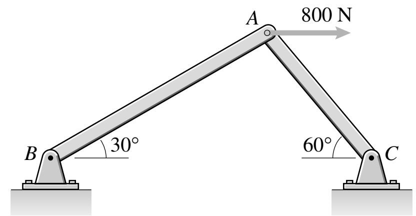
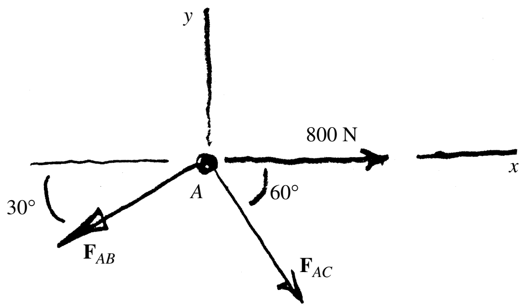
Problem 6.1 Determine the axial forces in the members of the truss and indicate whether they are in tension (T) or compression (C).

Strategy: Draw free-body diagram of joint A. By writing the equilibrium equations for the joint, you can determine the axial forces in the two members.



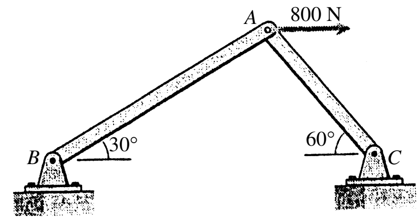
Solution:



Assume the forces are in the directions shown (both in tension). If a force turns up negative, that force will be in compression.
Equilibrium Eqns.

$$\sum F_x: \quad -F_{AB}\cos 30° + F_{AC}\cos 60° + 800 = 0$$

$$\sum F_y: \quad -F_{AB}\sin 30° - F_{AC}\sin 60° = 0$$



Solving: We get

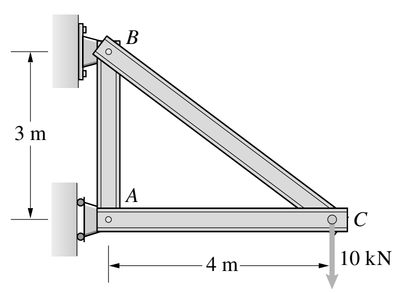
$F_{AB} = 693$ N (tension)

$F_{AC} = -400$ N (compression)

Problem 6.2 The truss supports a 10-kN load at C.

(a) Draw the free-body diagram of the entire truss, and determine the reactions at its supports.

(b) Determine the axial forces in the members. Indicate whether they are in tension (T) or compression (C).



Solution: (a) The free-body diagram of the system is shown. The sum of the moments about B is: $M_B = 3A_x - 4(10) = 0$, from which $A_x = 13.33$ kN. The sums of the forces:

$$\sum F_x = A_x + B_x = 0,$$

from which $\boxed{B_x = -A_x = -13.33 \text{ kN}}$.

$$\sum F_y = B_y - 10 = 0,$$

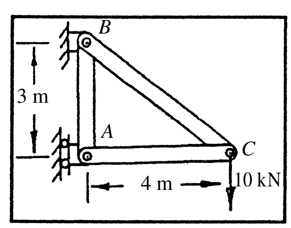
from which $\boxed{B_y = 10 \text{ kN}}$. (b) The interior angle ACB is $\alpha = \tan^{-1}(0.75) = 36.87°$. (b) Assume that the unknown forces act away from the joint. Denote the axial force in the member I, K by IK. The axial forces are $\mathbf{F}_{CB} = BC(-\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$, and $\mathbf{F}_{CA} = -AC\mathbf{i}$. Summing the forces:

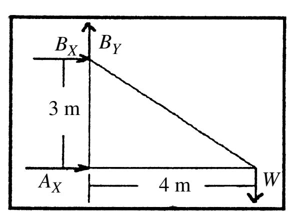
$$\sum F_y = BC\sin\alpha - 10 = 0,$$

from which $\boxed{BC = 16.67 \text{ kN } (T)}$.

$$\sum F_x = -BC\cos\alpha - AC = 0,$$

from which $\boxed{AC = -13.33 \text{ kN } (C)}$. For the joint A,



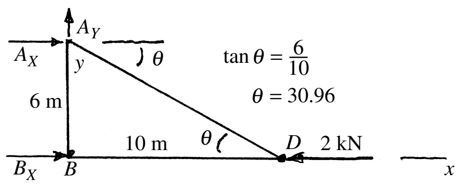


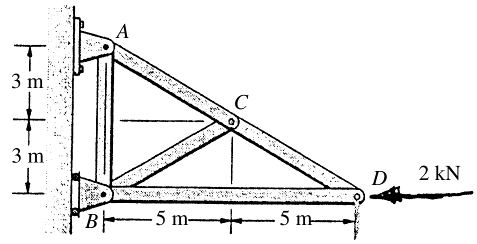
$$\sum F_y = AB = 0,$$

from which $\boxed{AB = 0}$

Problem 6.3 In Example 6.1, suppose that the 2-kN load is applied at D in the horizontal direction, pointing from D toward B. What are the axial forces in the members?

Solution: First, solve for the support forces and then use the method of joints at each joint to solve for the forces.





$$\sum F_x: \quad B_x + A_x - 2\text{ kN} = 0$$

$$\sum F_y: \quad A_y = 0$$

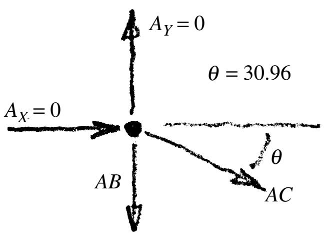
$$\sum M_B: \quad -6A_x = 0$$

Solving, we get

$$A_x = A_y = 0,$$

$$B_x = 2\text{ kN}$$

Joint A:



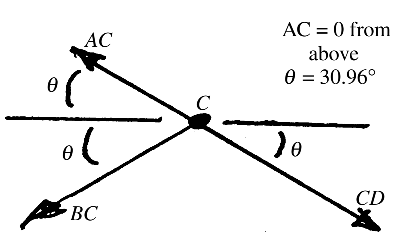
$$\theta = 30.96^\circ$$

$$\sum F_x: \quad AC\cos\theta = 0$$

$$\sum F_y: \quad -AB - AC\sin\theta = 0$$

Solving, we get $AB = AC = 0$

Joint C:



$$\sum F_x: \quad \overset{0}{\cancel{AC}}\cos\theta - BC\cos\theta + CD\cos\theta = 0$$

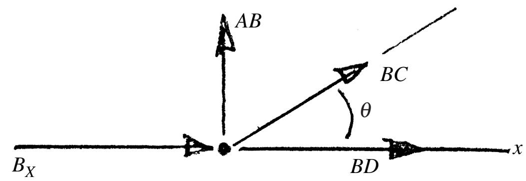
$$-BC + CD = 0$$

$$\sum F_y: \quad \overset{0}{\cancel{AC}}\sin\theta - BC\sin\theta - CD\sin\theta = 0$$

$$-BC - CD = 0$$

Solving, we get $BC = CD = 0$

Joint B:



We already know $AB = BC = 0$ and $B_x = 2$ kN

$$\sum F_x: \quad \overset{0}{\cancel{BC}}\cos\theta + BD + B_x = 0$$

$$BD + 2\text{ kN} = 0$$

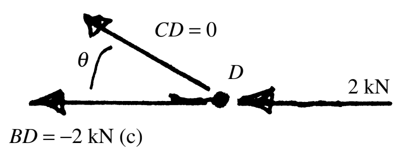
$$BD = -2\text{ kN (compression)}$$

$$\sum F_y: \text{(all forces zero)} = 0$$

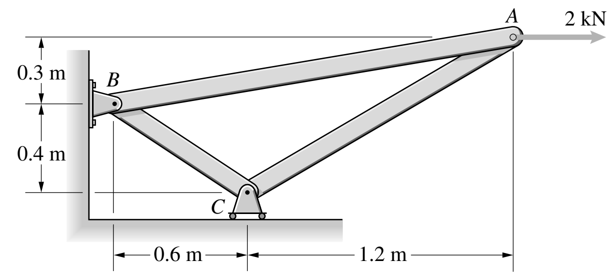
we have $AB = AC = BC = CD = 0$

$$BD = -2\text{ kN } (c)$$

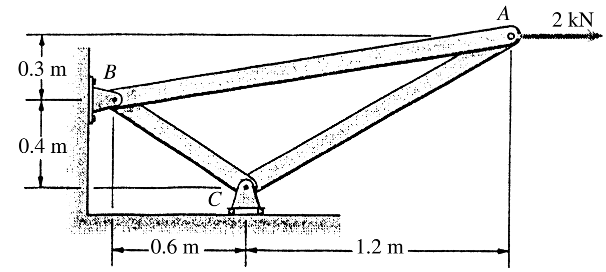
Note that we did not have to use joint D as we had already solved for the forces there. The FBD at D is BD, with the $(-)$ sign, is opposite the direction shown.

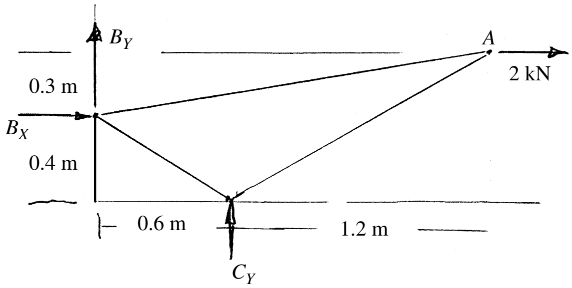


Problem 6.4 Determine the axial forces in the members of the truss.



Solution: First, solve for the support reactions at B and C, and then use the method of joints to solve for the forces in the members.





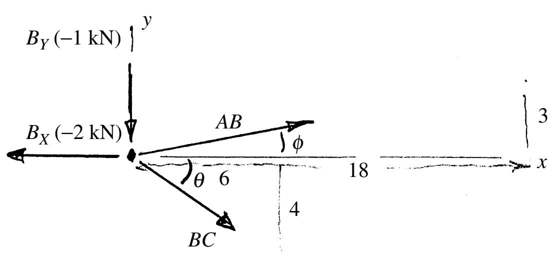
$$\sum F_x: \quad B_x + 2\text{ kN} = 0$$

$$\sum F_y: \quad B_y + C_y = 0$$

$$\circlearrowleft + \sum M_B: \quad 0,6C_y - (0,3)(2\text{ kN}) = 0$$

Solving, $B_x = -2$ kN $C_y = 1$ kN $B_y = -1$ kN

Joint B:

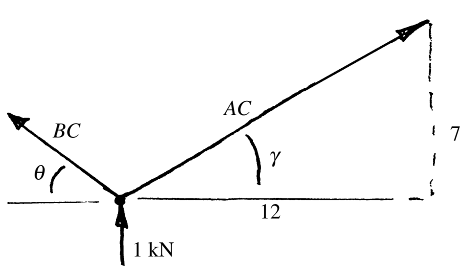


$$\sum F_x: \quad AB\cos\phi + BC\cos\theta - 2 = 0$$

$$\sum F_y: \quad AB\sin\phi - BC\sin\theta - 1 = 0$$

Solving, we get $AB = 2.839$ kN, $BC = -0.961$ kN

Joint C:



$$\tan\gamma = \frac{7}{12}$$

$$(BC = -0.961\text{ kN})$$

$$\gamma = 30.26°$$

$$\sum F_x = -BC\cos\theta + AC\cos\gamma = 0$$

$$\sum F_y = BC\sin\theta + AC\sin\gamma + 1 = 0$$

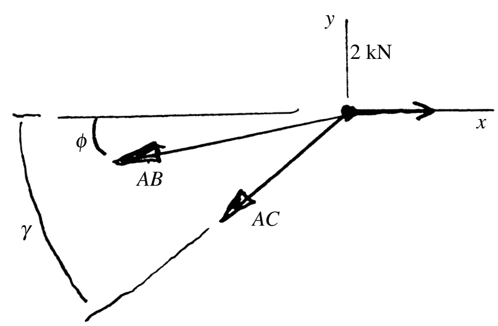
Solving, we get $AC = -0.926$ kN

We have $AB = 2.839$ kN (T)

$BC = -0.961$ kN (C)

$AC = -0.926$ kN (C)

Check: Look at Joint A

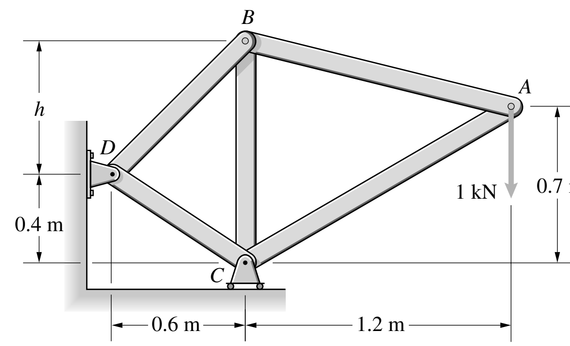


$$\sum F_x: \quad -AB\cos\phi - AC\cos\gamma + 2 = 0$$

$$\sum F_y: \quad -AB\sin\phi - AC\sin\gamma = 0$$

Substituting in the known values, the equations are satisfied: ∴ *Check*!

Problem 6.5 (a) Let the dimension $h = 0.1$ m. Determine the axial forces in the members, and show that in this case this truss is equivalent to the one in Problem 6.4. (b) Let the dimension $h = 0.5$ m. Determine the axial forces in the members. Compare the results to (a), and observe the dramatic effect of this simple change in design on the maximum tensile and compressive forces to which the members are subjected.



Solution: To get the force components we use equations of the form $\mathbf{T}_{PQ} = T_{PQ}\mathbf{e}_{PQ} = T_{PQX}\mathbf{i} + T_{PQY}\mathbf{j}$ where P and Q take on the designations A, B, C, and D as needed.

Equilibrium yields

At joint A:

$$\sum F_x = T_{ABX} + T_{ACX} = 0,$$

and $\sum F_y = T_{ABY} + T_{ACY} - 1 \text{ kN} = 0.$

At joint B:

$$\sum F_x = -T_{ABX} + T_{BCX} + T_{BDX} = 0,$$

and $\sum F_y = -T_{ABY} + T_{BCY} + T_{BDY} = 0.$

At joint C:

$$\sum F_x = -T_{BCX} - T_{ACX} + T_{CDX} = 0,$$

and $\sum F_y = -T_{BCY} - T_{ACY} + T_{CDY} + C_Y = 0.$

At joint D:

$$\sum F_x = -T_{CDX} - T_{BDX} + D_X = 0,$$

and $\sum F_y = -T_{CDY} - T_{BDY} + D_Y = 0.$

Solve simultaneously to get

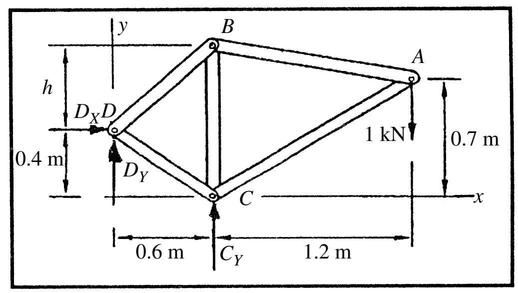
$$T_{AB} = T_{BD} = 2.43 \text{ kN},$$

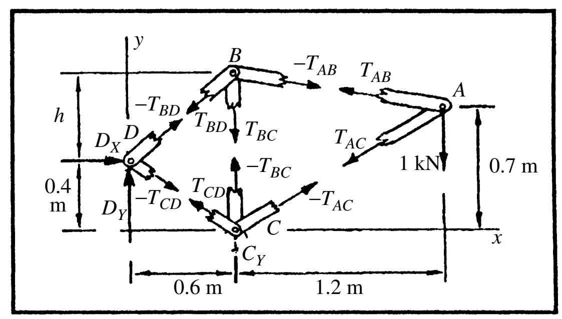
$$T_{AC} = -2.78 \text{ kN},$$

$$T_{BC} = 0, \; T_{CD} = -2.88 \text{ kN}.$$

Note that with appropriate changes in the designation of points, the forces here are the same as those in Problem 6.4. This can be explained by noting from the unit vectors that AB and BC are parallel. Also note that in this configuration, BC carries no load. This geometry is the same as in Problem 6.4 except for the joint at B and member BC which carries no load. Remember member BC in this geometry—we will encounter things like it again, will give it a special name, and will learn to recognize it on sight.

(b) For this part of the problem, we set $h = 0.5$ m. The unit vectors change because h is involved in the coordinates of point B. The new unit vectors are





$$\mathbf{e}_{AB} = -0.986\mathbf{i} + 0.164\mathbf{j},$$

$$\mathbf{e}_{AC} = -0.864\mathbf{i} - 0.504\mathbf{j},$$

$$\mathbf{e}_{BC} = 0\mathbf{i} - 1\mathbf{j},$$

$$\mathbf{e}_{BD} = -0.768\mathbf{i} - 0.640\mathbf{j},$$

and $\mathbf{e}_{CD} = -0.832\mathbf{i} + 0.555\mathbf{j}.$

We get the force components as above, and the equilibrium forces at the joints remain the same. Solving the equilibrium equations simultaneously for this situation yields

$$T_{AB} = 1.35 \text{ kN},$$

$$T_{AC} = -1.54 \text{ kN},$$

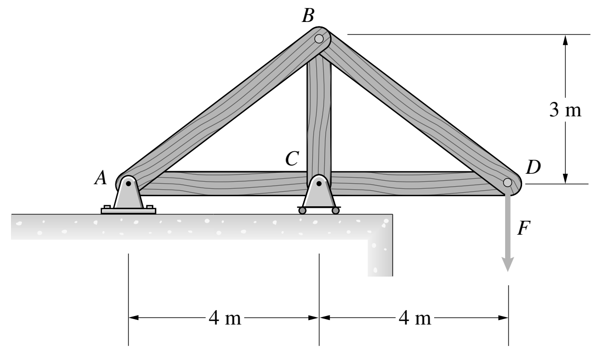
$$T_{BC} = -1.33,$$

$$T_{BD} = 1.74 \text{ kN},$$

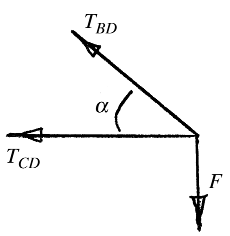
and $T_{CD} = -1.60$ kN.

These numbers differ significantly from (a). Most significantly, member BD is now carrying a compressive load and this has reduced the loads in all members except member BD. "Sharing the load" among more members seems to have worked in this case.

Problem 6.6 The load $F = 10$ kN. Determine the axial forces in the members.



Solution: The free-body diagram of joint D is



where $\alpha = \arctan(3/4) = 36.9°$. From the equations

$\sum F_x = -T_{CD} - T_{BD}\cos\alpha = 0,$

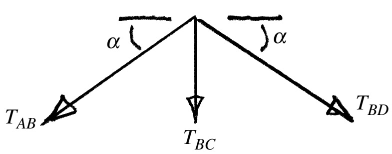
$\sum F_y = T_{BD}\sin\alpha - F = 0,$

we obtain

$T_{BD} = 1.67F = 16.7$ kN,

$T_{CD} = -1.33F = -13.3$ kN.

Joint B



From the equations

$\sum F_x = -T_{AB}\cos\alpha + T_{BD}\cos\alpha = 0,$

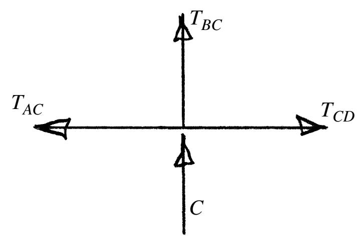
$\sum F_y = -T_{BC} - T_{AB}\sin\alpha - T_{BD}\sin\alpha = 0,$

we obtain

$T_{AB} = 1.67F = 16.7$ kN,

$T_{BC} = -2F = -20$ kN.

Joint C



we see that

$T_{AC} = T_{CD} = -1.33F = -13.3$ kN.

Problem 6.7 Consider the truss in Problem 6.6. Each member will safely support a tensile force of 150 kN and a compressive force of 30 kN. What is the largest downward load F that the truss will safely support at D?

Solution: See the solution of Problem 6.6. The largest tensile load is $1.67F$ in members BD and AB. Setting

$1.67F = 150$ kN

gives $F = 90$ kN. The largest compressive load is $2F$ in member BC. Setting

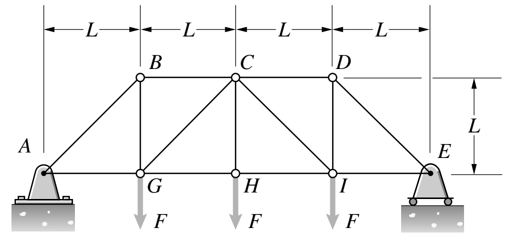
$2F = 30$ kN

gives $F = 15$ kN. The largest load is $F = 15$ kN.

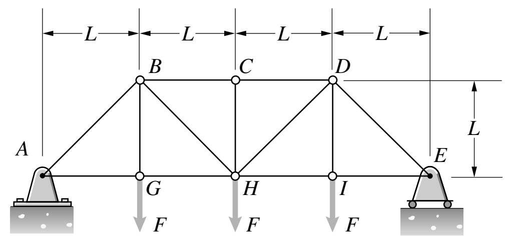
Problem 6.8 The Howe and Pratt bridge trusses are subjected to identical loads.

(a) In which truss does the largest tensile force occur? In what member(s) does it occur, and what is its value?

(b) In which truss does the largest compressive force occur? In what member(s) does it occur, and what is its value?



Howe



Pratt

Solution: (a) **Howe Bridge:** The moment about A is $M_A = -6F + 4E = 0$, from which $E = \frac{3}{2}F$. Denote the axial force in the member I, K by IK.

(1) *Joint E*: $DE = -\frac{E}{\sin 45^\circ} = -\frac{3\sqrt{2}}{2}F\ (C)$

$$EI = DE\cos 45^\circ = \frac{3}{2}F(T)$$

(2) *Joint D*: $CD = DE\cos 45^\circ = -\frac{3}{2}F\ (C)$,

$$DI = -DE\sin 45^\circ = \frac{3}{2}F(T)$$

(3) *Joint I*: $CI = \frac{F-|DI|}{\sin 45^\circ} = -\frac{\sqrt{2}}{2}F(C)$,

$$HI = EI - CI\sin 45^\circ = 2F(T)$$

(4) *Joint H*: $CH = F(T)$, $GH = HI = 2F(T)$.
By symmetry (the reaction at A has no x-component) the axial forces in the other members are $HG = HI$, $CG = CI$, $BG = DI$, $CD = BC$, $AG = EI$, and $AB = DE$. In the Howe truss, the members HI and GH have the highest tensile force $\boxed{GH = HI = 2F(T)}$ and the members DE and AB have the highest compressive force $\boxed{AB = DE = -\frac{3\sqrt{2}}{2}F(C)}$

(b) **Pratt Bridge:** The moment about A is $M_A = -6F + 4E = 0$, from which $E = \frac{3}{2}F$.

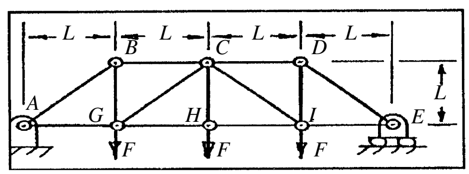
(1) *Joint E*: $DE = -\frac{3\sqrt{2}}{2}F(C)$, $EI = \frac{-DE}{\sqrt{2}} = \frac{3}{2}F(T)$

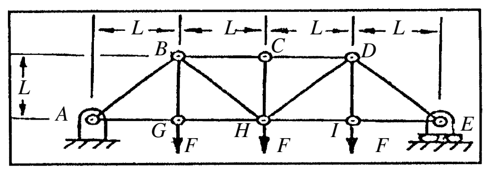
(2) *Joint I*: $DI = F(T)$, $HI = EI = \frac{3}{2}F(T)$

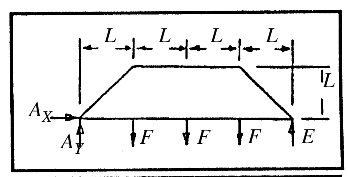
(3) *Joint D*: $DH = -\sqrt{2}DI - DE = \frac{\sqrt{2}}{2}F(T)$, $DC = \frac{DE}{\sqrt{2}} - \frac{DH}{\sqrt{2}} = -2F(C)$

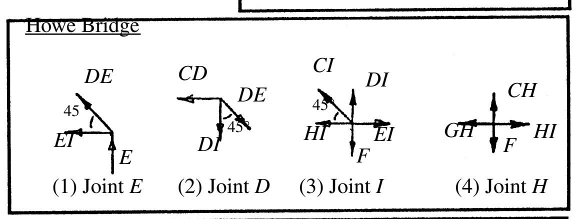
(4) *Joint C*: $BC = DC = -2F(C)$, $CH = 0$

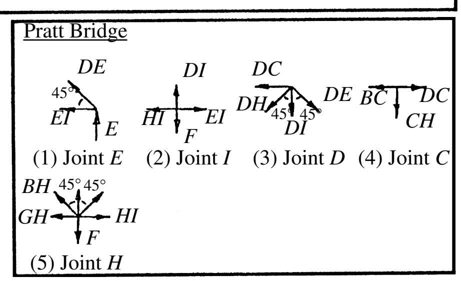
(5) *Joint H*: $GH = HI = \frac{3}{2}F(T)$. The axial forces in the remaining members are determined from symmetry. In the Pratt Bridge, the highest tensile force occurs in members EI, HI, GH, and AG $\boxed{= \frac{3}{2}F(T)}$, and the highest compressive force occurs in members DE and AB $\boxed{= -\frac{3\sqrt{2}}{2}F(C)}$. Thus (a) the Howe bridge has the highest tensile force in a member, and (b) the value of the compressive force is the same in members DE and $AB = \frac{3\sqrt{2}}{2}F(C)$ for both bridges.





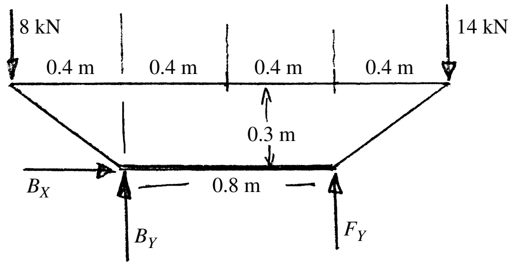






Problem 6.9 The truss shown is part of an airplane's internal structure. Determine the axial forces in members BC, BD, and BE.

Solution: First, solve for the support reactions and then use the method of joints to solve for the reactions in the members.



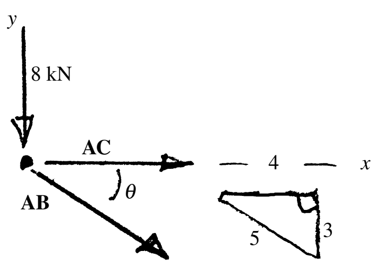
$$\sum F_x: \quad B_x = 0$$

$$\sum F_y: \quad B_y + F_y - 8 - 14 = 0 \text{ (kN)}$$

$$\circlearrowleft + \sum M_B: \quad (0.4)(8) + 0.8F_y - 1.2(14) = 0$$

Solving, we get $B_x = 0$, $B_y = 5.00$ kN $F_y = 17.00$ kN.
The forces we are seeking are involved at joints B, C, D, and E. The method of joints allows us to solve for two unknowns at a joint. We need a joint with only two unknowns. Joints A and H qualify. Joint A is nearest to the members we want to know about, so let us choose it. Assume tension in all members.

Joint A:



$\sin\theta = 0.6$ $\cos\theta = 0.8$ $\theta = 36.87^\circ$

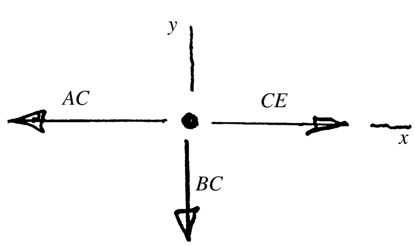
$$\sum F_x = AC + AB\cos\theta = 0$$

$$\sum F_y = -8 - AB\sin\theta = 0$$

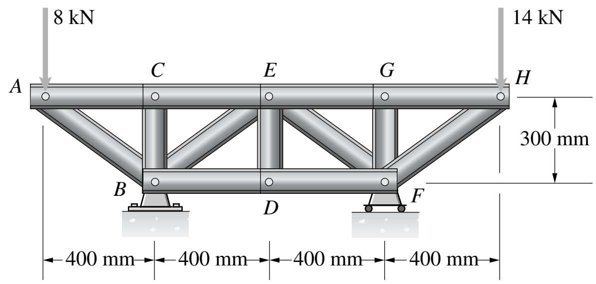
Solving, we get $AC = 10.67$ kN (T)

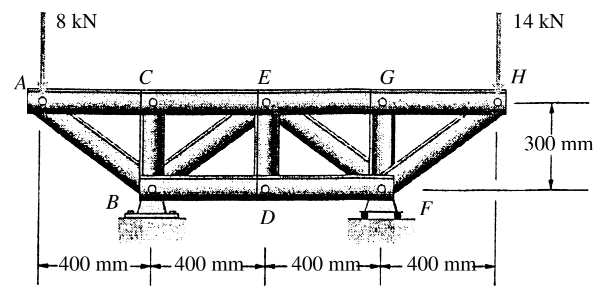
$AB = -13.33$ kN (C)

Joint C: (Again, assume all forces are in tension)



$[AC = 10.67 \text{ kN } (T)]$





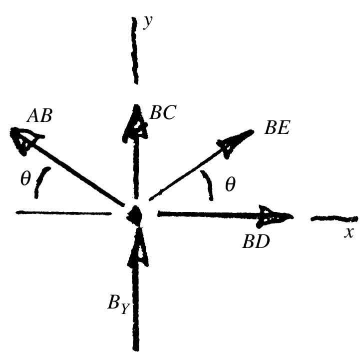
$$\sum F_x: \quad -BC = 0$$

$$\sum F_y: \quad -AC + CE = 0$$

Solving, we get $BC = 0$,

$CE = 10.67$ kN (T)

Joint B:



We know $AB = -13.33$ kN $BC = 0$ $B_y = 5.00$ kN.
We know 3 of the 5 forces at B Hence, we can solve for the other two.

$$\sum F_x: \quad BD + BE\cos\theta - AB\cos\theta = 0$$

$$\sum F_y: \quad BC + B_y + BE\sin\theta + AB\sin\theta = 0$$

Solving, we get $BD = -14.67$ kN (C)

$BE = 5.00$ kN (T)

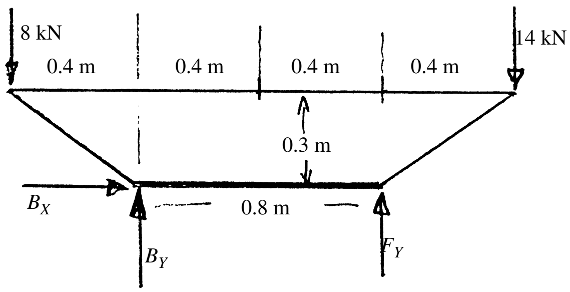
From Joint C, we had $BC = 0$

Thus

$BC = 0$,	$BD = -14.67$ kN (C)
$BE =$	5.00 kN (T)

Problem 6.10 For the truss in Problem 6.9, determine the axial forces in members DF, EF, and FG.

Solution: First, solve for the support reactions and then use the method of joints to solve for the reactions in the members.



$$\sum F_x: \quad B_x = 0$$

$$\sum F_y: \quad B_y + F_y - 8 - 14 = 0 \text{ (kN)}$$

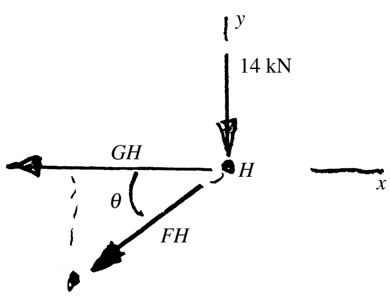
$$\circlearrowleft + \sum M_B: \quad (0.4)(8) + 0.8F_y - 1.2(14) = 0$$

Solving, we get $B_x = 0$,

$$B_y = 5.00 \text{ kN}$$

$$F_y = 17.00 \text{ kN}$$

The forces we are seeking are involved with joints D, E, F, and G The method of joints allows us to solve for two unknown forces at a joint. We need to start with a joint with only two unknowns. Joints A and H qualify. Joint H is nearest to the members we want to know about, so let us start there. Assume all unknown forces are tensions. If we get a negative force in a solution, this will then imply compression in that member.



Joint H

$$\sin\theta = \frac{3}{5} = 0.6$$

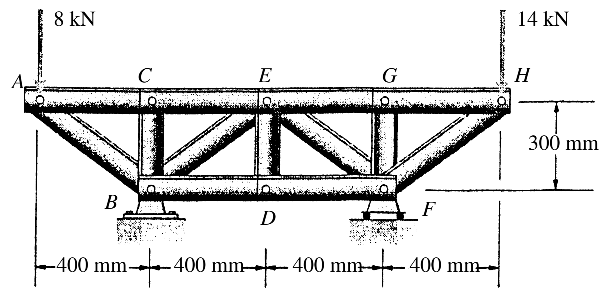
$$\cos\theta = \frac{4}{5} = 0.8$$

$$\sum F_x: \quad -GH - FH\cos\theta = 0$$

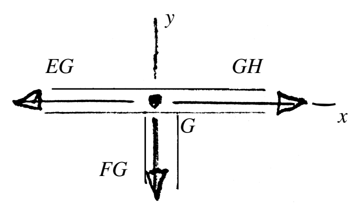
$$\sum F_y: \quad -14 - FH\sin\theta = 0$$

Solving $GH = 18.67$ kN (T),

$FH = -23.33$ kN (C)



Joint G:



$[GH = 18.67 \text{ kN } (T)]$

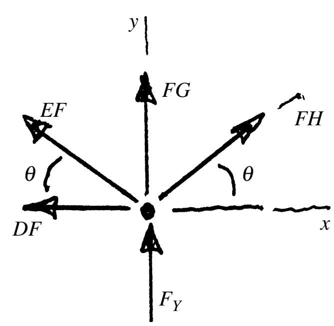
$$\sum F_x: \quad GH - EG = 0$$

$$\sum F_y: \quad FG = 0$$

Solving $EG = 18.67$ kN (T)

$FG = 0$

Joint F:



We know $FH = -23.33$ kN (C)

$FG = 0$

$F_y = 17.00$ kN

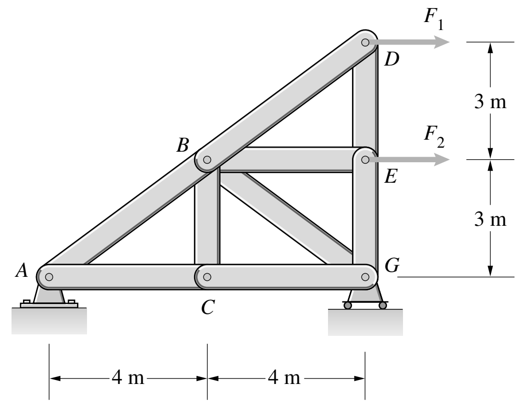
$$\sum F_x: \quad FH\cos\theta - EF\cos\theta - DF = 0$$

$$\sum F_y: \quad FG + F_y + FH\sin\theta + EF\sin\theta = 0$$

Solving, we get

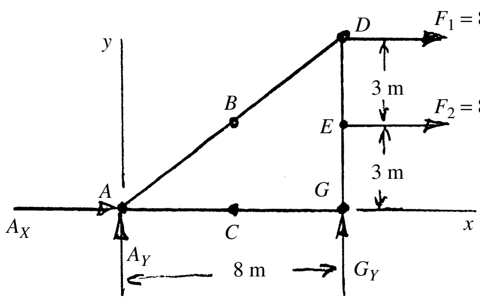
$EF = -5.00$ kN (C)
$DF = -14.67$ kN (C)
and from above $FG = 0$

Problem 6.11 The loads $F_1 = F_2 = 8$ kN. Determine the axial forces in members BD, BE, and BG.



Solution: First find the external support loads and then use the method of joints to solve for the required unknown forces. (Assume all unknown forces in members are tensions).

External loads:



$$\sum F_x: \quad A_x + F_1 + F_2 = 0 \text{ (kN)}$$

$$\sum F_y: \quad A_y + G_y = 0$$

$$\circlearrowleft + \sum M_A: \quad 8G_y - 3F_2 - 6F_1 = 0$$

Solving for the external loads, we get

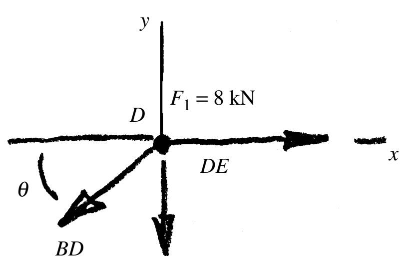
$A_x = -16$ kN (to the left)

$A_y = -9$ kN (downward)

$G_y = 9$ kN (upward)

Now use the method of joints to determine BD, BE, and BG. Start with joint D.

Joint D:



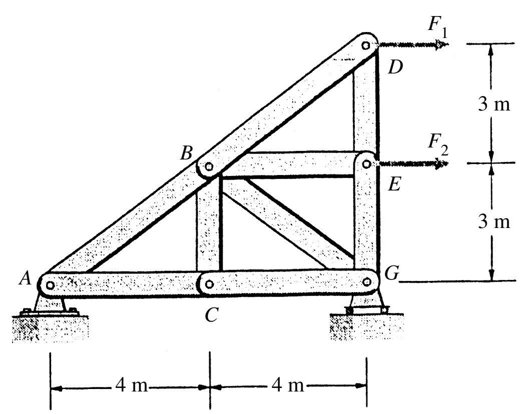
$\cos\theta = 0.8$

$\sin\theta = 0.6$

$\theta = 36.87°$

$$\sum F_x: \quad F_1 - BD\cos\theta = 0$$

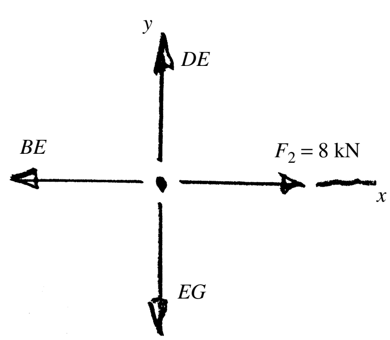
$$\sum F_y: \quad -BD\sin\theta - DE = 0$$



Solving, $BD = 10$ kN (T)

$DE = -6$ kN (C)

Joint E:



$DE = -6$ kN

$$\sum F_x = DE - EG = 0$$

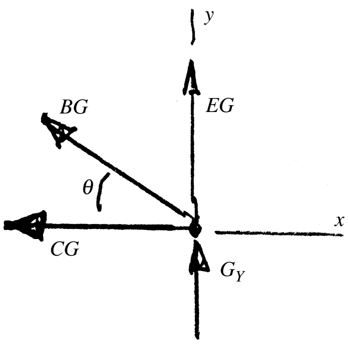
$$\sum F_y = -BE + F_2 = 0$$

Solving: $EG = -6$ kN (C)

$BE = 8$ kN (T)

6.11 *Contd.*

Joint G:



$(EG = -6 \text{ kN } (C))$

$G_y = 9$ kN

$$\sum F_x: \quad -CG - BG\cos\theta = 0$$

$$\sum F_y: \quad BG\sin\theta + EG + G_y = 0$$

Solving, we get

$BG = -5$ kN (C)

$CG = 4$ kN (T)

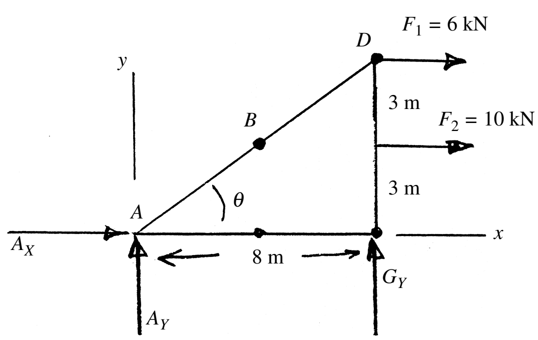
Thus, we have

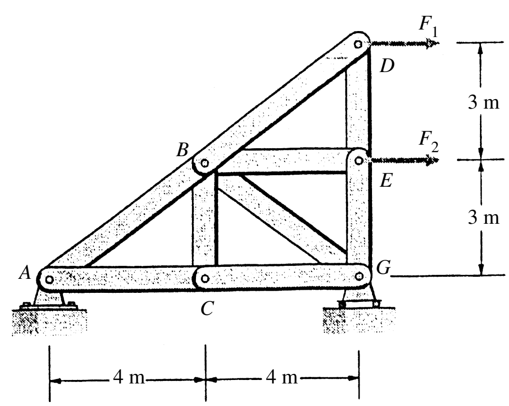
$$\boxed{\begin{aligned} BD &= 10 \text{ kN } (T) \\ BE &= 8 \text{ kN } (T) \\ BG &= -5 \text{ kN } (C) \end{aligned}}$$

Problem 6.12 If the loads on the truss shown in Problem 6.11 are $F_1 = 6$ kN and $F_2 = 10$ kN, what are the axial forces in members AB, BC, and BD?

Solution: Find the external support loads and then use the method of joints to determine loads in members. (Assume all loads in members to be tensions).

External Loads:





$\sin\theta = 0.6$ $\cos\theta = 0.8$ $\theta = 36.87°$

$$\sum F_x: \quad A_x + F_1 + F_2 = 0$$

$$\sum F_y: \quad A_y + G_y = 0$$

$$\circlearrowleft + \sum M_A: \quad 8G_y - 3F_2 - 6F_1 = 0$$

Solving, the external loads are

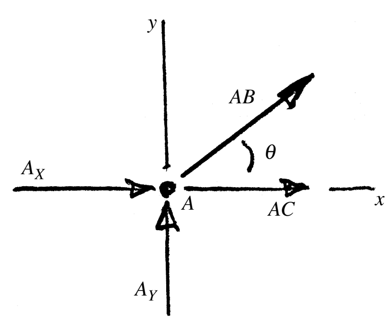
$A_x = -16$ kN,

$A_y = -8.25$ kN,

$G_y = 8.25$ kN.

Now use the method of joints to determine AB, BC, and BD. Start with Joint A:

Joint A:



$A_y = -8.25$ kN

$A_x = -16$ kN

$$\sum F_x: \quad AC + A_x + AB\cos\theta = 0$$

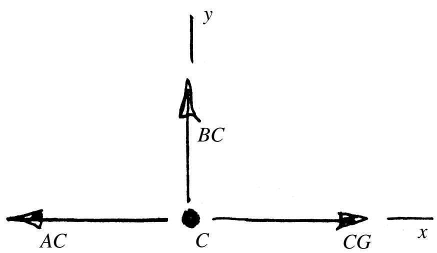
$$\sum F_y: \quad A_y + AB\sin\theta = 0$$

6.12 *Contd.*

Solving, $AC = 5$ kN (T)

$AB = 13.75$ kN (T)

Joint C:



$(AC = 5$ kN$)$

$\sum F_x: \quad CG - AC = 0$

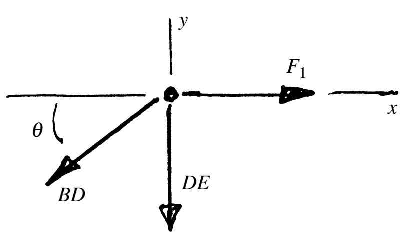
$\sum F_y: \quad BC = 0$

Solving,

$BC = 0$,

$CG = 5$ kN (T)

Joint D:



$F_1 = 6$ kN

$\sum F_x: \quad F_1 - BD\cos\theta = 0$

$\sum F_y: \quad -BD\sin\theta - DE = 0$

Solving, we get

$DE = -4.5$ kN (C)

and $BD = 7.5$ kN (T)

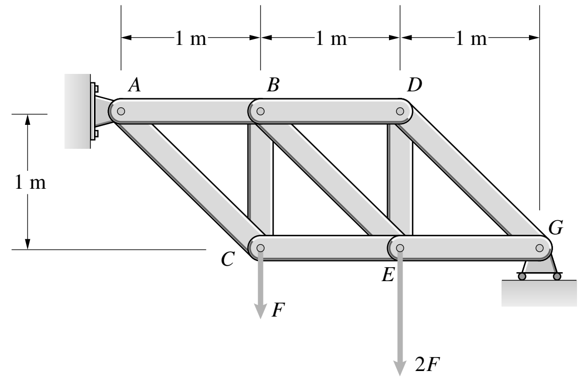
Thus, we have

$AB = 13.75$ kN (T)

$BC = 0$

$BD = 7.5$ kN (T)

Problem 6.13 The truss supports loads at C and E. If $F = 3$ kN, what are the axial forces in members BC and BE?



Solution: The moment about A is

$$\sum M_A = -1F - 4F + 3G = 0,$$

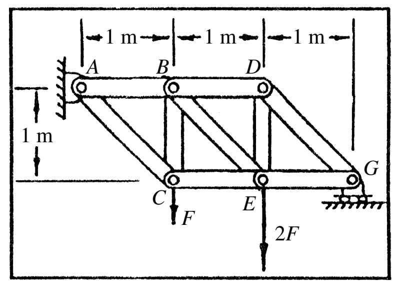
from which $G = \frac{5}{3}F = 5$ kN. The sums of forces:

$$\sum F_Y = A_Y - 3F + G = 0,$$

from which $A_Y = \frac{4}{3}F = 4$ kN.

$$\sum F_X = A_X = 0,$$

from which $A_X = 0$. The interior angles GDE, EBC are 45°, from which $\sin\alpha = \cos\alpha = \frac{1}{\sqrt{2}}$.



Denote the axial force in a member joining I, K by IK.

(1) *Joint G*:

$$\sum F_y = \frac{DG}{\sqrt{2}} + G = 0,$$

from which

$$DG = -\sqrt{2}G = -\frac{5\sqrt{2}}{3}F = -5\sqrt{2}\text{ kN } (C).$$

$$\sum F_x = -\frac{DG}{\sqrt{2}} - EG = 0,$$

from which

$$EG = -\frac{DG}{\sqrt{2}} = \frac{5}{3}F = 5\text{kN } (T).$$

(2) *Joint D*:

$$\sum F_y = -DE - \frac{DG}{\sqrt{2}} = 0,$$

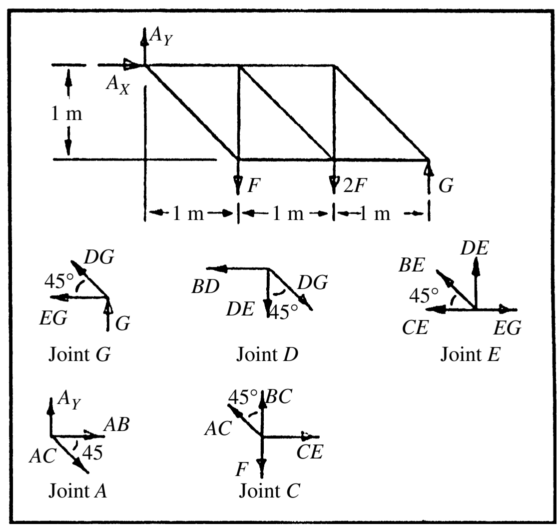
from which

$$DE = \frac{5}{3}F = 5\text{ kN } (T).$$

$$\sum F_x = -BD + \frac{DG}{\sqrt{2}} = 0,$$

from which

$$BD = -\frac{5}{3}F = -5\text{ kN } (C).$$



(3) *Joint E*:

$$\sum F_y = \frac{BE}{\sqrt{2}} - 2F + DE = 0,$$

from which $\boxed{BE = 2\sqrt{2}F - \sqrt{2}DE = \frac{\sqrt{2}}{3}F = \sqrt{2}\text{ kN } (T).}$

$$\sum F_x = -CE - \frac{BE}{\sqrt{2}} + EG = 0,$$

from which

$$CE = EG - \frac{BE}{\sqrt{2}} = \frac{4}{3}F = 4\text{ kN } (T).$$

(4) *Joint A:*

$$\sum F_y = A_y - \frac{AC}{\sqrt{2}} = 0,$$

from which $AC = \frac{4\sqrt{2}}{3}F = 4\sqrt{2}$ kN (T).

$$\sum F_x = AB + \frac{AC}{\sqrt{2}} = 0,$$

from which $AB = -\frac{4}{3}F = -4$ kN (C).

(5) *Joint C*:

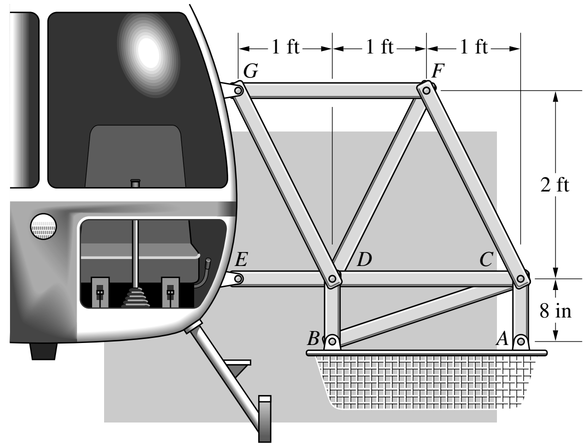
$$\sum F_y = BC + \frac{AC}{\sqrt{2}} - F = 0,$$

from which $\boxed{BC = F - \frac{AC}{\sqrt{2}} = -\frac{1}{3}F = -1\text{ kN } (C).}$

Problem 6.14 Consider the truss in Problem 6.13. Each member will safely support a tensile force of 28 kN and a compressive force of 12 kN. Taking this criterion into account, what is the largest safe (positive) value of F?

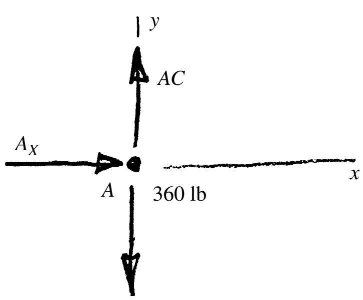
Solution: From the solution to Problem 6.14, the member with the largest tensile force is $EG = \frac{5}{3}F$, from which $F = \frac{3}{5}EG = 16.8$ kN. The member with the largest compressive force is DG, $DG = \frac{-5\sqrt{2}}{3}F$, from which $\boxed{F = \frac{3}{5\sqrt{2}}DG = \frac{36}{5\sqrt{2}} = 5.09 \text{ kN}}$ is the largest safe value.

Problem 6.15 The truss is a preliminary design for a structure to attach one end of a stretcher to a rescue helicopter. Based on dynamic simulations, the design engineer estimates that the downward forces the stretcher will exert will be no greater than 360 lb at A and at B. What are the resulting axial forces in members CF, DF, and FG?



Solution: Assume loads of 360 lbs at A and at B. Use the method of joints, starting with A and B, to work through the structure.

Joint A:



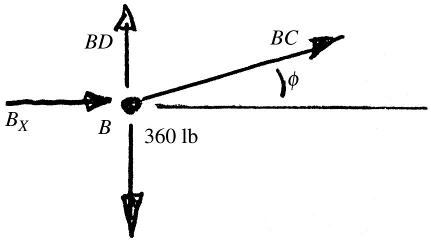
$$\sum F_y: \quad AC - 360 \text{ lb} = 0$$

$$AC = 360 \text{ lb}$$

$$\sum F_x: \quad A_x = 0$$

If $A_x = 0$, then $B_x = 0$ because the stretcher must be in equilibrium

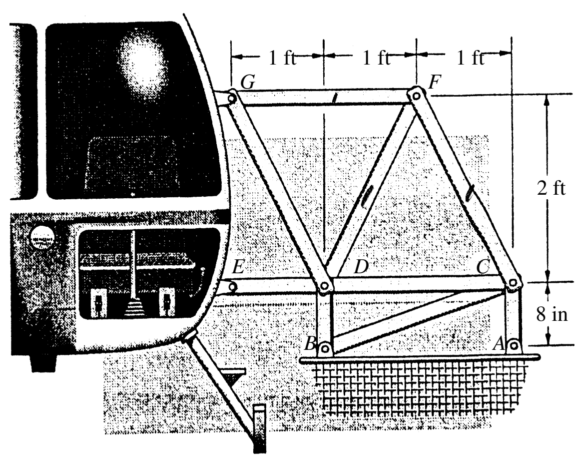
Joint B:



$$\tan\phi = \frac{8}{24}$$

$$\phi = 18.43^\circ$$

$$B_x = 0$$



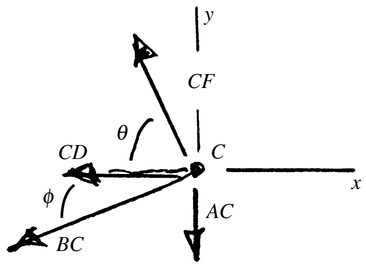
$$\sum F_x: \quad B_x + BC\cos\phi = 0$$

$$\sum F_y: \quad BC\sin\phi + BD - 360 = 0$$

Solving, $BD = 360$ lb, $BC = 0$

6.15 *Contd.*

Joint C:



$$\tan\theta = \frac{2}{1}$$

$$\theta = 63.43°$$

$$BC = 0,$$

$$AC = 360 \text{ lb}$$

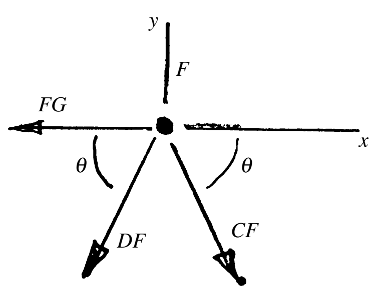
$$\sum F_x: \quad -CD - BC\cos\phi - CF\cos\theta = 0$$

$$\sum F_y: \quad CF\sin\theta - BC\sin\phi - AC = 0$$

Solving, $CD = -180$ lb (C)

$CF = 402$ lb (T)

Joint F:



$$\theta = 63.43°$$

$(CF = 402$ lbs $(T))$

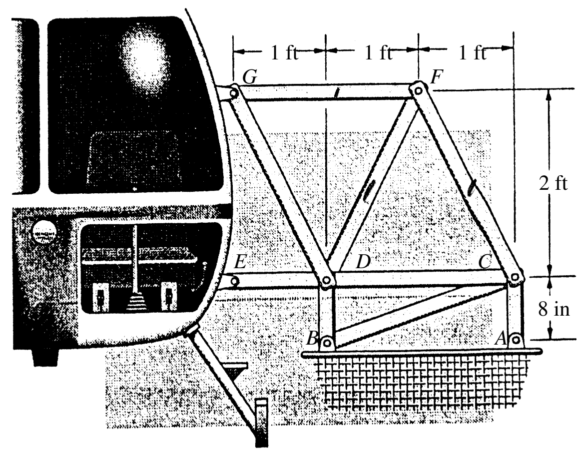
$$\sum F_x: \quad CF\cos\theta - DF\cos\theta - FG = 0$$

$$\sum F_y: \quad -CF\sin\theta - DF\sin\theta = 0$$

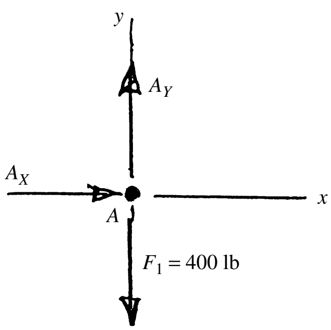
Solving; we get

$DF = -402$ lb (C)
$FG = 360$ lb (T)
and from earlier
$CF = 402$ lb (T)

Problem 6.16 Upon learning of an upgrade in the helicopter's engine, the engineer designing the truss shown in Problem 6.15 does new simulations and concludes that the downward forces the stretcher will exert at A and at B may be as large as 400 lb. What are the resulting axial forces in members DE, DF, and DG?



Solution: Assume loads of 400 lb at A and B. Use the method of Joints, starting with A and B, and work through the structure.
Joint A:



$$\sum F_x : \quad A_x = 0$$

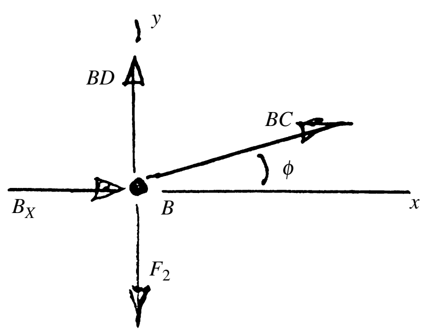
$$\sum F_y : \quad A_y - F_1 = 0$$

$$F_1 = 400 \text{ lb}$$

Solving, $A_y = 400$ lb.

$$A_x = 0$$

If $A_x = 0$, then $B_x = 0$ for the stretcher not to move horizontally. $(A_x + B_x = 0)$
Joint B:



$$B_x = 0$$

$$F_2 = 400 \text{ lb}$$

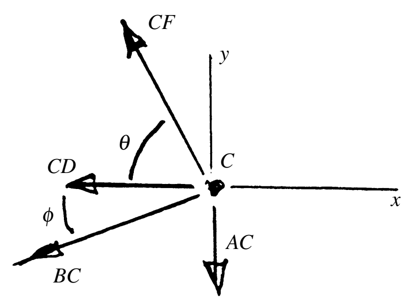
$$\tan\phi = \frac{8}{24}$$

$$\phi = 18.43^\circ$$

$$\sum F_x : \quad B_x + BC\cos\phi = 0$$

$$\sum F_y : \quad BC\sin\phi + BD - F_2 = 0$$

Solving, $BC = 0,\ BD = 400$ lb(T)
Joint C:



$$\tan\theta = \frac{2}{1}$$

$$\theta = 63.43^\circ$$

$BC = 0,\ AC = 400$ lb.

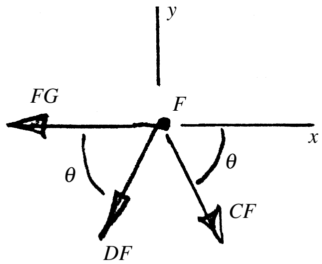
$$\sum F_x : \quad -CD - BC\cos\phi - CF\cos\theta = 0$$

$$\sum F_y : \quad CF\sin\theta - BC\sin\phi - AC = 0$$

Solving, $CD = -200$ lb (C)

6.16 *Contd.*

Joint F:



$\theta = 63.43°$

$CF = 44.7$ lb (T)

$\sum F_x : \quad CF\cos\theta - DF\cos\theta - FG = 0$

$\sum F_y : \quad -CF\sin\theta - DF\sin\theta = 0$

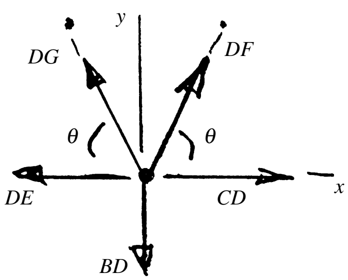
Solving, we get

$CF = 447$ lb (T)

$DF = -447$ lb (C)

$FG = 400$

Joint D:



$CD = -200$ lb (C)

$BD = 400$ lb (T)

$DF = -447$ lb (C)

$\sum F_x : \quad CD - DE + DF\cos\theta - DG\cos\theta = 0$

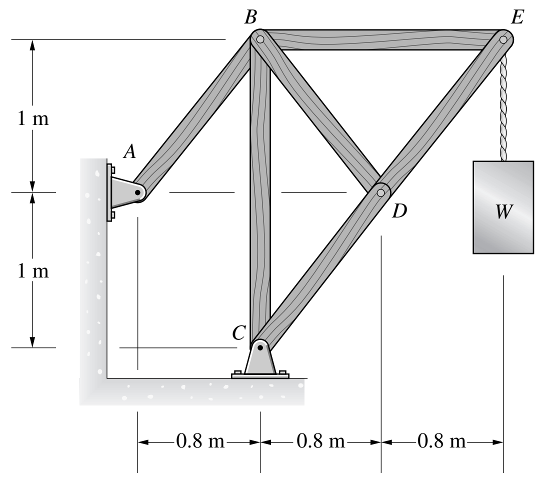
$\sum F_y : \quad DF\sin\theta + DG\sin\theta - BD = 0$

Solving $\quad DE = -968$ lb (C)

$\qquad DG = 894$ lb (T)

Thus, $DE = -800$ lb (C)
$DF = -447$ lb (C)
$DG = 894$ lb (T)

Problem 6.17 Determine the axial forces in the members in terms of the weight W.



Solution: Denote the axial force in a member joining two points I, K by IK. The angle between member DE and the positive x axis is $\alpha = \tan^{-1} 0.8 = 38.66°$. The angle formed by member DB with the positive x axis is $90° + \alpha$. The angle formed by member AB with the positive x-axis is α.

Joint E:

$$\sum F_y = -DE \cos\alpha - W = 0,$$

from which $\boxed{DE = -1.28W\ (C)}$.

$$\sum F_y = -BE - DE \sin\alpha = 0,$$

from which $\boxed{BE = 0.8W\ (T)}$

Joint D:

$$\sum F_x = DE \cos\alpha + BD \cos\alpha - CD \cos\alpha = 0,$$

from which $BD - CD = -DE$.

$$\sum F_y = -BD \sin\alpha + DE \sin\alpha - CD \sin\alpha = 0,$$

from which $BD + CD = DE$.

Solving these two equations in two unknowns:

$\boxed{CD = DE = -1.28W\ (C)}$, $\boxed{BD = 0}$

Joint B:

$$\sum F_x = BE - AB \sin\alpha - BD \sin\alpha = 0,$$

from which $\boxed{AB = \frac{BE}{\sin\alpha} = 1.28W(T)}$

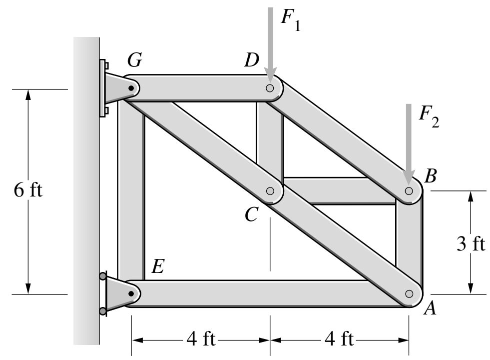
$$\sum F_y = -AB \cos\alpha - BC = 0,$$

from which $\boxed{BC = -AB \cos\alpha = -W(C)}$

Problem 6.18 Consider the truss in Problem 6.17. Each member will safely support a tensile force of 6 kN and a compressive force of 2 kN. Use this criterion to determine the largest weight W the truss will safely support.

Solution: From the solution to Problem 6.17, the largest tensile force is in member AB, $AB = 1.28W(T)$, from which $W = \frac{6}{1.28} = 4.69$ kN is the maximum safe load for tension. The largest compressive forces occur in members DE and CD, $DE = CD = 1.28W(C)$, from which $\boxed{W = \frac{2}{1.28} = 1.56 \text{ kN}}$ is the largest safe load for compression.

Problem 6.19 The loads $F_1 = 600$ lb and $F_2 = 300$ lb. Determine the axial forces in members AE, BD, and CD.



Solution: The reaction at E is determined by the sum of the moments about G:

$$M_G = +6E - 4F_1 - 8F_2 = 0,$$

from which

$$E = \frac{4F_1 + 8F_2}{6} = 800 \text{ lb}.$$

The interior angle EAG is

$$\alpha = \tan^{-1}\left(\frac{6}{8}\right) = 36.87°.$$

From similar triangles this is also the value of the interior angles ACB, CBD, and CGD. *Method of joints:* Denote the axial force in a member joining two points I, K by IK.

Joint E:

$$\sum F_y = E + AE = 0,$$

from which $\boxed{AE = -E = -800 \text{ lb } (C)}$.

$$\sum F_y = EG = 0,$$

from which $EG = 0$.

Joint A:

$$\sum F_y = -AE - AC\cos\alpha = 0,$$

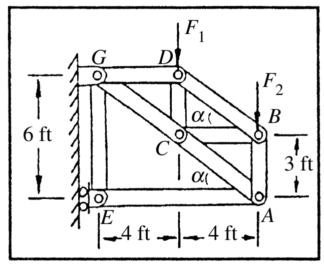
from which $AC = -\dfrac{AE}{0.8} = 1000 \text{ lb}(T)$.

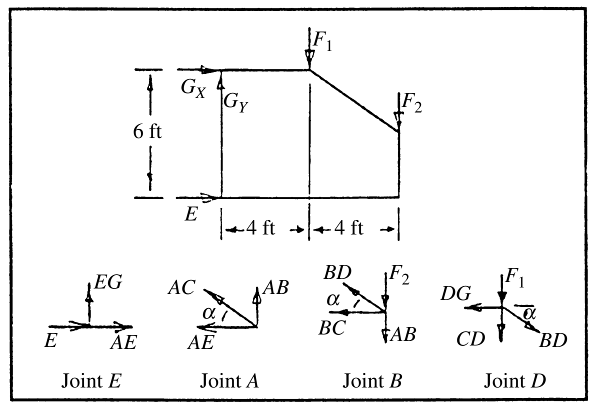
$$\sum F_y = AC\sin\alpha + AB = 0,$$

from which $AB = -AC(0.6) = -600 \text{ lb}(C)$.

Joint B:

$$\sum F_y = BD\sin\alpha - AB - F_1 = 0,$$





from which $\boxed{BD = \frac{F_2+AB}{0.6} = \frac{-300}{0.6} = -500 \text{ lb}(C)}$.

$$\sum F_x = -BC - BD\cos\alpha = 0,$$

from which $BC = -BD(0.8) = 400 \text{ lb}(T)$.

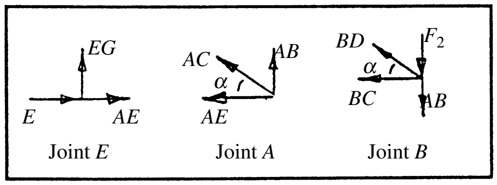
Joint D:

$$\sum F_y = -BD\sin\alpha - CD - F_1 = 0,$$

from which $\boxed{CD = -F_1 - BD(0.6) = -300 \text{ lb}(C)}$

Problem 6.20 Consider the truss in Problem 6.19. The loads $F_1 = 450$ lb and $F_2 = 150$ lb. Determine the axial forces in members AB, AC, and BC.

Solution: From the solution to Problem 6.19 the angle $\alpha = 36.87°$ and the reaction at E is $E = \frac{4F_1+8F_2}{6} = 500$ lb. Denote the axial force in a member joining two points I, K by IK.



Joint E:

$$\sum F_y = EG = 0.$$

$$\sum F_x = AE + E = 0,$$

from which $AE = -E = -500$ lb(C).

Joint A:

$$\sum F_x = -AE - AC\cos\alpha = 0,$$

from which $\boxed{AC = -\frac{AE}{0.8} = 625 \text{ lb}(T)}$.

$$\sum F_y = AC\sin\alpha + AB = 0,$$

from which $\boxed{AB = -AC(0.6) = -375 \text{ lb}(C)}$

Joint B:

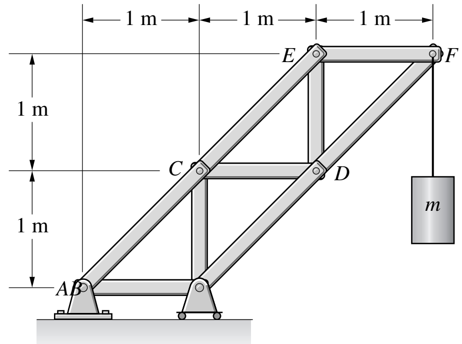
$$\sum F_y = BD\sin\alpha - F_2 - AB = 0,$$

from which $BD = \frac{F_2+AB}{0.6} = -375$ lb(C)

$$\sum F_x = -BC - BD\cos\alpha = 0,$$

from which $\boxed{BC = -BD(0.8) = 300 \text{ lb}(T)}$

Problem 6.21 Each member of the truss will safely support a tensile force of 4 kN and a compressive force of 1 kN. Determine the largest mass m that can safely be suspended.



Solution: The common interior angle $BAC = DCE = EFD = CDB$ is $\alpha = \tan^{-1}(1) = 45°$.
Note $\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}$. Denote the axial force in a member joining two points I, K by IK.

Joint F:

$$\sum F_y = -\frac{DF}{\sqrt{2}} - W = 0,$$

from which $DF = -\sqrt{2}W(C)$.

$$\sum F_x = -EF - \frac{DF}{\sqrt{2}} = 0,$$

from which $EF = W(T)$.

Joint E:

$$\sum F_x = -\frac{CE}{\sqrt{2}} + EF = 0$$

from which $CE = \sqrt{2}W(T)$.

$$\sum F_y = -ED - \frac{CE}{\sqrt{2}} = 0,$$

from which $ED = -W(C)$.

Joint D:

$$\sum F_Y = ED + \frac{DF}{\sqrt{2}} - \frac{BD}{\sqrt{2}} = 0,$$

from which $BD = -2\sqrt{2}W(C)$.

$$\sum F_X = \frac{DF}{\sqrt{2}} - \frac{BD}{\sqrt{2}} - CD = 0,$$

from which $CD = W(T)$

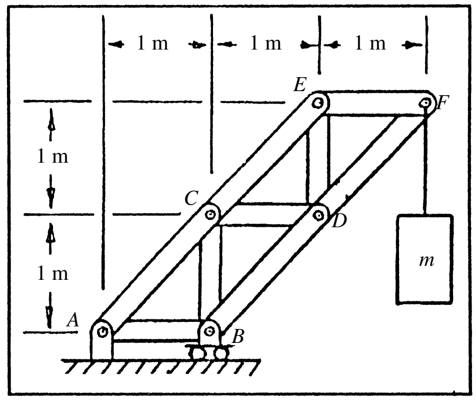
Joint C:

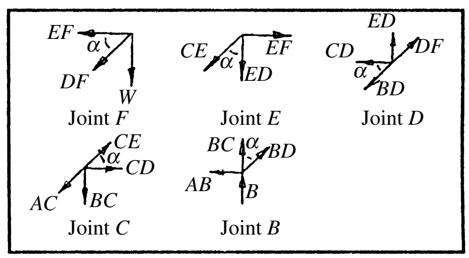
$$\sum F_x = -\frac{AC}{\sqrt{2}} + \frac{CE}{\sqrt{2}} + CD = 0,$$

from which $AC = 2\sqrt{2}W(T)$

$$\sum F_y = -\frac{AC}{\sqrt{2}} + \frac{CE}{\sqrt{2}} - BC = 0,$$

from which $BC = -W(C)$





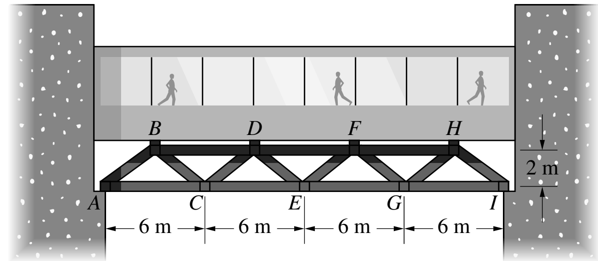
Joint B:

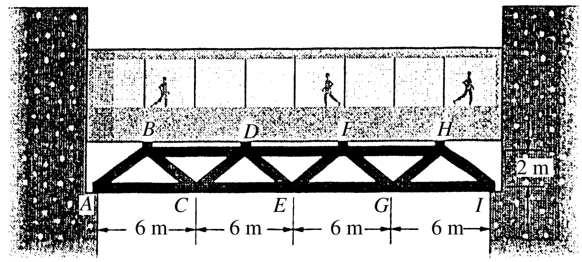
$$\sum F_x = -AB + \frac{BD}{\sqrt{2}} = 0,$$

from which $AB = -2W(C)$

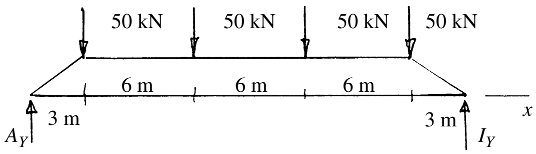
This completes the determination of the axial forces in all nine members. The maximum tensile force occurs in member AC, $AC = 2\sqrt{2}W(T)$, from which the safe load is $W = \frac{4}{2\sqrt{2}} = \sqrt{2} = 1.414$ kN. The maximum compression occurs in member BD, $BD = -2\sqrt{2}W\ (C)$, from which the maximum safe load is $W = \frac{1}{2\sqrt{2}} = 0.3536$ kN. The largest mass m that can be safely supported is $\boxed{m = \frac{353.6}{9.81} = 36.0 \text{ kg}}$

Problem 6.22 The Warren truss supporting the walkway is designed to support vertical 50-kN loads at B, D, F, and H. If the truss is subjected to these loads, what are the resulting axial forces in members BC, CD, and CE?





Solution: Assume vertical loads at A and I Find the external loads at A and I, then use the method of joints to work through the structure to the members needed.



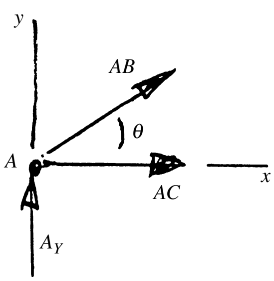
$\sum F_y$: $\quad A_y + I_y - 4(50) = 0$ (kN)

$\sum M_A$: $\quad -3(50) - 9(50) - 15(50) - 21(50) + 24\, I_y = 0$

Solving $\quad A_y = 100$ kN

$\quad I_y = 100$ kN

Joint A:



$$\tan\theta = \frac{2}{3}$$

$\theta = 33.69°$

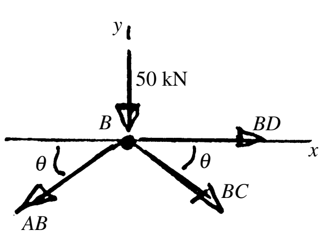
$\sum F_x$: $\quad AB\cos\theta + AC = 0$

$\sum F_y$: $\quad AB\sin\theta + A_y = 0$

Solving, $\quad AB = -180.3$ kN (C)

$\quad AC = 150$ kN (T)

Joint B:



$AB = -180.3$ kN

$\theta = 33.69°$

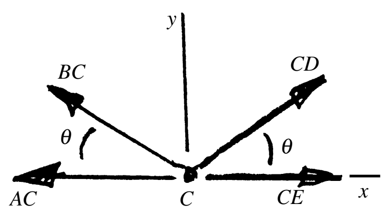
$\sum F_x$: $\quad BC\cos\theta + BD - AB\cos\theta = 0$

$\sum F_y$: $\quad -50 - AB\sin\theta - BC\sin\theta = 0$

Solving, $\quad BC = 90.1$ kN (T)

$\quad BD = -225$ kN (C)

Joint C:



$\theta = 33.69°$

$AC = 150$ kN (T)

$BC = 90.1$ kN (T)

$\sum F_x$: $\quad CE - AC + CD\cos\theta - BC\cos\theta = 0$

$\sum F_y$: $\quad CD\sin\theta + BC\sin\theta = 0$

Solving,

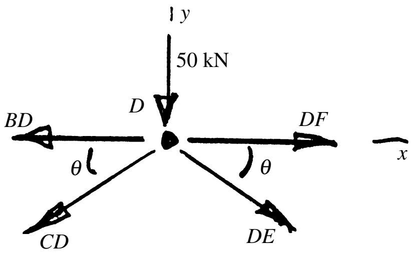
$CE = 300$ kN (T)

$CD = -90.1$ kN (C)

Hence $\quad BC = 90.1$ kN (T)
$\quad CD = -90.1$ kN (C)
$\quad CE = 300$ kN (T)

Problem 6.23 For the Warren truss in Problem 6.22, determine the axial forces in members DF, EF, and FG.

Solution: In the solution to Problem 6.22, we solved for the forces in AB, AC, BC, BD, CD, and CE. Let us continue the process. We ended with Joint C. Let us continue with Joint D.
Joint D:



$\theta = 33.69^\circ$

$BD = -225$ kN (C)

$CD = -90.1$ kN (C)

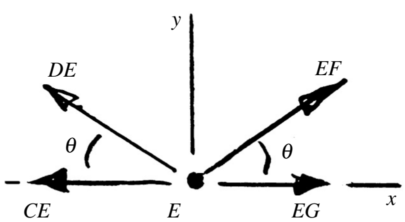
$\sum F_x: \quad DF - BD + DE\cos\theta - CD\cos\theta = 0$

$\sum F_y: \quad -50 - CD\sin\theta - DE\sin\theta = 0$

Solving, $DF = -300$ kN (C)

$DE = 0$

At this point, we have solved half of a symmetric truss with a symmetric load. We could use symmetry to determine the loads in the remaining members. We will continue, and use symmetry as a check.
Joint E:



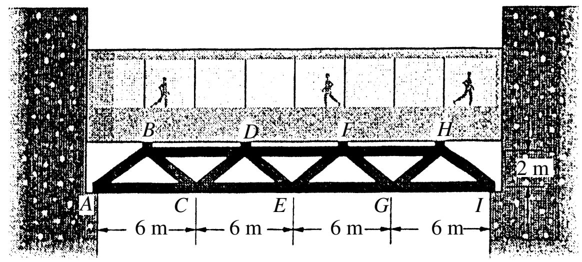
$\theta = 33.69^\circ$

$CE = 300$ kN (T)

$DE = 0$

$\sum F_x: \quad EG - CE + EF\cos\theta - DE\cos\theta = 0$

$\sum F_y: \quad DE\sin\theta + EF\sin\theta = 0$

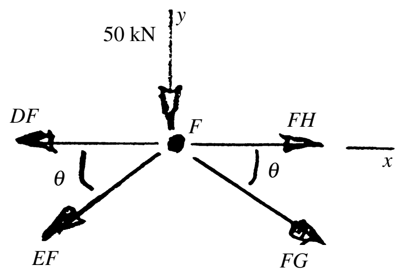


Solving, we get

$EF = 0$

$EG = 300$ kN (T)

Note: The results are symmetric to this point!
Joint F:



$\theta = 33.69^\circ$

$DF = -300$ kN (C)

$EF = 0$

$\sum F_x: \quad FH - DF + FG\cos\theta - EF\cos\theta = 0$

$\sum F_y: \quad -50 - EF\sin\theta - FG\sin\theta = 0$

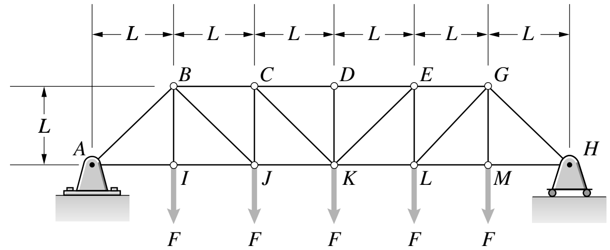
Solving: $FH = -225$ kN (C)

$FG = -90.1$ kN (C)

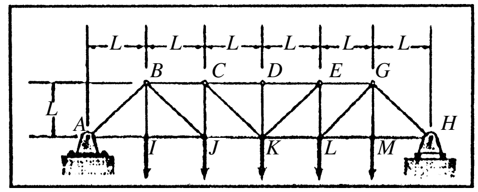
Thus, we have
$DF = -300$ kN (C)
$EF = 0$
$FG = -90.1$ kN (C)

Note-symmetry holds!

Problem 6.24 The Pratt bridge truss supports five forces ($F = 300$ kN). The dimension $L = 8$ m. Determine the axial forces in members BC, BI, and BJ.



Solution: Find support reactions at A and H. From the free body diagram,

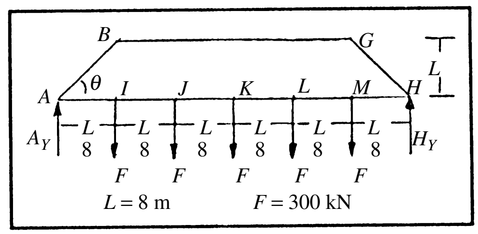


$$\sum F_x = A_X = 0,$$

$$\sum F_y = A_Y + H_Y - 5(300) = 0,$$

and $\sum M_A = 6(8)H_Y - 300(8 + 16 + 24 + 32 + 40) = 0.$

From these equations, $A_Y = H_Y = 750$ kN.
From the geometry, the angle $\theta = 45°$
Joint A: From the free body diagram,



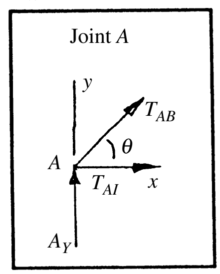
$$\sum F_x = A_X + T_{AB}\cos\theta + T_{AI} = 0,$$

$$\sum F_y = T_{AB}\sin\theta + A_Y = 0.$$

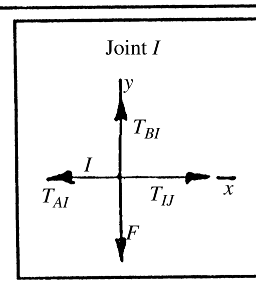
From these equations,

$$T_{AB} = -1061 \text{ kN}$$

and $T_{AI} = 750$ kN.



Joint I: From the free body diagram,



$$\sum F_x = T_{IJ} - T_{AI} = 0,$$

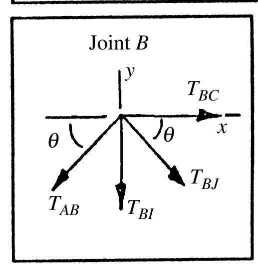
$$\sum F_y = T_{BI} - 300 = 0.$$

From these equations,

$$T_{BI} = 300 \text{ kN}$$

and $T_{IJ} = 750$ kN.

Joint B: From the free body diagram,



$$\sum F_x = T_{BC} + T_{BJ}\cos\theta - T_{AB}\cos\theta = 0,$$

$$\sum F_y = -T_{BI} - T_{BJ}\sin\theta - T_{AB}\sin\theta = 0.$$

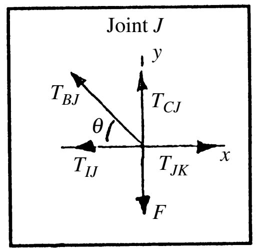
From these equations,

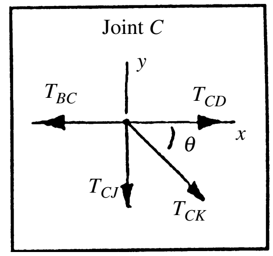
$$T_{BC} = -1200 \text{ kN}$$

and $T_{BJ} = 636$ kN.

Problem 6.25 For the Pratt bridge truss in Problem 6.24, determine the axial forces in members CD, CJ, and CK.

Solution: Use all of the known values from Problem 6.24, and start with Joint J.





Joint J: From the free body diagram,

$$\sum F_x = T_{JK} - T_{BJ}\cos\theta - T_{IJ} = 0,$$
$$\sum F_y = T_{CJ} + T_{BJ}\sin\theta - 300 = 0.$$

From these equations,

$$T_{JK} = 1200 \text{ kN}$$
and $T_{CJ} = -150$ kN.

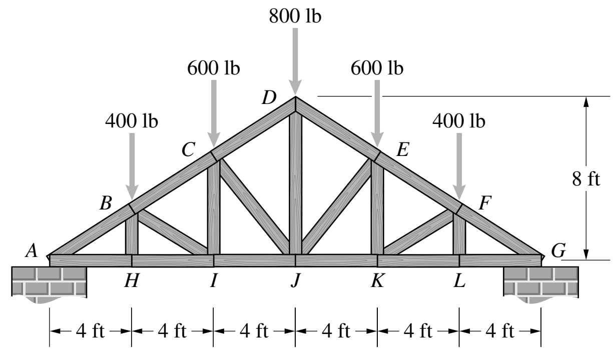
Joint C: From the free body diagram,

$$\sum F_x = T_{CD} + T_{CK}\cos\theta - T_{BC} = 0,$$
$$\sum F_y = -T_{CJ} - T_{CK}\sin\theta = 0.$$

From these equations,

$$T_{CD} = -1350 \text{ kN}$$
and $T_{CK} = 212$ kN.

Problem 6.26 The Howe truss helps support a roof. Model the supports at A and G as roller supports. Determine the axial forces in members AB, BC, and CD.



Solution: The strategy is to proceed from end A, choosing joints with only one unknown axial force in the x- and/or y-direction, if possible, and if not, establish simultaneous conditions in the unknowns. The interior angles HIB and HJC differ. The pitch angle is

$$\alpha_{\text{Pitch}} = \tan^{-1}\left(\frac{8}{12}\right) = 33.7°.$$

The length of the vertical members:

$$\overline{BH} = 4\left(\frac{8}{12}\right) = 2.6667 \text{ ft},$$

from which the angle

$$\alpha_{HIB} = \tan^{-1}\left(\frac{2.6667}{4}\right) = 33.7°.$$

$$\overline{CI} = 8\frac{8}{12} = 5.3333 \text{ ft},$$

from which the angle

$$\alpha_{IJC} = \tan^{-1}\left(\frac{5.333}{4}\right) = 53.1°.$$

The moment about G:

$$M_G = (4+20)(400) + (8+16)(600) + (12)(800) - 24A = 0,$$

from which $A = \frac{33600}{24} = 1400$ lb. *Check*: The total load is 2800 lb. From left-right symmetry each support A, G supports half the total load. *check*.

The method of joints: Denote the axial force in a member joining two points I, K by IK.

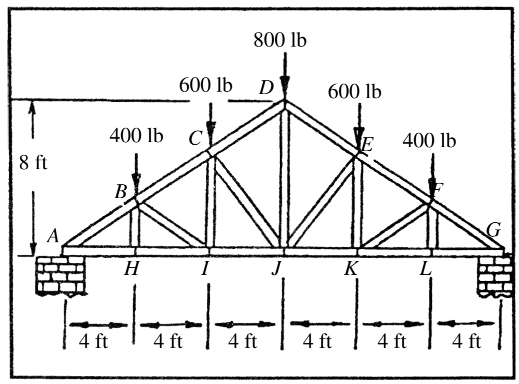
Joint A:

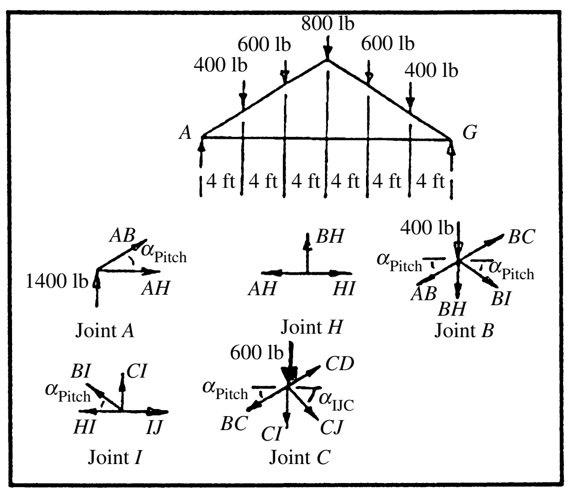
$$\sum F_y = AB\sin\alpha_P + 1400 = 0,$$

from which $\boxed{AB = -\frac{1400}{\sin\alpha_p} = -2523.9 \text{ lb } (C)}$

$$\sum F_x = AB\cos\alpha_{\text{Pitch}} + AH = 0,$$

from which $AH = (2523.9)(0.8321) = 2100$ lb (T)





Joint H:

$$\sum F_y = BH = 0, \text{ or, } BH = 0.$$

$$\sum F_x = -AH + HI = 0,$$

from which $HI = 2100$ lb (T)

Joint B:

$$\sum F_x = -AB\cos\alpha_{\text{Pitch}} + BC\cos\alpha_{\text{Pitch}} + BI\cos\alpha_{\text{Pitch}} = 0,$$

from which $BC + BI = AB$

6.26 *Contd.*

$$\sum F_y = -400 - AB\sin\alpha_{\text{Pitch}} + BC\sin\alpha_{\text{Pitch}} - BI\sin\alpha_{\text{Pitch}} = 0,$$

from which $BC - BI = AB + \frac{400}{\sin\alpha_{\text{Pitch}}}$.

Solve the two simultaneous equations in unknowns BC, BI:

$$BI = -\frac{400}{2\sin\alpha_{\text{Pitch}}} = -360.56 \text{ lb } (C),$$

and $\boxed{BC = AB - BI = -2163.3 \text{ lb } (C)}$

Joint I:

$$\sum F_x = -BI\cos\alpha_{\text{Pitch}} - HI + IJ = 0,$$

from which $IJ = 1800$ lb (T)

$$\sum F_y = +BI\sin\alpha_{\text{Pitch}} + CI = 0,$$

from which $CI = 200$ lb (T)

Joint C:

$$\sum F_x = -BC\cos\alpha_{\text{Pitch}} + CD\cos\alpha_{\text{Pitch}} + CJ\cos\alpha_{IJC} = 0,$$

from which $CD(0.8321) + CJ(0.6) = -1800$

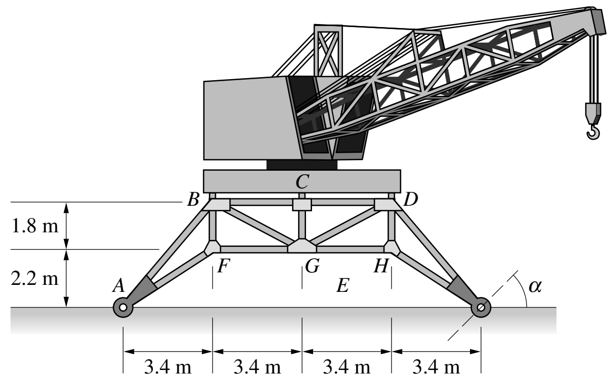
$$\sum F_y = -600 - CI - BC\sin\alpha_{\text{Pitch}} + CD\sin\alpha_{\text{Pitch}} - CJ\sin\alpha_{IJC} = 0,$$

from which $CD(0.5547) - CJ(0.8) = -400$

Solve the two simultaneous equations to obtain $CJ = -666.67$ lb (C),

and $\boxed{CD = -1682.57 \text{ lb } (C)}$

Problem 6.27 The plane truss forms part of the supports of a crane on an offshore oil platform. The crane exerts vertical 75-kN forces on the truss at B, C, and D. You can model the support at A as a pin support and model the support at E as a roller support that can exert a force normal to the dashed line but cannot exert a force parallel to it. The angle $\alpha = 45^\circ$. Determine the axial forces in the members of the truss.



Solution: The included angles

$$\gamma = \tan^{-1}\left(\frac{4}{3.4}\right) = 49.64^\circ,$$

$$\beta = \tan^{-1}\left(\frac{2.2}{3.4}\right) = 32.91^\circ,$$

$$\theta = \tan^{-1}\left(\frac{1.8}{3.4}\right) = 27.9^\circ.$$

The complete structure as a free body: The sum of the moments about A is

$$M_A = -(75)(3.4)(1 + 2 + 3) + (4)(3.4)E_y = 0.$$

with this relation and the fact that $E_x \cos 45^\circ + E_y \cos 45^\circ = 0$, we obtain $E_x = -112.5$ kN and $E_y = 112.5$ kN. From

$$\sum F_x^A = A_x + E_x = 0,\ A_X = -E_X = 112.5\ kN.$$

$$\sum F_y^A = A_y - 3(75) + E_y = 0,$$

from which $A_y = 112.5$ kN. Thus the reactions at A and E are symmetrical about the truss center, which suggests that symmetrical truss members have equal axial forces.

The method of joints: Denote the axial force in a member joining two points I, K by IK.

Joint A:

$$\sum F_x = AB\cos\gamma + A_x + AF\cos\beta = 0,$$

$$\sum F_y = AB\sin\gamma + A_y + AF\sin\beta = 0,$$

from which two simultaneous equations are obtained.

Solve: $\boxed{AF = -44.67 \text{ kN } (C)}$,

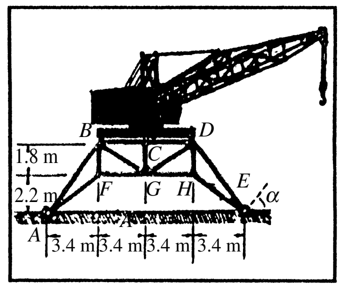
and $\boxed{AB = -115.8 \text{ kN } (C)}$

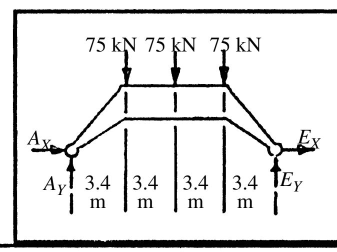
Joint E:

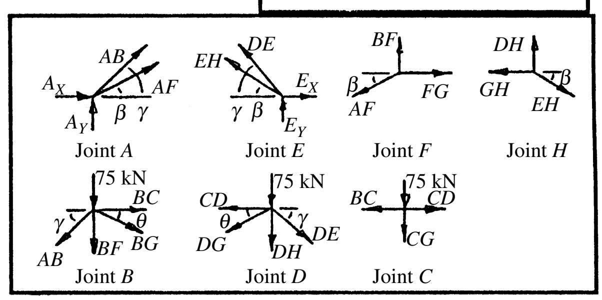
$$\sum F_y = -DE\cos\gamma + E_x - EH\cos\beta = 0.$$

$$\sum F_y = DE\sin\gamma + E_y + EH\sin\beta = 0,$$

from which two simultaneous equations are obtained.







Solve: $\boxed{EH = -44.67 \text{ kN}(C)}$,

and $\boxed{DE = -115.8 \text{ kN}(C)}$

Joint F:

$$\sum F_x = -AF\cos\beta + FG = 0,$$

from which $\boxed{FG = -37.5 \text{ kN } (C)}$

$$\sum F_y = -AF\sin\beta + BF = 0,$$

from which $\boxed{BF = -24.26 \text{ kN } (C)}$

Joint H:

$$\sum F_x = EH\cos\beta - GH = 0,$$

6.27 *Contd.*

from which $\boxed{GH = -37.5 \text{ kN } (C)}$

$\sum F_y = -EH \sin\beta + DH = 0,$

from which $\boxed{DH = -24.26 \text{ kN } (C)}$

Joint B:

$\sum F_y = -AB \sin\gamma - BF + BG \sin\theta - 75 = 0,$

from which $\boxed{BG = 80.1 \text{ kN } (T)}$

$\sum F_x = -AB \cos\gamma + BC + BG \cos\theta = 0,$

from which $\boxed{BC = -145.8 \text{ kN } (C)}$

Joint D:

$\sum F_y = -DE \sin\gamma - DH - DG \sin\theta - 75 = 0,$

from which $\boxed{DG = 80.1 \text{ kN } (T)}$

$\sum F_x = DE \cos\gamma - CD - DG \cos\theta = 0,$

from which $\boxed{CD = -145.8 \text{ kN } (C)}$

Joint C:

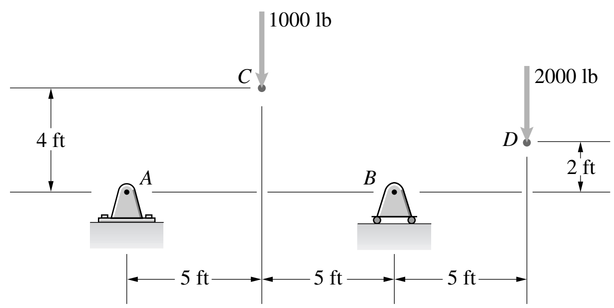
$\sum F_x = CD - BC = 0,$

from which $\boxed{CD = BC}$ *Check.*

$\sum F_y = -CG - 75 = 0,$

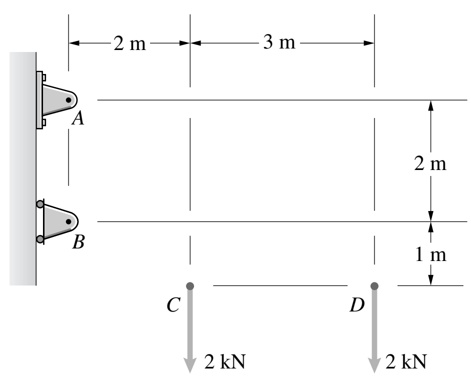
from which $\boxed{CG = -75 \text{ kN } (C)}$

Problem 6.28 (a) Design a truss attached to the supports A and B that supports the loads applied at points C and D.
(b) Determine the axial forces in the members of the truss you designed in (a)



Solution: Problem 6.28 don't have unique solution

Problem 6.29 (a) Design a truss attached to the supports A and B that supports the loads applied at points C and D.
(b) Determine the axial forces in the members of the truss you designed in (a).

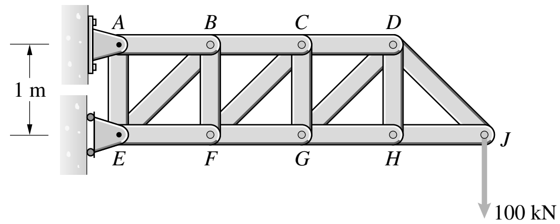


Solution: Problem 6.29 don't have unique solution

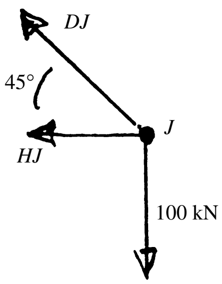
Problem 6.30 The truss supports a 100-kN load at J. The horizontal members are each 1 m in length.

(a) Use the method of joints to determine the axial force in member DG.

(b) Use the method of sections to determine the axial force in member DG.



Solution: (a) Start with Joint J



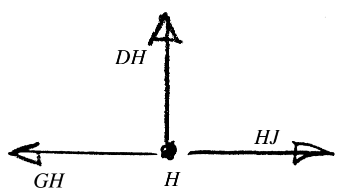
$\sum F_x$: $\quad -HJ - DJ\cos 45^\circ = 0$

$\sum F_y$: $\quad DJ\sin 45^\circ - 100 = 0$

Solving $\quad DJ = 141.4$ kN (T)

$HJ = -100$ kN (C)

Joint H:



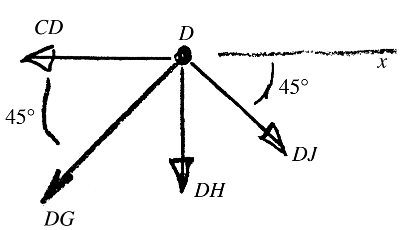
$\sum F_x$: $\quad HJ - GH = 0$

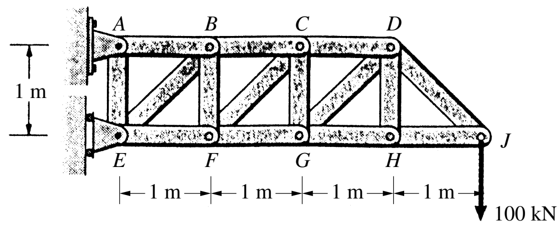
$\sum F_y$: $\quad DH = 0$

$DH = 0,$

$GH = -100$ kN (C)

Joint D





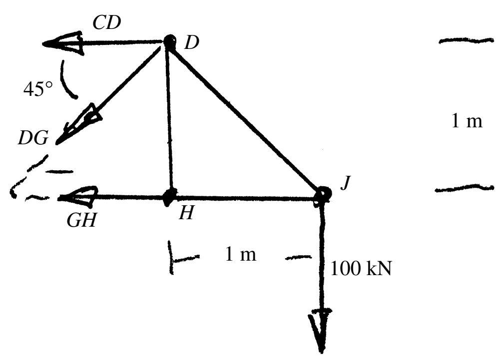
$\sum F_x$: $\quad -CD - DG\cos 45^\circ + DJ\cos 45^\circ = 0$

$\sum F_y$: $\quad -DG\sin 45^\circ - DJ\sin 45^\circ - DH = 0$

Solving, $\quad CD = 200$ kN

$DG = -141.4$ kN (C)

(b) Method of Sections



$\sum F_x$: $\quad -CD - DG\cos 45^\circ - GH = 0$

$\sum F_y$: $\quad -DG\sin 45^\circ - 100 = 0$

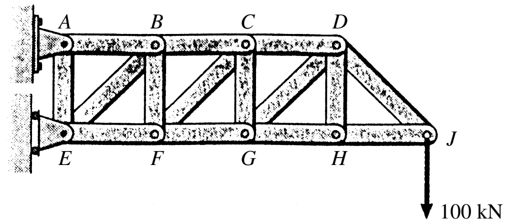
$\sum M_D$: $\quad -(1)GH - (1)(100) = 0$

Solving, $\quad GH = -100$ kN (C)

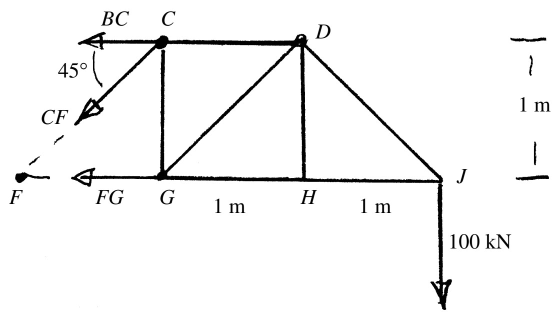
$CD = 200$ kN (T)

$DG = -141.4$ kN (C)

Problem 6.31 For the truss in Problem 6.30, use the method of sections to determine the axial forces in members BC, CF, and FG.



Solution:



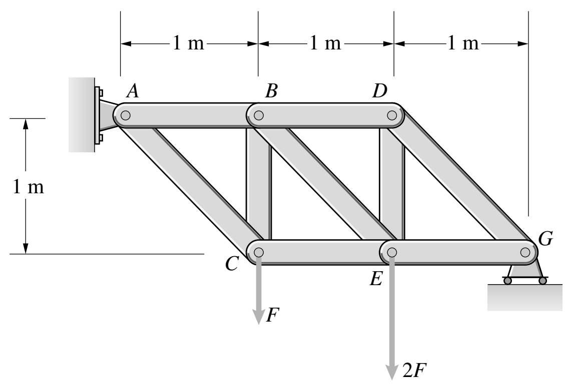
$\sum F_x$: $-BC - CF\cos 45 - FG = 0$

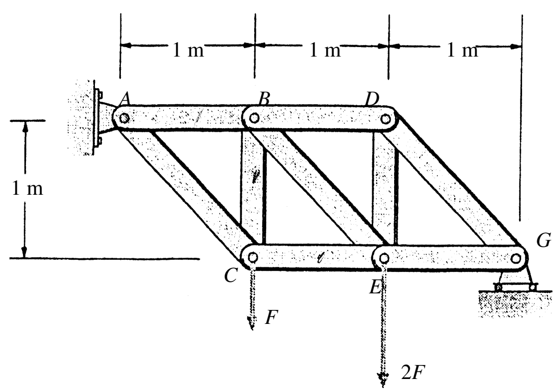
$\sum F_y$: $-CF\sin 45^\circ - 100 = 0$

$\sum M_C$: $-(1)FG - 2(100) = 0$

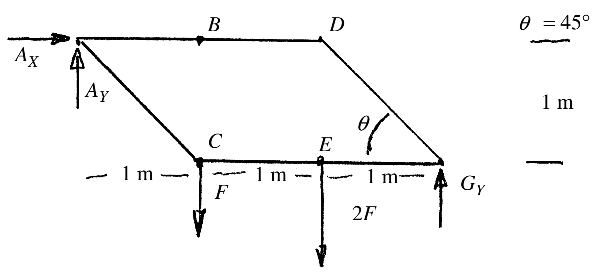
Solving
$$BC = 300 \text{ kN } (T)$$
$$CF = -141.4 \text{ kN } (C)$$
$$FG = -200 \text{ kN } (C)$$

Problem 6.32 Use the method of sections to determine the axial forces in members AB, BC, and CE.





Solution: First, determine the forces at the supports



$\sum F_x$: $A_x = 0$

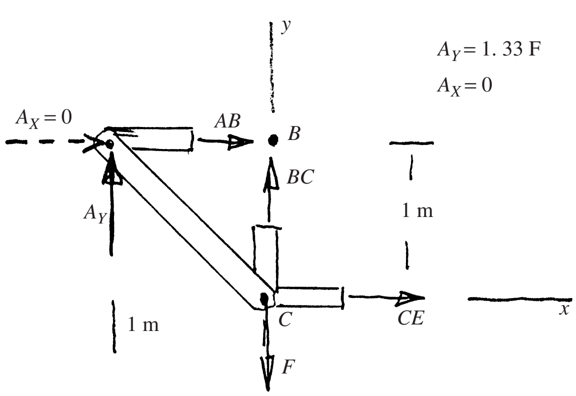
$\sum F_y$: $A_y + G_y - 3F = 0$

$\circlearrowleft + \sum M_A$: $-1(F) - 2(2F) + 3G_y = 0$

Solving $A_x = 0$ $G_y = 1.67F$

$A_y = 1.33F$

Method of Sections:



$\sum F_x$: $CE + AB = 0$

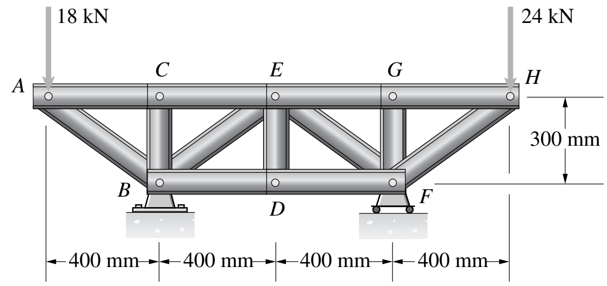
$\sum F_y$: $BC + A_y - F = 0$

$\circlearrowleft + \sum M_B$: $(-1)A_y + (1)CE = 0$

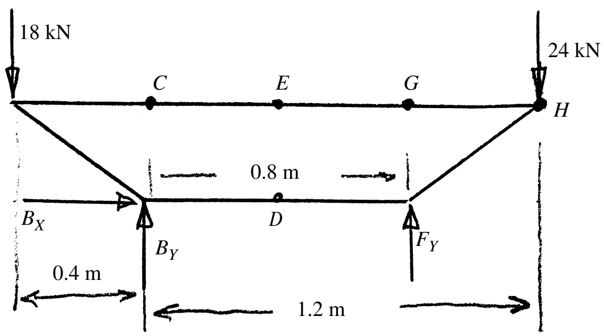
Solving, we get

$$AB = -1.33F \ (C)$$
$$CE = 1.33F \ (T)$$
$$BC = -0.33F \ (C)$$

Problem 6.33 The truss supports loads at A and H. Use the method of sections to determine the axial forces in members CE, BE, and BD.



Solution: First find the external support loads on the truss



$\sum F_x$: $B_x = 0$

$\sum F_y$: $B_y + F_y - 18 - 24 = 0$ (kN)

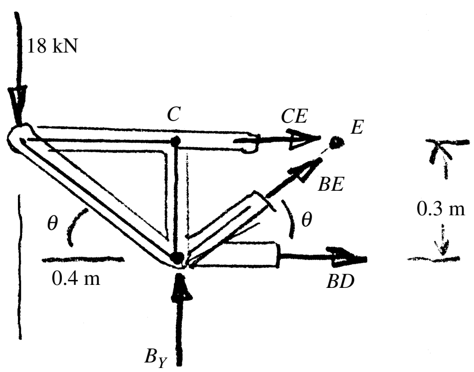
$\sum M_B$: $0.8F_y - (1.2)(24) + (0.4)18 = 0$

Solving: $B_x = 0$

$B_y = 15$ kN

$F_y = 27$ kN

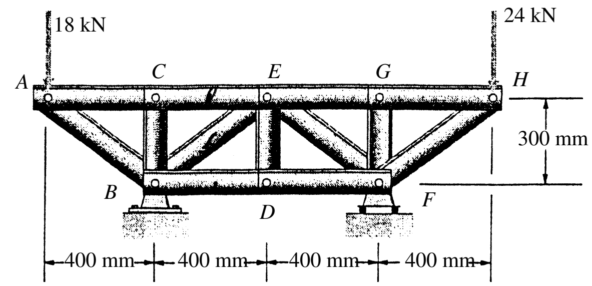
Method of sections:



$\tan\theta = \dfrac{3}{4}$

$\theta = 36.87°$

$B_y = 15$ kN



$\sum F_x$: $CE + BE\cos\theta + BD = 0$

$\sum F_y$: $B_y - 18 + BE\sin\theta = 0$

$\circlearrowleft + \sum M_B$: $+(0.4)(18) - (0.3)(CE) = 0$

Solving,

$CE = 24$ kN (T)
$BE = 5$ kN (T)
$BD = -28$ kN (C)

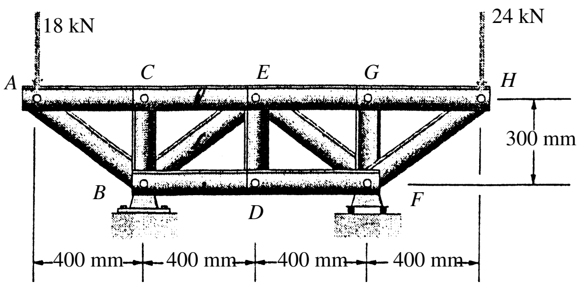
Problem 6.34 For the truss in Problem 6.33, use the method of sections to determine the axial forces in members EG, EF, and DF.

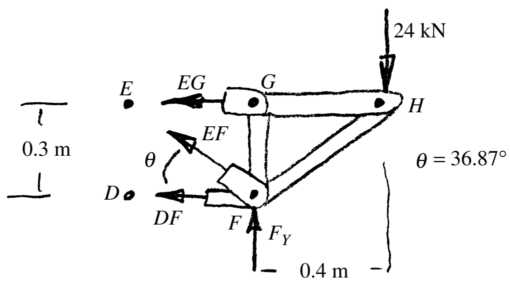
Solution: From the solution to Problem 6.33, the external forces at B and F are

$B_x = 0,$

$B_y = 15$ kN,

$F_y = 27$ kN.





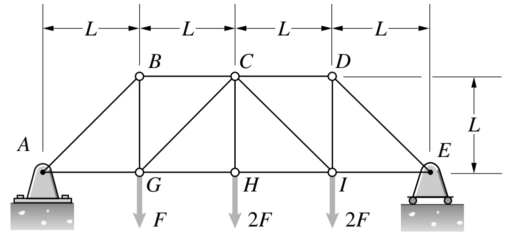
$$\sum F_x: \quad -EG - EF\cos\theta - DF = 0$$

$$\sum F_y: \quad -24 + F_y + EF\sin\theta = 0$$

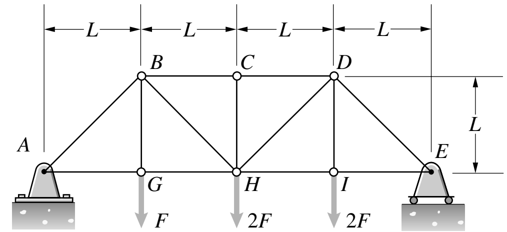
$$\sum M_F: \quad -(0.4)(24) + (0.3)EG$$

Solving:
$$\boxed{\begin{aligned} EG &= 32 \text{ kN } (T) \\ EF &= -5 \text{ kN } (C) \\ DF &= -28 \text{ kN } (C) \end{aligned}}$$

Problem 6.35 For the Howe and Pratt trusses, use the method of sections to determine the axial force in member BC.



Howe



Pratt

Solution: From the free body diagram of the whole truss, the equations of equilibrium are

$$\sum F_x = A_X = 0,$$

$$\sum F_y = A_Y + E_Y - 5F = 0,$$

and $\sum M_A = 4LE_Y - LF - (2L)2F - (3L)2F = 0.$

From these equations, we get $A_X = 0$, $A_Y = 2.25F$, and $E_Y = 2.75F$. Note that the support forces are the same for the Howe and Pratt trusses.

Howe Section: From the Howe truss section, we see that if we sum moments about G, we get one equation in one unknown, i.e.,

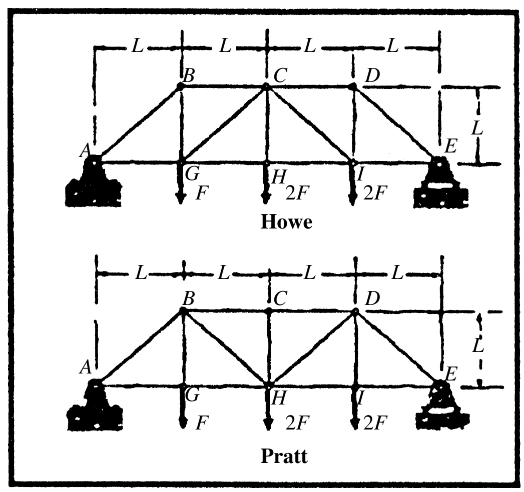
$$\sum M_G = -LA_Y - LT_{BC\text{Howe}} = 0,$$

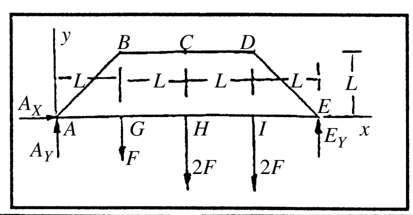
or $T_{BC\text{Howe}} = -2.25F$ (compression).

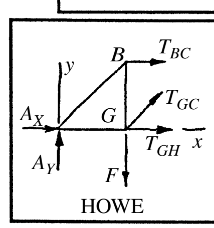
Pratt Section: From the Pratt truss section we see that summing moments about H is advantageous. Hence,

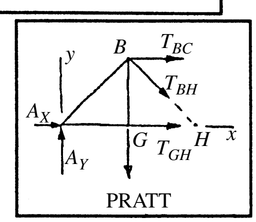
$$\sum M_H = -2LA_Y + LF - LT_{BC\text{Pratt}} = 0,$$

or $T_{BC\text{Pratt}} = -3.5F$ (compression).









Problem 6.36 For the Howe and Pratt trusses in Problem 6.35, determine the axial force in member HI.

Solution: *Howe Section:* From the Howe truss section, we see that if we sum moments about C, we get one equation in one unknown, i.e.,

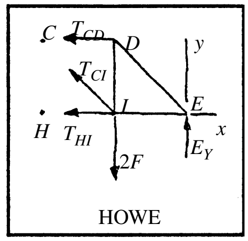
$$\sum M_C = 2LE_Y - 2LF - LT_{HI\text{Howe}} = 0,$$

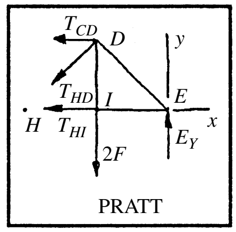
or $T_{HI\text{Howe}} = 3.5F$ (tension).

Pratt Section: From the Pratt truss section we see that summing moments about D is advantageous. Hence,

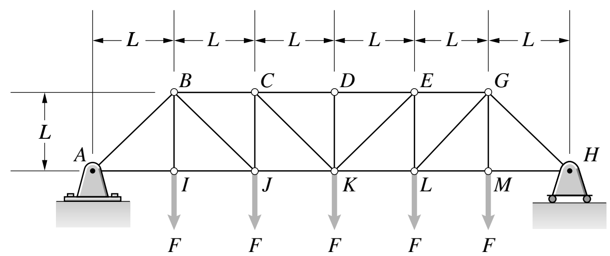
$$\sum M_D = LE_Y - LT_{HI\text{Pratt}} = 0$$

or $T_{HI\text{Pratt}} = 2.75F$ (tension).

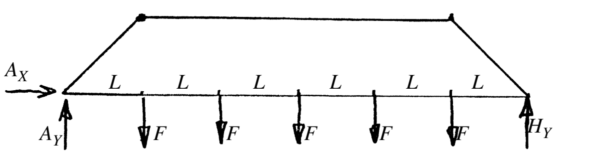




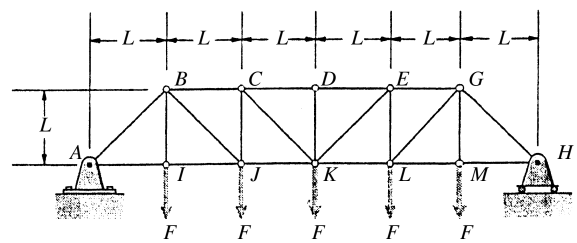
Problem 6.37 The Pratt bridge truss supports five forces $F = 340$ kN. The dimension $L = 8$ m. Use the method of sections to determine the axial force in member JK.



Solution: First determine the external support forces.



$F = 340$ kN, $L = 8$ M



$$\sum F_x: \quad A_x = 0$$

$$\sum F_y: \quad A_y - 5F + H_y = 0$$

$$\circlearrowleft + \sum M_A: \quad 6LH_y - LF - 2LF - 3LF - 4LF - 5LF = 0$$

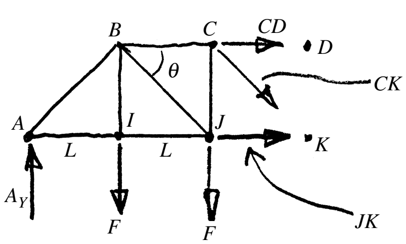
Solving: $A_x = 0,$

$A_y = 850$ kN

$H_y = 850$ kN

Note the symmetry:

Method of sections to find axial force in member JK.



$\theta = 45°$

$L = 8M$

$F = 340$ kN

$A_y = 850$ kN

$$\sum F_x: \quad CD + JK + CK\cos\theta = 0$$

$$\sum F_y: \quad A_y - 2F - CK\sin\theta = 0$$

$$\circlearrowleft + \sum M_C: \quad L(JK) + L(F) - 2L(A_y) = 0$$

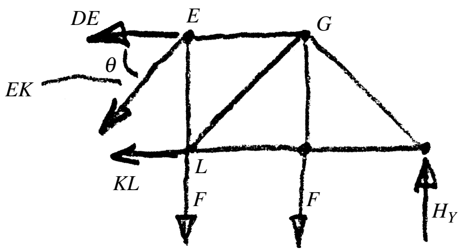
Solving, $JK = 1360$ kN (T)

Also, $CK = 240.4$ kN (T)

$CD = -1530$ kN (C)

Problem 6.38 For the Pratt bridge truss in Problem 6.41, use the method of sections to determine the axial force in member EK.

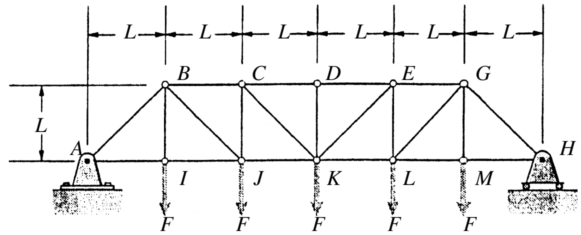
Solution: From the solution to Problem 6.37, the support forces are $A_x = 0,\ A_y = H_y = 850$ kN.
Method of Sections to find axial force in EK.



$$\sum F_x: \quad -DE - EK\cos\theta - KL = 0$$

$$\sum F_y: \quad H_y - 2F - EK\sin\theta = 0$$

$$\sum M_E: \quad -(L)(KL) - (L)(F) + (2L)H_y = 0$$

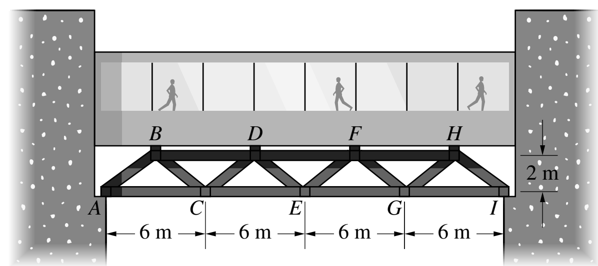


Solution: $EK = 240.4$ kN (T)

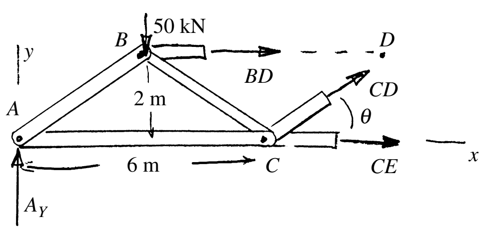
Also, $KL = 1360$ kN (T)

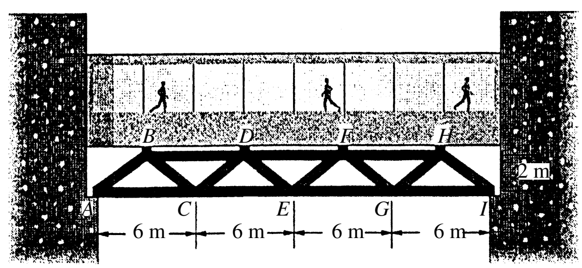
$DE = -1530$ kN (C)

Problem 6.39 The walkway exerts vertical 50-kN loads on the Warren truss at B, D, F, and H. Use the method of sections to determine the axial force in member CE.



Solution: First, find the external support forces. By symmetry, $A_y = I_y = 100$ kN (we solved this problem earlier by the method of joints).





$$\tan\theta = \frac{2}{3}$$

$$\theta = 33.69^\circ$$

$$\sum F_x: \quad BD + CD\cos\theta + CE = 0$$

$$\sum F_y: \quad A_y - 50 + CD\sin\theta = 0$$

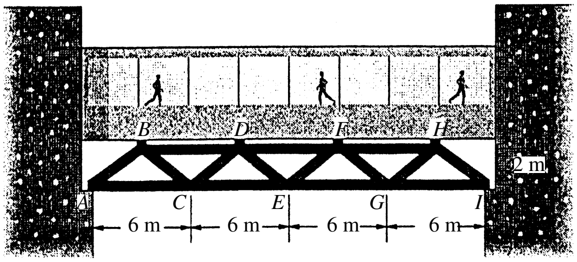
$$\sum M_C: \quad -6A_y + 3(50) - 2BD = 0$$

Solving: $CE = 300$ kN (T)

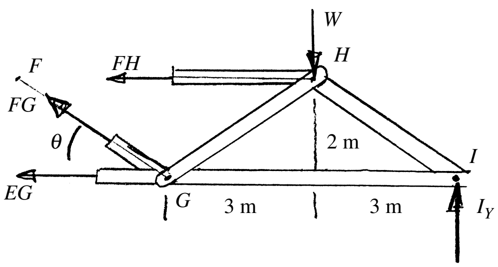
Also, $BD = -225$ kN (C)

$CD = -90.1$ kN (C)

Problem 6.40 The walkway in Problem 6.39 exerts equal vertical loads on the Warren truss at B, D, F, and H. Use the method of sections to determine the maximum allowable value of each vertical load if the magnitude of the axial force in member FG is not to exceed 100 kN.



Solution: Let the loads at B, D, F, and H be denoted by W. By summetry $A_y = I_y = 2W$.
Method of Sections



$$\tan\theta = \frac{2}{3}$$

$$\theta = 33.69^\circ$$

$$\sum F_x: \quad -EG - FH - FG\cos\theta = 0$$

$$\sum F_y: \quad I_y - W + FG\sin\theta = 0$$

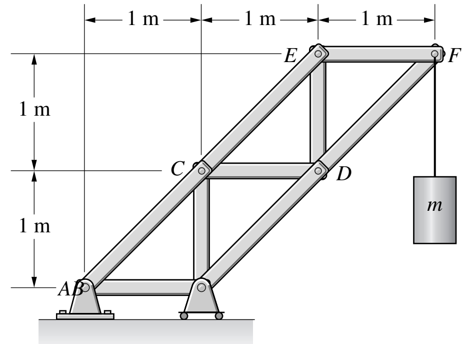
$$\sum M_G: \quad 2FH + 6I_y - 3W = 0$$

We set $FG = \pm 100$ kN and solve:
For $FG = +100$ kN, $W = -55.5$ kN (this implies an upward load on the bridge)
For $FG = -100$ kN (in compression)

$W = 55.5$ kN.

This is the load limit on the bridge based on the load in member FG.

Problem 6.41 The mass $m = 120$ kg. Use the method of sections to determine the axial forces in members BD, CD, and CE.



Solution: First, find the support reactions using the first free body diagram. Then use the section shown in the second free body diagram to determine the forces in the three members.

Support Reactions: Equilibrium equations are

$$\sum F_x = A_X = 0,$$

$$\sum F_y = A_Y + B_Y - mg = 0,$$

and summing moments around A,

$$\sum M_A = -3mg + (1)B_Y = 0.$$

Thus, $A_X = 0,\ A_Y = -2.35$kN, and $B_Y = 3.53$ kN

Section: From the second free body diagram, the equilibrium equations for the section are

$$\sum F_x = A_X + T_{CD} + T_{CE}\cos(45°) + T_{BD}\cos(45°) = 0,$$

$$\sum F_y = A_Y + B_Y + T_{CE}\sin(45°) + T_{BD}\sin(45°) = 0,$$

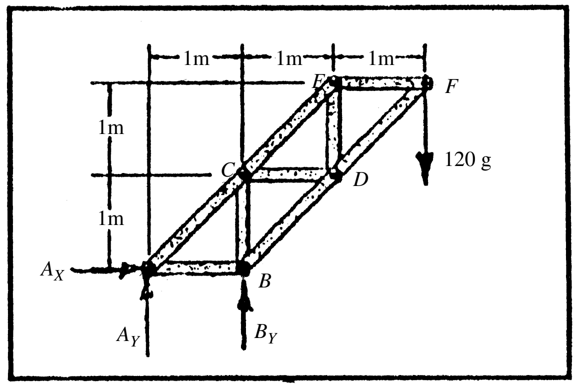
and, summing moments around C,

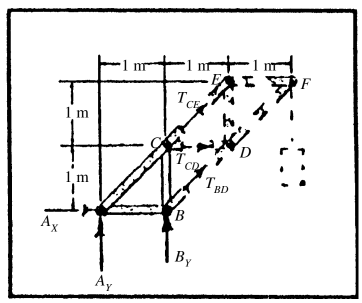
$$\sum M_C = (1)T_{BD}\cos(45°) - (1)A_Y + (1)A_X = 0.$$

Solving, we get $T_{BD} = -3.30$ kN,

$T_{CD} = 1.18$ kN,

$T_{CE} = 1.66$ kN.





Problem 6.42 For the truss in Problem 6.41, use the method of sections to determine the axial forces in members AC, BC, and BD.

Solution: Use the support reactions found in Problem 6.41. The free body diagram for the section necessary to find the three unknowns is shown at right. The equations of equilibrium are

$$\sum F_x = A_X + T_{AC}\cos(45°) + T_{BD}\cos(45°) = 0,$$

$$\sum F_y = A_Y + B_Y + T_{BC} + T_{AC}\sin(45°) + T_{BD}\sin(45°) = 0,$$

and, summing moments around B,

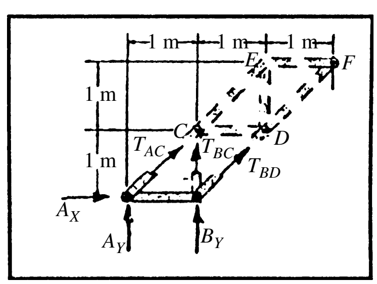
$$\sum M_B = (-1)A_Y - (1)T_{AC}\sin(45°) = 0.$$

The results are

$T_{AC} = 3.30$ kN,

$T_{BC} = -1.18$ kN,

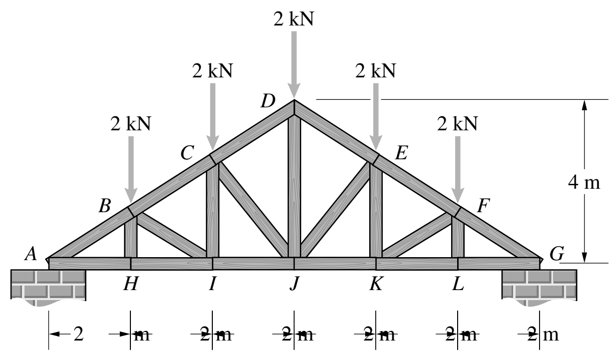
and $T_{BD} = -3.30$ kN.



Problem 6.43 The Howe truss helps support a roof. Model the supports at A and G as roller supports.

(a) Use the method of joints to determine the axial force in member BI.

(b) Use the method of sections to determine the axial force in member BI.



Solution: The pitch of the roof is

$$\alpha = \tan^{-1}\left(\frac{4}{6}\right) = 33.69°.$$

This is also the value of interior angles HAB and HIB. *The complete structure as a free body:* The sum of the moments about A is

$$M_A = -2(2)(1+2+3+4+5) + 6(2)G = 0,$$

from which $G = \frac{30}{6} = 5$ kN. The sum of the forces:

$$\sum F_Y = A - 5(2) + G = 0,$$

from which $A = 10 - 5 = 5$ kN.

The method of joints: Denote the axial force in a member joining I, K by IK.

(a) *Joint A:*

$$\sum F_y = A + AB\sin\alpha = 0,$$

from which $AB = \frac{-A}{\sin\alpha} = \frac{-5}{0.5547} = -9.01$ kN (C).

$$\sum F_x = AB\cos\alpha + AH = 0,$$

from which $AH = -AB\cos\alpha = 7.5$ kN (T).

Joint H:

$$\sum F_y = BH = 0.$$

Joint B:

$$\sum F_x = -AB\cos\alpha + BI\cos\alpha + BC\cos\alpha = 0,$$

$$\sum F_y = -2 - AB\sin\alpha - BI\sin\alpha + BC\sin\alpha = 0.$$

Solve: $\boxed{BI = -1.803 \text{ kN } (C)}$, $BC = -7.195$ kN (C)

(b) Make the cut through BC, BI and HI. *The section as a free body:* The sum of the moments about B:

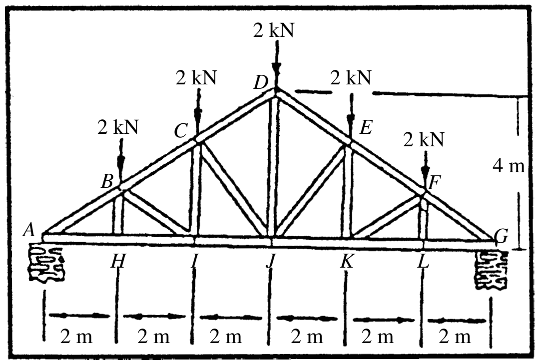
$$M_B = -A(2) + HI(2\tan\alpha) = 0,$$

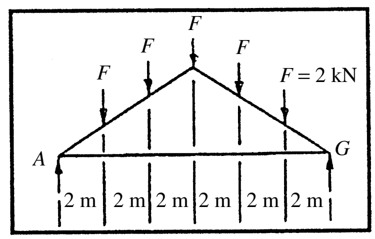
from which $HI = \frac{3}{2}A = 7.5$ kN(T). The sum of the forces:

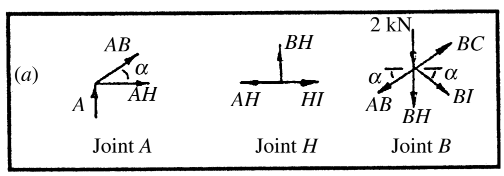
$$\sum F_x = BC\cos\alpha + BI\cos\alpha + HI = 0,$$

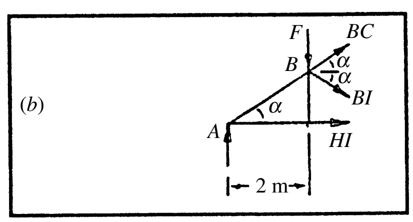
$$\sum F_y = A - F + BC\sin\alpha - BI\sin\alpha = 0.$$

Solve: $\boxed{BI = -1.803 \text{ kN } (C)}$.

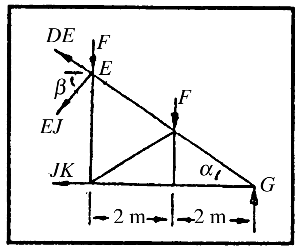








Problem 6.44 Consider the truss in Problem 6.43. Use the method of sections to determine the axial force in member EJ.



Solution: From the solution to Problem 6.43, the pitch angle is $\alpha = 36.69°$, and the reaction $G = 5$ kN. The length of member EK is

$$L_{EK} = 4 \tan\alpha = \frac{16}{6} = 2.6667 \text{ m}.$$

The interior angle KJE is

$$\beta = \tan^{-1}\left(\frac{L_{EK}}{2}\right) = 53.13°.$$

Make the cut through ED, EJ, and JK. Denote the axial force in a member joining I, K by IK. *The section as a free body:* The sum of the moments about E is

$$M_E = +4G - 2(F) - JK(2.6667) = 0,$$

from which $JK = \frac{20-4}{2.6667} = 6$ kN (T).

The sum of the forces:

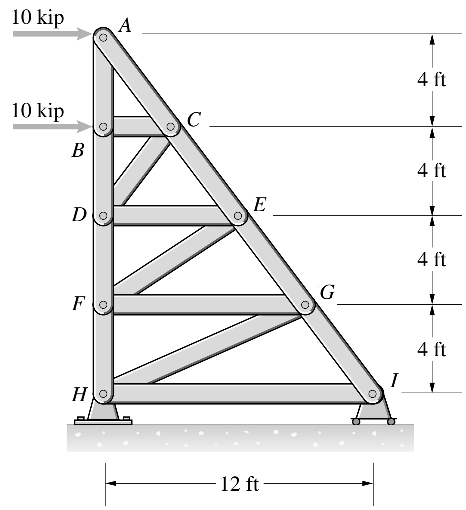
$$\sum F_x = -DE\cos\alpha - EJ\cos\beta - JK = 0.$$
$$\sum F_y = DE\sin\alpha - EJ\sin\beta - 2F + G = 0,$$

from which the two simultaneous equations:

$$0.8321DE + 0.6EJ = -6,$$
$$0.5547DE - 0.8EJ = -1.$$

Solve: $\boxed{EJ = -2.5 \text{ kN } (C)}$.

Problem 6.45 Use the method of sections to determine the axial force in member EF.



Solution: The included angle at the apex BAC is

$$\alpha = \tan^{-1}\left(\frac{12}{16}\right) = 36.87^{\circ}.$$

The interior angles BCA, DEC, FGE, HIG are $\gamma = 90^{\circ} - \alpha = 53.13^{\circ}$. The length of the member ED is $L_{ED} = 8\tan\alpha = 6$ ft. The interior angle DEF is

$$\beta = \tan^{-1}\left(\frac{4}{L_{ED}}\right) = 33.69^{\circ}.$$

The complete structure as a free body: The moment about H is $M_H = -10(12) - 10(16) + I(12) = 0$, from which $I = \frac{280}{12} = 23.33$ kip. The sum of forces:

$$\sum F_y = H_y + I = 0,$$

from which $H_y = -I = -23.33$ kip.

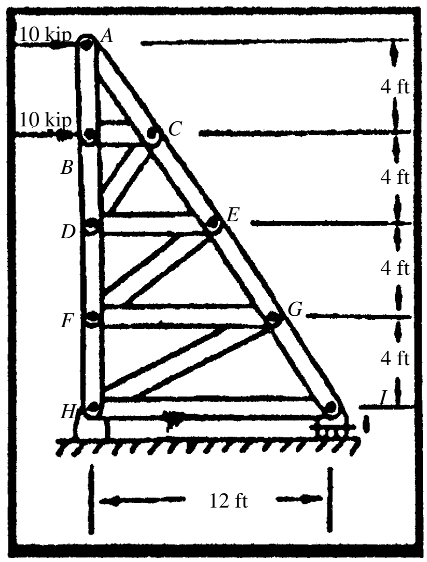
$$\sum F_x = H_x + 20 = 0,$$

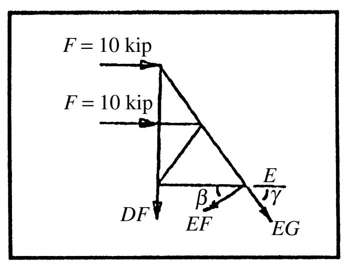
from which $H_x = -20$ kip. Make the cut through EG, EF, and DE. Consider the upper section only. Denote the axial force in a member joining I, K by IK. *The section as a free body:* The sum of the moments about E is $M_E = -10(4) - 10(8) + DF(L_{ED}) = 0$, from which $DF = \frac{120}{6} = 20$ kip.
The sum of forces:

$$\sum F_y = -EF\sin\beta - EG\sin\gamma - DF = 0,$$

$$\sum F_x = -EF\cos\beta + EG\cos\gamma + 20 = 0,$$

from which the two simultaneous equations: $0.5547EF + 0.8EG = -20$, and $0.8320EF - 0.6EG = 20$. Solve: $EF = 4.0$ kip (T)



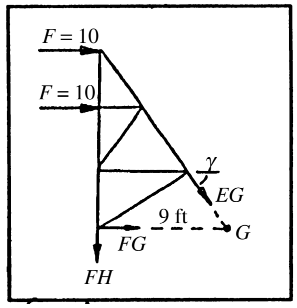


Problem 6.46 Consider the truss in Problem 6.45. Use the method of sections to determine the axial force in member FG.

Solution: From the solution of Problem 6.45, the apex A included angle is $\alpha = 36.87°$. The length of the member FG is $L_{FG} = 12\tan\alpha = 9$ ft. Make the cut through EG, GF, and FH, and consider the upper section. Denote the axial force in a member joining I, K by IK. *The section as a free body:* The cut in EG is made very near the point G; the moment about this cut by $M_G = -(8 + 12)F + L_{FG}FH = 0$ (where $F = 10$ kip from Problem 6.45), from which $FH = 22.22$ kip (T). The sum of the forces, from which $EG = -27.78$ kip(C).

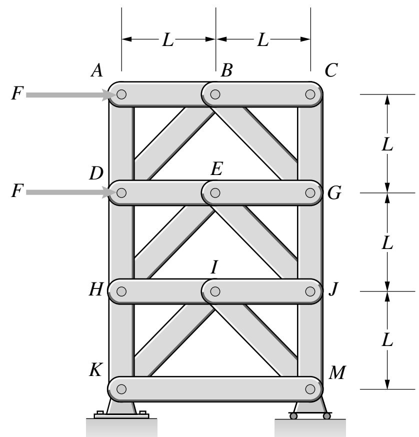
$$\sum F_x = FG + 2F + EG\cos\beta = 0,$$

from which $\boxed{FG = -3.33 \text{ kip } (C)}$.



Problem 6.47 The load $F = 20$ kN and the dimension $L = 2$ m. Use the method of sections to determine the axial force in member HK.

Strategy: Obtain a section by cutting members HK, HI, IJ, and JM. You can determine the axial forces in members HK and JM even though the resulting free-body diagram is statically indeterminate.



Solution: *The complete structure as a free body:* The sum of the moments about K is $M_K = -FL(2 + 3) + ML(2) = 0$, from which $M = \frac{5F}{2} = 50$ kN. The sum of forces:

$$\sum F_Y = K_Y + M = 0,$$

from which $K_Y = -M = -50$ kN.

$$\sum F_X = K_X + 2F = 0,$$

from which $K_X = -2F = -40$ kN.

The section as a free body: Denote the axial force in a member joining I, K by IK. The sum of the forces:

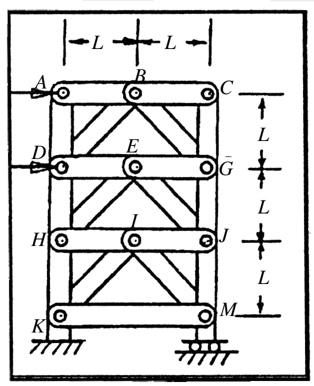
$$\sum F_x = K_x - HI + IJ = 0,$$

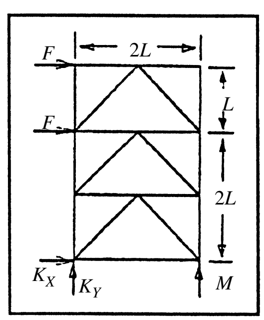
from which $HI - IJ = K_x$. Sum moments about K to get $M_K = M(L)(2) + JM(L)(2) - IJ(L) + HI(L) = 0$.

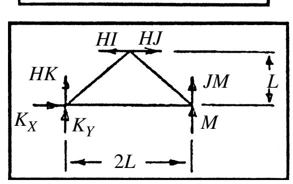
Substitute $HI - IJ = K_x$, to obtain $JM = -M - \frac{K_x}{2} = -30$ kN (C).

$$\sum F_y = K_y + M + JM + HK = 0,$$

from which $\boxed{HK = -JM = 30 \text{ kN}(T)}$



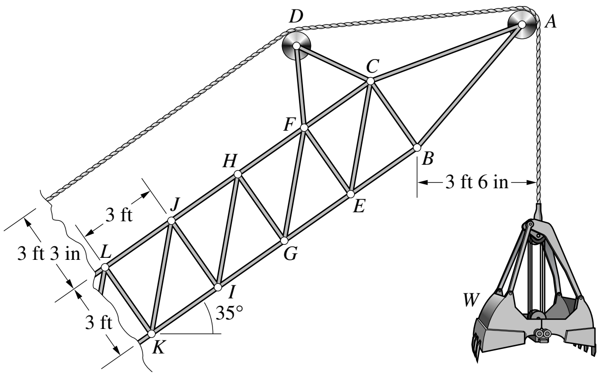




Problem 6.48 The weight of the bucket is $W = 1000$ lb. The cable passes over pulleys at A and D.

(a) Determine the axial forces in member FG and HI.

(b) By drawing free-body diagrams of sections, explain why the axial forces in members FG and HI are equal.



Solution: The truss is at angle $\alpha = 35°$ relative to the horizontal. The angles of the members FG and HI relative to the horizontal are $\beta = 45° + 35° = 80°$. (a) Make the cut through FH, FG, and EG, and consider the upper section. Denote the axial force in a member joining, α, β by $\alpha\beta$.

The section as a free body: The perpendicular distance from point F is $L_{FW} = 3\sqrt{2}\sin\beta + 3.5 = 7.678$ ft.

The sum of the moments about F is $M_F = -WL_{FW} + W(3.25) - |EG|(3) = 0$, from which $EG = -1476.1$ lb (C).

The sum of the forces:

$$\sum F_Y = -FG\sin\beta - FH\sin\alpha - EG\sin\alpha - W\sin\alpha - W = 0,$$

$$\sum F_X = -FG\cos\beta - FH\cos\alpha - EG\cos\alpha - W\cos\alpha = 0,$$

from which the two simultaneous equations: $-0.9848FG - 0.5736FH = 726.9$, and $-0.1736FG - 0.8192FH = -389.97$.

Solve: $\boxed{FG = -1158.5 \text{ lb } (C)}$, and $FH = 721.64$ lb (T). Make the cut through JH, HI, and GI, and consider the upper section.

The section as a free body: The perpendicular distance from point H to the line of action of the weight is $L_{HW} = 3\cos\alpha + 3\sqrt{2}\sin\beta + 3.5 = 10.135$ ft. The sum of the moments about H is $M_H = -W(L) - |GI|(3) + W(3.25) = 0$, from which $|GI| = -2295$ lb (C).

$$\sum F_Y = -HI\sin\beta - JH\sin\alpha - GI\sin\alpha - W\sin\alpha - W = 0,$$

$$\sum F_X = -HI\cos\beta - JH\cos\alpha - GI\cos\alpha - W\cos\alpha = 0,$$

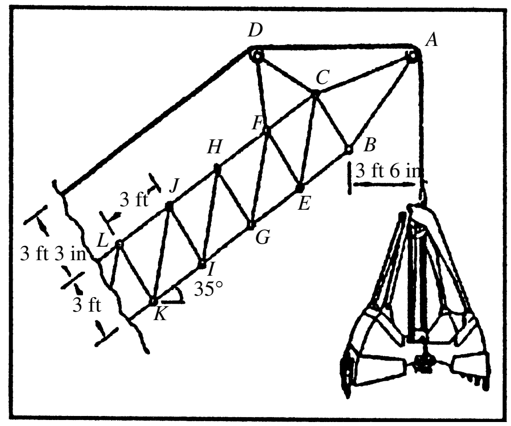
from which the two simultaneous equations:

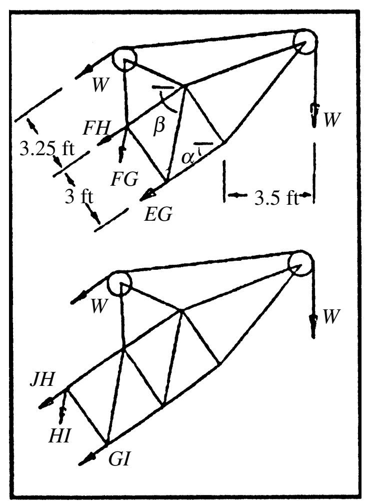
$$-0.9848HI - 0.5736JH = 257.22,$$

and $$-0.1736HI - 0.8192JH = -1060.8.$$

Solve: $\boxed{HI = -1158.5 \text{ lb}(C)}$,

and $\boxed{JH = 1540.6 \text{ lb}(T)}$.



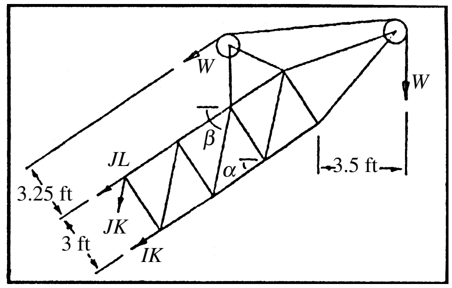


(b) Choose a coordinate system with the y-axis parallel to JH. Isolate a sectio by making cuts through FH, FG, and EG, and through HJ, HI, and G. The free section of the truss is shown. The sum of the forces in the x- a y-direction are each zero; since the only external x-components of axial forc are those contributed by FG and HI, the two axial forces must be equal:

$$\sum F_x = HI\cos 45° - FG\cos 45° = 0,$$

from which $HI = FG$

Problem 6.49 Consider the truss in Problem 6.48. The weight of the bucket is $W = 1000$ lb. The cable passes over pulleys at A and D. Determine the axial forces in members IK and JL.



Solution: Make a cut through JL, JK, and IK, and consider the upper section. Denote the axial force in a member joining, α, β by $\alpha\beta$. *The section as a free body:* The perpendicular distance from point J to the line of action of the weight is $L = 6\cos\alpha + 3\sqrt{2}\sin\beta + 3.5 = 12.593$ ft. The sum of the moments about J is $M_J = -W(L) + W(3.25) - IK(3) = 0$, from which $IK = -3114.4$ lb(C).
The sum of the forces:

$$\sum F_x = JL\cos\alpha - IK\cos\alpha - W\cos\alpha - JK\cos\beta = 0,$$

and $$\sum F_y = -JL\sin\alpha - IK\sin\alpha - W\sin\alpha - W - JK\sin\beta = 0,$$

from which two simultaneous equations:

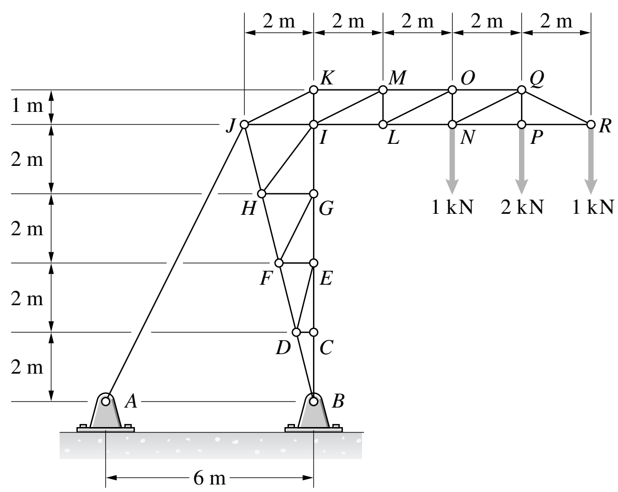
$$0.8192JL + 0.1736JK = -1732$$

and $$0.5736JL + 0.9848JK = 212.75.$$

Solve: $\boxed{JL = 2360 \text{ lb}(T)}$,

and $\boxed{JK = -1158.5 \text{ lb}(C)}$.

Problem 6.50 The truss supports loads at N, P, and R. Determine the axial forces in members IL and KM.



Solution: The strategy is to make a cut through KM, IM, and IL, and consider only the outer section. Denote the axial force in a member joining, α, β by $\alpha\beta$.
The section as a free body: The moment about M is

$$M_M = -IL - 2(1) - 4(2) - 6(1) = 0,$$

from which $\boxed{IL = -16 \text{ kN } (C)}$.

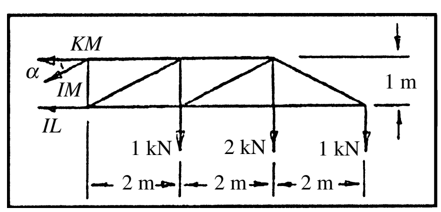
The angle of member IM is $\alpha = \tan^{-1}(0.5) = 26.57°$.
The sums of the forces:

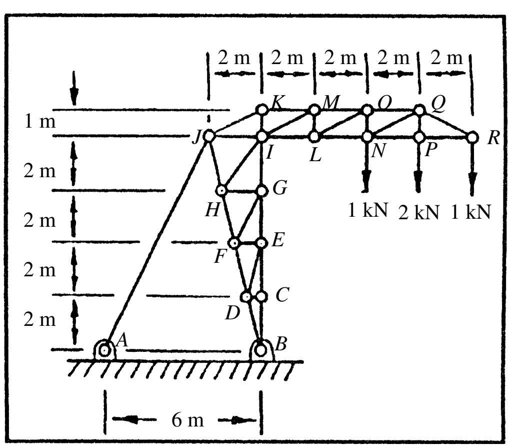
$$\sum F_y = -IM\sin\alpha - 4 = 0,$$

from which $IM = -\frac{4}{\sin\alpha} = -8.944$ kN (C).

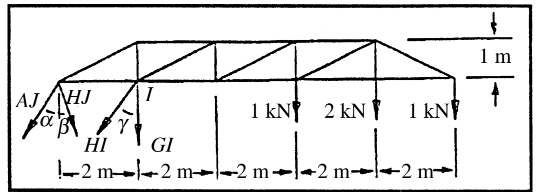
$$\sum F_x = -KM - IM\cos\alpha - IL = 0,$$

from which $\boxed{KM = 24 \text{ kN}(T)}$





Problem 6.51 Consider the truss in Problem 6.50. Determine the axial forces in members HJ and GI.



Solution: The strategy is to make a cut through the four members AJ, HJ, HI, and GI, and consider the upper section. The axial force in AJ can be found by taking the moment of the structure about B. *The complete structure as a free body:* The angle formed by AJ with the vertical is $\alpha = \tan^{-1}\left(\frac{4}{8}\right) = 26.57°$. The moment about B is $M_B = 6AJ\cos\alpha - 24 = 0$, from which $AJ = 4.47$ kN (T). *The section as a free body:* The angles of members HJ and HI relative to the vertical are $\beta = \tan^{-1}\left(\frac{2}{8}\right) = 14.0°$, and $\gamma = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$ respectively. Make a cut through the four members AJ, HJ, HI, and GI, and consider the upper section. The moment about the point I is $M_I = -24 + 2AJ\cos\alpha + 2HJ\cos\beta = 0$. From which $\boxed{HJ = 8.25 \text{ kN } (T)}$. The sums of the forces:

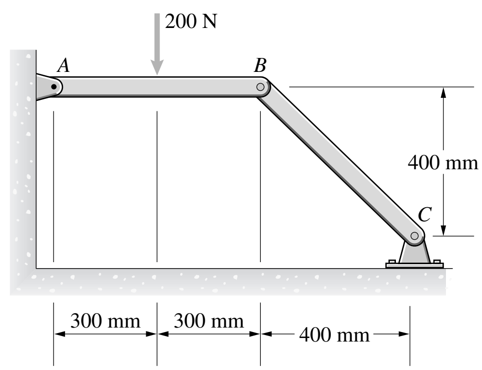
$$\sum F_x = -AJ\sin\alpha + HJ\sin\beta - HI\sin\gamma = 0,$$

from which $HI = \frac{AJ\sin\alpha - HJ\sin\beta}{\sin\gamma} = \frac{2-2}{\sin\gamma} = 0.$

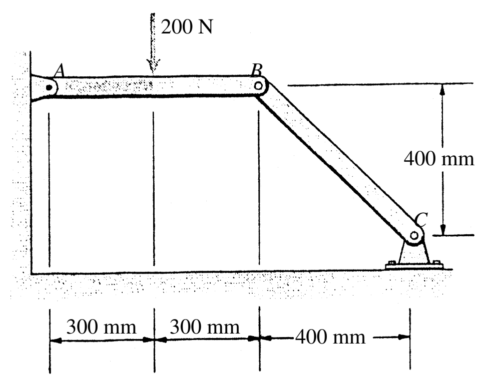
$$\sum F_Y = -AJ\cos\alpha - HJ\cos\beta - HI\cos\gamma - GI - 4 = 0,$$

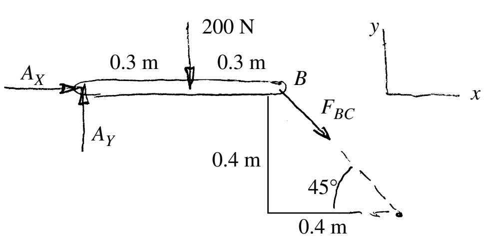
from which $\boxed{GI = -16 \text{ kN } (C)}$

Problem 6.52 Determine the reactions on member AB at A. (Notice that BC is a two-force member.)



Solution: Since BC is a two force member, the force in BC must be a long the line between B and C.





$$\sum F_x: \quad A_x + F_{BC}\cos 45° = 0$$

$$\sum F_y: \quad A_y - F_{BC}\sin 45° - 200 = 0$$

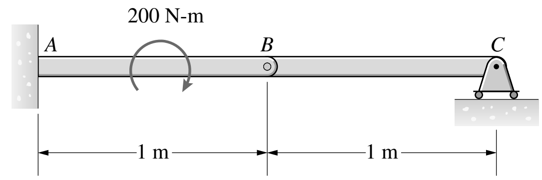
$$\sum M_A: \quad -(0.3)(200) - (0.6)CF_{BC}\sin 45° = 0$$

Solving: $A_x = 100$ N,

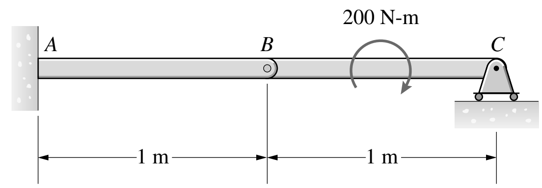
$A_y = 100$ N

$F_{BC} = 141.2$ N (compression)

Problem 6.53 (a) Determine the forces and couples on member AB for cases (1) and (2).
(b) You know that the moment of a couple is the same about any point. Explain why the answers are not the same in cases (1) and (2).



(1)



(2)

Solution: Case (a) *Element BC:* The moment about B is $M_B = (1)C_y = 0$, hence $\boxed{C_y = 0}$. The sum of the forces:

$$\sum F_y = B_y + C_y = 0,$$

from which $\boxed{B_y = 0}$.

$$\sum F_x = B_x = 0.$$

Element AB: The sum of the moments about A: $M = M_A - 200 = 0$, from which $\boxed{M_A = 200 \text{ N-m}}$. The sum of forces:

$$\sum F_y = B_y + A_y = 0,$$

from which $\boxed{A_y = 0}$,

$$\sum F_x = A_x = 0.$$

Case (b) *Element BC:* The sum of the moments about B:

$$\sum M_B = (1)C_y - 200 = 0,$$

from which $C_y = 200$ N. The sum of the forces:

$$\sum F_y = B_y^{BC} + C_y = 0,$$

from which $B_y^{BC} = -200$ N.

$$\sum F_x = B_x^{BC} = 0.$$

Element AB: The moments about A:

$$\sum M = M_A + (1)B_y^{AB} = 0,$$

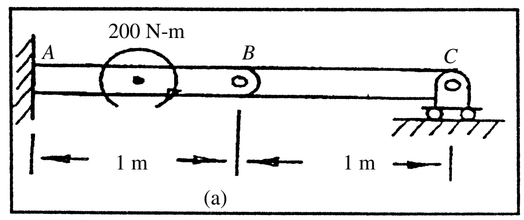
from which, since the reactions across the joint are equal and opposite: $\boxed{B_y^{AB} = -B_y^{BC} = 200 \text{ N}}$, $\boxed{M_A = -200 \text{ N-m}}$. The sum of the forces:

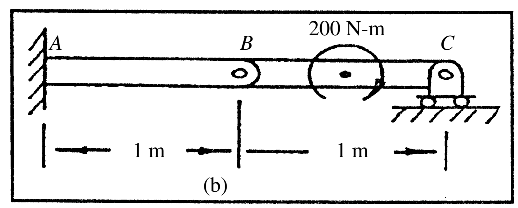
$$\sum F_y = A_y + B_y^{AB} = 0,$$

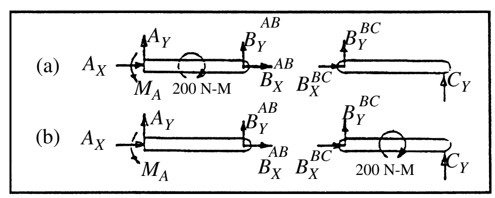
from which $\boxed{A_y = -200 \text{ N}}$.

$$\sum F_x = A_x + B_x^{AB} = 0,$$

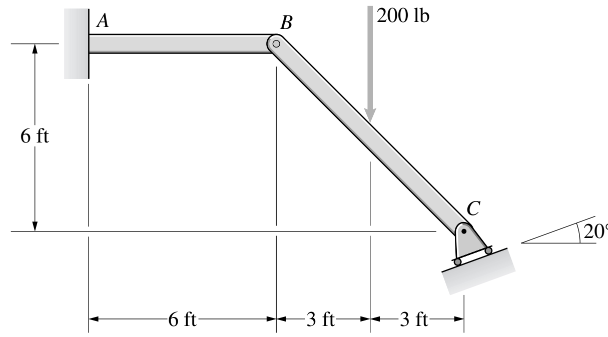
from which $A_x = 0$.
Explanation of difference: The forces are equal and opposite across the joint B, so it matters on which side of B the couple is applied.



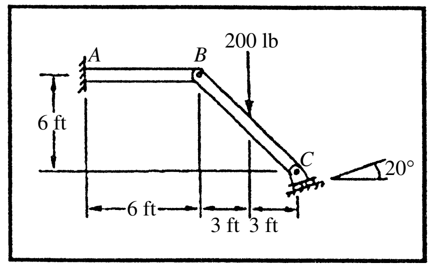




Problem 6.54 For the frame shown, determine the reactions at the built-in support A and the force exerted on member AB at B.



Solution: *Element AB:* The equilibrium equations are:



$$\sum F_X = A_X + B_X = 0,$$

$$\sum F_Y = A_Y + B_Y = 0,$$

and $\sum M_A = N_A + (6)B_Y = 0.$

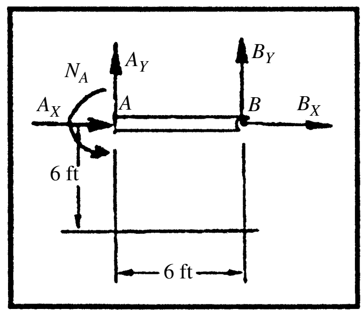
Element BC: The equilibrium conditions are

$$\sum F_X = -B_X - C\sin(20^\circ) = 0,$$

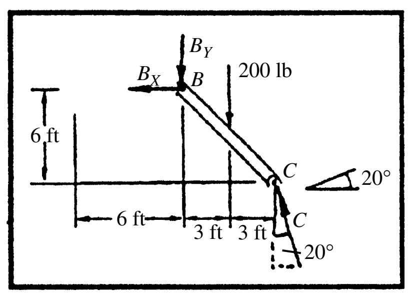
$$\sum F_Y = -B_Y - 200 + C\cos(20^\circ) = 0,$$

and, summing moments around B,

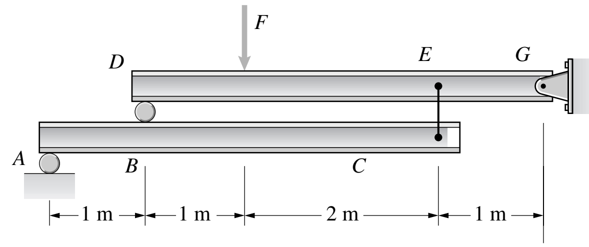
$$\sum M_B = -(3)200 - (6)C\sin(20^\circ) + (6)C\cos(20^\circ) = 0.$$



We have six equations in six unknowns. Solving simultaneously yields $A_X = 57.2$ lb, $A_Y = 42.8$ lb, $B_X = -57.2$ lb, $B_Y = -42.8$ lb, $C = 167.3$ lb, and $N_A = 256.6$ ft-lb.



Problem 6.55 The force $F = 10$ kN. Determine the forces on member ABC, presenting your answers as shown in Fig. 6.35.



Solution: *The complete structure as a free body:* The sum of the moments about G:

$$\sum M_G = +3F - 5A = 0,$$

from which $A = \frac{3F}{5} = 6$ kN which is the reaction of the floor. The sum of the forces:

$$\sum F_y = G_y - F + A = 0,$$

from which $G_y = F - A = 10 - 6 = 4$ kN.

$$\sum F_x = G_x = 0.$$

Element DEG: The sum of the moments about D

$$\sum M = -F + 3E + 4G_y = 0,$$

from which $E = \frac{F-4G_y}{3} = \frac{10-16}{3} = -2$ kN.
The sum of the forces:

$$\sum F_y = G_y - F + E + D = 0,$$

from which $D = F - E - G_y = 10 + 2 - 4 = 8$ kN.
Element ABC: Noting that the reactions are equal and opposite:

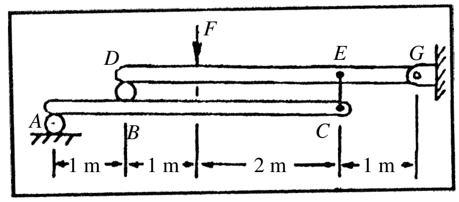
$$\boxed{B = -D = -8 \text{ kN}},$$

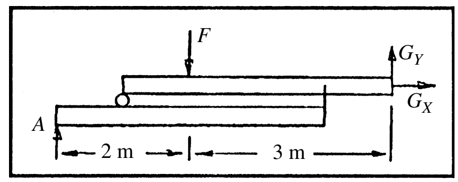
and $\boxed{C = -E = 2 \text{ kN}}$.

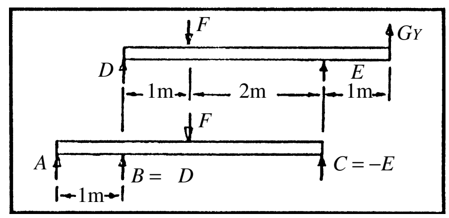
The sum of the forces:

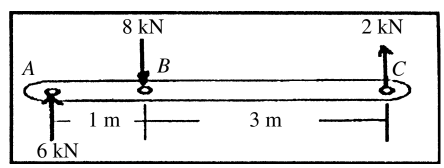
$$\sum F_y = A + B + C = 0,$$

from which $A = 8 - 2 = 6$ kN. *Check*





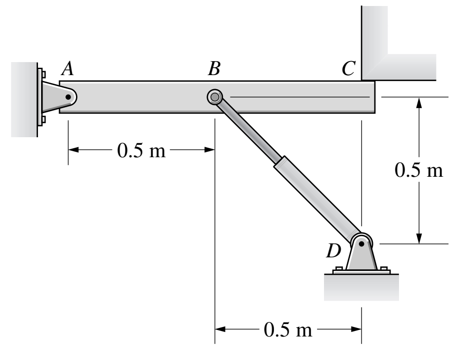




Problem 6.56 Consider the frame in Problem 6.55. The cable CE will safely support a tension of 10 kN. Based on this criterion, what is the largest downward force F that can be applied to the frame?

Solution: From the solution to Problem 6.55: $E = \frac{F-4G_y}{3}$, $G_y = F - A$, and $A = \frac{3}{5}F$. Back substituting, $E = -\frac{F}{5}$ or $F = -5E$, from which, for $E = 10$ kN, $F = -50$ kN

Problem 6.57 The hydraulic actuator BD exerts a 6-kN force on member ABC. The force is parallel to BD, and the actuator is in compression. Determine the forces on member ABC, presenting your answers as shown in Fig. 6.35.



Solution: The surface at C is smooth.

Element ABC: The sum of the moments about A is

$$\sum M = (0.5)B\sin 45^\circ + (1)C = 0,$$

from which $\boxed{C = -3(0.707) = -2.121 \text{ kN}}$.

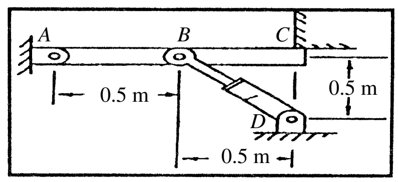
The sum of the forces:

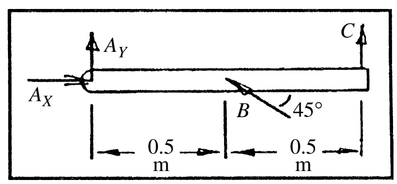
$$\sum F_y = A_y + B\sin 45^\circ + C = 0,$$

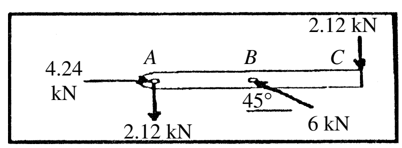
from which $\boxed{A_y = -B\,(0.707) - C = -2.121 \text{ kN}}$.

$$\sum F_x = A_x - B\cos 45^\circ = 0,$$

from which $\boxed{A_x = 4.24 \text{ kN}}$







Problem 6.58 The simple hydraulic jack shown in Problem 6.57 is designed to exert a vertical force at point C. The hydraulic actuator BD exerts a force on the beam ABC that is parallel to BD. The largest lifting force the jack can exert is limited by the pin support A, which will safely support a force of magnitude 20 kN. What is the largest lifting force the jack can exert at C, and what is the resulting axial force in the hydraulic actuator?

Solution: From the solution to Problem 6.??

$$A_y = -\frac{B}{\sqrt{2}} - C, \quad A_x = \frac{B}{\sqrt{2}},$$

and $C = -\dfrac{B}{2\sqrt{2}}$.

Substituting, $A_y = -\frac{B}{2\sqrt{2}}$,

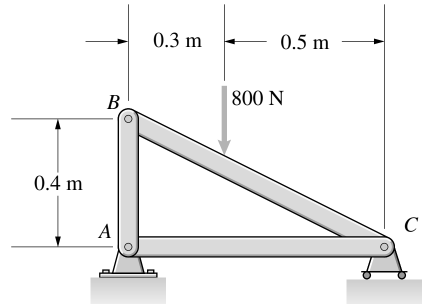
and $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2} = \dfrac{B}{\sqrt{2}}\sqrt{1^2 + \left(\dfrac{1}{2}\right)^2} = \dfrac{\sqrt{5}B}{2\sqrt{2}}$.

For $|\mathbf{A}| = 20$ kN,

$\boxed{B = \frac{2\sqrt{2}(20)}{\sqrt{5}} = 25.3 \text{ kN}}$ is the largest axial force,

and $\boxed{C = -\frac{B}{2\sqrt{2}} = -8.944 \text{ kN}}$ is the largest lifting force.

Problem 6.59 Determine the forces on member BC and the axial force in member AC.



Solution: *Element BC:* The sum of the moments about B:

$$\sum M = -(0.3)800 + (0.8)C = 0,$$

from which $\boxed{C = 300 \text{ N}}$. The sum of the forces

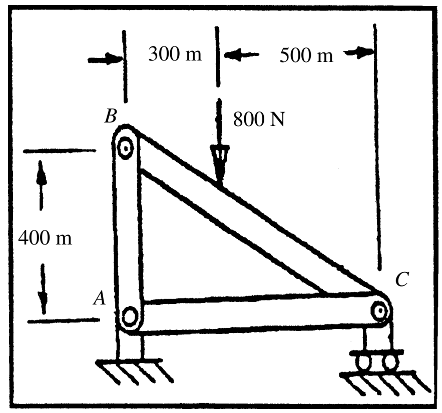
$$\sum F_y = B - 800 + C = 0,$$

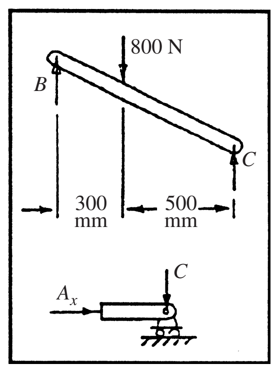
from which $\boxed{B = 500 \text{ N}}$.

$$\sum F_x = C_x = 0.$$

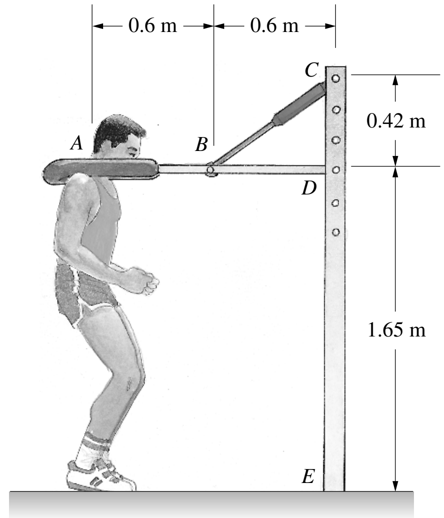
(The roller support prevents an x-direction reaction in C.) *Element AC:* The sum of the forces

$$\boxed{\sum F_x = A_x = 0}$$

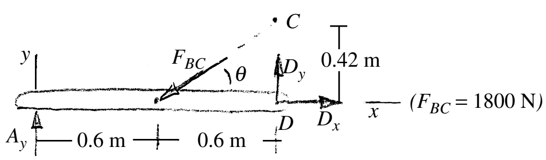


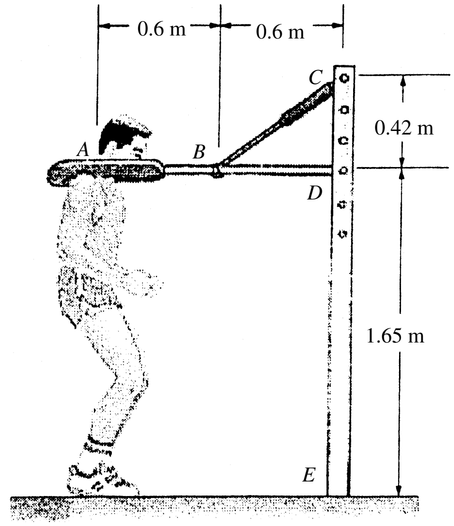


Problem 6.60 An athlete works out with a squat thrust machine. To rotate the bar ABD, he must exert a vertical force at A that causes the magnitude of the axial force in the two-force member BC to be 1800 N. When the bar ABD is on the verge of rotating, what are the reactions on the vertical bar CDE at D and E?



Solution: Member BC is a two force member. The force in BC is along the line from B to C.





$(F_{BC} = 1800 \text{ N})$

$$\tan\theta = \frac{0.42}{0.6} \quad \theta = 34.99^\circ.$$

$$\sum F_x: \quad D_x - F_{BC}\cos\theta = 0$$

$$\sum F_y: \quad A_y - F_{BC}\sin\theta + D_y = 0$$

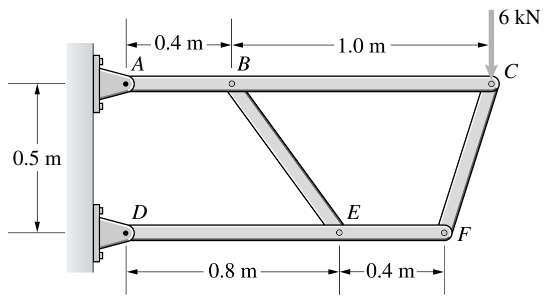
$$\circlearrowleft + \sum M_D: \quad -1.2A_y + 0.6F_{BC}\sin\theta = 0$$

Solving, we get $D_x = 1475$ N

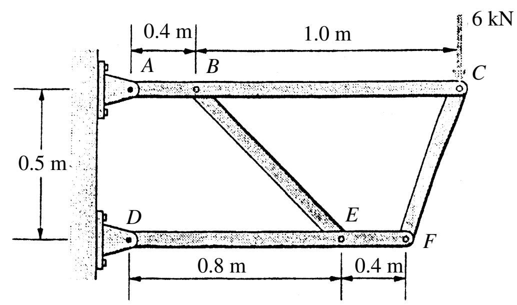
$D_y = 516$ N

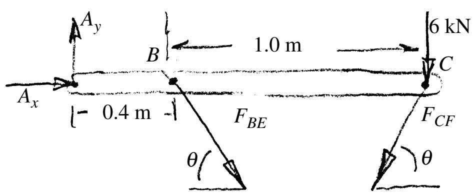
$A_y = 516$ N

Problem 6.61 The frame supports a 6-kN load at C. Determine the reactions on the frame at A and D.



Solution: Note that members BE and CF are two force members. Consider the 6 kN load as being applied to member ABC.

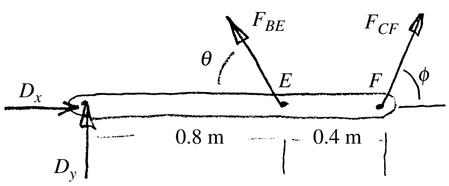




$$\tan\theta = \frac{0.5}{0.4} \quad \theta = 51.34^\circ$$

$$\tan\phi = \frac{0.5}{0.2} \quad \phi = 68.20^\circ$$

Member DEF



Equations of equilibrium:
Member ABC:

$$\sum F_x: \quad A_x + F_{BE}\cos\theta - F_{CF}\cos\phi = 0$$

$$\sum F_y: \quad A_y - F_{BE}\sin\theta - F_{CF}\sin\phi - 6 = 0$$

$$\circlearrowleft + \sum M_A: \quad -(0.4)F_{BE}\sin\theta - (1.4)F_{CF}\sin\phi - 1.4(6) = 0$$

Member DEF:

$$\sum F_x: \quad D_x - F_{BE}\cos\theta + F_{CF}\cos\phi = 0$$

$$\sum F_y: \quad D_y + F_{BE}\sin\theta + F_{CF}\sin\phi = 0$$

$$\circlearrowleft + \sum M_D: \quad (0.8)(F_{BE}\sin\theta) + 1.2F_{CF}\sin\phi = 0$$

Unknowns $A_x,\ A_y,\ D_x,\ D_y,\ F_{BE},\ F_{CF}$ we have 6 eqns in 6 unknowns.

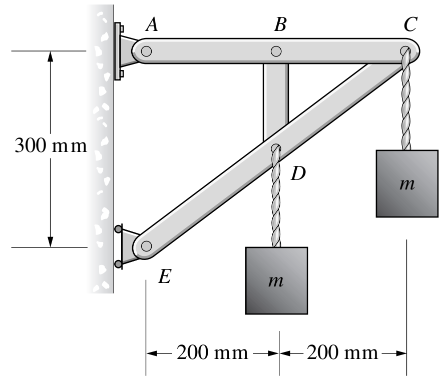
Solving, we get

$A_x = -16.8$ kN
$A_y = 11.25$ kN
$D_x = 16.3$ kN
$D_y = -5.25$ kN

Also, $F_{BE} = 20.2$ kN (T)

$F_{CF} = -11.3$ kN (C)

Problem 6.62 The mass $m = 120$ kg. Determine the forces on member ABC, presenting your answers as shown in Fig. 6.35.



Solution: The equations of equilibrium for the entire frame are

$$\sum F_X = A_X + E_X = 0,$$

$$\sum F_Y = A_Y - 2mg = 0,$$

and summing moments at A,

$$\sum M_A = (0.3)E_X - (0.2)mg - (0.4)mg = 0.$$

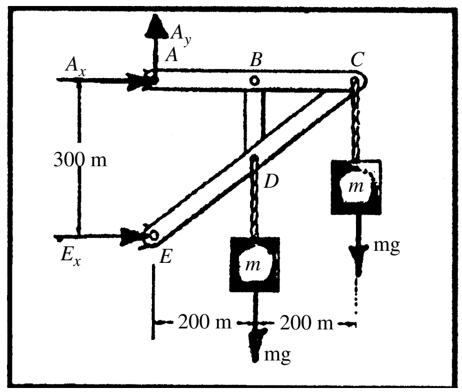
Solving yields $A_X = -2354$ N, $A_Y = 2354$ N, and $E_X = 2354$ N.
Member ABC: The equilibrium equations are

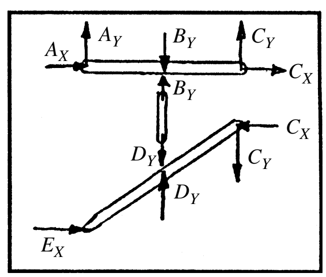
$$\sum F_X = A_X + C_X = 0,$$

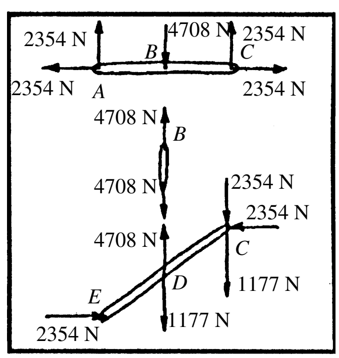
$$\sum F_Y = A_Y - B_Y + C_Y = 0,$$

and $\sum M_A = -(0.2)B_Y + (0.4)C_Y = 0.$

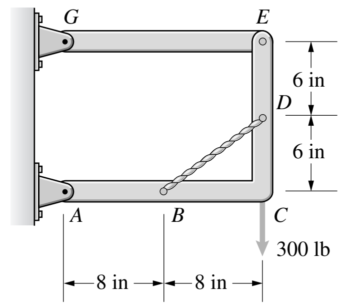
We have three equations in the three unknowns B_Y, C_X, and C_Y. Solving, we get $B_Y = 4708$ N, $C_X = 2354$ N, and $C_Y = 2354$ N. This gives all of the forces on member ABC. A similar analysis can be made for each of the other members in the frame. The results of solving for all of the forces in the frame is shown in the figure.



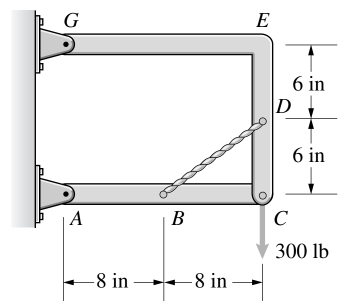




Problem 6.63 The tension in cable BD is 500 lb. Determine the reactions at A for cases (1) and (2).



(1)



(2)

Solution: Case (a) *The complete structure as a free body:* The sum of the moments about G:

$$\sum M_G = -16(300) + 12A_x = 0,$$

from which $\boxed{A_x = 400 \text{ lb}}$. The sum of the forces:

$$\sum F_x = A_x + G_x = 0,$$

from which $G_x = -400$ lb.

$$\sum F_y = A_y - 300 + G_y = 0,$$

from which $\boxed{A_y = 300 - G_y}$. *Element GE*: The sum of the moments about E:

$$\sum M_E = -16G_y = 0,$$

from which $G_y = 0$, and from above $A_y = 300$ lb.
Case (b) *The complete structure as a free body:* The free body diagram, except for the position of the internal pin, is the same as for case (a). The sum of the moments about G is

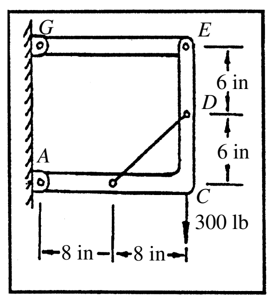
$$\sum M_C = -16(300) + 12A_x = 0,$$

from which $\boxed{A_x = 400 \text{ lb}}$.
Element ABC: The tension at the lower end of the cable is up and to the right, so that the moment exerted by the cable tension about point C is negative. The sum of the moments about C:

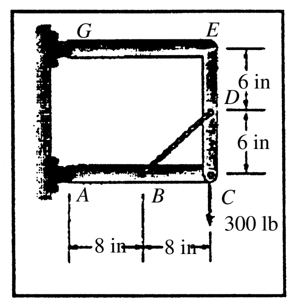
$$\sum M_C = -8B \sin\alpha - 16A_y = 0,$$

noting that $B = 500$ lb and $\alpha = \tan^{-1}\left(\frac{6}{8}\right) = 36.87°$,

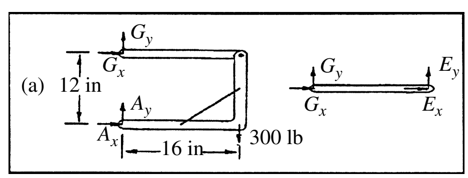
then $\boxed{A_y = -150 \, lb.}$

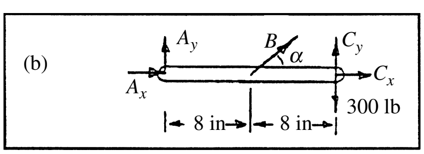


(a)

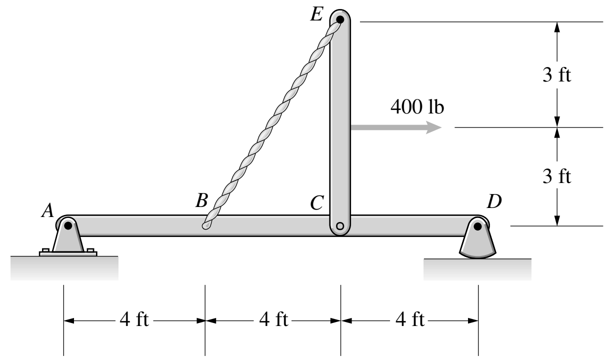


(b)





Problem 6.64 Determine the forces on member $ABCD$, presenting your answers as shown in Fig. 6.35.



Solution: *The complete structure as a free body:* The sum of the moments about A:

$$\sum M_A = -400(3) + 12D_y = 0,$$

from which $\boxed{D_y = 100 \text{ lb}}$. The sum of the forces:

$$\sum F_x = A_x + 400 = 0,$$

from which $\boxed{A_x = -400 \text{ lb}}$.

$$\sum F_y = A_y + D_y = 0,$$

from which $\boxed{A_y = -100 \text{ lb}}$.

Element EC: The sum of the moments about C:

$$\sum M_C = 6E \cos\alpha - 3(400) = 0.$$

The angle of cable element EB is

$$\alpha = \tan^{-1}\left(\frac{6}{4}\right) = 56.31°,$$

from which the cable tension is $\boxed{E = 360.6 \text{ lb}}$. The sum of the forces:

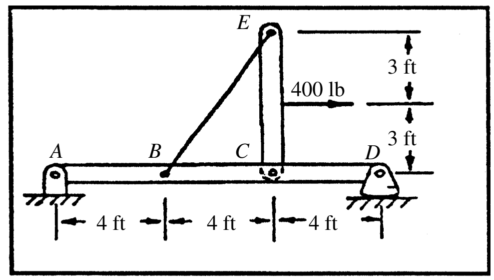
$$\sum F_x = -C_x + 400 - E\cos\alpha = 0,$$

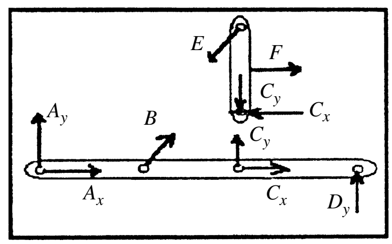
from which $C_x = 200$ lb.

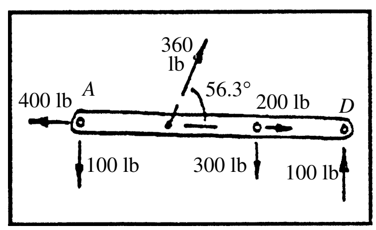
$$\sum F_y = -C_y - E\sin\alpha = 0,$$

from which $C_y = -300$ lb.

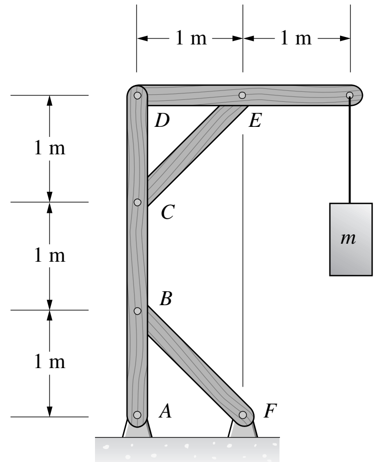
Element ABCD: The tension in the cable acts on element $ABCD$ with equal and opposite tension to the reaction on element EB, up and to the right at an angle of 56.31°, $\boxed{C_x = 200 \text{ lb}}$ to the right, and $\boxed{C_y = -300 \text{ lb}}$ downward.







Problem 6.65 The mass $m = 50$ kg. Determine the forces on member $ABCD$, presenting your answers as shown in Fig. 6.35.



Solution: The weight of the mass hanging is $W = mg = 50(9.81) = 490.5$ N *The complete structure as a free body:* The sum of the moments about A:

$$\sum M_A = -2W + F_y = 0,$$

from which $F_y = 981$ N. The sum of the forces:

$$\sum F_y = A_y + F_y - W = 0,$$

from which $A_y = -490.5$ N,

$$\sum F_x = A_x + F_x = 0,$$

from which $A_x = -F_x$. *Element BF:* The sum of the moments about F:

$$\sum M_F = -B_x - B_y = 0,$$

from which $B_y = -B_x$. The sum of the forces:

$$\sum F_y = B_y + F_y = 0,$$

from which $B_y = -981$ N, and $B_x = 981$ N.

$$\sum F_x = B_x + F_x = 0,$$

from which $F_x = -981$ N, and from above, $\boxed{A_x = 981 \text{ N}}$, *Element DE:* The sum of the moments about D:

$$\sum M_D = -E_y - 2W = 0,$$

from which $E_y = -981$ N. The sum of the forces:

$$\sum F_y = -D_y - E_y - W = 0,$$

from which $\boxed{D_y = 490.5 \text{ N}}$.

$$\sum F_x = -D_x - E_x = 0,$$

from which $D_x = -E_x$. *Element CE*: The sum of the moments about C:

$$\sum M_C = E_y - E_x = 0,$$

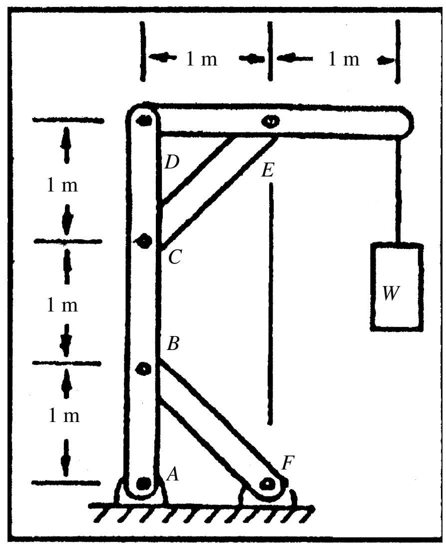
from which $E_x = -981$ N, and from above $\boxed{D_x = 981 \text{ N}}$.

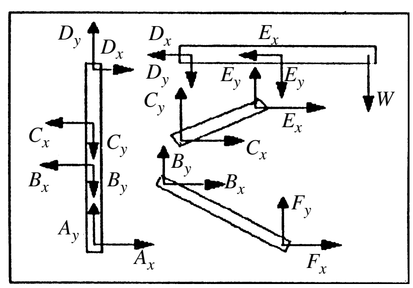
$$\sum F_y = E_y + C_y = 0,$$

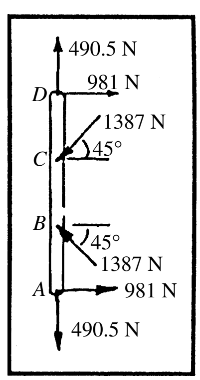
from which $C_y = 981$ N.

$$\sum F_x = E_x + C_x = 0,$$

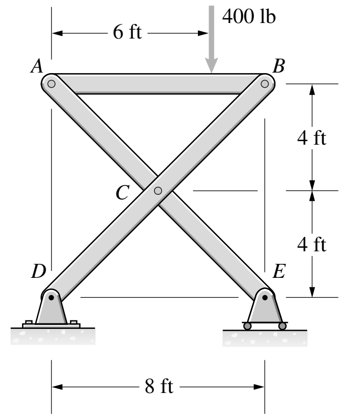
from which $C_x = 981$ N, and *Element ABCD:* All reactions on $ABCD$ have been determined above. The components at B and C have the magnitudes $\boxed{B = C = \sqrt{981^2 + 981^2} = 1387 \text{ N}}$, at angles of 45°.







Problem 6.66 Determine the forces on member BCD.



Solution: The following is based on free body diagrams of the elements: *The complete structure as a free body:* The sum of the moments about D:

$$\sum M_D = -(6)400 + 8E_y = 0,$$

from which $E_y = 300$ lb. The sum of the forces:

$$\sum F_x = D_x = 0.$$

$$\sum F_y = E_y + D_y - 400 = 0,$$

from which $D_y = 100$ lb. *Element AB:* The sum of the moments about A:

$$\sum M_A = -8B_y - (6)400 = 0,$$

from which $B_y = -300$ lb. The sum of forces:

$$\sum F_y = -B_y - A_y - 400 = 0,$$

from which $A_y = -100$ lb.

$$\sum F_x = -A_x - B_x = 0,$$

from which (1) $A_x + B_x = 0$ *Element ACE:* The sum of the moments about E:

$$\sum M_E = -8A_x + 4C_x - 8A_y + 4C_y = 0,$$

from which (2) $-2A_x + C_x - 2A_y + C_y = 0$. The sum of the forces:

$$\sum F_y = A_y + E_y - C_y = 0,$$

from which $\boxed{C_y = 200 \text{ lb}}$.

$$\sum F_x = A_x - C_x = 0,$$

from which (3) $A_x = C_x$. The three numbered equations are solved: $A_x = -400$ lb, $\boxed{C_x = 400 \text{ lb}}$, and $\boxed{B_x = -400 \text{ lb}}$.

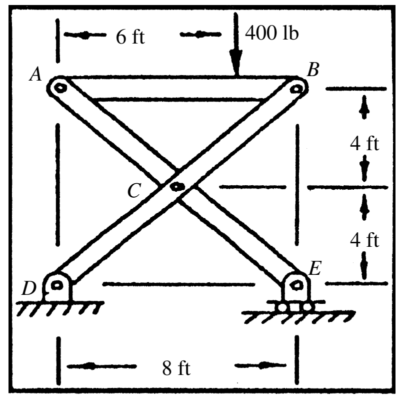
Element BCD:

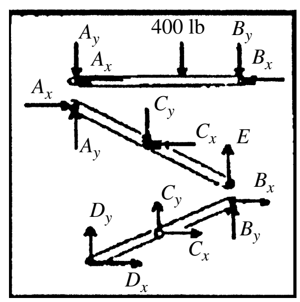
The reactions are now known:

$$\boxed{B_y = -300 \text{ lb}}, \boxed{B_x = -400 \text{ lb}}, \boxed{C_y = 200 \text{ lb}},$$

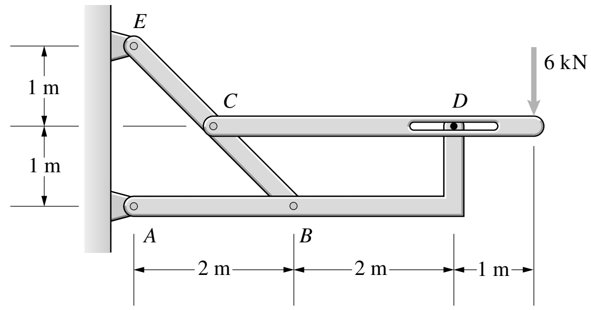
$$\boxed{D_x = 0}, \boxed{D_y = 100 \text{ lb}},$$

where negative sign means that the force is reversed from the direction shown on the free body diagram.





Problem 6.67 Determine the forces on member ABC.



Solution: *The frame as a whole:* The equations of equilibrium are

$$\sum F_X = A_X + E_X = 0,$$

$$\sum F_Y = A_Y + E_Y - 6000 \text{ N} = 0,$$

and, with moments about E,

$$\sum M_E = 2A_X - (5)6000 = 0.$$

Solving for the support reactions, we get $A_X = 15{,}000$ N and $E_X = -15{,}000$ N. We cannot yet solve for the forces in the y direction at A and E.

Member ABC: The equations of equilibrium are

$$\sum F_X = A_X - B_X = 0,$$

$$\sum F_Y = A_Y - B_Y - C_Y = 0,$$

and summing moments about A,

$$\sum M_A = -2B_Y - 4C_Y = 0.$$

Member BDE: The equations of equilibrium are

$$\sum F_X = E_X + D_X + B_X = 0,$$

$$\sum F_Y = E_Y + D_Y + B_Y = 0,$$

and, summing moments about E,

$$\sum M_E = (1)D_Y + (1)D_X + (2)B_Y + (2)B_X = 0.$$

Member CD: The equations of equilibrium are

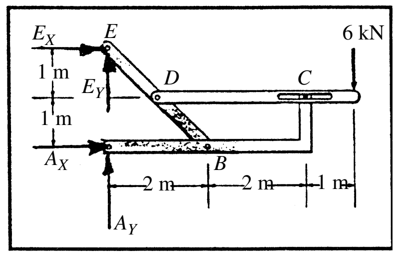
$$\sum F_X = -D_X = 0,$$

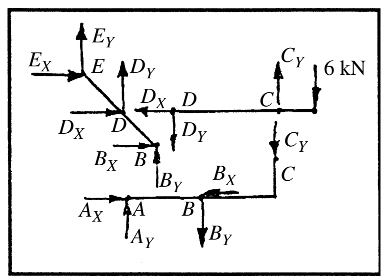
$$\sum F_Y = -D_Y + C_Y - 6000 = 0,$$

and summing moments about D,

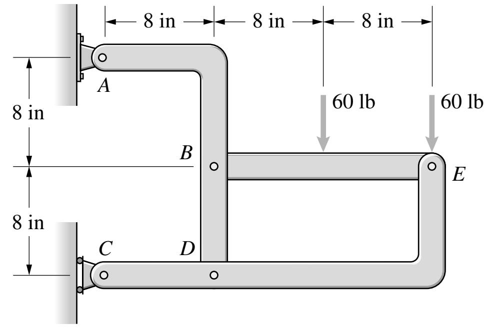
$$\sum M_D = -(4)6000 + 3C_Y = 0.$$

Solving these equations simultaneously gives values for all of the forces in the frame. The values are $A_X = 15{,}000$ N, $A_Y = -8{,}000$ N, $B_X = 15{,}000$ N, $B_Y = -16{,}000$ N, $C_Y = 8{,}000$ N, $D_X = 0$, and $D_Y = 2{,}000$ N.





Problem 6.68 Determine the forces on member ABD.



Solution: The equations of equilibrium for the truss as a whole are

$$\sum F_X = A_X + C_X = 0,$$

$$\sum F_Y = A_Y - 60 - 60 = 0,$$

and $\sum M_A = 16C_X - 16(60) - 24(60) = 0.$

Solving these three equations yields

$A_X = -150$ lb,

$A_Y = 120$ lb,

and $C_X = 150$ lb.

Member ABD: The equilibrium equations for this member are:

$$\sum F_X = A_X - B_X - D_X = 0,$$

$$\sum F_Y = A_Y - B_Y - D_Y = 0,$$

and $\sum M_A = -8B_Y - 8D_Y - 8B_X - 16D_X = 0.$

Member BE: The equilibrium equations for this member are:

$$\sum F_X = B_X + E_X = 0,$$

$$\sum F_Y = B_Y + E_Y - 60 - 60 = 0,$$

and $\sum M_B = -8(60) - 16(60) + 16E_Y = 0.$

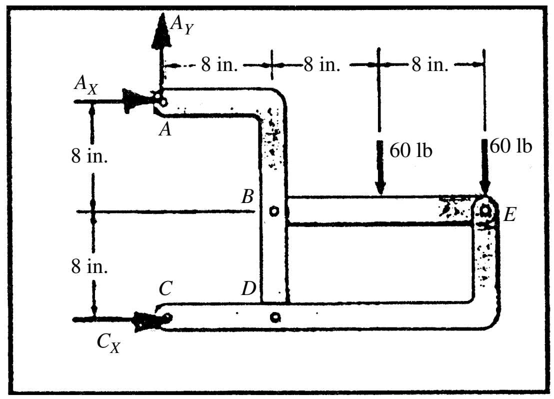
Member CDE: The equilibrium equations for this member are:

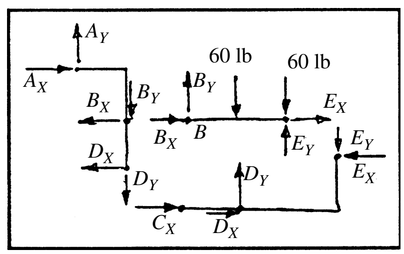
$$\sum F_X = C_X + D_X - E_X = 0,$$

$$\sum F_Y = D_Y - E_Y = 0,$$

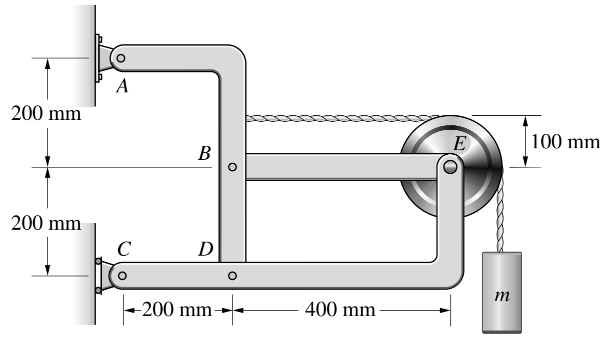
and $\sum M_D = 8E_X - 16E_Y = 0.$

Solving these equations, we get $B_X = -180$ lb, $B_Y = 30$ lb, $D_X = 30$ lb, $D_Y = 90$ lb, $E_X = 180$ lb, and $E_Y = 90$ lb. Note that we have 12 equations in 9 unknowns. The extra equations provide a check.





Problem 6.69 The mass $m = 12$ kg. Determine the forces on member CDE.



Solution: The equations of equilibrium for the entire truss are:

$$\sum F_X = A_X + C_X = 0,$$

$$\sum F_Y = A_Y - mg = 0,$$

and $\sum M_A = 0.4C_X - 0.7mg = 0.$

From these equations we get

$$A_X = -206.0 \text{ N}, \; A_Y = 117.7 \text{ N},$$

and $Cx = 206.0$ N.

Member ABD: The equations are

$$\sum F_X = A_X + B_X + D_X + T = 0,$$

$$\sum F_Y = A_Y + B_Y + D_Y = 0,$$

and $\sum M_A = 0.2B_Y + 0.2B_X + 0.2D_Y + 0.4D_X + 0.1T = 0.$

Member CDE: The equations are

$$\sum F_X = C_X - D_X + E_X = 0,$$

$$\sum F_Y = -D_Y + E_Y = 0,$$

and $\sum M_D = 0.4E_Y - 0.2E_X = 0.$

Member BE: The equations are

$$\sum F_X = -B_X + P_X - E_X = 0,$$

$$\sum F_Y = -B_Y + P_Y - E_Y = 0,$$

and $\sum M_E = 0.4B_Y = 0.$

The Pulley: The equations are

$$\sum F_X = -T - P_X = 0,$$

and $\sum F_Y = -T - P_Y = 0.$

The Weight: The equation is

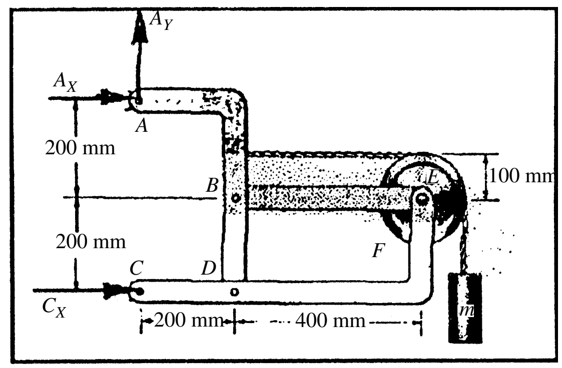
$$\sum F_Y = T - mg = 0.$$

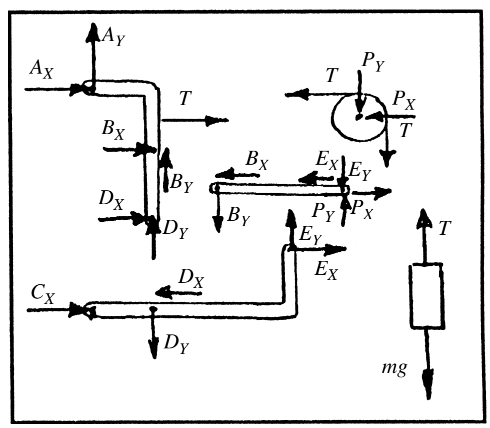
Solving the equations simultaneously, we get

$B_X = 117.7$ N, $B_Y = 0, D_X = -29.4$ N, $D_Y = -117.7$ N,

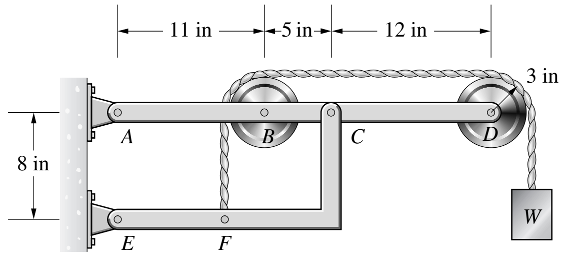
$E_X = -235.4$ N, $E_Y = -117.7$ N, $T = 117.7$ N,

$P_X = -117.7$ N, $P_Y = -117.7$ N

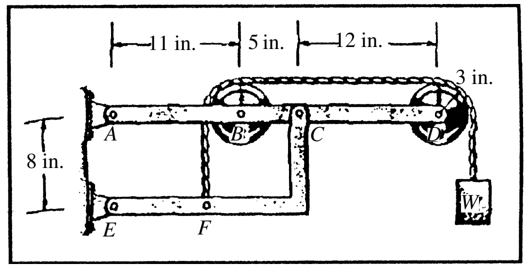




Problem 6.70 The weight $W = 80$ lb. Determine the forces on member $ABCD$.



Solution: *The complete structure as a free body:* The sum of the moments about A:



$$\sum M_A = -31W + 8E_x = 0,$$

from which $E_x = 310$ lb. The sum of the forces:

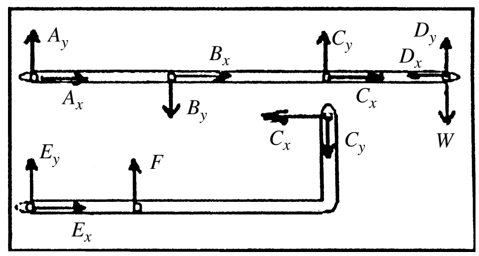
$$\sum F_x = E_x + A_x = 0,$$

from which $\boxed{A_x = -310 \text{ lb}}$.

$$\sum F_y = E_y + A_y - W = 0,$$

from which (1) $E_y + A_y = W$.

Element CFE: The sum of the forces parallel to x:



$$\sum F_x = E_x - C_x = 0,$$

from which $\boxed{C_x = 310 \text{ lb}}$. The sum of the moments about E:

$$\sum M_E = 8F - 16C_y + 8C_x = 0.$$

For frictionless pulleys, $F = W$, and thus $\boxed{C_y = 195 \text{ lb}}$. The sum of forces parallel to y:

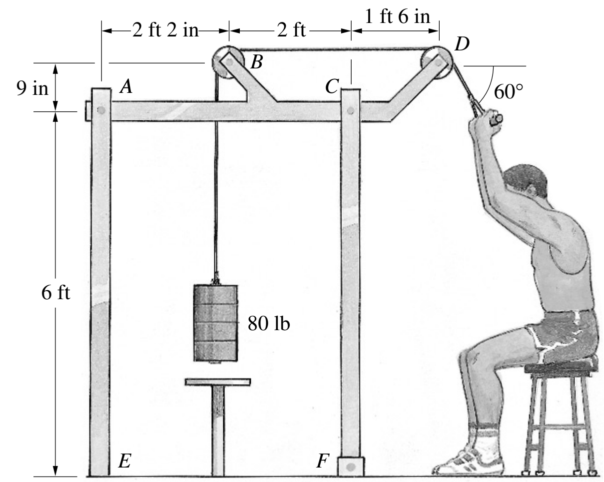
$$\sum F_y = E_y - C_y + F = 0,$$

from which $\boxed{E_y = 115 \text{ lb}}$.

Equation (1) above is now solvable: $\boxed{A_y = -35 \text{ lb}}$.

Element ABCD: The forces exerted by the pulleys on element $ABCD$ are, by inspection: $\boxed{B_x = W = 80 \text{ lb}}$, $\boxed{B_y = 80 \text{ lb}}$, $\boxed{D_x = 80 \text{ lb}}$, and $\boxed{D_y = -80 \text{ lb}}$, where the negative sign means that the force is reversed from the direction of the arrows shown on the free body diagram.

Problem 6.71 The man using the exercise machine is holding the 80-lb weight stationary in the position shown. What are the reactions at the built-in support E and the pin support F? (A and C are pinned connections.)



Solution: *The complete structure as a free body:* The sum of the moments about E:

$$\sum M = -26W - 68W \sin 60^\circ + 50F_y - 81W \cos 60^\circ + M_E = 0$$

from which (1) $50F_y + M_E = 10031$. The sum of the forces:

$$\sum F_x = F_x + W \cos 60^\circ + E_x = 0,$$

from which (2) $F_x + E_x = -40$.

$$\sum F_y = -W - W \sin 60^\circ + E_y + F_y = 0,$$

from which (3) $E_y + F_y = 149.28$

Element CF: The sum of the moments about F:

$$\sum M = -72C_x = 0,$$

from which $C_x = 0$. The sum of the forces:

$$\sum F_x = C_x + F_x = 0,$$

from which $\boxed{F_x = 0}$. From (2) above, $\boxed{E_x = -40 \text{ lb}}$

Element AE: The sum of the moments about E:

$$\sum M = M_E - 72A_x = 0, .$$

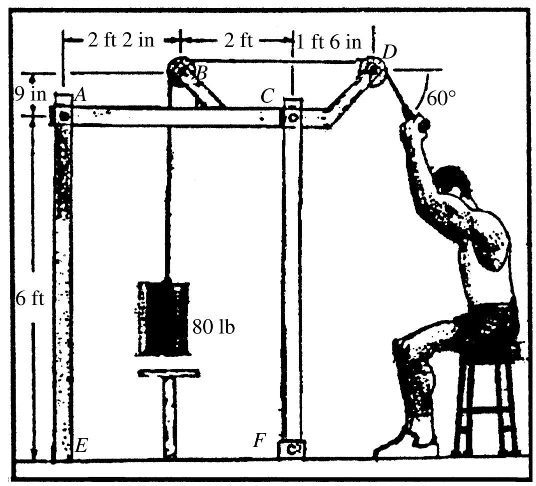
from which (4) $M_E = 72A_x$. The sum of the forces:

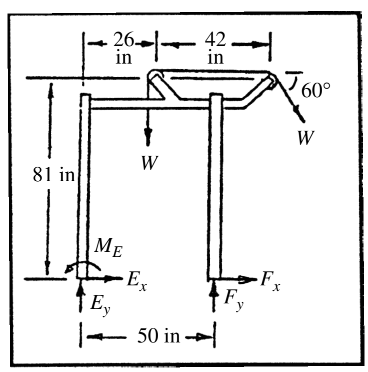
$$\sum F_y = E_y + A_y = 0,$$

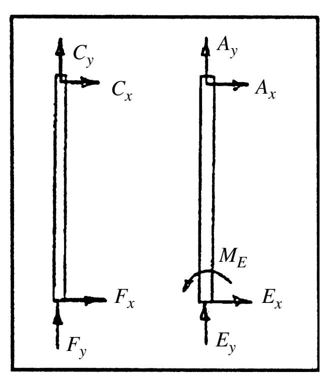
from which (5) $E_y + A_y = 0$.

$$\sum F_x = A_x + E_x = 0;$$

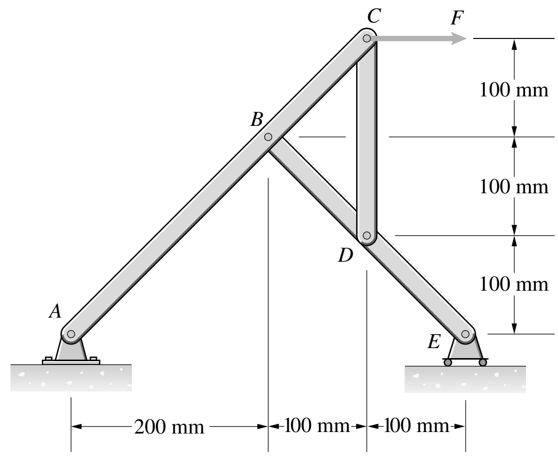
from which $A_x = 40$ lb, and from (4) $\boxed{M_E = 2880 \text{ in lb} = 240 \text{ ft lb}}$. From (1) $\boxed{F_y = 143.0 \text{ lb}}$, and from (2) $\boxed{E_y = 6.258 \text{ lb}}$. This completes the determination of the 5 reactions on E and F.



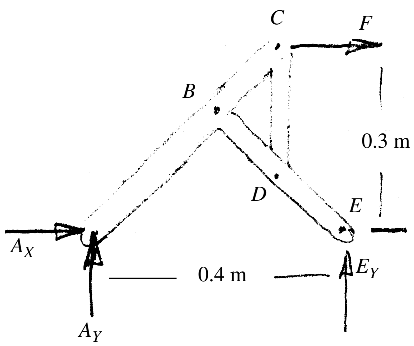




Problem 6.72 The frame supports a horizontal load F at C. The resulting compressive axial force in the two-force member CD is 2400 N. Determine the magnitude of the reaction exerted on member ABC at B.



Solution: First, write eqns to determine the support reactions at A and E (CD is a two force member)

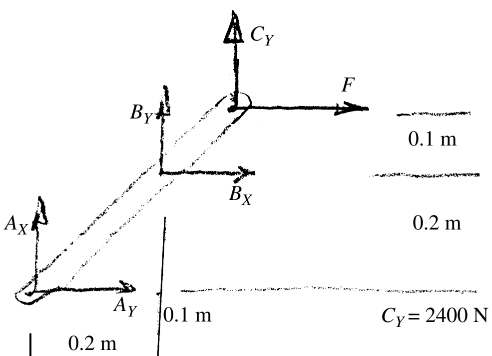


$$\sum F_x: \quad A_x + F = 0 \quad (1)$$

$$\sum F_y: \quad A_y + E_y = 0 \quad (2)$$

$$\circlearrowleft + \sum M_A: \quad 0.4E_y - 0.3F = 0 \ (3)$$

We have these eqns in four Unknowns (A_x, A_y, E_y, and F) Now write eqns for member ABC



$C_y = 2400$ N

$$\sum F_x: \quad A_x + B_x + F = 0 \quad (4)$$

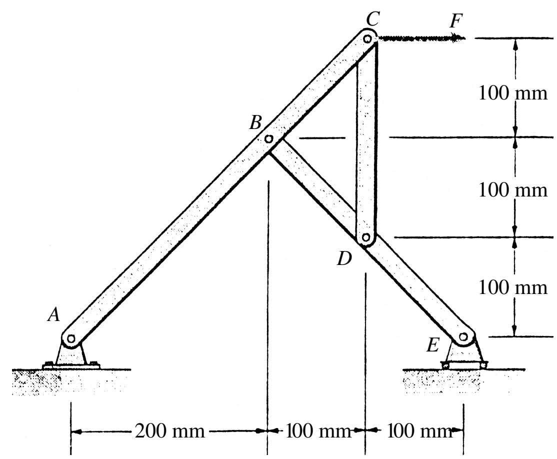
$$\sum F_y: \quad A_y + B_y + C_y = 0 \quad (5)$$

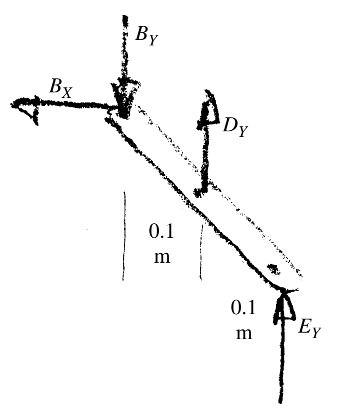
$$\circlearrowleft + \sum M_A: \quad 0.2B_y - 0.2B_x + 0.3C_y - 0.3F = 0 \ (6)$$

We now have 6 eqns in unknowns: (A_x, A_y, B_x, B_yE_y, F) Next, write the equations for member BDE.

Solving, we get $B_x = 0$ $B_y = -1200$ N

$|\mathbf{B}| = 1200$ N





$C_y = -D_y$

(two force member)

$$\sum F_x: \quad -B_x = 0 \quad (7)$$

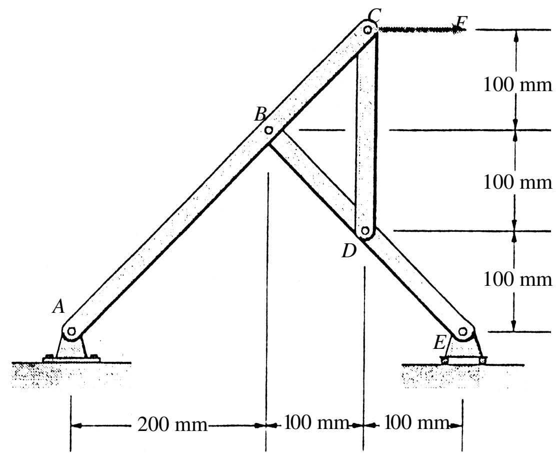
$$\sum F_y: \quad -B_y + D_y + E_y = 0 \ (8)$$

$$\sum M_B: \quad 0.1D_y + 0.2E_y = 0 \quad (9)$$

We now have 9 equations in 8 unknowns. Obviously, if they are compatible, one is a linear combination of the others. We could also have more than one redundant equation and still need another equation.

Combining Eqs. (4) and (7) gives Eq. (1). Thus, one of these three equations is not need.

Problem 6.73 The two-force member CD of the frame shown in Problem 6.72 will safely support a compressive axial load of 3 kN. Based on this criterion, what is the largest safe magnitude of the horizontal load F?



Solution: In the solution to Problem 6.72, we derived the equation's listed below for the loads shown on the frame.

$$\left.\begin{aligned} A_x + F &= 0 \\ A_y + E_y &= 0 \\ 0.4E_y - 0.3F &= 0 \end{aligned}\right\} \text{Entire Frame}$$

$$\left.\begin{aligned} A_x + B_x + F &= 0 \\ A_y + B_y + C_y &= 0 \\ 0.2B_y - 0.2B_x + 0.3C_y - 0.3F &= 0 \end{aligned}\right\} \text{Member } ABC$$

$C_y = -D_y$ – 2 force member

Set $C_y = 3000$ N and solve.

We get $F = 2000 \text{ N} = 2 \text{ kN}$

$A_x = -2$ kN

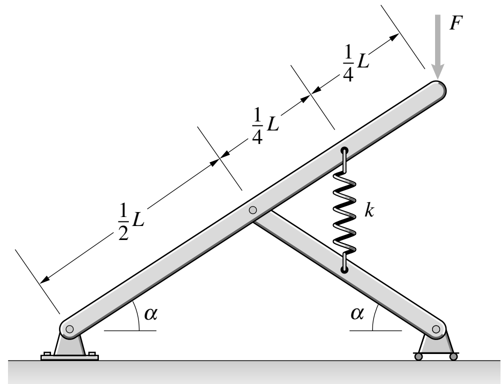
$A_y = -1.5$ kN

$E_y = 1.5$ kN

$B_x = 0$

$B_y = -1.5$ kN

Problem 6.74 The unstretched length of the string is L_O. Show that when the system is in equilibrium the angle α satisfies the relation $\sin\alpha = 2(L_O - 2F/k)L$.



Solution: Since the action lines of the force F and the reaction E are co-parallel and coincident, the moment on the system is zero, and the system is always in equilibrium, for a non-zero force F. The object is to find an expression for the angle α for any non-zero force F. *The complete structure as a free body:* The sum of the moments about A

$$\sum M_A = -FL\sin\alpha + EL\sin\alpha = 0,$$

from which $E = F$. The sum of forces:

$$\sum F_x = A_x = 0,$$

from which $A_x = 0$.

$$\sum F_y = A_y + E - F = 0,$$

from which $A_y = 0$, which completes a demonstration that F does not exert a moment on the system. *The spring C:* The elongation of the spring is $\Delta s = 2\frac{L}{4}\sin\alpha - L_O$, from which the force in the spring is

$$T = k\left(\frac{L}{2}\sin\alpha - L_O\right)$$

Element BE: The strategy is to determine C_y, which is the spring force on BE. The moment about E is

$$\sum M_E = -\frac{L}{4}C_y\cos\alpha - \frac{L}{2}B_y\cos\alpha - \frac{L}{2}B_x\cos\alpha = 0,$$

from which $\frac{C_y}{2} + B_y = -B_x$. The sum of forces:

$$\sum F_x = B_x = 0,$$

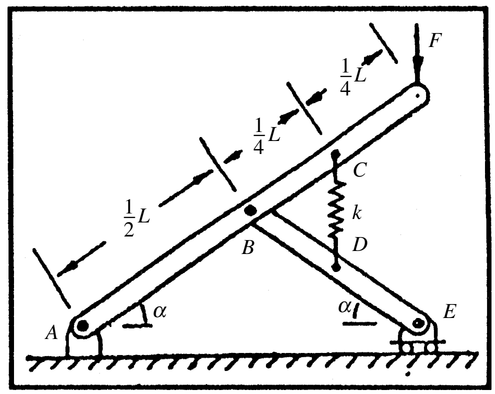
from which $B_x = 0$.

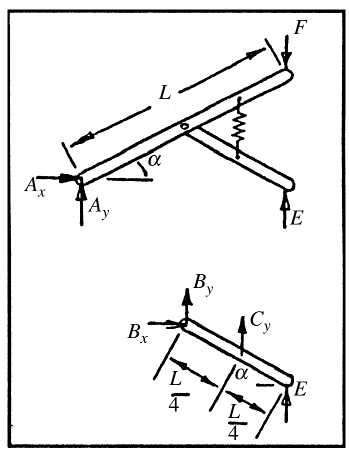
$$\sum F_y = C_y + B_y + E = 0,$$

from which $C_y + B_y = -E = -F$. The two simultaneous equations are solved: $C_y = -2F$, and $B_y = F$.
The solution for angle α: The spring force is

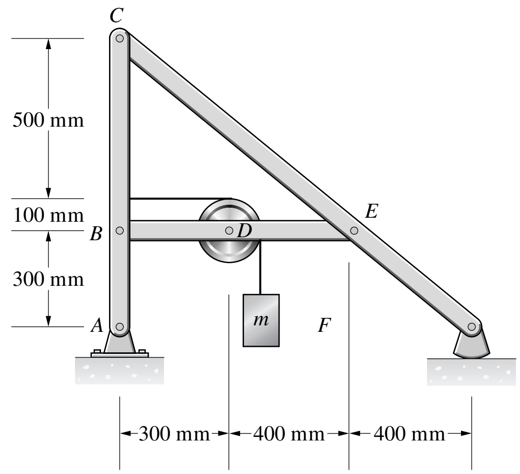
$$C_y = T = k\left(\frac{L}{2}\sin\alpha - L_O\right),$$

from which $k\left(\frac{L}{2}\sin\alpha - L_O\right) = -2F$.
Solve: $\sin\alpha = \frac{2\left(L_O - \frac{2F}{k}\right)}{L}$





Problem 6.75 The pin support B will safely support a force of 24-kN magnitude. Based on this criterion, what is the largest mass m that the frame will safely support?



Solution: The weight is given by $W = mg = 9.81$ g
The complete structure as a free body:
Sum the forces in the x-direction:

$$\sum F_x = A_x = 0,$$

from which $A_x = 0$
Element ABC: The sum of the moments about A:

$$\sum M_A = +0.3B_x + 0.9C_x - 0.4W = 0,$$

from which (1) $0.3B_x + 0.9C_x = 0.4W$. The sum of the forces:

$$\sum F_x = -B_x - C_x + W + A_x = 0,$$

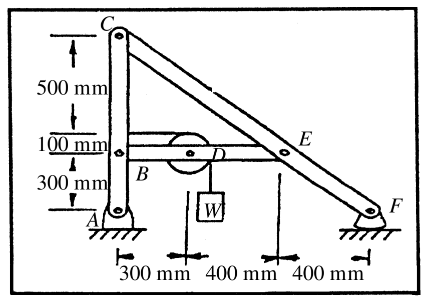
from which (2) $B_x + C_x = W$. Solve the simultaneous equations (1) and (2) to obtain $B_x = \frac{5}{6}W$
Element BE: The sum of the moments about E:

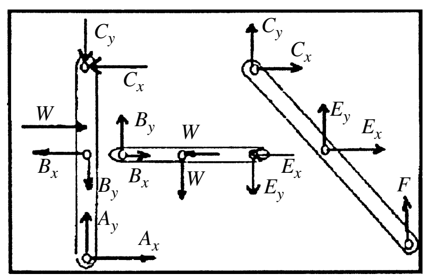
$$\sum M_E = 0.4W - 0.7B_y = 0,$$

from which $B_y = \frac{4}{7}W$. The magnitude of the reaction at B is

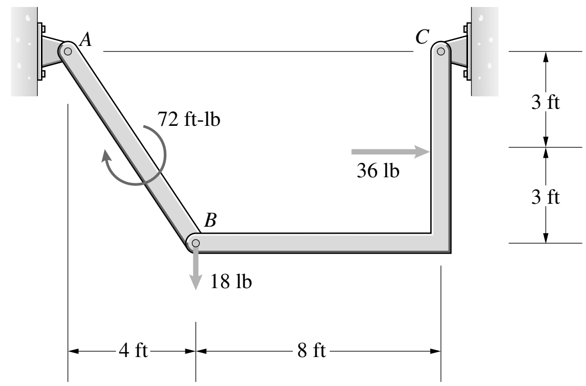
$$|\mathbf{B}| = W\sqrt{\left(\frac{5}{6}\right)^2 + \left(\frac{4}{7}\right)^2} = 1.0104W.$$

For a safe value of $|\mathbf{B}| = 24$ kN, $W = \frac{24}{1.0104} = 23.752$ kN is the maximum load that can be carried. Thus, the largest mass that can be supported is $m = W/g = 23752\ \text{N}/9.81\ \text{m/s}^2 = 2421$ kg.





Problem 6.76 Determine the reactions at A and C.



Solution: *The complete structure as a free body:* The sum of the moments about A:

$$\sum M_A = -4(18) + 3(36) + 12C_y - 72 = 0,$$

from which $C_y = 3$ lb. The sum of the forces:

$$\sum F_y = A_y + C_y - 18 = 0,$$

from which $A_y = 15$ lb.

$$\sum F_x = A_x + C_x + 36 = 0,$$

from which (1) $C_x = -A_x - 36$

Element AB: The sum of the forces:

$$\sum F_y = A_y - B_y - 18 = 0,$$

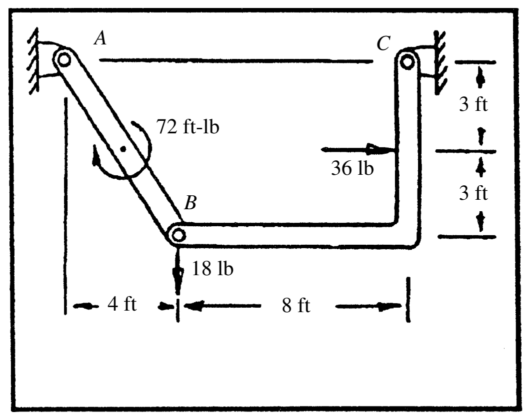
from which $B_y = -3$ lb. The sum of the moments:

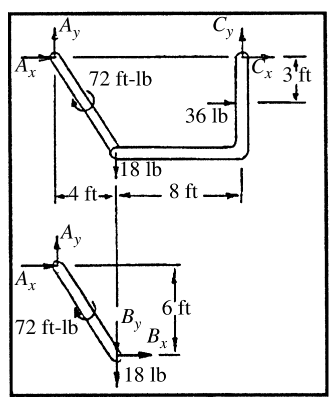
$$\sum M_A = 6B_x - 4(18) - 4B_y - 72 = 0,$$

from which $B_x = 22$ lb. The sum of the forces:

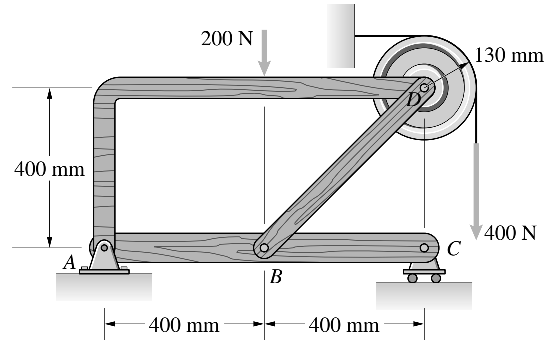
$$\sum F_x = A_x + B_x = 0,$$

from which $A_x = -22$ lb From equation (1) $C_x = -14$ lb

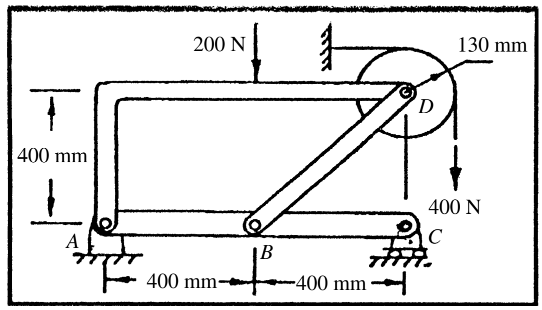


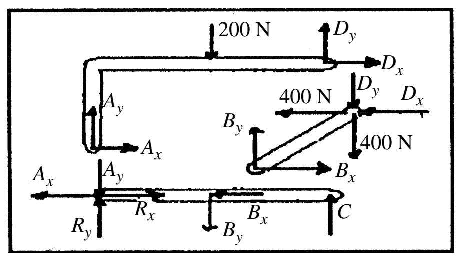


Problem 6.77 Determine the forces on member AD.



Solution: Denote the reactions of the support by R_x and R_y. *The complete structure as a free body:*





$$\sum F_x = R_x - 400 = 0,$$

from which $R_x = 400$ N. The sum of moments:

$$\sum M_A = 800C - 400(800) - 400(400) - 400(200) = 0,$$

from which $C = 300$ N.

$$\sum F_y = C + R_y - 400 - 200 = 0,$$

from which $R_y = 300$ N. *Element ABC*: The sum of the moments:

$$\sum M_A = -4B_y + 8C = 0,$$

from which $B_y = 600$ N. *Element BD*: The sum of the forces:

$$\sum F_y = B_y - D_y - 400 = 0,$$

from which $D_y = 200$ N.
Element AD: The sum of the forces:

$$\sum F_y = A_y + D_y - 200 = 0,$$

from which $A_y = 0$: *Element AD:* The sum of the forces:

$$\sum F_x = A_x + D_x = 0$$

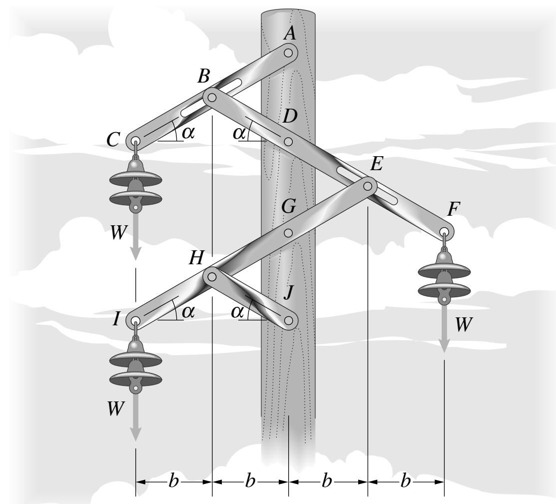
and $\sum M_A = -400(200) + 800D_y - 400D_x = 0$

$A_x = -200$ N, and $D_x = 200$ N.
Element BD: The sum of forces:

$$\sum F_x = B_x - D_x - 400 = 0$$

from which $B_x = 600$ N. This completes the solution of the nine equations in nine unknowns, of which A_x, A_y, D_x, and D_y are the values required by the Problem.

Problem 6.78 The frame shown is used to support high-tension wires. If $b = 3$ ft, $\alpha = 30°$, and $W = 200$ lb, what is the axial force in member HJ?



Solution: Joints B and E are sliding joints, so that the reactions are normal to AC and BF, respectively. Member HJ is supported by pins at each end, so that the reaction is an axial force. The distance $h = b \tan\alpha = 1.732$ ft

Member ABC. The sum of the forces:

$$\sum F_x = A_x + B\sin\alpha = 0,$$

$$\sum F_y = A_y - W - B\cos\alpha = 0.$$

The sum of the moments about B:

$$\sum M_B = bA_y - hA_x + bW = 0.$$

These three equations have the solution: $A_x = 173.21$ lb, $A_y = -100$ lb, and $B = -346.4$ lb.

Member BDEF: The sum of the forces:

$$\sum F_x = D_x - B\sin\alpha - E\sin\alpha = 0,$$

$$\sum F_y = D_y - W + B\cos\alpha - E\cos\alpha = 0.$$

The sum of the moments about D:

$$\sum M_D = -2bW - bE\cos\alpha - hE\sin\alpha - bB\cos\alpha + hB\sin\alpha = 0.$$

These three equations have the solution: $D_x = -259.8$ lb, $D_y = 350$ lb, $E = -173.2$ lb.

Member EGHI: The sum of the forces:

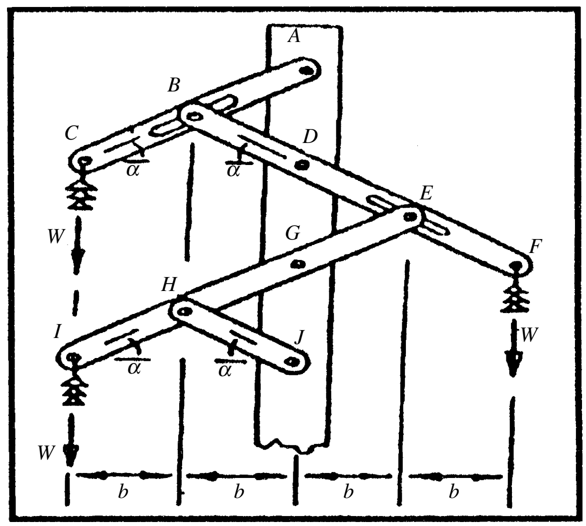
$$\sum F_x = G_x + E\sin\alpha - H\cos\alpha = 0,$$

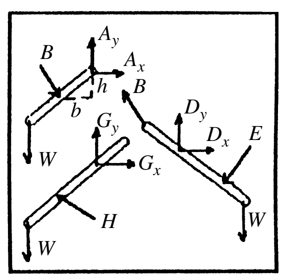
$$\sum F_y = G_y - W + E\cos\alpha + H\sin\alpha = 0.$$

The sum of the moments about H:

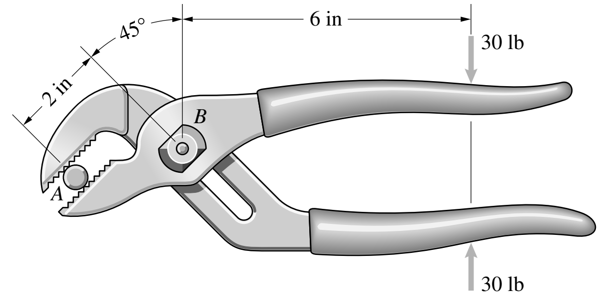
$$\sum M_H = bG_y - hG_x + bW + 2bE\cos\alpha - 2hE\sin\alpha = 0.$$

These three equations have the solution: $G_x = 346.4$ lb, $G_y = 200$ lb, and $H = 300$ lb. This is the axial force in HJ.





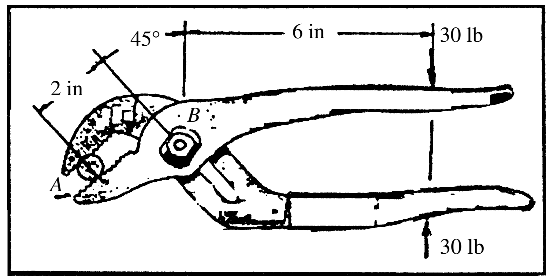
Problem 6.79 What are the magnitudes of the forces exerted by the pliers on the bolt at A when 30-lb forces are applied as shown? (B is a pinned connection.)

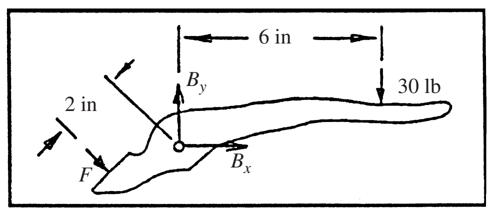


Solution: *Element AB:* The sum of the moments about B:

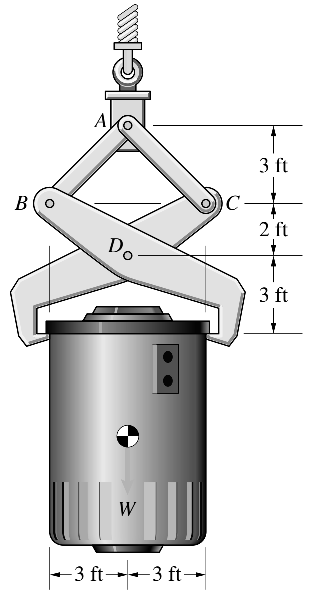
$\sum M_B = 2F - (6)30 = 0,$

from which $F = 90$ lb.





Problem 6.80 The weight $W = 60$ kip. What is the magnitude of the force the members exert on each other at D?



Solution: Assume that a tong half will carry half the weight, and denote the vertical reaction to the weight at A by R. *The complete structure as a free body:* The sum of the forces:

$$\sum F_y = R - W = 0,$$

from which $R = W$

Tong-Half ACD:

Element AC: The sum of the moments about A:

(1) $\sum M_A = 3C_y + 3C_x = 0.$

The sum of the forces:

(2) $\sum F_y = \frac{R}{2} + C_y + A_y = 0$, and

(3) $\sum F_x = C_x + A_x = 0.$

Element CD: The sum of the forces:

(4) $\sum F_x = D_x - P - C_x = 0$, and

(5) $\sum F_y = D_y - C_y - \frac{W}{2} = 0.$

The sum of the moments:

(6) $\sum M_D = 2C_x - 3C_y - 3P + \frac{3}{2}W = 0$

Element AB: The sum of the forces:

(7) $\sum F_y = -A_y + \frac{R}{2} - B_y = 0$, and

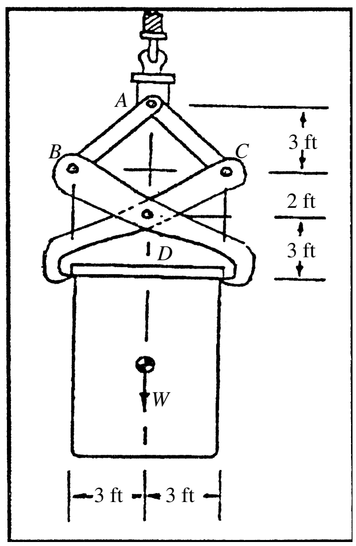
(8) $\sum F_x = -A_x - B_x = 0.$

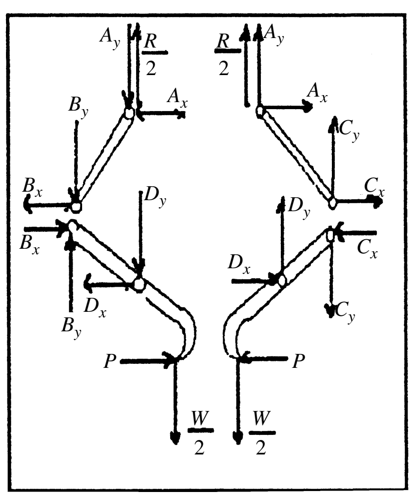
Element BD: The sum of the forces:

(9) $\sum F_y = B_y - D_y - \frac{W}{2} = 0$, and

(10) $\sum F_x = B_x - D_x + P = 0.$

These are ten equations in ten unknowns. These have the solution $R = 60$ kip. *Check*, $A_x = -30$ kip, $A_y = 0$, $B_x = 30$ kip, $B_y = 30$ kip, $C_x = 30$ kip, $C_y = -30$ kip, $D_x = 110$ kip, $D_y = 0$, and $P = 80$ kip. The magnitude of the force the members exert on each other at D is $D = 110$ kip.

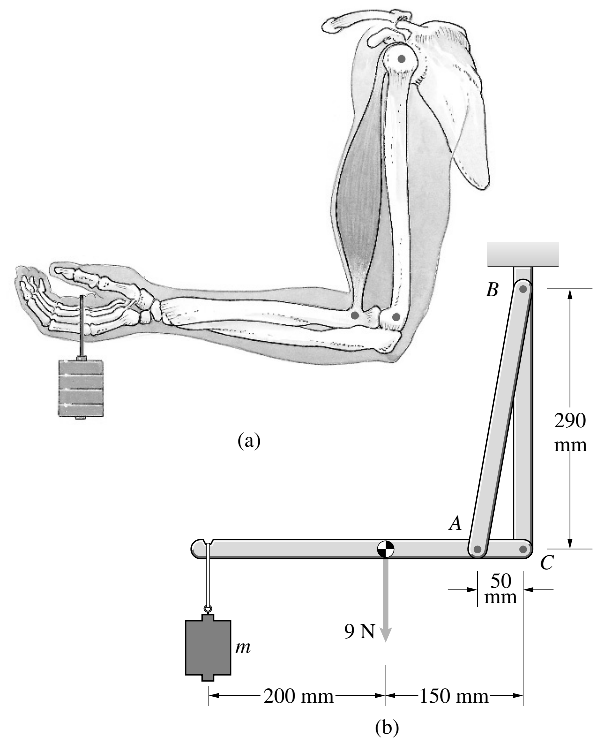




Problem 6.81 Figure a is a diagram of the bones and biceps muscle of a person's arm supporting a mass. Tension in the biceps muscle holds the forearm in the horizontal position, as illustrated in the simple mechanical model in Fig. b. The weight of the forearm is 9 N, and the mass $m = 2$ kg.

(a) Determine the tension in the biceps muscle AB.

(b) Determine the magnitude of the force exerted on the upper arm by the forearm at the elbow joint C.



Solution: Make a cut through AB and BC just above the elbow joint C. The angle formed by the biceps muscle with respect to the forearm is $\alpha = \tan^{-1}\left(\frac{290}{50}\right) = 80.2°$. The weight of the mass is $W = 2(9.81) = 19.62$ N.

The section as a free body: The sum of the moments about C is

$$\sum M_C = -50T\sin\alpha + 150(9) + 350W = 0,$$

from which $T = 166.76$ N is the tension exerted by the biceps muscle AB. The sum of the forces on the section is

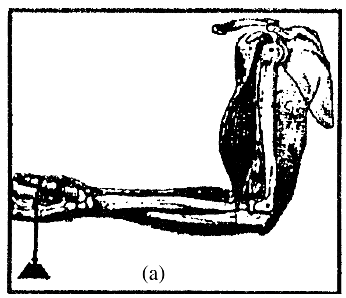
$$\sum F_X = C_x + T\cos\alpha = 0,$$

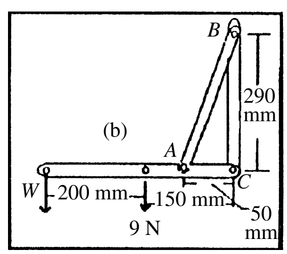
from which $C_x = -28.33$ N.

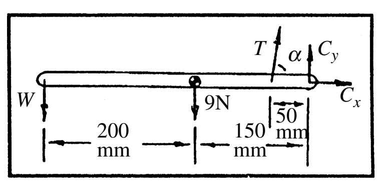
$$\sum F_Y = C_y + T\sin\alpha - 9 - W = 0,$$

from which $C_y = -135.72$. The magnitude of the force exerted by the forearm on the upper arm at joint C is

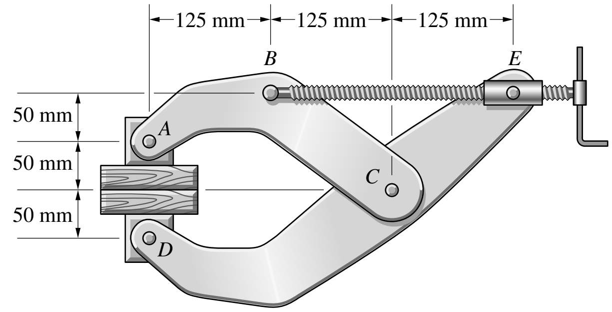
$$F = \sqrt{C_x^2 + C_y^2} = 138.65 \text{ N}$$







Problem 6.82 The clamp presses two blocks of wood together. Determine the magnitude of the force the members exert on each other at C if the blocks are pressed together with a force of 200 N.



Solution: Consider the upper jaw only.

The section ABC as a free body:

The sum of the moments about C is

$$\sum M_C = 100B - 250A = 0,$$

from which, for $A = 200$ N, $B = 500$ N. The sum of the forces:

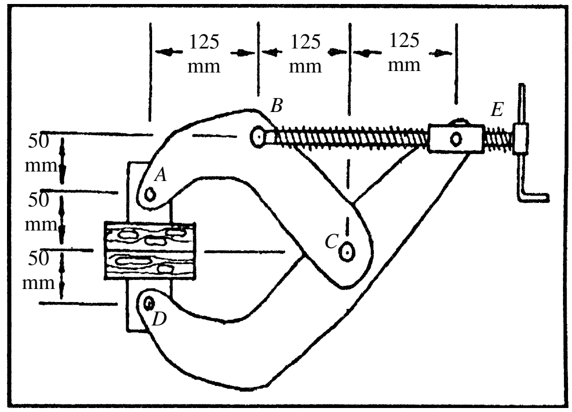
$$\sum F_x = C_x - B = 0,$$

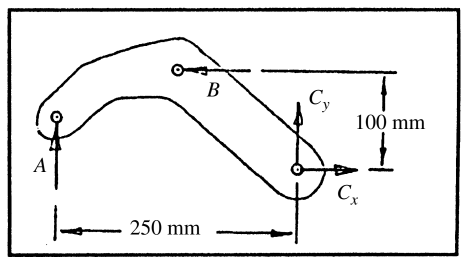
from which $C_x = 500$ N,

$$\sum F_y = C_y + A = 0,$$

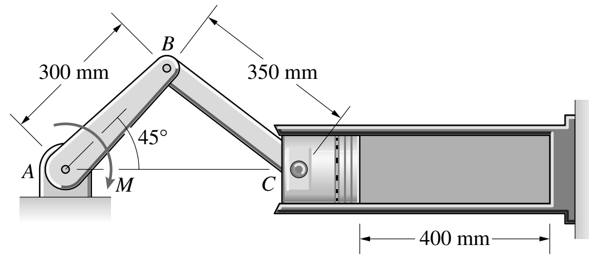
from which $C_y = -200$ N. The magnitude of the reaction at C:

$$C = \sqrt{C_x^2 + C_y^2} = 538.52 \text{ N}$$

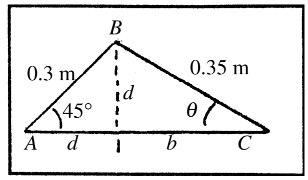




Problem 6.83 The pressure force exerted on the piston is 2 kN toward the left. Determine the couple M necessary to keep the system in equilibrium.



Solution: From the diagram, the coordinates of point B are (d, d) where $d = 0.3\cos(45°)$. The distance b can be determined from the Pythagorean Theorem as $b = \sqrt{(0.35)^2 - d^2}$. From the diagram, the angle $\theta = 37.3°$. From these calculations, the coordinates of points B and C are B (0.212, 0.212), and C (0.491, 0) with all distances being measured in *meters*. All forces will be measured in *Newtons*.

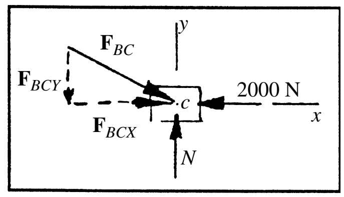


The unit vector from C toward B is $\mathbf{u}_{CB} = -0.795\mathbf{i} + 0.606\mathbf{j}$.

The equations of force equilibrium at C are

$$\sum F_X = F_{BC}\cos\theta - 2000 = 0,$$

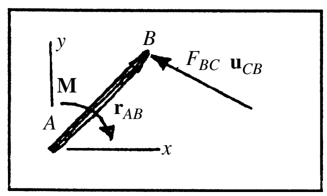
and $\sum F_Y = N - F_{BC}\sin\theta = 0.$



Solving these equations, we get $N = 1524$ *Newtons(N)*, and $F_{BC} = 2514$ N.

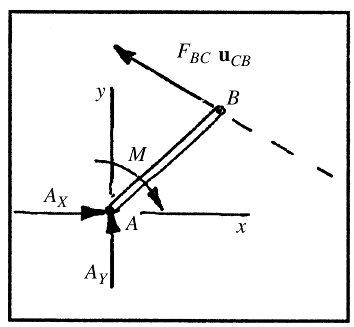
The force acting at B due to member BC is $F_{BC}\mathbf{u}_{BC} = -2000\mathbf{i} + 1524\mathbf{j}$ N.

The position vector from A to B is $\mathbf{r}_{AB} = 0.212\mathbf{i} + 0.212\mathbf{j}$ m, and the moment of the force acting at B about A, calculated from the cross product, is given by $\mathbf{M}_{FBC} = 747.6\mathbf{k}$ N-m (counter - clockwise). The moment $\mathbf{M}$ about A which is necessary to hold the system in equilibrium, is equal and opposite to the moment just calculated. Thus, $\mathbf{M} = -747.6\mathbf{k}$ N-m (clockwise).



Problem 6.84 In Problem 6.83, determine the forces on member AB at A and B.

Solution: In the solution of Problem 6.83, we found that the force acting at point B of member AB was $F_{BC}\mathbf{u}_{BC} = -2000\mathbf{i} + 1524\mathbf{j}$ **N**, and that the moment acting on member BC about point A was given by $\mathbf{M} = -747.6\mathbf{k}$ N-m (clockwise). Member AB must be in equilibrium, and we ensured moment equilibrium in solving Problem 6.83.



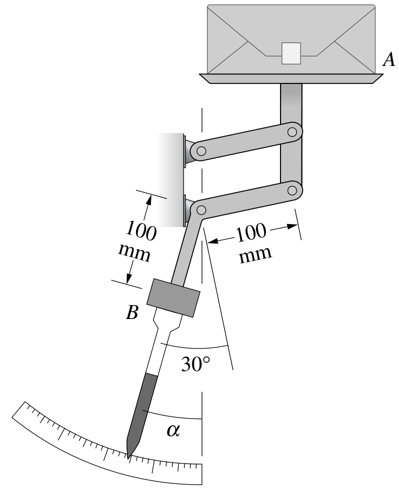
From the free body diagram, the equations for force equilibrium are

$$\sum F_X = A_X + F_{BC}u_{BCX} = A_X - 2000 \text{ N} = 0,$$

and $\sum F_Y = A_Y + F_{BC}u_{BCY} = A_Y + 1524 \text{ N} = 0.$

Thus, $A_X = 2000$ N, and $A_Y = -1524$ N.

Problem 6.85 The mechanism is used to weigh mail. A package placed at A causes the weighted point to rotate through an angle α. Neglect the weights of the members except for the counterweight at B, which has a mass of 4 kg. If $\alpha = 20°$, what is the mass of the package at A?



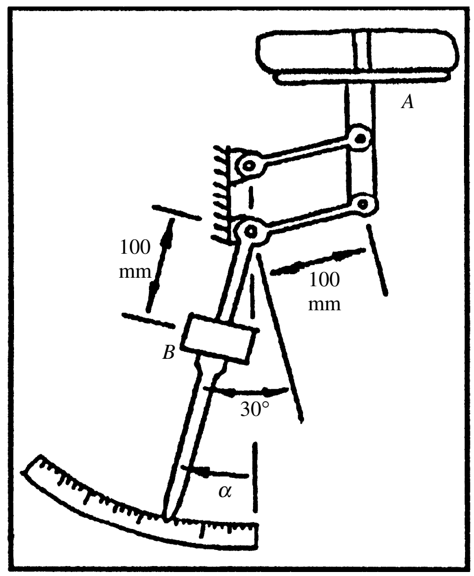
Solution: Consider the moment about the bearing connecting the motion of the counter weight to the motion of the weighing platform. The moment arm of the weighing platform about this bearing is $100\cos(30 - \alpha)$. The restoring moment of the counter weight is 100 mg $\sin\alpha$. Thus the sum of the moments is

$$\sum M = 100\ \mathrm{m}_B g \sin\alpha - 100\ \mathrm{m}_A g\cos(30 - \alpha) = 0.$$

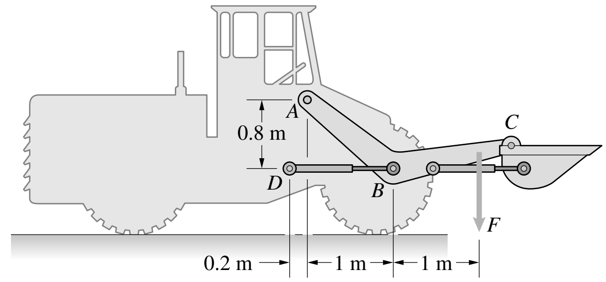
Define the ratio of the masses of the counter weight to the mass of the package to be $R_M = \frac{m_B}{m_A}$. The sum of moments equation reduces to

$$\sum M = R_M \sin\alpha - \cos(30 - \alpha) = 0,$$

from which $R_M = \frac{\cos(30-\alpha)}{\sin\alpha} = 2.8794$, and the mass of the package is $m_A = \frac{4}{R_M} = 1.3892 = 1.39$ kg



Problem 6.86 The scoop C of the front-end loader is supported by two identical arms, one on each side of the loader. One of the two arms (ABC) is visible in the figure. It is supported by a pin support at A and the hydraulic actuator BD. The sum of the other loads exerted on the arm, including its own weight, is $F = 1.6$ kN. Determine the axial force in the actuator BD and the magnitude of the reaction at A.



Solution: *The section ABC as a free body:* The sum of the moments about A:

$$\sum M_A = 0.8BD - 2F = 0,$$

from which $BD = 4$ kN.

The sum of the forces:

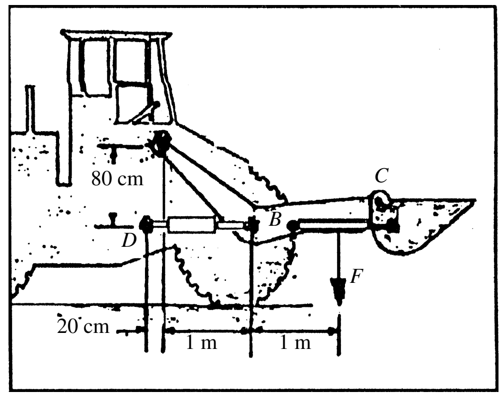
$$\sum F_x = A_x + BD = 0,$$

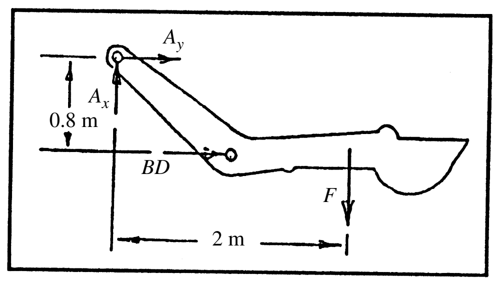
from which $A_x = -4$ kN.

$$\sum F_y = A_y - F = 0,$$

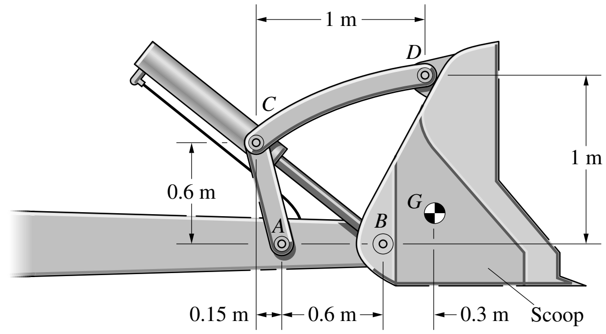
from which $A_y = 1.6$ kN. The magnitude of the reaction at A is

$$A = \sqrt{A_x^2 + A_y^2} = 4.308 \text{ kN}$$





Problem 6.87 The mass of the scoop is 220 kg, and its weight acts at G. Both the scoop and the hydraulic actuator BC are pinned to the horizontal member at B. The hydraulic actuator can be treated as a two-force member. Determine the forces exerted on the scoop at B and D.



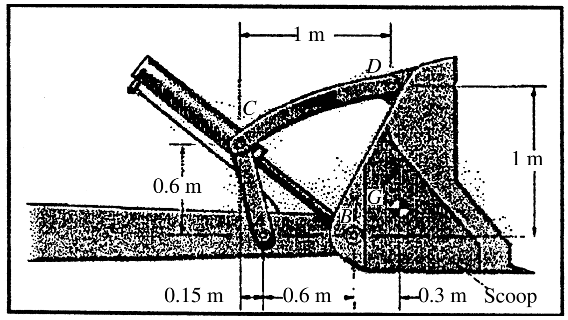
Solution: We need to know the locations of various points in the Problem . Let us use horizontal and vertical axes and define the coordinates of point A as (0,0). All distances will be in *meters (m)* and all forces will be in *Newtons (N)*. From the figure in the text, the coordinates in meters of the points in the problem are A (0, 0), B (0.6, 0), C (−0.15, 0.6), D (0.85, 1), and the x coordinate of point G is 0.9 m. The unit vector from C toward D is given by $\mathbf{u}_{CD} = 0.928\mathbf{i} + 0.371\mathbf{j}$, and the force acting on the scoop at D is given by $\mathbf{D} = D_X\mathbf{i} + D_Y\mathbf{j} = 0.928D\mathbf{i} + 0.371D\mathbf{j}$. From the free body diagram of the scoop, the equilibrium equations are

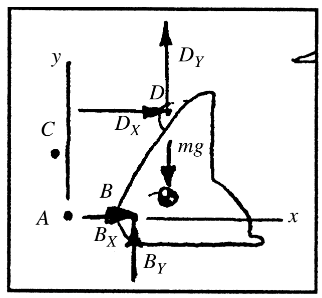
$$\sum F_X = B_X + D_X = 0,$$

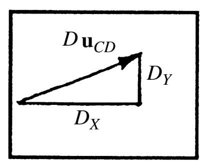
$$\sum F_Y = B_Y + D_Y - mg = 0,$$

and $\sum M_B = -0.3\,\text{mg} + x_{BD}D_Y - y_{BD}D_X = 0.$

From the geometry, $x_{BD} = 0.25$ m, and $y_{BD} = 1$ m. Solving the equations of equilibrium, we obtain $B_X = 719.4$ N, $B_Y = 2246$ N, and $D = -774.8$ N (member CD is in tension).







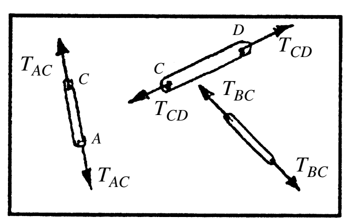
Problem 6.88 In Problem 6.87, determine the axial force in the hydraulic actuator BC.

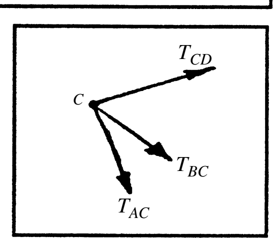
Solution: The unit vectors in the directions of the forces acting at C are $\mathbf{u}_{CD} = 0.928\mathbf{i} + 0.371\mathbf{j}$, $\mathbf{u}_{CA} = 0.243\mathbf{i} - 0.970\mathbf{j}$, and $\mathbf{u}_{CB} = 0.781\mathbf{i} - 0.625\mathbf{j}$. The force equilibrium equations at C are

$$\sum F_X = T_{BC}u_{CBX} + T_{AC}u_{CAX} + T_{CD}u_{CDX} = 0,$$

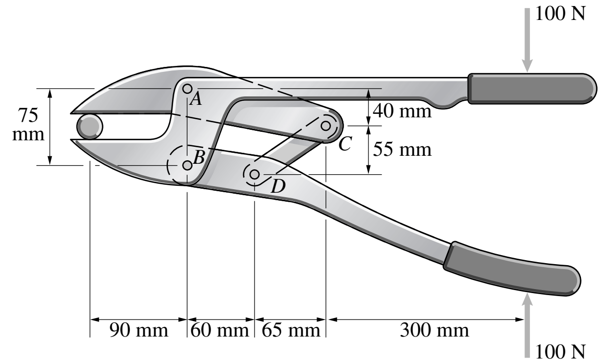
and $\sum F_Y = T_{BC}u_{CBY} + T_{AC}u_{CAY} + T_{CD}u_{CDY} = 0.$

Solving these equations, we get $T_{BC} = -1267$ N(*compression*), and $T_{AC} = 1112$ N(*tension*).





Problem 6.89 Determine the force exerted on the bolt by the bolt cutters.



Solution: The equations of equilibrium for each of the members will be developed.

Member AB: The equations of equilibrium are:

$$\sum F_X = A_X + B_X = 0,$$

$$\sum F_Y = A_Y + B_Y = 0,$$

and $\sum M_B = 90F - 75A_X - 425(100) = 0$

Member BD: The equations are

$$\sum F_X = -B_X + D_X = 0,$$

$$\sum F_Y = -B_Y + D_Y + 100 = 0,$$

and $\sum M_B = 15D_X + 60D_Y + 425(100) = 0.$

Member AC: The equations are

$$\sum F_X = -A_X + C_X = 0,$$

$$\sum F_Y = -A_Y + C_Y + F = 0,$$

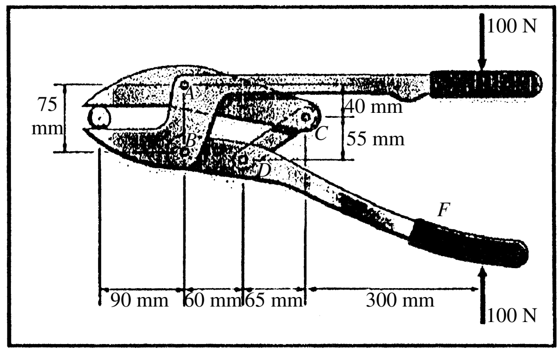
and $\sum M_A = -90F + 125C_Y + 40C_X = 0.$

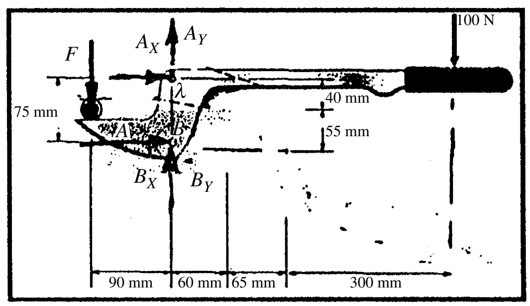
Member CD: The equations are:

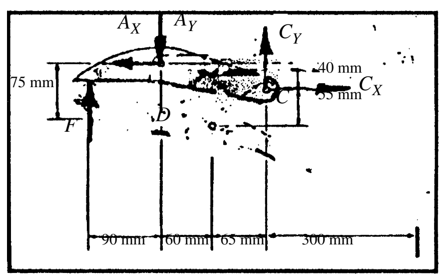
$$\sum F_X = -C_X - D_X = 0,$$

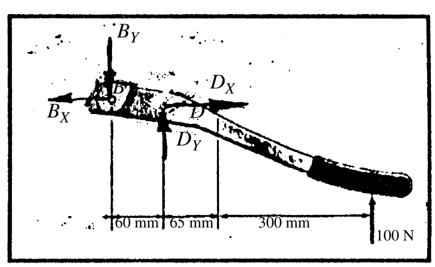
$$\sum F_Y = -C_Y - D_Y = 0.$$

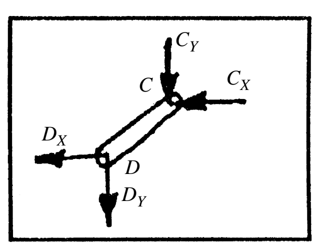
Solving the equations simultaneously (we have extra (but compatible) equations, we get $F = 1051$ N, $A_X = 695$ N, $A_Y = 1586$ N, $B_X = -695$ N, $B_Y = -435$ N, $C_X = 695$ N, $C_Y = 535$ N, $D_X = -695$ N, and $D_y = -535$ N







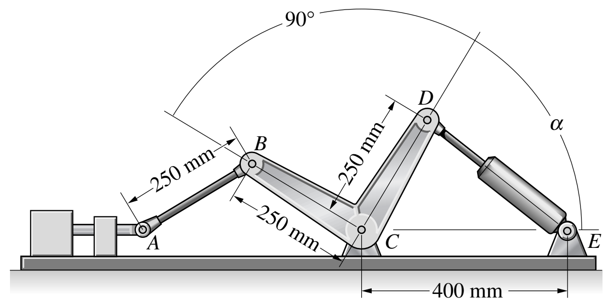




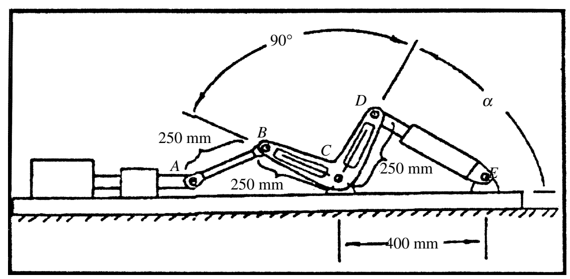
Problem 6.90 For the bolt cutters in Problem 6.89, determine the magnitude of the force the members exert on each other at the pin connection B and the axial force in the two-force member CD.

Solution: From the solution to 6.107, we know $B_X = -695$ N, and $B_Y = -435$ N. We also know that $C_X = 695$ N, and $C_Y = 535$ N, from which the axial load in member CD can be calculated. The load in CD is given by $T_{CD} = \sqrt{C_X^2 + C_Y^2} = 877$ N

Problem 6.91 The device is designed to exert a large force on the horizontal bar at A for a stamping operation. If the hydraulic cylinder DE exerts an axial force of 800 N and $\alpha = 80°$, what horizontal force is exerted on the horizontal bar at A?



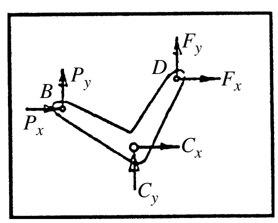
Solution: Define the x-y coordinate system with origin at C. The projection of the point D on the coordinate system is



$$R_y = 250 \sin\alpha = 246.2 \text{ mm},$$

and $R_x = 250 \cos\alpha = 43.4$ mm.

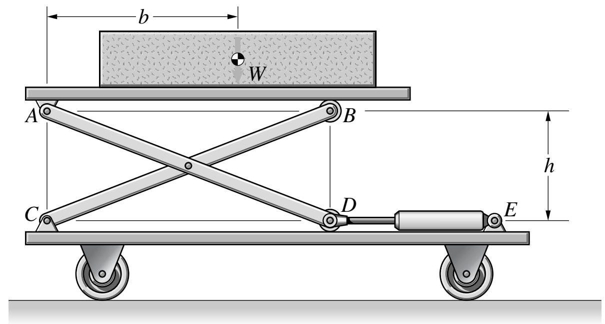
The angle formed by member DE with the positive x axis is $\theta = 180 - \tan^{-1}\left(\frac{R_y}{400 - R_x}\right) = 145.38°$. The components of the force produced by DE are $F_x = F\cos\theta = -658.3$ N, and $F_y = F\sin\theta = 454.5$ N. The angle of the element AB with the positive x axis is $\beta = 180 - 90 - \alpha = 10°$, and the components of the force for this member are $P_x = P\cos\beta$ and $P_y = P\sin\beta$, where P is to be determined. The angle of the arm BC with the positive x axis is $\gamma = 90 + \alpha = 170°$. The projection of point B is $L_x = 250\cos\gamma = -246.2$ mm, and $L_y = 250\sin\gamma = 43.4$ mm. Sum the moments about C:



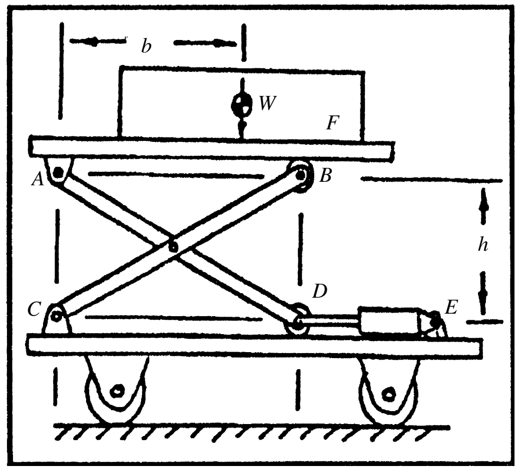
$$\sum M_C = R_x F_y - R_y F_x + L_x P_y - L_y P_x = 0.$$

Substitute and solve: $P = 2126.36$ N, and $P_x = P\cos\beta = 2094$ N is the horizontal force exerted at A.

Problem 6.92 This device raises a load W by extending the hydraulic actuator DE. The bars AD and BC are 4 ft long, and the distances $b = 2.5$ ft and $h = 1.5$ ft. If $W = 300$ lb, what force must the actuator exert to hold the load in equilibrium?



Solution: The angle ADC is $\alpha = \sin^{-1}\left(\frac{h}{4}\right) = 22.02°$. The distance CD is $d = 4\cos\alpha$.



The complete structure as a free body: The sum of the forces:

$$\sum F_y = -W + C_y + D_y = 0.$$

$$\sum F_x = C_x + D_x = 0.$$

The sum of the moments about C:

$$\sum M_C = -bW + dD_y = 0.$$

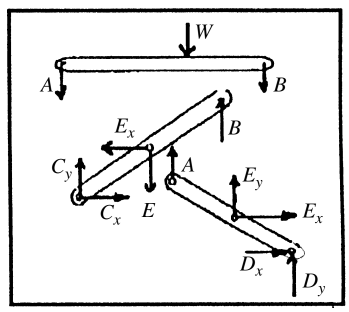
These have the solution:

$C_y = 97.7$ lb,

$D_y = 202.3$ lb,

and $C_x = -D_x$.

Divide the system into three elements: the platform carrying the weight, the member AB, and the member BC.



The Platform: (See Free body diagram) The moments about the point A:

$$\sum M_A = -bW - dB = 0.$$

The sum of the forces:

$$\sum F_y = A + B + W = 0.$$

These have the solution:

$B = -202.3$ lb,

and $A = -97.7$ lb.

Element BC: The sum of the moments about E is

$$\sum M_C = -\left(\frac{h}{2}\right)C_y + \left(\frac{d}{2}\right)C_x + \left(\frac{d}{2}\right)B = 0,$$ from which

(1) $dC_x - hC_y - dB = 0$. The sum of the forces:

$\sum F_x = C_x - E_x = 0$, from which

(2) $E_x - C_x = 0$,

$\sum F_y = C_y - E_y + B = 0$,

from which

(3) $C_y - E_y + B = 0$

Element AD: The sum of the moments about E:

$$\sum M_E = \left(\frac{d}{2}\right)D_y + \left(\frac{h}{2}\right)D_x - \left(\frac{d}{2}\right)A = 0,$$

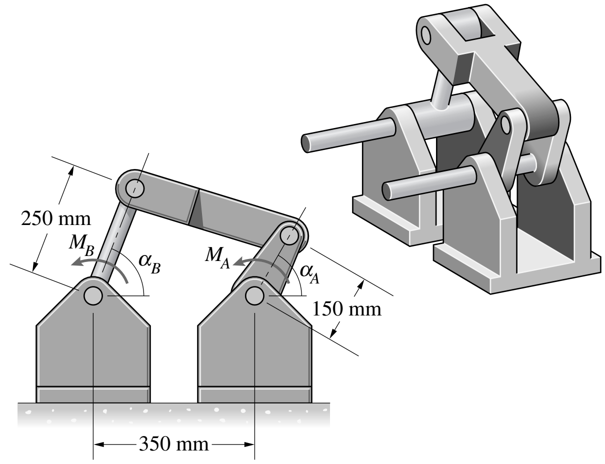
from which

(4) $dD_y + hD_x - dA = 0$.

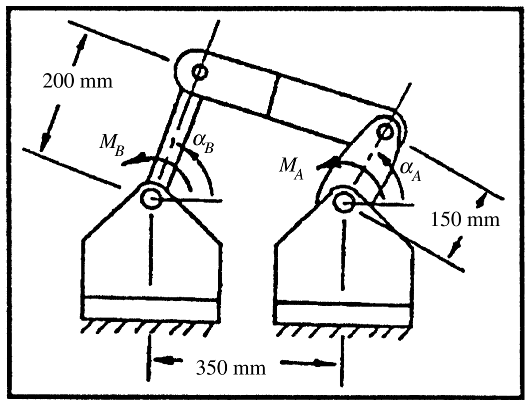
These are four equations in the four unknowns: E_X, E_Y, D_x, C_X and D_X

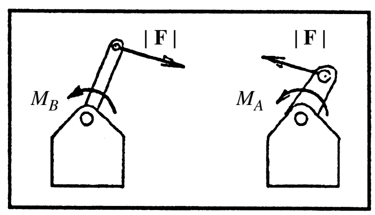
Solving, we obtain $D_x = -742$ lb.

Problem 6.93 The linkage is in equilibrium under the action of the couples M_A and M_B. If $\alpha_A = 60°$ and $\alpha_B = 70°$, what is the ratio M_A/M_B?



Solution: Make a cut through the linkage connecting the two cranks, and treat each system as a free body. The equilibrium condition occurs when the reaction forces in the linkage are equal and opposite.





The position vector of the end of the system B crank is

$$\mathbf{r}_B = R_B\,(\mathbf{i}\cos\alpha_B + \mathbf{i}\sin\alpha_B) = 85.51\mathbf{i} + 234.92\mathbf{j} \text{ (mm)}.$$

The position vector at the end of the system A crank is

$$\mathbf{r}_A = R_A(\mathbf{i}\cos\alpha_A + \mathbf{j}\sin\alpha_A) = 75\mathbf{i} + 129.9\mathbf{j} \text{ (mm)}.$$

The angle of the linkage from the end of the system B crank with respect to the horizontal is

$$\beta = \tan^{-1}\left(\frac{y_A - y_B}{x_A - x_B + 350}\right) = -17.19°.$$

The unit vector parallel to the linkage, originating at the B crank, is

$$\mathbf{e}_{BA} = \mathbf{i}\cos\beta + \mathbf{j}\sin\beta = 0.9553\mathbf{i} - 0.2955\mathbf{j}.$$

The unit vector originating at A crank is $\mathbf{e}_{AB} = -\mathbf{e}_{BA}$. The components of the forces in the linkage are $|\mathbf{F}|\mathbf{e}_{AB}$, and $|\mathbf{F}|\mathbf{e}_{BA}$.
System B: When the system is in equilibrium,

$$M_B + |\mathbf{F}|\begin{vmatrix} 0 & 0 & 1 \\ 85.5 & 234.9 & 0 \\ 0.9553 & -0.2955 & 0 \end{vmatrix} = 0,$$

from which $M_B = 249.7|\mathbf{F}|$.
System A: When the system is in equilibrium:

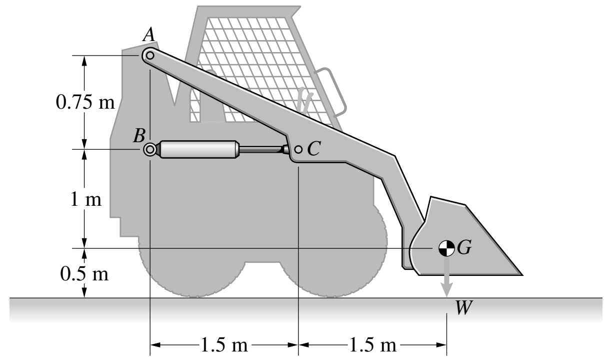
$$M_A + |\mathbf{F}|\begin{vmatrix} 0 & 0 & 1 \\ 75 & 129.9 & 0 \\ -0.9553 & 0.2955 & 0 \end{vmatrix} = 0,$$

from which $M_A = -146.27|\mathbf{F}|$.

Complete system: Both systems are in equilibrium for the value $|\mathbf{F}|$. Take the ratio of the two moments to eliminate $|\mathbf{F}|$.

$$\frac{M_A}{M_B} = -\frac{146.27}{249.7} = -0.5858$$

Problem 6.94 A load $W = 2$ kN is supported by the members ACG and the hydraulic actuator BC. Determine the reactions at A and the compressive axial force in the actuator BC.



Solution: The sum of the moments about A is

$$\sum M_A = 0.75BC - 3(2) = 0,$$

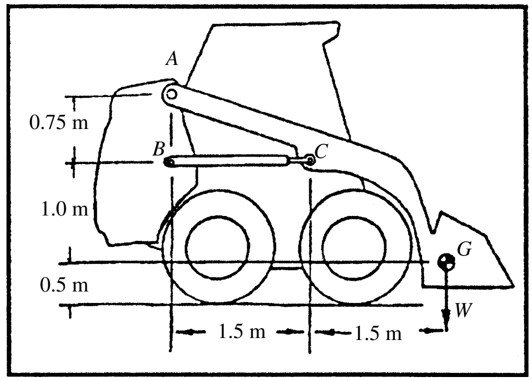
from which $BC = 8$ kN is the axial force. The sum of the forces

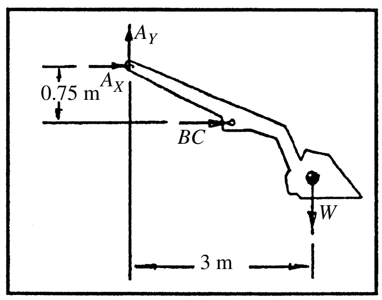
$$\sum F_X = A_X + BC = 0,$$

from which $A_X = -8$ kN.

$$\sum F_Y = A_Y - 2 = 0,$$

from which $A_Y = 2$ kN.





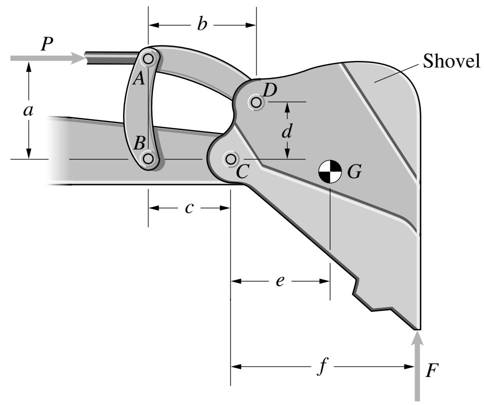
Problem 6.95 The dimensions are $a = 260$ mm, $b = 300$ mm, $c = 200$ mm, $d = 150$ mm, $e = 300$ mm, and $f = 520$ mm. The ground exerts a vertical force $F = 7000$ N on the shovel. The mass of the shovel is 90 kg and its weight acts at G. The weights of the links AB and AD are negligible. Determine the horizontal force P exerted at A by the hydraulic piston and the reactions on the shovel at C.

Solution: The free-body diagram of the shovel is from which we obtain the equations

$$\sum F_x = C_x - T\cos\beta = 0, \quad (1)$$

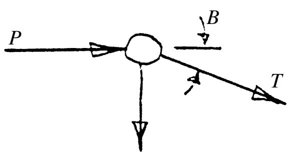
$$\sum F_y = C_y + T\sin\beta + F - mg = 0, \quad (2)$$

$$\sum M_{(\text{pt}C)} = fF - emg + (b-c)T\sin\beta + dT\cos\beta = 0. \quad (3)$$



The angle $\beta = \arctan[(a-d)/b]$.

From the free-body diagram of joint A,



we obtain the equation

$$\sum F = P + T\cos\beta = 0. \quad (4)$$

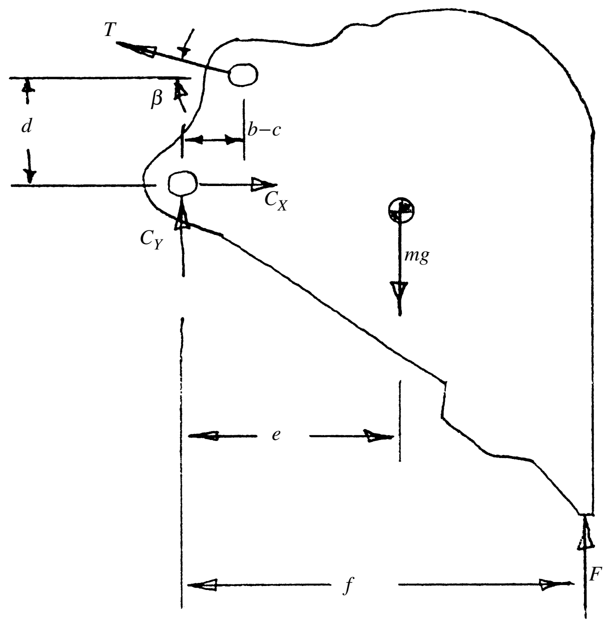
Substituting the given information into Eqs. (1)–(4) and solving, we obtain

$T = -19,260$ N,

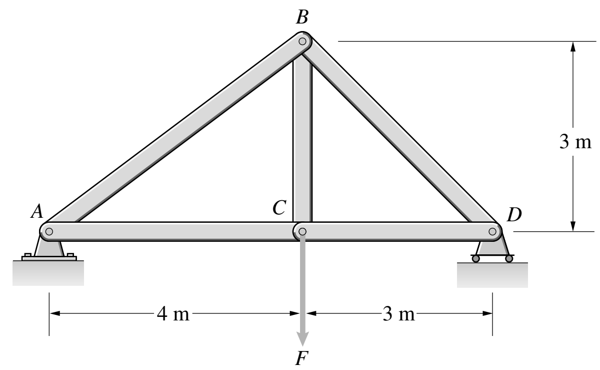
$P = 18,080$ N,

$C_x = -18,080$ N,

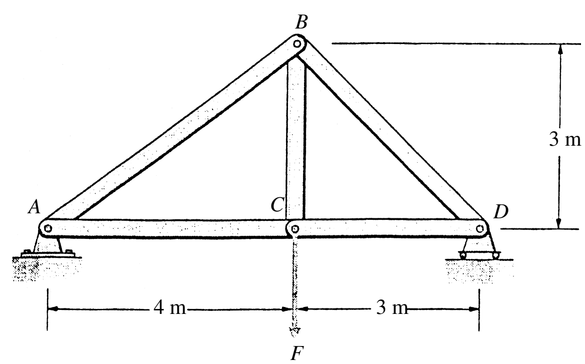
and $C_y = 513$ N.



Problem 6.96 The truss supports a load $F = 10$ kN. Determine the axial forces in the members AB, AC, and BC.



Solution: Find the support reactions at A and D.



$$\sum F_x: \quad A_x = 0$$

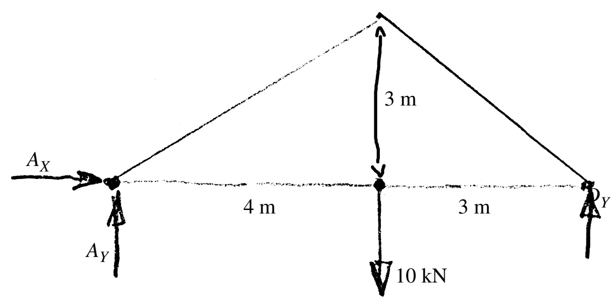
$$\sum F_y: \quad A_y + D_y - 10 = 0$$

$$\circlearrowleft + \sum M_A: \quad (-4)(10) + 7D_y = 0$$

Solving, $A_x = 0,$

$A_y = 4.29$ kN

$D_y = 5.71$ kN

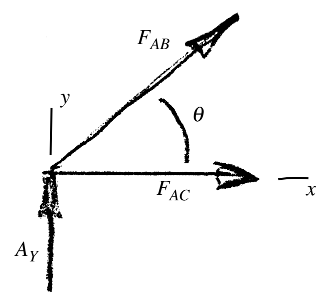


Joint A:

$$\tan\theta = \frac{3}{4}$$

$$\theta = 36.87^\circ$$

$$(A_y = 4.29 \text{ kN})$$



$$\sum F_x: \quad F_{AB}\cos\theta + F_{AC} = 0$$

$$\sum F_y: \quad A_y + F_{AB}\sin\theta = 0$$

Solving, $F_{AB} = -7.14$ kN (C)

$F_{AC} = 5.71$ kN (T)

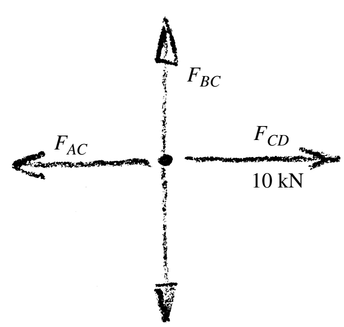
Joint C:

$$\sum F_x: \quad F_{CD} - F_{AC} = 0$$

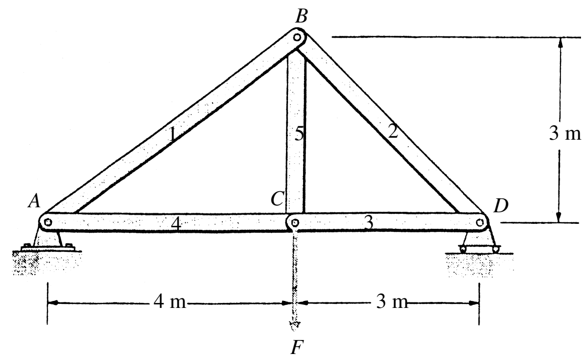
$$\sum F_y: \quad F_{BC} - 10 \text{ kN} = 0$$

Solving $F_{BC} = 10$ kN (T)

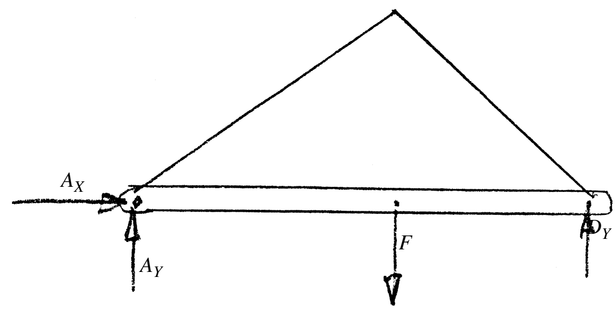
$F_{CD} = +5.71$ kN (T)



Problem 6.97 Each member of the truss shown in Problem 6.96 will safely support a tensile force of 40 kN and a compressive force of 32 kN. Based on this criterion, what is the largest downward load F that can safely be applied at C?



Solution: Assume a unit load F and find the magnitudes of the tensile and compressive loads in the truss. Then scale the load F up (along with the other loads) until either the tensile limit or the compressive limit is reached.



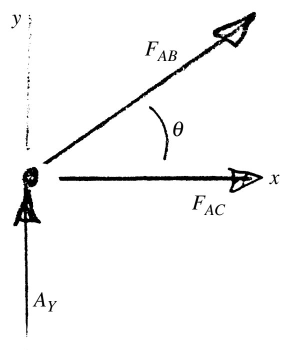
External Support Loads:

$$\sum F_x: \quad A_x = 0 \quad (1)$$

$$\sum F_y: \quad A_y + D_y - F = 0 \; (2)$$

$$\sum M_A: \quad -4F + 7D_y = 0 \quad (3)$$

Joint A:



$$\tan\theta = \frac{3}{2}$$

$$\theta = 36.87^\circ$$

$$\sum F_x: \quad F_{AC} + F_{AB}\cos\theta = 0 \; (4)$$

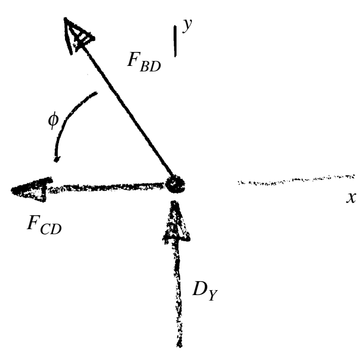
$$\sum F_y: \quad F_{AB}\sin\theta + A_y = 0 \quad (5)$$

Joint C

$$\sum F_x: \quad F_{CD} - F_{AC} = 0 \; (6)$$

$$\sum F_y: \quad F_{BC} - F = 0 \quad (7)$$

Joint D



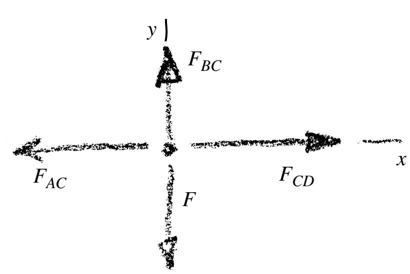
$$\tan\phi = \frac{3}{3}$$

$$\phi = 45^\circ$$

$$\sum F_x: \quad -F_{CD} - F_{BD}\cos\phi = 0 \; (8)$$

$$\sum F_y: \quad F_{BD}\sin\phi + D_y = 0 \quad (9)$$

Setting $F = 1$ and solving, we get the largest tensile load of 0.571 in AC and CD. The largest compressive load is 0.808 in member BD.



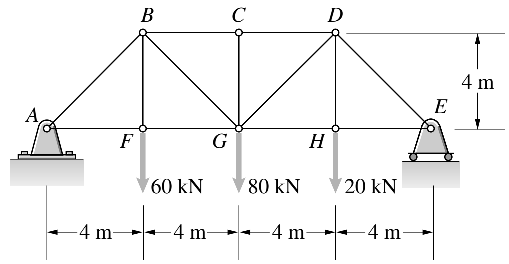
Largest Tensile is in member BC. $BC = F = 1$

The compressive load will be the limit

$$\frac{F_{\max}}{1} = \frac{32}{0.808}$$

$$F_{\max} = 40 \text{ kN}$$

Problem 6.98 The Pratt bridge truss supports loads at F, G, and H. Determine the axial forces in members BC, BG, and FG.



Solution: The angles of the cross-members are $\alpha = 45°$.

The complete structure as a free body:

The sum of the moments about A:

$$\sum M_A = -60(4) - 80(8) - 20(12) + 16E = 0,$$

from which $E = 70$ kN. The sum of the forces:

$$\sum F_x = A_x = 0.$$

$$\sum F_y = A_y - 60 - 80 - 20 + E = 0,$$

from which $A_y = 90$ kN

The method of joints: Joint A:

$$\sum F_Y = A_y + AB \sin\alpha = 0,$$

from which $AB = -127.3$ kN (C),

$$\sum F_x = AB\cos\alpha + AF = 0,$$

from which $AF = 90$ kN (T). *Joint F:*

$$\sum F_x = -AF + FG = 0,$$

from which $\boxed{FG = 90 \text{ kN } (T)}$.

$$\sum F_y = BF - 60 = 0,$$

from which $BF = 60$ kN (C). *Joint B:*

$$\sum F_x = -AB\cos\alpha + BC + BG\cos\alpha = 0,$$

and $\sum F_y = -AB\sin\alpha - BF - BG\sin\alpha = 0,$

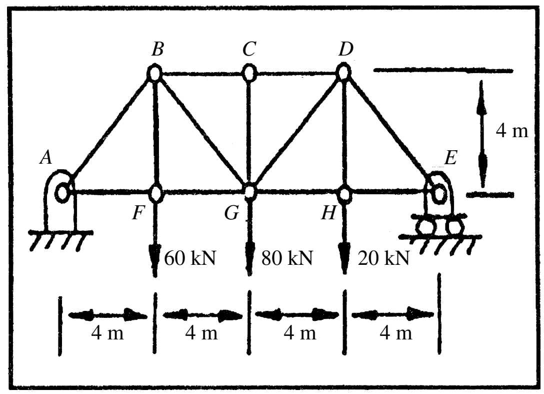
from which:

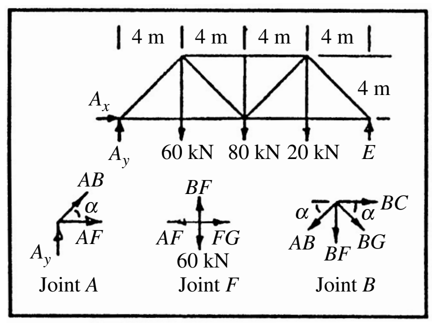
$$-AB\sin\alpha - BF - BG\sin\alpha = 0.$$

Solve: $\boxed{BG = 42.43 \text{ kN } (T)}$,

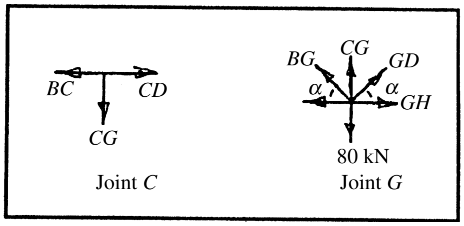
and $-AB\cos\alpha + BC + BG\cos\alpha = 0,$

from which $\boxed{BC = -120 \text{ kN } (C)}$





Problem 6.99 Consider the truss in Problem 6.98. Determine the axial forces in members CD, GD, and GH.



Solution: Use the results of the solution of Problem 6.98:

$BC = -120$ kN (C),

$BG = 42.43$ kN (T),

and $FG = 90$ kN (T).

The angle of the cross-members with the horizontal is $\alpha = 45°$.

Joint C:

$$\sum F_x = -BC + CD = 0,$$

from which $\boxed{CD = -120 \text{ kN } (C)}$

$$\sum F_Y = -CG = 0,$$

from which $CG = 0$.

Joint G:

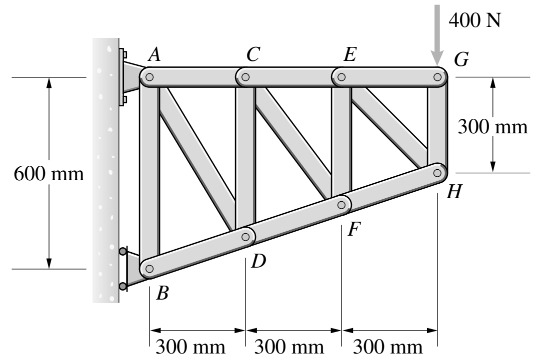
$$\sum F_y = BG \sin\alpha + GD \sin\alpha + CG - 80 = 0,$$

from which $\boxed{GD = 70.71 \text{ kN } (T)}$.

$$\sum F_y = -BG \cos\alpha + GD \cos\alpha - FG + GH = 0,$$

from which $\boxed{GH = 70 \text{ kN } (T)}$

Problem 6.100 The truss supports a 400-N load at G. Determine the axial forces in members AC, CD, and CF



Solution: *The complete structure as a free body:* The sum of the moments about A:

$$\sum M_A = -900(400) + 600B = 0,$$

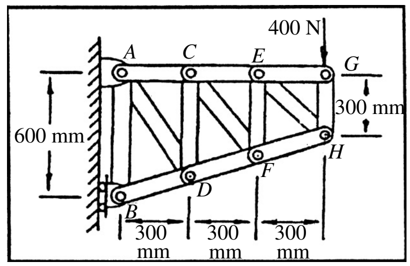
from which $B = 600$ N. The sum of forces:

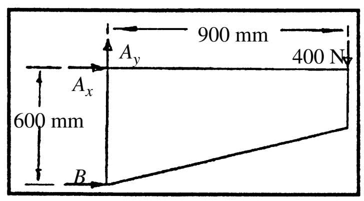
$$\sum F_x = A_x + B = 0,$$

from which $A_x = -600$ N.

$$\sum F_y = A_y - 400 = 0,$$

from which $A_y = 400$ N.





The method of joints: The angle from the horizontal of element BD is

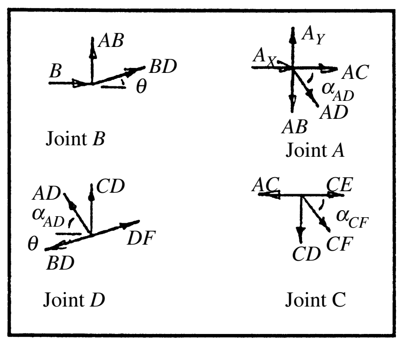
$$\theta = \tan^{-1}\left(\frac{300}{900}\right) = 18.43°.$$

The angle from the horizontal of element AD is

$$\alpha_{AD} = 90 - \tan^{-1}\left(\frac{300}{600 - 300\tan\theta}\right) = 59.04°.$$

The angle from the horizontal of element CF is

$$\alpha_{CF} = 90 - \tan^{-1}\left(\frac{300}{600(1 - \tan\theta)}\right) = 53.13°.$$



Joint B:

$$\sum F_x = B + BD\cos\theta = 0,$$

from which $BD = -632.5$ N (C)

$$\sum F_y = AB + BD\sin\theta = 0,$$

from which $AB = 200$ N (T)

Joint A:

$$\sum F_y = A_y - AD\sin\alpha_{AD} - AB = 0,$$

from which $AD = 233.2$ N (T)

$$\sum F_x = A_x + AC + AD\cos\alpha_{AD} = 0,$$

from which $\boxed{AC = 480 \text{ N } (T)}$

Joint D:

$$\sum F_x = -AD\cos\alpha_{AD} - BD\cos\theta + DF\cos\theta = 0,$$

from which $DF = -505.96$ N (C)

$$\sum F_y = AD\sin\alpha_{AD} + CD - BD\sin\theta + DF\sin\theta = 0,$$

from which $\boxed{CD = -240 \text{ N } (C)}$

Joint C:

$$\sum F_y = -CD - CF\sin\alpha_{CF} = 0,$$

from which $\boxed{CF = 300 \text{ N } (T)}$

Problem 6.101 Consider the truss in Problem 6.100. Determine the axial forces in members CE, EF, and EH.

Solution: Use the results of the solution of Problem 6.100:

$$AC = 480\text{ N }(T),$$

$$CF = 300\text{ N }(T),$$

$$DF = -505.96\text{ N }(C),$$

$$\theta = 18.4^\circ,$$

$$\alpha_{CF} = 53.1^\circ.$$

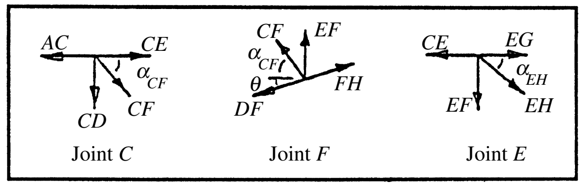
The method of joints: The angle from the horizontal of element EH is

$$\alpha_{EH} = 90 - \tan^{-1}\left(\frac{300}{600 - 900\tan\theta}\right) = 45^\circ$$

Joint C:

$$\sum F_x = -AC + CE + CF\cos\alpha_{CF} = 0,$$

from which $\boxed{CE = 300\text{ N }(T)}$



Joint F:

$$\sum F_y = -CF\cos\alpha_{CF} - DF\cos\theta + FH\cos\theta = 0,$$

from which $FH = -316.2\text{ N }(C)$

$$\sum F_y = EF + CF\sin\alpha_{CF} - DF\sin\theta + FH\sin\theta = 0,$$

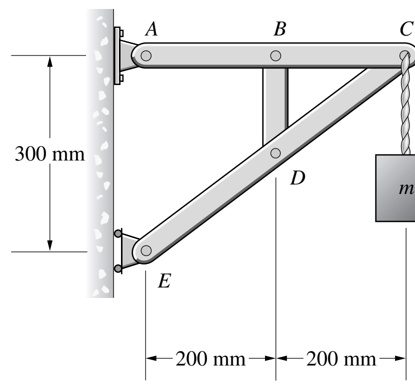
from which $\boxed{EF = -300\text{ N }(C)}$

Joint E:

$$\sum F_y = -EH\sin\alpha_{EH} - EF = 0,$$

from which $\boxed{EH = 424.3\text{ N }(T)}$

Problem 6.102 The mass $m = 120$ kg. Determine the forces on member ABC.



Solution: The weight of the hanging mass is given by

$$W = mg = 120\text{ kg}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 1177\text{ N}.$$

The complete structure as a free body: The equilibrium equations are:

$$\sum F_X = A_X + E_X = 0,$$

$$\sum F_Y = A_Y - W = 0,$$

and $\sum M_A = 0.3E_X - 0.4W = 0.$

Solving, we get

$$A_X = -1570\text{ N},$$

$$A_Y = 1177\text{ N},$$

and $E_X = 1570$ N.

Element ABC: The equilibrium equations are

$$\sum F_X = A_x + C_X = 0,$$

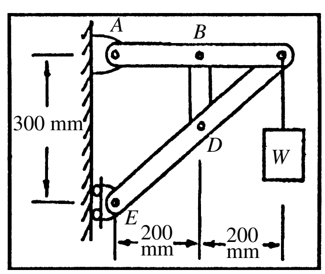
$$\sum F_Y = A_Y + C_Y - B_Y - W = 0,$$

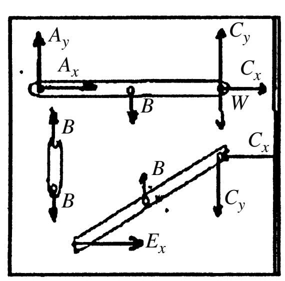
and: $\sum M_A = -0.2B_Y + 0.4c_Y - 0.4W = 0.$

Solution gives $B_Y = 2354$ N (member BD is in tension),

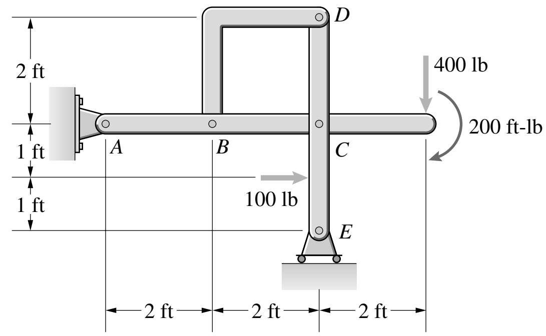
$$C_X = 1570\text{ N},$$

and $C_Y = 2354$ N.





Problem 6.103 Determine the forces on member ABC, presenting your answers as shown in Fig. 6.35.



Solution: *The complete structure as a free body:* The sum of the moments:

$$\sum M_A = 100(1) - 400(6) - 200 + 4E = 0,$$

from which $E = 625$ lb. The sum of the forces:

$$\sum F_y = A_y + E - 400 = 0,$$

from which $A_y = -225$ lb.

$$\sum F_x = A_x + 100 = 0,$$

from which $A_x = -100$ lb. These results are used as a check on the solution below.

Element ECD: (See the free body diagram.) The sum of the moments about E:

$$\sum M_E = -4D_x - 2C_x - 100 = 0,$$

from which (1) $4D_x + 2C_x = -100$. The sum of the forces:

$$\sum F_x = D_x + C_x + 100 = 0,$$

from which (2) $D_x + C_x = -100$.

$$\sum F_y = E + C_y + D_y = 0,$$

thus (3) $D_y + C_y + E = 0$.

Element BD: The sum of the moments about B:

$$\sum M_B = 2D_x - 2D_y = 0,$$

from which (4) $D_x - D_y = 0$. The sum of the forces:

$$\sum F_x = B_x - D_x = 0,$$

from which (5) $B_x - D_x = 0$.

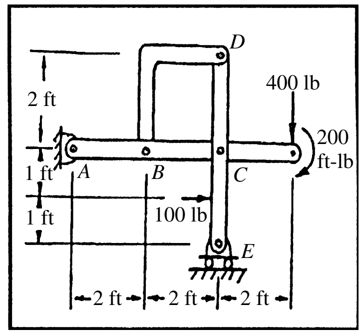
$$\sum F_y = B_y - D_y = 0,$$

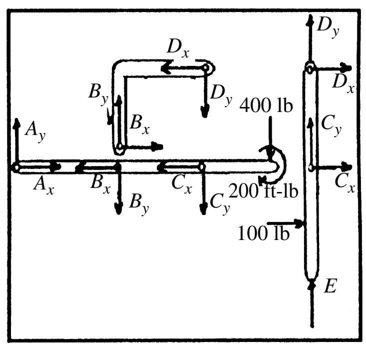
from which (6) $B_y - D_y = 0$

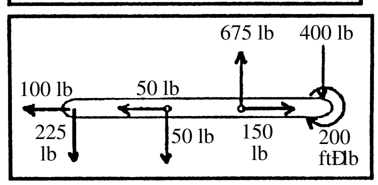
Element ABC: The sum of the moments about A:

$$\sum M_A = -2B_y - 4C_y - 200 - 6(400) = 0,$$

from which (7) $B_y + 2C_y = -1300$.







The sum of the forces:

$$\sum F_x = A_x - B_x - C_x = 0,$$

from which (8) $A_x - B_x - C_x = 0$.

$$\sum F_y = A_y - B_y - C_y - 400 = 0,$$

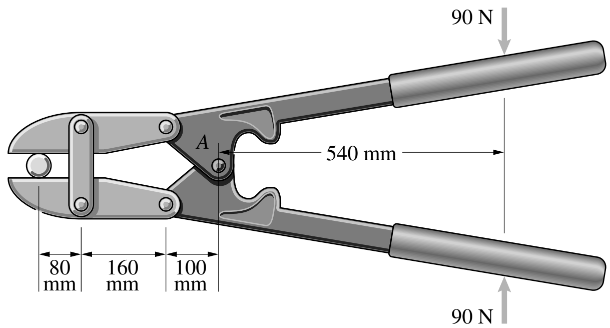
from which (9) $A_y - B_y - C_y = 400$. These nine equations are solved for the nine reactions The reactions are $D_X = 50$ lb, $D_Y = 50$ lb,:

$$C_X = -150 \text{ lb}, \ C_Y = -675 \text{ lb}, \ B_X = 50 \text{ lb},$$

$$B_Y = 50 \text{ lb}, \ A_X = -100 \text{ lb}, \ A_Y = -225 \text{ lb},$$

and $E = 625$ lb.

Problem 6.104 Determine the force exerted on the bolt by the bolt cutters and the magnitude of the force the members exert on each other at the pin connection A.



Solution: *Element AB:* The moment about A is

$$\sum M_A = -10B - 54F = 0,$$

where $F = 90$ N. From which $B = -486$ N. The sum of the forces:

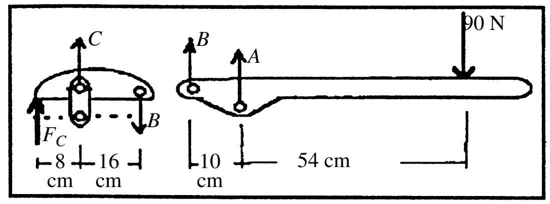
$$\sum F_y = A + B - F = 0,$$

from which $\boxed{A = 576 \text{ N}}$

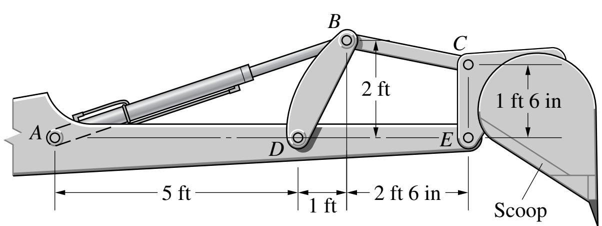
Element BC: The moment about C:

$$\sum M_C = -16B - 8F_C = 0,$$

from which the cutting force is $\boxed{F_C = 972 \text{ N}}$



Problem 6.105 The 600-lb weight of the scoop acts at a point 1 ft 6 in. to the right of the vertical line CE. The line ADE is horizontal. The hydraulic actuator AB can be treated as a two-force member. Determine the axial force in the hydraulic actuator AB and the forces exerted on the scoop at C and E.

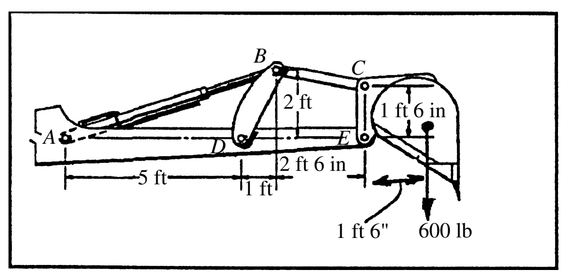


Solution: The free body diagrams are shown at the right. Place the coordinate origin at A with the x axis horizontal. The coordinates (in ft) of the points necessary to write the needed unit vectors are A (0, 0), B (6, 2), C (8.5, 1.5), and D (5, 0). The unit vectors needed for this problem are

$$\mathbf{u}_{BA} = -0.949\mathbf{i} - 0.316\mathbf{j},$$

$$\mathbf{u}_{BC} = 0.981\mathbf{i} - 0.196\mathbf{j},$$

and $\mathbf{u}_{BD} = -0.447\mathbf{i} - 0.894\mathbf{j}$.

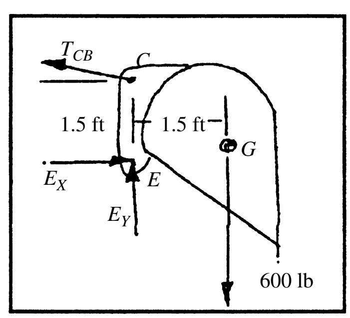


The scoop: The equilibrium equations for the scoop are

$$\sum F_X = -T_{CB} u_{BCX} + E_X = 0,$$

$$\sum F_Y = -T_{CB} u_{BCY} + E_Y - 600 = 0,$$

and $\sum M_C = 1.5E_X - 1.5(600 \text{ lb}) = 0$.



Solving, we get

$$E_X = 600 \text{ lb},$$

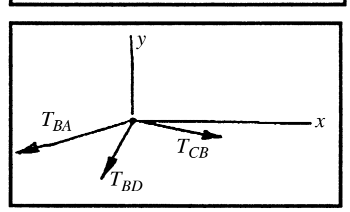
$$E_Y = 480 \text{ lb},$$

and $T_{CB} = 611.9$ lb.

Joint B: The equilibrium equations for the scoop are

$$\sum F_X = T_{BA} u_{BAX} + T_{BD} u_{BDX} + T_{CB} u_{BCX} = 0,$$

and $\sum F_Y = T_{BA} u_{BAY} + T_{BD} u_{BDY} + T_{CB} u_{BCY} = 0$.



Solving, we get

$$T_{BA} = 835 \text{ lb},$$

and $T_{BD} = -429$ lb.