InQu 4010: Momentum Transfer Operations
(Fluid Mechanics)

Chapter 3:
Fluid Statics

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Hydrostatic Force on Submerged Surfaces

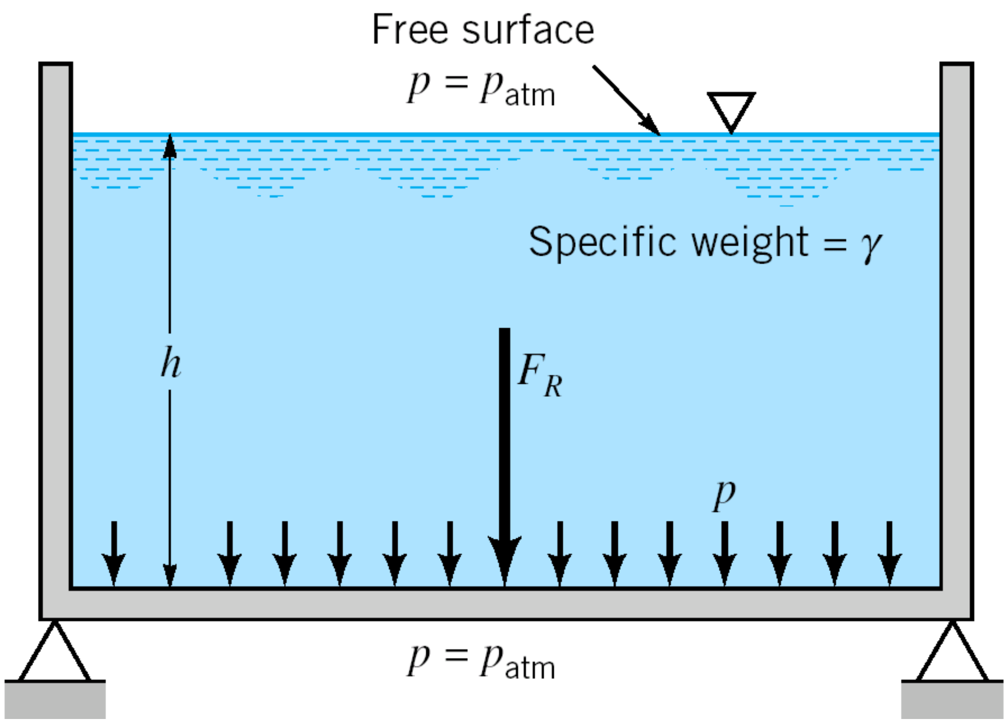
- We need to specify:
  - ✓Magnitude of the force
  - ✓Direction
  - ✓Line of action

- We will consider **plane s**ubmerged surfaces.

Hydrostatic Force on Submerged Surfaces

- **Static fluid** – no shear stresses
  - ***Hydrostatic force*** on any element on the surface acts ***normal to the surface***.

$$dF = pdA \quad \rightarrow \quad F_R = \int_A pdA$$



Submerged horizontal surface

$$F_R = pdA$$

$$p = \gamma h$$

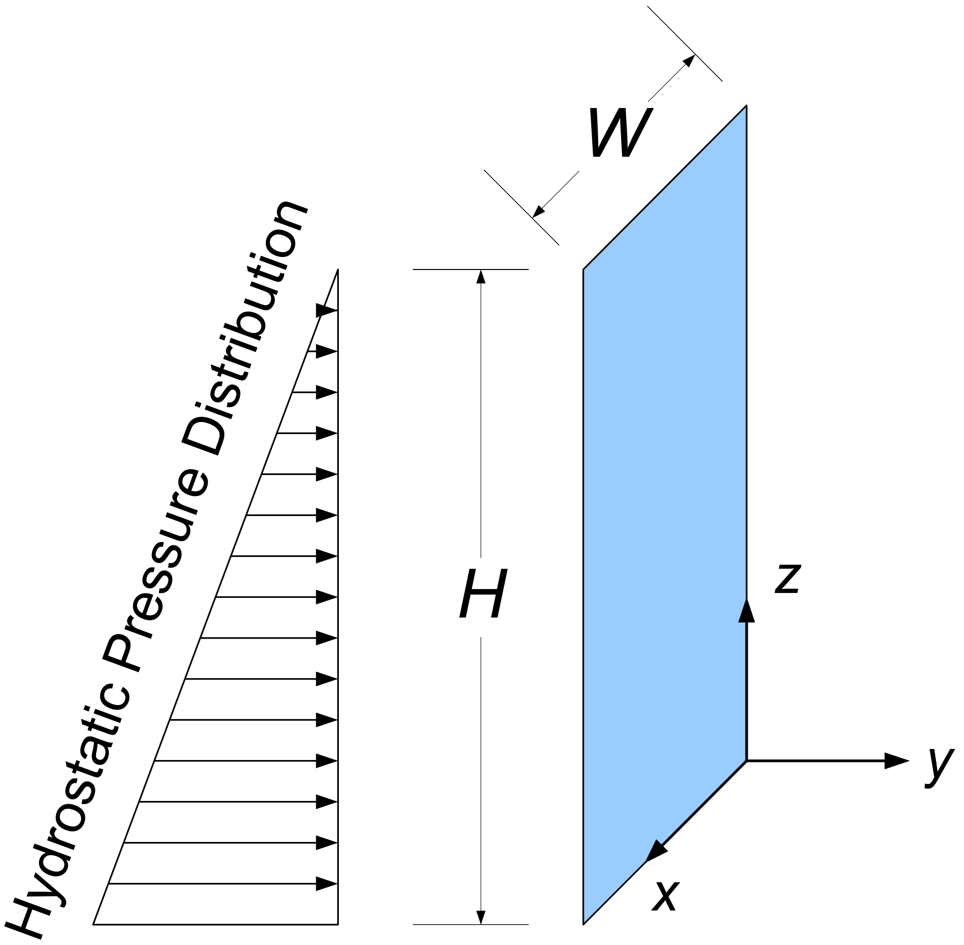
Forces due to Hydrostatic Pressure Distributions

In general:

$$\mathbf{F} = \iint_S p\mathbf{n}dS$$

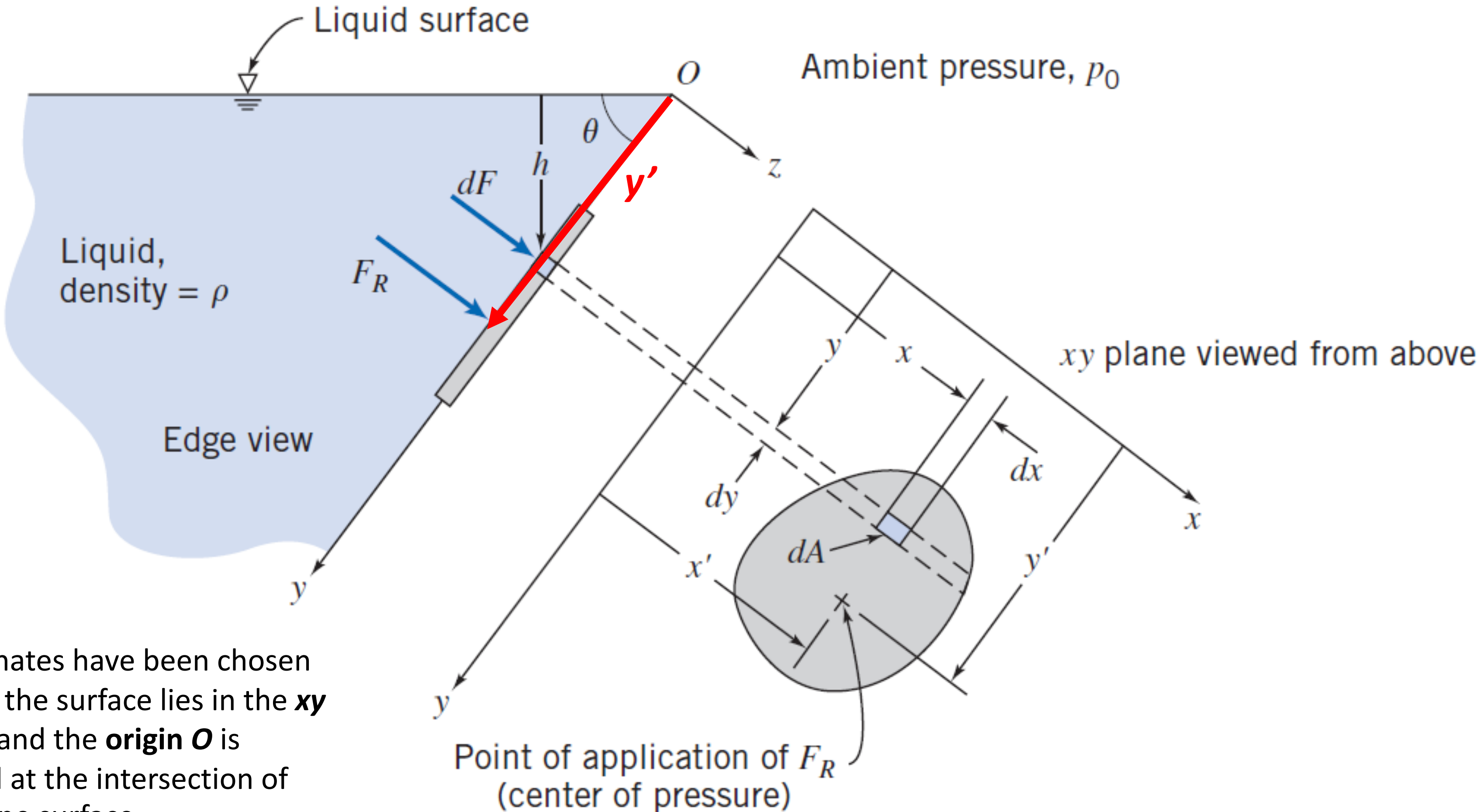
$$F_y = \int_0^H \int_0^W p\,dx\,dz$$

If the surface is rectangular:



Plane Submerged Surface

- We wish to determine the **resultant hydrostatic force** on the upper face of the submerged plane.
- ***We will find the magnitude of the force F_R and will locate the point through which it acts on the surface.***



Coordinates have been chosen so that the surface lies in the ***xy* plane**, and the **origin *O*** is located at the intersection of the plane surface.

Plane Submerged Surface

Pressure force acting on element *dA:* $dF = p\,dA$

Resultant Force: $F_R = \int_A p\,dA$ $p = p_0 + \rho g h$ $h = y \sin\theta$

$$F_R = \int_A p\,dA = \int_A (p_0 + \rho g h)dA = \int_A (p_0 + \rho g y \sin\theta)dA$$

$$F_R = p_0 \int_A dA + \rho g \sin\theta \int_A y\,dA = p_0 A + \rho g \sin\theta \int_A y\,dA$$

$\int_A y\,dA = y_c A$ → $F_R = p_0 A + \rho g \sin\theta\, y_c A = (p_0 + \rho g h_c)A$

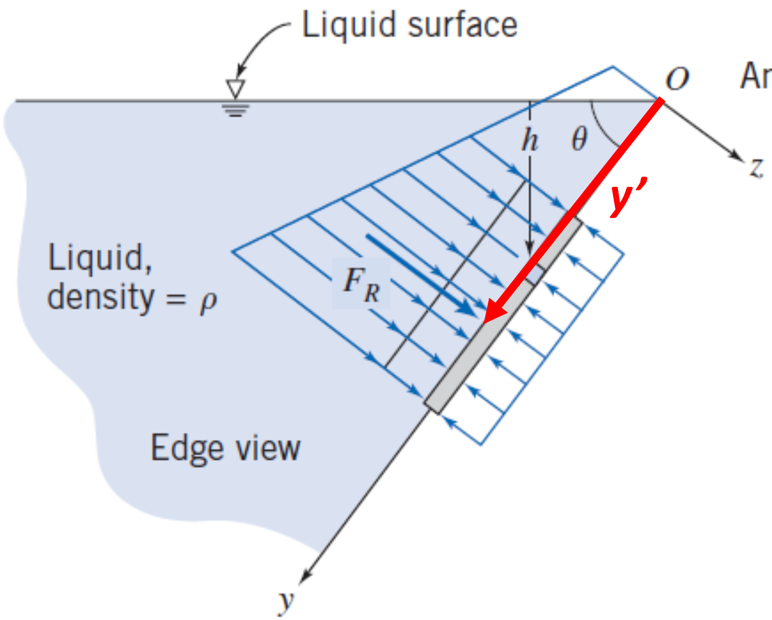
y_c is the *y* coordinate of the centroid of the area

$$F_R = p_c A$$

Total Pressure Force

Plane Submerged Surface – Location of the Force

Pressure distribution on plane submerged surface



$$y'F_R = \int_A yp\,dA$$

→ The vertical location of the line of action is determined by equating the **moment of the resultant force to the moment of the distributed pressure force** about the *x*-axis.

We can integrate by expressing *p* as a function of *y*:

$$y'F_R = \int_A yp\,dA = \int_A y(p_0 + \rho g h)\,dA = \int_A (p_0 y + \rho g y^2 \sin\theta)\,dA$$

$$= p_0 \underbrace{\int_A y\,dA}_{y_c A} + \rho g \sin\theta \underbrace{\int_A y^2 dA}_{I_{xx} = I_{\hat{x}\hat{x}} + A y_c^2}$$

Plane Submerged Surface – Location of the Force

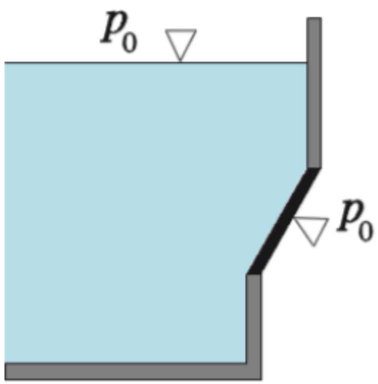
Now we find:

$$y'F_R = p_0 y_c A + \rho g \sin\theta (I_{\hat{x}\hat{x}} + A y_c^2) = y_c(p_0 + \rho g y_c \sin\theta)A + \rho g \sin\theta\, I_{\hat{x}\hat{x}}$$
$$= y_c(p_0 + \rho g h_c)A + \rho g \sin\theta I_{\hat{x}\hat{x}} = y_c F_R + \rho g \sin\theta I_{\hat{x}\hat{x}}$$

Finally, we obtain for y':

$$y' = y_c + \frac{\rho g \sin\theta\, I_{\hat{x}\hat{x}}}{F_R}$$

Neglecting p_0 to compute the net force (when the same p_0 acts at the free surface and on the other side of the submerged surface):



Net Pressure Force → $F_R = p_{c_{\text{gage}}} A = \rho g h_c A = \rho g y_c \sin\theta\, A$

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}$$

Plane Submerged Surface – Location of the Force

To find the ***x*** location of the force in the plane:

$$x' F_R = \int_A x\, p\, dA$$

$$x' F_R = \int_A xp\, dA = \int_A x(p_0 + \rho g h)\, dA = \int_A (p_0 x + \rho g x y \sin\theta)\, dA$$
$$= p_0 \int_A x\, dA + \rho g \sin\theta \int_A xy\, dA$$

$$x_c A \qquad\qquad I_{xy} = I_{\hat{x}\hat{y}} + A x_c y_c$$

We obtain for x':

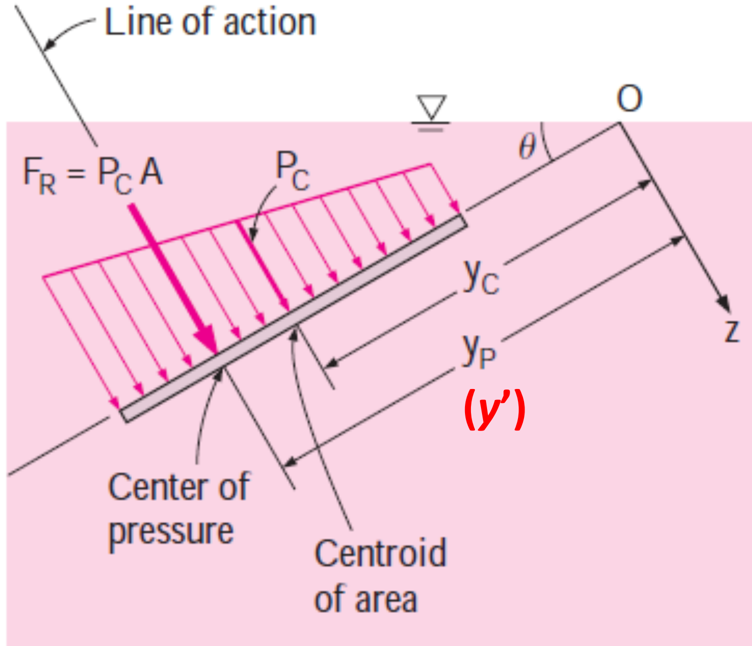
$$x' = x_c + \frac{\rho g \sin\theta\, I_{\hat{x}\hat{y}}}{F_R}$$

Neglecting p_0 to compute the net force (when the same p_0 acts at the free surface and on the other side of the submerged surface):

$$x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}$$

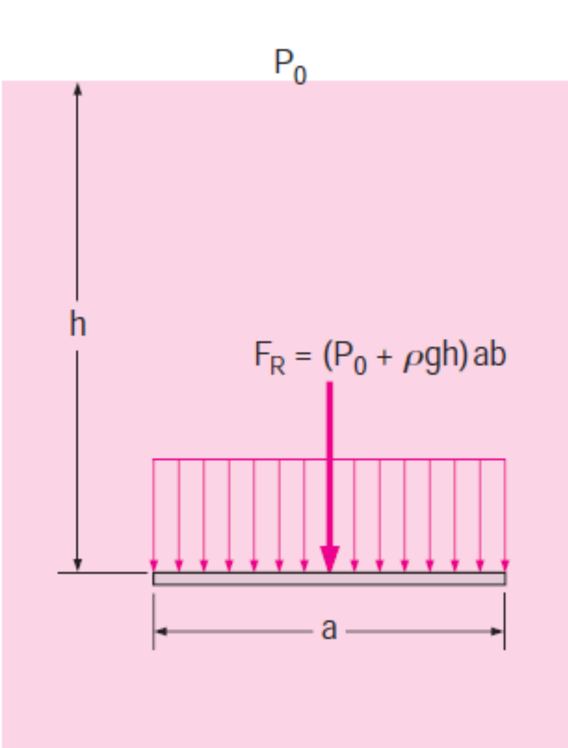
Note: I_{xy} is *zero* if symmetric, and x = x_c

Plane Submerged Surface

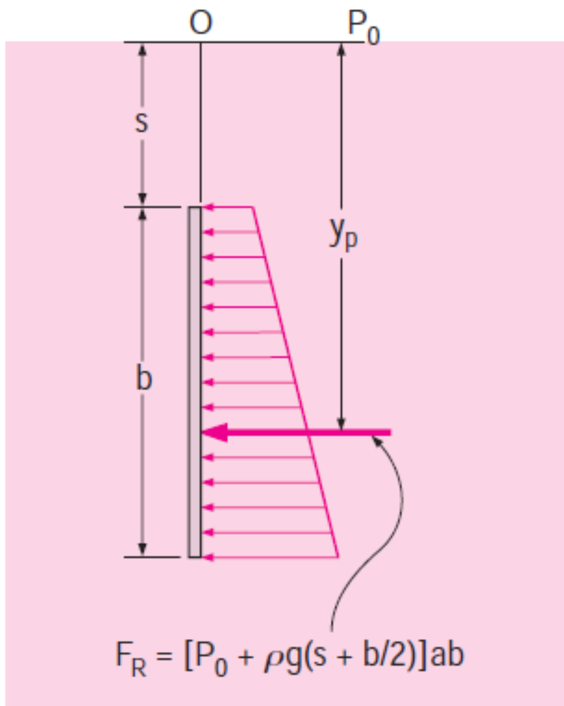


- The **magnitude of the resultant force** acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the **product of the pressure P_C at the centroid of the surface and the area A of the surface**.
- Its line of action passes through the **center of pressure.**

Plane Submerged Surface

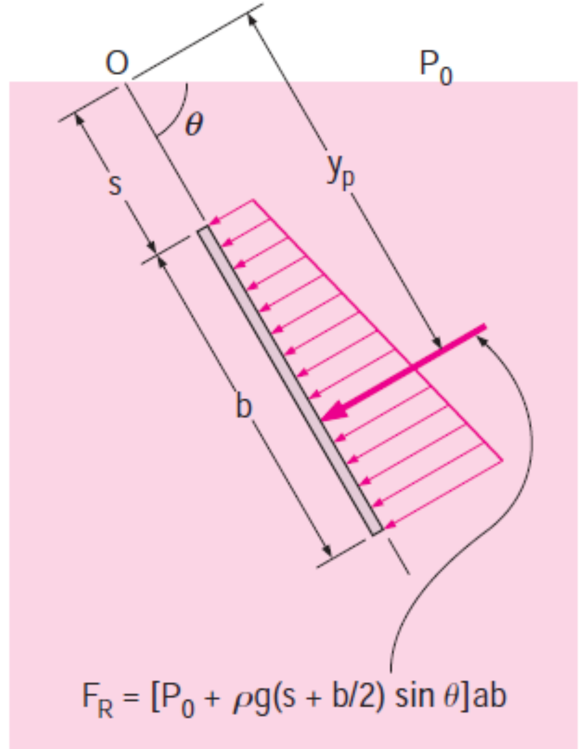


Horizontal plate



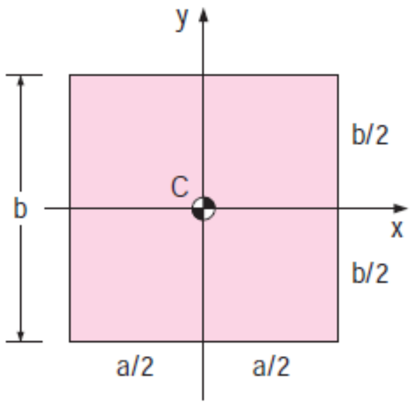
Vertical plate

(θ = 90° , sin θ = 1)



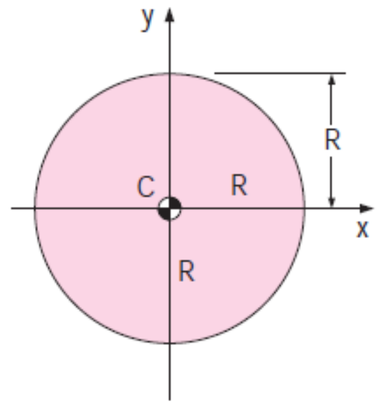
Tilted plate

Geometric Properties



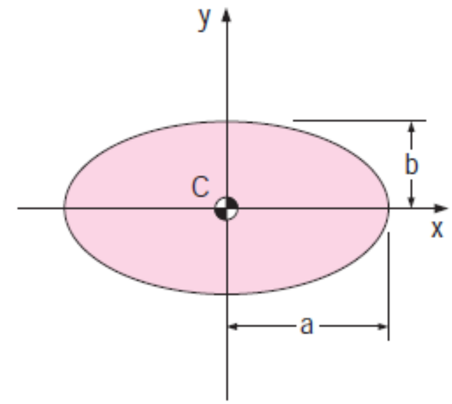
$A = ab,\ I_{xx,C} = ab^3/12$

(a) Rectangle



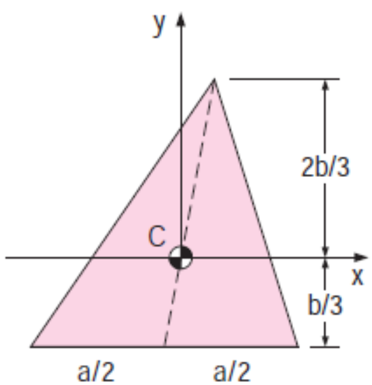
$A = \pi R^2,\ I_{xx,C} = \pi R^4/4$

(b) Circle



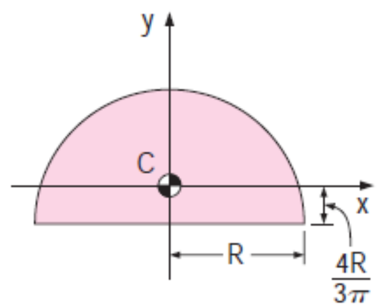
$A = \pi ab,\ I_{xx,C} = \pi ab^3/4$

(c) Ellipse



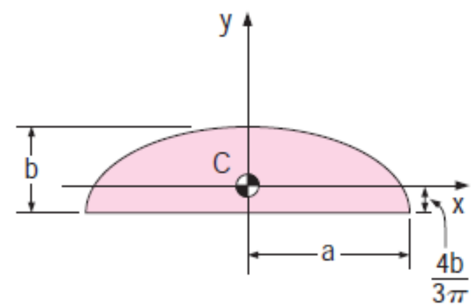
$A = ab/2,\ I_{xx,C} = ab^3/36$

(d) Triangle



$A = \pi R^2/2,\ I_{xx,C} = 0.109757R^4$

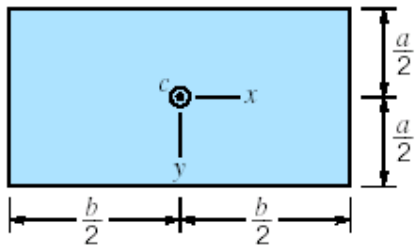
(e) Semicircle



$A = \pi ab/2,\ I_{xx,C} = 0.109757ab^3$

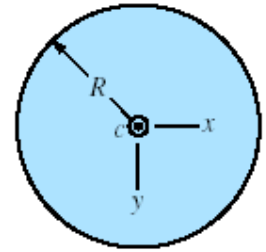
(f) Semiellipse

Geometric Properties



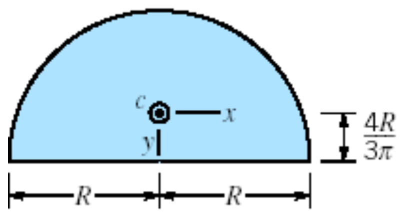
$$A = ba$$
$$I_{xc} = \frac{1}{12} ba^3$$
$$I_{yc} = \frac{1}{12} ab^3$$
$$I_{xyc} = 0$$

(*a*)



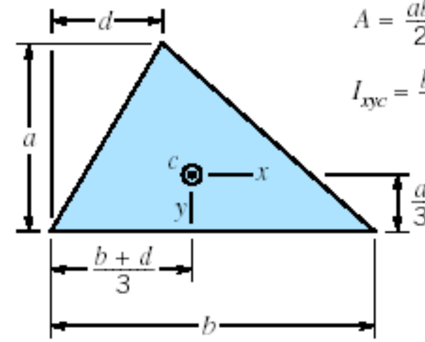
$$A = \pi R^2$$
$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$
$$I_{xyc} = 0$$

(*b*)



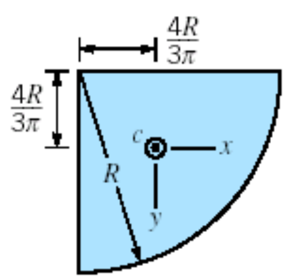
$$A = \frac{\pi R^2}{2}$$
$$I_{xc} = 0.1098R^4$$
$$I_{yc} = 0.3927R^4$$
$$I_{xyc} = 0$$

(*c*)



$$A = \frac{ab}{2} \qquad I_{xc} = \frac{ba^3}{36}$$
$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

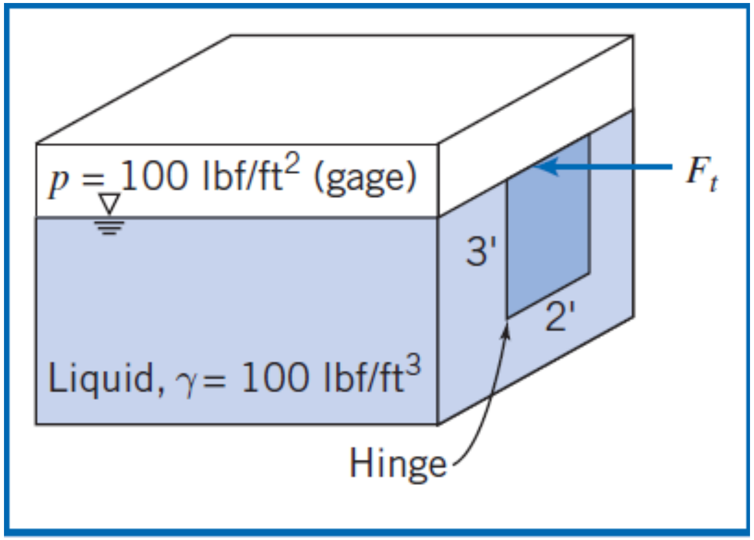
(*d*)



$$A = \frac{\pi R^2}{4}$$
$$I_{xc} = I_{yc} = 0.05488R^4$$
$$I_{xyc} = -0.01647R^4$$

(*e*)

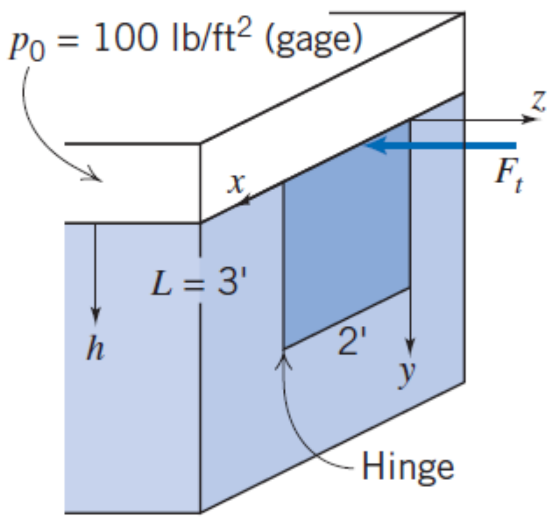
Example 3.6

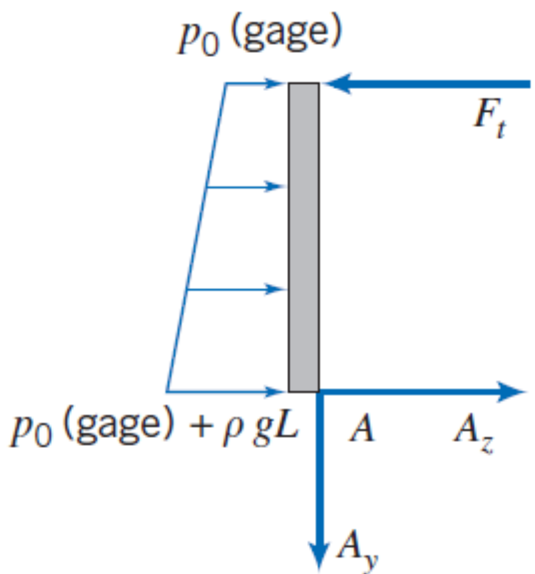


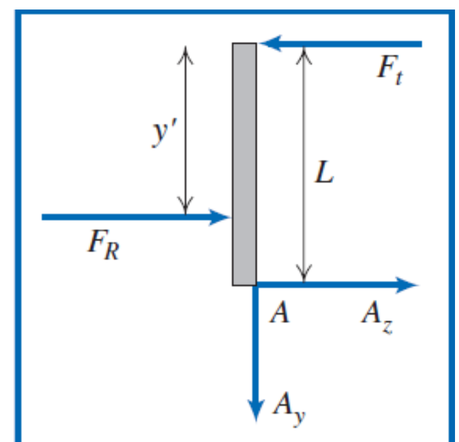
The door shown in the side of the tank is hinged along its bottom edge. A pressure of 100 lb_f/ft_2 (gage) is applied to the liquid free surface. **Find the force, F_t, required to keep the door closed.**

Equations:

$$F_R = p_c A \qquad y' = y_c + \frac{\rho g \sin\theta \, I_{\hat{x}\hat{x}}}{F_R} \qquad \sum M_A = 0$$





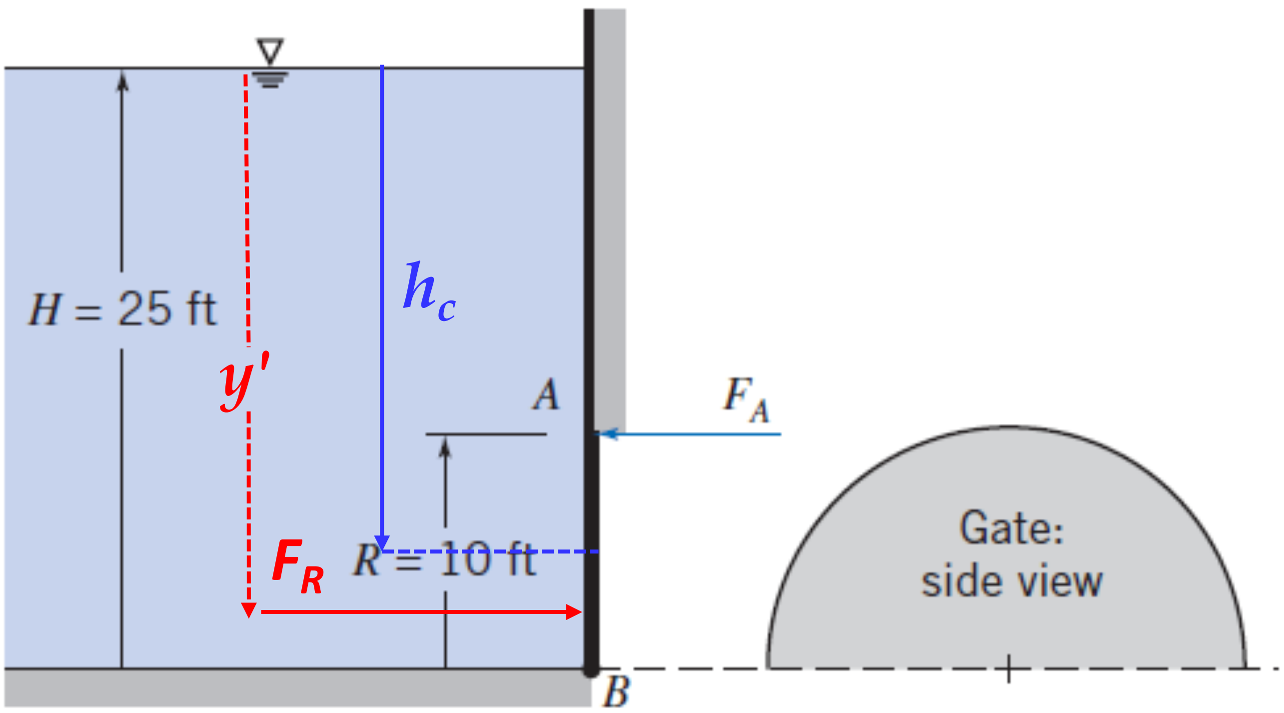


Force free-body diagram

Problem 3.41

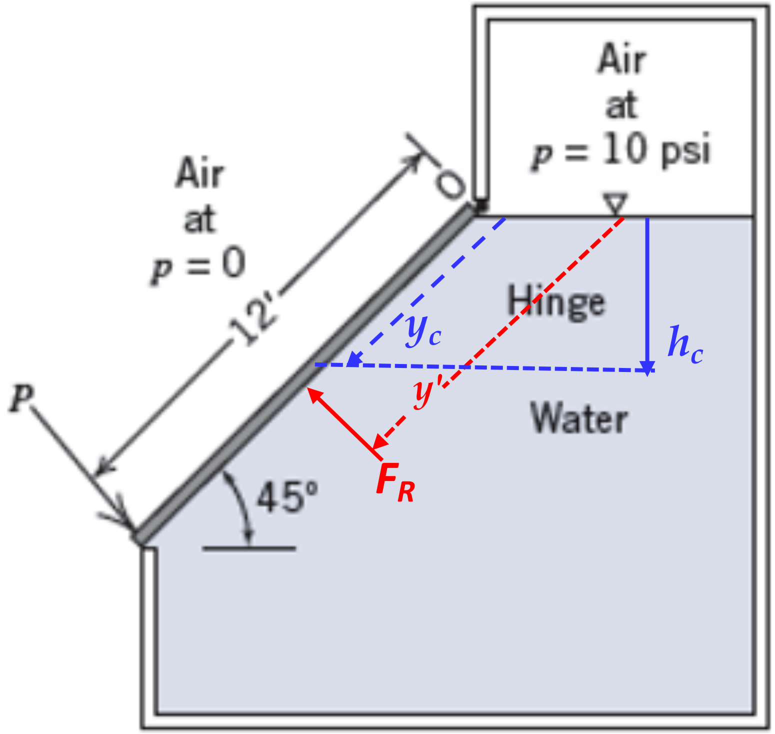
Semicircular plane gate **AB** is hinged along B and held by horizontal force F_A applied at A. The liquid to the left of the gate is water.

Calculate the force F_A required for equilibrium.



Problem 3.48

Calculate the minimum force *P* necessary to hold a uniform 12 ft square gate weighing 500 lb closed on a tank of water under a pressure of 10 psi. Draw a free body diagram of the gate as part of your solution.



Problem 3.55

The gate shown is hinged at H. The gate is 3 m wide normal to the plane of the diagram. **Calculate the force required at *A* to hold the gate closed.**