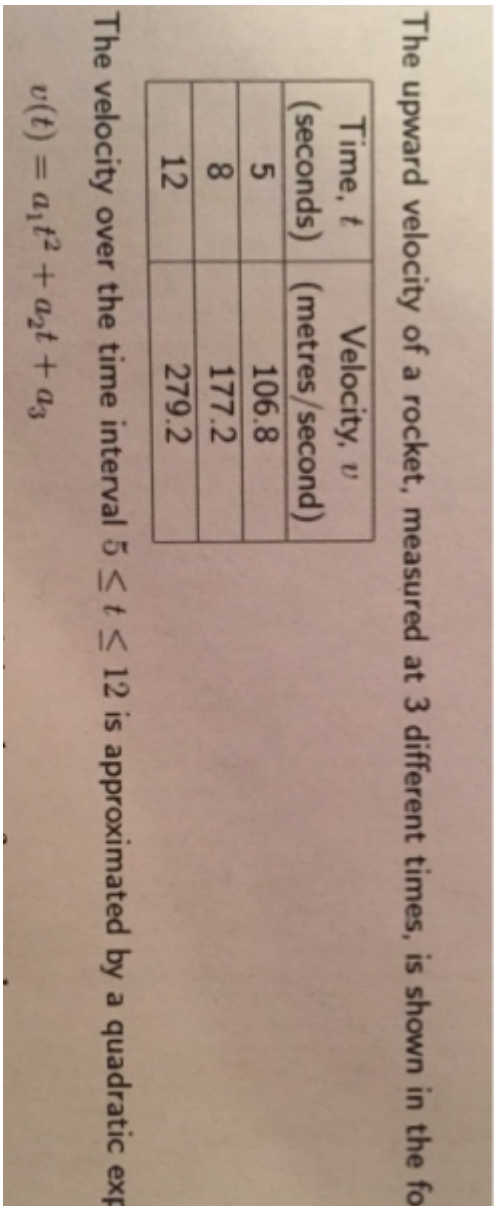
Question: The upward velocity of a rocket, measured at 3 different times, is shown in the following table T...

ME Analysis, Need help with questions 12-17 please!



The upward velocity of a rocket, measured at 3 different times, is shown in the fo

Time, t (seconds)	Velocity, v (metres/second)
5	106.8
8	177.2
12	279.2

The velocity over the time interval $5 \leq t \leq 12$ is approximated by a quadratic exp

$$v(t) = a_1 t^2 + a_2 t + a_3$$

llowing table

pression as

Q12-Q17: Hand calculation

Q12) Use an inverse matrix method to find the values of a_1, a_2 and a_3.

Q13) Use a Cramer's rule to find the values of a_1, a_2 and a_3.

Q14) Use a Gauss Elimination method to find the values of a_1, a_2 and a_3.

Q15) Use an LU factorization method to find the values of a_1, a_2 and a_3.

Q16) Use a Gauss-Seidel method to find the values of a_1, a_2 and a_3.

Q17) Use a Jacobi method to find the values of a_1, a_2 and a_3.

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Expert Answer

Anonymous
answered this

$v(t) = a_1 t^2 + a_2 t + a_3$

$106.8 = a_1 \times 5^2 + a_2 \times 5 + a_3$

$106.8 = 25a_1 + 5a_2 + a_3$ — (i)

Similarly

$177.2 = 64a_1 + 8a_2 + a_3$ — (ii)

$279.2 = 144a_1 + 12a_2 + a_3$ — (iii)

writing down eq^n (i) - (ii) - (iii) in matrix form

$$\begin{Bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{Bmatrix} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

① $[V] = [K][a]$

$[a] = [K]^{-1} V$ — inverse matrix method

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = [K]$$

$[K]^{-1} = \frac{1}{\det K}$ adjoint of K

$\det A = 25(8-12) - 5(64-144) + 1(64\times12 - 144\times8)$

$= -100 + 400 + (-384) = -84$

$$\text{adj } |A| = \begin{bmatrix} -4 & 7 & -120 \\ +80 & -119 & 39 \\ -384 & 420 & -120 \end{bmatrix}$$

$$[A]^{-1} = [K]^{-1} = \begin{bmatrix} 1/21 & -1/21 & 1/28 \\ -20/21 & 17/12 & -13/28 \\ 32/7 & -5 & 10/7 \end{bmatrix}$$

$$[a] = \begin{bmatrix} \frac{1}{21} & -\frac{1}{21} & \frac{1}{28} \\ -\frac{20}{21} & \frac{17}{12} & -\frac{13}{28} \\ \frac{32}{7} & -5 & 10/7 \end{bmatrix} \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} =$$

Matrix Multiply

$a_1 = 6.619047$

$a_2 = 19.69047$

$a_3 = 1.08571$

② Cramer rule

$$\begin{Bmatrix} 106.8 \\ 107.2 \\ 279.2 \end{Bmatrix} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

$[D]$

$\text{Det}[D] = -84$

$$D_1 = \begin{bmatrix} 106.8 & 5 & 1 \\ 107.2 & 8 & 1 \\ 279.2 & 12 & 1 \end{bmatrix}$$

$\det D_1 = -514.4$

$$D_2 = \begin{bmatrix} 25 & 106.8 & 1 \\ 64 & 107.2 & 1 \\ 144 & 279.2 & 1 \end{bmatrix}$$

$\det D_2 = 6676$

$$D_3 = \begin{bmatrix} 25 & 5 & 106.8 \\ 64 & 8 & 107.2 \\ 144 & 12 & 279.2 \end{bmatrix}$$

$\det D_3 = -29491.1999$

$a_1 = \frac{-514.4}{-84} = 6.123$

$a_2 = \frac{6676}{-84} = -79.476$

$a_3 = \frac{-29491.1999}{-84} = 351.08571$

Gauss Elemination

$$\left[\begin{array}{ccc|c} 25 & 5 & 1 & 106.8 \\ 64 & 8 & 1 & 107.2 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

$R_1 \to R_1/25$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 64 & 8 & 1 & 107.2 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

$R_2 \to R_2 - 64R_1$, $R_3 \to R_3 - 144R_1$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 0 & -4.8 & -1.56 & -166.208 \\ 0 & -16.8 & -4.76 & -335.965 \end{array}\right]$$

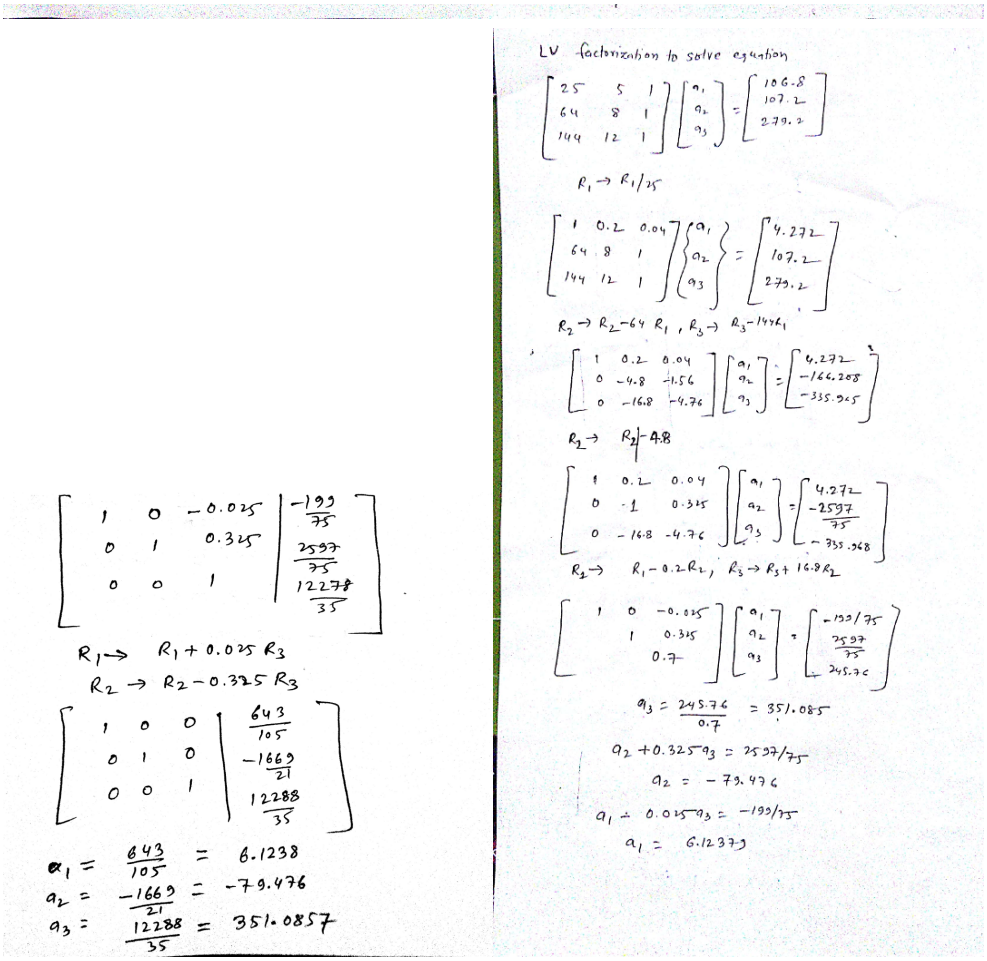
$R_2 \to R_2/-4.8$

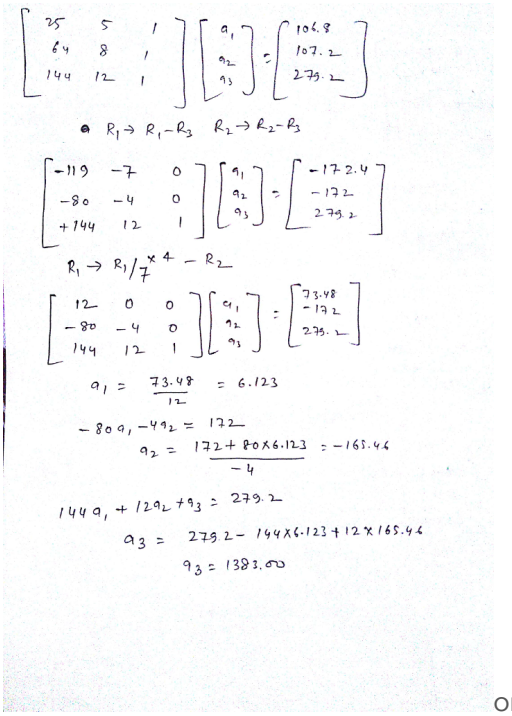
$$\left[\begin{array}{ccc|c} 1 & 0.2 & 0.04 & 4.272 \\ 0 & 1 & 0.325 & \frac{2597}{75} \\ 0 & -16.8 & -4.76 & -335.968 \end{array}\right]$$

$R_1 \to R_1 - 0.2R_2$, $R_3 \to R_3 + 16.8R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.025 & \frac{-109}{75} \\ 0 & 1 & 0.325 & \frac{2597}{75} \\ 0 & 0 & 0.7 & 245.76 \end{array}\right]$$

$R_3/0.7 \to R_3$





only first four part is answered due to short time

0 Comments

Was this answer helpful? 0 0