

Answer & Explanation

The average of 4 numbers is **4.5**.

Please see the explanation below.

Step-by-step explanation

Let the numbers be a, b, c & d . Now let's write the given statements in the form of equations.

(i) Sum of the first, second, third exceeds the fourth by 6: $a+b+c = d+6$. -----> equation (1),

(ii) Sum of the squares of the third and fourth exceeds the sum of the squares of first and second by 36: $c^2+d^2 = a^2+b^2 + 36$, -----> equation (2),

(iii) Sum of the product of first and second to the product of third and fourth is 42: $ab + cd = 42$. -----> equation(3),

(iv) Cube of the fourth equals the sum of the cubes of the other numbers: $d^3 = a^3+b^3+c^3$. -----> equation(4).

From equation (1): $(a+b) = (d-c)+6$, now squaring on both sides: $(a+b)^2 = [(d-c)+6]^2$,

$\Rightarrow a^2+b^2+2ab = d^2-2cd+c^2 + 12(d-c) + 36$, Now use $c^2+d^2 = a^2+b^2+36$ in this equation.

$\Rightarrow a^2+b^2+2(ab+cd) = a^2+b^2 + 36 + 12(d-c) + 36$, Now we can use equation (3) here,

$\Rightarrow 2(42) = 72 + 12(d-c)$,

$\Rightarrow 12(d-c) = 12$, and that implies $(d-c) = 1$. So now from equation (1): $(a+b) = 7$.

From equation (4): $d^3-c^3 = a^3+b^3$, We know that $x^3-y^3 = (x-y)(x^2+xy+y^2)$ & $x^3+y^3 = (x+y)(x^2-xy+y^2)$

So that implies $(d-c)(d^2+cd+c^2) = (a+b)(a^2+b^2-ab)$, Now use equations (2) & (3) here,

$\Rightarrow 1(d^2+cd+c^2) = 7(c^2+d^2-36+cd-42)$,

$\Rightarrow (d^2+cd+c^2) = 7c^2+7d^2+7cd-546$,

$\Rightarrow 6(d^2+cd+c^2) = 546$,

$\Rightarrow (d^2+cd+c^2) = 91$. -----> equation (5).

$\Rightarrow (a^2+b^2+36 + 42-ab) = 91$, $\Rightarrow (a^2+b^2+2ab -3ab) = 13$, $\Rightarrow (a+b)^2 - 3ab = 13$, $\Rightarrow 49 - 3ab = 13$, $\Rightarrow 3ab = 36$, $\Rightarrow ab = 12$. So $cd = 42-ab = 30$.

Now using equation (5): $(d^2+cd+c^2) = 91$,

$\Rightarrow (d^2+2cd+c^2 - cd) = 91$,

$\Rightarrow (c+d)^2 - 30 = 91$,

\Rightarrow So $(c+d)^2 = 121$, $\Rightarrow (c+d) = 11$.

Now $(a+b+c+d) = (a+b) + (c+d) = 7 + 11 = 18$.

The average of the 4 numbers is $\text{Avg} = \frac{a+b+c+d}{4} = \frac{18}{4} = 4.5$.