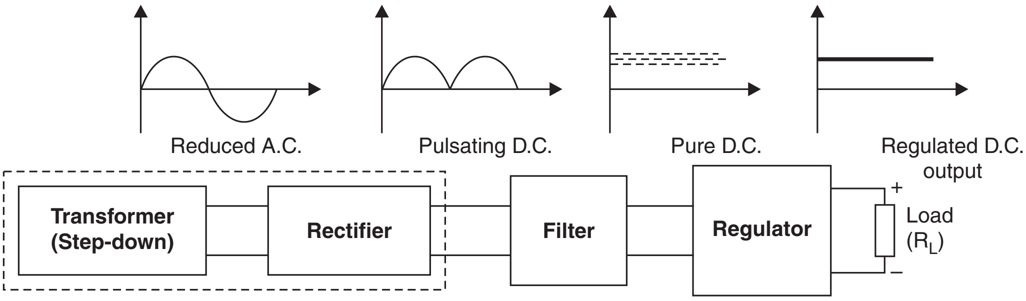


Fig. 55. Block diagram of a regulated power supply.

Transformer. Since normally D.C. voltages required for the operation of various electronic equipments are 6 V, 9 V or 12 V, therefore, a *step-down transformer* is employed before rectification to reduce the voltage to the required level.

Rectifier. The function of the rectifier is to convert A.C. into pulsating D.C. Usually bridge type full wave rectifier is used for the purpose.

Filter. Its function is to remove the ripples (A.C. component of pulsating D.C.) from the output of rectifier and smoothes it out.

Regulator. Its function is to keep the D.C. voltage constant even if the A.C. mains voltage or load varies. Usually, zener or glow-tube voltage regulator is used.

- **Voltage regulation.** *The change in D.C. output voltage from no load to full load with respect to full load voltage of a power supply is known as its* ***voltage regulation.***

Mathematically, % voltage regulation = $\frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$

where, V_{NL} = No-load D.C. voltage at the output terminals of the power supply, and

V_{FL} = Full load D.C. voltage at the output terminals of the power supply.

In a well designed power supply *percentage regulation should not be more than 1 percent.*

6.3. Voltage Regulators

A **voltage regulator** *is a device (an electronic circuit) which maintains the output voltage of an ordinary power supply constant irrespective of the change in input A.C. mains voltage or load variations.*

The regulator may be constructed from a zener diode, and/or discrete transistors, and/or integrated circuits. All voltage regulators must have a stable voltage reference source which is provided by a special type of diode operated in reverse breakdown called a *zener diode*.

The various types of voltage regulators are :

1. *Zener diode voltage regulator*

2. *Discrete transistor voltage regulators :*

(*i*) series voltage regulator.

(*ii*) Shunt voltage regulator.

3. *Zener controlled transistor voltage regulators :*

(*i*) Transistor series voltage regulator or Emitter follower voltage regulator.

(*ii*) Transistor shunt voltage regulator.

(*iii*) Controlled transistor series regulator.

(*iv*) Controlled transistor series regulator with over load and short circuit protection.

(*v*) Fold back current limiting.

(*vi*) Transistor current regulator.

4. *I.C. voltage regulators :*

(*i*) Fixed output voltage regulators : Positive and/or negative output voltage.

(*ii*) Adjustable output voltage regulators : Positive or negative output voltage.

(*iii*) Switching regulators.

(*iv*) Special regulators.

6.4. Zener Diode Voltage Regulator

Fig. 56 shows a simple zener diode voltage regulator circuit. The zener diode is selected in such a way that its zener voltage V_Z is *less* than $V_{DC(in)}$ (*i.e.,* output voltage of an ordinary power supply)

As long as input voltage is more than V_Z, the zener operates in the breakdown region and maintains constant voltage (V_Z) across it and the load. The value of output voltage is given by :

$$V_{DC\,(out)} = V_{DC\,(in)} - IR_S$$

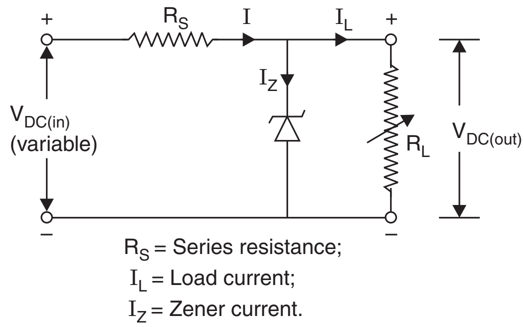


Fig. 56. Simple zener diode voltage regulator circuit.

As the load current increases, the zener current decreases and vise-versa which maintains constant current in series resistance R_S. Thus the output voltage $V_{DC\,(out)}$ remains unchanged.

The *limitations* of the zener diode voltage regulator are :

(*i*) Poor efficiency at heavy loads (due to large losses in R_S)

(*ii*) Slight changes in output voltage with changes in load.

6.5. Transistor Series Voltage Regulator

Fig. 57 shows an improved zener diode voltage regulator circuit called *transistor series voltage regulator*.

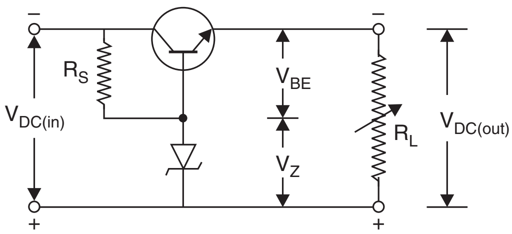


Fig. 57. Transistor series voltage regulator.

Here output voltage appearing across the load will be : $V_{DC(out)} = V_Z - V_{BE}$

As V_{BE} is very small, therefore,

$$V_{DC\,(out)} \simeq V_Z$$

HIGHLIGHTS

1. *Semiconductors* are solid materials, either non-metallic elements or compounds, which allow electrons to pass through them so that they conduct electricity in much the same way as a metal.

2. A pure semiconductor is called *instrinsic semiconductor*.

3. The process of adding impurity (extremely in small amounts, about 1 part in 10^8) to a semiconductor to make it extrinsic (impure) semiconductor is called *doping*.

4. The *N*- and *P*-type materials represent the basic building blocks of semiconductor devices.

5. The outstanding property of *P-N* junction diode to conduct current in one direction only permits it to be used as a *rectifier*.

6. *P-N* junction diodes usually made of germanium or silicon, are commonly used as *power rectifiers*.

7. A properly doped *P-N* junction diode which has a sharp breakdown voltage is known as *Zener diode*.

8. *Tunnel diode* is a heavily doped *P-N* junction type germanium having an extremely narrow junction.

9. A "*transistor*" is a semiconductor device having both rectifying and amplifying properties.

10. The two basic types of transistors are :
   (*i*) Bipolar junction transistor (BJT)
   (*ii*) Field-effect transistor (FET).

11. *Transistor circuit configurations :*
   (*i*) Common-base (CB) configuration
   (*ii*) Common-emitter (CE) configuration
   (*iii*) Common-collector (CC) configuration.

12. A *rectifier* is a circuit which uses one or more diodes to convert A.C. voltage into pulsating D.C. voltage. A rectifier may be half-wave or full-wave.

13. The ratio of D.C. power output to the applied A.C. input power is known as *rectifier efficiency*.

14. The A.C. component present in the output is called a *ripple*.

15. *Half-wave rectifier :*
   Max. efficiency = 40.6%
   Ripple factor = 1.21
   Full-wave rectifier :
   Max. efficiency = 81.2%
   Ripple factor = 0.48.

THEORETICAL QUESTIONS

Semiconductor Devices

1. Define a 'semiconductor'.
2. List the important characteristics of semiconductors.
3. Give examples of semiconducting materials.
4. What is the difference between a semiconductor and an insulator ?
5. What is an intrinsic semiconductor ?
6. What do you mean by the term *doping* ?
7. How does an extrinsic semiconductor differ from an intrinsic semiconductor ?
8. Explain the structure of a *P*-type semiconductor with help of neat sketches.
9. Explain briefly about 'atomic binding in semiconductors'.
10. How are holes formed in semiconductors ?
11. Derive an expression for electron conductivity of a metal.
12. Derive expressions for conductivity of *N*-type and *P*-type semiconductors.

13. What do you mean by conductivity modulation ?

14. Explain briefly the following :

(*i*) Thermistors and sensitors (*ii*) Photoconductors.

15. List the applications of semiconductor materials.

16. How is germanium prepared ?

P-N junction diode

17. What is a *P-N* junction diode ? How its terminals are identified ?

18. Draw the *V-I* characteristics of a junction diode when it is (*a*) forward biased and (*b*) reverse biased.

19. Draw the graphical symbol of a crystal diode and explain its significance. How the polarities of function diode are identified ?

20. Draw the equivalent circuit of a crystal diode.

21. What is an ideal diode and real diode ?

22. Explain the following terms :

(*i*) Static resistance

(*ii*) Bulk resistance

(*iii*) Junction resistance

(*iv*) A.C. or dynamic resistance

(*v*) Reverse resistance of a diode.

23. What are the important applications of a diode.

24. Write a short note on the power and current ratings of a diode.

25. What is a Zener diode ? Draw its equivalent circuit

26. Explain briefly the applications of a Zener diode.

27. What do you understand by Zener voltage ?

28. Explain why Zener diode is always operated in reverse biasing.

29. Explain how a Zener diode can stabilize the voltage across the load.

30. Explain the process of Zener breakdown.

31. Draw and explain a Zener diode voltage regulator.

Transistors

32. Define the term 'Transistor'.

33. What are the various types of transistors ?

34. Explain the function of emitter in the operation of a junction transistor.

35. What is the significance of arrow in the transistor symbol ?

36. Why is emitter wider than collector and base ?

37. Why is base made thin ?

38. Draw *N-P-N* and *P-N-P* transistors.

39. Explain the working of a *P-N-P* transistor.

40. Differentiate between *P-N-P* and *N-P-N* transistors. Why are collector and emitter currents nearly equal in these transistors ?

41. Define α and β of a transistor and derive the relationship between them.

42. Define three basic configurations of *N-P-N* transistor.

43. Draw input and output characteristics of *CB* transistor configuration.

44. Draw the circuits of the various transistor configurations. List their important features. Why *CE* configuration is mainly used ?

Regulated power supply

1. What is an ordinary power supply ? What are its limitations ?

2. Draw the block diagram of a regulated power supply and mention the function of its various component elements.
3. Explain briefly the term 'voltage regulation'.
4. What is a voltage regulator ?
5. Name the various types of voltage regulators
6. Explain briefly following voltage regulators :
(*i*) Zener diode voltage regulator.
(*ii*) Transistor series voltage regulator.

EXERCISE

1. A silicon diode has a bulk resistance of 2 Ω and a forward current of 12 mA. What is the actual value of V_F for the device ? [**Ans.** 0.624 V]
2. The current flowing in a certain *P-N* junction diode at room temperature is 2×10^{-7} A, when large reverse voltage is applied. Calculate the current flowing, when 0.1 V forward bias is applied at room temperature. [**Ans.** 10.7 μA]
3. Determine the germanium *P-N* junction diode current for the forward bias voltage of 0.22 V at room temperature 25°C with reserve saturation current equal to 1 mA. Take $\eta = 1$. [**Ans.** 5.22 A]
4. For the circuit shown in Fig. 58, find the maximum and minimum values of Zener diode current.

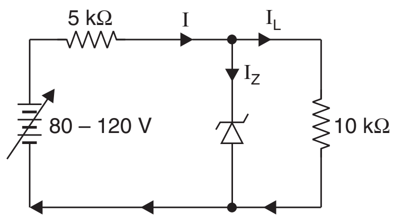


Fig. 58 [**Ans.** 9 mA, 1 mA]

5. The zener diode shown in Fig. 58 has V_Z = 18 V. The voltage across the load stays as 18 V as long as I_Z is maintained between 200 mA and 2A. Find the value of series resistance R_S so that V_{out} remains 18 V while input voltage is free to vary between 22 V to 28 V. [**Ans.** 3.33 Ω]
6. In a common-base configuration, the emitter current is 1 mA. If the emitter circuit is open, the collector current is 50 μA. Find the total collector current. Given that $\alpha = 0.92$. [**Ans.** 0.97 mA]
7. In a common-base configuration, $\alpha = 0.95$. The voltage drop across 2 kΩ resistance which is connected in the collector is 2 V. Find the base current. [**Ans.** 0.05 mA]
8. Calculate I_E in a transistor for which $\beta = 50$, I_B = 20 μA. [**Ans.** 1.02 mA]
9. For a transistor, $\beta = 45$ and voltage drop across 1 kΩ which is connected in the collector circuit is 1 volt. Find the base current for common-emitter connection. [**Ans.** 0.022 mA]
10. A transistor is connected in *CE* configuration in which collector supply is 8 V and the voltage drop across resistance R_C connected in the collector circuit is 0.5 V. The value of R_C = 800 Ω. If $\alpha = 0.96$, determine the collector emitter voltage and base current. [**Ans.** 7.5 V, 0.026 mA]
11. A full-wave rectifier uses two diodes, the internal resistance of each diode may be assumed constant at 20 Ω. The transformer r.m.s. secondary voltage from centre tap of each end of secondary is 50 V and load resistance is 980 Ω. Calculate :
(*i*) The mean load current.
(*ii*) The r.m.s. value of load current. [**Ans.** (*i*) 45 mA, (*ii*) 50 mA]
12. The four diodes used in a bridge rectifier circuit have forward resistances which may be considered constant at 1 Ω and infinite reverse resistance. The alternating supply voltage is 240 r.m.s. and load resistance is 480 Ω. Calculate :
(*i*) Mean load current.
(*ii*) Power dissipated in each diode. [**Ans.** (*i*) 0.45 A, (*ii*) 0.123 W]

UNIT-IV : *DIGITAL ELECTRONICS*

Chapter :

9. Digital Electronics

9

Digital Electronics

1. INTRODUCTION

The *branch of electronics which deals with digital circuits is called* **digital electronics.**

- A *continuously varying signal (voltage or current) is called an* **"analog signal."**

Example. A *sinusoidal voltage*.

In an analog electronic circuit, the output voltage changes continuously according to the input voltage variations *i.e.*, the output voltage can have an *infinite number of values*.

- A *signal (voltage or current) which can have only two discrete values is called a* **"digital signal."**

Example. A *square wave*.

- *An electronic circuit that is designed for two-state operation is called a* **digital circuit.**

These days digital circuits are being used in many electronic products such as *video games, microwave ovens, oscilloscopes etc.*

2. ADVANTAGES AND DISADVANTAGES OF DIGITAL ELECTRONICS

The advantages and disadvantages of digital electronics are listed below :

Advantages :

1. Digital system can be normally *easily designed*.
2. Digital circuits are less *affected by noise*.
3. Storage of information is easy with digital circuits.
4. Digital circuits provide greater accuracy and precision.
5. More digital circuits can be fabricated on integrated chips.

Disadvantages :

1. The digital circuits can handle only digital signals ; it requires encoders and decoders, due to which cost *of the equipment is increased*.

2. Under certain situations the use of only the analog techniques is simpler and economical (*e.g.,* the process of signal amplification).

However, since the advantages outweigh the disadvantages, therefore, we are switching to digital techniques at a faster pace.

3. DIGITAL CIRCUIT

An *electronic circuit that handles only a digital signal is called a* **digital circuit**.

Or

An *electronic circuit in which a state switches between the two states with time or with the change of the input states, and it is its state at the inputs and the outputs which has a significance is called a* **digital circuit**.

"*Digital*" is derived from "*digitus*". In Latin, the later means "finger". A finger is either up or down. Similarly an electronic circuit may have one of the states as :

(*i*) '*ON* ' (conduction) or '*OFF*' (poor conduction), or

(*ii*) '*High*' voltage or '*Low*' voltage between two terminals, or

(*iii*) '*High*' current through a circuit or '*Low*' current through a circuit, or

(*iv*) '*High*' frequency signal or '*Low*' frequency signal, or

(*v*) 'Negative' potential difference or '*Positive*' potential difference, or

(*vi*) "**1**" or "**0**" etc.

Therefore, *digital circuit is one that expresses the values in digits 1's or 0's, hence the name "digital*". The number concept that uses only the two digits 1 and 0 is the *binary numbering system*.

- As a digital circuit is based upon the two states, it is used in dealing with binary numbers ; digital circuit is therefore used in *computers*.

Advantages of Digital circuit :

1. *Noise free* as output is measured in terms of its state, not in terms of a voltage, or a current, or a frequency. A *state has a definiteness*.

2. *Capabilities of logical decision, arithmetic and Boolean operation on the binary numbers*.

Disadvantages :

1. *Slower speed* due to greater number of components to represent a state.

2. The circuits have *complexities* also. To represent a big decimal number, a *large number of components needed*.

Advantages of "Analog circuit" :

1. More close to physical system values.

2. A voltage level may represent temperature, wind, speed etc.

Disadvantages :

Lack of definiteness, preciseness and reliability.

4. LOGIC GATES

4.1. General Aspects

A *digital circuit with one or more input signals but only one output signal is called a* **logic gate**.

Or

A **logic gate** *is an electronic circuit which makes logic decision.*

- Logic gates are the *basic building blocks* from which most of the digital systems are built up. They implement the hardware logic function based on the logical algebra developed by George Boolean which is called *Boolean algebra* in his honour.
  — A unique characteristic of Boolean algebra is that variables used in it can assume only one of the two values *i.e., either 0 or 1*. Hence, every variable is either a 0 or a 1 (Fig. 1—limits on TTLIC's).

- Each gate has *distinct graphic symbol and its operation can be described by means of Boolean algebraic function*.

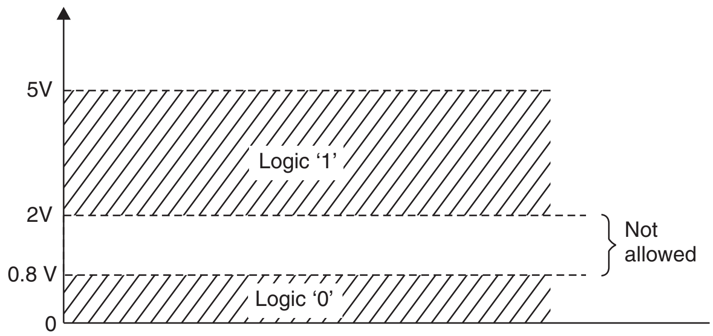


Fig. 1. Voltage assignment in a digital system.

- The *table which indicates output of gate for all possible combinations of input is known as a* **truth table**.
- These gates are available today in the form of various IC families. The *most popular families* are :

(*i*) Transistor-transistor logic (TTL)

(*ii*) Emitter-coupled logic (ECL)

(*iii*) Metal-oxide-semiconductor (MOS)

(*iv*) Complementary metal-oxide-semiconductor (CMOS).

4.2. Applications of Logic Gates

The following are the *fields of application of logic gates* :

1. Calculators and computers
2. Digital measuring techniques
3. Digital processing of communications
4. Musical instruments
5. Games and domestic appliances, etc.
6. The logic gates are also employed for decision making in *automatic control of machines and various industrial processes and for building more complex devices such as binary counters* etc.

4.3. Positive and Negative Logic

The number symbols 0 and 1 represent, in computing systems, two possible states of a circuit or device. It does not make any difference if these two states are referred to as '*ON*' and '*OFF*', '*Closed*' and '*Open*', '*High*' and '*Low*', '*Plus*' and '*Minus*' or '*True*' and '*False*' depending upon the situations. The main point is they must be symbolized by *two opposite conditions*. In *positive logic* a '**1**' *represents* : an '*ON circuit*' ; a '*Closed switch*' ; a '*High voltage*' , a '*Plus sign*', a '*True statement*'. Consequently, a *O represent* : an 'OFF circuit' ; an '*Open switch*', a '*Low voltage*' ; a '*Minus sign*', a 'False statement'.

In **negative logic**, the just opposite conditions prevail.

Example. A digital system has two voltage levels of 0 V and 5 V. If we say that symbol 1 stands for 5 V and symbol 0 for 0 V, then we have *positive logic system*. If on the other hand, we decide that a 1 should represent 0 V and 0 should represent 5 V, then we will get negative logic system.

Main point is that in '*positive logic*' the ***more positive*** of the two voltage levels represents the 1 while in '*negative logic*' the ***more negative*** voltage represents the 1.

4.4. Types of Logic Gates

In the *complex circuits*, the following *six* different digital electronics gates are used as *basic elements* :

1. NOT Gate	2. NAND Gate
3. AND Gate	4. OR Gate
5. NOR Gate	6. XOR Gate.

— A truth table has 2^n rows. It gives in each of its row m outputs for a given combination of n inputs.

1. NOT gate :

- *Not operation means that the output is the complement of input*. If input is logic '1' , the output is logic '0' and if input is logic '0', the output is logic '1',
- Fig. 2 shows the *symbol of NOT Gate*. It is generally represented by a *triangle followed by a bubble* (or a bubble followed by a triangle).
- NOT gate is *used when an output is desired to be complement of the input*.
- If all inputs of NAND gates are joined it shall act as NOT gate.
- NOT gate is also *called 'inverting logic circuit'*. It is also *called a 'complementing circuit'*.

2. NAND gate :

- A NAND gate can said to be *basic building block* of the *all digital TTL logic gates and other digital circuits*.
- It is represented by the symbol shown in Fig. 3.
- Its *unique property is that output is high '1' if any of the input is at low '0' logic level*.

Let us consider two inputs with the states A and B at the NAND gate. The answer (output) $X = \overline{A\,.\,B}$. Bar denotes a NOT log operation on *A.B.* The meaning of *A.B,* called AND operation, is given in 3 below.

3. AND gate :

- *A NAND gate followed by a NOT gate gives us AND gate.*
- It is represented by a symbol in Fig. 4. Its symbol differs from NAND only by *omission of a bubble (circle)*.
- Its *unique property is that its output is '0' unless all the inputs to it are at the logic 1's*.
- A two inputs, AND gate has $X = A.B.$ *Dot* between the two states indicates 'AND' logic operation using these.

4. OR gate :

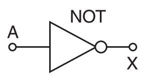
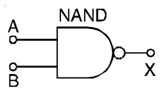
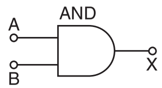
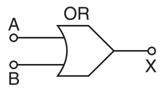
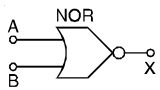
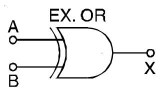
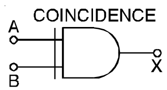
- An OR operation means that the *output is '0' only if all the input are '0'*.
- It is represented by a symbol shown in Fig. 5.
- If any of the inputs is '1' the output is '1' . A two inputs 'OR' gate has $X = A + B$. Sign + between the two states indicates an 'OR' logic operation.

5. NOR gate :

- An 'OR' circuit followed by a NOT circuit gives a 'NOR' gate (Fig. 6).
- Its *unique property is that its output is '0' if any of input is '1'*.
- A NOR gate is a *basic building block for other types of the logic gates than TTLs*. In the TTL circuits, a NOR is fabricated in an IC by the several NANDs.

A two inputs NOR has $X = \overline{A + B}$.

Table 1. Symbols, Boolean Expressions and Truth Tables of Various Logic Gates

Logic	NOT	NAND	AND	OR	NOR	EX . OR	COINCIDENCE
Symbol	 Fig. 2	 Fig. 3	 Fig. 4	 Fig. 5	 Fig. 6	 Fig. 7	 Fig. 8
Boolean Expression	$X = \overline{A}$	$X = \overline{A . B}$	$X = A.B$	$X = A + B$	$X = \overline{A + B}$	$X =$ $A . \overline{B} + \overline{A} . B$ $= A \oplus B$	$X = \overline{A} . \overline{B} + A.B$
Truth table	*A X* 0 1 1 0	*A B X* 0 0 1 0 1 1 1 0 1 1 1 0	*A B X* 0 0 0 0 1 0 1 0 0 1 1 1	*A B X* 0 0 0 0 1 1 1 0 1 1 1 1	*A B X* 0 0 1 0 1 0 1 0 0 1 1 0	*A B X* 0 0 0 0 1 1 1 0 1 1 1 0	*A B X* 0 0 1 0 1 0 1 0 0 1 1 1
Definition	Output available—when there is no input	Output available— to all states except when all the inputs are available	Output available—when all inputs available	Output available—when only one or more inputs available	Output available—when no input is available	Output available—when the inputs are *not* identical	Output available—to those states when the inputs are identical

6. XOR gate :

- A XOR gate (Fig. 7) is called 'Exclusive OR' gate.
- Its *unique property is that the output is '1' only if odd number of the inputs at it are '1's.*
- The 'Exclusive OR' can be written as : $X = A \cdot \overline{B} + \overline{A} \cdot B$ or $A \oplus B$.
- Exclusive OR gate is important in the circuits *for addition of two binary numbers.*

7. Coincidence gate :

- This gate (Fig. 8) can be written as : $X = \overline{A} \cdot \overline{B} + A \cdot B$.
- Output available to those states when the inputs are identical.

Basic building blocks. AND, OR and NOT gates are called *basic* building blocks or basic gates because they are *essential to realize any boolean expression*.

Universal gates. NAND and NOR gates are known as *universal gates* because any logic gate can be constructed either by using NAND gates only or by using NOR gates only.

5. UNIVERSAL GATES

NAND and NOR gates are known as **universal gates.**

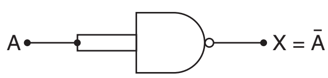
The AND, OR, NOT gates can be realized using only NAND or NOR gates.

— Demorgan's theorem afford a convenient method to use these two gates in *logic design.* The entire logic system can be implemented by using any of these two gates.

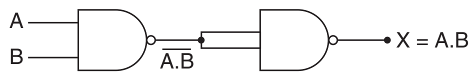
— These two gates are *easier to realize and consume less power than other gates.*

(*i*) **Realization of logic gates using NAND gates :**

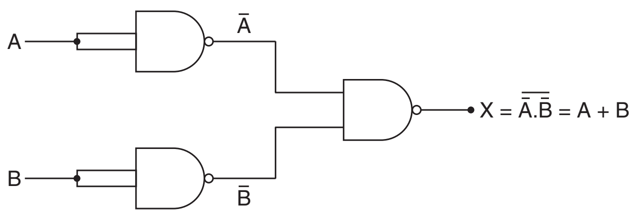
Fig. 9 (*a*), (*b*), (*c*) shows realization of NOT, AND, OR gates using NAND gates respectively, which is self explanatory.



(a) Realization of NOT gate using NAND gate



(b) Realization of AND gate using NAND gates

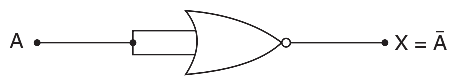


(c) Realization of OR gate using NAND gates

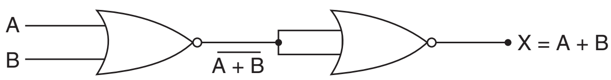
Fig. 9. Realization of NOT, AND and OR gates using NAND gates.

(*ii*) **Realization of logic gates using NOR gates :**

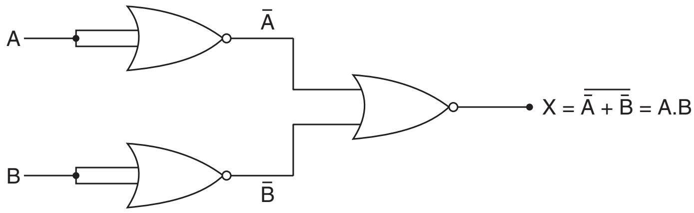
The realization of NOT, OR and AND gates using NOR gates is shown in Fig. 10 (*a*), (*b*), (*c*) respectively.



(a) Realization of NOT gate using NOR gates



(b) Realization of OR gate using NOR gates



(c) Realization of AND gate using NOR gates

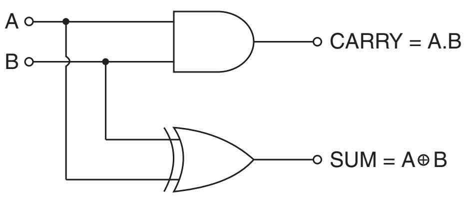
Fig. 10. Realization of NOT, OR and AND gates using NOR gates.

6. HALF ADDER

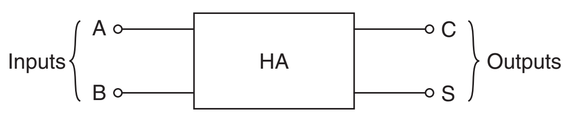
It is a *1-bit adder* and carries out binary addition with the help of XOR and AND gates. It has **two** *inputs* and **two** *outputs*.

It can add 2 binary digits at a time and produce a 2-bit data *i.e.,* SUM and CARRY according to binary addition rules.

The circuit of a half adder is shown in Fig. 11. (*a*). It consists of an Ex-OR gate and AND gate. The outputs of the Ex-OR gate is called the SUM (S), while the output of the AND gate is known as CARRY (C). As the AND gate produces a *high* output only when both inputs are *high* and Ex-OR gate produces a *high* output if either input (not both) is high, the truth table of a half adder is developed by writing the truth table output of AND gate in the CARRY column and the output truth table of Ex-OR gate in SUM column. Truth table for half adder is given in table.



(a) Logic circuit



(b) Logic symbol

Fig. 11. Half adder.

Table 2. Truth table for Half Adder

Inputs		Outputs	
A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

The logical expressions for CARRY and SUM can be written from the truth table for a half adder as follows :

$$\text{CARRY, C} = \text{A.B}$$

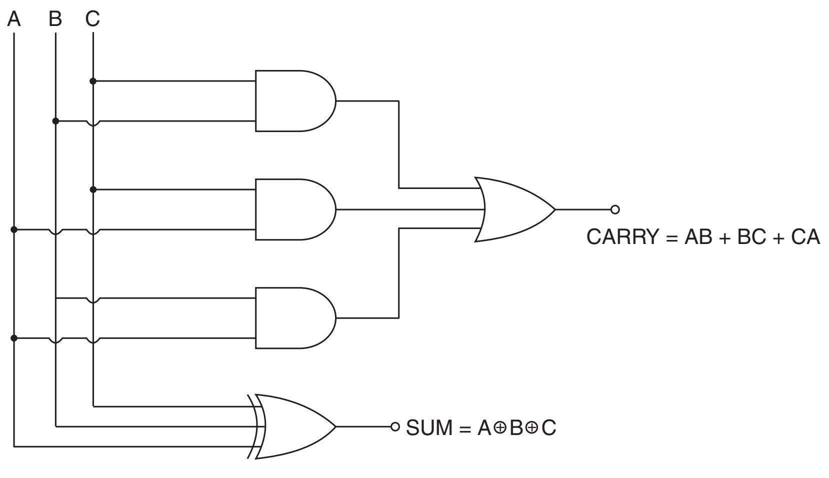
$$\text{SUM, S} = \text{A} \oplus \text{B}$$

- This circuit is called *half-adder,* because it *cannot* accept a CARRY-IN from previous additions. Owing to this reason the half-adder circuit can be used for binary addition of lower most bit only.

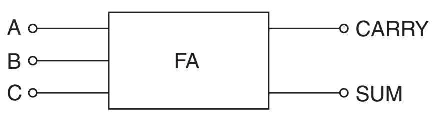
For higher-order columns, 3-input adder called *full adder* are used.

7. FULL ADDER

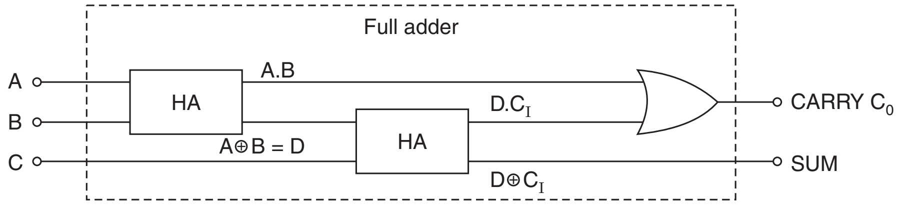
A full adder has *three inputs* and *two outputs.* It *can add 3 digits* (*or bits*) *at a time.* The bits A and B which are to be added come from the two registers and the third input C curves from the *'carry'* generated by the previous addition. It produces two outputs, SUM and CARRY-OUT (going to next higher column).



(a) Logic circuit



(b) Logic symbol



(c) Full adder circuit

Fig. 12. Full adder.

Table 3. Truth table for Full Adder

A	B	C	CARRY	SUM
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

A simple circuit of a full adder is shown in Fig. 12 (*a*), though other designs are also possible. It uses 3 AND gates, one Ex-OR gate and one OR gate. The final CARRY is given by the OR gate while the final SUM is given out by the Ex-OR gate.

Fig. 12 (*b*) shows the logic symbol for a full adder.

Truth table for full adder for all passible inputs/outputs is given in Table 3. Truth table can be checked easily for its validity.

A full adder can be made by using two half adders and an OR gate. The circuit is shown in Fig. 12 (*c*).

- The full adder can do more than a million additions per second. Besides that, it never get tired or bored or asks for a rest.

Note : *Binary additions* : Following are the four rules/cases for addition of binary numbers :

(1) 0 + 0 = 0

(2) 0 + 1 = 1

(3) 1 + 0 = 1

(4) 1 + 1 = 10 (This sum is *not ten* but *one-zero*).

8. BOOLEAN ALGEBRA

George Boolean in 1854 developed a mathematics now referred as *Boolean algebra*. It is the *algebra of logic presently applied to the operation of computer devices*. The rules of this algebra are based on human reasoning.

Digital circuits perform the binary arithmetic operations with binary digits 1 and 0. These operations are called logic functions or logic operations. *The algebra used to symbolically describe logic functions is called* **Boolean algebra**. Boolean algebra is *a set of rules and theorems by which logical operations can be expressed symbolically in equation form and be manipulated mathematically*.

Boolean algebra differs from ordinary algebra in that Boolean constant and variables *can have only two values* ; '0' and '1' :

In Boolean algebra the following *four connecting symbols* are used :

1. **Equal sign (=).** In Boolean algebra the 'equal sign' refers to the standard mathematical equality. In other words, the logical value on one side of the sign is identical to the logical value on the other side of the sign.

Example. We are given two logical variables such that $A = B$. Then if $A = 1$, then $B = 1$ and if $A = 0$ then $B = 0$.

2. **Plus sign (+).** In Boolean algebra the 'plus sign' refers to logical *OR operation*.

The statement $A + B = 1$ means A ORed with B equals 1. Consequently, either $A = 1$ or $B = 1$ or both equal to 1.

3. **Multiply sign (.).** In Boolean algebra the 'multiply sign' refers to *AND operation*.

The statement $A.B = 1$ means A ANDed with B equals 1. Consequently, $A = 1$ and $B = 1$.

The function $A \,.\, B$ often written as *AB, omitting the dot* for convenience.

4. **Bar sign (–).** In Boolean algebra the 'bar sign' refers to *NOT operations*. The NOT has the effect of inverting (complementing) the logic value.

Thus if $A = 1$, then $\overline{A} = 0$.

9. BOOLEAN LAWS (FOR OUTPUTS FROM LOGIC INPUTS)

The following Laws can said to be associated with Boolean algebra :

1. **'OR' Laws**

The 'OR' Laws are described by the following equations :

$$A + 1 = 1 \quad \text{...[1(a)]}$$

$$A + 0 = A \quad \text{...[1(b)]}$$

$$A + A = A \quad \text{...[1(c)]}$$

$$A + \overline{A} = 1 \quad \text{...[1(d)]}$$

- An 'OR' operation is denoted by *plus sign*.
- 'OR' Law means :

(*i*) Any number (0 or 1) is a first input to an OR gate and another member at the second input is 1 then answer is 1,

(*ii*) If another is 0 then answer is as first input, and

(*iii*) If two inputs to an OR gate *complements* then output is '1'.

2. **'AND' Laws**

'AND' operation is denoted by the *dot* sign.

- True and true make true
- True and false make false
- False and false make false.

$$A \,.\, 1 = A \quad \text{...[2(a)]}$$

$$A \,.\, 0 = 0 \quad \text{...[2(b)]}$$

$$A \,.\, A = A \quad \text{...[2(c)]}$$

$$A \,.\, \overline{A} = 0 \quad \text{...[2(d)]}$$

3. **'NOT' Laws (Laws of Complementation)**

A NOT operation is denoted by putting a *bar* over a number.

- The NOT true means false.
- The NOT false means true.

$$\overline{1} = 0 \qquad \text{...[3(a)]}$$

$$\overline{\overline{A}} = A \qquad \text{...[3(b)]}$$

Eqn. [3 (*c*)] means that if *A is inverted (complemented) and then again inverted, we get the original number*.

4. **Commutative Laws**

These Laws mean that *order of a logical operation is immaterial*.

$$A + B = B + A \qquad \text{...[4(a)]}$$

$$A \,.\, B = B \,.\, A \qquad \text{...[4(b)]}$$

5. **Associative Laws**

These laws *allow a grouping of the Boolean variables*.

$$A + (B + C) = (A + B) + C \qquad \text{...[5(a)]}$$

$$A \,.\, (B \,.\, C) = (A.\, B) \,.\, C \qquad \text{...[5(b)]}$$

6. **Distributive Laws**

These laws *simplify the problems in the logic designs*.

$$A \,.\, (B + C) = (A \,.\, B) + (B \,.\, C) \qquad \text{...[6(a)]}$$

$$A + (B \,.\, C) = (A + B) \,.\, (A + C) \qquad \text{...[6(b)]}$$

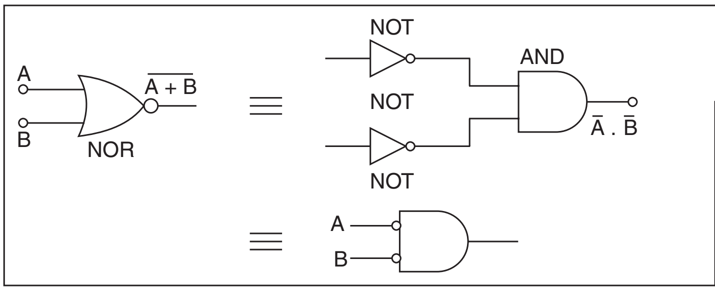
$$A + (\overline{A} \,.\, B) = A + B \qquad \text{...[6(c)]}$$

The last two equations are typical to the Boolean algebra, and are not followed in the usual algebra.

10. DE MORGAN'S THEOREMS

First theorem shows an equivalence of a NOR gate with an AND gate having bubbled inputs (Fig. 13), and is given by the equation :

$$\overline{A + B} = \overline{A} \,.\, \overline{B} \qquad \text{...(7)}$$

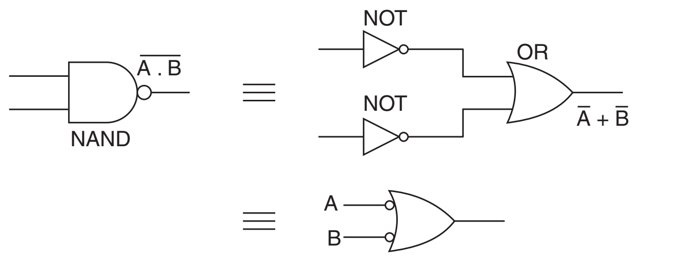


A	B	X
0	0	1
1	0	0
0	1	0
1	1	0

Fig. 13. De Morgan's First theorem showing an equivalence of a NOR gate (same holds for multiple inputs).

Second theorem shows an equivalence of a NAND gate with an OR having bubbled inputs as shown in Fig. 14 and is given by the equation :

$$\overline{A \,.\, B} = \overline{A} + \overline{B} \qquad \text{...(8)}$$



A	B	X
0	0	1
1	0	1
0	1	1
1	1	0

Fig. 14. De Morgan's Second theorem showing the equivalence of a NAND gate (same holds for the multiple inputs).

In fact with the eqns. (7) and (8) also hold for the cases of the multiple (more than two) inputs.

$$\overline{A+B+C} + ... = \overline{A}\,.\,\overline{B}\,.\,\overline{C} \qquad ...[9(a)]$$

$$A\,.\,B\,.\,C\,....... = \overline{A}+\overline{B}+\overline{C} \qquad ...[9(b)]$$

The *purpose of these theorems is to enable digital circuit designers to implement all the other logic gates with the help* of *either NOR gates only or NAND gates only*. For example, a NOT gate is implementable by a NAND or a NOR as shown in the left part or lower right part of Fig. 13 respectively. This theorem finds wide use in the digital logic circuits as these are implementable on one single basic logic gate considered as a basic building unit.

- The *'first statement'* (De Morgan's) says that *the complement of a sum equals the product of the complements*. The *'second statement'* says that the *complement of a product equals the sum of the complements*. In fact, it allows transformation from a sum-of-products form to a product-of-sum form.

— The procedure required for *taking out an expression from under a NOT sign is as follows* :

1. *Complement the given expression i.e., remove the overall NOT sign.*
2. *Change all and ANDs to ORs and all the ORs to ANDs*.
3. *Complement or negate all individual variables.*

Examples : (*i*) $\overline{A+BC} = A + BC$ ...*Step 1*

$= A(B + C)$ ...*Step 2*

$= \overline{A}\,(\overline{B}+\overline{C})$ ...*Step 3*

(*ii*) $\overline{(\overline{A}+B+\overline{C})(\overline{A}+B+C)} = (\overline{A}+B+\overline{C})(\overline{A}+B+C)$

$= \overline{A}B\overline{C} + \overline{A}BC = \overline{\overline{A}}\,\overline{B}\,\overline{\overline{C}} + \overline{\overline{A}}\,\overline{B}\,\overline{C} = A\overline{B}C + A\overline{B}\,\overline{C}$

This *process is called* **demorganization.**

— It may be noted that the opposite procedure would be followed to *bring an expression under the NOT sign* :

Example : $\overline{A}+\overline{B}+\overline{C} = \overline{\overline{A}}+\overline{\overline{B}}+\overline{\overline{C}}$ ...*Step 3*

$= A + B + C$

$= ABC$ ...*Step 2*

$= \overline{ABC}$ ...*Step 1*

11. OPERATOR PRECEDENCE

For evaluating Boolean expression, the operator precedence is :

(*i*) parenthesis, (*ii*) NOT, (*iii*) AND and (*iv*) OR. In other words :

— The expression inside the *parenthesis* must be evaluated before all other operations,

— The next operation that holds precedence is the *complement*,

— Then follows the AND, and

— Finally the OR.

Example. In the Boolean expression $A + \overline{B}\ (C + D)$, and expression inside the parenthesis will be evaluated first, then $\overline{B}$ will be evaluated, then the results of the two [*i.e.*, $\overline{B}$ and $(C + D)$] will be ANDed and finally, the result of the product ORed with A.

Example 1. *Prove the following identity :* $AC + ABC = AC$.

Solution. Taking the left hand expression as X, we get

$$X = AC + ABC = AC(1 + B)$$

Now, $1 + B = 1$ [Eqn. 1 (*a*)]

$\therefore$ $X = AC.1 = AC$

$\therefore$ $AC + ABC = AC$ ...**Proved.**

Example 2. *Prove the following Boolean identity :* $(A + B)(A + C) = A + BC$.

Solution. Putting the left hand side expression equal to X, we get

$$X = (A + B)(A + C)$$
$$= AA + AC + AB + BC \quad \text{...[Eqn. 6 (a)]}$$
$$= A + AC + AB + BC \quad [AA = A \text{ ...(2 (c))]}$$
$$= A + AB + AC + BC$$
$$= A(1 + B) + AC + BC \quad (\because\ 1 + B = 1)$$
$$= A + AC + BC$$
$$= A(1 + C) + BC$$
$$= A + BC \quad (\because\ 1 + C = 1)$$

$\therefore$ $(A + B)(B + C) = A + BC$. ...**Proved.**

Example 3. *Prove the following identity :* $A + \overline{A}B = A + B$.

Solution. Putting the left hand expression equal to X, we get

$$X = A + \overline{A}B = A\ .\ 1 + \overline{A}B \quad \text{[Eqn. 2 (a)]}$$
$$= A(1 + B) + \overline{A}B \quad \text{[Eqn. 1 (a)]}$$
$$= A.1 + AB + \overline{A}B \quad \text{[Eqn. 6 (a)]}$$
$$= A + BA + B\overline{A}$$
$$= A + B(A + \overline{A}) \quad \text{[Eqn. 6 (a)]}$$
$$= A + B\ .\ 1 \quad \text{[Eqn. 1 (d)]}$$
$$= A + B \quad \text{[Eqn. 2 (a)]}$$

$\therefore$ $A + \overline{A}B = A + B$. ...**Proved.**

Example 4. *Simplify the following Boolean expression to a minimum of literals :*

$$X = AB + \overline{A}C + BC.$$

Solution. $X = AB + \overline{A}C + BC$

$$= AB + \overline{A}C + BC(A + \overline{A}) \quad \text{...[Eqn. 1 (d)]}$$
$$= AB + \overline{A}C + ABC + \overline{A}BC$$
$$= AB(1 + C) + \overline{A}C(1 + B)$$
$$= AB + \overline{A}C \quad \text{[Eqn. 1 (a)]}$$

$\therefore$ $X = AB + \overline{A}C$. **(Ans.)**

Example 5. *Simplify the following Boolean expression :*

$$AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}BC + ABC + A\overline{B}C.$$

Solution. Let, $X = AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}BC + ABC + A\overline{B}C$

Bringing together those terms which have two common letters, we get

$$X = ABC + AB\,\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C + \overline{A}BC$$

$$= AB(C + \overline{C}) + A\overline{B}\,(\overline{C} + C) + \overline{A}BC$$

$$= AB + A\overline{B} + \overline{A}BC \qquad \text{...[Eqn. 1}(d)]$$

$$= A(B + \overline{B}) + \overline{A}BC$$

$$= A + \overline{A}BC = A + BC. \quad \textbf{(Ans.)} \qquad \text{...[Eqn. 6}(c)]$$

Example 6. *Using Boolean algebra techniques, simplify the following expression :*

$$X = A.B.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.B.C.\overline{D} + A.B.C.\overline{D}\,.$$

Solution. $X = B\overline{C}\,\overline{D}\,(A + \overline{A}) + BC\overline{D}\,(A + \overline{A})$ ...Taking out the common factors

$$= B\overline{C}\,\overline{D} + BC\overline{D} \qquad \text{...[Eqn. 1}(d)]$$

$$= B\overline{D} + (C + \overline{C}) \qquad \text{...Again factorize}$$

$$= B\overline{D}\;.\;1 = B\overline{D} \qquad \text{...(Simplified form)} \quad \textbf{(Ans.)} \qquad \text{...[Eqn. 1}(d)]$$

Example 7. *Simplify the following expression and show the minimum gate implementation.*

$$X = A\,.\,B\,.\overline{C}\,.\,\overline{D} + \overline{A}\,.B\,.\,\overline{C}\,.\,\overline{D} + B\,.\overline{C}\,.\,D$$

Solution. $X = B\,.\overline{C}\,.\,\overline{D}\;(\overline{A} + B) + B\,.\overline{C}\,.\,D$

$$= B\,.\overline{C}\,.\,\overline{D}\;.\;1 + B\,.\overline{C}\,.\,D \qquad \text{...[Eqn. 1}(d)]$$

$$= B\,.\overline{C}\,.\,\overline{D} + B\,.\overline{C}\,.\,D$$

$$= B\,.\overline{C}\;.\;(D + \overline{D}) = B\,.\overline{C}\;.\;1 = B\,.\,\overline{C} \qquad \text{...[Eqn. 1}(d)]$$

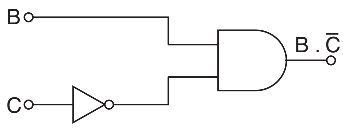


Fig. 15

Minimum gate implementation is shown in Fig. 15.

Example 8. *Determine output expression for the circuit shown in Fig. 16.*

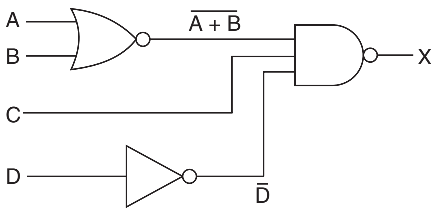


Fig. 16

Solution. The output expression for the circuit shown in Fig. 16 is :

$$X = \overline{[(\overline{A} + \overline{B}) . C . \overline{D}]}.$$

Example 9. *Simplify the following Boolean expression and draw the logic circuit for simplified expression :*

$$X = \overline{B}(A + C) + C(\overline{A} + B) + AC.$$

Solution.

$$X = \overline{B}\ (A + C) + C(\overline{A} + B) + AC$$

$$= A\overline{B} + \overline{B}C + \overline{A}C + BC + AC$$

$$= A\overline{B} + C(\overline{B} + \overline{A} + B + A)$$

$$= A\overline{B} + C.1 = A\overline{B} + C \quad \text{...Simplified expression.} \quad \textbf{(Ans.)}$$

Logic circuit for the simplified expression is shown in Fig. 17.

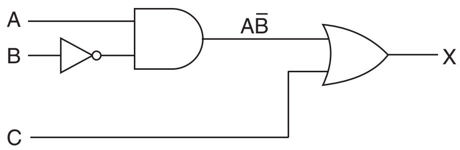


Fig. 17

Example 10. *Simplify the expression : (AB + C) (AB + D).*

Solution. Let $X = (AB + C)(AB + D)$

$= AB\,AB + ABD + ABC + CD$ ...[Eqn. 6(*a*)]

$= AABB + ABD + ABC + CD$

$= AB + ABD + ABC + CD$ ...[Eqn. 2(*c*)]

$= AB(1 + D) + ABC + CD$

$= AB + ABC + CD$ ...[Eqn. 1(*a*)]

$= AB(1 + C) + CD$

$\therefore \quad (AB + C)(AB + D) = AB + CD.$ **(Ans.)**

Example 11. *Draw the logic circuit represented by the expression :*

$$X = AB + \overline{A}.\overline{B} + \overline{A}.B.C.$$

Solution. A circuit using gates can simply be designed by looking at the expression and finding out the basic gates which can be used to realize the various terms and then correct these gates appropriately.

In the given expression there are three input logical variables and X is the output.

- The *first* term $A . B$ is obtained by ANDing A with B as shown in Fig. 18(*i*).
- The *second* term $\overline{A}.\overline{B}$ is obtained by using two INVERTERs and one AND gate and connecting them as shown in Fig. 18 (*ii*).
- The *last* term is used by using one INVERTER one AND gate and connecting them as shown in Fig. 18 (*iii*).

Now, the complete logic expression is realised by ORing the three outputs of the arrangements explained above *i.e.*, by ORing $A.B$, $\overline{A}.\overline{B}$ and $\overline{A}.B.C$. The logic gate implementation for the given expression is shown in Fig. 19.

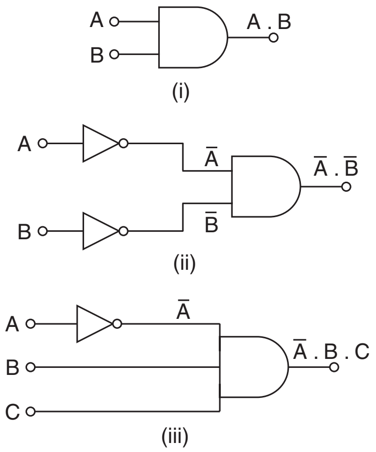


Fig. 18

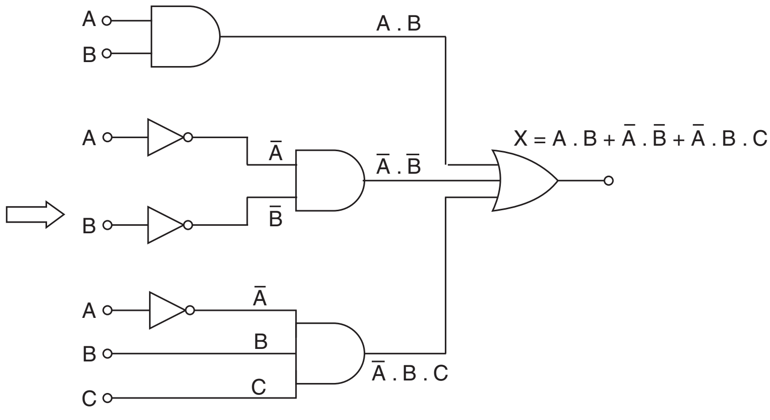


Fig. 19. Logic gate implementation of expression $A \,.\, B + \overline{A} \,.\, \overline{B} + \overline{A} \,.\, B \,.\, C$.

Example 12. *Show that :*

(i) $(A + B)(\overline{A} + C) = AC + \overline{A}B$ *(ii)* $AB + ABC + \overline{A}B + A\overline{B}C = B + AC$

(iii) $ABC + A\overline{B}C + AB\overline{C} = A(B + C)$.

Solution. (*i*) $(A + B)(\overline{A} + C)$

$= A\overline{A} + AC + B\overline{A} + BC$ $(\because \; A\,.\,\overline{A} = 0)$

$= AC + B\overline{A} + BC$

Multiplying the third term by $(A + \overline{A})$, we get

$= AC + B\overline{A} + BC(A + \overline{A})$ [$A + \overline{A}$, being equal to 1 does not make any effect]

$= AC + B\overline{A} + ABC + \overline{A}\,BC$ ...[Eqn. 1 (*d*)]

$= AC(1 + B) + B\overline{A}(1 + C)$

$= AC + B\overline{A}$ ...[Eqn. 1 (*d*)]

$= AC + \overline{A}B$. ...**Proved.**

(*ii*) $AB + ABC + \overline{A}B + A\overline{B}C$

$= AB + \overline{A}B + AC(B + \overline{B})$

$= B(A + \overline{A}) + AC(B + \overline{B})$

$= B + AC$. ...**Proved.** ...[Eqn. 1 (*d*)]

(*iii*) $ABC + A\overline{B}C + AB\overline{C}$

$= AC(B + \overline{B}) + AB\overline{C}$

$= AC + AB\overline{C}$ ...[Eqn. 1 (*d*)]

$= A(C + B\overline{C})$

$= A(C + B)$ ...[Eqn. 6 (*c*)]

$= A(B + C)$. ...**Proved.**

Example 13. *Simplify the expression* $A\overline{A} + C(\overline{A + C}) + AC$.

Solution. $A\overline{A} + C(\overline{A + C}) + AC$

$= 0 + C(\overline{A + C}) + AC$ ...[Eqn. 2 (*d*)]

$= C(\overline{A} . \overline{C}) + AC$ ...[Eqn. (7)]

$= C\overline{A}\,\overline{C} + AC$

$= 0 + AC$ ...[Eqn.2 (*d*)]

= **AC. (Ans.)**

12. DUALS

In Boolean algebra *each expression has its dual which is as true as the original expression*. For getting the dual of a given Boolean expression, the procedure involves conversion of

(*i*) all 1s to 0s and all 0s to 1s.

(*ii*) all ANDs to ORs and all ORs to ANDs.

The dual so obtained is *also found to be true.*

Some of the Boolean relations and their duals are given in Table 4.

Table 4

Relation	Dual Relation
$A . 0 = 0$	$A + 1 = 1$
$A . A = 1$	$A + A = A$
$A . \overline{A} = 0$	$A + \overline{A} = 1$
$A . 1 = A$	$A + 0 = A$
$A . (A + B) = A$	$A + AB = 1$
$A . (\overline{A} + B) = AB$	$A + \overline{A}B = A + B$

Example 14. *Determine the Boolean expression for the logic circuit shown in Fig. 20. Simplify the Boolean expression using Boolean laws and De Morgan's theorem. Redraw the logic circuit using the simplified Boolean expression.*

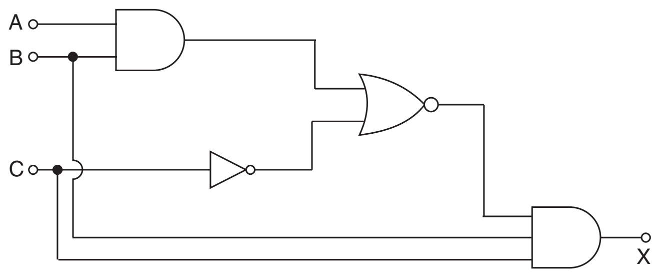


Fig. 20

Solution. The output of a given circuit can be obtained by *determining the output of each logic gate while working from left to right.*

With reference to Fig. 21, the output of the circuit is :

$$X = BC\ (\overline{AB + \overline{C}})$$

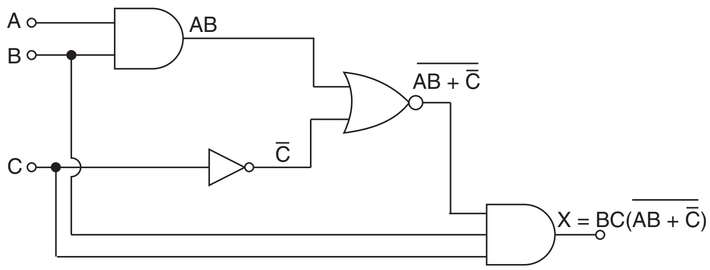


Fig. 21

The output X can be simplified by *De Morganizing* the term $(\overline{AB + \overline{C}})$ as shown below :

$$BC(\overline{AB + \overline{C}}) = BC(AB + \overline{C}) \quad \text{...Step-1}$$
$$= BC(A + B)\ .\ \overline{C} \quad \text{...Step-2}$$
$$= BC(\overline{A} + \overline{B})\ .\ \overline{\overline{C}} \quad \text{...Step-3}$$
$$= BC\ (\overline{A} + \overline{B})C \quad \text{...[Eqn. 3(b)]}$$
$$= BC(\overline{A} + \overline{B}) \quad \text{...[Eqn. 2(c)]}$$
$$= \overline{A}BC + BC\overline{B}$$
$$= \overline{A}BC + 0 \quad \text{...[Eqn. 2(d)]}$$
$$= \overline{A}BC \quad \text{...[Eqn. 1(b)]}$$

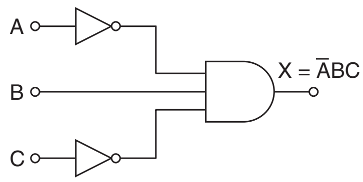


Fig. 22

The logic circuit with a simplified Boolean expression $X = \overline{A}BC$ is as shown in Fig. 22.

Example 15. *Determine the output X of a logic circuit shown in Fig. 23. Simplify the output expression using Boolean Laws and theorems. Redraw the logic circuit with the simplified expression.*

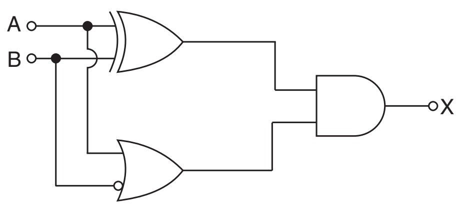


Fig. 23

Solution. The output of the given logic circuit can be obtained by determining the output of each logic gate while working from left to right.

As seen from Fig. 24, the output ;

$$X = (\overline{A}B + A\overline{B})\ (A + \overline{B})$$
$$= \overline{A}BA + A\overline{B}A + \overline{A}B\overline{B} + A\overline{B}\,\overline{B}$$
$$= 0 + A\overline{B} + 0 + A\overline{B}\,\overline{B} \quad \text{...[Eqn. 2(d)]}$$
$$= A\overline{B} + A\overline{B} \quad \text{...[Eqn. 1(b), 2(c)]}$$
$$= A\overline{B} \quad \text{...[Eqn. 1(c)]}$$

Using the simplified Boolean expression, the logic circuit is as shown in Fig. 25.

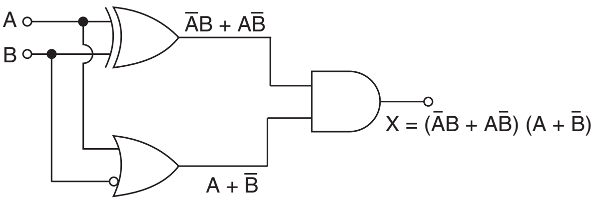


Fig. 24

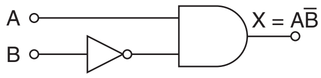


Fig. 25

13. LOGIC SYSTEM

The logic system may be of the following two types :

1. Combinational
2. Sequential.

The essential characteristics of combinational and sequential logic systems are compared below :

Combinational	Sequential
1. Possesses no memory or storage capacity.	Possesses memory or storage capacity.
2. Simple logic gates only carry out the implementation.	To carry out the implementation along-with the logic gates, flip-flops, counters, registers memory cores are also used.
3. The system is described by a set of output functions only.	It is described by a set of output functions and also next state functions.
4. Output of the system depends only on the present input.	Output of the system depends on the present input as well as on the present state of the system.

13.1. Combinational Circuits

A ***combinational circuit*** *consists of logic gates whose outputs at any time are determined directly from the combination of inputs without regard for previous input*.

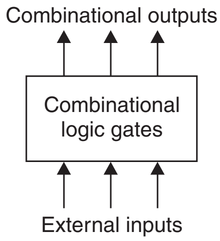


Fig. 26. Combinational logic circuit.

The circuit possesses a set of *inputs,* a *memoryless logic network* to operate on the inputs and a *set of outputs* as shown in Fig. 26. Moreover, output combinational networks are used to make logical decisions and control the operation of different circuits in digital electronic systems. For a given set of input conditions, the output of such a circuit is the same. Consequently, truth table can fully describe the operation of such a circuit.

Examples. Examples of a combinational circuit are :

(*i*) Decoders (*ii*) Adders

(*ii*) Multiplexers (*iv*) Demultiplexers etc.

- **Multiplexers and demultiplexers**
  — *Transmission of a large number of information units over a small number of lines is known as* "**Multiplexing**".
  — "**Demultiplexing**" is a *reverse operation and denotes receiving information from a small number of channels and distributing it over a large number of destinations.*

Design procedure of combinational circuit :

Following *operations* are involved in the design procedure :

1. To state the problem.
2. To determine the number of available input variables and required output variables.
3. To assign letter symbol to each input and output variable.
4. To derive the truth table that defines the required relationship between inputs and outputs.
5. To obtain the simplified Boolean function for each output.
6. To draw the logic diagram.

- A circuit that adds two bits is called a **half adder**.
- A **full adder** consists of three inputs and two outputs. The outputs are designated by the symbol S for sum and C for carry.
- A **two bit subtractor** has two inputs X (minuend) and Y (subtrahend).
- A **full subtractor (FS)** is a *combinational circuit that performs a subtraction between two bits*. This circuit has *three inputs and two outputs*.

Code conversion :

- A variety of codes are used by different digital systems. It is sometimes necessary to use the output of one system as input to the other.
- A conversion circuit must be inserted between the two systems if each uses different codes for the same information.
- To convert from binary code A to binary code B, the input lines must supply the bit combinations of elements as specified by code A and the output lines must generate the corresponding bit combination of code B. A combination circuit performs this transformation by means of *logic gates*.

Comparator. A comparator is a combinational circuit that compares two numbers A and B and determine their relative magnitude. The outcome of the comparison is displayed in three outputs that indicate $A > B \equiv X$, $A = B = Y$, $A < B \equiv Z$.

Decoders and encoders :

— A **decoder** is a combination circuit that converts a binary code of n variables into m output lines, one for each discrete element of information.

— An **encoder** is a combination circuit that accepts m input lines, one for each element of information, and generates a binary code of n output lines.

13.2. Sequential Circuits

Such circuits have *inputs, logic network, outputs* and a *memory*, as shown in Fig. 27. There present output depends not only on their present inputs but also on the previous logic states of the outputs.

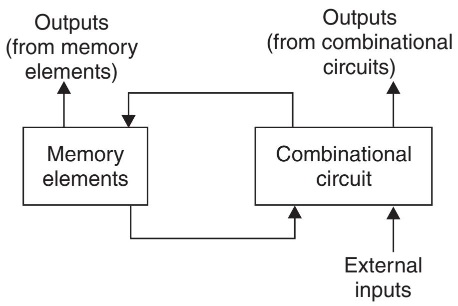


Fig. 27. Block diagram of a sequential circuit.

Examples. Examples of sequential circuits are :

(*i*) *Latches* (*ii*) *Flip-flops*.

The two main *types* of sequential circuits are :

1. Synchronous sequential circuits .... referred to as *clocked-sequential circuits*
2. Asynchronous sequential circuits.

- The **synchronous sequential circuits** are *built to operate* at a *clocked rate whereas* **asynchronous ones** *are without clocking*.

14. FLIP-FLOP CIRCUITS

The memory elements used in clocked sequential circuits are called **flip-flops**. These circuits are binary cells capable of storing one bit of information. It has two *outputs*, *one for the normal value and one for the complement value of the bit stored in it* : Binary information can enter a flip-flop in a variety of ways. Hence there different types of flip-flops.

A number of flip-flops are available in *IC* form. Some of these are *SR* (Set-Reset), *J-K* and *D* flip-flops. They are widely used as *switches, latches, counters, registers* and *memory cells* in computers.

- A salient feature of the flip-flop is that *output can exist in one of the two stable states, logic 1 and logic 0, simultaneously*. This is ensured by the appropriate crossed feedback connections associated with the most elementary form of the flip-flop known as a *latch*.

The following flip-flops will be discussed in the following articles :

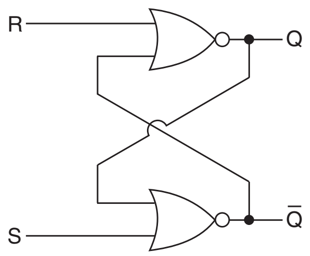
1. *R-S* flip-flop.
2. Clocked *R-S* flip-flop.
3. *D* flip-flop.
4. *J-K* flip flop.
5. *T flip-flop*.

14.1. R-S flip-flop

Fig. 28 shows a *R-S flip-flop using* **NOR** *gates*. There are two inputs to the flip-flops called *S* (set) and *R (reset). The* cross-coupled connection from the output of one gate and input of the other constitutes a feedback path. For that reason, the *circuit is classified as synchronous circuit.*

- A low *R* and a high *S* results in the *set* state.
- A high *R* and a low *S* give the *reset* state.
- If both *R* and *S* are high, the output becomes *indeterminate* and this is known as *'race condition'*. This condition is *avoided by proper design.*

The truth table is shown in Table 5.



(a) Circuit diagram

Table 5. Truth table for NOR latch

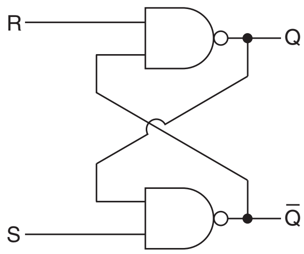
R	S	Q	Comment
0	0	NC	No change
0	1	1	Set
1	0	0	Reset
1	1		Race

(b) Truth table

Fig. 28. *R-S* flip-flop using NOR gates.

Fig. 29 shows a *R-S* flip-flop using **NAND** *gates.* Table 6 shows the truth table. It is seen that the inactive and race conditions are **reversed**.

- When *R* is *low*, output *Q* is *high*.
- When *R* is *high*, output *Q* is *low*.
- When both *R* and *S* are low, we get race condition which must be *avoided.*
- When both *R* and *S* are high - *no change condition*.



(a) Circuit diagram

Table 6. Truth table for NAND latch

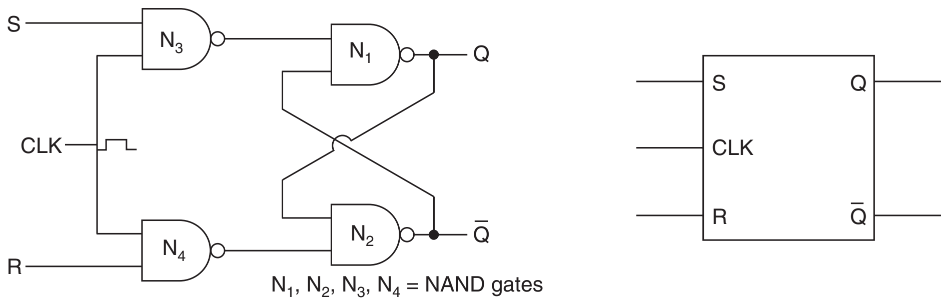
R	S	Q	Comment
0	0		Race
0	1	1	Set
1	0	0	Reset
1	1	NC	No change

(b) Truth table

Fig. 29. *R-S* flip-flop using NAND gates.

14.2. Clocked R-S Flip-Flop

A large number of flip-flops are used in a computer. In order to coordinate their working a square wave signal known as *clock* is applied to the flip-flop. This *clock signal (indicated as CLK) prevents the flip-flop changing state till the right instant occurs.*



(a) Circuit diagram (b) Symbol

Fig. 30. Clocked *R-S* flip-flop.

Fig. 30(*a*) shows a clocked *R-S* flip-flop using NAND gates (N_1 and N_2). This circuit uses two NAND gates N_3 and N_4 to apply CLK signal.

- When CLK is *low* the flip-flop *output Q indicates no change*.
- If *S is high* and *R is low,* the *flip-flop must wait till CLK becomes high before Q can be set on 1.*
- If *S* is low and *R* is low, the flip-flop must wait for CLK to be high before Q is reset to low (0).

Clocked *R-S* flip-flop is a *synchronous sequential logic circuit* because output state of the circuit changes at *discrete clocked instant of time*.

Fig. 30 (*b*) shows a symbol for clocked *R-S* flip-flop.

Level clocking and edge triggering :

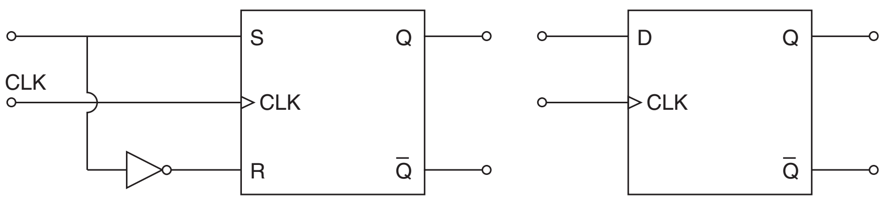
In a clocked flip-flop, the output can change state when CLK is high. When CLK is low, the output remains in the same state. Thus, the output can change state during the entire half cycle when CLK is high. This may be a *disadvantage in several situations*. *It is necessary that the output should change state only at one instant in the positive half cycle of the clock.* This *is known as* ***edge triggering*** *and the resulting flip-flop is known as* ***edge triggered flip-flop***.

Edge triggering can be made feasible by the use of an *RC* circuit. The time constant *RC* is made much smaller than the width of the clock pulse. Therefore, the capacitor can charge fully when CLK is high. The exponential charging produces a narrow positive voltage strike across the resistor. *The input gates are activated at the instant of this positive strike.*

14.3. D Flip-Flop

A *D* flip-flop is an improvement over the *R-S* filp-flop to *avoid race condition*. It can be *level clocked or edge triggered.* The edge triggered one causes the change in output state at a unique instant.

In a *clocked R-S* flip-flop two input signals are required to drive the flip-flop which is a disadvantage with many digital circuits. In some events, both input signals become *high* which is again an undesirable condition. So these shortcomings/drawbacks of clocked *R-S* flip-flop are overcome in *D* flip-flop.



(a) Circuit diagram (b) Symbol

Input D_n	Output Q_{n+1}
0	0
1	1

(c) Truth-table

Fig. 31. D flip-flop

Fig. 31(*a*), (*b*), (*c*) show the circuit diagram, symbol and truth table of *D* flip-flop respectively. It may be observed that only single data bit, *D* is required to drive the flip-flop.

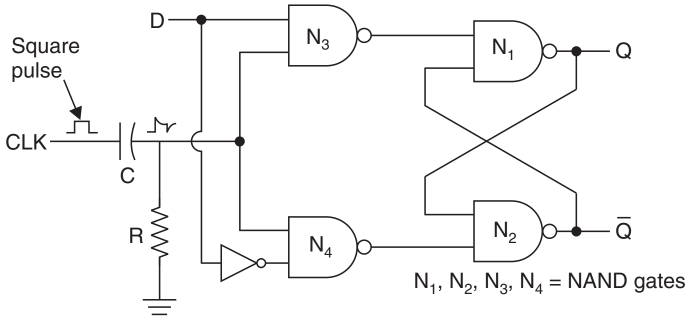
— When the clock signal is at low level, data bit *D* is prevented to reach at output *Q* until clock signal becomes *high* at next pulse.

— It may be noted from the truth table that when data bit D_n is *high*, output Q_{n+1} gets at *high* level and when data bit D_n is low, Q_{n+1} gets at low level. Thus *D* flip-flop transfers the data bit *D* to *Q* as it is, and *Q* remains in the same state until the next pulse of the clock arrives.

- The flip-flop is named (*D*) flip-flop since *the transfer* of *data from the input to output is delayed.*
- The *D*-type flip-flop is *either used as a delay device* or *as a latch to store 1-bit of binary information.*

Edge Triggered D Flip-Flop

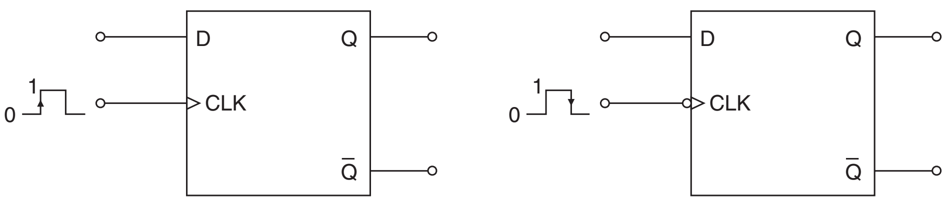
Fig. 32(*a*) shows the circuit diagram and symbols of an edge triggered *D* flip-flop. The *clock provides the square wave signal. RC* circuit *converts* this signal *into strikes* so that triggering occurs at the instant of positive strike. The data bit D *drives one of the inputs*. Because of inverter, the complement $\overline{D}$ *drives the other output*. At the instant of positive strike, input D and its complement $\overline{D}$ cause the output Q to set or reset. Fig. 32(b) shows the truth table.



(a) Circuit diagram

CLK	D	Q
0	×	No change
1	×	No change
↓	×	No change
↑	0	0
↑	1	1

(b) Truth table



Positive or leading — Symbols — Negative or trailing

Fig. 32. Edge triggered D flip-flop.

- When CLK is 0 or 1, the D input is not there and there is no change in state of Q.
- On the *negative* edge of the clock (marked ↓) the output remains in the same state.
- One the *positive* edge of the clock (marked ↑) Q changes to 0 if D is 0 and to 1 if D is 1.

14.4. Edge Triggered J-K Flip-Flop

— *J-K* (fip-flop is *very versatile* and is perhaps the most *widely used type of flip-flop.*

— The J and K designations for the inputs have no known significance except that they are adjacent letters in the alphabet.

— *J-K* flip-flop *functions identically to R-S flip-flop.*

The difference is that the *J-K* flip-flop has no *invalid state* as does the *R-S* flip-flop.

— It is widely used in digital devices such as *counters*, *registers, arithmetic logic units, and other digital systems.*

Fig. 33 (a) shows the circuit diagram of a edge triggered *J-K* flip-flop used in *digital counters.* The CLK input is through an *RC* circuit with a short time constant. The *RC* circuit converts the rectangular clock pulse to narrow spikes as shown. Due to double inversion through NAND gates, the circuit is positive edge triggered.

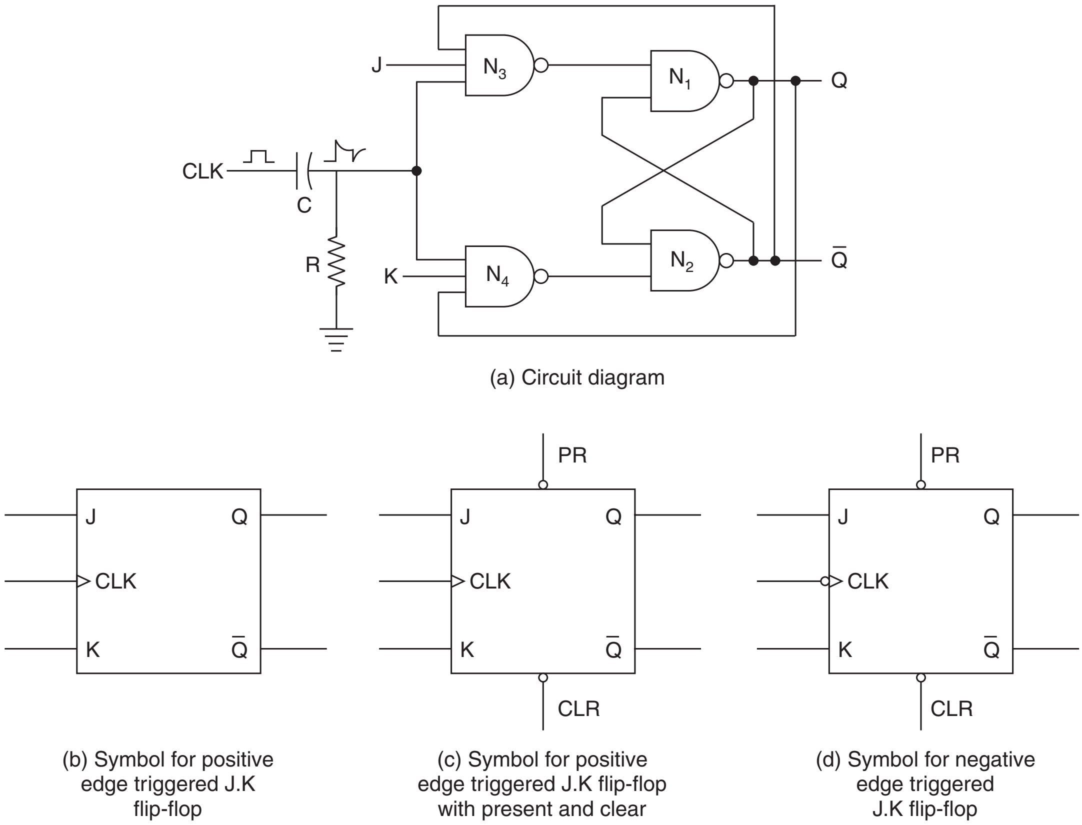


Fig. 33. Edge triggered *J-K* flip-flop.

- When both inputs *J* and *K* are low, the circuit is inactive at all times irrespective of the presence of CLK pulse.
- When *J* is low (*i.e.*, 0) and *K* is high (*i.e.*, 1), the circuit will be reset when positive CLK edge strikes the circuit and $Q = 0$. The flip-flop will remain in reset state if it is already in reset state.
- When $J = 1$ and $K = 0$, the circuit sets at the arrival of next positive clock edge.
- When $J = 1$ and $K = 1$, the flip-flop will toggle (means to switch to opposite state) on the next positive CLK edge. The action is illustrated in the table 7 :

Table 7. Positive edge triggered J-K flip-flop

CLK	J	K	Q
0	×	×	No change
1	×	×	No change
↓	×	×	No change
X	0	0	No change
↑	0	1	0 (reset)
↑	1	0	1 (set)
↑	1	1	toggle

Use of *RC* circuit for edge triggering is not very convenient for fabrication. Actual circuits use additional NAND gates for edge triggering, such circuits are known as *direct coupled circuit*.

— Fig. 33(*b*) shows the symbol for positive edge triggered *J-K* flip-flop.

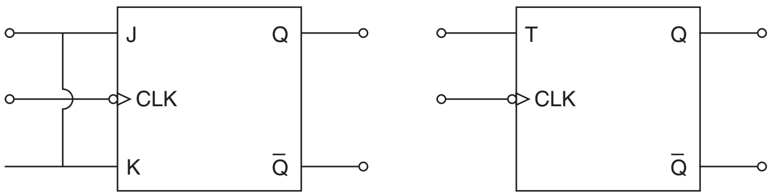
— Fig. 33(*c*) shows a positive edge *J-K* flip-flop with present (*PR*) and clear (CLR)

— Fig. 33(*d*) shows the symbol for negative edge triggered *J-K* flip-flop with *PR* and CLR. The *small bubble at CLR indicates negative triggering*.

14.5. T flip-flop

T flip-flop is basically a *J-K* flip-flop, in this circuit input terminals *J* and *K* are connected with each other and this input is named as *T*.

Fig. 34(*a*) and (*b*) show the circuit diagram and symbol respectively of a trailing edge triggered *T* flip-flop.



(a) Circuit diagram (b) Symbol

Input T_n	Output Q_{n+1}
0	Q_n
1	$\bar{Q}_n$

(c) Truth table

Fig. 34. Trailing edge triggered T flip-flop.

— When **low** *level signal* is applied to the input terminal *T*, then *initial state of output of flip-flop remains the same.*

— When **high** level signal is applied to the input terminal *T*, then output of the flip-flop toggles after arrival of every new clock pulse. So the frequency of output signal is half of the clock signal frequency.

- This flip-flop can be treated as frequency divider or a device which takes the input frequency at the clock terminal and divide it by *two*.

15. COUNTERS

— A *counter* is a *sequential circuit that goes through a prescribed sequence of states upon the application of input pulses.*

— The input pulses, called *count pulses,* may be clock pulse or may originate from an external source and may occur at prescribed intervals of true or random.

— The sequence of states in a counter may follow a binary count or any other sequence of states.

— They are used for counting the number of occurrences of an event and are useful for generating time sequences to control operations in a digital system.

Straight binary sequence counter. It is the simple and most straight forward. An *n*-bit binary counter has *n* flip-flops and can count in binary from 0 to $2^n - 1$.

Binary ripple counter. It consists of a series connections of *T* flip-flops without any logic gates. Each flip-flop is triggered by the output of its preceding flip-flop goes from 1 to 0. The signal propagates through the counter in a *Ripple* manner, *i.e.*, the flip-flop essentially changes once at a time in rapid succession. It is the most simplest and most straight forward. It, however, has speed limitations ; an increase in speed can be obtained by the use of a parallel or a synchronous counter.

Synchronous 3-bit binary counter. In this all flip-flops are triggered simultaneously by count pulse. The flip-flop is complemented only if its T input is equal to 1.

Counter-decoder circuits :

— Counters together with decoders are used to generate timing and sequencing signals that control the operation of digital systems.

— The counter-decoder can be designated to give any desired number of repeated timing sequence.

Applications of counters :

The fundamental applications of counters are given below :

1. Measurement of time interval.
2. Direct counting.
3. Measurement of speed.
4. Measurement of frequency.
5. Measurement of distance.
6. Gating a counter.

16. REGISTERS

A **register** *is a group of memory elements which work together as one unit. The simple resisters only store a binary word. The other registers modify the stored word by shifting its bits to left or right.*

The number of registers differs from processor to processor. A *counter* is a special kind of register to count the number of clock pulses aiming at the input.

The registers can be *classified* as :

(*i*) Accumulator ;

(*ii*) General purpose registers ;

(*iii*) Special purpose registers.

In a digital computer, the programs are executed in central processing unit (CPU). The various instructions of the program are executed in proper sequence. The *computer has a number of registers to store data temporarily during the execution of the program.*

— The ALV requires two numbers on which it operates (*e.g.,* adds, subtracts etc.) and produces the results. These two numbers are known as *'operands'*. One of the operands is obtained from memory and the other from accumulator (an accumulator is a one word memory). The result is placed back into the accumulator. Thus *'accumulator' is the most frequently used register.*

— The *'general purpose registers'* store data and intermediate results during program execution.

— The *'special purpose registers'* include *program counter* (to store the address to the memory location which contains the next instruction to be obtained from the memory), *status register* (to hold indications like vary sign, parity etc.), *index register* (for addressing) etc.

- A *"buffer register"* is the simplest kind of register, it is *used only to store a digital word.*
- A "*shift register*" is *an array of flip-flops designed to store and shift the data.*

HIGHLIGHTS

1. The branch of electronics which deals with digital circuits is called *digital electronics*.
2. An electronic circuit that handles only a digital signal is called a *digital circuit.* In digital circuits the following four systems of arithmetic are often used :
   Decimal, Binal, Octal, Hexadecimal.
3. A digital circuit with one or more input signals but only one output signal is called a *logic gate*.
   In the complex circuits, the following *six* different digital electronics gates are used as basic elements :
   (*i*) NOT gate ; (*ii*) NAND gate (*iii*) AND gate ; (*iv*) OR gate ; (*v*) NOR gate ; (*vi*) XOR gate.
4. The algebra used to symbolically describe logic functions is called *Boolean algebra.*
5. A *combinational circuit* consists of logic gates whose outputs at any time are determined directly from the combination of inputs without regard for previous input.
6. The *synchronous sequential circuits* are built to operate at *a clocked rate* whereas *asynchronous ones* are *without clocking.*
7. The memory elements used in clocked sequential circuits are called *flip-flops.*
8. A *counter* is a sequential circuit that goes through a prescribed sequence of states upon the application of input pulses.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. Which of the following statements is *correct* regarding a pure sine wave ?
   (*a*) It is a digital signal (*b*) It is a digital signal at higher frequencies
   (*c*) It is an analog signal (*d*) It is neither digital nor analog signal.
2. Boolean algebra is essentially based on ....
   (*a*) numbers (*b*) truth
   (*c*) logic (*d*) symbols.
3. In a digital signal the number of levels is
   (*a*) two (*b*) four
   (*c*) six (*d*) ten.
4. The binary system uses powers of ..... for positional values.
   (*a*) 2 (*b*) 4
   (*c*) 6 (*d*) 8.
5. In binary numbers, shifting the binary point one place to the right
   (*a*) multiplies by 2 (*b*) multiplies by 4
   (*c*) divides by 4 (*d*) increases by 2.
6. Binary 111 represents
   (*a*) decimal 4 (*b*) decimal 7
   (*c*) decimal 8 (*d*) decimal 222.
7. In binary system, decimal 1 can be written as
   (*a*) 0001 (*b*) 0010
   (*c*) 0100 (*d*) 1000.
8. After counting 0, 1, 10, 11 the next binary number is
   (*a*) 15 (*b*) 100
   (*c*) 110 (*d*) 120.
9. ..... is *not* an octal number
   (*a*) 15 (*b*) 19
   (*c*) 77 (*d*) 101.

10. Which of the following binary relations is *invalid* ?
(*a*) 1 + 1 = 10 (*b*) 0 × 0 = 0
(*c*) 1 × 1 = 1 (*d*) 0 + 1 = 1.

11. The cumulative addition of the four binary bits (1 + 1 + 1 + 1) gives
(*a*) 100 (*b*) 111
(*c*) 1001 (*d*) 1111.

12. If decimal 10 in binary is 1010, then decimal 100 in binary will be
(*a*) 1111111 (*b*) 1100100
(*c*) 1110100 (*d*) 1000100.

13. The binary equivalent of A_{16} is
(*a*) 1000 (*b*) 1010
(*c*) 1011 (*d*) 1110.

14. The number 1000_2 is equivalent to decimal number
(*a*) four (*b*) eight
(*c*) sixteen (*d*) one thousand.

15. Binary 111111 represents
(*a*) decimal 51 (*b*) decimal 63
(*c*) decimal 87 (*d*) decimal 99.

16. In binary system decimal 0.875 is represented by
(*a*) 0.001 (*b*) 0.0101
(*c*) 0.011 (*d*) 0.111.

17. In binary system decimal 10.75 is represented by
(*a*) 10.1010 (*b*) 101.1110
(*c*) 111.1111 (*d*) 1010.11.

18. Binary 1000 when subtracted from binary 1111, the result will be
(*a*) 111 (*b*) 1000
(*c*) 1010 (*d*) 1110.

19. The result of binary subtraction (100 – 011) is
(*a*) 001 (*b*) 011
(*c*) 111 (*d*) – 111.

20. Due to which of the following main reasons the digital computers use complemental subtraction ?
(*a*) It avoids direct subtraction (*b*) It is a very simple process
(*c*) It simplifies their circuitry (*d*) It can handle negative numbers easily.

21. BCD code is ......
(*a*) a binary code (*b*) an alphanumeric code
(*c*) non-weighted (*d*) the same thing as binary numbers.

22. A logic gate is an electronic circuit which
(*a*) works on binary algebra (*b*) makes logic decisions
(*c*) alternates between 0 and 1 values (*d*) allows electron flow only in one direction.

23. Binary 1000 will be the result of which of the following subtraction in binary system ?
(*a*) 11111 – 1110 (*b*) 1011 – 1110
(*c*) 1111 – 111 (*d*) 1010 – 101.

24. Logic state 1, in positive logic, corresponds to
(*a*) zero voltage (*b*) lower voltage level
(*c*) positive voltage (*d*) higher voltage level.

25. Logic state 1, in negative logic, corresponds to
(*a*) zero voltage (*b*) lower voltage level
(*c*) negative voltage (*d*) more negative voltage.

26. For AND gate the Boolean expression is
(*a*) $A = B$ (*b*) $A + B = Y$
(*c*) $A - B \neq Y$ (*d*) $A \,.\, B = Y$.

27. $A + B = Y$ is the Boolean expression for which of the following ?
(*a*) NOR gate (*b*) AND gate
(*c*) XNOR gate (*d*) None of the above.

28. An XOR gate produces an output only when its two inputs are .....
(*a*) same (*b*) different
(*c*) low (*d*) high.

29. In Boolean algebra different variables used can have values of
(*a*) true or false (*b*) low or high
(*c*) ON or OFF (*d*) 0 or 1.

30. For getting an output from an XNOR gate, its both inputs must be ......
(*a*) at the same logic level (*b*) at the opposite logic level
(*c*) high (*d*) low.

31. ...... gate is formed by inversion of the output of the AND gate.
(*a*) XNOR (*b*) NOR
(*c*) OR (*d*) NAND.

32. ...... gate corresponds to the action of parallel switches.
(*a*) NAND (*b*) OR
(*c*) AND (*d*) NOR.

33. A NOR gate is ON only when all its inputs are
(*a*) OFF (*b*) ON
(*c*) positive (*d*) high.

34. A combination of AND function and NOT function will results in .... gate.
(*a*) NAND (*b*) AND
(*c*) XNOR (*d*) NOR.

35. $A = \overline{A}$ is the Boolean expression for
(*a*) multiplier (*b*) inverter
(*c*) adder (*d*) subtractor.

36. ...... logic function has the output low only when both inputs are high.
(*a*) NOR (*b*) OR
(*c*) AND (*d*) NAND.

37. The dual of the statement $(A + 1) = 1$ is
(*a*) $A + A = A$ (*b*) $A \,.\, 1 = A$
(*c*) $A \,.\, A = 1$ (*d*) $A \,.\, 0 = 0$.

38. In Boolean algebra, $1 + A + B + C$ is equal to ....
(*a*) $1 + 3A$ (*b*) $1 + A$
(*c*) A (*d*) 1.

39. The radix for binary system is
(*a*) 10 (*b*) 2
(*c*) 1 (*d*) 0.

40. In Boolean algebra, $A + A + A + \ldots\ldots + A$ is the same as
(*a*) zero (*b*) A
(*c*) nA (*d*) A^n.

41. The expression $\overline{ABC}$ can be simplified to
(*a*) $\overline{A} + \overline{B} + \overline{C}$ (*b*) $\overline{A} \,.\, \overline{B} \,.\, \overline{C}$
(*c*) $AB + \overline{C}$ (*d*) $AB + BC + CA$.

42. According to Boolean algebra if $A = 0$, $B = 0$, then $\overline{A}\ \overline{B}$ is
(*a*) 10 (*b*) 1
(*c*) 0 (*d*) none of the above.

43. In a certain 2-input logic gate, when $A = 0$, $B = 0$ and $C = 1$, it must be ......
(*a*) NOR gate (*b*) XOR gate
(*c*) AND gate (*d*) NAND gate.

44. Integrated circuit logic gates contain the properties of
(*a*) diodes (*b*) bipolar junction transistors
(*c*) resistors (*d*) all of the above.

45. Saturated logic circuits have inherently ....
(*a*) higher power dissipation (*b*) low switching speed
(*c*) short saturation delay time (*d*) lower noise immunity.
(*e*) none of the above.

46. The abbreviation TTL stands for
(*a*) Transistor-transformer logic (*b*) Transistor-transistor logic
(*c*) Transmitter-transistor logic (*d*) Tuned transistor loop.

47. IGFET stands for
(*a*) Insulated gate field effect transistor (*b*) Integrated gain field effect transistor
(*c*) Infinite gain field effect transistor (*d*) Imaginary grid field effect transistor.

48. DTL stands for
(*a*) Delayed tracking logic (*b*) Diode transistor logic
(*c*) Digital timing logic (*d*) Dynamic transient logic.

49. The basic DTL configuration is ...... gate.
(*a*) OR (*b*) NOR
(*c*) NAND (*d*) AND.

50. The basic circuit configuration for TTL resembles that of a .... gate.
(*a*) OR (*b*) NOR
(*c*) AND (*d*) NAND.

51. The flip-flop circuit is ......
(*a*) unstable (*b*) multistable
(*c*) monostable (*d*) bistable.

52. A digital counter consists of a group of
(*a*) flip-flops (*b*) half adders
(*c*) full adders (*d*) none of the above.

53. For LCD display which of the following liquid crystals is used ?
(*a*) Aqua regia (*b*) Nemanic fluid
(*c*) Liquid boron (*d*) Mercury.

54. .... needs D.C. forward voltage to emit light.
(*a*) LCD (*b*) LED
(*c*) Both (*a*) and (*b*) (*d*) None of the above.

55. A digital voltmeter has .... input and ......output.
(*a*) digital, digital (*b*) digital, analog
(*c*) analog, analog (*d*) analog, digital.

56. ...... display consumes least amount of power.
(*a*) LED (*b*) LCD
(*c*) Fluorescent display (*d*) All display consumes same power.

57. Which of the following memories has both read and write capabilities ?
(*a*) ROM only (*b*) RAM only
(*c*) Both (*a*) and (*b*) (*d*) None of the above.

58. A 4-bit counter with four flip-flops will count upto decimal ....
(*a*) 8 (*b*) 15
(*c*) 31 (*d*) 63.

59. PROM stands for
(*a*) Positive read only memory (*b*) Permanent read only memory
(*c*) Polarized read only memory (*d*) Programmable read only memory.

60. In a *RS* flip-flop no change occurs during
(*a*) set mode (*b*) reset mode
(*c*) disabled mode (*d*) prohibited mode.

61. Compounds of .... are generally used for LED.
(*a*) sulphur (*b*) silica
(*c*) phosphorus (*d*) gallium.

62. ...... family of logic circuits uses field effect transistors.
(*a*) CMOS (*b*) TTL
(*c*) Both (*a*) and (*b*) (*d*) None of the above.

63. Power is drawn by a CMOS circuit only when
(*a*) in static state (*b*) its output is high
(*c*) its output is low (*d*) its switches logic levels.

64. CMOS circuits are extensively used for one-chip computers mainly because of their extremely ......
(*a*) high noise immunity (*b*) low power dissipation
(*c*) low cost (*d*) large paching density.

65. .... multivibrator can be used as a clock timer.
(*a*) Astable (*b*) Bistable
(*c*) Either of the above (*d*) None of the above.

66. The output of a 2-input OR gate is zero only when its
(*a*) either input is 0 (*b*) either input is 1
(*c*) both inputs are 0 (*d*) both inputs are 1.

67. Which of the following 4-bit combination(s) is/are invalid in the BCD code ?
(*a*) 0010 (*b*) 0101
(*c*) 1000 (*d*) 1010.

68. The number 100101_2 is equal to octal .....
(*a*) 25 (*b*) 37
(*c*) 45 (*d*) 54.

69. A unique advantageous feature of CMOS logic family is its
(*a*) speed (*b*) use of NMOS circuits
(*c*) dependence on frequency (*d*) power dissipation is nanowatt range.

70. The two outputs of *RS* flip-flop are
(*a*) always high (*b*) always low
(*c*) either low or high (*d*) always complementary.

71. An *A/D* converter uses for reference purposes
(*a*) a flip-flop (*b*) a saw tooth generator
(*c*) d.c. voltage (*d*) set of keys.

72. ...... converter can be used to change analog voltage to binary data.
(*a*) *A/D* (*b*) *D/A*
(*c*) Both (*a*) and (*b*) (*d*) None of the above.

73. The number of full adders in a 4-bit parallel adder will be
(*a*) two (*b*) three
(*c*) four (*d*) six.

74. Which of the following binary additions is *incorrect* ?
(*a*) 1 + 1 = 0 (*b*) 0 + 1 = 1
(*c*) 0 + 0 = 0 (*d*) 1 + 0 = 1.

75. ...... is synchronous.
(*a*) Full adder (*b*) Half adder
(*c*) Clocked *R-S* flip-flop (*d*) *R-S* flip-flop.

76. A *D*-flip-flop is .... flip-flop.
(*a*) digital (*b*) delayed
(*c*) dial type (*d*) differential.

77. The output of basic DTL configuration is
(*a*) high when all inputs are low (*b*) high when all inputs are high
(*c*) low when all inputs are high (*d*) low when one of the inputs is high.

78. In Schottky TTL, a Schottky diode is used primarily to
(*a*) act as a switch (*b*) act as a controlling switch
(*c*) prevent saturation of the transistor (*d*) saturate the transistor.

79. .... flip-flop is used as latch.
(*a*) *T* (*b*) *D*
(*c*) *JK* (*d*) *RS*.

80. Which of the following codes is used to reduce the error due to ambiguity in reading of a binary optical encoder ?
(*a*) BCD code (*b*) Gray code
(*c*) Octal code (*d*) Excess-3 code.

81. For digital *ICs* which of the following parameters is not specified ?
(*a*) Noise margin (*b*) Bandwidth
(*c*) Gate dissipation (*d*) Propagation delay.

82. A gate in which all inputs must be low to get a high output is called
(*a*) an AND gate (*b*) a NAND gate
(*c*) an inverter (*d*) a NOR gate.

83. Which of the following is the simplified versions of the Boolean expression $\overline{A}B + A\overline{B}C + \overline{(A + B + C)}$
(*a*) $\overline{A}B + \overline{B}C$ (*b*) $A\overline{B} + \overline{B}C$
(*c*) $AB + BC$ (*d*) $A\overline{B} + B\overline{C}$.

84. For implementation of all functions of the basic logic functions, it suffices to have
(*a*) NOT (*b*) ANDNOT
(*c*) OR (*d*) none of the above.

85. The memory element in magnetic film memory consists of
(*a*) doped aluminium (*b*) plated wires
(*c*) nickle iron alloy (*d*) superconductive material.

86. To solve differential equations numerically which of the following methods is used ?
(*a*) Newton-Raphson method (*b*) Gauss-elimination method
(*c*) Runga-Kutta method (*d*) Any of the above.

87. .... is used for storing binary information
(*a*) A latch (*b*) A register
(*c*) A flip-flop (*d*) All of the above.

88. BCD expresses each decimal digit as
(*a*) a byte (*b*) a string of 8 bits
(*c*) a string of 4 bits (*d*) a string of 2 bits.

89. Which of the following statements is *incorrect* ?
(*a*) $Z X + Z \overline{X} Y = Z X + Z Y$ (*b*) $X(\overline{X} + Y) = X Y$
(*c*) $X + \overline{X} Y = X$ (*d*) $X Y + X\overline{Y} = X$.

90. The output states in sequential circuits are function of
(*a*) present and past input (*b*) presents input states
(*c*) past input states (*d*) none of the above.

91. When a binary adder is used as BCD adder, the sum is *correct* when it is
(*a*) less than 9 (*b*) greater than 9
(*c*) less than 16 (*d*) none of the above.

92. During instructions execution read cycle is always followed by
(*a*) delete signal (*b*) read cycle
(*c*) write cycle (*d*) none of the above.

93. Schmitt trigger can be used as a
(*a*) flip-flop (*b*) comparator
(*c*) square-wave generator (*d*) all of the above.

94. For digital *ICs* the most widely used 'Bipolar Technology' is
(*a*) ECL (*b*) DTL
(*c*) TTL (*d*) none of the above.

95. .... circuit can be used as parallel-to-series converter
(*a*) Multiplexer (*b*) Digital counter
(*c*) Decoder (*d*) De-multiplexer.

96. A half adder has which of the following ?
(*a*) Two inputs and two outputs (*b*) Three inputs and two outputs
(*c*) Two inputs and one output (*d*) One input and one output.

97. .... flip-flop does not have race problem.
(*a*) *D* (*b*) *T*
(*c*) *JK* (*d*) Master-slave.

98. A ring counter is same as
(*a*) shift resister (*b*) parallel counter
(*c*) up-down counter (*d*) none of the above.

99. A gate in which all inputs must be high to get a low input is called
(*a*) A NAND gate (*b*) An AND gate
(*c*) A NOR gate (*d*) An inverter.

100. The schmitt trigger, for a sinusoidal input, gives output as
(*a*) sinusoidal itself (*b*) square wave
(*c*) saw tooth (*d*) none of the above.

101. .... error can be usually detected by a parity check.
(*a*) One-bit (*b*) Double-bit
(*c*) Three-bit (*d*) Any-bit.

102. BCD numbers are obtained by
(*a*) converting binary to decimal
(*b*) each decimal digit is represented by a four bit binary
(*c*) converting decimal number to binary
(*d*) converting decimal to octal numbers.

103. A BYTE stands for a string of .... BITS.
(*a*) two (*b*) four
(*c*) eight (*d*) twelve.

104. ..... is an unweighted code.
(*a*) 63210 (*b*) 2421
(*c*) 8421 (*d*) Excess-3 code.

105. Semiconductor memories are
(*a*) non-volatile, small size (*b*) volatile, small size
(*c*) volatile (*d*) non-volatile.

106. Due to which of the following reasons a NAND gate is called a universal logic element ?
(*a*) Many digital computers use NAND gates
(*b*) All the minimizing techniques are applicable for optimum NAND gate realization
(*c*) Any logic function can be realized by NAND get alone
(*d*) All of the above.

107. *K*-map method of simplification can only be applied when the given function is in
(*a*) canonical form (*b*) product of sum form
(*c*) sum of product form (*d*) any of the above form.

108. In which of the following the power dissipation is the lowest ?
(*a*) ECL (*b*) MOS
(*c*) TTL (*d*) None of the above.

109. NAND operation is
(*a*) $\overline{X} + \overline{Y}$ (*b*) $\overline{X} \,.\, \overline{Y}$
(*c*) $(\overline{X} + \overline{Y})(\overline{X} + Y)$ (*d*) $\overline{X + Y}$.

110. Which of the following are the most widely used universal gates ?
(*a*) NAND and OR gates (*b*) NOR and AND gates
(*c*) OR and AND gates (*d*) NOR and NAND gates.

111. As compared to analog computers, digital computers are more widely used because they are
(*a*) easier to maintain (*b*) useful over wider ranges of problem types
(*c*) less expensive (*d*) always more accurate and faster.

112. In a full adder, there are
(*a*) three binary digit inputs and three binary digit outputs
(*b*) three binary digit inputs and binary outputs
(*c*) two binary number inputs and two outputs (*d*) none of the above.

113. Generally ..... flip-flops are used in shift registers.
(*a*) *D* (*b*) *T*
(*c*) *SR* (*d*) *JK*.

114. In octal system the value of 2^5 is
(*a*) 20 (*b*) 40
(*c*) 200 (*d*) 400.

115. Which of the following circuits exhibits memory ?
(*a*) Ex. OR gate (*b*) NAND gate
(*c*) Astable multivibrator (*d*) Bistable multivibrator.

116. A simple flip-flop is a .... bit storage cell.
(*a*) one (*b*) two
(*c*) three (*d*) four.

117. An AND gate is a .... circuit.
(*a*) relaxation (*b*) memory
(*c*) sequential (*d*) combinational.

118. The ASCII code is for information interchange by a binary code for
(*a*) alpha numerics and other common symbols (*b*) alphabet only
(*c*) numbers only (*d*) none of the above.

119. .... time is shortest interval between pulses with which a binary will switch from one state to another.
(*a*) Delay (*b*) Resolving
(*c*) Rise (*d*) Relaxation.

120. State diagram and state tables are used for
(*a*) pulse operation analysis (*b*) level operation analysis
(*c*) any of the above (*d*) none of the above.

121. Which of the following statements is *correct* ?
(*a*) Race condition is not desirable at all (*b*) Race condition is only dangerous if critical
(*c*) Race condition is always desirable feature of digital circuit
(*d*) Race condition is sometime desirable in digital systems.

122. For a TTL gate the recommended fan-out is
(*a*) 5 (*b*) 10
(*c*) 15 (*d*) 20.

123. The basic universal gate in integrated circuit electronics is .... gate.
(*a*) OR (*b*) NAND
(*c*) AND (*d*) NOR.

124. Truth table can only be used for ...... circuit.
(*a*) asynchronous (*b*) synchronous
(*c*) combinational (*d*) digital.

125. Dynamic memory cells are constructed using
(*a*) FETs (*b*) MOSFETs
(*c*) flip-flops (*d*) transistors.

126. Storage time of a transistor is the time taken for the collector current to fall to
(*a*) 10 percent from maximum value (*b*) 20 percent to 80 percent from maximum value
(*c*) 60 percent from maximum value (*d*) 90 percent from maximum value.

127. The *D* flip-flop and *T* flip-flops are respectively used as
(*a*) delay and toggle switch (*b*) toggle switch and delay gate
(*c*) both used as toggle switch (*d*) both used as delay gates.

128. Which of the following is an advantage of self-correcting code ?
(*a*) It has even parity (*b*) It is a weighted code
(*c*) It is easy to decode electronically (*d*) All of the above.

129. Counters can be used to measure
(*a*) distance (*b*) time
(*c*) frequency.

130. The typical power consumption of low power TTL is
(*a*) 2 mW (*b*) 1 mW
(*c*) 10 mW (*d*) 100 μW.

131. The .... logic is mostly used in microprocessor chips.
(*a*) TTL (*b*) DTL
(*c*) CMOS (*d*) NMOS.

132. If $A + B + C = 1$, then which of the following is *true* ?
(*a*) $A + B = \overline{C}$ (*b*) $B + C = \overline{A}$
(*c*) $C + A = \overline{B}$ (*d*) All of the above.

133. Odd out the *false* statement :
(*a*) The Schottky TTL is used whenever high switching speed is not required
(*b*) The typical power dissipation incase of low power TTL is 1 mW
(*c*) The typical propagation delay in case of low power TTL is 35 ns
(*d*) The standard TTL will be implemented whenever high output current is required.

134. The EX-OR gate is well used in
(*a*) controlled invertor logic circuits (*b*) comparator circuits
(*c*) parity checker/generators (*d*) all of the above.

135. If $A + B + C = 0$ then $A + B =$
(*a*) $\overline{C}$ (*b*) C
(*c*) 0 (*d*) 8.

136. If the number of inputs to a multiplexer are 22, the number of control signals needed are
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 8.

137. The AND gate in positive logic is equivalent to .... gate in negative logic.
(*a*) NAND (*b*) OR
(*c*) AND (*d*) NOT.

138. Odd out the *false* statement :
(*a*) The output of an EX-NOR gate is low only when odd number of inputs to it are high
(*b*) The output of an EX-OR gate is low only when even number of inputs are present at it are high
(*c*) The EX-OR gate is extensively used in parity checkers
(*d*) The EX-OR is not an universal gate.

139. If the number of outputs of a demultiplexer are 34, then the number of control signals needed are
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 8.

140. Any circuit that simulates mental processes is called the
(*a*) processing circuit (*b*) digital simulator
(*c*) logic circuit (*d*) analog simulator.

141. $A + B + A =$
(*a*) $A + B$ (*b*) B
(*c*) A (*d*) 1.

142. $A + \overline{A}B =$
(*a*) B (*b*) A
(*c*) $\overline{A} + B$ (*d*) $A + B$.

143. The minimum number of NAND gates required to construct a half adder is ....
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7.

144. Which of the following logic families has high fanout parameter ?
(*a*) TTL (*b*) DTL
(*c*) CMOS (*d*) PMOS.

145. $\overline{A} + 1 =$
(*a*) $\overline{A}$ (*b*) 0
(*c*) 1 (*d*) A.

146. The digital circuits are mostly reliable because
(*a*) they will not make any erroneous results
(*b*) all electronic devices will operate reliably in two-state operation
(*c*) they can be designed with high accuracy (*d*) of all the above reasons.

147. The minimum number of NAND gates required to constructed an Ex-OR gate is
(*a*) 2 (*b*) 4
(*c*) 5 (*d*) 6.

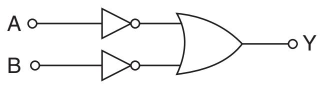
148. If a logical expression is true then its dual is
(*a*) true (*b*) false
(*c*) either true or false (*d*) none of the above.

149. The *ICs* containing more than 100 gates per chip and less than 1000 gates per chip will come under
(*a*) SSI (*b*) MSI
(*c*) LSI (*d*) VLSI.

150. $A + A =$
(*a*) A (*b*) 1
(*c*) 0 (*d*) none of the above.

151. The OR gate is also known as
(*a*) any-or-nothing gate (*b*) any-or-all gate
(*c*) all-or-nothing gate (*d*) none of the above.

152. The figure given below represents .... operation.



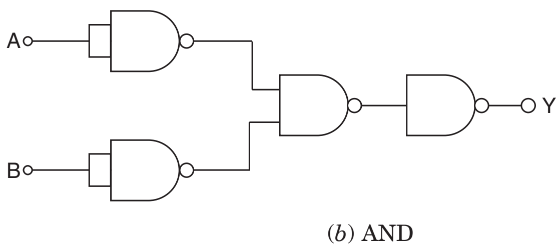
(*a*) AND (*b*) OR
(*c*) EX-OR (*d*) NAND.

153. Each appearance of Boolean variable either in complemented or in uncomplemented form is called
(*a*) variable (*b*) complement
(*c*) literal (*d*) dual.

154. The OR gate in positive logic is equivalent to .... gate in negative logic.
(*a*) AND (*b*) NOT
(*c*) NAND (*d*) OR.

155. The dual of the Boolean expression $\overline{AB} + AB + \overline{BD}$ is
(*a*) $(A + \overline{B})(A + B)(B + D)$ (*b*) $(\overline{A} + B)(\overline{A} + \overline{B})(B + D)$
(*c*) $(A + \overline{B})(A + B)(\overline{B + D})$ (*d*) $(\overline{A} + B)(\overline{A} + \overline{B})(\overline{B + D})$.

156. The figure given below performs ....



(*a*) NAND (*b*) AND
(*c*) OR (*d*) NOR.

157. $\overline{XY} + \overline{X}\,\overline{Y} + \overline{X}Y + \overline{XY} =$
(*a*) X (*b*) 1
(*c*) 0 (*d*) Y.

158. Which of following logic families has low noise generation parameter ?

(*a*) DTL (*b*) NMOS

(*c*) TTL (*d*) CMOS.

159. Which of the following statements is *true* ?

(*a*) The dual of a function is false if that logical function is true

(*b*) The dual of complement of a function equals the complement of dual of that function

(*c*) The dual of complement of a function is not equal to the complement of dual of a function

(*d*) None of the above.

160. The bubbled OR gate in any logic circuit can be replaced by

(*a*) NAND gate (*b*) NOR gate

(*c*) EX-OR gate (*d*) AND-OR circuit.

161. $X + \overline{X}YZ + \overline{YZ}X =$

(*a*) $X + YZ$ (*b*) $X + \overline{YZ}$

(*c*) $\overline{X} + YZ$ (*d*) $\overline{X} + \overline{YZ}$.

162. The result of $A + AB + ABC + ABCD + \ldots\ldots \infty$ is

(*a*) A (*b*) $\overline{A}$

(*c*) $A + B + C + D + \ldots\ldots \infty$ (*d*) 1.

163. The EX-OR gate is also called

(*a*) difference circuit (*b*) 1-bit comparator

(*c*) Inequality circuit (*d*) all of the above.

164. The reduced form of the Boolean expression $\overline{XYZ + XY} + YZ$ is

(*a*) $\overline{X} + \overline{Y} + \overline{Z} + \overline{X} \,.\, \overline{Y}$ (*b*) $\overline{X}\,\overline{Y}\,\overline{Z} + \overline{X} \,.\, \overline{Y}$

(*c*) $\overline{X}\,Y\overline{Z} + \overline{XY}$ (*d*) $XYZ + \overline{XY}$.

165. Which of the following is a sequential logic circuit ?

(*a*) Flip-flop (*b*) OR gate

(*c*) AND gate (*d*) EX-OR gate.

166. $\overline{X}\,YZ + X\overline{YZ} + \overline{YZ} + YZ =$

(*a*) $\overline{X} + \overline{YZ}$ (*b*) $X + YZ$

(*c*) 1 (*d*) 0.

167. Which of the following logic families is having the lowest power consumption ?

(*a*) DTL (*b*) CMOS

(*c*) PMOS (*d*) NMOS.

168. The minimum number of NOR gates required to construct an EX-NOR gate is

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6.

169. $\overline{A + B} =$

(*a*) $AB + \overline{A\,B}$ (*b*) $\overline{A\,B} + \overline{A\,B}$

(*c*) $AB + \overline{A\,B}$ (*d*) $\overline{A\,B} + \overline{A}B$.

170. The minimum number of NAND gates required to perform NOT operation is

(*a*) 2 (*b*) 3

(*c*) 1 (*d*) 4.

171. Odd out the *false* statement :
(*a*) The parallel adder requires more number of full adder circuits
(*b*) The parallel adder can add the bits very fastly
(*c*) The speed of operation of serial adder when compared to parallel adder is better
(*d*) The hardware requirement of serial adder is less.

172. The OR gate can be constructed with two .... gates.
(*a*) EX-NOR (*b*) NAND
(*c*) NOR (*d*) EX-OR.

173. If the output of a logic circuit can be any of its inputs depending on control bit pattern, then that circuit is the
(*a*) demultiplexer (*b*) decoder
(*c*) encoder (*d*) multiplexer.

174. A .... gate recognizes the input word whose all bits are low.
(*a*) NAND (*b*) OR
(*c*) AND (*d*) NOR.

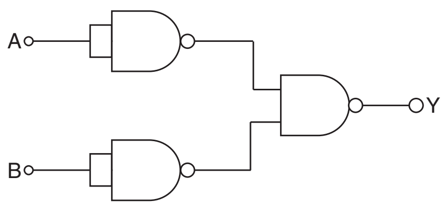
175. The logic that is generally employed in pocket calculators is
(*a*) DTL (*b*) NMOS
(*c*) CMOS (*d*) TTL.

176. The basic gate function of CMOS family is
(*a*) NAND/NOR function (*b*) NOR/EX-OR function
(*c*) NOR/OR function (*d*) NAND/AND function.

177. The bubbled AND gate in any logic circuit can be replaced by
(*a*) AND-OR circuit (*b*) NOR gate
(*c*) EX-OR gate (*d*) NAND gate.

178. $(A + B)(A + C) =$
(*a*) $A + BC$ (*b*) $AC + BC$
(*c*) $AB + AC$ (*d*) $(A + B)C$.

179. The figure given below is equivalent to



(*a*) EX-OR gate (*b*) AND gate
(*c*) OR gate (*d*) NOT gate.

180. The clocked flip-flop is used because
(*a*) the propagation delay can be reduced (*b*) clock can be used to control the timing
(*c*) it is convention (*d*) clock input is available.

181. The latch can be developed using
(*a*) NOT gates (*b*) OR gates
(*c*) AND gates (*d*) NAND gates.

182. When both the J and K inputs of an edge triggered JK flip-flop are 1, then the flip-flop is in
(*a*) asynchronous condition (*b*) synchronous condition
(*c*) race condition (*d*) toggle condition.

183. A simple flip-flop is a .... cell.
(*a*) 1 bit storage (*b*) 4 bit storage
(*c*) 3 bit storage (*d*) 2 bit storage.

184. The number of outputs and inputs of a basic *RS* flip-flop is
(*a*) 2, 2 (*b*) 3, 2
(*c*) 2, 3 (*d*) 1, 2.

185. A *JK* flip-flop can be used as a *D* flip-flop if
(*a*) $J = 0$ and $K = 0$ (*b*) an invertor is added such that $J = K$
(*c*) $J = 0$ and $K = 1$ (*d*) the *JK* inputs are tied together.

186. A delay flip-flop is
(*a*) *JK* flip-flop (*b*) *D* flip-flop
(*c*) *RS* flip-flop (*d*) *T* flip-flop.

187. In a *RS* flip-flop the input conditions for *R* and *S* for which the flip-flop is reset is
(*a*) $R = 0, S = 0$ (*b*) $R = 1, S = 0$
(*c*) $R = 1, S = 1$ (*d*) $R = 0, S = 1$.

188. To reset *RS* flip-flop, the *R* and *S* inputs respectively must be
(*a*) 1 and 0 (*b*) 0 and 0
(*c*) 1 and 1 (*d*) 0 and 1.

189. A flip-flop is the fundamental building block of .... logic circuit.
(*a*) sequential (*b*) arithmetic
(*c*) combinational (*d*) none of the above.

190. A negative edge triggered flip-flop transfers data from input to output
(*a*) during leading edge of the clock (*b*) when clock is high
(*c*) during trailing edge of the clock (*d*) when clock is low.

191. In a *RS* flip-flop using NOR gates when $R = S = 0$, the output *Q* when the flip-flop is clocked is
(*a*) 0 (*b*) 1
(*c*) indeterminate (*d*) no change.

192. A level triggered flip-flop transfers data from input to output
(*a*) when the clock is either high or low (*b*) during the leading edge of clock
(*c*) during the trailing edge of clock (*d*) none of the above.

193. In a sequential circuit the output depends on
(*a*) the interconnections in the circuit (*b*) the combination of inputs
(*c*) the input at that particular instant (*d*) the inputs and the previous outputs.

194. To operate a clocked *RS* flip-flop, we need
(*a*) both *R* and *S* inputs (*b*) clock enable input
(*c*) clock disable input (*d*) *S* input only.

195. If one wants to design a binary counter, preferred type of flip-flop is
(*a*) *D* type (*b*) *JK* type
(*c*) Latch (*d*) *SR* type.

196. A flip-flop that operates in step with the clock operates
(*a*) asynchronously (*b*) synchronously
(*c*) both (*a*) and (*b*) (*d*) none of the above.

197. In a *JK* flip-flop the output state will toggle only when
(*a*) $J = 1, K = 0$ (*b*) $J = 0, K = 1$
(*c*) $J = 0, K = 0$ (*d*) $J = 1, K =1$.

198. Basic *RS* flip-flop can be constructed from
(*a*) two OR gates (*b*) two AND gates
(*c*) two cross coupled NAND gates (*d*) none of the above.

199. If $R = 1$ and $S = 1$, the output of *RS* flip-flop lies in
(*a*) transient state (*b*) race condition
(*c*) triggered state (*d*) latched state.

200. A group of flip-flops connected in cascade to store a binary word is termed as a
(*a*) register (*b*) decoder
(*c*) multiplexer (*d*) counter.

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)
6. (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*b*)	**10.** (*d*)
11. (*a*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)
16. (*d*)	**17.** (*d*)	**18.** (*a*)	**19.** (*a*)	**20.** (*c*)
21. (*a*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)	**25.** (*b*)
26. (*d*)	**27.** (*d*)	**28.** (*b*)	**29.** (*d*)	**30.** (*a*)
31. (*d*)	**32.** (*b*)	**33.** (*a*)	**34.** (*a*)	**35.** (*b*)
36. (*d*)	**37.** (*d*)	**38.** (*d*)	**39.** (*b*)	**40.** (*b*)
41. (*a*)	**42.** (*c*)	**43.** (*d*)	**44.** (*d*)	**45.** (*b*)
46. (*b*)	**47.** (*a*)	**48.** (*b*)	**49.** (*c*)	**50.** (*d*)
51. (*d*)	**52.** (*a*)	**53.** (*b*)	**54.** (*b*)	**55.** (*d*)
56. (*b*)	**57.** (*b*)	**58.** (*b*)	**59.** (*d*)	**60.** (*c*)
61. (*d*)	**62.** (*a*)	**63.** (*d*)	**64.** (*d*)	**65.** (*a*)
66. (*c*)	**67.** (*d*)	**68.** (*c*)	**69.** (*d*)	**70.** (*d*)
71. (*b*)	**72.** (*a*)	**73.** (*b*)	**74.** (*d*)	**75.** (*c*)
76. (*b*)	**77.** (*c*)	**78.** (*c*)	**79.** (*d*)	**80.** (*b*)
81. (*b*)	**82.** (*d*)	**83.** (*a*)	**84.** (*b*)	**85.** (*c*)
86. (*c*)	**87.** (*d*)	**88.** (*c*)	**89.** (*c*)	**90.** (*a*)
91. (*a*)	**92.** (*c*)	**93.** (*d*)	**94.** (*c*)	**95.** (*a*)
96. (*a*)	**97.** (*d*)	**98.** (*a*)	**99.** (*a*)	**100.** (*b*)
101. (*a*)	**102.** (*b*)	**103.** (*c*)	**104.** (*d*)	**105.** (*b*)
106. (*b*)	**107.** (*c*)	**108.** (*b*)	**109.** (*a*)	**110.** (*d*)
111. (*b*)	**112.** (*b*)	**113.** (*d*)	**114.** (*b*)	**115.** (*d*)
116. (*a*)	**117.** (*d*)	**118.** (*a*)	**119.** (*b*)	**120.** (*c*)
121. (*b*)	**122.** (*b*)	**123.** (*c*)	**124.** (*b*)	**125.** (*b*)
126. (*d*)	**127.** (*a*)	**128.** (*c*)	**129.** (*d*)	**130.** (*b*)
131. (*a*)	**132.** (*d*)	**133.** (*a*)	**134.** (*d*)	**135.** (*c*)
136. (*b*)	**137.** (*b*)	**138.** (*b*)	**139.** (*c*)	**140.** (*b*)
141. (*a*)	**142.** (*d*)	**143.** (*b*)	**144.** (*c*)	**145.** (*c*)
146. (*b*)	**147.** (*b*)	**148.** (*a*)	**149.** (*c*)	**150.** (*a*)
151. (*b*)	**152.** (*a*)	**153.** (*c*)	**154.** (*a*)	**155.** (*c*)
156. (*d*)	**157.** (*b*)	**158.** (*d*)	**159.** (*b*)	**160.** (*a*)
161. (*b*)	**162.** (*a*)	**163.** (*d*)	**164.** (*d*)	**165.** (*a*)
166. (*c*)	**167.** (*b*)	**168.** (*b*)	**169.** (*c*)	**170.** (*c*)
171. (*c*)	**172.** (*c*)	**173.** (*d*)	**174.** (*a*)	**175.** (*c*)
176. (*a*)	**177.** (*b*)	**178.** (*a*)	**179.** (*c*)	**180.** (*b*)
181. (*d*)	**182.** (*d*)	**183.** (*a*)	**184.** (*a*)	**185.** (*b*)
186. (*b*)	**187.** (*b*)	**188.** (*a*)	**189.** (*a*)	**190.** (*c*)
191. (*c*)	**192.** (*a*)	**193.** (*d*)	**194.** (*b*)	**195.** (*a*)
196. (*b*)	**197.** (*d*)	**198.** (*c*)	**199.** (*b*)	**200.** (*a*).

EXERCISE

1. What is 'digital electronics' ?
2. State the advantages and disadvantages of digital electronics ?
3. What is a 'digital circuit' ?
4. Why binary system is preferred in 'digital system' ?
5. Discuss the importance of 1's and 2's complement numbers.
6. Define the terms 'logic function' and 'logic gate'.
7. What is Boolean algebra ? How is it different from ordinary algebra ?
8. Draw a NOT gate. Write its truth table.
9. Differentiate between OR and NOR gates. NOR is a combination of which gates ?
10. Which gates combine to form a NAND gate ?
11. What is exclusive OR gate ? Write its Boolean expression.
12. Show symbols of OR, AND, NOT, NAND, NOR, exclusive OR gates.
13. Simplify the Boolean expression : (*a*) $B \cdot 1 + A$, (*b*) $A + B + 1$, (*c*) $(A + B) \cdot 1$

    [**Ans.** (*a*) $A + B$, (*b*) 1, (*c*) $A + B$]

14. Draw a network to generate the function $Y = \overline{A \cdot B + C}$.
15. Simplify the Boolean expression

    $$Y = A \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot \overline{B} \cdot C + A \cdot B \cdot \overline{C}.$$ [**Ans.** A]

16. Simplify the following Boolean function and draw a network to generate the original function and simplified function

    $$Y = \overline{\overline{A \cdot B} + \overline{A \cdot C}}.$$ [**Ans.** $A \cdot B \cdot C$]

17. Draw the truth tables for the following functions : (*a*) $X = A \cdot \overline{B} + A \cdot B$, (*b*) $Y = \overline{A} \cdot B \cdot C + \overline{A} \cdot \overline{B}$. Draw circuits to generate above function.
18. Prove that :

    $$(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot \overline{B}$$

    $$A \cdot B \cdot C + A \cdot B \cdot \overline{C} = A. B.$$

19. Using only NAND gates draw network to generate $Y = A + \overline{B} \cdot C$.
20. If $A = 1$, $B = 0$ and $C = 1$, find the values of

    (*a*) $\overline{A} \cdot \overline{B} + A \cdot B + \overline{B} + C$ (*b*) $A \cdot B \cdot \overline{C} + \overline{A} \cdot B + A \cdot \overline{B}$. [**Ans.** (*a*) 1, (*b*) 1]

21. Differentiate between combinational and sequential logic circuits.
22. Distinguish between a 'half adder' and a 'full adder'.
23. State De-Morgan's theorems.
24. What is the importance of binary numbers ? What are the advantages of digital signals over the analog signals ?
25. Show that a multiplexer may be used as a sequential data selector.
26. What are demultiplexers ? What is the difference between a demultiplexer and a decoder ? Show connection diagram of a demultiplexer and a decoder.
27. With the aid of a neat sketch explain the operation of a BCD to decimal decoder.
28. Draw the diagram of a clocked *S-R* flip-flop and give the truth table.
29. Show that a *R-S* flip-flop results when two NOR gates are cross-coupled.
30. What is a flip-flop ? Explain the principle of operation of *S-R* flip-flop with truth table.
31. With the aid of a neat sketch, explain the operation of *J-K* flip-flop.
32. Explain the operation of a *R-S* flip-flop with the help of waveform and truth table.
    How does the master-slave action in a *J-K* flip-flop improves its operation ?
33. Briefly describe *R-S*, *J-K, D-* and *T*-type flip-flops.

UNIT-V : *FUNDAMENTALS OF COMMUNICATION ENGINEERING*

Chapter :

10

Telecommunication Systems

1. Communication. 2. Types of signals. 3. Transmission paths. 4. Modulation and demodulation—Introduction—modulation—amplitude modulation (AM)—frequency modulation (FM)—pulse modulation—digital modulation techniques—demodulation or detection. 5. Noise. 6. Modems. 7. Radio communication systems—radio transmitter—AM transmitters—FM transmitters—radio receiver. 8. Television systems. 9. Microwaves—Introduction—applications of microwaves—waveguides—cavity resonators—microstrip—microwave vacuum tubes—monolithic microwave integrated circuits (MMICs)—microwave antennas. 10. Satellite communication—Introduction—communication satellite—general aspects—types of communication satellites—satellite description—satellite space craft systems—launching, power supplies and control of satellites. 11. Radar system—Introduction—radar applications—types of radar systems. 12. Fiber optic communications—Introduction—advantages and limitations of fibre optic system—optical fibre communication system—optical fibre construction. 13. Cellular mobile communications—Introduction—block diagram and principle of operation of a cell phone—*Highlights—Objective Type Questions—Theoretical Questions.*

1. COMMUNICATION

- **Communication** *means to share one's thought with others. It is a bi-directional process.*
- The term communication refers to the *sending, processing and reception of information by electrical means.*

Depending upon the types of information to be sent and received following systems have been developed over the years.

(*i*) Radio (*ii*) Telephony
(*iii*) Telegraphy (*iv*) Broadcasting
(*v*) Radar (*vi*) Radio telemetry
(*vii*) Radio aids to navigation etc.

Fig. 1 shows the *block diagram* for any communication system.

Information source. It provides information to be communicated.

Transmitter. It processes the information and makes it fit for transmission.

Receiver. Its functions is that of *demodulation or detection or decoding*.

Destination. The receiver separates the original information from the high frequency wave and feeds it to the loudspeaker, punched cards, radar displays or TV picture tube etc. known as destination.

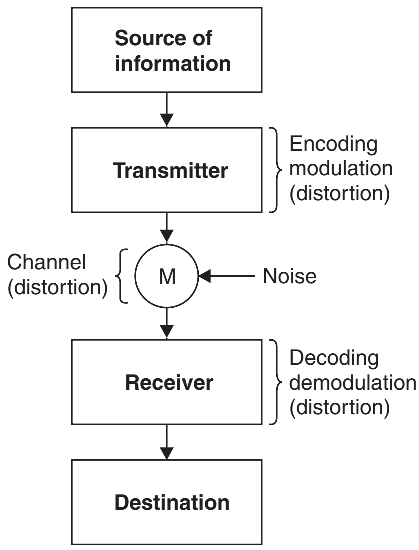


Fig. 1. Block diagram of communication system.

- Any communication system must fulfill the following *two basic requirements*.

(*i*) Accurate communication.

(*ii*) Fast communication.

- The ratio S/N $\left(= \dfrac{\text{useful signal power}}{\text{noise signal power}}\right)$ is an important parameter in evaluating the performance of a system and for accurate transmission, this should be as *high as possible.*

2. TYPES OF SIGNALS

Encoders produce the following two types of electrical signals :

1. Analog signals :

— A *telephone, radio broadcast or TV signals* are very common types of signals for use of general public.

— These are represented by voltage waveforms that have different amplitudes at different instants of time.

2. Digital signals :

— These signals comprise of pulses occurring at discrete intervals of time.

— The pulses may occur singly at a definite period of time or as a coded group.

— These signals play a very important role in the transmission and reception of coded messages.

— Simplest types of digital signals are the Telegraph and Teleprinter signals but sometimes analog messages are also converted into digital form before being transmitted for certain reasons.

3. TRANSMISSION PATHS

The transmission of messages can be carried out by means of the following :

1. **Transmission lines** ... termed as *Line Communications.*
2. **Radio waves** ... termed as *Radio Communications.*

Line Communications :

- The most common form of line communication is *telephone* and *telex networks.*
- These are carried out using overhead lines on poles and also by use of buried cables.
- In UHF range and above line communications are carried out by the use of **wave guides.** The following points are with noting.

(*i*) In *wave guides,* the signal propagation takes place by means of *radio waves* provided the cross-sectional dimensions of the proper wave guide rectangular hollow pipe maintain a proper relation with a wavelength of the wave.

(*ii*) *Wave guides,* are employed as transmission lines for microwave communication in the frequency range 2 GHz to 11 GHz.

(*iii*) *Wave guides* do not transmit signals *below cut-off frequency* and can have band-width in excess of 20% of their operating frequencies.

(*iv*) *Optical wave guides* have been developed in the form of glass fibres to carry light modulated signals at frequencies of the order of 100 THz (1 Tera hertz = 10^{12} hertz) from *lasers.* This will have a very vast capacity to carry signals.

Radio Communications :

- In this type of communication propagation of signals through atmosphere is used.
- Non-directional antennas are employed for broadcast transmission and reception while directional antennas are employed in radio telephony and other point-to-point links.
- Radio waves are commonly termed as unbounded waves but generally they are bounded by the surface of earth and various ionospheric conducting layers that lie between a height of 50 km and 400 km above ground.

Major Radio Frequency Bands

Name	*Frequencies*	
1. Very low frequency (VLF)	Below 30 kHz	... Radio location equipment
2. Low frequency (LF)	30 kHz to 300 kHz	... Martime radio navigation
3. Medium frequency (MF)	300 kHz to 3 MHz	... Includes AM radio broadcast band
4. High frequency (HF)	3 MHz to 30 MHz	... Radio
5. Very high frequency (VHF)	30 MHz to 300 MHz	... Includes FM broadcast band and television VHF channels
6. Ultra high frequency (UHF)	300 MHz to 3 GHz	... Includes television UHF channels
7. Super high frequency (SHF)	3 GHz to 30 GHz	... Satellite communications
8. Extremely high frequency (EHF)	30 GHz to 300 GHz	... Satellite communications

In a *radio receiver* by providing an *RF amplifier* the following advantages are obtained :

(*i*) Signal-to-noise ratio is improved.
(*ii*) Availability of greater gain and better sensitivity.
(*iii*) Image frequency rejection is improved.
(*iv*) Spurious frequencies are prevented.
(*v*) Better selectivity.
(*vi*) Improved coupling of the receiver to the antenna.

4. MODULATION AND DEMODULATION

4.1. Introduction

Several signals out of different types of signals that are generally encountered in communication systems have frequency spectra that is not suitable for direct transmission especially when atmosphere is used as the transmission channel. In such a case, the frequency spectra of the signal may be translated by modulating high frequency carrier wave with signal.

In order to transmit and receive the intelligence (code, voice, music etc.) successfully, the following two processes are essential :

(*i*) Modulation; (*ii*) Demodulation.

4.2. Modulation

4.2.1. Definition

Modulation may be defined as follows :

- ***Modulation*** is *the process of combining the low-frequency signal with a very high-frequency radio wave called 'carrier wave* (*CW*)'. The resultant wave is called *modulated carrier wave.* This job is done at the transmitting station.

Or

Modulation is *a process in electronic circuits by which the characteristics of one waveform (carrier) is modified by the variations in another wave (audio signal).*

Or

Modulation is *the process of combining an audio-frequency (AF) signal with a radio frequency (RF) carrier wave.* AF signal is called a *modulating wave* and the resulting wave produced is called *modulated wave.* During modulation, some characteristic of the carrier wave is varied in time with the modulating signal and is accomplished by combining the two.

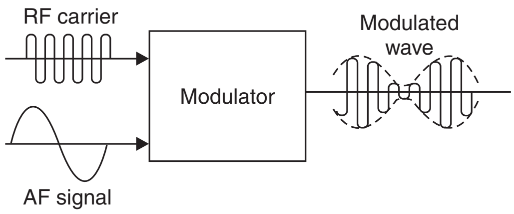


Fig. 2.

4.2.2. Need of modulation

In electronic communication, the modulation is necessary for the following *reasons* :

1. Modulation increases operating range.
2. It reduces the size of transmitting and receiving antennas.
3. It permits transmission without wire.
4. It is extremely difficult to radiate low frequency signals through earth's atmosphere in the form of electromagnetic energy.

4.2.3. Methods of modulation

For a sinusoidal carrier wave, the mathematical expression is given as

$$e = E_c \sin (\omega_c t + \phi) = E_c \sin (2\pi f_c t + \phi) \quad ...(1)$$

Thus, the waveform can be varied by any of its following three factors or parameters :

(*i*) E_c – The *amplitude;*

(*ii*) f_c – The *frequency;*

(*iii*) ϕ – The *phase.*

Accordingly, there are three types of sine-wave modulations known as :

1. Amplitude Modulation (AM)
2. Frequency Modulation (FM)
3. Phase Modulation (FM).

4.3. Amplitude Modulation (AM)

The process by which the amplitude of a carrier wave is varied in accordance with the modulating signal is called **amplitude modulation.**

The process of amplitude modulation is shown graphically in Fig. 3.

— For the sake of simplicity, the AF/message signal has been assumed sinusoidal [Fig. 3 (*a*)].

— The carrier wave by which it is desired to transmit the AF signal is shown in [Fig. 3 (*b*)].

— The resultant wave called *modulated wave* is shown in [Fig. 3 (*c*)].

The function of the modulation is to mix these two waves in such a way that (*a*) is transmitted along with (*b*).

— The fluctuations in the amplitude of the carrier wave depend on the *signal amplitude* and the rate at which these fluctuations take place depends on the *frequency* of the audio signal.

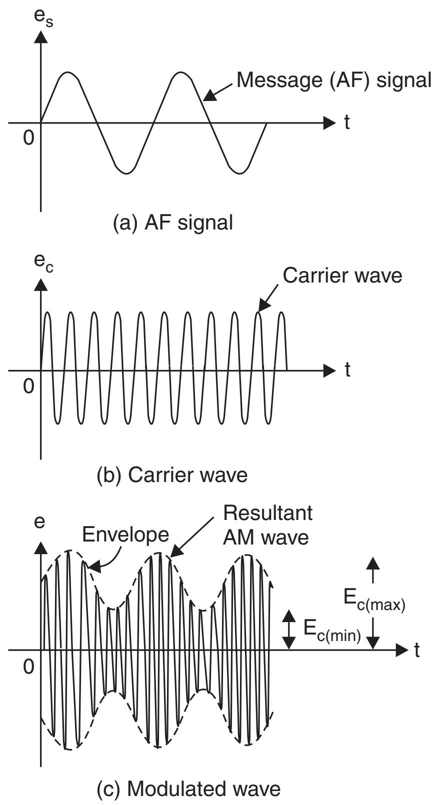


Fig. 3. Amplitude modulation.

- *All stations broadcasting on the standard broadcast band* (*550 – 1550 kHz*) *use AM modulation.*)
- **Percent modulation** *indicates the degree to which the AF signal modulates the carrier wave.*
- *Methods of amplitude modulation are :*

(*i*) Amplifier modulation.

(*ii*) Oscillator modulation.

Limitations of amplitude modulation :

Following are the **limitations** of amplitude modulation :

1. Smaller operating range.
2. Poor efficiency.
3. Poor audio quality.
4. Noisy reception.

4.4. Frequency Modulation (FM)

The process by which the frequency of a carrier wave is varied in accordance with the modulating signal is called **frequency modulation.**

The process of frequency modulation is shown graphically in Fig. 4.

The three waves shown in the figure are : (*a*) Signal ; (*b*) Carrier ; (*c*) The resultant frequency modulated wave.

When a signal of frequency f_s is modulated with carrier wave of frequency f_c, a resultant modulated wave is produced.

The following points are worth noting :

(*i*) The *amplitude* of the modulated wave is the *same* as that of the carrier wave.

(*ii*) The *frequency* of the modulated wave *varies in accordance* with the message signal.

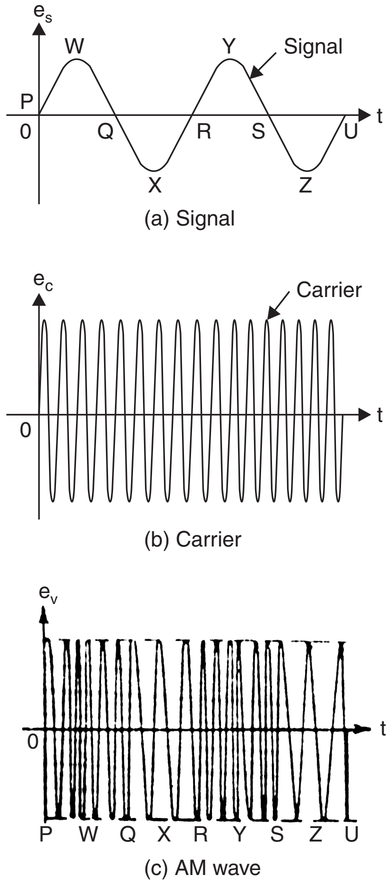


Fig. 4. Frequency modulation.

Advantages :

1. Better audio quality.
2. High transmission efficiency.
3. Noiseless reception.

Limitations :

1. Smaller area of reception.
2. Wider channel is needed.
3. Equipment used is more complex and costly.

4.5. Pulse Modulation

Pulse modulation is *a technique of modulating the analog signal and converting it into corresponding values. In this process the instantaneous voltage of the analog signal is sampled at regular intervals and transmitted during these sampling periods only.*

— **In analog modulation,** the *amplitude or frequency or phase* (*any one parameter*) *of carrier is varied according to the instantaneous voltage of signal*.

— **In pulse modulation,** either amplitude or width or position of the pulse is varied according to the instantaneous voltage of signal (except for PCM).

- Pulse modulation is *classified* as follows :

  Pulse modulation :

  A. *Analog :*

  (*i*) PAM

  (*ii*) PTM ⟶ PDM, PWM; PPM

  B. *Digital :*

  (*i*) PCM, (*ii*) Delta-modulation.

- **Sampling theorem :** This theorem was developed by *Nyquist*.

  It states : "*If sampling rate in any pulse modulating system exceeds twice the maximum signal frequency, the original signal can be constructed at the receiver with minimum distortion.*"

  — If f_m is the maximum signal frequency then sampling rate is greater than $2f_m$.

  — The minimum samplings rate is called the *Nyquist rate.*

  — *The reciprocal of the sampling rate* (or sampling frequency) is called **sampling line** or **Nyquist interval.**

- **Pulse Amplitude Modulation (PAM) :**

  — PAM can be *generated* by using an AND gate.

  — PAM can be *demodulated* by passing through low pass filter with cut-off frequency as the highest signal frequency.

  — Generally FM is used to modulate the pulses so that it can be transmitted ; such a system is called PAM-FM.

- **Pulse Width Modulation (PWM) :**

  — It is also called Pulse Duration Modulation (PDM) or Pulse Length Modulation (PLM).

  — It can be generated using a monostable multivibrator.

  — It can be demodulated by feeding the PWM signal to an integrating circuit.

  — In PWM the width of the pulse is changed in accordance with the instantaneous value of the modulating signal with amplitude remaining constant.

  — The most oftenly used integrating circuit is the loudspeaker itself.

- **Pulse Position Modulation (PPM) :**

  — It can be generated in the similar way as PWM but the pulse width is kept *constant* from the starting point of occurrence of pulse.

  — It can be *demodulated* by converting into PWM using a flip-flop.

  — In PPM the position of the pulse or the time of occurrence of the pulses is changed in accordance with the instantaneous magnitude of the modulating signal.

- **Pulse Code Modulation (PCM) :**

  — In this type of modulation, digital equivalents of the instantaneous voltage levels of the signals are transmitted in the form of pulses.

  — The total amplitude range which the signal may occupy is divided into a number of standard levels ; this process is called *Quantizing.* The level actually sent is the nearest standard level. The *quantization levels depend upon number of bits per sample.*

— PCM encoder functions similar to A/D converter ; PCM decoder functions similar to D/A convertor.

4.6. Digital Modulation Techniques

In digital communications, the modulating signal consists of binary data or an *M*-ary encoded version of it. This data is used to modulate a carrier wave (usually sinusoidal) with fixed frequency. In fact the input data may represent the digital computer outputs or PCM waves, generated by digitizing voice or video signals. The channel may be a telephone channel, microwave radio link, satellite channel or an optical fibre.

In **digital communication**, *the modulation process involves switching or keying the amplitude, frequency or phase of the carrier in accordance with the input data.* Thus, there are three *basic* modulation techniques for the transmission of digital data ; they are known as :

(*i*) Amplitude-shift keying (ASK).

(*ii*) Frequency-shift keying (FSK).

(*iii*) Phase-shift keying (PSK).

The above techniques can be viewed as special cases of amplitude modulation, frequency modulation and phase modulation respectively.

Types of digital modulation techniques :

Various types of digital modulation techniques are :

I. *Coherent digital modulation techniques :*

1. Coherent binary modulation techniques.
2. Coherent binary amplitude shift keying or on-off keying.
3. Coherent demodulation of binary ASK.
4. Binary phase shift keying (BPSK).
5. Coherent binary frequency shift keying (BFSK).

II. *Non-coherent binary modulation techniques :*

1. Non-coherent binary ASK.
2. Non-coherent detection of FSK.
3. Differential phase shift keying (DPSK).
4. Quadrature phase shift keying (QPSK).

4.7. Demodulation or Detection

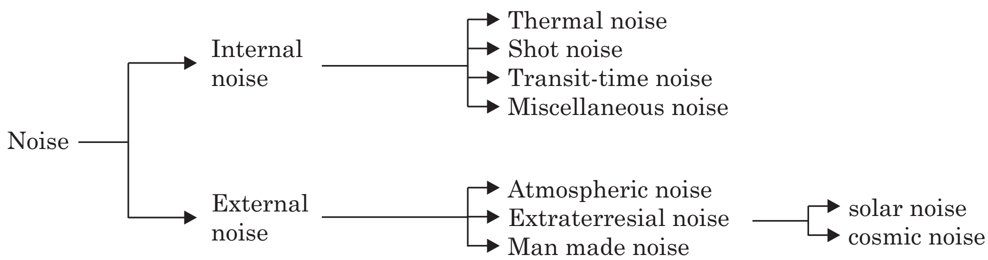
The process of extracting the low frequency modulating signal from the modulated wave is known as **demodulation** *or* **detection.**

- The demodulation of an '*AM wave*' involves *two operations* :
  (*i*) Rectification of the modulated wave;
  (*ii*) Elimination of the RF component of the modulated wave.
- The demodulation of an '*FM wave*' involves *three operations* :
  (*i*) Conversion of frequency changes produced by modulating signal into corresponding amplitude changes;
  (*ii*) Rectification of the modulating signal;
  (*iii*) Elimination of RF component of the modulated wave.

5. NOISE

- **Noise** may be defined, in an electrical sense, *as any extraneous form of energy tending to interfere with the proper and easy reception and reproduction of those signals which it is desirable to receive.*

- **Noise power :**
  — Noise power,
  (*i*) is *directly proportional to temperature;*
  (*ii*) is *directly proportional to the bandwidth of system;*
  (*iii*) is *independent of resistance value.*
  — Unit of *noise power is watts*.
  — Units of *noise power density are watts/hertz.*
  — The *average thermal noise power is constant as long as the bandwidth is fixed.*
  — **Thermal noise voltage** is *proportional to temperature, bandwidth and resistance.*
- **Noise figure :** *It is the ratio of S/N ratio at input to S/N ratio at output.* Ideal noise figure is '1' or 0 dB. Noise figure is *always greater than 1. Higher the noise figure of the amplifier, noiser the amplifier*.
- **Classification of noise :**



All receivers have their basic components consisting of tubes or transistors. The different types of noise present in these devices are thermal noise, shot noise, partition noise etc.

— In superheterodyne receivers, mixers are much noiser than amplifiers. It is so, because, the plate current of the partition noise is greatly increased in mixers.

— At short wave frequencies, the image frequency rejection is very poor and this introduces noise.

— A low value of transductance also helps in increasing the noise.

Thus, the effect of noise is maximum in mixers of all the receivers.

6. MODEMS

- Modem is an *acronym for MODulator DEModulator.*
- *A* **modem** *is a device that converts data from digital computer signals to analog signals that can be sent over a phone line.* This is called *"modulation".* The *analog signals are then converted back into digital data by the receiving modem.* This is called *"demodulation".*
- Fig. 5 shows a modem interfacing block diagram :
  — A modem is fed with digital information in the form of ones and zeros, from the CPU (central processing unit). The modem then analyzes this information and converts it into analog signals, that can be sent over a telephone line.
  — Another modem then receives these signals, converts them back into digital data, and sends the data to the receiving CPU.

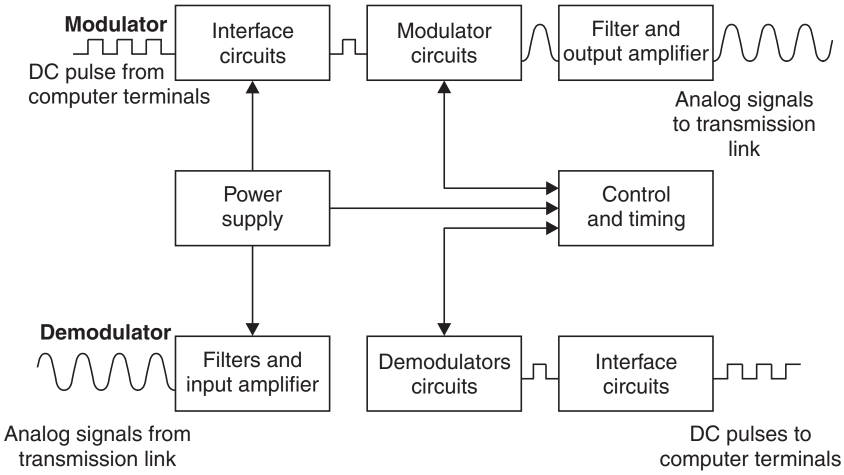


Fig. 5. Modem interfacing block diagram.

Types of Modems :

Various types of modems are :

1. Voice-grade modems and telephone couplers
2. Wide-band modems
3. Short-haul modems
4. Modem eliminators or line diverters/receivers

(Items 1–4: Binary serial DCES)

5. Frequency shift keying (FSK) modems
   — Bell 103 modem
   — V 21 modem
   — Bell 202 modem
6. Phase shift keying (PSK) modems
   — 201 modem
   — V.26 modem
   — Bell 212 modem
   — V.22 modem
   — V.22 bis modem
   — V.27 modem
   — V.29 modem
   — V.32 modem
   — V.32 bis modem
   — V.FC modem
   — V.34 modem
7. Data modems
8. Smart modems
9. Multiport modems
10. Radio modems.

Modem modulation methods :

The various modem modulation methods are :

1. Amplitude modulation (AM)
2. Frequency shift keying (FSK)
3. Phase modulation (FM)
4. Differential phase shift keying
5. Multilevel modulation.

7. RADIO COMMUNICATION SYSTEMS

7.1. Radio Transmitter

A **radio transmitter** is a *device that transmits information by means of radio waves.*

The signal intelligence is translated in terms of a high frequency wave commonly termed as *Carrier wave* and the process of intelligence *translation into high frequency is termed as Modulation.* All radio transmitters use one form of modulation or the other for transmission of intelligence.

All radio transmitting systems must have the following :

1. A section for generation of high frequency carrier wave;

2. A section for converting information into electrical impulses and amplifying them to the required level ; and

3. A section for modulating the carrier with signal intelligence amplification stages for increasing the level of the modulated wave to the desired power and antenna system for transmitting these signals into free space.

A basic radio transmitter system is shown in Fig. 6.

Transmitters are usually named depending upon

(*i*) The type of signal to be transmitted;

(*ii*) The type of modulation employed;

(*iii*) The carrier frequency used ; or

(*iv*) The type of radio waves radiated by the system.

— A transmitter, may, therefore, be named as *broadcast transmitter, telephony or telegraphy* transmitter depending upon whether the signal is an entertainment programme, speech or code signal or a picture signal.

— It may be termed as *AM* (amplitude-modulated) or *FM* (frequency-modulated) *transmitter* depending upon the modulation process employed.

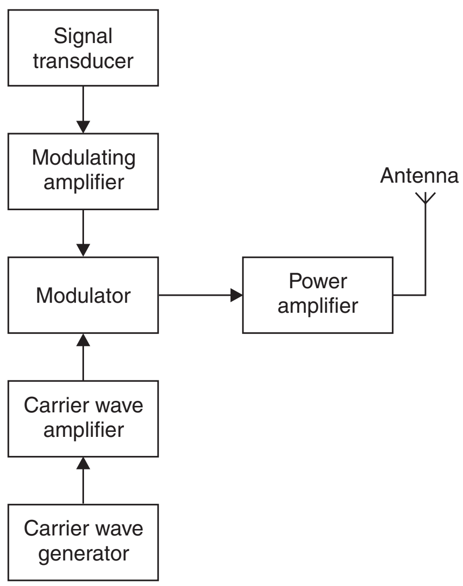


Fig. 6. A basic radio transmission system.

— Similarly, it may be termed as *Medium wave, 'Short wave', VHF, UHF or Microwave transmitter* depending upon the carrier frequency employed.

— Lastly, a transmitter may be termed as *long distance transmitter* or a *line of sight transmitter* depending upon whether the transmission is by sky waves or space waves.

7.2. AM Transmitters

- These transmitters are generally employed for radio broadcasts over long, medium or short waves, point-to-point communication systems using radio telephony/telegraphy signals over short waves or VHF waves and picture transmission over the VHF or UHF ranges.
- These transmitters employ one or the other method of producing modulated waves such that the depth of modulation is directly proportional to the magnitude of the modulation signal.
- Antenna systems for AM transmitters are large and must be located at some point remote from the studio operations. All the studio signal operations are performed at relatively low levels and transmitted to the main transmitter location either over telephone wire lines or a radio link such as a microwave system.

7.3. FM Transmitters

Direct FM :

- In frequency modulation, the amplitude of r-f carrier remains constant but its frequency is continuously varied in accordance with the instantaneous amplitude of an audio signal. Thus the original sound waves are converted into frequency deviation of the r-f carrier frequency that is proportional to the intensity of the sound waves.
- In FM transmitter, we make use of reactance modulator which changes the amplitude variations of an audio signal into frequency variations.

FM transmitter is much more efficient than an equivalent AM transmitter.

Indirect FM :

- Phase modulation may be used to achieve frequency modulation by indirect method. It is only necessary to integrate the modulating signal prior to applying it to the phase modulator.
- This transmitter is widely used in VHF and UHF radio telephone equipment.

7.4. Radio Receivers

A **radio receiver** *is a device that picks up the desired signal from the numerous signals propagating at that time through the atmosphere, amplifies the desired signal to the requisite level, recovers from it the original modulating signal and eventually displays it in the desired manner.*

Basic functions of a receiver are shown in Fig. 7.

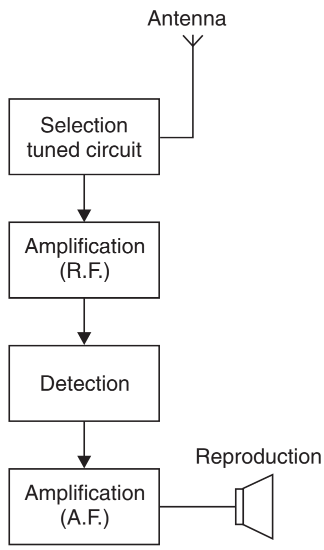


Fig. 7. Basic functions of a receiver

Requirements of radio receivers :

Sensitivity. It is the capability of a radio receiver to detect weak signals. It is defined as *the amount of r-f input voltage needed to produce a specified amount of audio output power.*

Selectivity. It is the ability of a *receiver to reject unwanted signals and amplify the desired one.*

In addition to the above requirements, every radio receiver should have :

— stability in frequency

— constant band-width of operation.

Types of receivers :

Following two types of receivers have practical or commercial significance :

1. Tuned Radio Frequency (TRF) receiver.
2. Superheterodyne receiver.

1. ***Tuned radio frequency receiver :***

— It consists of several stages of R-F amplification with tuned LC tank circuit at the input of each stage, an AM detector, one or more audio frequency amplifier stages driving loud-speaker or headphones as shown in Fig. 8.

— The provision of r-f stage at the input of the receiver provides the following advantages :

(*i*) Greater gain.

(*ii*) Prevention of re-radiation of the local oscillator (in case of superheterodyne receiver).

(*iii*) Improved rejection of adjacent unwanted signals.

(*iv*) Better coupling of antenna with the receiver.

● A tuned radio frequency receiver possesses the following good *features* :

1. High sensitivity 2. Simplicity.

It has the following *shortcomings* :

1. Bandwidth variations 2. Instability
3. Insufficient adjacent frequency rejection.

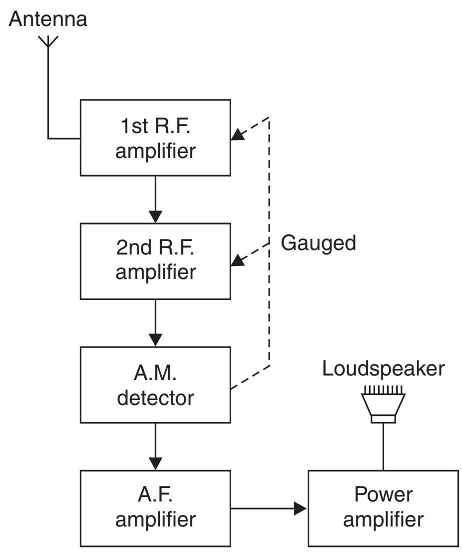


Fig. 8. Tuned radio frequency receiver.

2. ***Superheterodyne receiver :***

These days, practically, all receivers are superheterodyne.

— The superhet has the same essential components as that of a TRF receiver in addition to the mixer, local oscillator and interfrequency amplifier (Fig. 9).

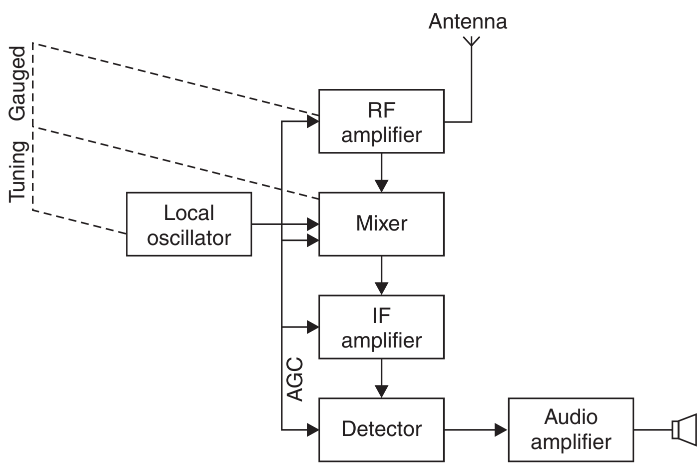


Fig. 9. Superheterodyne receiver.

— The RF amplifier is tuned to the required incoming frequency.

— The output of RFA is combined with the local oscillator voltage and normally converted into a signal of a lower fixed frequency called *intermediate frequency*. The signal at intermediate frequency (IF) contains the same modulation as the original carrier.

— IF signal is then amplified and detected to obtain the original information.

— A fixed frequency difference is maintained between the local oscillator and RF frequency with the help of capacitance tuning, in which all the capacitors are ganged together and operated in unison.

— IF stage consists of a number of transformers, each consisting of a pair of mutually coupled tuned circuits, providing a large gain. The characteristics of the IF amplifier are kept independent of the frequency to which the receiver is tuned, so that sensitivity and selectivity of supperhet usually remain fairly uniform throughout its tuning range.

— The radio frequency section provides coupling from the antenna input terminals of the receiver to the first stage of RF amplifier so as to amplify the incoming signal before its frequency is changed.

— The RF section carries out the following *main functions :*

(*i*) To provide discrimination or selectivity against image and interacted frequency signals.

(*ii*) To provide an efficient coupling between the antenna and first stage of RF amplifier.

(*iii*) To reduce the noise figure of the receiver.

8. TELEVISION SYSTEMS

- **Television** *means seeing at a distance.*
- To be successful, a television system may be required to reproduce faithfully the following :

(*i*) The shape of each object, or structural content.

(*ii*) The relative brightness of each object, or tonal content.

(*iii*) Motion, or kinematic content.

(*iv*) Sound.

(*v*) Colour, or chromatic content.

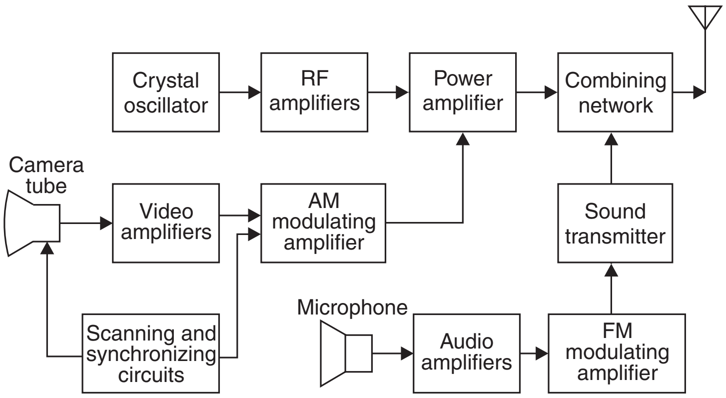
(*vi*) Perspective, or stereoscopic content.

- *Television system* deals with the transmission and reception of visual 'live' scene by means of radio broadcasting. Alongwith the signals, the sound signals associated with the scene are also transmitted at the same time to provide a complete sight and reproduction at the receiver of the televised programme.

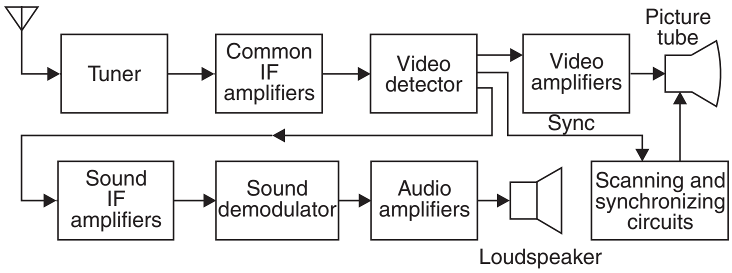
  Fig. 10 shows a block diagram of a rudimentary television system, indicating basically how the requirements of monochrome TV transmission and reception may be met.

Principles of Colour Television :

- The basic principles of colour television are the same as for black and white with the main difference that colour TV system requires the transmission and reception of specific colour information alongwith monochrome (black and white) signals.
- *Colour television* must have two way *compatibility* with monochrome television. That is, a colour transmission can be reproduced in black-and-white shades by a *monochrome receiver* and *monochrome transmission* is reproduced in black-and-white transmission by a colour receiver.



(*a*) Transmitter



(*b*) Receiver

Fig. 10. Basic monochrome television system.

Selected Standards of Major Television Systems

Standard	*American System*	*European System (India)*
Number of frames per second	30	25
Number of lines per frame	525	625
Field frequency, Hz	60	50
Line frequency, Hz	15750	15625
Channel width, MHz	6	7
Video bandwidth, MHz	4.2	5.5
Colour subcarrier, MHz	3.58	4.43
Sound system	FM	FM
Maximum sound deviation, kHz	25	50
Intercarrier frequency	4.5	5.5

9. MICROWAVES

9.1. Introduction

The waves of high frequencies usually above 300 MHz are termed as **microwaves.**

- The wavelength of such waves is *less than 1 metre.*

- The microwaves frequencies span the following three major bands at the highest end of RF spectrum :

(*i*) Ultra High Frequency (UHF) ... 0.3 to 3 GHz
(*ii*) Super High Frequency (SHF) ... 3 to 30 GHz
(*iii*) Extra High Frequency (EHF) ... 30 to 300 GHz.

At such high frequencies, the components depend on the varying electromagnetic field rather than on the current in a wire conductor or the voltage across two points. Instead of resonant LC circuits and conventional wire conductors, therefore, *resonant cavities and waveguides are often used at microwave frequencies.*

9.2. Applications of Microwaves

Microwaves are widely used in the following fields :

1. Telecommunications
2. Radar
3. Televisions for relaying signals
4. Satellite communications
5. Industrial heating
6. Research
7. Cooking.

9.3. Waveguides

— ***Waveguides** are hollow metal tubes used to propagate microwave energy in the form of electric and magnetic fields.*

— The purpose of waveguide is to *guide the wave from source end to load end.* The fundamental difference is that the propagation in the waveguides is in *the form of electric and magnetic fields,* whereas propagation in transmission line or free space is *in the form of voltage and currents*.

— These are usually made of brass or aluminium. The inner surface is usually silver plated to minimise the losses at higher frequencies. As the frequency increases, the size of waveguide reduces.

Advantages :

The waveguides entail the following *advantages* :

1. These are simpler to manufacture than co-axial lines.
2. Improved power handling ability.
3. Power losses lower in comparison to transmission lines.
4. These have mechanical simplicity and a much higher maximum operating frequency as compared to co-axial lines.

Applications :

1. It is observed that waveguides have dimensions that are convenient in the *3 to 100 GHz* range and somewhat inconvenient much outside the range. *Within this range, waveguides are generally superior to co-axial transmission lines for a whole spectrum of micro-wave application, for either power or low level signals.*

2. Waveguides as well as transmission lines can pass several signals simultaneously, but in waveguides it is sufficient for them to be propagated in different modes to be separated. They do not have to be of different frequencies.

Guided wavelength :

The wavelength in the waveguide is different from the wavelength in air or free space. It can be shown that the relationship among guide wavelength, cut off wavelength, and free space wavelength is :

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

where, λ_g = Guide wavelength,

λ = Free space wavelength, and

λ_c = Cut off wavelength

The above equation is true for any mode in waveguide of any cross-section provided the value of λ_c *corresponds to the mode and cross-section*.

Modes :

The two kinds of modes in the waveguides are :

1. *Transverse electric mode* (*TE*) ... Electric field is always transverse to the direction of propagation.
2. *Transverse magnetic mode* (*TM*) ... Magnetic filed is always transverse to the direction of propagation.

9.4. Cavity Resonators

- Cavity resonators are used as tuned circuits at microwave frequencies. This results in lower losses and Q of a resonate frequency as high as 20000.
- The cavity resonator can be used to select one frequency and reject the frequencies above and below a function similar to that of a band pass filter.

Mainly a resonant cavity is *used to control the frequency of oscillations when it is used with an active device to form a microwave oscillator.*

9.5. Microstrip

A ***microstrip*** *is a co-axial transmission line made as a printed circuit for application in an amplifier or oscillator in which microwave diodes and transistors are used.*

The characteristic impedance for microstrip is determined by :

(*i*) Thickness of the substrate;

(*ii*) Dielectric constant of the substrate;

(*iii*) Width of the conductor strip.

9.6. Microwave Vacuum Tubes

The mechanism of operation of various microwave tubes differ in details, but all of them involve transfer of power from a *source of D.C. voltage to a source of A.C. voltage by means of a current density modulated electron beam.* This is achieved by accelerating electrons in a static electric field and retarding them in an A.C. field. The density modulation of the electron beam allows more electrons to be retarded by the A.C. field than accelerated by A.C. field, and therefore, makes possible a net energy to the delivered to the A.C. electric field.

1. **Klystron :**

- A klystron has a thermionic cathode which releases a stream of electrons. The stream passes through two cavities called a buncher and a catcher.
- It is used as microwave amplifier.

2. **Magnetron :**

- It consists of an anode and a heated cathode. The anode is a metal block with machined cavities.

- The cavity dimensions determine the frequency of oscillations. The path of electrons is cycloidal.
- The process of accelerating or decelerating the electrons is called *velocity modulation.*
- Used in radar and *linear particle accelerators. Current efficiencies are of order of 50 per-cent.*

3. **Travelling Wave Tubes (TWT) :**

- It is a wide band device compared with the klystron and magnetron.
- Travelling wave tubes are employed when larger bandwidth (both klytrons and magnetrons have very high Q resulting in narrow bandwidth) is required.
- These are used as *linear amplifiers of microwave signals* in *satellite communications.*

The characteristics of TWT are :

(*i*) Power gain : 60 db
(*ii*) Power output : 10 kW average
(*iii*) Efficiency : 20 % to 40%
(*iv*) Bandwidth : about 0.8 GHz
(*v*) Frequency range : greater than 3 GHz.

4. **Crossed-field Amplifier (CFA) :**

- It is microwave power amplifier, based on the magnetron.
- It is a cross between the TWT and the megnetron in its operation.
- Pulsed CFAs are available for the frequency range from 1 to 50 GHz.
- The maximum powers available are well over 10 MW in UHF range, 1 MW at 10 GHz and 400 kW in the S-band.

Power gain ... upto 20 db
Efficiency ... upto 70%
Band width ... 25% of centre frequency

Applications. Used widely for *radar and electronic counter-measures.*

5. **Tunnel diode :**

- A tunnel diode is a thin junction diode.
- It exhibits negative resistance under low forward bias conditions.
- It is highly suitable for *microwave frequencies,* because of thin junction and short transit time.
- It is a low noise device.
- The tunnel diode amplifier (TDA) is a broadband, high gain microwave amplifier.

Applications :

(*i*) Suitable for *space work* (because TDAs are immune to the ambient radiation encountered in inter-planetary space)
(*ii*) Used as self-excited mixers.
(*iii*) Used for high speed switching and logic operations.
(*iv*) Due to their simplicity, frequency stability and immunity to radiation these are used as low power oscillators upto 100 GHz.

6. **Gunn diodes :**

- In bulk semiconductor materials the gunn effect is instrumental in the generation of microwave oscillators.

- Gunn diodes are grown epitaxially out of GaAs or Inp doped with silicon.
- Gunn diodes have an efficiency of the order of 2.5 to 5 percent.

7. **Impatt diodes :**

- **IMPATT** diode exhibits negative resistance and delivers high pulsed power at the lower microwave frequencies.
- A typical IMPATT diode works at about 50 GHz.
- These diodes are used at higher frequencies and for the highest output powers.

8. **Trapatt diodes :**

- These diodes also exhibit negative resistance and deliver high pulsed powers at lower microwave frequencies.
- These have an efficiency of 30 percent.
- Most TRAPATT oscillators and amplifiers are still in laboratory stage.
- These diodes may find application in airborne and marine radars.

9. **Laser :**

- It is an oscillator.
- Optical fibre communication using laser operates at wavelength of about 0.85 μm.

Non-communication application of lasers include the following :

(*i*) Industrial welding;
(*ii*) Distance and speed measuring equipment;
(*iii*) Formation of three-dimensional holograms;
(*iv*) Optical and other surgery etc.

10. **Parametric Amplifiers :**

- The name of the amplifier stems from the fact that capacitance is a parameter of a tuned circuit.
- It is a low-noise microwave amplifier.

Parametric amplifiers find application in the following :

(*i*) Radio-telescopes.
(*ii*) Space probe tracking and communications.
(*iii*) Tropospheric scatter receivers.

9.7. Monolithic Microwave Integrated Circuits (MMICs)

These circuits claim the following *advantages over discrete circuits :*

1. Small size	2. Light-weight
3. Improved performance	4. Improved reproducibility
5. Low cost	6. High reliability.

Applications. Monolithic microwave integrated circuits are suitable for *space* and *military applications since they meet the requirements for shock, temperature conditions, and severe vibrations.*

9.8. Microwave Antennas

An **antenna** *is a structure capable of radiating or receiving electromagnetic waves ; their function is to couple the transmitter or receiver to space.* In case of microwaves the transmitting and receiving antennas should be *highly directive.* Some of the essential requirements are given below :

(*i*) Omnidirectional antennas are not required because no broadcasting is done at these frequencies.

(*ii*) The signals at the input terminals of the receiver should be as large as possible.
(*iii*) Directional application is often required.
(*iv*) Need not be physically large in size.
(*v*) Power gain should be high.

Horn antennas :

— An horn antenna is just an open section of a rectangular guide. It is flared out to match the characteristic impedance Z_0.
— The flare angle of the horn is usually between 40° and 66° (changing over to Horn from open ended waveguide increases the directivity and also diffraction is reduced).

This antenna has the following features :

(*i*) Fairly good directivity.
(*ii*) Adequate bandwidth.
(*iii*) Simple mechanical construction.

Antenna with parabolloidal reflectors :

— A parabola is a very suitable reflector for light rays or microwaves.
— In these antennas all the waves are in phase as a result, radiation is very strong and concentrated.
— The gain is influenced by the aperture ratio and the uniformity of illumination.

Lens antenna :

— A lens antenna is used as a collimator at frequencies greater than 3 GHz and works just like a glass lens in optics.
— These antennas are oftenly made of polystyrene.
— It is used to correct the curved wave front from a horn that has to be made a plane wave front.

Advantages :

(*i*) Greater design tolerances.
(*ii*) No obstruction of radiation.

Disadvantages :

(*i*) Greater bulk.
(*ii*) Costly.
(*iii*) Design difficulties.

Helical antenna :

— It consists of a loosely wound helix backed up by a ground plane which is simply a screen made of "chicken wire".
— This is a broad band VHF and UHF antenna which is used to provide circular polarization characteristics.
— It is used either singly or in an array for transmission and reception of VHF signals through the ionosphere.
— It is employed for satellite and probe communications, particularly for radio telemetry.

Discone antenna :

— This is low gain antenna, but it is omnidirectional.
— It is a combination of disc and cone and has enormous bandwidth for both input impedance and variation pattern.

— It is used for VHF and UHF signals, specially at airports where communication must be maintained with aircraft coming from any direction.

Log periodic antenna :

— Directive gains are low to moderate.

— Radiation pattern way be unidirectional or bidirectional.

— Used in HF range.

10. SATELLITE COMMUNICATION

10.1. Introduction

The need for satellite communication was realised when the other conventional systems became overcrowded. A number of channels can simultaneously be operated using a satellite. The process of sending information around the world has been revolutionised by the advent of satellites.

— The real beginning in the field of satellite was made in October 1957 when first man-made satellite Sputnik I was launched.

— Sputnik I was followed by a rally man-made satellites. Satellites TELESTAR I and II of AT and T launched in July 1962 and May 1963 formed the basis of communication satellites. TELESTAR I was the first satellite to demonstrate a television link between USA and Europe. This was followed by a series of communication satellites by various companies.

10.2. Communication Satellite—General Aspects

- A **communication satellite** is basically a *microwave-link repeater.* It receives the energy beamed up from an earth station, amplifies and returns it to the earth at a frequency of a couple of GHz away to prevent interference between the uplink and the downlink.
- Satellite is an artificial body that is projected from earth, which revolves around the earth in a *particular orbit.* It keeps on revolving in the orbit due to *balance of force between inertia of the revolving satellite and gravitational pull of the oriented body.*
- The satellites remain in geostationary orbit, *i.e,* they have the same angular velocity as the earth and hence they appear to be stationed over one spot on the globe.
- A satellite 35,800 km from the earth will complete a revolution in 24 hours.
- These stationary satellites present no tracking problems, but require large antennas, high power and high sensitivity.
- Satellite communication come under the *Transionospheric wave propagation.*
- Satellites must be launched into the space with a certain velocity so that it can overcome the gravitational pull of the earth and reach the orbit. That velocity with which the satellite is launched is called **Escape velocity.** The escape velocity of earth is 25,000 miles/hour.
- The satellites revolve in particular orbits allotted to them ; they may be *circular orbits, elliptical orbits, polar orbits, equatorial orbits* or any *combination of them.*

  — If the direction of the satellite's revolution is the same as the earth rotation, then the orbit is said to a **Posigrade orbit.**

  — If the direction of the satellite's revolution in an orbit is in opposite direction to that of the earth's rotation, then the orbit is said to be a **Retrograde orbit.**

  — When the satellite is in an elliptical orbit, the *highest point from the earth of that orbit is called* ***Apogee.***

  — In an elliptical orbit, the *lowest point of that orbit from the earth is called* ***Perigee.***

- The rocket that carries the satellite to the required height and puts it into the orbit is called **Satellite Launch Vehicle [SLV].**
- The place on the earth where the transmitting and receiving equipment used to communicate with satellite is installed is called **Earth station.**
- **Transponder.** It is a combination of transmitter and receiver. It receives signal, amplifies and retransmits the signal with different frequency. They are used in "*Active satellites*".
- The orbit in which Geostationary satellite is kept is called *Geostationary orbit and is most widely used.*
- The circular orbits are used for *Navigational purposes* for ships and for *Air traffic control.*
- The inclined orbit is *not* widely used. It is mainly used to cover the *polar regions.*
- The position of the satellite may be changed due to magnetic field of earth, sun and moon. The changed position is corrected by the earth station ; *this process of correcting the satellite position is called* **Station keeping.**
- *The satellite in position with transmitter, power supply and control circuit is called* **On-Board equipment.**
- The power supply to the satellite is generally provided through *solar cells* (main source). In case of an availability of solar energy they are powered from *rechargeable Nickel-Cadmium* (*Ni-Cd*) *cells.*

10.3. Types of Communication Satellites

The communication satellites can be of *two basic types* :

1. Passive satellites :

A passive satellite acts merely as a *reflector of signals.* These satellites need very powerful ground systems. These satellites do not have electronic parts (in comparison to active satellites) which may fail and render the satellite useless.

Passive communication satellites can be of two types :

(*i*) *Discrete structure satellites ;* or

(*ii*) *Where a large volume of space is filled with a large number of tiny passive satellites* (*dipoles*).

The large structure usually takes the form of a sphere. When a radio signal is beamed at a sphere much large than the signals wavelength, it gets reflected in all directions. This kind of sphere is called an *Isotropic reflector,* because the power in the signal is equally distributed in all directions. The sphere, a balloon, can be made of a thin plastic material ; the reflector surface can be made of a thin coating of metal. The sphere is folded into the nose cone of the booster rocked and later inflated in orbit.

2. Active satellites :

An active satellite *receives the signal, amplifies it and transmits it back to other points* on the earth.

Active satellites can be of the following two types :

(*i*) *Store and forward type* : It is a rather primitive concept in satellite technology ; the message is stored in tape-recorder in the satellite, and then played back when the satellite has moved over the appropriate receiving point on the earth.

(*ii*) *Line of sight repeater type* : It is the type being used today.

Like the microwave repeaters which must keep each other in view, the satellites, placed at a high altitude, must be visible to both transmitting and receiving stations.

Synchronous or Geostationary satellite :

— If a satellite can be lifted into orbit 35,680 km above the earth in circular orbit lying in the plane of the equator, it is termed as *synchronous satellite.*

— In this satellite the satellite period of revolution is the same as the earth's period for one complete rotation about its axis. This means, satellite remains fixed relative to a point on the earth, thus to a ground observer it appears stationary.

Synchronous systems claim the following *advantages* over lower altitude systems :

1. As few as three properly positioned satellites can entirely cover the earth where as 50 lower altitude satellites are required to serve the same purpose.

2. A single synchronous satellite can provide uninterrupted communication service 24 hours daily. Lower altitude systems require that ground station track and follow the satellites as the space craft crosses the sky, and the stations switch from satellite to satellite as one dips below the horizon and another comes within the range.

Problems encountered with synchronous communication satellites :

(*i*) Time delay concerning telephone.

(*ii*) Difficulty in establishing synchronous orbit.

● The *space communication field,* according to the requirements, can be subdivided into *three parts :*

1. Communication with and tracking of fast moving satellite orbit of about 145 km radius.
2. Communication via geostationary satellites.
3. Communication and tracking connected with interplanetary probes.

Orbiting period of satellite, $T = 2\pi \sqrt{\dfrac{r^2}{kM}}$

where, r = Radius of circular orbit,

k = *Gravitational constant,* and

M = Mass of earth.

The height (h) of orbit above the earth in which the satellite moves is given by :

$$h = (rkM - 6372) \text{ km}$$

— The operating frequency for the satellite should be such that these signals are able to pass the ionosphere without getting reflected by its layer and should also not get absorbed or attenuated by the troposphere.

— At lower limit frequencies used are 10 MHz to 100 MHz and the upper limit is 100 GHz.

10.4. Satellite Description

A satellite is an extremely complex and sophisticated electronic and mechanical device. A satellite typically consists of many subsystems functions, all carefully integrated into a single system.

The **"subsystems"** include :

1. Electrical power subsystem.
2. Telemetry tracking and control subsystem.
3. Main and auxiliary propulsion subsystem.
4. Communication channel subsystem.
5. Antennas.

10.5. General Structure of a Satellite Communication System

Fig. 11 shows the general structure of a satellite communication system. A satellite in space links many earth stations.

— The *user* is connected to the earth station through *terrestrial network.* This network may assume various configurations including a telephone switch or a dedicated link to the earth station.

— Signal generated by the user is processed and transmitted from the earth station to the satellite. The satellite receives the modulated RF carriers at the predetermined uplink (earth-to-satellite) frequencies from all the earth stations in the network, amplifies these carriers and then re-transmits them back to earth at downlink (satellite to earth) frequencies. The downlink frequencies are kept different from the uplink frequencies in order to avoid interference.

— The modulated carrier received at receiving earth station is processed to get back the original base band signal. This signal is then sent to the user through a terrestrial network.

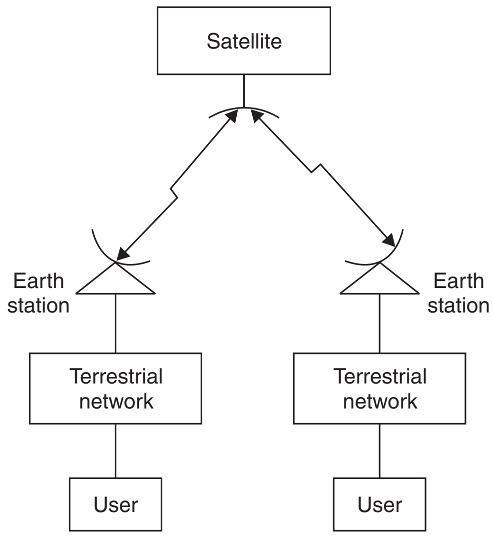


Fig. 11. General structure of a satellite communication system.

10.6. Satellite Spacecraft Systems

A complete satellite consists of several *subsystems,* but most important of these are :

1. Power supply system :

(*i*) Solar array (*ii*) Battery

(*iii*) Power supply control circuit.

2. Altitude and orbit control :

(*i*) Sensor (*ii*) Propulsion system

(*iii*) Altitude control (*iv*) Antenna control

(*v*) Orbit control.

3. Telemetry and telecontrol

4. Communications.

10.7. Launching, Power Supplies and Control of Satellites

Launching. The *launcher* places the satellite first into a low altitude circular induced orbit and then into an elliptical transfer orbit with apogee at the altitude of geostationary orbit. At an approximate apogee, the apogee boost motor is fired to circularise the orbit and remove most of the remaining orbit inclinations.

Power supplies. Silicon solar cells are the accepted source of primary power except during eclipse when power is maintained by Nickel-Cadmium cell batteries (for about 70 minutes).

Control :

— Body-stabilized designs generally employ an internal momentum wheels with axis perpendicular to the orbit plane.

— Control about the pitch axis is through the wheel's drive motor electrodes while control about the yaw and roll axes may be by gimballing the wheel or by use of hydrazine mono-propellant thrusters correct the axis direction.

— The orbit of satellite is controlled by ground command of hydrazine thrusters.

10.8. International Regulation of Satellite Service

- The regulations concerning frequency allocation for different purposes, standard of communication etc. are laid down by *International Telecommunication Union* (*ITU*), a body of United Nations Organization.
- *World Administrative Radio Conference* (*WARC*) held in 1979 has allocated frequency band for satellite communication servicewise under 17 categories namely (1) fixed (2) intersatellite (3) mobile (4) land mobile (5) maritime mobile (6) acronautical mobile (7) broadcasting (8) earth exploration (9) space research (10) meterological (11) space operation (12) amateur (13) radio transmission (14) radio navigation (15) aeronautical radio navigation (16) maritime radio navigation (17) standard frequency and time signal.

WARC further divided the globe into the following three geographic regions for frequency allocation :

Region I : Europe, Africa, USSR and Mongolia.

Region II : North America, South America, Greenland.

Region III : Asia (except USSR and Mongolia), Australia and South West Pacific.

11. RADAR SYSTEMS

11.1. Introduction

RADAR (**RA**dio **D**etection **A**nd **R**anging) is an *electronic device for detection and location of objects.*

Radar was developed during World War II.

Radar system *utilizes the fact that electromagnetic waves are reflected from the objects.* If a radio wave encounters sudden change in conductivity (σ), permittivity (ε) or permeability (μ) in the medium, a part of the electromagnetic energy gets absorbed by the second medium and is reradiated. This *sudden change in the electrical properties of the medium then constitutes the target. This reradiated energy on being received back at the Radar station gives information about the location of target*. For satisfactory location of the target, the power received back must be appreciable. Accordingly, the amount of energy required to be radiated by the Radar transmitting antenna is tremendous, typically being a *few megawatts. High power magnetrons are used for generation of such large amounts of power at high frequencies.*

- '*Practical radar systems*' require very high powered transmitters operating at GHz frequencies to produce large enough reflections to be detected by receiver for measurement accuracies. The shape and nature of the target also play a large roll in radar's ability to function over required distances.

— Some radars are designed to *detect all targets*, while others are designed to show only moving targets. Some of the advanced radars *use transmitters and receivers which are not located at the same site*.

11.2. Radar Applications

Following are the fields of application of radar :

1. Tracking and guidance.
2. Air surveillance.
3. Space and missile surveillance.
4. Surface search and battlefield.
5. Astronomy and geodesy.
6. Weather radar.

11.3. Types of Radar Systems

Radar systems may be put into the following *two basic categories* :

1. Pulsed radar system.
2. Continuous Wave Radar (CW radar) system.

Out of these two, the pulse radar system has proved to be more useful during war for detection of targets such as enemy aircraft etc.

1. Pulsed radar system :

Fig. 12 shows the block diagram of pulsed radar system. The various components elements are briefly discussed below :

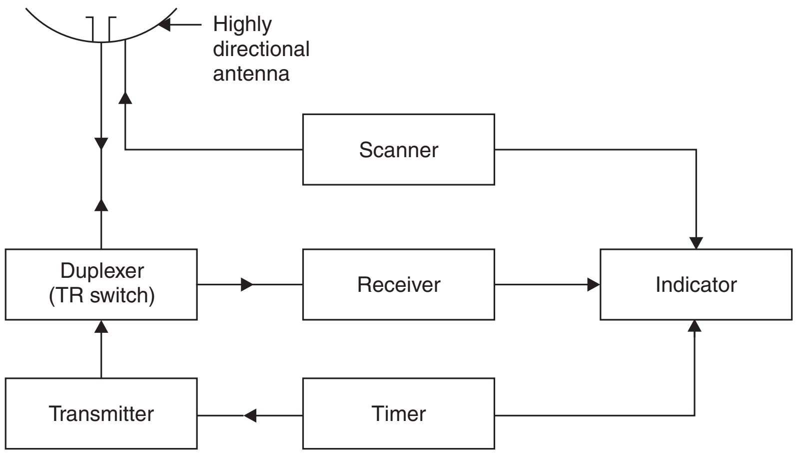


Fig. 12. Block diagram of pulsed radar system.

Construction :

(*i*) ***Transmitter.*** It consists of a radio frequency oscillator which is controlled by a modulator (pulser) in such a way as to produce pulses of high power but of short duration.

(*ii*) ***Antenna systems :***

— The output of transmitter is fed to the antenna which radiates it efficiently in the desired direction.

— The antenna system is highly directional and is usually capable of rotating in azimuth or in vertical plane or both and is thus able to direct the beam as desired.

— In most of the radar systems, a single antenna system is used for both transmitting and receiving.

(*iii*) ***Duplexer*** or ***Trans-receive switch (T.R. Switch) :***

— It controls the transmitting and receiving operations, permitting the use of only one antenna for both transmitting and receiving purposes.

— During the pulse interval, this duplexer connects the transmitter to antenna and isolates the sensitive receiver from the damaging effects of high power transmitter pulses. In the interval between the pulses, during which the reflected energy is being received, the duplexer connects the antenna to the receiver.

(*iv*) ***Timer :***

— It controls the generation of pulses in the transmitter.

— The receiver as well as the indicator are synchronized to the timer.

(*v*) ***Indicator :***

— The receiver output is usually fed to the indicator which displays the output on a cathode ray tube screen in such a manner as to show the difference between the outgoing pulse and the returning echoes. For this purpose, the voltage of the cathode ray tube (CRT) display is synchronized with the transmitted pulses.

(*vi*) ***Scanner :***

— The scanning system rotates both the antenna system and the indicator deflection coils (in the case of magnetic deflection coil).

— However, in case of electrostatic deflection system, a rotating sweep voltage is generated and controlled by the scanning circuit.

(*vii*) ***Receiver :***

— The radar receiver is a high sensitivity UHF or microwave receiver having lowest possible noise figure and having a bandwidth adequate for handling the pulses involved.

— The receiver is generally capable of detecting signals of small power of the order of 10^{-12} W.

Working/Operation :

- The transmitter sends a train of high-power, high frequency short duration pulses through the TR switch to antenna. After reflection from the target, the pulse (the echo pulse) comes back to antenna and through the TR switch to the receiver.

The TR switch is so designed that for the outgoing pulse from the transmitter, it connects the transmitter to the antenna and little energy is permitted to reach the receiver. On the other hand, for the incoming echo pulse from the antenna, the TR switch connects the aerial to the receiver and little fraction of the echo pulse energy reaches the transmitter.

- The echo pulse after amplification and detection in the receiver is displayed on the indicator screen.

2. Continuous wave radar (CW radar) system :

- It is *used to know the speed of vehicles or wind speed :*
- It *uses the doppler effect phenomenon.*
- It consists of a *circulator* (which acts as a duplexer) ; it gives some part of *Transmitter* output to *Receiver.*

RADAR Beacons :

- Also called *Transponders.*
- It consists of a *Receiver, Transmitter, Frequency Translator and Omni-directional antenna.* It is used for identification of beacon by say an aircraft.
- This system is used *on airports for traffic control, ship navigation and also for military purposes.*

Monopulse :

- It is a system which we can get the exact position of target with one pulse, which is transmitted in four directions.
- It is the most popular method of tracking after acquiring the position of the target.

Definitions :

1. **Pulse Repetition Frequency (PRF) :** It is defined as the *number of pulses transmitted per second.*

2. **Time period (T) :** It is the *reciprocal* of PRF $\left(i.e., T = \frac{1}{PRF}\right)$

3. **Peak power :** The *output power of the transmitter is called "Peak power".*

4. **Pulse length :** It *is the period for which the transmitter transmits microwave frequencies.*

5. **Duty cycle :** It *is the ratio of ON time to the total time of transmitter.*

6. **Average power :** It is defined as *product* of duty cycle and peak power (*i.e.,* Average power = duty cycle × peak power).

7. **Range :** *The maximum distance at which the radar can detect the objects is called RADAR range.* If PRF increases, the RADAR range decreases.

— Range is measured in *Nautical miles.*

— The best method to increase RADAR range is by increasing the diameter of antenna.

— The RADAR range *depends upon pulse energy and not on peak power.*

12. FIBRE OPTIC COMMUNICATIONS

12.1. Introduction

Owing to rapidly increasing demands for telephone communications throughout the world, multiconductor copper cables have become not only very expensive but also an inefficient way to meet these information requirements. The frequency limitations inherent in copper conductor system (approximately 1 MHz) make a conducting medium for high-speed communications necessary. The **optical fibre,** *with its low weight and high frequency characteristics (approximately 40 GHz) and its imperviousness to interference from electromagnetic radiation, has become the choice for all heavy-demand long-line telephone communications systems.*

12.2. Advantages and Limitations of Fibre Optic System

Advantages. The advantages of fibre optic system over conventional co-axial cable or microwave systems are as follows :

1. Light weight.
2. Larger information carrying capacity (A single fibre can handle as many voice channels as 1500-pair cable can.
3. Less space and easy installation.
4. Information tapping from a fibre cable is virtually impossible.

5. Fibre cables are cheaper.
6. No capacitance and inductance formation.
7. The fibre cables can operate over larger temperature and are less affected by corrosive liquids and gases.
8. Immunity to interference (from lightning, cross talk and electromagnetic radiation).

Limitations :

1. Higher initial cost.
2. Switching and routing of fiber optic signals is difficult.
3. Difficult to splice optical fibres to make them longer or to repair breaks.
4. Connectors for fibre optics are more complex to attach to cable and require precise physical alignment.

12.3. Optical Fibre Communication System

Fig. 13 shows the simplified block diagram for an optical fiber communication system. *This system converts electrical signal into light, transmits the light through fibre cable and converts light back to electrical signal.*

— The *information source* provides the electrical signal to a *transmitter* consisting of an electrical stage which drives an *optical source* to give modulation of the light wave carrier. The amount of light emitted by the optical source is proportional to the drive current.

— The transmission medium consists of an **optical fibre cable** *with plastic or glass core, a cladding and protective jacket.*

— The *receiver* consists of an optical detector which drives a further electrical stage and hence provides *demodulation of the optical carrier.*

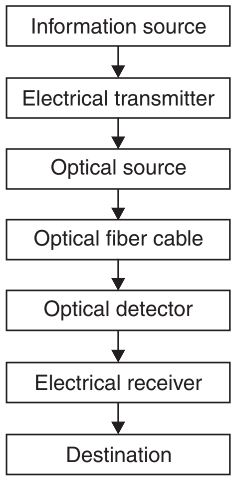


Fig. 13. Optical fibre communication system.

Optical fibre construction :

Refer to Fig. 14.

- An optical fibre consists of a *central core* and an *outer cladding*. The material of core has higher index of refraction as compared to cladding. The optical fibres are available either as a single fibre [Fig. 14 (*a*)] or as several fibers bundled together [Fig. 14 (*b*)].
- Materials used for optical fibres are *glass* (fused silica) and *plastic*.

— *Glass fibres* are more costly as compared to plastic fibres. They propagate light more efficiently than plastic. These are used for high speed application and also used for long transmission paths.

— Plastic fibres are more flexible and rugged than glass. These fibres are less expensive.

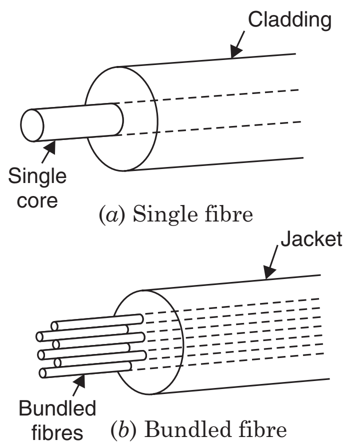


Fig. 14. Optical fibre construction.

13. CELLULAR MOBILE COMMUNICATIONS

13.1. Introduction

Mobile phone :

- *A **mobile phone** is a portable device which acts as a normal telephone whilst being able to move over a wide area.* Mobile phones allow connections to be made to the telephone network, normally by direct dialling the other party's number on inbuilt keypad.
- Most current mobile phones use a combination of radio wave transmission and conventional telephone circuit switching, though packet switching is already in use for some parts of the mobile phone network especially for services such as push-to-talk mobile is operated by the use of sim-card.

Cell phone :

- *A* **cell phone** *is a portable telephone which receives or makes calls through a Cell site or transmitting tower.*
- *Radio waves are used to transfer signals to and from the cell phone.*
- Large geographic areas (representing the coverage range of a service provider) are split up into smaller cells to deal with line-of-sight signal loss and large number of active phones in an area. Each cell site has a range of 3 to 15 miles and overlaps other cell sites. *All of the cell sites are connected to one or more cellular switching exchanges which can detect the strength of the signal received from the telephone.*

13.2. Block Diagram and Principle of Operation of a Cell Phone

Block diagram :

The cell phone is a compact and small piece which can be easily handled by a user.

The inside of a cell phone can be broadly divided into the following *three units* :

1. Transreceiver unit.
2. Frequency synthesizer.
3. System microprocessor unit along with a keypad and display system.

Fig. 15 shows the block diagram of a cell phone handset.

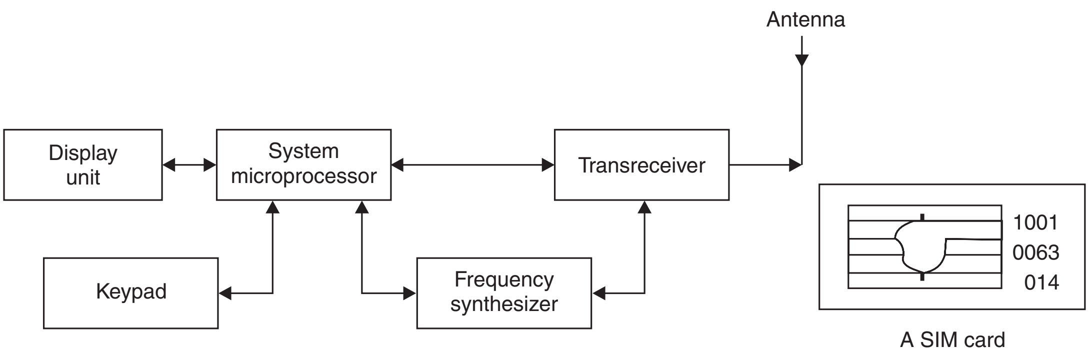


Fig. 15. Block diagram of a cell phone.

— SIM (Subscriber Identity Mobile) Card is the heart of the cell phone. It is directly associated with the system microprocessor. It is a 'smart card' which contains all the subscriber related information like cellular identification number and other related functions. It also stores messages and phone numbers.

Principle Operation/Working :

- When a user dials a number on the keypad of the cell phone, the called number is placed into the organising register in the cell, and checks to see that number is correct and pushes the 'second button'.
- The handset transmits the digits through the built-in radio receiver, to a nearby cell site. The nearby cell site receives it and selects the best directive antenna for the voice channel and sends the request to the mobile telephone switching office (MTSO).
- The MTSO selects an appropriate voice channel for the call, and the cell site acts on it through the best directive antenna to link the cell phone. All the switching functions within a group special mobile (GSM) are handled by MTSO.
- Once the request is forwarded to the MTSO, it determines how to route the call and set up the required link to enable the conversation. If the call is destined for a fixed line phone then network sends it to the public telephone exchange, over a leased line, which then switches the call to the desired phone.

  However, if call is destined for another mobile phone, then MTSO has to find out where the desired mobile phone is, and then forward the call to the nearest radio base station (or cell).

— Cellular phones use frequencies from 825 MHz to 890 MHz (UHF TV Channels 73-83) ; the transmit band frequencies from 825 MHz to 840 MHz and receive band frequencies from 870 MHz to 890 MHz.

— These days all the cellular operators provide the calling line identification source which displays the incoming caller's number. Once the number is displayed the user can decide if he or she want to take the call on their cell phone.

— Voice mail facility is also available in cell phone.

— SMS (Short Message Service) is also available, with this the users send or receive the message upto 160 characters on their cell phones.

HIGHLIGHTS

1. The term *communication* refers to the sending, processing and reception of information by electrical means.
2. Modulation is the process of combining the low-frequency signal with a very high-frequency radio wave called carrier wave (CW).
3. The process of extracting the low-frequency modulating signal from the modulated wave is known as *demodulation* or *detection.*
4. A *modem* is a device that converts data from digital computer signals to analog signals that can be sent over a phone line.
5. Television means seeing at a distance.
6. The waves of high frequencies usually above 300 MHz are termed as *microwaves*.
7. A *communication satellite* is basically a microwave-link repeater.
8. *Radar* is an electronic device for detection and location of the objects.
9. A *mobile phone* is a portable device which acts as a normal telephone.

OBJECTIVE TYPE QUESTIONS

Fill in the Blanks or Say 'Yes' or 'No' :

1. Communication is a bi-directional process.
2. The ratio S/N should be as .... as possible.

3. .... signals comprise of pulses occurring at discrete intervals of time.
4. Transmission lines are termed as .... communications.
5. .... is a process in electronic circuits by which the characteristics of one wave-form (carrier) is modified by the variations in another wave (audio signal).
6. The process by which the amplitude of a carrier wave is varied in accordance with the modulating signal is called .... modulation.
7. The process by which the frequency of a carrier wave is varied in accordance with the modulating signal is called .... modulation.
8. In analog modulation the amplitude or width or position of the pulse is varied according to the instantaneous voltage of the signal.
9. Sampling theorem was developed by Faraday.
10. Noise power is inversely proportional to temperature.
11. .... is a device that converts data from digital computer signals to analog signals that can be sent over a phone line.
12. A radio transmitter is a device that transmits information by means of radio waves.
13. FM transmitter is less efficient than an equivalent AM transmitter.
14. .... is the ability of a receiver to reject unwanted signals and amplify the desired one.
15. .... means seeing at a distance.
16. .... are hollow metals tubes used to propagate microwave energy in the form of electric and magnetic fields.
17. Cavity resonators are used as timed circuits at microwave frequencies.
18. A communication satellite is basically a microwave-link repeater.
19. .... is an electronic device for detection and location of objects.
20. .... waves are used to transfer signals to and from the cell phone.

ANSWERS

1. Yes	**2.** high	**3.** Digital	**4.** line	**5.** Modulatin
6. amplitude	**7.** frequency	**8.** Yes	**9.** No	**10.** No
11. Modem	**12.** Yes	**13.** No	**14.** Selectivity	**15.** Television
16. Waveguides	**17.** Yes	**18.** Yes	**19.** Radar	**20.** Radio.

THEORETICAL QUESTIONS

1. What do you mean by the term 'Communication' ?
2. Explain briefly the following :
(*i*) Analog signals ; (*ii*) Digital signals.
3. Define the term 'Modulation' ?
4. What is the need of modulation ?
5. What are the various methods of modulation ?
6. Explain briefly the following :
(*i*) Amplitude modulation (AM)
(*ii*) Frequency modulation (FM).
7. What are the limitations of amplitude modulation ?
8. What is frequency modulation ? What are its advantages and limitations ?
9. What is 'Pulse modulation ' ? Explain.
10. Name the various types of digital modulation techniques.
11. What is 'Noise' ? Explain.
12. What is a 'Modem' ?
13. How are modems classified ?
14. List the various modem modulation methods.

15. What is a radio transmitter ?

16. Give the block diagram of a basic radio transmission system.

17. Explain briefly the following :

(*i*) AM transmitters

(*ii*) FM transmitters.

18. What is a radio receiver ? What are its basic functions ?

19. Explain briefly the following receivers :

(*i*) Tuned Radio Frequency (TRF) receiver.

(*ii*) Superheterodyne receiver.

20. Write a short note on 'Television system' ?

21. What are 'Microwaves' ?

22. What are applications of microwaves ?

23. What are 'waveguides' ? What are its advantages and applications ?

24. What is a communication satellite ?

25. Explain briefly the following communication satellites :

(*i*) Passive satellites ; (*ii*) Active satellites.

26. What is a 'Synchronous satellite' ? Explain.

27. Give the general structure of a satellite communication system.

28. What is a 'Radar' ? What are its applications ?

29. Explain briefly 'Pulse radar system', with the help of a block diagram.

30. What is a 'Continuous wave radar system' ?

31. What are the advantages and limitations of fibre optic system ?

32. Explain briefly 'optical fibre communication system', with the help of a block diagram.

33. Write a short note on optical fibre construction.

34. What is mobile phone ?

35. Explain the principle of operation of a cell phone, with the help of a block diagram.

ABOUT THE BOOK

This book on **"Basic Electrical and Electronics Engineering"** has been specifically written for B.E. 1st Year ***(Common to Mechanical, Production, Automobile, Aeronautical and Marine Engineering disciplines)*** of **Anna University, Tamil Nadu,** strictly according to the latest syllabus. It consists of ten chapters in all, covering exhaustively the various topics in different chapters of the complete syllabus.

Salient Features :

- The presentation of the subject matter is very systematic and the language of the text is lucid, direct and easy to understand.
- Each chapter of the book is saturated with much needed text supported by neat and self-explanatory diagrams to make the subject self-speaking to a great extent.
- A large number of solved examples, properly graded, have been added in various chapters to enable the students to attempt different types of questions in the examination without any difficulty.
- At the end of each chapter ***Short Answer Questions, Highlights, Objective Type Questions, Theoretical Questions*** and ***Unsolved Examples*** have been added to make the book a complete unit in all respects.

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He has more than 35 years of experience in teaching different subjects of Mechanical and Electrical Engineering disciplines. He has published/presented a large number of technical papers. He is the author of several books on the important subjects of Mechanical as well as Electrical Engineering disciplines.

He has earned, by dint of hard work and devotion to duty, the following **awards/honours :**

**** Best Teacher (Academic) Award.***

**** Jawahar Lal Nehru Memorial Gold Medal for an outstanding research paper (Institution of Engineers).***

**** Distinguished Author Award.***

**** Man of Achievement Award.***

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