The Distribution Normality

Summary	Goals	Participant Handouts
This is a basic overview of what is a normal distribution and how to use the tables to find probabilities.	• Understand what is a normal distribution • Practice using the normal distribution tables	1. The distribution Normality 2. The Standard Normal Distribution Table

Materials	Technology	Source	Estimated Time
Paper Rulers Colored Pencils	LCD Projector Facilitator Laptop Excel	Math is Fun website	60 minutes

Mathematics Standards

Common Core State Standards for Mathematics
MAFS.912.S-ID.1: Summarize, represent, and interpret data on a single count or measurement variable 1.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

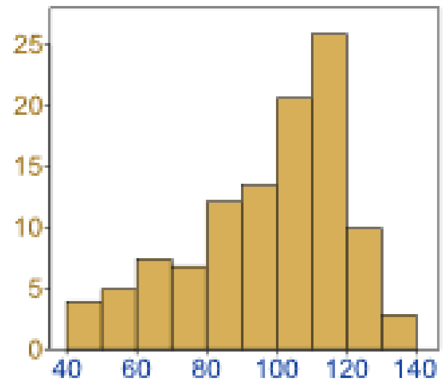
Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use tools appropriately

Instructional Plan

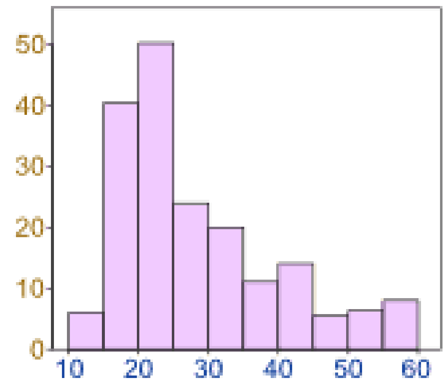
This is a basic overview of what a normal distribution is and how to use the tables to find probabilities. Use the Power point to go over some of the main concepts of a normal distribution

Data can be "distributed" (spread out) in different ways.

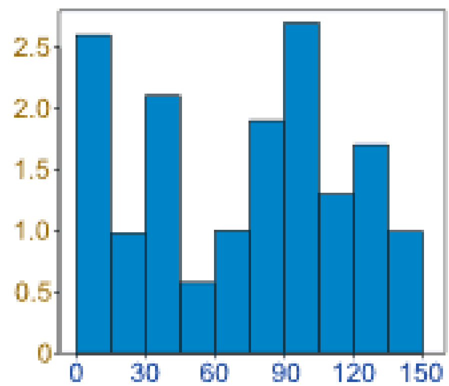
It can be spread out more on the left



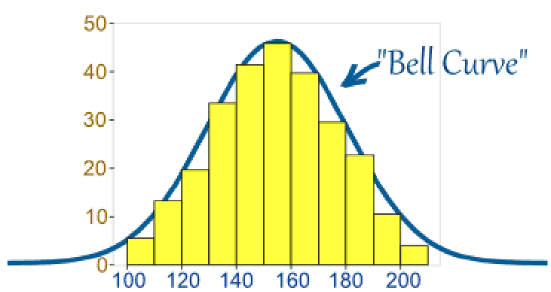
Or more on the right



Or it can be all jumbled up



But there are many cases where the data tends to be around a central value with no bias left or right, and it gets close to a "Normal Distribution" like this:



A Normal Distribution

The "Bell Curve" is a Normal Distribution. The yellow histogram shows some data that follows it closely, but not perfectly (which is usual).

It is often called a "Bell Curve" because it looks like a bell.

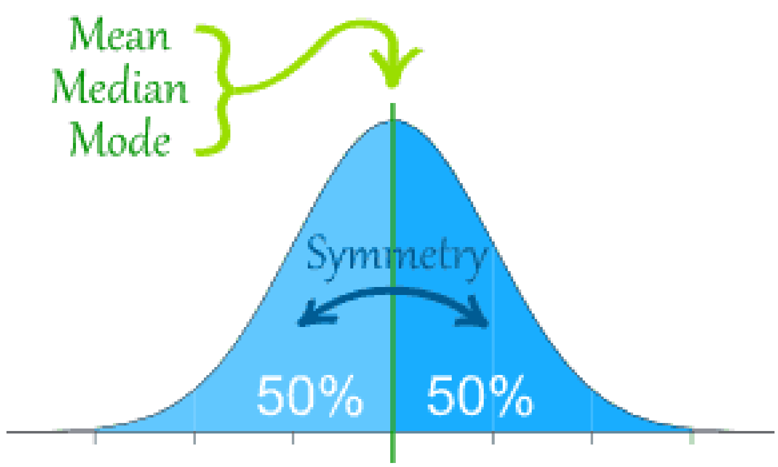
Many things closely follow a Normal Distribution:

- heights of people
- size of things produced by machines
- errors in measurements
- blood pressure
- marks on a test

We say the data is "normally distributed".

The Normal Distribution has:

- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean

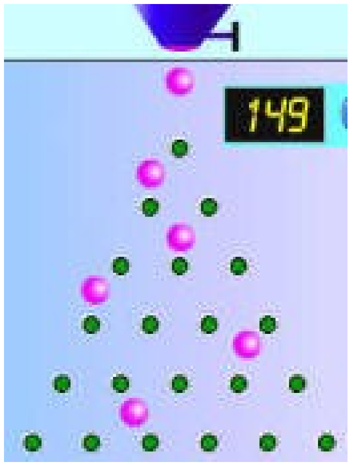


Quincunx (http://www.mathsisfun.com/data/quincunx.html)

You can see a normal distribution being created by random chance!

It is called the [Quincunx](http://www.mathsisfun.com/data/quincunx.html) and it is an amazing machine.

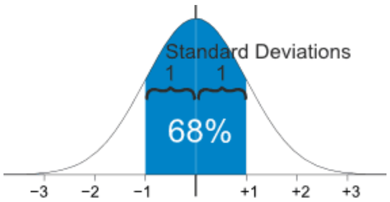
Have a play with it!



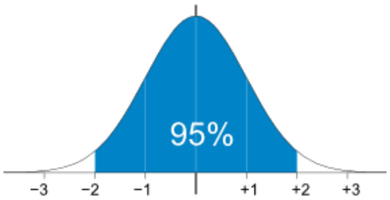
Standard Deviations

The Standard Deviation is a measure of how spread out numbers are.

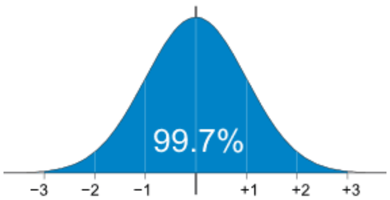
When you calculate the standard deviation of your data, you will find that (generally):



68% of values are within
1 standard deviation of the mean



95% of values are within
2 standard deviations of the mean



99.7% of values are within
3 standard deviations of the mean

Example: 95% of students at school are between 1.1m and 1.7m tall.

Assuming this data is **normally distributed** can you calculate the mean and standard deviation?

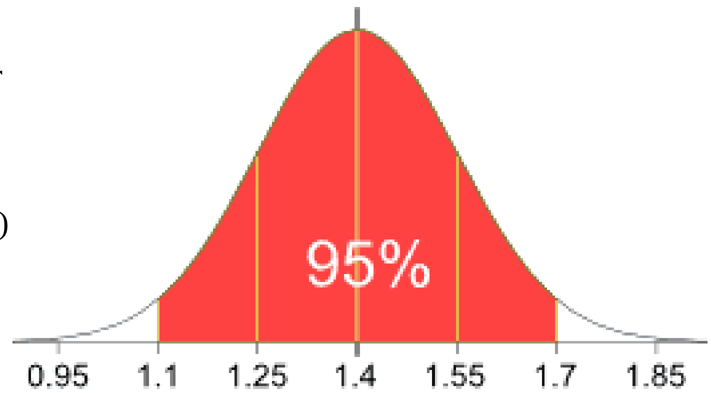
The mean is halfway between 1.1m and 1.7m:

Mean = (1.1m + 1.7m) / 2 = **1.4m**

95% is 2 standard deviations either side of the mean (a total of 4 standard deviations) so:

1 standard deviation = (1.7m-1.1m) / 4
= 0.6m / 4 = **0.15m**

And this is the result:



It is good to know the standard deviation, because we can say that any value is:

- **likely** to be within 1 standard deviation (68 out of 100 should be)
- **very likely** to be within 2 standard deviations (95 out of 100 should be)
- **almost certainly** within 3 standard deviations (997 out of 1000 should be)

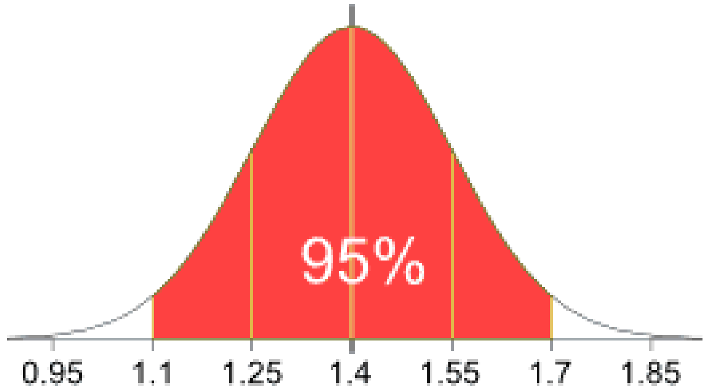
Standard Scores

The number of **standard deviations from the mean** is also called the "Standard Score" or "z-score".

Example: In that same school one of the students is 1.85m tall

You can see on the bell curve that 1.85m is **3 standard deviations** from the mean of 1.4, so:

The student's height has a "z-score" of 3.0



It is also possible to **calculate** how many standard deviations 1.85 is from the mean

How far is 1.85 from the mean?

It is 1.85 - 1.4 = **0.45m from the mean**

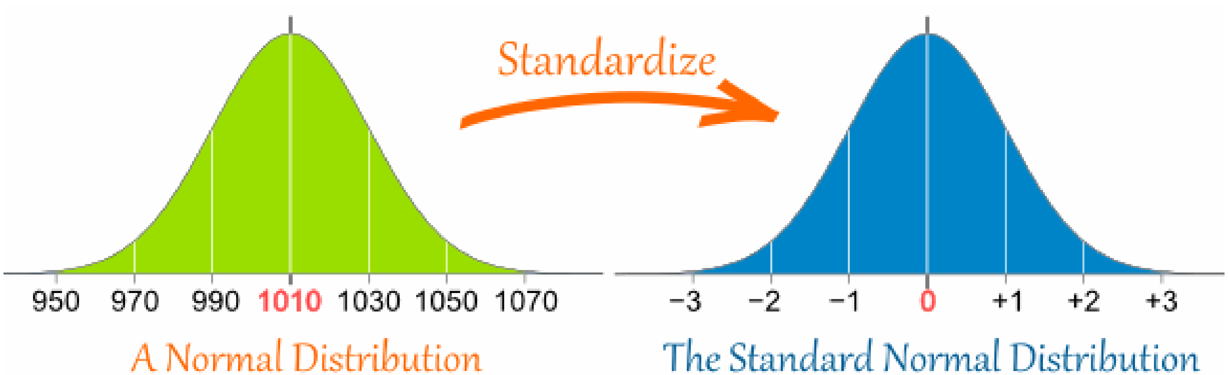
How many standard deviations is that? The standard deviation is 0.15m, so:

0.45m / 0.15m = **3 standard deviations**

So to convert a value to a Standard Score ("z-score"):

- first subtract the mean,
- then divide by the Standard Deviation

And doing that is called "Standardizing":



You can take any Normal Distribution and convert it to The Standard Normal Distribution.

Example: Travel Time

A survey of daily travel time had these results (in minutes):

26, 33, 65, 28, 34, 55, 25, 44, 50, 36, 26, 37, 43, 62, 35, 38, 45, 32, 28, 34

The **Mean is 38.8 minutes**, and the **Standard Deviation is 11.4 minutes** (you can copy and paste the values into the Standard Deviation Calculator if you want). (http://www.mathsisfun.com/data/standard-deviation-calculator.html)

Convert the values to z-scores ("standard scores").

To convert **26**:

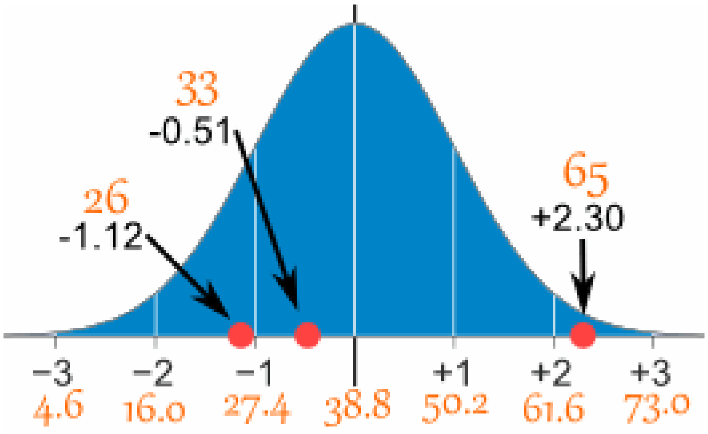
first subtract the mean: 26 - 38.8 = -12.8,
then divide by the Standard Deviation: -12.8/11.4 = -1.12

So **26** is **-1.12 Standard Deviations** from the Mean

Here are the first three conversions

Original Value	Calculation	Standard Score (z-score)
26	(26-38.8) / 11.4 =	**-1.12**
33	(33-38.8) / 11.4 =	**-0.51**
65	(65-38.8) / 11.4 =	**+2.30**
...	...	...

And here they are graphically:



You can calculate the rest of the z-scores yourself!

Here is the formula for z-score that we have been using:

$$z = \frac{x - \mu}{\sigma}$$

- **z** is the "z-score" (Standard Score)
- **x** is the value to be standardized
- **μ** is the mean
- **σ** is the standard deviation

Why Standardize ... ?

It can help you make decisions about your data.

Example: Professor Willoughby is marking a test.

Here are the students' results (out of 60 points):

20, 15, 26, 32, 18, 28, 35, 14, 26, 22, 17

Most students didn't even get 30 out of 60, and **most will fail**.

The test must have been really hard, so the Prof decides to Standardize all the scores and only fail people 1 standard deviation below the mean.

The **Mean is 23**, and the **Standard Deviation is 6.6**, and these are the Standard Scores:

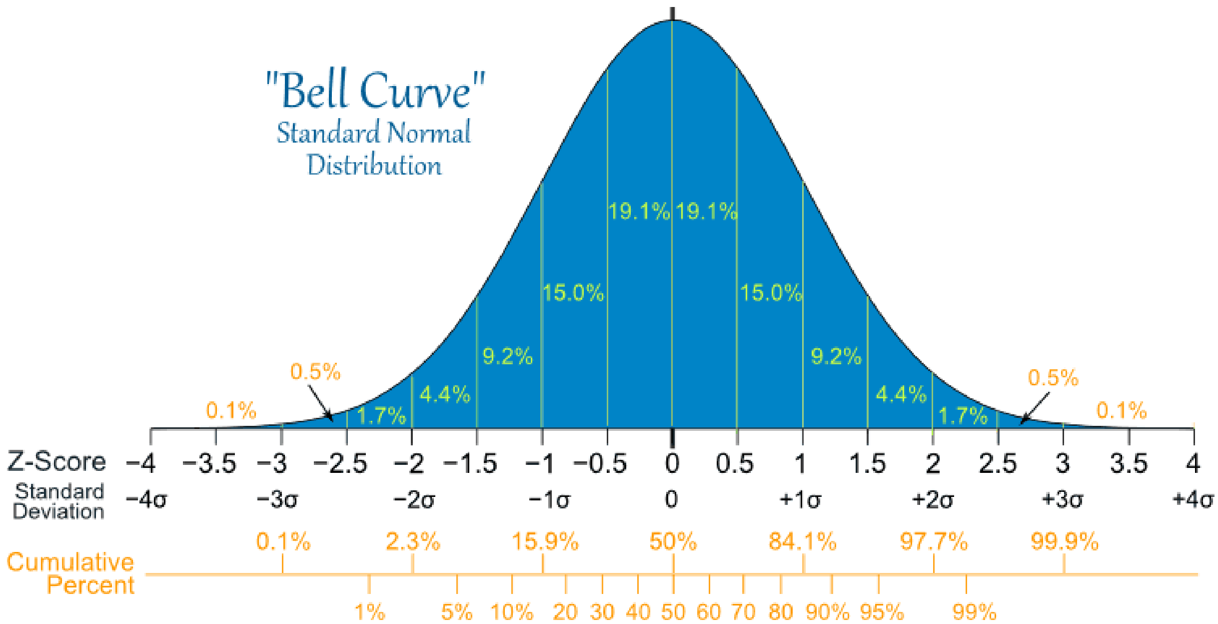
-0.45, -1.21, 0.45, 1.36, -0.76, 0.76, 1.82, -1.36, 0.45, -0.15, -0.91

Only 2 students will fail (the ones who scored 15 and 14 on the test)

It also makes life easier because we only need one table (the Standard Normal Distribution Table discuss shortly), rather than doing calculations individually for each value of mean and standard deviation.

In More Detail

Here is the Standard Normal Distribution with percentages for every **half of a standard deviation**, and cumulative percentages:



Example: Your score in a recent test was **0.5 standard deviations** above the average, how many people scored **lower** than you did?

- Between 0 and 0.5 is **19.1%**
- Less than 0 is **50%** (left half of the curve)

So the total less than you is:

50% + 19.1% = 69.1%

In theory **69.1% scored less** than you did (but with real data the percentage may be different)

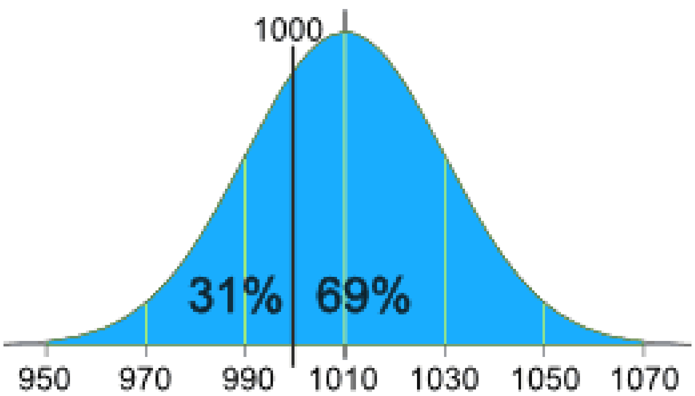
A Practical Example: Your company packages sugar in 1 kg bags.

When you weigh a sample of bags you get these results:

- 1007g, 1032g, 1002g, 983g, 1004g, ... (a hundred measurements)
- Mean = 1010g
- Standard Deviation = 20g

Some values are less than 1000g ... can you fix that?

The normal distribution of your measurements looks like this:



31% of the bags are less than 1000g,
which is cheating the customer!

Because it is a random thing we can't **stop** bags having less than 1000g, but we can try to **reduce it** a lot ...

- if 1000g was at -3 standard deviations there would be only **0.1%** (very small)
- at -2.5 standard deviations we can calculate:

  Below 3 is 0.1% and between 3 and 2.5 standard deviations is 0.5%, together that is 0.1%+0.5% = **0.6%**

So let us adjust the machine to have **1000g at 2.5 standard deviations** from the mean.

We could adjust it to:

- increase the amount of sugar in each bag (this would change the mean), or
- make it more accurate (this would reduce the standard deviation)

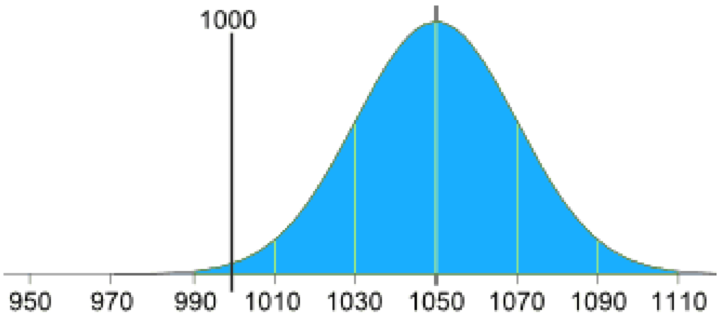
Let us try both:

Adjust the mean amount in each bag

The standard deviation is 20g, and we need 2.5 of them:

2.5 × 20g = 50g

So the machine should average **1050g**, like this:

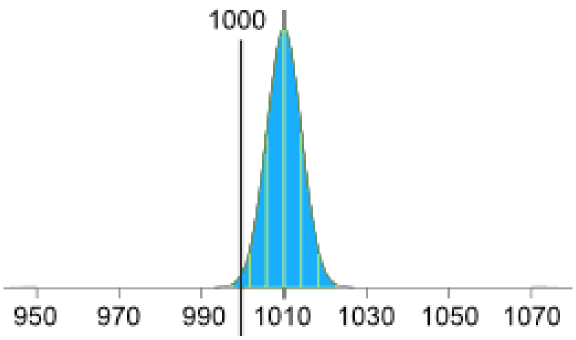


Adjust the accuracy of the machine

Or we can keep the same mean (of 1010g), but then we
need 2.5 standard deviations to be equal to 10g:

10g / 2.5 = 4g

So the standard deviation should be **4g**, like this:



(We hope the machine is that accurate!)

Or perhaps we could have some combination of better accuracy and slightly larger average size, I will leave that up to you!

<u>Practice Questions</u>

1. 95% of students at school weigh between 62 kg and 90 kg. Assuming this data is normally distributed, what are the mean and standard deviation?

A	Mean = 66 kg S.D. = 7 kg
B	Mean = 76 kg S.D. = 7 kg
C	Mean = 86 kg S.D. = 7 kg
D	Mean = 76 kg S.D. = 14 kg

2. A machine produces electrical components. 99.7% of the components have lengths between 1.176 cm and 1.224 cm.
Assuming this data is normally distributed, what are the mean and standard deviation?

A	Mean = 1.210 cm S.D. = 0.008 cm
B	Mean = 1.190 cm S.D. = 0.008 cm
C	Mean = 1.200 cm S.D. = 0.004 cm
D	Mean = 1.200 cm S.D. = 0.008 cm

3. 68% of the marks in a test are between 51 and 64. Assuming this data is normally distributed, what are the mean and standard deviation?

A	Mean = 57 S.D. = 6.5
B	Mean = 57 S.D. = 7
C	Mean = 57.5 S.D. = 6.5
D	Mean = 57.5 S.D. = 13

4. The Fresha Tea Company pack tea in bags marked as 250 g. A large number of packs of tea were weighed and the mean and standard deviation were calculated as 255 g and 2.5 g respectively.
Assuming this data is normally distributed, what percentage of packs are underweight?

A	2.5 %
B	3.5 %
C	4%
D	5%

5. Students pass a test if they score 50% or more. The marks of a large number of students were sampled and the mean and standard deviation were calculated as 42% and 8% respectively.

Assuming this data is normally distributed, what percentage of students pass the test?

A	5%
B	16 %
C	24 %
D	32 %

6. A company makes parts for a machine. The lengths of the parts must be within certain limits or they will be rejected. A large number of parts were measured and the mean and standard deviation were calculated as 3.1 m and 0.005 m respectively.

   Assuming this data is normally distributed and 99.7% of the parts were accepted, what are the limits?

A	Between 3.075 m and 3.125 m
B	Between 3.080 m and 3.120 m
C	Between 3.085 m and 3.115 m
D	Between 3.090 m and 3.110 m

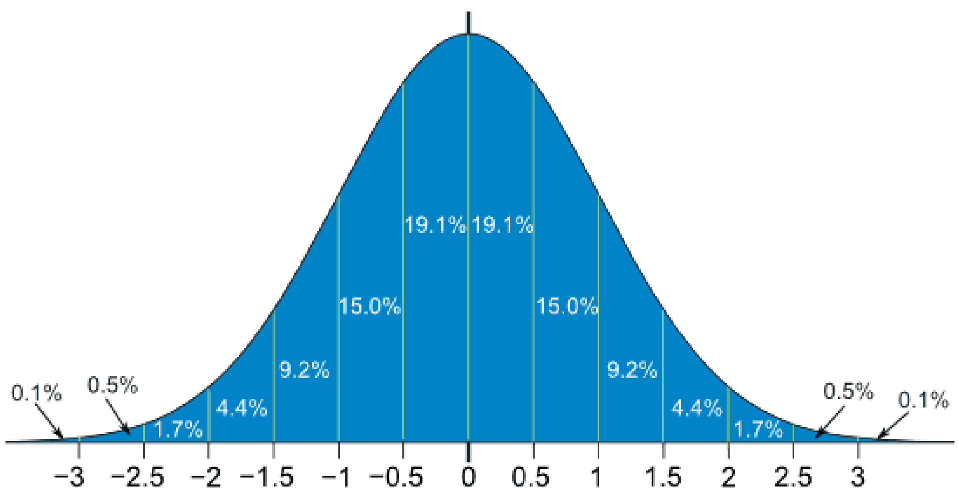
7. The mean June midday temperature in Desertville is 36°C and the standard deviation is 3°C.

Assuming this data is normally distributed, how many days in June would you expect the midday temperature to be between 39°C and 42°C?

A	3
B	4
C	7
D	1 4

8. The heights of male adults are Normally distributed with mean 1.7 m and standard deviation 0.2 m
In a population of 400 male adults, how many would you expect to have a height between 1.4 m and 1.6 m?

You can use this Standard Normal Distribution curve:



A	24
B	54

C	97
D	10 0

9. The mean July daily rainfall in Waterville is 10 mm and the standard deviation is 1.5 mm

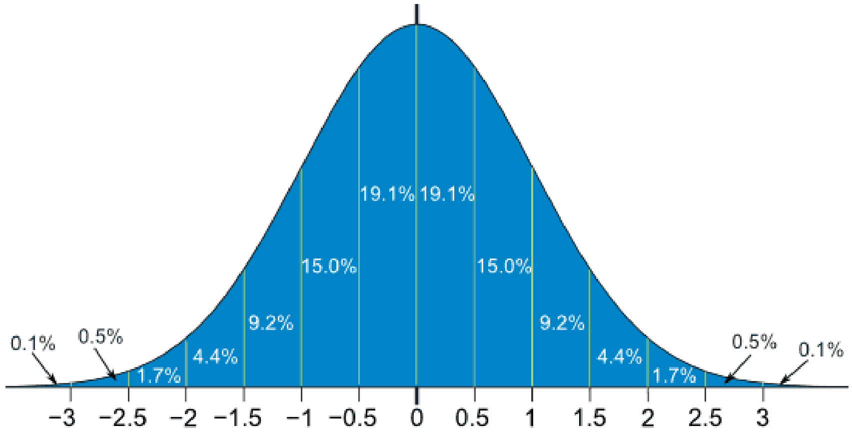
   Assume that this data is normally distributed.
   How many days in July would you expect the daily rainfall to be less than 8.5 mm?

A	5
B	6
C	7
D	1 0

10. The ages of the population of a town are Normally distributed with mean 43 and standard deviation 14

    The town has a population of 5,000.
    How many would you expect to be aged between 22 and 57?

You can use this Standard Normal Distribution curve:



A	1,750
B	3,410
C	3,870
D	4,330

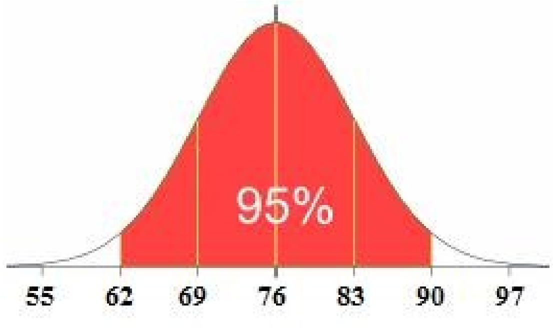
Answers:

1. The mean is halfway between 62 kg and 90 kg:

   Mean = (62 kg + 90 kg)/2 = 76 kg

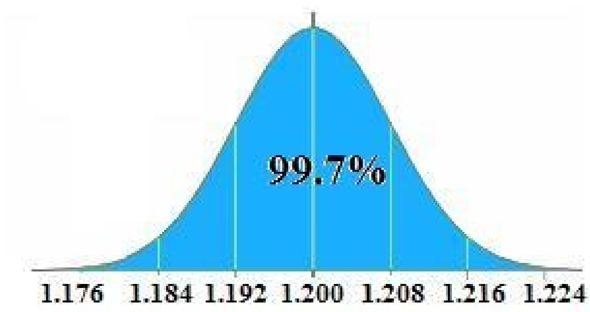
   95% is 2 standard deviations either side of the mean (a total of 4 standard deviations) so:
   1 standard deviation = (90 kg - 62 kg)/4 = 28 kg/4 = 7 kg

   And this is the result :



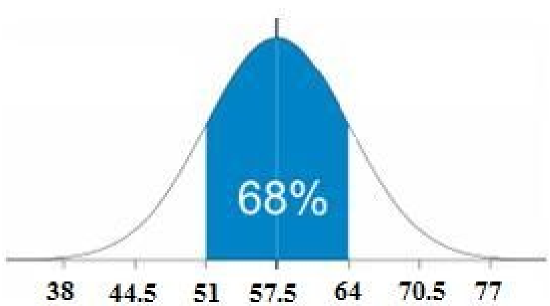
2. The mean is halfway between 1.176 cm and 1.224 cm:

   Mean = (1.176 cm + 1.224 cm)/2 = 1.200 cm
   99.7% is 3 standard deviations either side of the mean (a total of 6 standard deviations) so:
   1 standard deviation = (1.224 cm - 1.176 cm)/6 = 0.048 cm/6 = 0.008 cm
   And this is the result:

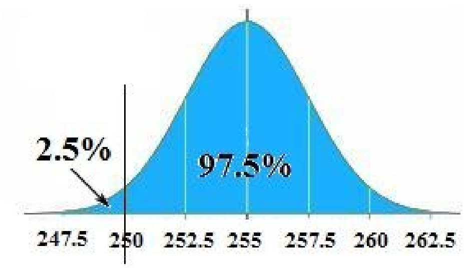


3. The mean is halfway between 51 and 64:

   Mean = (51 + 64)/2 = 57.5
   68% is 1 standard deviation either side of the mean (a total of 2 standard deviations) so:
   1 standard deviation = (64 - 51)/2 = 13/2 = 6.5
   And this is the result:

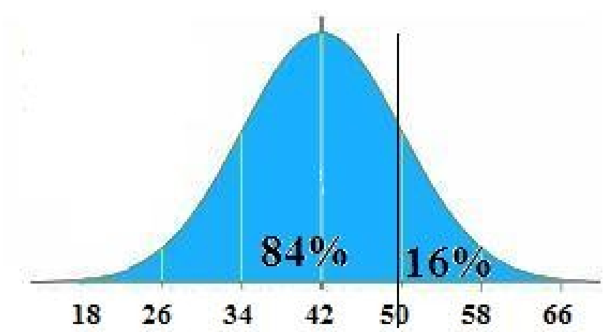


4. The following diagram shows 1, 2 and 3 standard deviations on either side of the mean:



250 g is two standard deviations below the mean
Since 95% of packs are within 2 standard deviations of the mean, it follows that 5% ÷ 2 = 2.5% of packs were underweight.

5. The following diagram shows 1, 2 and 3 standard deviations on either side of the mean:



50% is one standard deviation above the mean

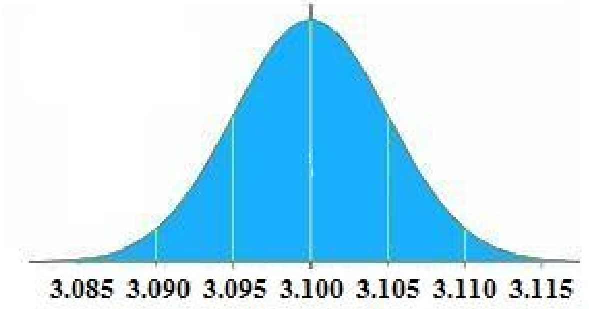
Since 68% of marks are within 1 standard deviation of the mean, it follows that 32% will be more than 1 standard deviation from the mean.

And the half of those that are on the high side will pass.

So 32% ÷ 2 = 16% of students pass.

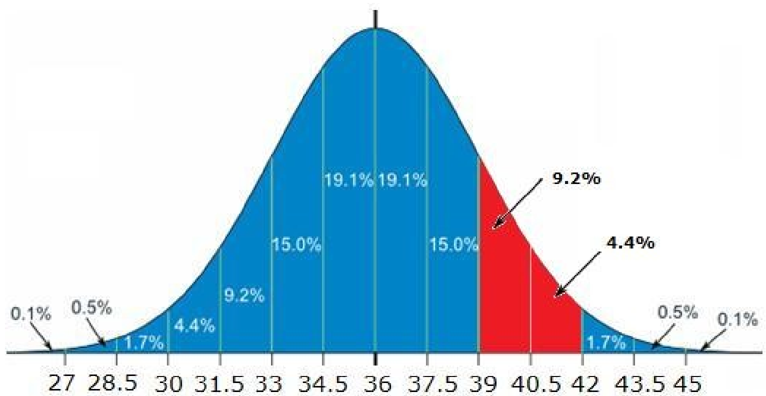
6. 99.7% is 3 standard deviations on either side of the mean.

The following diagram shows 1, 2 and 3 standard deviations on either side of the mean:



So the lengths of the parts must be between 3.085 m and 3.115 m

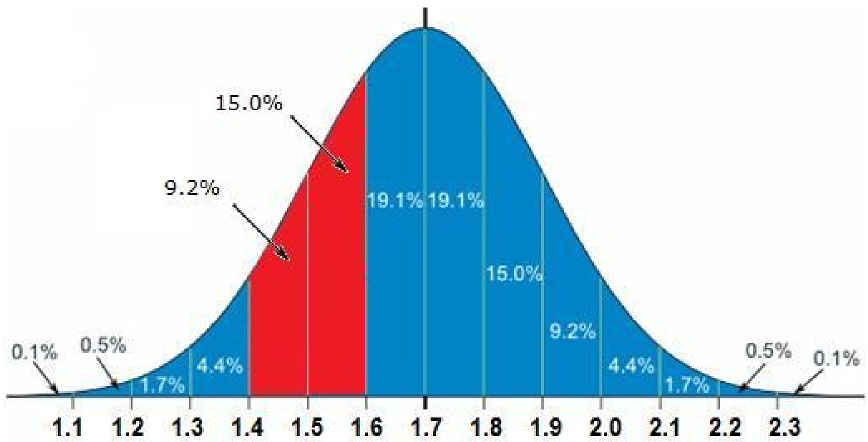
7. 39°C is one standard deviation above the mean and 42°C is two standard deviations above the mean:



Therefore we would expect the temperature to be between 39°C and 42°C on 13.6% of the days
(9.2% + 4.4% = 13.6%)

There are 30 days in June
13.6% of 30 = 4.08 = 4 to the nearest day

8. 1.6 m is half a standard deviation below the mean and 1.4 m is one and a half standard deviations below the mean:

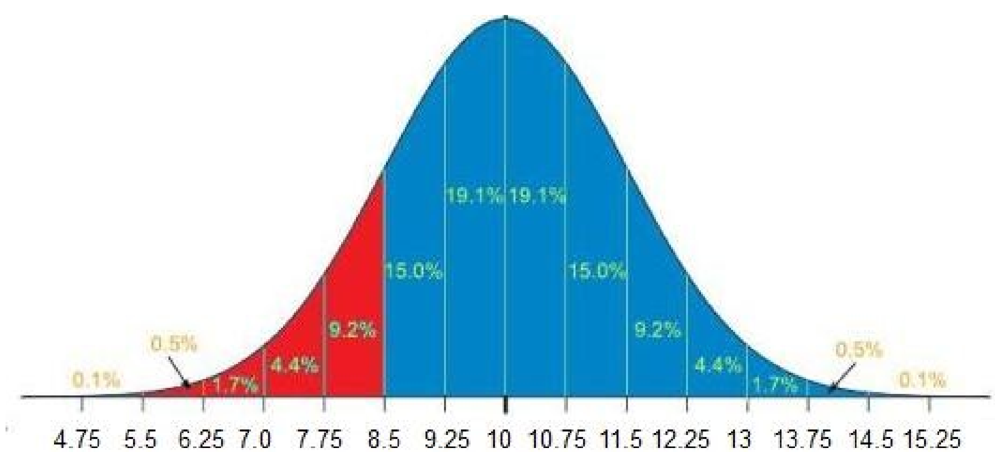


Therefore the percentage of male adults with heights between 1.4 m and 1.6 m = 15.0% + 9.2% = 24.2%

24.2% of 400 = 96.8

Therefore we would expect 97 male adults with heights between 1.4 m and 1.6 m

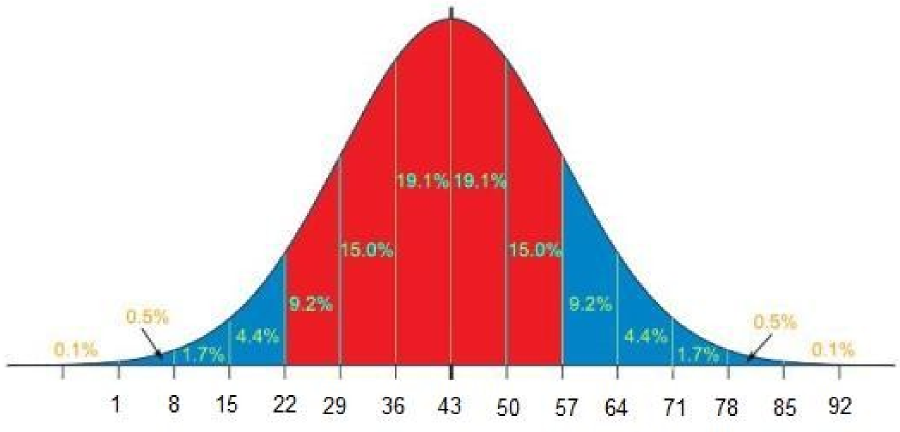
9. 8.5 mm is one standard deviation below the mean:



Therefore we would expect the rainfall to be less than 8.5 mm on 15.9% of the days
(9.2% + 4.4% + 1.7% + 0.5% + 0.1% = 15.9%)

There are 31 days in July
15.9% of 31 = 4.929 = 5 to the nearest day

10. 22 is one and a half a standard deviation below the mean and 57 is one standard deviations above the mean:



So, the percentage of the population aged between 22 and 57
= 9.2% + 15.0% + 19.1% + 19.1% + 15.0% = 77.4%

77.4% of 5,000 = 3,870

Finally, we would expect about 3,870 people to be aged between 22 and 57

The Standard Normal Distribution Table

You can get more accurate values from the table below. The table shows the area from 0 to Z.

Instead of one LONG table, we have put the "**0.1**"s running down, then the "**0.01**"s running along. (Example of how to use is below)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852

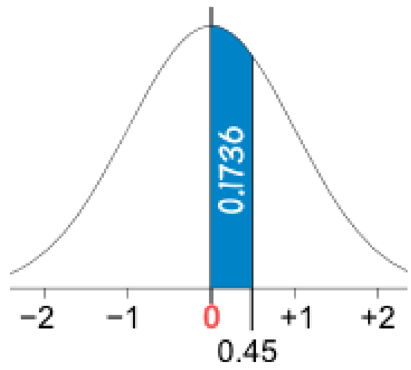
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Example: Percent of Population Between 0 and 0.45

Start at the row for 0.4, and read along until 0.45: there is the value 0.1736

And 0.1736 is **17.36%**

So 17.36% of the population are between 0 and 0.45 Standard Deviations from the Mean.



Because the curve is symmetrical, the same table can be used for values going either direction, so a negative 0.45 also has an area of 0.1736

Example: Percent of Population Z Between -1 and 2

From **−1 to 0** is the same as from **0 to +1**:

At the row for 1.0, first column 1.00, there is the value **0.3413**

From **0 to +2** is:

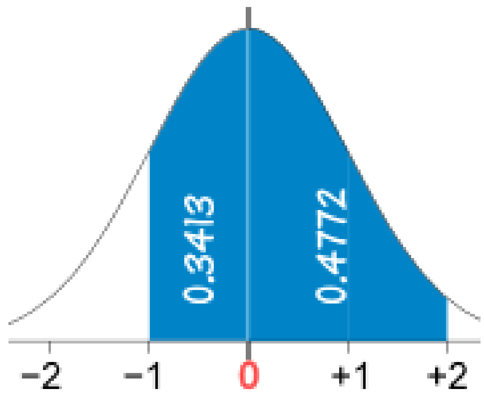
At the row for 2.0, first column 2.00, there is the value **0.4772**

Add the two to get the total between -1 and 2:

0.3413 + 0.4772 = **0.8185**

And **0.8185** is **81.85%**

So 81.85% of the population are between -1 and +2 Standard Deviations from the Mean.



Practice Problems

1. Use the Standard Normal Distribution table to find $P(0 < Z \leq 1)$

A	0.039 8
B	0.166 6
C	0.315

	9
D	0.341 3

2. Use the Standard Normal Distribution table to find P(Z ≤ 2)

A	0.022 8
B	0.477 2
C	0.977 2
D	1

3. Use the Standard Normal Distribution table to find P(-1.65 < Z ≤ 1.93)

A	0.923 7
B	0.922 8
C	0.916 5
D	0.022 7

4. Use the Standard Normal Distribution table to find P(0.85 < Z ≤ 2.23

A	0.1103
B	0.184 8
C	0.289 4
D	0.789 4

5. Use the Standard Normal Distribution table to find P(Z > 1.75)

A	0.540 1
B	0.459 9
C	0.040 9
D	0.040 1

6. Use the Standard Normal Distribution table to find P(Z ≤ -0.69)

A	0.245 1
B	0.254 9

C	0.745 1
D	0.755 1

7. Use the Standard Normal Distribution table to find P(-1.27 < Z ≤ 0).

A	0.102 0
B	0.398 0
C	0.602 0
D	0.898 0

8. Use this Standard Normal Distribution table to find P(Z > -2.64)

A	0.004 1
B	0.495 9
C	0.504 1
D	0.995 9

9. Use the Standard Normal Distribution table to find P(Z ≤ 0.96).

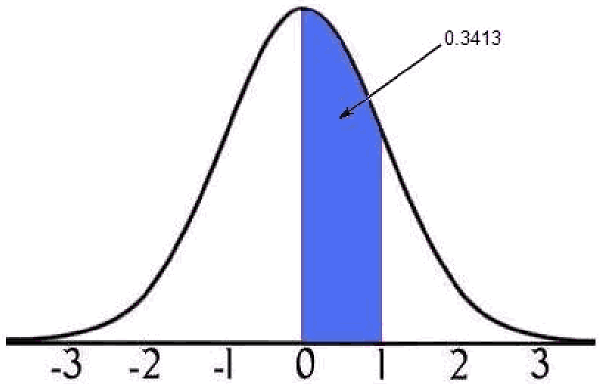
A	0.168 5
B	0.331 5
C	0.668 5
D	0.831 5

10. Use the Standard Normal Distribution table to find P(-2.31 < Z ≤ 0.82).

A	0.195 7
B	0.216 5
C	0.783 5
D	0.804 3

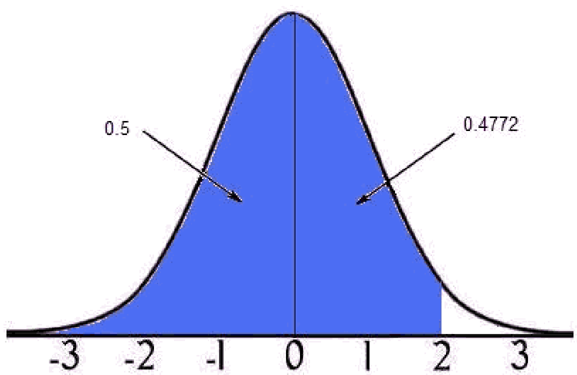
Answers:

1. $P(0 < Z \le 1) = 0.3413$



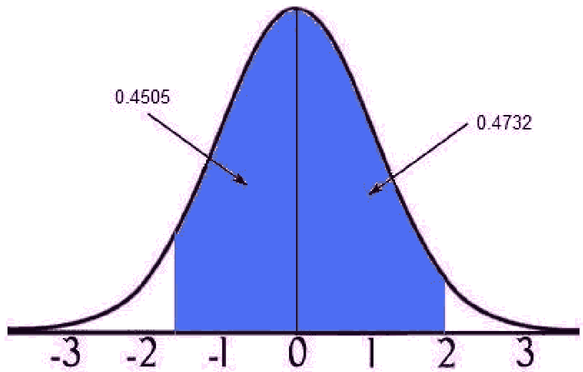
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621

2. P(Z ≤ 2) = P(-∞ < Z ≤0) + P(0 < Z ≤ 2) = 0.5 + 0.4772 = 0.9772



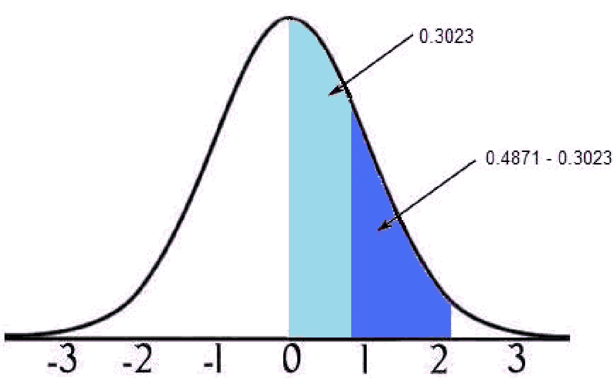
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817

3. P(-1.65 < Z ≤ 1.93) = 0.4505 + 0.4732 = 0.9237



1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817

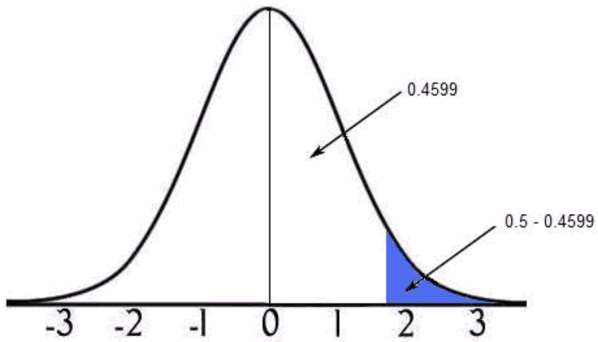
4. P(0.85 < Z ≤ 2.23) = 0.4871 - 0.3023 = 0.1848



0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890

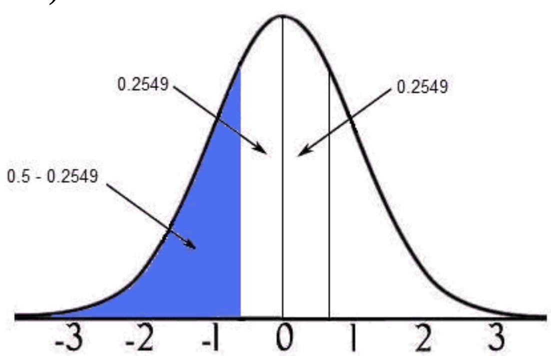
5. P(Z > 1.75) = 0.5 - 0.4599 = 0.0401

   (The 0.5 is the entire right half, and we need to take 0.4599 away from it to leave 0.0401)



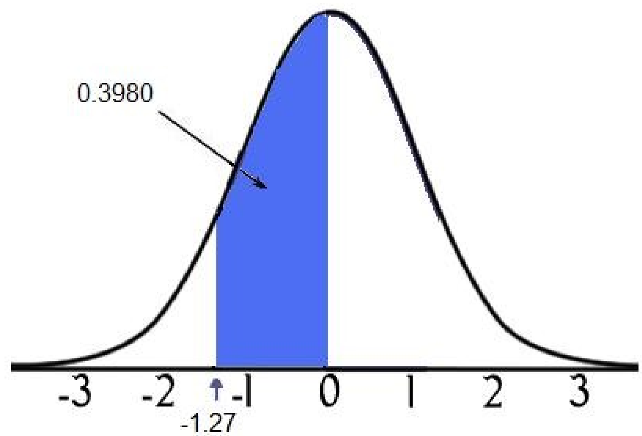
1.7	0.4554	0.4564	0.4573	0.4582	0.459	0.4599	0.4608	0.4616	0.4625	0.4633

6. P(Z ≤ -0.69) = 0.5 - 0.2549 = 0.2451



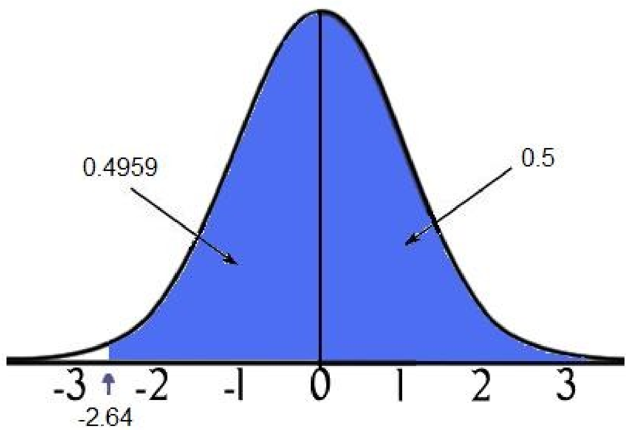
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.251	0.2549

7. P(-1.27 < Z ≤ 0) = P(0 < Z ≤ 1.27) = 0.3980



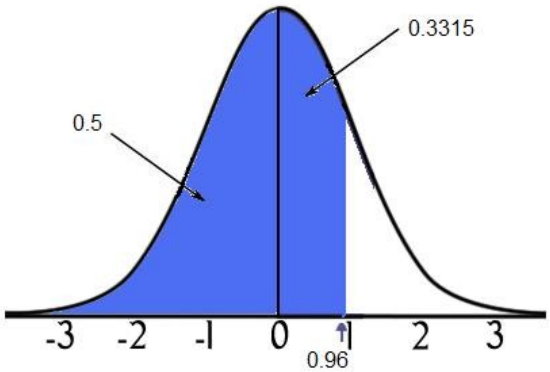
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

8. $P(Z > -2.64) = P(0 < Z \leq 2.64) + P(Z > 0) = 0.4959 + 0.5 = 0.9959$



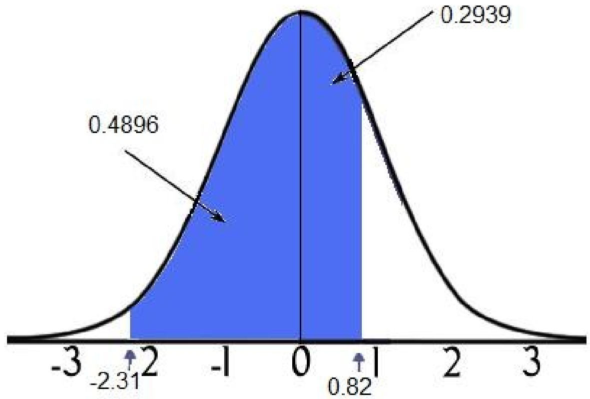
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974

9. $P(Z \leq 0.96) = P(0 < Z \leq 0.96) + P(Z \leq 0) = 0.3315 + 0.5 = 0.8315$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621

10. P(-2.31 < Z ≤ 0.82) = P(0 < Z ≤ 2.31) + P(0 < Z ≤ 0.82) = 0.4896 + 0.2939 = 0.7835



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936