AMPLITUDE MODULATION

- **AM Wave**

$$V(t) = V_c \sin \omega_c t + \frac{V_m}{2}\cos(\omega_c - \omega_m)t - \frac{V_m}{2}\cos(\omega_c + \omega_m)t$$

where: V_c = maximum voltage of the carrier signal
V_m = maximum voltage of the original modulating signal
$\omega_c = 2\pi f_c$ = frequency of the carrier signal
$\omega_m = 2\pi f_m$ = frequency of the modulating signal

$$m = \frac{V_m}{V_c} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

where: m = modulation index
V_{max} = maximum peak-to-peak voltage swing of AM wave
V_{min} = minimum peak-to-peak voltage swing of AM wave

AM wave equation in terms of modulation index

$$V(t) = V_c \sin \omega_c t + \frac{mV_c}{2}\cos(\omega_c - \omega_m)t - \frac{mV_c}{2}\cos(\omega_c + \omega_m)t$$

- **AM Bandwidth**

$$BW = 2f_m$$

where: BW = bandwidth
f_m = modulating signal frequency

- **AM Power and Current**

$$P_t = \frac{V_{carr}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \qquad P_c = \frac{V_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{m^2 V_c^2}{8R} = \frac{V_m^2}{8R} = P_c\frac{m^2}{4}$$

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2} \qquad \left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m^2}{2}$$

where: P_t = total transmitted power
P_c = unmodulated carrier power
I_t = total transmitted current
I_c = unmodulated carrier current
m = modulation index

Note: The voltage should be in rms

Amplitude Modulation with Multiple Signals

$$P_t = P_c\left(1 + \frac{m_t^2}{2}\right)$$

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 ......}$$

where: m_t = total modulation index
m_1, m_2, m_3 = modulation index of signal having index 1, 2, 3 respectively

Power Savings

a. **Single Sideband (SSB)**

$$PS = \frac{P_{LSB/USB} + P_C}{P_t}$$

b. **Single Sideband full carrier (SSBFC)**

$$PS = \frac{P_{LSB/USB}}{P_t}$$

c. **Two independent Sidebands**

$$PS = \frac{P_C}{P_t}$$

- **Tuned Radio-Frequency (TRF)AM Receiver**

TRF Design formulas

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{f_r}{BW}$$

where: Q = quality factor
f_r = frequency
BW = Bandwidth

- **Superheterodyne Receiver**

$$f_{si} = f_s + 2f_i$$

where: f_{si} = image frequency
f_s = signal frequency
f_i = intermediate frequency

$$\alpha = \sqrt{1 + Q^2\rho^2}$$

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{f_{image}}{f_{RF}} - \frac{f_{RF}}{f_{image}}$$

where: α = image-frequency rejection ratio(IFRR)
Q = quality factor of the circuit

TELEVISION

Details of Horizontal Blanking

Period	Time, µsec
Total line (H)	63.5
H blanking	0.15H-0.18H or 9.5-11.5
H sync pulse	0.08H, or 4.75 ± 0.5
Front porch	0.02H, or 1.27
Back porch	0.06H, 3.81
Visible line time	52-54

Details of Vertical Blanking

Period	Time
Total field (V)	1/60s 0.0167s
V blanking	0.05V-0.08V or 9.5-11.5
Each V sync pulse	27.35 µs
Total of 6 V sync pulse	3H = 190.5 µs
Each equalizing pulse	0.04H = 2.54 µs
Each serration	0.07H = 4.4 µs
Visible field time	0.92V-0.95V, 0.015-0.016s

Picture Information Encoding

$$Y = 0.30R + 0.59G + 0.11B$$

$$I = 0.60R - 0.28G - 0.32B$$

$$Q = 0.21R - 0.52G + 0.31B$$

Relative amplitude for the AM RF picture signal

Tip of sync = 100%
Blanking level = 75%
Black setup = 67.5%
Maximum white = 10 to 15% or 12.5% (typical)

NAVIGATIONAL AIDS

Directional Gain

$$G_{dir} = \frac{4p}{qf} \qquad q = \frac{l}{L}$$

where: θ = horizontal beam-width (radians)
λ = the wavelength of the radar
L = the dimension of the antenna in the direction of interest (i.e. width or height)
f = vertical beam-width (radians)

RADAR Pulse (Waveform)

$$PRT = PW + RT$$

$$PRF = \frac{1}{PRT} \qquad DR = \frac{PW}{PRT}$$

$$P_{AV} = P_{PEAK} \times DR \qquad P_{PEAK} = \frac{P_{AV}\,PRT}{PW}$$

where: PRT = Pulse Repetition Time
PW = Pulse Width (µs)
RT = Rest Time (µs)
PRF = Pulse Repetition Frequency
DR = Duty Cycle or Duty Ratio
P_{AV} = Average Power
P_{PEAK} = Peak Power

Maximum unambiguous range

$$R_{unamb} = c\frac{PRT}{2}$$

Minimum displayed range

$$R_{\min} = c\frac{PW}{2}$$

where: c = speed of light (3×10^8 m/s)

Radar Range

$$R = \sqrt[4]{\frac{P_T A_P S A_O}{(4p)^2 P_{R\min}}}$$

$$A_P = \frac{4pA_O}{l^2}$$

$$R = \sqrt[4]{\frac{P_T A_O{}^2 S}{4p\,l^2 P_{R\min}}}$$

where: R = Radar Range
P_T = Transmitted Power
A_P = antenna gain
S = cross-sectional area of the target
A_0 = captured area of an antenna
P_{Rmin} = detected signal level in W

Doppler Effect

$$F_D = \frac{2v\cos q}{l}$$

where: F_D = frequency change between transmitter and reflected signal
v = relative velocity between RADAR and target
λ = wavelength of the transmitted wave
θ = angle between target direction and RADAR system

TRANSMISSION LINES

- **Electrical Characteristics**

Characteristic Impedance

$$Z_O = \sqrt{\frac{Z}{Y}} \qquad \Omega$$

where: Z = R +jωL Ω/m
Y = G +jωC S/m

$$Z_O = \sqrt{\frac{R}{G}} \qquad \text{at low frequency, } \Omega$$

$$Z_O = \sqrt{\frac{L}{C}} \qquad \text{at high frequency, } \Omega$$

Also,

$$Z_O = \sqrt{Z_{SC}Z_{OC}} \qquad \Omega$$

where: Z_{SC} = short circuit impedance
Z_{OC} = open circuit impedance

For Parallel-wire line:

$$L = \frac{m}{p}\ln\frac{2D}{d} \qquad \frac{H}{m}$$

$$C = \frac{pe}{\ln\frac{2D}{d}} \qquad \frac{F}{m}$$

where: L = Inductance
C = Capacitance
D = Separation between center to center
d = diameter of the wire

Alternate formulas:

$$L = 1.016Z_O\sqrt{e_r}\times10^{-3} \qquad \mu\text{H/ft}$$

$$C = 1.016\frac{\sqrt{e_r}}{Z_O}\times10^{-3} \qquad \mu\text{F/ft}$$

Characteristic Impedance, Z_0

$$Z_O = \sqrt{\frac{L}{C}}$$

$$Z_O = \frac{120}{\sqrt{e_r}}\ln\frac{2D}{d}$$

$$Z_O = \frac{276}{\sqrt{e_r}}\log\frac{2D}{d}$$

Note: $150\Omega \le Z_0 \le 600\Omega$

Resistance, R

$$R = 8.34\times10^{-8}\frac{\sqrt{f}}{a} \qquad \frac{\Omega}{m}$$

$$R = \frac{\sqrt{f}}{5d} \qquad \frac{\Omega}{100-ft}$$

where: a = radius (m)
f = frequency (MHz)
d = diameter (inches)

For coaxial line:

$$L = \frac{m}{2p}\ln\frac{D}{d} \qquad \frac{H}{m}$$

$$C = \frac{2pe}{\ln\frac{D}{d}} \qquad \frac{F}{m}$$

where: D = diameter of the outer conductor
d = diameter of the inner conductor

Alternate formulas:

$$L = 1.016Z_O\sqrt{e_r}\times10^{-3} \qquad \mu\text{H/ft}$$

$$C = 1.016\frac{\sqrt{e_r}}{Z_O}\times10^{-3} \qquad \mu\text{F/ft}$$

Characteristic Impedance, Z_0

$$Z_O = \frac{60}{\sqrt{e_r}}\ln\frac{D}{d}$$

$$Z_O = \frac{138}{\sqrt{e_r}}\log\frac{D}{d}$$

Note: $40\Omega \le Z_0 \le 150\Omega$

Resistance, R

$$R = 8.34\times10^{-8}\sqrt{f}\left(\frac{1}{D}+\frac{1}{d}\right) \qquad \frac{\Omega}{m}$$

where: D = diameter of the outer conductor (m)
d = diameter of the inner conductor (m)
f = frequency (MHz)

$$R = 0.1\sqrt{f}\left(\frac{1}{D}+\frac{1}{d}\right) \qquad \frac{\Omega}{100-ft}$$

where: D = diameter of the outer conductor (inches)
d = diameter of the inner conductor (inches)
f = frequency (MHz)

Complex Propagation constant, γ

$$g = a + jb = \sqrt{ZY}$$

where: α = attenuation constant or coefficient (Nepers/length)
β = phase constant or coefficient (Radians/length)

$$a = 4.343\left(\frac{R}{Z_O}\right) \qquad \text{dB/length}$$

$$b = w\sqrt{LC} = \frac{w}{V_P} = \frac{2p}{l} \qquad \text{radians/length}$$

$$V_P = \frac{1}{\sqrt{LC}} \qquad \text{m/s}$$

where: V_p = propagation velocity

- **Loading Conditions**

Note: The zero reference is at the load not on the generator.

1. $Z_L = Z_0$ (match load)

with $Z_L = Z_0$ then $Z_{in} = Z_0$

$$I_R = I_S e^{-gL} \qquad I_S = I_R e^{gL}$$

$$V_R = V_S e^{-gL} \qquad V_S = V_R e^{gL}$$

$$P_R = P_S e^{-2gL} \qquad P_S = P_R e^{2gL}$$

where: I_R, I_S, V_R, V_S = receiving and sending end current and voltages respectively
P_R, P_S = power at the receiving and sending end
γ = complex propagation constant
L = length of the transmission lone
Z_{in} = input impedance
Z_L = load impedance

2. $Z_L \neq Z_0$ (Mismatch)

$$Z_{in} = \frac{Z_O^2}{Z_L} \qquad \text{for } \lambda/4 \text{ line}$$

$$Z_{in} = Z_O\left(\frac{Z_L + Z_O \tanh gL}{Z_O + Z_L \tanh gL}\right) \qquad \text{for L} > \lambda/4$$

where: Z_{in} = the equivalent impedance representing the entire line terminated by the load

Load boundary characteristics

$$V(d) = V^+ e^{jbd} + V^- e^{-jbd}$$

$$I(d) = \frac{1}{Z_O}\left(V^+ e^{jbd} - V^- e^{-jbd}\right)$$

Loss-less transmission line

$$V(d) = V^+ e^{jgd} + V^- e^{-jgd}$$

$$I(d) = \frac{1}{Z_O}\left(V^+ e^{jgd} - V^- e^{-jgd}\right)$$

Loss-less transmission line

where: V(d) = line voltage at point d
I(d) = line current at point d
Z_0 = characteristic impedance of the line
V^+ = incident voltage
V^- = reflected voltage
γ = complex propagation constant for lossy-line
β = complex propagation constant for loss-less line
d = distance from the load

Four Cases (loss-less transmission line)

1. $Z_L \to 0$ (short circuit)

$$V(d) = 2jV^+ \sin(bd)$$

$$I(d) = \frac{2V^+ \cos(bd)}{Z_0}$$

$$Z(d) = \frac{V(d)}{I(d)} = jZ_0 \tan(bd)$$

$$\Gamma_R = -1$$

2. $Z_L \to \infty$ (open circuit)

$$V(d) = 2V^+ \cos(bd)$$

$$I(d) = \frac{2jV^+ \sin(bd)}{Z_0}$$

$$Z(d) = \frac{V(d)}{I(d)} = -jZ_0 \cot(bd)$$

$$\Gamma_R = 1$$

3. $Z_L = Z_0$ (matched load)

$$V(d) = V^+ e^{jbd}$$

$$I(d) = \frac{V^+ e^{jbd}}{Z_0}$$

$$Z(d) = Z_0$$

$$\Gamma_R = 0$$

4. $Z_L = jX$ (pure reactance)

- Reactive impedance can be realized with transmission lines terminated by a short or by an open circuit.

$$Z_{in} = jZ_0 \tan(bL)$$

- Reflection coefficient has a unitary magnitude, as in the case of short and open circuit load.

Shorted Transmission Line – Fixed Frequency

$L = 0$	$Z_{in} = 0$	Series Resonance
$0 < L < \frac{l}{4}$	$\mathrm{Im}(Z_{in}) > 0$	Inductance
$L = \frac{l}{4}$	$Z_{in} \to \infty$	Parallel Resonance
$\frac{l}{4} < L < \frac{l}{2}$	$\mathrm{Im}(Z_{in}) < 0$	Capacitance
$L = \frac{l}{2}$	$Z_{in} = 0$	Series Resonance
$\frac{l}{2} < L < \frac{3l}{4}$	$\mathrm{Im}(Z_{in}) > 0$	Inductance
$L = \frac{3l}{4}$	$Z_{in} \to \infty$	Parallel Resonance
$\frac{3l}{4} < L < l$	$\mathrm{Im}(Z_{in}) < 0$	Capacitance

Shorted Transmission Line – Fixed Frequency

$L = 0$	$Z_{in} \to \infty$	Parallel Resonance
$0 < L < \frac{l}{4}$	$\mathrm{Im}(Z_{in}) < 0$	Capacitance
$L = \frac{l}{4}$	$Z_{in} = 0$	Series Resonance
$\frac{l}{4} < L < \frac{l}{2}$	$\mathrm{Im}(Z_{in}) > 0$	Inductance
$L = \frac{l}{2}$	$Z_{in} \to \infty$	Parallel Resonance
$\frac{l}{2} < L < \frac{3l}{4}$	$\mathrm{Im}(Z_{in}) < 0$	Capacitance
$L = \frac{3l}{4}$	$Z_{in} = 0$	Series Resonance
$\frac{3l}{4} < L < l$	$\mathrm{Im}(Z_{in}) > 0$	Inductance

- **Degree of Mismatch**

A. Standing Wave Ratio (SWR)

$$SWR = \frac{Z_0}{R_L} = \frac{R_L}{Z_0} \qquad \text{(whichever is larger)}$$

$$SWR = \frac{1+\Gamma}{1-\Gamma}$$

Note: The greater the SWR, the greater the mismatch

B. Voltage Standing Wave Ratio (SWR)

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{V_{inc} + V_{ref}}{V_{inc} - V_{ref}}$$

where: $V^+ = V_{inc}$ = incident (forward) voltage
$V^- = V_{ref}$ = reflected (reverse) voltage

C. Current Standing Wave Ratio (SWR)

$$ISWR = \frac{I_{\max}}{I_{\min}} = \frac{I_{inc} + I_{ref}}{I_{inc} - I_{ref}}$$

where: I_{inc} = incident (forward) current
I_{ref} = reflected (reverse) current

Note: SWR = VSWR = ISWR
In dB:

$$SWR_{dB} = 20 \log SWR$$

Coefficient of reflection, Γ

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{I_{ref}}{I_{inc}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{SWR - 1}{SWR + 1}$$

Solutions to mismatch condition:

1. Quarter-wave transformer matching

- for purely resistive

$$Z_0' = \sqrt{Z_0 Z_L} \quad \Omega$$

where: Z_0' = Characteristic impedance of the quarter-wave matching transformer

2. Stub

Procedure of using stubs:

a. Calculate the load admittance
b. Calculate the stub susceptance
c. Connect the stub to the load, the resulting admittance being the load conductance G.
d. Transform conductance to resistance, and calculate Z_0' of the quarter-wave transformer.

ANTENNA

- **Antenna Characteristics**

$$G = 10 \log \frac{P_2}{P_1}$$

where: G(dB) = antenna gain in decibels
P_1 = power of unidirectional antenna
P_2 = power of reference antenna

$$ERP = P_{in} G$$

$$ERP = P_{rad} D$$

where: G = power gain (unitless)
P_{in} = power delivered to the feedpoint

For an isotropic antenna: $P_T = P_{rad}$

But for a unidirectional antenna: $P_T = ERP$

$$R_{rad} = \frac{P_{rad}}{I^2}$$

where: R_{rad} = radiation resistance
P_{rad} = power radiated by the antenna
I = current at the feedpoint

Radiation resistance for l not in excess of λ/8

$$R_{rad} = 790\left(\frac{l}{l}\right)^2$$

$$P_d = P_{in} - P_{rad}$$

where: R_{rad} = radiation resistance
P_{rad} = power radiated by the antenna

$$h = \frac{R_{rad}}{R_T}$$

$$P_{rad} = hP_{in}$$

$$G = hD$$

where: η = antenna efficiency (1 for lossless ant.)
R_{rad} = antenna radiation resistance
R_T = antenna radiation resistance
= R_{rad} and R_d (ohmic resistance)
D = directivity (maximum directive gain)

$$BW = \frac{f_r}{Q}$$

$$f = 70\frac{l}{D}$$

where: BW = bandwidth
f_r = antenna resonant frequency
Q = antenna quality factor
f = beamwidth

$$A_{FB} = 10\log\frac{P_F}{P_B} = 20\log\frac{x_F}{x_B}$$

where: A_{FB} = front-to-back ratio (dB)
P_F = power output in the most optimum direction
P_B = power output in the opposite direction
ξ_F = field strength in the most optimum direction
ξ_B = field strength in the opposite direction

$$x = \frac{60pL_e I}{l\,r}\sin q = \frac{\sqrt{30P_T}}{r}$$

where: ξ_F = magnitude of field strength
r = distance
L_e = antenna length
I = current amplitude
θ = the angle of the axis of the wire and the point of maximum radiation

Isotropic Antenna

Gain over isotropic = 0 dB
Beamwidth = 360°

- **Types of Antenna**

A. Dipole Antenna

a. Half-wave dipole

Gain over isotropic = 2.14 dB
Beamwidth = 55°

b. Folded half-wave dipole

Gain over isotropic = 5.64 dB
Beamwidth = 45°

B. Beam Antenna

a. Yagi-Uda Antenna

Gain over isotropic = 7.14 dB
Beamwidth = 25°

b. Rhombic Antenna

Gain over isotropic = 5.14 dB

C. Loop Antenna

Gain over isotropic = 3.14 dB
Beamwidth = 200°

$$V = k(2pf)BAN$$

where: V = voltage induced in a loop antenna
k = physical proportional factor
B = field strength flux, V/m
A = loop area, m^2
N = number of turns

D. Antenna with parabolic reflector

$$G = \frac{A_{eff}}{A_{iso}} = \frac{kA_s}{A_{iso}} = p^2 k\left(\frac{D}{l}\right)^2$$

$$A_{iso} = \frac{l^2}{4p} \qquad A_s = \frac{pD^2}{4}$$

where: A_{eff} = effective aperture or antenna capture area
A_{iso} = isotropic area
k = illumination factor
D = diameter of parabolic reflector

$$G = 6\left(\frac{D}{l}\right)^2 \quad \text{with } k = 0.65$$

Parabolic dipole: $D = \frac{5l}{2}$

Horn Antenna (Pyramidal)

Elevation Pattern: $-3\text{dB}\ beamwidth = \frac{56l}{h}$

Azimuth Pattern: $-3\text{dB}\ beamwidth = \frac{70l}{w}$

$$G = 7.5\left(\frac{D}{l}\right)^2$$

E. Helical Antenna

$$G = 15\left(\frac{D}{l}\right)^2 \frac{NS}{l}$$

$$f = \frac{52}{\frac{pD}{l}\sqrt{\frac{NS}{l}}}$$

where: G = Power gain
f = beamwidth
D = helix diameter
N = number of turns
S = pitch between turns
λ = wavelength
L = center-line axis length ≈ NS

Note: If pitch is not given S = λ/4

F. Log-Periodic Antenna

Design factor formulas:

$$r = \frac{l_2}{l_1} = \frac{l_3}{l_2} = \frac{l_4}{l_3} \qquad r = \frac{d_2}{d_1} = \frac{d_3}{d_2} = \frac{d_4}{d_3}$$

$$f_H = \frac{c}{l_n} \qquad f_L = \frac{c}{l_1}$$

where: $l_1 = \frac{l_1}{2}$ = the length of the longest element
d_1 = the distance between the longest element and the second element
r = design factor which is between 0.7 and 0.98

Antenna Height

For a straight vertical antenna with h ≤ λ/4

$$h_e = \frac{l}{p \sin\frac{2ph}{l}} \sin^2\frac{ph}{l}$$

where: h_e = effective height
h = actual height

Note: h_e the antenna effective height is ½ to ⅔ of the actual height.

FIBER OPTICS

- **Nature of Light**

$$E_P = hf$$

where: E_p = energy of a photon; Joules (J)
h = Planck's constant, 6.625×10^{-34} J-s
f = frequency, Hz

frequency of red light = 4.4×10^{14} Hz
frequency of violet light = 7×10^{14} Hz

- **Snell's Law**

$$n_1 \sin q_1 = n_2 \sin q_2$$

where: n_1 = refractive index of material 1
n_2 = refractive index of material 2
θ_1 = angle of incidence
θ_2 = angle of refraction

Note: 1 Å = 10^{-10} m
1 micron = 10^{-6} m
n_{air} = 1.0003 ≈ 1

$$n = \frac{c}{v}$$

where: n = refractive index
c = speed of light
v = velocity of light at material with refractive index of n

Note: Angle of incidence and refraction are measured from normal

$$\sin q_C = \frac{n_2}{n_1}$$

where: θ_C = critical angle

- **Propagation of Light Through a Fiber**

$\theta_1 < \theta_C \rightarrow$ light is refracted
$\theta_1 > \theta_C \rightarrow$ light is reflected
$\theta_1 = \theta_C \rightarrow$ reflected or refracted

$$\sin q_{in(\max)} = n_1 \cos q_C$$

where: $\theta_{in(max)}$ = acceptance angle
= acceptance cone half angle

$$NA = \sqrt{n_1^2 - n_2^2}$$

Where: NA = Numerical Aperture

- **Mode of Propagation**

$$N = \frac{1}{2} V^2$$

$$V = p \frac{d}{l_O} \sqrt{n_1^2 - n_2^2} = p \frac{d}{l_O} NA$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

where: N = number of modes
V = V number
d = diameter
λ = wavelength
NA = numerical Aperture
n_1 = refractive index of core
n_2 = refractive index of cladding
Δ = fractional index difference

- **Optical Fiber System Design**

Mathematical Analysis

The power budget is the basis of the design of an optical fiber link.

$$\textit{Total gain} - \textit{Total losses} \geq 0$$

Therefore

$$(P_t + P_r) - (\alpha_f + \alpha_c + \alpha_s + f_m) \geq 0$$

Thus,

$$L = P_t - P_r = (\alpha_f + \alpha_c + \alpha_s + f_m)$$

where: P_t = transmitted power
P_r = receiver sensitivity (minimum received power)
α_f = fiber attenuation
α_c = connector attenuation
α_s = total splice losses
f_m = fiber margin
L = distance between repeaters

$$Z = \frac{1}{5} B \Delta t$$

where: Z = system length
B = maximum bit rate
Δt = total fiber dispersion

RADIO WAVE PROPAGATION

- **The Electromagnetic Wave**

Velocity of propagation

$$V_p = \frac{1}{\sqrt{me}} \qquad \text{m/s}$$

$$m = m_r m_0 \qquad\qquad e = e_r e_0$$

where: μ = permeability of the medium (H/m)
ε = permittivity of the medium (F/m)

The Power Density

$$\wp = \frac{ERP}{A} = \frac{P_T G_T}{4pr^2} \qquad \text{W/m}^2$$

The Electric Field Intensity or Strength

$$x = aH = \frac{\sqrt{30 P_T G_T}}{r} \qquad \text{V/m}$$

where: α = characteristic impedance of free space, Ω
H = rms value of magnetic field intensity or strength (A/m)

The characteristic impedance of a medium

$$a = \sqrt{\frac{m}{e}} \qquad \Omega$$

Characteristic impedance in free space

$$m_o = 4p \times 10^{-7} = 1.26 \times 10^{-6} \quad \text{H/m}$$

$$e_o = \frac{10^{-9}}{36p} = 8.854 \times 10^{-12} \qquad \text{F/m}$$

$$a = \sqrt{\frac{4p \times 10^{-7}}{\frac{10^{-9}}{36p}}} = 120p\Omega = 377\Omega$$

The Attenuation of Power Density and Electric Field Intensity

$$A_{\wp}(dB) = 10\log\frac{\wp_1}{\wp_2} = 20\log\frac{r_2}{r_1}$$

$$A_x(dB) = 10\log\frac{x_1}{x_2} = 20\log\frac{r_2}{r_1}$$

- **The effects of environment to propagation of radio waves**

Refractive indices of different materials

H_2O	1.33
Glass	1.50
Quartz Crystal	1.54
Glycerin	1.47
Diamond	2.42

Snell's Law

$$\frac{n_1}{n_2} = \frac{\sin q_1}{\sin q_2} = \frac{V_2}{V_1} = \sqrt{\frac{k_1}{k_2}}$$

where: θ_2 = angle of refraction
θ_1 = angle of incidence
V_2 = refracted wave velocity in medium 2
V_1 = incident wave velocity in medium 1
k_1 = dielectric constant of medium 1
k_2 = dielectric constant of medium 2
n_1 = refractive index of medium 1
n_2 = refractive index of medium 2

$$n = \frac{c}{V_p} = \sqrt{k}$$

where: n = refractive index
c = velocity of light in free space
V_p = velocity of light in a given medium

Resultant field strength between waves traveling in different (direct and reflected paths)

$$x_r = 2x_d \sin 2p\,\frac{d}{2l} \qquad \text{V/m}$$

$$d = \frac{2h_{at}h_{ar}}{d}$$

where: ξ_d = direct radio wave field strength (V/m)
δ = the geometrical length difference between the direct and reflected paths
h_{at} and h_{ar} = the heights of transmitting and receiving antenna above the reflecting plane tangent to the effective earth

- **The Propagation Modes**

The Radio Frequency Spectrum

Band Name	Frequency (MHz)	Propagation
VLF	0.01 – 0.03	Ground Wave
LF	0.03 – 0.3	Ground Wave
MF	0.3 – 3.0	Ground Wave
HF	3.0 – 30	Sky Wave
VHF	30 – 300	Space Wave
UHF	300 – 3,000	Space Wave
SHF	3,000 – 30,000	Space Wave
EHF	30,000 – 300,000	Space Wave

A. The Ground (Surface) Wave Method

The field strength at a distance (ξ)

$$x = \frac{ah_t I}{lr}$$

The signal receive at that distance if a receiving antenna is in place

$$V = xh_r$$

where: α = characteristic impedance of free space
h_t and h_r = effective height of the transmitting and receiving antennas
I = antenna current
r = distance from transmitting antenna

B. The Ionosphere

The refractive index of the ionosphere

$$n = \frac{\sin q_i}{\sin q_r} = \sqrt{1 - \frac{81N}{f^2}}$$

where: N = number of free electrons per m^3
f = frequency of radio wave (Hz)

The Ionospheric Layers

D Layer – average height 70 km, with an average thickness of 10 km.

E Layer – existing at a height about 100 km, with a thickness of 25 km.

F_1 Layer – exists at a height 180 km, daytime thickness is about 20 km.

F_2 Layer – height ranges from 250 – 400 km in daytime and at night it falls to a height of 300 km where it combines with F_1 layer, approximate thickness at about 200 km.

The height of the ionospheric layer

$$h = \frac{d}{2\tan q}$$

The critical frequency (fc)

$$f_c = MUF\cos q = 9\sqrt{N_{\max}}$$

The Maximum Usable Frequency (MUF)

$$MUF = \frac{f_c}{\cos q} = f_c \sec q$$

The Optimum Working Frequency (OWF) or Frequency of Optimum Transmission (FOT)

$$OWF = FOT = 0.85MUF$$

C. The Space Wave Propagation

The Radio Horizon Distance

$$EC = \frac{d_1 d_2}{k}$$

$$R_e = kR_0$$

where: EC = Earth's Curvature
R_e = effective earth's radius
R_0 = earth's radius ≈ 3960 mi
k = correction factor for relatively flat earth
k = 4/3

The maximum line of sight distance between transmitter and receiver towers is given by

$$d = d_1 + d_2 = 4\sqrt{h_t} + 4\sqrt{h_r}$$

where: h_t and h_r = in meters
d, d_1 and d_2 = in kilometers

$$d = \sqrt{2h_t} + \sqrt{2h_r}$$

where: h_t and h_r = in feet
d = in miles

The correction factor (k)

$$k = [1 - 0.04665e^{0.005577N_s}]^{-1}$$

where: N_s = surface refractivity

D. Tropospheric Scatter Wave (Troposcatter) Propagation

Operates at the UHF band (between n 350 MHz to 10 GHz (and used to link multi-channel telephone links). The common frequencies are 0.9 GHz, 2 GHz and 5 GHz.

NOISE

- **Noise Calculation**

$$N = kTB$$

where: N = noise power
k = Boltzmann's constant
T = resistor temperature
B = bandwidth of the system

Note: 17 °C/290 K is the typical noise temperature

$$V_n = \sqrt{4kTBR} \qquad \text{in } \mu V$$

where: V_n = noise voltage
R = resistance generating the noise

Series Resistors

$$V_{n_T} = \sqrt{V_{n_1}^2 + V_{n_2}^2 + V_{n_3}^2 + \ldots}$$

Parallel Resistors

$$I_{n_T} = \sqrt{I_{n_1}^2 + I_{n_2}^2 + I_{n_3}^2 + \ldots}$$

For a diode, the rms noise current

$$I_n = \sqrt{2eI_D B} \qquad \text{typically in } \mu A$$

where: e = charge of an electron (1.6×10^{-19} C)
I_D = direct diode current
B = bandwidth of the system

$$I_n = \sqrt{2e(I_D + 2I_o)B}$$

where: I_0 = negligible reverse saturated current

I. Addition of noise due to several sources

$$V_{n_T} = \sqrt{4kTBR_T}$$

II. Addition of noise due to several amplifiers in cascade

$$R_{eq} = R_1 + R_2' + R_3' + \ldots + R_n'$$

$$R_{eq} = R_1 + \frac{R_2}{(A_1)^2} + \frac{R_3}{(A_1)^2(A_2)^2} + \ldots + \frac{R_n}{(A_1)^2\ldots(A_{n-1})^2}$$

III. Signal-to-Noise Ratio

$$\frac{S}{N}(dB) = 10\log\frac{S}{R}$$

IV. Noise Factor (NF) or Noise Figure (F)

$$NF = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}}$$

$$F(dB) = 10\log NF$$

For a noiseless receiver,

NF = 1; F = 0 dB

V. Equivalent Noise Temperature (T_e)

$$T_e = T_o(NF - 1)$$

where: T_e = equivalent noise temperature
T_o = reference temperature = 290 K
NF = noise factor

For a noiseless receiver, T_e = 0 K

For attenuator elements

$$T_e = T_p(L - 1)$$

where: L = loss (absolute value)
T_p = physical temperature (K)

VI. Overall Noise Factor (Friis' Formula)

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \ldots + \frac{NF_n - 1}{G_1 G_2 G_3 \ldots G_{n-1}}$$

VII. Overall Noise Temperature

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_1} + \frac{T_{e_3}}{G_1 G_2} + \ldots + \frac{T_{e_n}}{G_1 G_2 G_3 \ldots G_{n-1}}$$

- **Information Theory**

Hartley Law

$$C = 2B\log_2 n \qquad \text{bps}$$

where: C = channel capacity
B = channel bandwidth (Hz)
n = number of coding levels (2 for binary, 8 for octal, 10 for decimal etc.)

Shannon-Hartley Law

$$C = B\log_2(1 + S/N) \qquad \text{bps}$$

$$C = 3.32B\log(1 + S/N) \qquad \text{bps}$$

where: S/N = signal-to-noise ratio (absolute value)

Note: For a practical telephone channel B = 3.1 kHz (300 – 3400 Hz).

Total information sent

$$H = Ct \qquad \text{bits}$$

Power required

$$\frac{P_n}{P_2} = (n - 1)^2$$

where: P_n = power required in the n-level code
P_2 = power level required in the binary code
n = number of levels in a code

- **Noise Measurements Units**

dBrn (dB above reference noise)

$$dBrn = 10\log\frac{N}{1\times 10^{-12}W}$$

$$dBrn = dBm + 90$$

dBa (dB above adjusted noise)

For a pure tone:

$$dBa = 10\log\frac{N}{1\times 10^{-11.5}}$$

$$dBa = dBm + 85$$

For F1A weighted:

$$dBa = dBm + 82$$

dBrnC (dB above reference noise, C-message weighted)

$$dBrnC = dBm + 90$$

pWp (picowatts, psophometrically weighted)

$$pWp = \frac{(\text{psophometric}\,V^2)}{600\Omega}\times 10^{-12}$$

$$dBmp = 10\log\frac{pWp}{10^{-3}}$$

Transmission level point

$$TLP(dB) = 10\log\frac{S}{S_{0TLP}}$$

$$TLP_{dB} = S_{dBm} - S_{dBm0}$$

$$S_{dBm0} = S_{dBm} - TLP_{dB}$$

dBa0 (dBa at 0 dBm level point)

$$dBa0 = dBa - TLP_{dB}$$

dBrnC0 (dBrnC at 0 dBm level point)

$$dBrnC0 = dBrnC - TLP_{dB}$$

ANGLE MODULATION

- **Angle Modulation Characteristics**

Phase Deviation/ Modulation Index

PM waveform:

$$m = \Delta q = K_1 V_m$$

where: m = Δθ = modulation index or peak phase deviation (radians)
K$_1$ = deviation sensitivity of the PM modulator (rad/V)
V$_m$ = peak modulating signal amplitude (V)

FM waveform:

$$m = \frac{K_2 V_m}{f_m}$$

where: K$_2$ = deviation sensitivity of the FM modulator (rad/V-s)
f$_m$ = modulating signal frequency (Hz)

Frequency Deviation

PM waveform:

$$\Delta f = K_1 V_m f_m$$

where: Δf = peak frequency deviation of PM waveform (Hz)

FM waveform:

$$\Delta f = K_2 V_m$$

where: Δf = peak frequency deviation of FM waveform (Hz)

Percent Modulation (FM or PM)

$$\% \mathrm{mod}\,ulation = \frac{\Delta f_{actual}}{\Delta f_{\max}} \times 100$$

where: Δf$_{actual}$ = actual frequency deviation of carrier in hertz
Δf$_{max}$ = maximum frequency deviation allowed for communication system

Deviation Ratio

$$D.R. = \frac{\Delta f_{\max}}{\Delta f_{m(\max)}}$$

where: D.R. = deviation ratio of an FM waveform
Δf$_{m(max)}$ = maximum modulating frequency

Power Relations in an Angle-Modulated Wave

$$P_t = \frac{V_c^2}{2R}$$

where: P$_t$ = total transmitted power in an angle-modulated waveform (modulation or no modulation)
V$_C$ = peak amplitude of the carrier signal
R = load resistor

Bandwidth Requirements for Angle-Modulated Waves

Low-index modulation (narrowband FM)

$$B \approx 2 f_m$$

High-index modulation

$$B \approx 2\Delta f$$

Using the Bessel Table (practical bandwidth)

$$B = 2(n \times f_m)$$

where: n = number of significant sidebands

Using Carson's Rule (approximate bandwidth)

$$B = 2(\Delta f + f_m)$$

Noise and Angle Modulation

Maximum phase deviation due to an interfering single-frequency sinusoid:

$$\Delta q \approx \frac{V_n}{V_c} \qquad \text{radians}$$

where: Δθ = peak phase deviation due to interfering signal
V$_n$ = peak amplitude of noise voltage
V$_c$ = peak amplitude of carrier voltage

Maximum frequency deviation due to an interfering single-frequency sinusoid:

$$\Delta f \approx \left(\frac{V_n}{V_c}\right) f_n \quad \text{Hertz}$$

where: Δf = peak frequency deviation due to interfering signal
f$_n$ = noise modulating frequency

FM Noise Analysis

$$d_N = \Phi f_m$$

$$\Phi = \sin^{-1}\left(\frac{N}{S}\right)$$

$$\frac{S}{N} = \frac{d_S}{d_N}$$

where: δ$_N$ = frequency deviation of the noise
Φ = phase shift (radians)
δ$_S$ = frequency deviation of the carrier

DIGITAL COMMUNICATION

- **Frequency Shift-Keying**

$$MI = \frac{\Delta F}{F_a}$$

where: MI = modulation index
ΔF = frequency deviation
F_a = modulating frequency

For worst-case condition (alternating 1's and 0's)

$$MI = \frac{F_m - F_s}{F_b}$$

where: F_m = mark frequency
F_s = space frequency
F_b = input bit rate

Condition for synchronization

$$F_m = F_s = \frac{nF_b}{2}$$

where: n = any odd whole integer

- **Phase Shift-Keying**

M-ary encoding

$$N = \log_2 M$$
$$M = 2^N$$

where: N = number of bits
M = number of output conditions possible with n bits

1. **Quaternary or Quadrature Phase Shift Keying (QPSK)**

Binary Input	QPSK Output Phase
00	–135°
01	–45°
10	+135°
11	+45°

2. **Eight PSK (8PSK)**

Binary Input	8PSK Output
000	–112.5°
001	–157.5°
010	–67.5°
011	–22.5°
100	+112.5°
101	+157.5°
110	+67.5°
111	+22.5°

- **Quadrature Amplitude Modulation**

8QAM

Binary Input	8QAM Output
000	0.795V –135°
001	1.848V –135°
010	0.795V –45°
011	1.848V –45°
100	0.795V +135°
101	1.848V +135°
110	0.795V +45°
111	1.848V +45°

$$B.E. = \frac{F_b}{BW}$$

where: B.E. = Bandwidth Efficiency
F_b = transmission rate
BW = bandwidth

Digital Modulation Summary

Modulation	No. of Bit(s)	BW	Baud	B.E.
PSK	1	f_b	f_b	≤1
BPSK	1	f_b	f_b	1
QPSK	2	$f_b/2$	$f_b/2$	2
8 PSK	3	$f_b/3$	$f_b/3$	3
8 QAM	3	$f_b/3$	$f_b/3$	3
16 PSK	4	$f_b/4$	$f_b/4$	4
16 QAM	4	$f_b/4$	$f_b/4$	4

- **Sampling**

Nyquist sampling theorem states that the minimum sampling rate that can be used for a given PCM code is twice the highest audio input frequency

$$f_s \geq 2 f_a$$

where: f_s = minimum Nyquist sampling rate
f_a = highest frequency to be sampled

- **PCM code**

$$q_{e_{\max}} = \frac{resolution}{2}$$

$$DR = \frac{V_{\max}}{V_{\min}} = \frac{V_{\max}}{resolution}$$

In dB:

$$DR_{db} = 20 \log \frac{V_{\max}}{V_{\min}}$$

where: q_{emax} = quantization error
DR = dynamic range

V_{min} = equal to the resolution
V_{max} = maximum voltage that can be decoded by the DAC

To determine the number of bits required for a PCM code

$$2^n - 1 \geq DR$$

where: n = number of PCM bits (excluding sign bit)

Coding Efficiency

$$Coding\_efficiency = \frac{Minimum\_no.\_of\_bits}{Actual\_no.\_of\_bits} \times 100$$

Analog Companding

a. **μ-law companding** – used in U.S. and Japan

$$V_{out} = \frac{V_{\max} \ln\left(1 + m \frac{V_{in}}{V_{\max}}\right)}{\ln(1+m)}$$

where: V_{max} = maximum uncompressed analog input amplitude
V_{min} = amplitude of the input signal at a particular distant of time
μ = parameter used to define amount of compression
V_{out} = compressed output amplitude

b. **A-law companding** – used in Europe

$$V_{out} = V_{\max} \frac{A \frac{V_{in}}{V_{\max}}}{1 + \ln A} \quad 0 \leq \frac{V_{in}}{V_{\max}} \leq \frac{1}{A}$$

$$V_{out} = V_{\max} \frac{1 + \ln\left(A \frac{V_{in}}{V_{\max}}\right)}{1 + \ln A} \quad \frac{1}{A} \leq \frac{V_{in}}{V_{\max}} \leq A$$

where: A = parameter used to define the amount of compression

ACOUSTICS

- **The Sound Generation**

Octave

$$f_n = f_a 2^{n-1}$$

where: f_n = frequency of the nth octave
f_a = fundamental frequency
n = 1, 2, 3 …

Phon

$$Phon = 40 + 10 \log_2 (sone)$$

The apparent loudness and loudness levels

0 – 15 dB	very faint
15 – 30 dB	faint
30 – 60 dB	moderate
60 – 80 dB	loud
80 – 130 dB	very loud
130 dB	deafening

Notes: 0 dB – threshold of hearing
60 dB – average conversation
120 dB – threshold of pain
150 dB – permanent damage to hearing

Sound Pressure Levels of common sound sources

Source	SPL (dB)
Faintest audible sound	0
Whisper	20
Quiet residence	30
Soft stereo in residence	40
Speech range	50 – 70
Cafeteria	80
Pneumatic jack hammer	90
Loud crowd noise	100
Accelerating motorcycle	100
Rock concert	120
Jet engine (75 feet away)	140

- **Basic Formulas**

Sound Velocity

$$v = fl$$

Sound Velocity in Gases

$$v = \sqrt{\frac{gP_O}{r_O}}$$

where: γ = ratio of the specific heat at constant volume
P_o = the steady pressure of the gas (N/m^2)
ρ_o = the steady or average density of the gas (kg/m^3)

In dry air (experimental)

$$v = 331.45 \pm 0.05 \quad \text{m/s}$$
$$v = 1087.42 \pm 0.16 \quad \text{ft/s}$$

Velocity of sound in air for a range of about 20° Celsius change on temperature

$$v = 331.45 \pm 0.607 T_C \quad \text{m/s}$$
$$v = 1052.03 \pm 1.016 T_F \quad \text{ft/s}$$

where: T_C = temperature in degrees Celsius

T_F = temperature in degrees Fahrenheit

For TC > 20°C,

$$v = 331.45\sqrt{\frac{T_K}{273}} \qquad \text{m/s}$$

where: T_K = temperature in Kelvin

Recall: $T_K = T_C + 273$

$$T_R = T_F + 460$$

$$T_F = \frac{9}{5}T_C + 32$$

$$T_C = \frac{5}{9}(T_F - 32)$$

Sound Pressure Level

$$SPL = 20\log\frac{P}{P_O} = 10\log\left(\frac{P}{P_O}\right)^2 = 10\log\frac{I}{I_O}$$

where: P = RMS sound pressure (N/m^2)

P_o = reference sound pressure

= 2×10^{-5} N/m^2 or Pascal (Pa)

= 0.0002 μbar

= 2.089 lb/ft^2

Sound Intensity

$$I = \frac{P^2}{rv} = \frac{P^2}{410} \qquad W/m^2$$

where: ρ = density of air

v = velocity of sound in air

ρv = characteristic impedance of air to sound

= 410 rayls in air

The total intensity, I_T

$$I_T = I_1 + I_2 + I_3 + \ldots + I_n$$

The total pressure, P_T

$$P_T = \sqrt{P_1^2 + P_2^2 + P_3^2 + \ldots + P_n^2}$$

Sound Intensity coming from

(a) a point source (isotropic) in free space

$$I = \frac{W}{4pr^2}$$

(b) a source at ground level

$$I = \frac{W}{2pr^2}$$

The Sound Intensity Level

$$I_L = 10\log\frac{I}{I_O} = 10\log\left(\frac{P}{P_O}\right)^2$$

where: I_o = threshold intensity (W/m^2)

= 10^{-12} W/m^2

The Sound Power Level (PWL)

$$PWL = 10\log\frac{W}{W_O}$$

$$PWL = 10\log W + 120$$

where: W = sound power in watts

W_o = reference sound power

= 10^{-12} W

The Relation of SPL and PWL

(a) for a sound produced in free space by an isotropic source

$$SPL = PWL - 20\log r - 11$$

(b) for a sound produced at ground level

$$SPL = PWL - 20\log r - 8$$

- **Room Acoustics**

Optimum reverberation (at 500 to 1000 Hz)

Room Function	Reverberation time (s)
Recording and broadcast studios	0.45 – 0.55
Elementary classrooms	0.6 – 0.8
Playhouses, intimate drama production	0.9 – 1.1
Lecture and conference rooms	0.9 – 1.1
Cinema	0.8 – 1.2
Small Theaters	1.2 – 1.4
High school auditoriums	1.5 – 1.6
General purpose auditoriums	1.5 – 1.6
Churches	1.4 – 3.4

Different ways in computing reverberation times

A. **Stephens and Bate formula** (for ideal reverberation time computation)

$$t_{60} = r(0.012\sqrt[3]{V} + 0.1070) \qquad \text{seconds}$$

where: V = room volume (m^3)

r = 4 for speech

= 5 for orchestra

= 6 for choir

Optimum volume/person for various types of hall

Types of halls	Optimum volume/person (m^3)
Concert halls	7.1
Italian-type opera houses	4.2 – 5.1
Churches	7.1 – 9.9
Cinemas	3.1
Rooms for speech	2.8

B. **Sabine's formula** (for actual reverberation time with average absorption less than or equal to 0.2)

$$t_{60} = \frac{0.161V}{a} \qquad \text{seconds}$$

where: V = room volume (m^3)
a = total absorption units (m^2 – metric Sabine) (for a room: the sum of all absorption of the ceiling, walls, floor, furnishings and occupants).

$$t_{60} = \frac{0.049V}{a} \qquad \text{seconds}$$

where: V = room volume (ft^3)
a = total absorption units (ft^2 – customary Sabine)

Coefficient of absorption is the ratio of the absorbed sound intensity to the incident sound intensity.

$$a = \frac{I_a}{I_i} (unitless)$$

Note: α = 1 for perfect absorbent material

$$I_a = I_i - I_r$$

where: I_r = reflected sound intensity

Average absorption coefficient $(\bar{a})$

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

Total absorption (a)

$$a = \bar{a}A \qquad (m^2 \text{ or } ft^2)$$

where: A = surface area of the absorbent structure (m^2 or ft^2)

C. **Norris-Eyring's formula** (for actual reverberation time with average absorption coefficient greater than 0.2)

$$t_{60} = \frac{0.161V}{-S\ln(1-\bar{a})} \qquad \text{seconds}$$

where: S = total surface area (m^2)
$\bar{a}$ = average absorption coefficient of the reflecting surface

$$t_{60} = \frac{0.049V}{-S\ln(1-\bar{a})} \qquad \text{seconds}$$

where: S = total surface area (ft^2)
$\bar{a}$ = average absorption coefficient of the reflecting surface

A further correction may need to be added for higher frequency to allow for air absorption.

$$t_{60} = \frac{0.161V}{-S\ln(1-\bar{a}) + xV} \qquad \text{seconds}$$

For values of α less than about 0.2 but frequencies above 1000 Hz then a modified form of Sabine's formula is considered.

$$t_{60} = \frac{0.161V}{a + xV} \qquad \text{seconds}$$

where: x = sound absorption/volume of air (m^2/m^3)

x per m³ at a temperature of 20°C

Freq (Hz)	30%RH $\times10^{-3}$	40%RH $\times10^{-3}$	50%RH $\times10^{-3}$	60%RH $\times10^{-3}$	70%RH $\times10^{-3}$	80%RH $\times10^{-3}$
1000	3.28	3.28	3.28	3.28	3.28	3.28
2000	11.48	8.2	8.2	6.56	6.56	6.56
4000	39.36	29.52	22.96	19.68	16.4	16.4

RH = Relative Humidity

Methods of measuring absorption coefficient

A. Reverberation Chamber Method

Note: The lowest frequency should not be lower than the computed frequency from the formula below to ensure a diffuse sound field where v is the volume of the room.

$$f_{lowest} = \sqrt[3]{\frac{180}{v}} \qquad \text{Hz}$$

Principle of reverberation chamber method

"A measurement of reverberation time is made first without, and then with the absorbent material in the chamber."

Without the absorbent material,

$$t_1 = \frac{0.161V}{a}$$

With the absorbent material,

$$t_2 = \frac{0.161V}{a + da}$$

Therefore:

$$da = 0.161V\left(\frac{1}{t_2} - \frac{1}{t_1}\right)$$

In practice some slight correction needs to be made for the behavior of sound in the chamber which can make a difference of nearly 5%.

$$da = \left(55.3\frac{V}{v}\right)\left(\frac{1}{t_2} - \frac{1}{t_1}\right)$$

Absorption coefficient

$$a = \frac{da}{S}$$

where: V = volume of reverberation chamber
t_1 = reverberation of the chamber without absorbent material
t_2 = reverberation of the chamber with absorbent material
a = absorption of the chamber without absorbent material
δa = extra absorption due to the material
v = velocity of sound in air
S = surface area under measurement, which should be a single area between 10 and 12 m^2

B. Impedance Tube Method
Absorption coefficient

$$a = \frac{4A_1A_2}{(A_1 + A_2)^2}$$

where: A_1 and A_2 are the maximum and minimum amplitudes of the resultant standing wave pattern reverberation of the chamber without absorbent material

Note: α of impedance tube method is less than α of reverberation chamber method.

Types of absorbents
A. Membrane or Panel absorbers

The absorption is highly dependent upon frequency and is normally in the range of 50 to 500 Hz. They are often used in recording.

$$f = \frac{60}{\sqrt{md}}$$

where: f = approximate resonant frequency
m = mass of the panel in kg/m^2
d = depth of the air space in m

B. Helmholtz or Cavity or Volume Resonators

Resonant frequency (f) for a narrow-neck resonator is approximately

$$f = \frac{vr}{2p}\sqrt{\frac{2p}{(2l + pr)V}}$$

If there is no neck, l = 0

$$f = \frac{v}{2p}\sqrt{\frac{2r}{V}}$$

where: v = velocity of sound in air
r = radius of the neck
l = length of the neck
V = volume of cavity

SATELLITE COMMUNICATIONS

- **Communications Satellite**

Orbit Location (Satellite Elevation category)

(a) Low Earth Orbit (LEO) Satellite

Orbital height : 100 – 300 mi
Orbital velocity : 17,500 mph
Orbital time (period) : 1.5 hours
Satellite Availability : 15 min per orbit
Typical operating frequency : 1.0 GHz – 2.5 GHz

(b) Medium Earth Orbit (MEO) Satellite

Orbital height : 6,000 – 12,000 mi
Orbital velocity : 9,580 mph
Orbital time (period) : 5 – 12 hours
Satellite Availability : 2 – 4 hours per orbit
Typical operating frequency : 1.2 GHz – 1.66 GHz

(c) Geostationary or Geosynchronous (GEO) Satellite

Orbital height : 22,300 mi
Orbital velocity : 6,879 mph
Orbital time (period) : 24 hours
Satellite Availability : 24 hours per orbit
Typical operating frequency : 2 GHz – 18 GHz

THE GEOSYNCHRONOUS SATELLITE

Altitude : 19,360 nmi
: 22,284 smi
: 35,855 km
Period : 23 hr, 56 min, 4.091 s (one sidereal day)
Orbit inclination : 0°

Velocity : 6879 mph
Coverage : 42.5% of earth's surface (0° elevation)
Number of satellites : Three for global coverage with some areas of overlap (120° apart)
Areas of no coverage : Above 81° north and south latitude
Advantages : Simpler ground station tracking
: No handover problem
: Nearly constant range
: Very small Doppler shift
Disadvantages : Transmission delay
: Range loss (free space loss)
Spatial separation : 3° – 6° [Typically 4° (equivalent to at least 1833 miles of separation distance) or more]

Satellite classification according to size

Size	Mass	Cost
Large Satellite	> 1,000 kg	> $ 100 M
Small Satellite	500 – 1,000 kg	$ 50 – 100 M
Mini-Satellite	100 – 500 kg	$ 5 – 20 M
Micro-Satellite	10 – 100 kg	$ 2 – 3 M
Nano-Satellite	< 10 kg	< $ 1 M

- **Satellite Orbital Dynamics**

$$a = AP^{\frac{2}{3}}$$

where: α = semi-major axis (km)
A = constant (unitless)
A = 42241.0979 for earth
P = mean solar earth days [ratio of the time of one sidereal day (23 hours and 56 minutes) to the time of one revolution of earth (24 hours)]
P = 0.9972

For a satellite to stay in orbit, the centrifugal force caused by its rotation around earth should be equal to the earth's gravitational pull.

$$F_c = F_g$$

$$F_g = G\frac{m_s m_e}{(R+h)^2} \qquad F_c = \frac{m_s v^2}{(R+h)}$$

where: F_c = centrifugal force
F_g = gravitational force
G = gravitational constant (6.670×10^{-11})
m_s = mass of satellite
m_e = mass of earth (5.98×10^{24} kg)
v = velocity
R = earth's radius (≈ 3960 mi ≈ 6371 km)
h = satellite height

Satellite velocity in orbit

$$v = \sqrt{\frac{4\times10^{11}}{(R_{km}+h_{km})}} \qquad \text{m/s}$$

Satellite height

$$h = \sqrt[3]{\frac{gT^2R^2}{4p^2}} - R \qquad \text{km}$$

where: T = satellite period
g = gravitational acceleration (9.81×10^{-3} km/s^2)

The escape velocity of earth is 25,000 mph or from the formula:

$$\text{Escape velocity} = \sqrt{2gR}$$

The minimum acceptable angle of elevation is 5°.

Satellite Range (distance from an earth station)

$$d = \sqrt{(R+h)^2 - R^2\cos^2 b} - R\sin b$$

where: β = angle of elevation

Note: β = 0°, d is maximum, satellite is farthest
β = 90°, d = h, satellite is nearest

- **Frequency Allocation**

The most common carrier frequencies used for SATCOM are the 6/4 and 14/12 GHz bands.

Frequency bands used in satellite communications

Frequency	Band
225 – 390 MHz	P
350 – 530 MHz	J
1530 – 2700 MHz	L
2500 – 2700 MHz	S
3400 – 6425 MHz	C
7250 – 8400 MHz	X
10.95 – 14.5 GHz	Ku
17.7 – 21.2 GHz	Ka
27.5 – 31 GHz	K
36 – 46 GHz	Q
46 – 56 GHz	V
56 – 100 GHz	W

Microwave frequency bands

Band designation	Frequency range (GHz)
L	1 – 2
S	2 – 4
C	4 – 8
X	8 – 12
Ku	12 – 18
K	18 – 27
Ka	27 – 40
Millimeter	40 – 300
Submillimeter	>300

Earth coverage is approximately one-third of the earth's surface with approximate antenna beamwidth of 17°.

- **The Satellite System Parameters**

1. Transmit Power and Bit Energy

$$E_b = P_t T_b = \frac{P_t}{f_b}$$

where: E_b = energy of a single bit (Joules/bit)
P_t = total carrier power (watts)
T_b = time of a singe bit (seconds)
f_b = bit rate (bps)

2. Effective Isotropic Radiated Power (EIRP)

$$EIRP = P_r G_t$$

where: P_r = total power radiated from an antenna
G_t = transmit antenna power gain

3. Equivalent Noise Temperature (T_e)

$$T_e = T_o(NF - 1)$$

where: T_o = temperature of the environment (K)
NF = noise factor (absolute value)

4. Noise Density (N_o)

$$N_o = \frac{N}{BW} = kT_e$$

5. Carrier-to-Noise Density Ratio

$$\frac{C}{N_o} = \frac{C}{kT_e}$$

6. Energy Bit-to-Noise Density Ratio

$$\frac{E_b}{N_o} = \frac{\frac{C}{f_b}}{\frac{N}{BW}} = \frac{CBW}{Nf_b}$$

7. Gain-to-Equivalent Noise Temperature Ratio

$$\frac{G}{T_e} = \frac{Gr + G(LNA)}{T_e}$$

The satellite system link equations

Uplink Equations

$$\frac{C}{N_o} = \frac{A_t P_r (L_p L_u) A_r}{kT_e} = \frac{A_t P_r (L_p L_u)}{k} \times \frac{G}{T_e}$$

Expressed in dB

$$\frac{C}{N_o} = 10\log A_t P_r - 20\log\left(\frac{4pD}{l}\right) + 10\log\left(\frac{G}{T_e}\right)$$
$$-10\log L_u - 10\log k$$

$$\frac{C}{N_o} = EIRP(dBW) - L_p(dB) + \left(\frac{G}{T_e}\right)(dBK^{-1})$$
$$-L_u(dB) - k(DBWK)$$

Uplink Equations

$$\frac{C}{N_o} = \frac{A_t P_r (L_p L_d) A_r}{kT_e} = \frac{A_t P_r (L_p L_d)}{k} \times \frac{G}{T_e}$$

Expressed in dB

$$\frac{C}{N_o} = 10\log A_t P_r - 20\log\left(\frac{4pD}{l}\right) + 10\log\left(\frac{G}{T_e}\right)$$
$$-10\log L_d - 10\log k$$

$$\frac{C}{N_o} = EIRP(dBW) - L_p(dB) + \left(\frac{G}{T_e}\right)(dBK^{-1})$$
$$-L_d(dB) - k(DBWK)$$

MULTIPLEXING

- **Frequency Division Multiplexing**

Voice band frequency (VF): 0 – 4 kHz
Basic voice band (VB) circuit is called 3002
Channel: 300 – 3000 Hz band

Note: The basic 3002 channel can be subdivided into 24 narrower 3001 (telegraph) channels that have been frequency-division multiplexed to form a single 3002 channel.

A. Basic Group

$$f_c = (112 - 4n) \qquad \text{kHz}$$

where: f_c = channel carrier frequency
n = channel number

Lower sideband

$$f_{LSB} = f_c - (0 \text{ to } 4\ kHz)$$

Upper sideband

$$f_{USB} = f_c + (0 \text{ to } 4\ kHz)$$

B. Basic Supergroup

$$f_c = (372 + 48n) \qquad \text{kHz}$$

where: f_c = group carrier frequency
n = group number

Lower sideband

$$f_{LSB} = f_c - (60 \text{ to } 108\ kHz)$$

Upper sideband

$$f_{USB} = f_c + (60 \text{ to } 108\ kHz)$$

C. Basic Mastergroup

Two categories of mastergroups

U600 – may be further multiplexed and used for higher-capacity microwave radio.
L600 – used for low-capacity microwave systems.

Basic Mastergroup bandwidth:

L600 (60 – 2788 kHz)

$$BW = 2728\ kHz$$

U600 (564 – 3084 kHz)

$$BW = 2520\ kHz$$

The Supergroup Carrier Frequencies
L600 Mastergroup

Supergroup	Carrier frequency (kHz)
1	612
2	Direct
3	1116
4	1364
5	1612
6	1860
7	2108
8	2356
9	2724
10	3100

U600 mastergroup

Supergroup	Carrier frequency (kHz)
13	1116
14	1364
15	1612
16	1860
17	2108
18	2356
D25	2652
D26	2900
D27	3148
D28	3396

Summary of AT&T's FDM Hierarchy

Group = 12 VB channels
Supergroup = 5 Groups
= 60 VB channels
Mastergroup = 10 Supergroups
= 50 Groups
= 600 VB channels
Jumbogroup = 6 Mastergroups
= 60 Supergroups
= 300 Groups
= 3600 VB channels
Superjumbogroup = 3 Jumbogroups
= 18 Mastergroups
= 180 Supergroups
= 900 Groups
= 10800 VB channels

Summary of CCITT's FDM Hierarchy

Group = 12 VB channels
Supergroup = 5 Groups
= 60 VB channels
Mastergroup = 5 Supergroups
= 25 Groups
= 300 VB channels
Supermastergroup = 3 Mastergroups
= 15 Supergroups

- **Time Division Multiplexing**

Summary of Digital Multiplex Hierarchy (North American)

Line Type	Digital Signal	Bit rate (Mbps)	Channel Capacity	Services Offered	Medium
T1	DS – 1	1.544	24	VB telephone	Twisted pair
T1C	DS – 1C	3.152	48	VB telephone	Twisted pair
T2	DS – 2	6.312	96	VB tel, picturephone	Twisted pair, μwave
T3	DS – 3	44.736	672	VB tel, picturephone, TV	coax, μwave
T4M	DS – 4	274.176	4032	Same as T3 except more capacity	coax, optical fiber
T5	DS – 5	560.160	8064	Same as T3 except more capacity	optical fiber

Summary of CEPT 30 + 2 PCM Multiplex Hierarchy (European)

Level	Data Rate (Mbps)	Channel Capacity
1	2.048	30
2	8.448	120
3	34.368	480
4	139.264	1920
5	564.992	7680

Japanese Multiplex Hierarchy

Level	Data Rate (Mbps)	Channel Capacity
1	1.544	24
2	6.312	96
3	32.064	480
4	97.728	1440
5	565.148	7680

CCITT Time-Division Multiplexed Carrier System (European Standard PCM-TDM System)

With CCITT system, a 125-μs frame is divided into 32 equal time slots.

E – 1 Carrier

Framing and alarm channel	Voice channel 1	Voice Channel 2 – 15	Common Signaling channel	Voice Channel 16 – 29	Voice Channel 30
TS 0	TS 1	TS 2 – 16	TS 17	TS 18 – 30	TS 31

Line-Encoding Summary

Encoding Format	Minimum BW	Average DC	Clock Recovery	Error Detection
UPNRZ	$f_b/2$	+ V/2	Poor	No
BPNRZ	$f_b/2$	0 V	Poor	No
UPRZ	f_b	+ V/2	Good	No
BPRZ	f_b	0 V	Best	No
BPRZ-AMI	$f_b/2$	0 V	Good	Yes

TELEPHONY

- **Introduction**

1.) Typical sounds produced by humans: 100 to 1000 Hz.
2.) Peak sensitivity of human hearing: 4 kHz.
3.) Upper frequency limit for hearing: 18 to 20 kHz.
4.) Lower frequency limit for hearing: 18 to 20 Hz.

Nature of Speech

1.) Sound pressure wave of speech contains frequencies: 100 Hz to 10 kHz.
2.) Speech power range: 10 to 1,000 μW.
3.) Maximum intelligibility for voice frequency: 1,000 and 3,000 Hz.
4.) Maximum voice energy is located between 250 and 500 Hz.

Speech Measurement

For typical single talker average power in dBm:

$$P(dBm) = VU\ reading - 1.4\ dB$$

For more than one speaker over the channel

$$P(dBm) = VU\ reading - 1.4 + 10\log N$$

where: N = number of speakers

- **The telephone Set**

Pulse Dialing

To transmit a digit, it takes 0.1 second per pulse + 0.5 second inter-digital delay time.

DTMF Frequencies

Frequencies	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

Network Call Progress Tones

Tone	Frequency (Hz)
Dial Tone	350 + 440
Ringback	440 + 480
Busy Signal	480 + 620

- **Switching and Signaling**

$$N = \frac{n(n-1)}{2}$$

where: N = number of connections
n = number of subscribers

- **Traffic Engineering**

Measurement of Telephone Traffic

$$A = C \times T$$

where: A = traffic intensity in Erlangs
C = designates the number of calls originated during a period of 1 hr (calls/hr or calls/min)
T = the average holding time, usually given in hours (hr/call or min/call)

$$A = \frac{S}{t}$$

where: S = sum of all the holding time (min)
t = observation period (1 hr or 60 min)

Note: 1 Erlang = 36 ccs (Century Call Seconds or Hundred Call Seconds)

$$Grade\_of\_service = \frac{Number\_of\_lost\_calls}{Total\_no.\_of\_offered\_calls}$$

- **GSM Network**

Radio-Path Propagation Loss

$$\Delta P = 40\log\left(\frac{d_1}{d_2}\right)$$

(40 dB/decade path loss)

$$\Delta G = 20\log\left(\frac{h_1'}{h_1}\right)$$

(A base station antenna height gain of 6dB/octave)

where: ΔP = the difference in two receive signal strengths based on two different path lengths d_1 and d_2
ΔG = the difference in two receive signal strengths based on two different antenna heights h_1 and h_1'

Receive signal in decibels
For non-obstructive path

$$P_r = P_{ro} - g\ \log\left(\frac{r}{r_o}\right) + 20\log\left(\frac{h_e'}{h_1}\right) + a$$

For obstructive path

$$P_r = P_{ro} - g\ \log\left(\frac{r}{r_o}\right) + L + a$$

where: r = distance between the base and the mobile unit in mi or km
h_e' = effective antenna height
L = shadow loss
P_{ro} = received signal at a reference distance r_o
r_o = usually equal to 1 mi (1.6 km)
α = correction factor

Standard Condition:

Frequency (f_o)	900 MHz
Base-station antenna height (h_1)	30.46 m
Base-station power at the antenna	10 watts
Base-station antenna gain (G_t)	6 dBd
Mobile-unit antenna height	3 m
r_o	1.6 km
Mobile-unit antenna gain (G_m)	0 dBd

General Formula for Mobile radio Propagation Path Loss:

$$P_r = P_t - 134.4 - 38.4\log r_1 + 20\log h_1 + 20\log h_2 + G_t + G_m$$

where: P_t and P_r are in decibels above 1mW, r_1 is in kilometers, h_1 and h_2 are in meters, and G_t and G_m are in decibels

Cochannel Interference Reduction Factor (CIRF), q

$$q = \frac{D}{R}$$

Frequency Reuse factor, K

$$q = \sqrt{3K} \qquad K = \frac{q^2}{3}$$

Radio Capacity

A. Analog, FDMA and TDMA cellular system

$$m = \frac{B_t}{B_c\sqrt{\frac{2}{3}\left(\frac{C}{I}\right)}}$$

where: B_t = total allocated spectrum
B_c = channel bandwidth
(C/I) = required carrier-to-interference ratio in linear values

B. CDMA cellular system

$$m = \frac{M}{K}$$

where: M = total number of voice channels
K = frequency reuse factor

Antenna Separation Requirement

A. At the Base Station

$$\frac{h}{d} = 11$$

where: h = antenna height
d = spacing between two antennas

B. At the Mobile Unit

A separation of a half-wavelength between two mobile antennas is required at 850 MHz. Therefore, the separation between two antennas needs to be only 0.18 m (about 6 inches) at the cellular frequency of 850 MHz.

Cell Splitting Formulas

$$N = \frac{A}{3.464r^2}$$

where: N = number of cells
A = total area to be covered
r = radius inscribed in the hexagon

MICROWAVE COMMUNICATIONS

- **Microwave Passive Repeater**

Gain of a passive repeater

$$G_P = 20\log\frac{4pA\cos a}{l^2} \qquad A_P = \frac{A\sin q}{2}$$

where: A = actual surface area of the repeater (ft^2)
Acosα = A$_P$ or A$_{eff}$ = projected or effective area of the passive repeater (ft^2)

Also,

$$G_P = 22.1 + 20\log A_P + 40\log f_{GHz}$$

Beamwidth of a fully illuminated passive repeater

$$q = \frac{58.7\,l}{L}$$

where: L = effective linear dimension of the repeater in the direction in which the beamwidth is to be measured

Net Path Loss (NPL)

$$NPL(dB) = G_T - L_{P1} + G_P - L_{P2} + G_R$$

where: G$_T$ = transmit antenna gain
L$_{P1}$ = path loss on path 1
G$_P$ = passive repeater gain
L$_{P2}$ = path loss on path 2
G$_R$ = receive antenna gain

Near field and far field conditions

$$\frac{1}{k} = \frac{p\,l\,d'}{4A}$$

If $\frac{1}{k} < 2.5$, near-field condition exists
$\frac{1}{k} > 2.5$, far-field condition exists

where: d' = length of the path in question (i.e., the shorter distance)

- **Protection Switching**

The antenna separation required for optimum operation of space diversity system may be calculated using the formula:

$$S = \frac{3\,l\,R_e}{L}$$

where: S = separation
R$_e$ = effective earth's radius
L = path length

- **Path Characteristics**

Free-space attenuation or path Loss, L$_P$

$$L_P = \left(\frac{4pd}{l}\right)^2$$

$$L_P = 32.4 + 20\log d_{km} + 20\log f_{MHz}$$
$$L_P = 92.4 + 20\log d_{km} + 20\log f_{GHz}$$
$$L_P = 36.6 + 20\log d_{mi} + 20\log f_{MHz}$$
$$L_P = 96.6 + 20\log d_{mi} + 20\log f_{GHz}$$

Antenna Gain, G

$$G = 6\left(\frac{D}{l}\right)^2$$

$$G = -52.6 + 20\log D_{ft} + 20\log f_{MHz}$$
$$G = 7.5 + 20\log D_{ft} + 20\log f_{GHz}$$
$$G = -42.3 + 20\log D_m + 20\log f_{MHz}$$
$$G = 17.7 + 20\log D_m + 20\log f_{GHz}$$

Receive Signal Level (RSL)

$$RSL_{dBm} = P_{odBm} - L_{fTA} + G_T - L_P + G_R - L_{fTB}$$
$$RSL_{dBm} = (\text{minimum RF input})_{dBm} + FM_{dB}$$

where: P$_o$ = transmitter output
L$_{fTA}$ = total fixed losses at the transmitter side which includes feeder loss, connector loss, branching loss, waveguide loss etc.
L$_{fRB}$ = total fixed losses at the receiver side
G$_T$ = transmitter antenna gain
G$_R$ = receiver antenna gain
min. RF input = practical receiver threshold
FM = fade margin

Fade Margin – an attenuation allowance so that anticipated fading will still keep the signal above specified minimum RF input.

$$FM = 30\log d_{km} + 10\log 6abf_{GHz} - 10\log(1-R) - 70$$

where: a = roughness factor
= 4 for smooth terrain, including over water
= 1 for average terrain, with some roughness
= 0.25 for mountainous, very rough
b = 0.5 for hot, humid coastal areas

= 0.25 for normal interior temperature or subarctic areas
= 0.125 for mountainous or very dry but nonreflective areas

System Gain, G_S

$$G_S = P_{OdBm} - (\text{minimum RF input})_{dBm}$$

System Reliability Estimates

b) Based on Propagation

$$R = (1 - U_{ndp}) \times 100\%$$

where: Undp = non-diversity outage probability for a given path

$$U_{ndp} = abf^{1.5}d^{3}(1.25 \times 10^{-6})10^{\frac{-FM}{10}}$$

where: f = frequency in GHz
d = distance in miles

c) Based on equipment

$$R = (1 - U) \times 100\%$$

$$U = \frac{MTTR}{MTBF + MTTR}$$

where: U = unavailability or probability of outage
MTTR = Mean Time To Repair
MTBF = Mean Time Between Failures or Mean Time Before Failures

Also,

$$U = \frac{Outage}{8760\_hours}$$

Note: Downtime or Outage time (in hours per year)

$$A = \frac{MTBF}{MTBF + MTTR}$$

where: A = Availability

Fresnel Zone Radius/Clearance/Height

$$R_{ft} = 72.1\sqrt{\frac{nd_1d_2}{f_{GHz}d_{mi}}}$$

where: d_1 and d_2 are distances in miles
n = number of Fresnel Zone (n = 1 for 1st FZ; n = 2 for 2nd FZ, etc.)

$$R_m = 17.3\sqrt{\frac{nd_1d_2}{f_{GHz}d_{km}}}$$

where: d_1 and d_2 are distances in kilometers