

Question: The upward velocity of a rocket, measured at 3 different times, is shown in the following table T...

ME Analysis, Need help with questions 12-17 please!

The upward velocity of a rocket, measured at 3 different times, is shown in the fo

| Time, t (seconds) | Velocity, v (metres/second) |
|------------------------|----------------------------------|
| 5 | 106.8 |
| 8 | 177.2 |
| 12 | 279.2 |

The velocity over the time interval $5 \leq t \leq 12$ is approximated by a quadratic exp

$$v(t) = a_1 t^2 + a_2 t + a_3$$

Following table

expression as

Q12-Q17: Hand calculation

- Q12) Use an inverse matrix method to find the values of a_1 , a_2 and a_3 .
- Q13) Use a Cramer's rule to find the values of a_1 , a_2 and a_3 .
- Q14) Use a Gauss Elimination method to find the values of a_1 , a_2 and a_3 .
- Q15) Use an LU factorization method to find the values of a_1 , a_2 and a_3 .
- Q16) Use a Gauss-Seidel method to find the values of a_1 , a_2 and a_3 .
- Q17) Use a Jacobi method to find the values of a_1 , a_2 and a_3 .

Show transcribed image text

Expert Answer



Anonymous
answered this

$$v(t) = a_1 t^2 + a_2 t + a_3$$

$$106.8 = a_1 \times 5^2 + a_2 \times 5 + a_3$$

$$106.8 = 25a_1 + 5a_2 + a_3 \quad \text{--- (i)}$$

Similarly

$$177.2 = 64a_1 + 8a_2 + a_3 \quad \text{--- (ii)}$$

$$279.2 = 144a_1 + 12a_2 + a_3 \quad \text{--- (iii)}$$

writing down eq (i) - (ii) - (iii) in matrix form

$$\begin{cases} 106.8 \\ 177.2 \\ 279.2 \end{cases} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{cases} a_1 \\ a_2 \\ a_3 \end{cases}$$

$$\textcircled{1} [v] = [K][a]$$

$$[a] = [K]^{-1} v \quad \text{--- inverse matrix method}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = [K]$$

$$[K]^{-1} = \frac{1}{\det K} \text{adjoint of } K$$

$$\det A = 25(8-12) - 5(64-144) + 1(64 \times 12 - 144 \times 8)$$

$$= -100 + 400 + (-800) = -84$$

$$\text{adj } |A| = \begin{bmatrix} -4 & 7 & -120 \\ +80 & -119 & 39 \\ -384 & 420 & -120 \end{bmatrix}$$

$$[A]^{-1} = [K]^{-1} = \begin{bmatrix} \frac{1}{21} & -\frac{1}{21} & \frac{1}{28} \\ -\frac{20}{21} & \frac{13}{12} & -\frac{13}{28} \\ \frac{32}{7} & -5 & 19/3 \end{bmatrix}$$

$$[a] = \begin{bmatrix} \frac{1}{21} & -\frac{1}{21} & \frac{1}{28} \\ -\frac{20}{21} & \frac{13}{12} & -\frac{13}{28} \\ \frac{32}{7} & -5 & 19/3 \end{bmatrix} \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} =$$

Matrix Multiply

$$a_1 = 6.613047$$

$$a_2 = -79.4747$$

$$a_3 = 1.08571$$

$$a_1 = \frac{-514.4}{-84} = 6.123$$

$$a_2 = \frac{6676}{-84} = -79.474$$

$$a_3 = \frac{-29491.1999}{-84} = 351.08571$$

② Cramer's rule

$$\begin{cases} 106.8 \\ 107.2 \\ 279.2 \end{cases} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{cases} a_1 \\ a_2 \\ a_3 \end{cases}$$

$$[D]$$

$$\det [D] = -84$$

$$D_1 = \begin{bmatrix} 106.8 & 5 & 1 \\ 107.2 & 8 & 1 \\ 279.2 & 12 & 1 \end{bmatrix}$$

$$\det D_1 = -514.4$$

$$D_2 = \begin{bmatrix} 25 & 106.8 & 1 \\ 64 & 107.2 & 1 \\ 144 & 279.2 & 1 \end{bmatrix}$$

$$\det D_2 = 6676$$

$$D_3 = \begin{bmatrix} 25 & 5 & 106.8 \\ 64 & 8 & 107.2 \\ 144 & 12 & 279.2 \end{bmatrix}$$

$$\det D_3 = -29491.1999$$

Gauss Elimination

$$\left[\begin{array}{ccc|c} 25 & 5 & 1 & 106.8 \\ 64 & 8 & 1 & 107.2 \\ 144 & 12 & 1 & 279.2 \end{array} \right]$$

$$R_1 \rightarrow R_1/25$$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 64 & 8 & 1 & 107.2 \\ 144 & 12 & 1 & 279.2 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 64R_1, \quad R_3 \rightarrow R_3 - 144R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 0 & -4.8 & -1.56 & -166.208 \\ 0 & -16.8 & -4.76 & -335.968 \end{array} \right]$$

$$R_2 \rightarrow R_2 / -4.8$$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 0 & 1 & 0.325 & 25.97 \\ 0 & -16.8 & -4.76 & -335.968 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 0.2R_2, \quad R_3 \rightarrow R_3 + 16.8R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.025 & -1.09 \\ 0 & 1 & 0.325 & 25.97 \\ 0 & 0 & 0.7 & 245.76 \end{array} \right]$$

$$R_3 / 0.7 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.025 & -\frac{199}{35} \\ 0 & 1 & 0.325 & \frac{2597}{35} \\ 0 & 0 & 1 & \frac{12278}{35} \end{array} \right]$$

$$R_1 \rightarrow R_1 + 0.025 R_3$$

$$R_2 \rightarrow R_2 - 0.325 R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{643}{105} \\ 0 & 1 & 0 & -\frac{1669}{21} \\ 0 & 0 & 1 & \frac{12288}{35} \end{array} \right]$$

$$a_1 = \frac{643}{105} = 6.1238$$

$$a_2 = \frac{-1669}{21} = -79.476$$

$$a_3 = \frac{12288}{35} = 351.0857$$

LU factorization to solve equation

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 107.2 \\ 279.2 \end{bmatrix}$$

$$R_1 \rightarrow R_1/25$$

$$\begin{bmatrix} 1 & 0.2 & 0.04 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4.272 \\ 107.2 \\ 279.2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 64 R_1, R_3 \rightarrow R_3 - 144 R_1$$

$$\begin{bmatrix} 1 & 0.2 & 0.04 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4.272 \\ -164.208 \\ -335.265 \end{bmatrix}$$

$$R_2 \rightarrow R_2/4.8$$

$$\begin{bmatrix} 1 & 0.2 & 0.04 \\ 0 & -1 & -0.325 \\ 0 & -16.8 & -4.76 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4.272 \\ -2597/75 \\ -335.268 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 0.2 R_2, R_3 \rightarrow R_3 + 16.8 R_2$$

$$\begin{bmatrix} 1 & 0 & -0.025 \\ 0 & -1 & -0.325 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -199/75 \\ 2597/75 \\ 245.76 \end{bmatrix}$$

$$a_3 = \frac{245.76}{0.7} = 351.0857$$

$$a_2 + 0.325 a_3 = 2597/75$$

$$a_2 = -79.476$$

$$a_1 + 0.025 a_3 = -199/75$$

$$a_1 = 6.1237$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 107.2 \\ 279.2 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$$

$$\begin{bmatrix} -119 & -7 & 0 \\ -80 & -4 & 0 \\ +144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -172.4 \\ -172 \\ 279.2 \end{bmatrix}$$

$$R_1 \rightarrow R_1/7, R_2 \rightarrow R_2/4$$

$$\begin{bmatrix} 12 & 0 & 0 \\ -80 & -4 & 0 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 73.48 \\ -172 \\ 279.2 \end{bmatrix}$$

$$a_1 = \frac{73.48}{12} = 6.123$$

$$-80 a_1 - 4 a_2 = 172$$

$$a_2 = \frac{172 + 80 \times 6.123}{-4} = -165.44$$

$$144 a_1 + 12 a_2 + a_3 = 279.2$$

$$a_3 = 279.2 - 144 \times 6.123 + 12 \times 165.44$$

$$a_3 = 1383.00$$

only first four part is answered due to short time

0 Comments

Was this answer helpful?

