ALGEBRA 1

LOGARITHM

$$x = \log_b N \rightarrow N = b^x$$

Properties

$$\log(xy) = \log x + \log y$$
$$\log\left(\frac{x}{y}\right) = \log x - \log y$$
$$\log x^n = n \log x$$
$$\log_b x = \frac{\log x}{\log b}$$
$$\log_a a = 1$$

REMAINDER AND FACTOR THEOREMS

Given:

$$\frac{f(x)}{(x-r)}$$

Remainder Theorem: **Remainder = f(r)**
Factor Theorem: **Remainder = zero**

QUADRATIC EQUATIONS

$$Ax^2 + Bx + C = 0$$
$$Root = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Sum of the roots = - B/A
Products of roots = C/A

MIXTURE PROBLEMS

Quantity Analysis: A + B = C
Composition Analysis: Ax + By = Cz

WORK PROBLEMS

Rate of doing work = 1/ time
Rate x time = 1 (for a complete job)
Combined rate = sum of individual rates

Man-hours (is always assumed constant)

$$\frac{(Workers_1)(time_1)}{quantity.of.work_1} = \frac{(Workers_2)(time_2)}{quantity.of.work_2}$$

ALGEBRA 2

UNIFORM MOTION PROBLEMS

$$S = Vt$$

Traveling with the wind or downstream:

$$V_{total} = V_1 + V_2$$

Traveling against the wind or upstream:

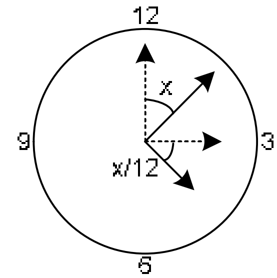
$$V_{total} = V_1 - V_2$$

DIGIT AND NUMBER PROBLEMS

$100h + 10t + u \rightarrow$ 2-digit number

where: **h** = hundred's digit
t = ten's digit
u = unit's digit

CLOCK PROBLEMS



where: **x** = distance traveled by the minute hand in minutes
x/12 = distance traveled by the hour hand in minutes

PROGRESSION PROBLEMS

a_1 = first term
a_n = n^{th} term
a_m = any term before a_n
d = common difference
S = sum of all "n" terms

ARITHMETIC PROGRESSION (AP)

- difference of any 2 no.'s is constant
- calcu function: **LINEAR (LIN)**

$$a_n = a_m + (n-m)d$$

n^{th} term

$$d = a_2 - a_1 = a_3 - a_2, ...etc$$

Common difference

$$S = \frac{n}{2}(a_1 + a_n)$$

Sum of ALL terms

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$

Sum of ALL terms

GEOMETRIC PROGRESSION (GP)

- RATIO of any 2 adj, terms is always constant
- Calcu function: **EXPONENTIAL (EXP)**

$$a_n = a_m r^{n-m}$$

n^{th} term

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

ratio

$$S = \frac{a_1(r^n - 1)}{r-1} \to r > 1$$

Sum of ALL terms, r >1

$$S = \frac{a_1(1-r^n)}{1-r} \to r < 1$$

Sum of ALL terms, r < 1

$$S = \frac{a_1}{1-r} \to r < 1 \,\&\, n = \infty$$

Sum of ALL terms, r < 1 , n = ∞

HARMONIC PROGRESSION (HP)

- a sequence of number in which their reciprocals form an AP
- calcu function: **LINEAR (LIN)**

Mean – middle term or terms between two terms in the progression.

COIN PROBLEMS

Penny = 1 centavo coin
Nickel = 5 centavo coin
Dime = 10 centavo coin
Quarter = 25 centavo coin
Half-Dollar = 50 centavo coin

DIOPHANTINE EQUATIONS

If the number of equations is less than the number of unknowns, then the equations are called "Diophantine Equations".

ALGEBRA 3

Fundamental Principle:
"If one event can occur in **m** different ways, and after it has occurred in any one of these ways, a second event can occur in **n** different ways, and then the number of ways the two events can occur in succession is **mn** different ways"

PERMUTATION

Permutation of **n** objects taken **r** at a time

$$nP_r = \frac{n!}{(n-r)!}$$

Permutation of **n** objects taken **n** at a time

$$nP_n = n!$$

Permutation of n objects with q,r,s, etc. objects are alike

$$P = \frac{n!}{q!r!s!...}$$

Permutation of n objects arrange in a circle

$$P = (n-1)!$$

COMBINATION

Combination of n objects taken r at a time

$$nC_r = \frac{n!}{(n-r)!r!}$$

Combination of n objects taken n at a time

$$nC_n = 1$$

Combination of n objects taken 1, 2, 3…n at a time

$$C = 2^n - 1$$

BINOMIAL EXPANSION

Properties of a binomial expansion: $\mathbf{(x + y)^n}$

1. The number of terms in the resulting expansion is equal to "**n+1**"
2. The powers of **x** decreases by **1** in the successive terms while the powers of **y** increases by **1** in the successive terms.
3. The sum of the powers in each term is always equal to "**n**"
4. The first term is $\mathbf{x^n}$ while the last term in $\mathbf{y^n}$ both of the terms having a **coefficient of 1**.

r^{th} term in the expansion $(x + y)^n$

$$r^{th}\ \text{term} = nC_{r-1}\,(x)^{n-r+1}\,(y)^{r-1}$$

term involving y^r in the expansion $(x + y)^n$

$$y^r\ \text{term} = nC_r\,(x)^{n-r}\,(y)^r$$

sum of coefficients of $(x + y)^n$

$$\text{Sum} = (\text{coeff. of } x + \text{coeff. of } y)^n$$

sum of coefficients of $(x + k)^n$

$$\text{Sum} = (\text{coeff. of } x + k)^n - (k)^n$$

PROBABILITY

Probability of an event to occur (P)

$$P = \frac{number\_of\_successful\_outcomes}{total\_outcomes}$$

Probability of an event not to occur (Q)

$$\mathbf{Q = 1 - P}$$

MULTIPLE EVENTS

Mutually exclusive events without a common outcome

$$P_{A\text{ or }B} = P_A + P_B$$

Mutually exclusive events with a common outcome

$$P_{A\text{ or }B} = P_A + P_B - P_{A\&B}$$

Dependent/Independent Probability

$$P_{AandB} = P_A \times P_B$$

REPEATED TRIAL PROBABILITY

$$P = nC_r\,p^r\,q^{n-r}$$

p = probability that the event happen
q = probability that the event failed

VENN DIAGRAMS

Venn diagram in mathematics is a diagram representing a set or sets and the logical, relationships between them. The sets are drawn as circles. The method is named after the British mathematician and logician John Venn.

PLANE TRIGONOMETRY

ANGLE, MEASUREMENTS & CONVERSIONS

1 revolution = 360 degrees

1 revolution = 2π radians

1 revolution = 400 grads

1 revolution = 6400 mils

1 revolution = 6400 gons

Relations between two angles (A & B)

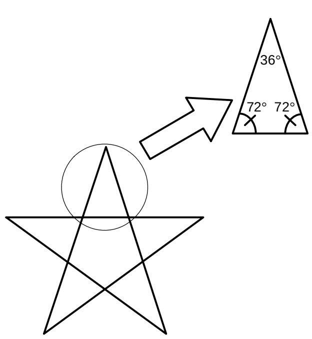
Complementary angles → **A + B = 90°**

Supplementary angles → **A + B = 180°**

Explementary angles → **A + B = 360°**

Angle (θ)	Measurement
NULL	θ = 0°
ACUTE	0° < θ < 90°
RIGHT	θ = 90°
OBTUSE	90° < θ < 180°
STRAIGHT	θ =180°
REFLEX	180° < θ < 360°
FULL OR PERIGON	θ = 360°

Pentagram – golden triangle (isosceles)



TRIGONOMETRIC IDENTITIES

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \cot^2 A = \csc^2 A$$

$$1 + \tan^2 A = \sec^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot A \pm \cot B}$$

$$\sin 2A = 2 \sin A \cos B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$$

SOLUTIONS TO OBLIQUE TRIANGLES

SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COSINE LAW

$$a^2 = b^2 + c^2 - 2\,b\,c\cos A$$

$$b^2 = a^2 + c^2 - 2\,a\,c\cos B$$

$$c^2 = a^2 + b^2 - 2\,a\,b\cos C$$

AREAS OF TRIANGLES AND QUADRILATERALS

TRIANGLES

1. Given the base and height

$$Area = \frac{1}{2}bh$$

2. Given two sides and included angle

$$Area = \frac{1}{2}ab\sin\theta$$

3. Given three sides

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

4. Triangle inscribed in a circle

$$Area = \frac{abc}{4r}$$

5. Triangle circumscribing a circle

$$Area = rs$$

6. Triangle escribed in a circle

$$Area = r(s-a)$$

QUADRILATERALS

1. Given diagonals and included angle

$$Area = \frac{1}{2} d_1 d_2 \sin q$$

2. Given four sides and sum of opposite angles

$$Area = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd \cos^2 q}$$

$$q = \frac{A+C}{2} = \frac{B+D}{2}$$

$$s = \frac{a+b+c+d}{2}$$

3. Cyclic quadrilateral – is a quadrilateral inscribed in a circle

$$Area = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{a+b+c+d}{2}$$

$$r = \frac{\sqrt{(ab+cd)(ac+bd)(ad+bc)}}{4(Area)}$$

$d_1 d_2 = ac + bd \rightarrow$ Ptolemy's Theorem

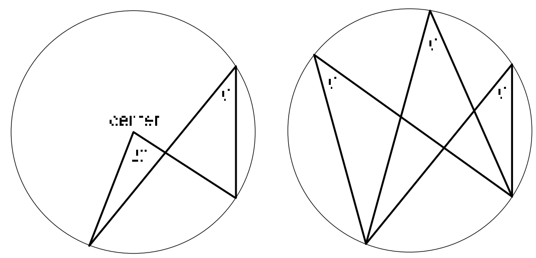
4. Quadrilateral circumscribing in a circle

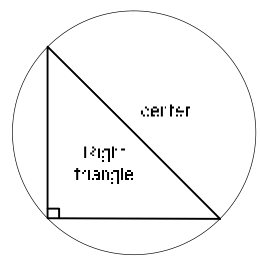
$$Area = rs$$

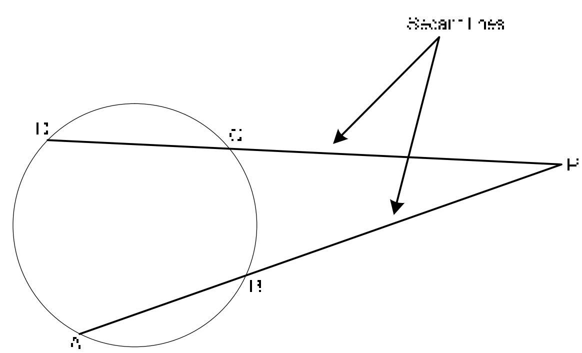
$$Area = \sqrt{abcd}$$

$$s = \frac{a+b+c+d}{2}$$

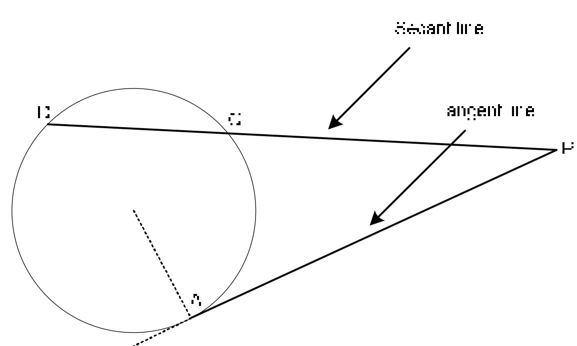
THEOREMS IN CIRCLES







(PA)(PB)=(PC)(PD)



(PA)^2=(PC)(PD)

SIMILAR TRIANGLES

$$\frac{A_1}{A_2} = \left(\frac{A}{a}\right)^2 = \left(\frac{B}{b}\right)^2 = \left(\frac{C}{c}\right)^2 = \left(\frac{H}{h}\right)^2$$

SOLID GEOMETRY

POLYGONS

3 sides – Triangle
4 sides – Quadrilateral/Tetragon/Quadrangle
5 sides – Pentagon
6 sides – Hexagon
7 sides – Heptagon/Septagon
8 sides – Octagon
9 sides – Nonagon/Enneagon
10 sides – Decagon
11 sides – Undecagon
12 sides – Dodecagon
15 sides – Quidecagon/ Pentadecagon
16 sides – Hexadecagon
20 sides – Icosagon
1000 sides – Chillagon

Let: **n** = number of sides
θ = interior angle
α = exterior angle

Sum of interior angles:

S = n θ = (n – 2) 180°

Value of each interior angle

$$q = \frac{(n-2)(180°)}{n}$$

Value of each exterior angle

$$a = 180° - q = \frac{360°}{n}$$

Sum of exterior angles:

S = n α = 360°

Number of diagonal lines (N):

$$N = \frac{n}{2}(n-3)$$

Area of a regular polygon inscribed in a circle of radius r

$$Area = \frac{1}{2}nr^2 \sin\left(\frac{360°}{n}\right)$$

Area of a regular polygon circumscribing a circle of radius r

$$Area = nr^2 \tan\left(\frac{180°}{n}\right)$$

Area of a regular polygon having each side measuring x unit length

$$Area = \frac{1}{4}nx^2 \cot\left(\frac{180°}{n}\right)$$

PLANE GEOMETRIC FIGURES

CIRCLES

$$A = \frac{pd^2}{4} = pr^2$$

$$Circumference = pd = 2pr$$

Sector of a Circle

$$A = \frac{1}{2}rs = \frac{1}{2}r^2q$$

$$A = \frac{pr^2q_{(deg)}}{360°}$$

$$s = rq_{(rad)} = \frac{prq_{(deg)}}{180°}$$

Segment of a Circle

$$A_{segment} = A_{sector} - A_{triangle}$$

ELLIPSE

$$A = \pi a b$$

PARABOLIC SEGMENT

$$A = \frac{2}{3}bh$$

TRAPEZOID

$$A = \frac{1}{2}(a+b)h$$

PARALLELOGRAM

$$A = ab\sin a$$

$$A = bh$$

$$A = \frac{1}{2}d_1 d_2 \sin q$$

RHOMBUS

$$A = \frac{1}{2}d_1 d_2 = ah$$

$$A = a^2 \sin a$$

SOLIDS WITH PLANE SURFACE

Lateral Area = (No. of Faces) (Area of 1 Face)

Polyhedron – a solid bounded by planes. The bounding planes are referred to as the faces and the intersections of the faces are called the edges. The intersections of the edges are called vertices.

PRISM

V = Bh

$A_{(lateral)}$ = PL
$A_{(surface)}$ = $A_{(lateral)}$ + 2B

where: P = perimeter of the base
L = slant height
B = base area

Truncated Prism

$$V = B\left(\frac{\sum heights}{number\ of\ heights}\right)$$

PYRAMID

$$V = \frac{1}{3}Bh$$

$$A_{(lateral)} = \sum A_{faces}$$

$$A_{(surface)} = A_{(lateral)} + B$$

Frustum of a Pyramid

$$V = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 A_2})$$

A_1 = area of the lower base
A_2 = area of the upper base

PRISMATOID

$$V = \frac{h}{6}(A_1 + A_2 + 4A_m)$$

A_m = area of the middle section

REGULAR POLYHEDRON

a solid bounded by planes whose faces are congruent regular polygons. There are five regular polyhedrons namely:

A. Tetrahedron
B. Hexahedron (Cube)
C. Octahedron
D. Dodecahedron
E. Icosahedron

Name	Type of FACE	No. of FACES	No. of EDGES	No. of VERTICES	Formulas for VOLUME
Tetrahedron	Triangle	4	6	4	$V = \frac{\sqrt{2}}{12}x^3$
Hexahedron	Square	6	12	8	$V = x^3$
Octahedron	Triangle	8	12	6	$V = \frac{\sqrt{2}}{3}x^3$
Dodecahedron	Pentagon	12	30	20	$V = 7.66x^3$
Icosahedron	Triangle	20	30	12	$V = 2.18x^3$

Where: x = length of one edge

SOLIDS WITH CURVED SURFACES

CYLINDER

V = Bh = KL

$A_{(lateral)} = P_k L = 2 \pi r h$

$A_{(surface)} = A_{(lateral)} + 2B$

P_k = perimeter of right section
K = area of the right section
B = base area
L= slant height

CONE

$$V = \frac{1}{3}Bh$$

$$A_{(lateral)} = \pi rL$$

FRUSTUM OF A CONE

$$V = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 A_2}$$

$$A_{(lateral)} = \pi (R + r)L$$

SPHERES AND ITS FAMILIES

SPHERE

$$V = \frac{4}{3}\pi r^3$$

$$A_{(surface)} = 4\pi r^2$$

SPHERICAL LUNE

is that portion of a spherical surface bounded by the halves of two great circles

$$A_{(surface)} = \frac{\pi r^2 \theta_{(deg)}}{90°}$$

SPHERICAL ZONE

is that portion of a spherical surface between two parallel planes. A spherical zone of one base has one bounding plane tangent to the sphere.

$$A_{(zone)} = 2\pi \; r \; h$$

SPHERICAL SEGMENT

is that portion of a sphere bounded by a zone and the planes of the zone's bases.

$$V = \frac{\pi h^2}{3}(3r - h)$$

$$V = \frac{\pi h}{6}(3a^2 + h^2)$$

$$V = \frac{\pi h}{6}(3a^2 + 3b^2 + h^2)$$

SPHERICAL WEDGE

is that portion of a sphere bounded by a lune and the planes of the half circles of the lune.

$$V = \frac{\pi r^3 \theta_{(deg)}}{270°}$$

SPHERICAL CONE

is a solid formed by the revolution of a circular sector about its one side (radius of the circle).

$$V = \frac{1}{3} A_{(zone)} r$$

$$A_{(surface)} = A_{(zone)} + A_{(lateralofcone)}$$

SPHERICAL PYRAMID

is that portion of a sphere bounded by a spherical polygon and the planes of its sides.

$$V = \frac{pr^3 E}{540°}$$

$$E = [(n-2)180°]$$

E = Sum of the angles
E = Spherical excess
n = Number of sides of the given spherical polygon

SOLIDS BY REVOLUTIONS

TORUS (DOUGHNUT)

a solid formed by rotating a circle about an axis not passing the circle.

$$V = 2\pi^2 R r^2$$
$$A_{(surface)} = 4\pi^2 R r$$

ELLIPSOID

$$V = \frac{4}{3} pabc$$

OBLATE SPHEROID

a solid formed by rotating an ellipse about its minor axis.

It is a special ellipsoid with **c = a**

$$V = \frac{4}{3} pa^2 b$$

PROLATE SPHEROID

a solid formed by rotating an ellipse about its major axis.

It is a special ellipsoid with **c=b**

$$V = \frac{4}{3} pab^2$$

PARABOLOID

a solid formed by rotating a parabolic segment about its axis of symmetry.

$$V = \frac{1}{2} pr^2 h$$

SIMILAR SOLIDS

$$\frac{V_1}{V_2} = \left(\frac{H}{h}\right)^3 = \left(\frac{R}{r}\right)^3 = \left(\frac{L}{l}\right)^3$$

$$\frac{A_1}{A_2} = \left(\frac{H}{h}\right)^2 = \left(\frac{R}{r}\right)^2 = \left(\frac{L}{l}\right)^2$$

$$\left(\frac{V_1}{V_2}\right)^2 = \left(\frac{A_1}{A_2}\right)^3$$

ANALYTIC GEOMETRY 1

RECTANGULAR COORDINATE SYSTEM

x = abscissa
y = ordinate

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope of a line

$$m = \tan q = \frac{y_2 - y_1}{x_2 - x_1}$$

Division of a line segment

$$x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2} \qquad y = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}$$

Location of a midpoint

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

STRAIGHT LINES

General Equation

$$Ax + By + C = 0$$

Point-slope form

$$y - y_1 = m(x - x_1)$$

Two-point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Slope and y-intercept form

$$y = mx + b$$

Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

Slope of the line, *Ax + By + C = 0*

$$m = -\frac{A}{B}$$

Angle between two lines

$$q = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

Note: Angle θ is measured in a **counterclockwise** direction. m_2 is the slope of the terminal side while m_1 is the slope of the initial side.

Distance of point (x_1,y_1) from the line

Ax + By + C = 0;

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

Note: The denominator is given the sign of B

Distance between two parallel lines

$$|d| = \frac{C_1 - C_2}{\sqrt{A^2 + B^2}}$$

Slope relations between parallel lines: $m_1 = m_2$

Line 1 → $Ax + By + C_1 = 0$
Line 2 → $Ax + By + C_2 = 0$

Slope relations between perpendicular lines: $m_1 m_2 = -1$

Line 1 → $Ax + By + C_1 = 0$
Line 2 → $Bx - Ay + C_2 = 0$

PLANE AREAS BY COORDINATES

$$A = \frac{1}{2}\left|\frac{x_1, x_2, x_3, \ldots. x_n, x_1}{y_1, y_2, y_3, \ldots. y_n, y_1}\right|$$

Note: The points must be arranged in a counter clockwise order.

LOCUS OF A MOVING POINT

The curve traced by a moving point as it moves in a plane is called the locus of the point.

SPACE COORDINATE SYSTEM

Length of radius vector r:

$$r = \sqrt{x^2 + y^2 + z^2}$$

Distance between two points $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

ANALYTIC GEOMETRY 2

CONIC SECTIONS

a two-dimensional curve produced by slicing a plane through a three-dimensional right circular conical surface

Ways of determining a Conic Section

1. By Cutting Plane
2. Eccentricity
3. By Discrimination
4. By Equation

General Equation of a Conic Section:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0 \ **$$

	Cutting plane	Eccentricity
Circle	**Parallel to base**	**$e \to 0$**
Parabola	**Parallel to element**	**$e = 1.0$**
Ellipse	**none**	**$e < 1.0$**
Hyperbola	**Parallel to axis**	**$e > 1.0$**

	Discriminant	**Equation****
Circle	**$B^2 - 4AC < 0$, $A = C$**	**$A = C$**
Parabola	**$B^2 - 4AC = 0$**	**$A \neq C$ same sign**
Ellipse	**$B^2 - 4AC < 0$, $A \neq C$**	**Sign of A opp. of B**
Hyperbola	**$B^2 - 4AC > 0$**	**A or C = 0**

CIRCLE

A locus of a moving point which moves so that its distance from a fixed point called the center is constant.

Standard Equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

General Equation:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Center at (h,k):

$$h = -\frac{D}{2A}; \quad k = -\frac{E}{2A}$$

Radius of the circle:

$$r^2 = h^2 + k^2 - \frac{F}{A} \quad \text{or} \quad r = \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$$

PARABOLA

a locus of a moving point which moves so that it's always equidistant from a fixed point called focus and a fixed line called directrix.

where: a = distance from focus to vertex
= distance from directrix to vertex

AXIS HORIZONTAL:

$$Cy^2 + Dx + Ey + F = 0$$

Coordinates of vertex (h,k):

$$k = -\frac{E}{2C}$$

Substitute k to solve for h

Length of Latus Rectum:

$$LR = \frac{D}{C}$$

AXIS VERTICAL:

$$Ax^2 + Dx + Ey + F = 0$$

Coordinates of vertex (h,k):

$$h = -\frac{D}{2A}$$

substitute h to solve for k

Length of Latus Rectum:

$$LR = \frac{E}{A}$$

STANDARD EQUATIONS:

Opening to the right:

$$(y - k)^2 = 4a(x - h)$$

Opening to the left:

$$(y - k)^2 = -4a(x - h)$$

Opening upward:

$$(x - h)^2 = 4a(y - k)$$

Opening downward:

$$(x - h)^2 = -4a(y - k)$$

Latus Rectum (LR)

a chord drawn to the axis of symmetry of the curve.

$LR = 4a$ for a parabola

Eccentricity (e)

the ratio of the distance of the moving point from the focus (fixed point) to its distance from the directrix (fixed line).

$e = 1$ for a parabola

ELLIPSE

a locus of a moving point which moves so that the sum of its distances from two fixed points called the foci is constant and is equal to the length of its major axis.

d = distance of the center to the directrix

STANDARD EQUATIONS:

Major axis is horizontal:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Major axis is vertical:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

General Equation of an Ellipse:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Coordinates of the center:

$$h = -\frac{D}{2A}; k = -\frac{E}{2C}$$

If **A > C**, then: $a^2 = A$; $b^2 = C$

If **A < C**, then: $a^2 = C$; $b^2 = A$

KEY FORMULAS FOR ELLIPSE

Length of major axis: 2a

Length of minor axis: 2b

Distance of focus to center:

$$c = \sqrt{a^2 - b^2}$$

Length of latus rectum:

$$LR = \frac{2b^2}{a}$$

Eccentricity:

$$e = \frac{c}{a} = \frac{a}{d}$$

HYPERBOLA

a locus of a moving point which moves so that the difference of its distances from two fixed points called the foci is constant and is equal to length of its transverse axis.

d = distance from **center to directrix**
a = distance from **center to vertex**
c = distance from **center to focus**

STANDARD EQUATIONS

Transverse axis is horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse axis is vertical:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

GENERAL EQUATION

$$Ax^2 - Cy^2 + Dx + Ey + F = 0$$

Coordinates of the center:

$$h = -\frac{D}{2A}; \quad k = -\frac{E}{2C}$$

If **C** is negative, then: $\mathbf{a^2 = C}$, $\mathbf{b^2 = A}$
If **A** is negative, then: $\mathbf{a^2 = A}$, $\mathbf{b^2 = C}$

Equation of Asymptote:

$$(y - k) = m(x - h)$$

Transverse axis is horizontal:

$$m = \pm\frac{b}{a}$$

Transverse axis is vertical:

$$m = \pm\frac{a}{b}$$

KEY FORMULAS FOR HYPERBOLA

Length of transverse axis: 2a

Length of conjugate axis: 2b

Distance of focus to center:

$$c = \sqrt{a^2 + b^2}$$

Length of latus rectum:

$$LR = \frac{2b^2}{a}$$

Eccentricity:

$$e = \frac{c}{a} = \frac{a}{d}$$

POLAR COORDINATES SYSTEM

$$\mathbf{x = r \cos \theta}$$

$$\mathbf{y = r \sin \theta}$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan q = \frac{y}{x}$$

SPHERICAL TRIGONOMETRY

Important propositions

1. If **two angles** of a spherical triangle are **equal**, the **sides opposite are equal**; and conversely.

2. If **two angels** of a spherical triangle are **unequal**, the **sides opposite are unequal**, and the **greater side lies opposite the greater angle**; and conversely.

3. The **sum of two sides** of a spherical triangle is **greater** than the **third side**.

$$a + b > c$$

4. The **sum of the sides** of a spherical triangle is **less than 360°.**

$$0° < a + b + c < 360°$$

5. **The sum of the angles** of a spherical triangle is **greater that 180° and less than 540°**.

$$180° < A + B + C < 540°$$

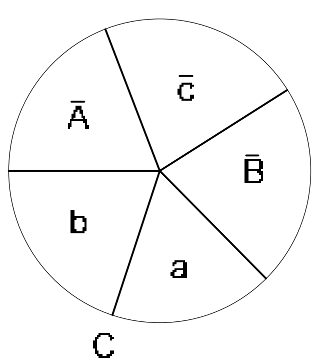
6. The **sum of any two angles** of a spherical triangle is **less than 180° plus the third angle**.

$$A + B < 180° + C$$

<u>SOLUTION TO RIGHT TRIANGLES</u>

NAPIER CIRCLE

Sometimes called Neper's circle or Neper's pentagon, is a mnemonic aid to easily find all relations between the angles and sides in a right spherical triangle.



Napier's Rules

1. The sine of any middle part is equal to the product of the cosines of the opposite parts.
   Co-op

2. The sine of any middle part is equal to the product of the tangent of the adjacent parts.
   Tan-ad

Important Rules:

1. In a right spherical triangle and oblique angle and the side opposite are of the same quadrant.

2. When the hypotenuse of a right spherical triangle is less than 90°, the two legs are of the same quadrant and conversely.

3. When the hypotenuse of a right spherical triangle is greater than 90°, one leg is of the first quadrant and the other of the second and conversely.

QUADRANTAL TRIANGLE

is a spherical triangle having a side equal to 90°.

SOLUTION TO OBLIQUE TRIANGLES

Law of Sines:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of Cosines for sides:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Law of Cosines for angles:

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

AREA OF SPHERICAL TRIANGLE

$$A = \frac{p\ R^2 E}{180°}$$

R = radius of the sphere
E = spherical excess in degrees,

E = A + B + C – 180°

TERRESTRIAL SPHERE

Radius of the Earth = 3959 statute miles

Prime meridian (Longitude = 0°)

Equator (Latitude = 0°)

Latitude = 0° to 90°

Longitude = 0° to +180° (eastward)
= 0° to –180° (westward)

1 min. on great circle arc = 1 nautical mile

1 nautical mile = **6080 feet**
= **1852 meters**

1 statute mile = **5280 feet**
= **1760 yards**

1 statute mile = **8 furlongs**
= **80 chains**

Derivatives

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\sqrt{u} = \frac{\frac{du}{dx}}{2\sqrt{u}}$$

$$\frac{d}{dx}\left(\frac{c}{u}\right) = \frac{-c\frac{du}{dx}}{u^2}$$

$$\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$\frac{d}{dx}(\ln_a u) = \frac{\log_a e \frac{du}{dx}}{u}$$

$$\frac{d}{dx}(\ln u) = \frac{\frac{du}{dx}}{u}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\csc^{-1} u) = \frac{-1}{u\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}(\tanh u) = \sec h^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\coth u) = -\csc h^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec hu) = -\sec hu \tanh u\frac{du}{dx}$$

$$\frac{d}{dx}(\csc hu) = -\csc hu \coth u\frac{du}{dx}$$

$$\frac{d}{dx}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\sinh^{-1} u) = \frac{-1}{u^2-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sec h^{-1} u) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\csc h^{-1} u) = \frac{-1}{u\sqrt{1+u^2}}\frac{du}{dx}$$

DIFFERENTIAL CALCULUS

LIMITS

Indeterminate Forms

$$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad (0)(\infty), \quad \infty - \infty, \quad 0^0, \quad \infty^0, \quad 1^\infty$$

L'Hospital's Rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} = \lim_{x\to a}\frac{f''(x)}{g''(x)}.....$$

Shortcuts

Input equation in the calculator

TIP 1: **if x → 1, substitute x = 0.999999**

TIP 2: **if x → ∞, substitute x = 999999**

TIP 3: **if Trigonometric, convert to RADIANS then do tips 1 & 2**

MAXIMA AND MINIMA

Slope (pt.)	Y'	Y"	Concavity
MAX	**0**	**(-) dec**	**down**
MIN	**0**	**(+) inc**	**up**
INFLECTION	**-**	**No change**	**-**

HIGHER DERIVATIVES

n^{th} derivative of x^n

$$\frac{d^n}{dx^n}(x^n) = n!$$

n^{th} derivative of xe^n

$$\frac{d^n}{dx^n}(xe^n) = (x+n)\,e^X$$

TIME RATE

the rate of change of the variable with respect to time

$$+\frac{dx}{dt}$$ = increasing rate

$$-\frac{dx}{dt}$$ = decreasing rate

APPROXIMATION AND ERRORS

If "dx" is the error in the measurement of a quantity x, then "dx/x" is called the RELATIVE ERROR.

RADIUS OF CURVATURE

$$R = \frac{[1+(y')^2]^{\frac{3}{2}}}{|y''|}$$

INTEGRAL CALCULUS 1

$$\int du = u + C$$

$$\int a du = au + C$$

$$\int [f(u) + g(u)]du = \int f(u)du + \int g(u)du$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C .... (n \neq 1)$$

$$\int \frac{du}{u} = \ln u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int e^u du = e^u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \tan u du = \ln|\sec u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \csc u du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(1 - \frac{u}{a}\right) + C$$

$$\int \sinh u du = \cosh u + C$$

$$\int \cosh u du = \sinh u + C$$

$$\int \sec h^2 u du = \tanh u + C$$

$$\int \csc h^2 u du = -\coth u + C$$

$$\int \sec hu \tanh u du = -\sec hu + C$$

$$\int \csc hu \coth u du = -\csc hu + C$$

$$\int \tanh u du = \ln|\cosh u| + C$$

$$\int \coth u du = \ln|\sinh u| + C$$

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a}\sinh^{-1}\frac{a}{u} + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C .... |u| < a$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{a}\coth^{-1}\frac{u}{a} + C .... |u| > a$$

$$\int u dv = uv - \int v du$$

PLANE AREAS

Plane Areas bounded by a curve and the coordinate axes:

$$A = \int_{x_1}^{x_2} y_{(curve)}\, dx$$

$$A = \int_{y_1}^{y_2} x_{(curve)}\, dy$$

Plane Areas bounded by a curve and the coordinate axes:

$$A = \int_{x_1}^{x_2} (y_{(up)} - y_{(down)})\, dx$$

$$A = \int_{y_1}^{y_2} (x_{(right)} - x_{(left)})\, dy$$

Plane Areas bounded by polar curves:

$$A = \frac{1}{2}\int_{q_1}^{q_2} r^2\, dq$$

CENTROID OF PLANE AREAS (VARIGNON'S THEOREM)

Using a Vertical Strip:

$$A \bullet \bar{x} = \int_{x_1}^{x_2} dA \bullet x$$

$$A \bullet \bar{y} = \int_{x_1}^{x_2} dA \bullet \frac{y}{2}$$

Using a Horizontal Strip:

$$A \bullet \bar{x} = \int_{y_1}^{y_2} dA \bullet \frac{x}{2}$$

$$A \bullet \bar{y} = \int_{y_1}^{y_2} dA \bullet y$$

CENTROIDS

Half a Parabola

$$\bar{x} = \frac{3}{8} b$$

$$\bar{y} = \frac{2}{5} h$$

Whole Parabola

$$\bar{y} = \frac{2}{5} h$$

Triangle

$$\bar{x} = \frac{1}{3} b = \frac{2}{3} b$$

$$\bar{y} = \frac{1}{3} h = \frac{2}{3} h$$

LENGTH OF ARC

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{z_1}^{z_2} \sqrt{\left(\frac{dx}{dz}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dz$$

INTEGRAL CALCULUS 2

TIP 1: Problems will usually be of this nature:

- "Find the area bounded by"
- "Find the area revolved around.."

TIP 2: Integrate only when the shape is IRREGULAR, otherwise use the prescribed formulas

VOLUME OF SOLIDS BY REVOLUTION

Circular Disk Method

$$V = p\int_{x_1}^{x_2} R^2 dx$$

Cylindrical Shell Method

$$V = 2p\int_{y_1}^{y_2} RL\, dy$$

Circular Ring Method

$$V = p\int_{x_1}^{x_2} (R^2 - r^2)dx$$

PROPOSITIONS OF PAPPUS

First Proposition: If a plane arc is revolved about a coplanar axis not crossing the arc, the area of the surface generated is equal to the product of the length of the arc and the circumference of the circle described by the centroid of the arc.

$$A = S \bullet 2p\bar{r}$$

$$A = \int dS \bullet 2p\bar{r}$$

Second Proposition: If a plane area is revolved about a coplanar axis not crossing the area, the volume generated is equal to the product of the area and the circumference of the circle described by the centroid of the area.

$$V = A \bullet 2p\bar{r}$$

$$V = \int dA \bullet 2p\bar{r}$$

CENTROIDS OF VOLUMES

$$V \bullet \bar{x} = \int_{x_1}^{x_2} dV \bullet x$$

$$V \bullet \bar{y} = \int_{y_1}^{y_2} dV \bullet y$$

WORK BY INTEGRATION

Work = force × distance

$$W = \int_{x_1}^{x_2} F dx = \int_{y_1}^{y_2} F dy$$; where **F = k x**

Work done on spring

$$W = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

k = spring constant
x_1 = initial value of elongation
x_2 = final value of elongation

Work done in pumping liquid out of the container at its top

Work = (density)(volume)(distance)

Force = (density)(volume) = ρv

Specific Weight:

$$g = \frac{Weight}{Volume}$$

γ_{water} = 9.81 kN/m^2 SI
γ_{water} = 45 lbf/ft^2 cgs

Density:

$$r = \frac{mass}{Volume}$$

ρ_{water} = 1000 kg/m^3 SI
ρ_{water} = 62.4 lb/ft^3 cgs
ρ_{subs} = (substance) (ρ_{water})
1 ton = 2000lb

MOMENT OF INERTIA

Moment of Inertia about the x- axis:

$$I_x = \int_{x_1}^{x_2} y^2 dA$$

Moment of Inertia about the y- axis:

$$I_y = \int_{y_1}^{y_2} x^2 dA$$

Parallel Axis Theorem

The moment of inertia of an area with respect to any coplanar line equals the moment of inertia of the area with respect to the parallel centroidal line plus the area times the square of the distance between the lines.

$$I_x = Ix_o = Ad^2$$

Moment of Inertia for Common Geometric Figures

Square

$$I_x = \frac{bh^3}{3}$$

$$I_{xo} = \frac{bh^3}{12}$$

Triangle

$$I_x = \frac{bh^3}{12}$$

$$I_{xo} = \frac{bh^3}{36}$$

Circle

$$I_{xo} = \frac{pr^4}{4}$$

Half-Circle

$$I_x = \frac{pr^4}{8}$$

Quarter-Circle

$$I_x = \frac{pr^4}{16}$$

Ellipse

$$I_x = \frac{pab^3}{4}$$

$$I_y = \frac{pa^3b}{4}$$

FLUID PRESSURE

$$F = w\overline{h}A = g\,\overline{h}A$$

$$F = \int w\overline{h}\,dA$$

F = force exerted by the fluid on one side of the area
h = distance of the c.g. to the surface of liquid
w = specific weight of the liquid (γ)
A = vertical plane area

Specific Weight:

$$g = \frac{Weight}{Volume}$$

γ_{water} = 9.81 kN/m^2 SI
γ_{water} = 45 lbf/ft^2 cgs

MECHANICS 1

VECTORS

Dot or Scalar product

$$P \bullet Q = |P||Q|\cos q$$
$$P \bullet Q = P_x Q_x + P_y Q_y + P_z Q_z$$

Cross or Vector product

$$P \times Q = |P||Q|\sin q$$
$$P \times Q = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

EQUILIBRIUM OF COPLANAR FORCE SYSTEM

Conditions to attain Equilibrium:

$$\sum F_{(x-axis)} = 0$$
$$\sum F_{(y-axis)} = 0$$
$$\sum M_{(po\,\mathrm{int})} = 0$$

Friction

$$F_f = \mu N$$

$$\tan\varphi = \mu$$

φ = angle of friction

if no forces are applied except for the weight,

$$\varphi = \theta$$

CABLES

PARABOLIC CABLES

the load of the cable of distributed horizontally along the span of the cable.

Uneven elevation of supports

$$H = \frac{wx_1^{\;2}}{2d_1} = \frac{wx_2^{\;2}}{2d_2}$$

$$T_1 = \sqrt{(wx_1)^2 + H^2}$$

$$T_2 = \sqrt{(wx_2)^2 + H^2}$$

Even elevation of supports

$$\frac{L}{d} > 10$$
$$H = \frac{wL^2}{8d}$$

$$T = \sqrt{\left(\frac{wL}{2}\right)^2 + H^2}$$

$$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$$

L = span of cable
d = sag of cable
T = tension of cable at support
H = tension at lowest point of cable
w = load per unit length of span
S = total length of cable

CATENARY
the load of the cable is distributed along the entire length of the cable.

Uneven elevation of supports

$$T_1 = wy_1$$
$$T_2 = wy_2$$
$$H = wc$$
$$y_1^{\,2} = S_1^{\,2} + c^2$$
$$y_2^{\,2} = S_2^{\,2} + c^2$$
$$x_1 = c\ln\left(\frac{S_1 + y_1}{c}\right)$$
$$x_2 = c\ln\left(\frac{S_2 + y_2}{c}\right)$$
$$Span = x_1 + x_2$$

Total length of cable = $S_1 + S_2$

Even elevation of supports

$$T = wy$$
$$H = wc$$
$$y^2 = S^2 + c^2$$
$$x = c\ln\left(\frac{S + y}{c}\right)$$
$$Span = 2x$$

Total length of cable = 2S

MECHANICS 2

RECTILINEAR MOTION

Constant Velocity

S = Vt

Constant Acceleration: Horizontal Motion

$$S = V_0 t \pm \frac{1}{2}at^2$$
$$V = V_0 \pm at$$
$$V^2 = V_0^{\,2} \pm 2aS$$

+ (sign) = body is speeding up
– (sign) = body is slowing down

Constant Acceleration: Vertical Motion

$$\pm H = V_0 t - \frac{1}{2}gt^2$$
$$V = V_0 \pm gt$$
$$V^2 = V_0^{\,2} \pm 2gH$$

+ (sign) = body is moving down
– (sign) = body is moving up

Values of g,

	SI (m/s^2)	English (ft/s^2)
general	9.81	32.2
estimate	9.8	32
exact	9.806	32.16

Variable Acceleration

$$V = \frac{dS}{dt}$$

$$a = \frac{dV}{dt}$$

PROJECTILE MOTION

$$x = (V_0 \cos q)t$$

$$\pm y = (V_0 \sin q)t - \frac{1}{2}gt^2$$

$$\pm y = x \tan q - \frac{gx^2}{2V_0^{\ 2} \cos^2 q}$$

Maximum Height and Horizontal Range

$$y = \frac{V_0^{\ 2} \sin^2 q}{2g} \text{ max } ht$$

$$x = \frac{V_0^{\ 2} \sin 2q}{g}$$

Maximum Horizontal Range

Assume: V_0 = fixed
θ = variable

$$R_{\max} = \frac{V_0^{\ 2}}{g} \Leftrightarrow q = 45°$$

ROTATION (PLANE MOTION)

Relationships between linear & angular parameters:

$$V = rw$$
$$a = ra$$

V = linear velocity
ω = angular velocity (rad/s)
a = linear acceleration
α = angular acceleration (rad/s^2)
r = radius of the flywheel

	Linear Symbol	Angular Symbol
Distance	S	θ
Velocity	V	ω
Acceleration	A	α
Time	t	t

Constant Velocity

θ = ωt

Constant Acceleration

$$q = w_0 t \pm \frac{1}{2}at^2$$

$$w = w_0 \pm at$$

$$w^2 = w_0^{\ 2} \pm 2aq$$

+ (sign) = body is speeding up
– (sign) = body is slowing down

D'ALEMBERT'S PRINCIPLE

"Static conditions maybe produced in a body possessing acceleration by the addition of an imaginary force called reverse effective force (REF) whose magnitude is (W/g)(a) acting through the center of gravity of the body, and parallel but opposite in direction to the acceleration."

$$REF = ma = \left(\frac{W}{g}\right)a$$

UNIFORM CIRCULAR MOTION

motion of any body moving in a circle with a constant speed.

$$F_c = \frac{mV^2}{r} = \frac{WV^2}{gr}$$

$$a_c = \frac{V^2}{r}$$

F_c = centrifugal force
V = velocity
m = mass
W = weight
r = radius of track
a_c = centripetal acceleration
g = standard gravitational acceleration

BANKING ON HI-WAY CURVES

Ideal Banking: The road is frictionless

$$\tan q = \frac{V^2}{gr}$$

Non-ideal Banking: With Friction on the road

$$\tan(q + f) = \frac{V^2}{gr}; \qquad \tan f = m$$

V = velocity
r = radius of track
g = standard gravitational acceleration
θ = angle of banking of the road
f = angle of friction
μ = coefficient of friction

Conical Pendulum

T = W secθ

$$\tan q = \frac{F}{W} = \frac{V^2}{gr}$$

$$f = \frac{1}{2p}\sqrt{\frac{g}{h}}$$ *frequency*

BOUYANCY

A body submerged in fluid is subjected by an unbalanced force called buoyant force equal to the weight of the displaced fluid

F_b = W

$F_b = \gamma V_d$

F_b = buoyant force
W = weight of body or fluid
γ = specific weight of fluid
V_d = volume displaced of fluid or volume of submerged body

Specific Weight:

$$g = \frac{Weight}{Volume}$$

γ_{water} = 9.81 kN/m^2 SI
γ_{water} = 45 lbf/ft^2 cgs

ENGINEERING MECHANICS 3

IMPULSE AND MOMENTUM

Impulse = Change in Momentum

$$F\Delta t = mV - mV_0$$

F = force
t = time of contact between the body and the force
m = mass of the body
V_0 = initial velocity
V = final velocity

Impulse, I

$$I = F\Delta t$$

Momentum, P

$$P = mV$$

LAW OF CONSERVATION OF MOMENTUM

"In every process where the velocity is changed, the momentum lost by one body or set of bodies is equal to the momentum gain by another body or set of bodies"

Momentum lost = Momentum gained

$$m_1V_1 + m_2V_2 = m_1V_1^{'} + m_2V_2^{'}$$

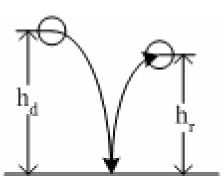
m_1 = mass of the first body
m_2 = mass of the second body
V_1 = velocity of mass 1 before the impact
V_2 = velocity of mass 2 before the impact
V_1' = velocity of mass 1 after the impact
V_2' = velocity of mass 2 after the impact

Coefficient of Restitution (e)

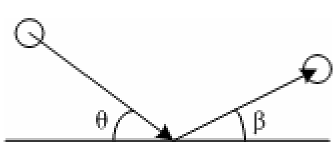
$$e = \frac{V_2^{'} - V_1^{'}}{V_1 - V_2}$$

Type of collision	e	Kinetic Energy
ELASTIC	100% conserved	$0 < e > 1$
INELASTIC	Not 100% conserved	$e = 0$
PERFECTLY INELASTIC	Max Kinetic Energy Lost	$e = 1$

Special Cases



$$e = \sqrt{\frac{h_r}{h_d}}$$



$$e = \cot q \tan b$$

Work, Energy and Power

Work

$$W = F \cdot S$$

Force	Distance	Work
Newton (N)	Meter	Joule
Dyne	Centimeter	ft-lb_f
Pound (lb_f)	Foot	erg

Potential Energy

$$PE = mgh = Wh$$

Kinetic Energy

$$KE_{linear} = \frac{1}{2}mV^2$$

$$KE_{rotational} = \frac{1}{2}Iw^2 \rightarrow V = r\omega$$

I = mass moment of inertia
ω = angular velocity

Mass moment of inertia of rotational INERTIA for common geometric figures:

Solid sphere: $I = \frac{2}{5}mr^2$

Thin-walled hollow sphere: $I = \frac{2}{3}mr^2$

Solid disk: $I = \frac{1}{2}mr^2$

Solid Cylinder: $I = \frac{1}{2}mr^2$

Hollow Cylinder: $I = \frac{1}{2}m(r_{outer}^2 - r_{inner}^2)$

m = mass of the body
r = radius

POWER

rate of using energy

$$P = \frac{W}{t} = F \cdot V$$

1 watt = **1 Newton-m/s**

1 joule/sec = **107 ergs/sec**

1 hp = **550 lb-ft per second**
= **33000 lb-ft per min**
= **746 watts**

LAW ON CONSERVATION OF ENERGY

"Energy cannot be created nor destroyed, but it can be change from one form to another"

Kinetic Energy = Potential Energy

WORK-ENERGY RELATIONSHIP

The net work done on an object always produces a change in kinetic energy of the object.

Work Done = ΔKE

Positive Work – Negative Work = ΔKE

Total Kinetic Energy = linear + rotation

HEAT ENERGY AND CHANGE IN PHASE

Sensible Heat is the heat needed to change the temperature of the body without changing its phase.

$$Q = mc\Delta T$$

Q = sensible heat
m = mass
c = specific heat of the substance
ΔT = change in temperature

Specific heat values

C_{water} = **1 BTU/lb–°F**
C_{water} = **1 cal/gm–°C**
C_{water} = **4.156 kJ/kg**
C_{ice} = **50% C_{water}**
C_{steam} = **48% C_{water}**

Latent Heat is the heat needed by the body to change its phase without changing its temperature.

$$Q = \pm mL$$

Q = heat needed to change phase
m = mass
L = latent heat (fusion/vaporization)
(+) = heat is entering (substance melts)
(–) = heat is leaving (substance freezes)

Latent heat of Fusion – solid to liquid
Latent heat of Vaporization – liquid to gas

Values of Latent heat of Fusion and Vaporization,

L_f = **144 BTU/lb**
L_f = **334 kJ/kg**
$L_{f\ ice}$ = **80 cal/gm**
$L_{v\ boil}$ = **540 cal/gm**

L_f = 144 BTU/lb
= 334 kJ/kg
L_v = 970 BTU/lb
= 2257 kJ/kg
1 calorie = **4.186 Joules**
1 BTU = **252 calories**
= **778 ft–lb_f**

LAW OF CONSERVATION OF HEAT ENERGY

When two masses of different temperatures are combined together, the heat absorbed by the lower temperature mass is equal to the heat given up by the higher temperature mass.

Heat gained = Heat lost

THERMAL EXPANSION

For most substances, the physical size increase with an increase in temperature and decrease with a decrease in temperature.

$$\Delta L = L\alpha\Delta T$$

ΔL = change in length
L = original length
α = coefficient of linear expansion
ΔT = change in temperature

$$\Delta V = V\beta\Delta T$$

ΔV = change in volume
V = original volume
β = coefficient of volume expansion
ΔT = change in temperature

Note: In case β is not given; $\beta = 3\alpha$

THERMODYNAMICS

In thermodynamics, there are four laws of very general validity. They can be applied to systems about which one knows nothing other than the balance of energy and matter transfer.

ZEROTH LAW OF THERMODYNAMICS
stating that thermodynamic equilibrium is an equivalence relation.
If two thermodynamic systems are in thermal equilibrium with a third, they are also in thermal equilibrium with each other.

FIRST LAW OF THERMODYNAMICS
about the conservation of energy

The increase in the energy of a closed system is equal to the amount of energy added to the system by heating, minus the amount lost in the form of work done by the system on its surroundings.

SECOND LAW OF THERMODYNAMICS
about entropy

The total entropy of any isolated thermodynamic system tends to increase over time, approaching a maximum value.

THIRD LAW OF THERMODYNAMICS,
about absolute zero temperature

As a system asymptotically approaches absolute zero of temperature all processes virtually cease and the entropy of the system asymptotically approaches a minimum value. This law is more clearly stated as: "the entropy of a perfectly crystalline body at absolute zero temperature is zero."

STRENGTH OF MATERIALS

SIMPLE STRESS

$$Stress = \frac{Force}{Area}$$

Axial Stress

the stress developed under the action of the force acting axially (or passing the centroid) of the resisting area.

$$s_{axial} = \frac{P_{axial}}{A}$$

P_{axial} ⊥ Area

σ_{axial} = axial/tensile/compressive stress
P = applied force/load at centroid of x'sectional area
A = resisting area (perpendicular area)

Shearing stress

the stress developed when the force is applied parallel to the resisting area.

$$s_s = \frac{P}{A}$$

$P_{appliedl}$ ‖ Area

σ_s = shearing stress
P = applied force or load
A = resisting area (sheared area)

Bearing stress

the stress developed in the area of contact (projected area) between two bodies.

$$s_b = \frac{P}{A} = \frac{P}{dt}$$

P ⊥ $A_{baering}$

σ_b = bearing stress
P = applied force or load
A = projected area (contact area)
d,t = width and height of contact, respectively

Units of σ

SI	mks/cgs	English
N/m^2 = Pa kN/m^2 = kPa MN/m^2 = MPa GN/m^2 = Gpa **N/mm^2 = MPa**	Kg/cm^2	lb_f/m^2 = psi 10^3 psi = ksi 10^3 lb_f = kips

Standard Temperature and Pressure (STP)

101.325 kPa	=	14.7 psi
	=	1.032 kg_f/cm^2
	=	780 torr
	=	1.013 bar
	=	1 atm
	=	780 mmHg
	=	29.92 in

Thin-walled Pressure Vessels

A. Tangential stress

$$s_T = \frac{r\ r}{t} = \frac{rD}{2t}$$

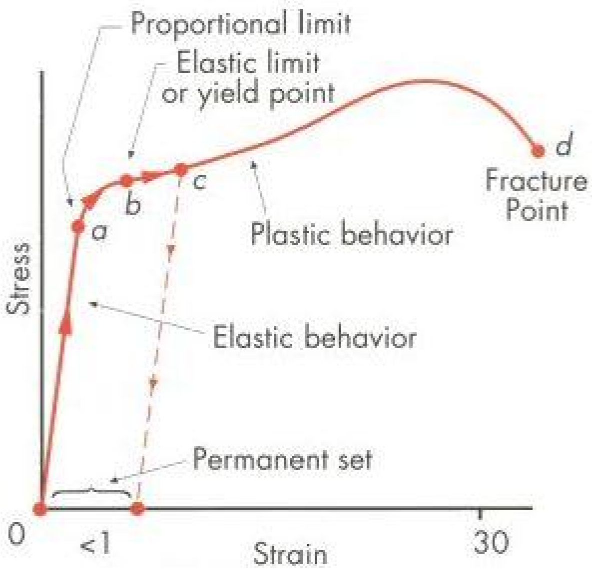
B. Longitudinal stress (also for Spherical)

$$s_L = \frac{r\ r}{2t} = \frac{rD}{4t}$$

σ_T = tangential/circumferential/hoop stress
σ_L = longitudinal/axial stress, used in spheres
r = outside radius
D = outside diameter
ρ = pressure inside the tank
t = thickness of the wall
F = bursting force

SIMPLE STRAIN / ELONGATION

Strain – ratio of elongation to original length



$$e = \frac{d}{L}$$

ε = strain
δ = elongation
L = original length

Elastic Limit – the range beyond which the material WILL NOT RETURN TO ITS ORIGINAL SHAPE when unloaded but will retain a permanent deformation

Yield Point – at his point there is an appreciable elongation or yielding of the material without any corresponding increase in load; ductile materials and continuous deformation

Ultimate Strength – it is more commonly called ULTIMATE STRESS; it's the hishes ordinate in the curve

Rupture Strength/Fracture Point – the stress at failure

Types of elastic deformation:

a. Due to axial load

HOOKE'S LAW ON AXIAL DEFORMATION

"Stress is proportional to strain"

$$s \;\; a \;\; e$$
$$s = Ye \quad Young's\ Modulus\ of\ Elasticity$$
$$s = Ee \quad Modulus\ of\ Elasticity$$
$$s_s = E_s e_s \quad Modulus\ in\ Shear$$
$$s_V = E_V e_V \quad Bulk\ Modulus\ of\ Elasticity$$
$$1/E_v \quad compressibility$$

$$d = \frac{PL}{AE}$$

δ = elongation
P = applied force or load
A = area
L = original length
E = modulus of elasticity
σ = stress
ε = strain

b. Due to its own mass

$$d = \frac{rgL^2}{2E} = \frac{mgL}{2AE}$$

δ = elongation
ρ = density or unit mass of the body
g = gravitational acceleration
L = original length
E = modulus of elasticity or Young's modulus
m = mass of the body

c. Due to changes in temperature

$$d = La(T_f - T_i)$$

δ = elongation
α = coefficient of linear expansion of the body
L = original length
$\mathbf{T_f}$ = final temperature
$\mathbf{T_i}$ = initial temperature

d. Biaxial and Triaxial Deformation

$$m = -\frac{e_y}{e_x} = -\frac{e_z}{e_x}$$

μ = Poisson's ratio
μ = 0.25 to 0.3 for steel
= 0.33 for most metals
= 0.20 for concrete
μ_{min} = 0
μ_{max} = 0.5

TORSIONAL SHEARING STRESS

Torsion – refers to twisting of solid or hollow rotating shaft.

Solid shaft

$$t = \frac{16T}{pd^3}$$

Hollow shaft

$$t = \frac{16TD}{p(D^4 - d^4)}$$

τ = torsional shearing stress
T = torque exerted by the shaft
D = outer diameter
d = inner diameter

Maximum twisting angle of the shaft's fiber:

$$q = \frac{TL}{JG}$$

θ = angular deformation (radians)
T = torque
L = length of the shaft
G = modulus of rigidity
J = polar moment of inertia of the cross

$$J = \frac{pd^4}{32} \rightarrow \text{Solid shaft}$$

$$J = \frac{p(D^4 - d^4)}{32} \rightarrow \text{Hollow shaft}$$

G_{steel} = 83 GPa;
E_{steel} = 200 GPa

Power delivered by a rotating shaft:

$$P = Tw$$

$$P_{rpm} = 2pTN \quad rps$$

$$P_{rpm} = \frac{2pTN}{60} \quad rpm$$

$$P_{hp} = \frac{2pTN}{550} \quad \frac{ft - lb}{\text{sec}}$$

$$P_{hp} = \frac{2pTN}{3300} \quad \frac{ft - lb}{\text{min}}$$

T = torque
N = revolutions/time

HELICAL SPRINGS

$$t = \frac{16PR}{pd^3}\left(1 + \frac{d}{4R}\right)$$

$$t = \frac{16PR}{pd^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

where,

$$m = \frac{D_{mean}}{d} = \frac{R_{mean}}{r}$$

elongation,

$$d = \frac{64PR^3n}{Gd^4}$$

τ = shearing stress
δ = elongation
R = mean radius
d = diameter of the spring wire
n = number of turns
G = modulus of rigidity

ENGINEERING ECONOMICS 1

SIMPLE INTEREST

$$I = Pin$$

$$F = P(1 + in)$$

P = principal amount
F = future amount
I = total interest earned
i = rate of interest
n = number of interest periods

Ordinary Simple Interest

$$n = \frac{days}{360} \qquad n = \frac{months}{12}$$

Exact Simple Interest

$$n = \frac{days}{365} \rightarrow \textit{ordinary year}$$

$$n = \frac{days}{366} \rightarrow \textit{leap year}$$

COMPOUND INTEREST

$$F = P(1+i)^n$$

Nominal Rate of Interest

$$i = \frac{NR}{m} \Leftrightarrow n = mN$$

Effective Rate of Interest

$$ER = (1+i)^m - 1$$

$$ER = \left(1 + \frac{NR}{m}\right)^m - 1$$

$$ER \geq NR\,;\ equal\ if\ \ Annual$$

i = rate of interest per period
NR = nominal rate of interest
m = number of interest periods per year
n = total number of interest periods
N = number pf years
ER = effective rate of interest

Mode of Interest	m
Annually	1
Semi-Annually	2
Quarterly	4
Semi-quarterly	8
Monthly	12
Semi-monthly	24
Bimonthly	6
Daily	360

Shortcut on Effective Rate

ANNUITY

Note: interest must be effective rate

Ordinary Annuity

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

A = uniform periodic amount or annuity

Perpetuity or Perpetual Annuity

$$P = \frac{A}{i}$$

ENGINEERING ECONOMICS 2

LINEAR / UNIFORM GRADIENT SERIES

$$P = P_A + P_G$$

$$P_G = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

$$F_G = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right]$$

$$A_G = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Perpetual Gradient

$$P_G = \frac{G}{i^2}$$

UNIFORM GEOMETRIC GRADIENT

$$P = C\left[\frac{(1+q)^n(1+i)^{-n} - 1}{q - i}\right] \quad \text{if } q \neq i$$

$$F = C\left[\frac{(1+q)^n - (1+i)^n}{q - i}\right] \quad \text{if } q \neq i$$

$$P = \frac{Cn}{1+q} \qquad P = \frac{Cn(1+i)^n}{1+q} \qquad \textit{if } q = i$$

$$q = \frac{second}{first} - 1$$

C = initial cash flow of the geometric gradient series which occurs one period after the present
q = fixed percentage or rate of increase

DEPRECIATION

Straight Line Method (SLM)

$$d = \frac{C_0 - C_n}{n}$$

$D_m = md$

$C_m = C_0 - D_m$

d = annual depreciation
C_0 = first cost
C_m = book value
C_n = salvage or scrap value
n = life of the property
D_m = total depreciation after m-years
m = m^{th} year

Sinking Fund Method (SFM)

$$d = \frac{(C_0 - C_n)i}{(1+i)^n - 1}$$

$$D_m = \frac{d[(1+i)^m - 1]}{i}$$

$C_m = C_0 - D_m$

i = standard rate of interest

Sum of the Years Digit (SYD) Method

$$d_m = (C_0 - C_n)\left[\frac{2(n - m + 1)}{n(n+1)}\right]$$

$$D_m = (C_0 - C_n)\left[\frac{(2n - m + 1)m}{n(n+1)}\right]$$

$$SYD = \frac{n(n+1)}{2}$$

$C_m = C_0 - D_m$

SYD = sum of the years digit
d_m = depreciation at year m

Declining Balance Method (DBM)

$$k = 1 - \sqrt[n]{\frac{C_n}{C_o}}$$

$$k = 1 - \sqrt[m]{\frac{C_m}{C_o}}$$ Matheson Formula

$$C_m = C_0 (1-k)^m$$

$$d_m = kC_0 (1-k)^{m-1}$$

$D_m = C_0 - C_m$

k = constant rate of depreciation

CAPITALIZED AND ANNUAL COSTS

$$CC = C_0 + P$$

CC = **Capitalized Cost**
C_0 = first cost
P = cost of perpetual maintenance (A/i)

$$AC = d + C_0(i) + OMC$$

AC = Annual Cost
d = Annual depreciation cost
i = interest rate
OMC = Annual operating & maintenance cost

BONDS

$$P = P_{anuity} = P_{cpd\ \mathrm{int}\,erest}$$

$$P = \frac{Zr[(1+i)^n - 1]}{(1+i)^n i} + \frac{C}{(1+i)^n}$$

P = present value of the bond
Z = par value or face value of the bond
r = rate of interest on the bond per period
Z_r = periodic dividend
i = standard interest rate
n = number of years before redemption
C = redemption price of bond

BREAK-EVEN ANALYSIS

Total income = Total expenses