

SPRINGS

Springs - elastic stored energy machine element that when released, will recover its basic form or position. It is a device made of an elastic material that undergoes a significant change in shape or deformation under an applied load.

Uses of Springs

1. To absorb energy or shock loads
2. To maintain contact between machine members
3. To act as a source of energy
4. To serve as a measuring device

Types of Springs

1. **Helical Springs** - are made up of wire coiled in the form of a helix and is primarily intended for compressive and tensile loads.
2. **Conical and Volute Springs** - are used in special applications where a telescoping spring or a spring with a spring rate that increases when the load is required.
3. **Torsion Springs** - are springs that work by torsion or twisting; that is, a flexible elastic object that stores mechanical energy when it is twisted.
4. **Laminated Leaf Springs** - consist of a number of flat plates of varying lengths held together by means of clamps or belts.
5. **Disc or Belleville Springs** - consist of a number of conical discs held together against slipping by a central bolt or tube.

Design of Helical Springs

- | | |
|---|---|
| 1. Length of wire before coiling | $L_w = \pi D_m N_c$ |
| 2. Length of coil if Pitch is given | $L_c = \sqrt{(\pi D_m)^2 + P^2}$ |
| 3. Mean Diameter | $D_m = D_o - D_w$
$D_m = D_i + D_w$ |
| 4. Total Shear Stress | $S = \frac{8K_w Fc}{\pi D_w^2}$ |
| 5. Spring Index | $c = \frac{D_m}{D_w}$ |
| 6. Wahl Correction Factor / Wahl's Factor | $K_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$ |
| 7. Factor for direct shear | $K_s = 1 + \frac{1}{2c}$ |
| 8. Stress Concentration Factor due to curvature of coil | $K_c = \frac{K_w}{K_s}$ |
| 9. Bergstrasser Factor | $K_B = \frac{4c+2}{4c-3}$ |
| 10. Spring Deformation | $\delta = \frac{8Fc^3 N_c}{GD_w}$ |

$$G = 83 \text{ GPa}$$

$$G = 12 \times 10^6 \text{ psi}$$

11. Spring Constant (Spring Rate, Spring Modulus, $k = \frac{F}{\delta}$

Spring Gradient, Stiffness) $k = \frac{GD_w}{8c^3N_c}$

Sample Problems:

- How much wire is needed to coil a spring having a mean diameter of 1 inch if there are 8 active coils?
- The spring index of a spring is 4.5. Determine the stress concentration factor due to curvature of coil.
- A spring has a load of 50 lbs with a spring index of 8. If the induced stress is 90 ksi, determine the wire diameter.
- Determine the maximum shear and elongation of a helical steel spring composed of 20 turns of 20 mm diameter wire on a mean radius of 90 mm when the spring is supporting a load of 1.5 kN.
- A spring has a wire diameter of 25 mm with 12 active coils. If a load of 10 kN is applied, it deflects by 75 mm. Determine the mean diameter of the spring. Use $G = 80$ GPa.
- A spring has a stiffness of 30 kN/m. If the wire diameter is 10 mm and the spring index is 7, determine the number of active coils. Use $G = 79.5$ GPa.
- It is found that a load of 50 lbs in an extension spring produces a deflection of 8.5 inches. What load deflects the spring by 2.5 inches?
- The load on a helical spring is 1600 lbs and the corresponding deformation is to be 4 inches. Rigidity modulus is 11×10^6 psi and the maximum intensity of safe shear stress is 60000 psi. If the wire diameter and the mean diameter are 0.625 inches and 3 inches, respectively, determine the Wahl's factor, number of effective coils, and the spring constant.
- A spring with 12 active coils and an index of 9, supports a static load of 220 N with a deflection of 12 mm. The shear modulus of elasticity of the spring material is 83 GPa. What is the theoretical wire diameter?

Types of Ends

	Plain	Plain and Ground	Squared/Closed	Squared and Ground
Number of End Coils	0	1	2	2
Total Number of Coils, N_t	N_c	$N_c + 1$	$N_c + 2$	$N_c + 2$
Free Length, FL	$PN_c + D_w$	$P(N_c + 1)$	$PN_c + 3D_w$	$PN_c + 2D_w$
Solid Height, SH	$D_w(N_t + 1)$	D_wN_t	$D_w(N_t + 1)$	D_wN_t
Pitch, P	$\frac{FL - D_w}{N_c}$	$\frac{FL}{N_c + 1}$	$\frac{FL - 3D_w}{N_c}$	$\frac{FL - 2D_w}{N_c}$

Sample Problems:

1. A spring with plain and ground ends has a pitch of 30 mm and a wire diameter of 15.2 mm. If there are 12 actual numbers of coils, determine the deflection of the spring at solid height.
2. A helical spring having closed and ground ends has a total of 18 coils and its material has a modulus of elasticity in shear of 79.81 GPa. If the spring has an outside diameter of 14.02 cm and a wire diameter of 0.725 cm, compute the maximum deflection and maximum shear that can be produced in the spring by a load of 50 kg.
3. A spring with squared and ground ends has a free length of 250 mm. There are 8 active coils with a wire diameter of 12.5 mm. If the spring rate is 150 kN/m and the mean coil diameter is 100 mm, determine the spring's solid stress.

Spring Arrangements

1. Springs arranged in Parallel

- a. Equal Lengths and Concentric

Forces: $F_T = F_o + F_i$

Deformation: $\delta_T = \delta_o = \delta_i$

Spring Rate: $k_T = k_o + k_i$

- b. Unequal Lengths and Concentric

Forces: $F_T = F_o + F_i$

Deformation: $\delta_T = \delta_o = \delta_i + e$

- c. Non-concentric

Forces: $F_T(a + b + c) = F_1(a) + F_2(a + b)$

Deformation: $\frac{\delta_1}{a} = \frac{\delta_2}{a+b}$

2. Springs arranged in Series

Forces: $F_T = F_1 = F_2 = \dots = F_n$

Deformation: $\delta_T = \delta_1 + \delta_2 + \dots + \delta_n$

Spring Rate: $\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$

Springs under Impact Loads

1. From a potential energy source

$$PE = SE$$

$$W(h + \delta) = \frac{1}{2}k\delta^2$$

2. From a kinetic energy source

$$KE = SE$$

$$mv^2 = k\delta^2$$

Sample Problems:

1. A spring sustains 200 ft-lb of energy with a deflection of 3 inches. Assume that the mean diameter is seven times the wire diameter and that the allowable stress is 100 ksi. Determine the wire diameter.
2. A large coil spring whose spring constant, $k = 1.75 \text{ N/mm}$, is elongated within the elastic range by 305 mm. The stored energy in N-m is nearest to?
3. A weight of 100 lbs strikes a coil spring from a height of 18 inches and deflects the spring by 6 inches. Find the maximum force applied to the spring.
4. A mass weighing 12 kg falls a distance of 1.52 m from the unsupported end of a vertical helical spring having a spring constant of 3.5 N/mm. What will be the velocity of the mass after it has compressed the spring by 203.2 mm?

Coil Clearance

$$CC = \frac{CL - SH}{N_c}$$

Criteria for Buckling

- a. From Mott

$$\frac{FL}{D_w} < 5.2 \text{ (Buckling will not occur)}$$

- b. From Shigley

$$FL < 2.63 \frac{D_m}{\alpha} \text{ (Buckling will not occur)}$$

Where:

$\alpha = 0.5$	<i>fixed ends</i>
$\alpha = 0.707$	<i>fixed - hinged ends</i>
$\alpha = 1.0$	<i>hinged ends</i>
$\alpha = 2.0$	<i>damped - free ends</i>

Actual Outside Diameter - outside diameter of the spring at solid length condition.

$$D_o' = \sqrt{D_m^2 + \frac{P^2 - D_w^2}{\pi^2}} + D_w$$

Sample Problem:

A spring made from wire ASTM A228 steel has the following data:

$FL = 1.75 \text{ inches}$	$N_t = 10$
$D_w = 0.055 \text{ inches}$	$G = 11.85 \times 10^6 \text{ psi}$
$D_o = 0.561 \text{ inches}$	Squared and ground ends
$F = 14.01 \text{ lbs}$	

Calculate the following:

- a. D_m and D_i
- b. c and K_w
- c. stress in the spring
- d. deflection at operating force
- e. spring rate
- f. force at solid height
- g. coil clearance
- h. check for buckling
- i. outside diameter if the spring is compressed to its solid length

PROBLEM SET

1. A coiled spring with 1 $\frac{3}{4}$ inches outside diameter is required to work under load of 140 lbs. Wire diameter used is 0.192 inches, spring is to have seven active coils, and the ends are to be closed and ground. Determine the safe shearing stress and the unit deflection. Assume G is equal to 12 million psi and the mean radius to be 0.779 inches.
2. Compute the maximum deflection of a 22-coil helical spring having a load of 120 kgs. The spring is squared and ground ends with modulus of elasticity in shear of 80 GPa, outside diameter of 8 cm and wire diameter of 8 mm.
3. A Mercedes Benz gas engine valve spring is to have a mean diameter of 3.81 cm. The maximum load it will have to sustain is 45.45 kgs with corresponding deflection of 1.27 cm. The spring is to be made of tempered steel wire. Since the material is to be subjected to repeated loading, and fatigue must be considered, a low working stress of 2820 kg/cm² will be used. Determine the size of the wire.
4. A high alloy spring has squared and ground ends, a total of 16 coils and modulus of rigidity of 85 GPa. Compute the Wahl's factor. The spring outside diameter is 9.66 cm and the wire diameter is 0.65 cm.
 - a. 1.058
 - b. 1.10
 - c. 1.185
 - d. 1.2
5. All four compression springs support one load of 700 kgs. All four are arranged in parallel and rated the same at 0.609 kg/mm. Compute the deflection in mm.
 - a. 64
 - b. 147
 - c. 178
 - d. 287
6. Compute the maximum deflection of a 20-coil helical spring having a load of 75 kgs. The spring is squared and ground with modulus of elasticity in shear of 79.84 GPa, outside diameter of 101.6 mm and wire diameter of 9.525 mm.
 - a. 140.7 mm
 - b. 112.7 mm
 - c. 15.17 mm
 - d. 126.7 mm
7. A three extension coil springs are hooked in series that support a single weight of 100 kgs. The first spring is rated at 0.400 kg/mm and the other two is rated at 0.64 kg/mm. Compute the total deflection.
 - a. 563 mm
 - b. 268 mm
 - c. 156 mm
 - d. 250 mm
8. A spring with 12 active coils and a spring index of 9 supports a static load of 220 N with a deflection of 12 mm. The shear modulus of the spring material is 83 GPa. What is the theoretical wire diameter?
 - a. 18 mm
 - b. 16 mm
 - c. 14 mm
 - d. 20 mm
9. A helical spring having squared and ground ends has a total of 24 coils and material modulus of elasticity in shear is 89 GPa. The spring has an outside diameter of 12 cm and a wire diameter of 0.65 cm. Compute the maximum deflection that can be produced in the spring due to the load of 60 kgs.
 - a. 870 mm
 - b. 954 mm
 - c. 999 mm
 - d. 1030 mm
10. A coiled spring with 5 cm outside diameter is required to work under a load of 190 N. The wire diameter is 5 mm, the spring is to have 6 active coils and the ends are to be closed and ground. Determine the total number of coils. The modulus of rigidity is 80 GPa and the mean radius is to be 23 mm, with 7 mm pitch of the spring.
 - a. 6.5
 - c. 8.5

22. A spring with plain ends has 15 active coils, diameter of 6 mm and a pitch of 10 mm. If the spring rate is 100 kN/m, determine the solid force.
- 4 kN
 - 5 kN
 - 6 kN
 - 7 kN
23. A helical coil spring has a mean coil diameter of 2.5 inch and a wire diameter of 3/8 inch. Determine the Wahl's factor of the spring.
- 1.2246
 - 1.2642
 - 1.4262
 - 1.4226
24. A spring has a rate of 30 kN/m. If the wire diameter is 10 mm with mean diameter of 70 mm, determine the number of active coils. $G = 80 \text{ GPa}$
- 7
 - 8
 - 9
 - 10
25. A helical coil spring has a mean coil diameter of 2.5 inch and a wire diameter of 3/8 inch. Determine the Bergstrasser factor of the spring.
- 1.2124
 - 1.1224
 - 1.4122
 - 1.4221
26. A three extension coil springs are hooked in series that support a single weight of 100 kgs. The first spring is rated at 4 kN/m and the other two is rated at 6 kN/m. Determine the equivalent stiffness of the springs.
- 1.71 kN/m
 - 2.71 kN/m
 - 5 kN/m
 - 3.71 kN/m
27. A helical coil spring has a mean coil diameter of 2.5 inch and a wire diameter of 3/8 inch. Determine the curve correction factor of the spring.
- 1.1391
 - 1.1913
 - 1.3191
 - 1.3119
28. A three extension coil springs are hooked in series that support a single weight of 100 kgs. The first spring is rated at 0.400 kg/mm and the other two is rated at 0.64 kg/mm. Compute the total deflection.
- 563 mm
 - 268 mm
 - 156 mm
 - 250 mm
29. Find the number of active coils of a No. 8 wire helical spring with index of 6, steady load with spring rate of 42.5 lb/in. Maximum allowable stress is 60 ksi.
- 25
 - 28
 - 29
 - 22
30. Compute the maximum deflection of a 20-coil helical spring having a load of 75 kgs. The spring is squared and ground with modulus of elasticity in shear of 79.84 GPa, outside diameter of 101.6 mm and wire diameter of 9.525 mm.
- 140.7 mm
 - 112.7 mm
 - 15.17 mm
 - 126.7 mm
31. Compute the deflection of an 18-coil helical spring having a load of 100 kgs. The modulus of rigidity is 96.62 GPa., OD of 9.256 cm and with a diameter of 9.525 mm. The spring has squared and ground ends.
- 91 mm
 - 101 mm
 - 112 mm
 - 104 mm
32. It is a device made of an elastic material that undergoes a significant change in shape or deformation under an applied load.
- springs
 - pulleys
 - chains
 - belts
33. They are made up of wire coiled in the form of a helix and is primarily intended for compressive and tensile loads.

- a. belleville springs
 - b. helical springs
 - c. volute springs
 - d. torsion springs
34. These are springs that work by torsion or twisting; that is, a flexible elastic object that stores mechanical energy when it is twisted.
- a. belleville springs
 - b. helical springs
 - c. volute springs
 - d. torsion springs
35. Springs that consist of a number of conical discs held together against slipping by a central bolt or tube.
- a. belleville springs
 - b. helical springs
 - c. conical springs
 - d. torsion springs
36. Springs that consist of a number of flat plates of varying lengths held together by means of clamps or belts.
- a. belleville springs
 - b. helical springs
 - c. conical springs
 - d. laminated leaf springs
37. Springs which are used in special applications where a telescoping spring or a spring with a spring rate that increases when the load is required.
- a. belleville springs
 - b. helical springs
 - c. conical and volute springs
 - d. laminated leaf springs

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