SEMICONDUCTOR THEORY

- **Atomic Structure**

Diameter of neutron = 10^{-13} cm

Maximum number of electrons per shell or orbit

$$N_e = 2n^2$$
$$n = 1,2,3,4$$

Letter designation

K shell – 1	O shell – 5
L shell – 2	P shell – 6
M shell – 3	Q shell – 7
N shell – 4	

Mass and Charge of different Particles

Particle	Mass (kg)	Charge (C)
Electron	9.1096×10^{-31}	-1.6022×10^{-19}
Proton	1.6726×10^{-27}	$+1.6022\times10^{-19}$
Neutron	1.6726×10^{-27}	No charge

A = no. of protons + no. of neutrons
Z = number of protons or electrons

Where: A = Atomic mass or weight (A)
Z = Atomic number (Z)

Note: Mass of proton or neutron is 1836 times that of electron.

Energy Gap Comparison

Element	No. of Valence Electrons (V_e)	Energy
Insulator	8	> 5eV
Semiconductor	4	Si = 1.1eV Ge = .67eV
Conductor	1	0eV

At room temperature: there are approximately **1.5×10^{10}** of free electrons in a cubic centimeter (cm^3) for intrinsic **silicon** and **2.5×10^{13}** for **germanium**.

- **Diode Theory**

$$Vth_{T1} = Vth_{T0} + k(T_1 - T_0)$$

where: Vth_{T1} = threshold voltage at T_1
Vth_{T0} = threshold voltage at T_0
k = –2.5 mV/°C for Ge
k = –2.0 mV/°C for Si

The diode current equation

$$I_d = I_s(e^{\frac{kV_d}{T_k}} - 1)$$

Where: I_d = diode current
I_s = reverse saturation current or leakage current
V_d = forward voltage across the diode
T_k = room temperature at °K
= °C + 273

$$k = \frac{11600}{n}$$

for low levels of diode current
n = 1 for Ge and n = 2 for Si

for higher levels of diode current
n = 1 for both Si and Ge

Temperature effects on Is

$$I_{sT_1} = I_{sT_0} e^{k(T_1 - T_0)}$$

Where: I_{sT1} = saturation current at temperature T_1
I_{sT0} = saturation current at room temperature
k = 0.07/°C
T_1 = new temperature
T_0 = room temperature (25°C)

Reverse Recovery Time (T_{rr})

$$T_{rr} = t_s + t_t$$

Where: T_{rr} = time elapsed from forward to reverse bias (ranges from a few ns to few hundreds of ps)
T_t = transition time
T_s = storage time

DC CIRCUITS 1

1 Coulomb = 6.24×10^{18} electrons

By definition: A wire of 1 mil diameter has a cross-sectional area of 1 Circular Mil (CM)

1 mil = 10^{-3} in
1 in = 1000 mils
A_{square} = 1 mil^2

$$A_{circle} = \frac{\pi D^2}{4} mil^2$$

$$1mil^2 = \frac{4}{\pi} CM$$

Type/Flavors of Quarks

Quark	Symbol	Charge	Baryon no.
Up	U	+2/3	1/3
Down	D	–1/3	1/3
Charm	C	+2/3	1/3
Strange	S	–1/3	1/3
Top	T	+2/3	1/3
Bottom	B	–1/3	1/3

Proton – 2 Up and 1 Down
Neutron – 1 Up and 2 Down

Types of Battery

Type	Height (in)	Diameter (in)
D	2¼	1¼
C	1¾	1
AA	1⅞	9⁄16
AAA	1¾	⅜

$$I = \frac{Q}{t} \qquad Ampere(A); \frac{Coulomb(C)}{\sec ond(s)}$$

$$V = \frac{W}{Q} \qquad Volt(V); \frac{Joule(J)}{Coulomb(C)}$$

$$r = \frac{RA}{L} \qquad \Omega - m; \Omega - cm; \frac{\Omega - CM}{ft}$$

Resistivities of common metals and alloys

Material	ρ (10^{-8}Ω-m)
Aluminum (Al)	2.6
Brass	6
Carbon	350
Constantan (60% Cu and 40% Ni)	50
Copper (Cu)	1.7
Manganin (84% Cu, 12%Mn & 4%Ni)	44
Nichrome	100
Silver (Ag)	1.5
Tungsten (W)	5.6

Absolute zero = 0 K = –273°C
ρ_{Cu} = 10.37 Ω–CM/ft

Temperature effects on resistance

$$\frac{R_2}{R_1} = \frac{|T| + t_2}{|T| + t_1} \qquad \alpha_1 = \frac{1}{|T| + t_1}$$

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

where: |T| = inferred absolute temperature, °C
R_2 = final resistance at final temp. t_2
R_1 = initial resistance at initial temp. t_1
α_1 = temp coefficient of resistance at t_1

American Wire Gauge (AWG)

AWG #10: A = 5.261 mm^2
AWG #12: A = 3.309 mm^2
AWG #14: A = 2.081 mm^2

Inferred Absolute Temp. for Several Metals

Material	Inferred absolute zero, °C
Aluminum	-236
Copper, annealed	-234.5
Copper, hard-drawn	-242
Iron	-180
Nickel	-147
Silver	-243
Steel, soft	-218
Tin	-218
Tungsten	-202
Zinc	-250

Temperature-Resistance Coefficients at 20 °C

Material	α_{20}
Nickel	0.006
Iron, commercial	0.0055
Tungsten	0.0045
Copper, annealed	0.00393
Aluminum	0.0039
Lead	0.0039
Copper, hard-drawn	0.00382
Silver	0.0038
Zinc	0.0037
Gold, pure	0.0034
Platinum	0.003
Bras	0.002
Nichrome	0.00044
German Silver	0.0004
Nichrome II	0.00016
Manganin	0.00003
Advance	0.000018
Constantan	0.000008

$$G = \frac{1}{R} = \frac{A}{rL} = s\ \frac{A}{L}$$

where: σ = specific conductance or conductivity of the material in siemens/m or mho/m.

Note: The best is **silver** with **1.68×10^{24}** free electrons per in^3. Next is **copper** with **1.64×10^{24}** free electrons per in^3 and then **aluminum** with **1.6×10^{24}** free electrons per in^3.

$$P = \frac{W}{t} = \frac{Q}{t}E = IE = \frac{E^2}{R} = I^2R$$

where: W = work in Joules (J)
t = time in seconds (s)
Q = charge in Coulomb (C)

Voltage Division Theorem
2 resistors in series with one

$$V_1 = \frac{R_1}{R_1 + R_2}E \qquad V_2 = \frac{R_2}{R_1 + R_2}E$$

Current Division Theorem

$$I_1 = \frac{R_2}{R_1 + R_2}I_T \qquad I_2 = \frac{R_1}{R_1 + R_2}I_T$$

Transformations or Conversations:
Delta (Δ) to Wye (Y)

$$R_Y = \frac{\mathrm{Pr}\,oduct\_of\_adjacent\_R\_in\_\Delta}{\sum of\_all\_R\_in\_\Delta}$$

Wye (Y) to Delta (Δ)

$$R_\Delta = \frac{\sum of\_cross\_products\_in\_Y}{Opposite\_R\_in\_Y}$$

Color Coding Table

Color	1st signifi-cant	2nd signifi-cant	Multiplier	Toler-rance (±%)	Temp Coef ppm/°C
Black	0	0	100	20	0
Brown	1	1	101	1	-33
Red	2	2	102	2	-75
Orange	3	3	103	3	-150
Yellow	4	4	104	GMV	-220
Green	5	5	105	5	-330
Blue	6	6	106	-	-470
Violet	7	7	107	-	-750
Gray	8	8	108	-	+30
White	9	9	109	-	+500
Gold	-	-	0.1	5	+100
Silver	-	-	0.01	10	Bypass
None	-	-	-	20	-

GMV = Guaranteed Minimum Value: -0%, +100%

Fifth band reliability color code

Color	Failures during 1000 hours of operation
Brown	1.0%
Red	0.1%
Orange	0.01%
Yellow	0.001%

Batteries

$$Battery\ life = \frac{Ampere - hour\_rating(Ah)}{Amperes\_drawn(A)}$$

Cell Types and Open-Circuit Voltage

Cell Name	Type	Nominal Open-Circuit Voltage
Carbon-zinc	Primary	1.5
Zinc-chloride	Primary	1.5
Manganese dioxide (alkaline)	Primary or Secondary	1.5
Mercuric oxide	Primary	1.35
Silver oxide	Primary	1.5
Lead-acid	Secondary	2.1
Nickel-cadmium	Secondary	1.25
Nickel-iron (Edison cell)	Secondary	1.2
Silver-zinc	Secondary	1.2
Silver-cadmium	Secondary	1.1
Nickel metal hydride (NiMH)	Secondary	1.2

DIODES

- **Diode Applications**

Half–wave Rectification

$$V_{DC} = \frac{V_m}{p} = 0.318V_m$$

$$PIV\ rating \geq V_m$$

Full–wave Rectification

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^t V(t)^2\,dt}$$

$$V_{DC} = 0.36V_m$$

$PIV\ rating \geq V_m$ *for bridge-type*

$PIV\ rating \geq 2V_m \qquad for\ center\text{-}tapped$

- **Other Semiconductor Devices**

Zener Diode

$$T_{CC} = \frac{\Delta V_Z}{V_Z(T_1 - T_0)}$$

where: T_{CC} = temperature coefficient
$T_1 – T_0$ = change in temperature
V_Z = Zener Voltage at T_0

Basic Zener Regulator

I. V_i and R_L fixed

(a) Determine the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.
(b) Substitute the appropriate equivalent circuit and solve for the desired unknown.

II. Fixed R_L, variable V_i

$$V_{i\min} = \frac{(R_L + R_S)V_Z}{R_L} \qquad V_{i\max} = I_{R\max}R_S + V_Z$$

III. Fixed V_i, variable R_L

$$R_{L\min} = \frac{RV_Z}{V_i - V_Z} \qquad R_{L\max} = \frac{V_Z}{I_{L\min}}$$

Varactor diode or Varicap diode

$$C_T = e\frac{A}{W_d}$$

where: C_T = transition capacitance which is due to the established covered charges on either side of the junction
A = pn junction area
W_d = depletion width

In terms of the applied reverse bias voltage:

$$C_T = \frac{k}{(V_T + V_R)^n}$$

where: C_T = transition capacitance which is due to the established covered charges on either side of the junction
k = constant determined by the semiconductor material and construction technique
V_T = knee voltage
V_R = reverse voltage
n = ½ for alloy junctions and ⅓ for diffused junctions

In terms of the applied reverse bias voltage:

$$C_T = \frac{C(0)}{\left(1 + \frac{V_R}{V_T}\right)^n}$$

where: C(0) = capacitance at zero-bias condition

Also,

$$TC_C = \frac{\Delta C}{C_O(T_1 - T_0)}$$

where: TC_C = temperature coefficient
$T_1 - T_0$ = change in temperature
C_0 = capacitance at T_0

Photodiode

$$W = hf = h\frac{c}{l};\ Joules$$

where: W = energy associated with incident light waves
h = Planck's constant (6.624×10^{-34} J-sec)
f = frequency

1eV = 1.6×10^{-9} J
1 Angstrom (Å) = 10^{-10} m

Solar Cell

$$h = \frac{P_O}{P_i} = \frac{P_{\max}}{(Area)\left(\frac{1W}{cm^2}\right)}$$

where: η = efficiency
P_0 = electrical power output
P_i = power provided by the light source
P_{max} = maximum power rating of the device
Area = in cubic centimeters

Note: The power density received from the sun at sea level is about 1000 mW/cm^2

BIPOLAR JUNCTION TRANSISTOR

$$Ratio = \frac{width_{total}}{width_{base}} = \frac{0.150}{0.001} = 150$$

- **Basic Operation**

Relationship between I_E, I_B and I_C:

$$I_E = I_B + I_C$$

I_C is composed of two components:

$$I_C = I_{majority} + I_{minority}$$

DC Transistor Parameters

$$a = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{Vcb=constant} \qquad b = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{Vce=constant}$$

$$a = \frac{I_C}{I_E} \qquad b = \frac{I_C}{I_B}$$

where: I_E = emitter current
I_B = base current
I_C = collector current
α = CB short-circuit amplification factor
β = CE forward-current amplification factor

Relationship between α and β:

$$a = \frac{b}{b+1} \qquad b = \frac{a}{1-a}$$

Stability Factor (S):

$$S(I_{CO}) = \frac{\Delta I_C}{\Delta I_{CO}} \qquad \textit{Unitless}$$

$$S(I_{CO}) = \frac{\Delta I_C}{\Delta V_{BE}} \qquad \textit{Siemens}$$

$$S(I_{CO}) = \frac{\Delta I_C}{\Delta b} \qquad \textit{Ampere}$$

Small Signal Analysis

A. Hybrid Model

$$V_i = h_{11} I_{in} + h_{12} V_0$$

$$I_o = h_{21} I_{in} + h_{22} V_0$$

If $V_o = 0$

$$h_{11} = \frac{V_i}{I_{in}} \qquad \textit{ohms}$$

If $I_{in} = 0$

$$h_{12} = \frac{V_i}{V_0} \qquad \textit{unitless}$$

If $V_o = 0$

$$h_{21} = \frac{I_0}{I_{in}} \qquad \textit{unitless}$$

If $I_{in} = 0$

$$h_{11} = \frac{I_0}{V_0} \qquad \textit{siemens}$$

where: h_{11} = input-impedance, h_i
h_{12} = reverse transfer voltage ratio, h_r
h_{21} = forward transfer current ratio, h_f
h_{22} = output conductance, h_o

H-Parameters typical values

	CE	CB	CC
h_i	1kΩ	20Ω	1kΩ
h_r	2.5×10-4	3×10^{-4}	≈1
h_f	50	-0.98	-50
h_o	25μS	0.5μS	25μS

Comparison between 3 transistor configurations

	CB	CE	CC
Z_i	low	moderate	high
Z_o	high	moderate	low
A_i	low	high	moderate
A_v	high	high	low
A_p	moderate	high	low
shift	none	180°	none

B. R_e Model

Note:
Common Base : $h_{ib} = r_e$; $h_{fb} = -1$
Common Emitter: $\beta = h_{fe}$; $\beta r_e = h_{ie}$

FIELD EFFECT TRANSISTORS

- **JFET**

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \qquad g_{mo} = \frac{2I_{DSS}}{|V_P|}$$

$$g_m = \frac{2I_{DSS}}{|V_P|}\left(1 - \frac{V_{GS}}{V_P}\right) = \left.\frac{\Delta I_d}{\Delta V_{gs}}\right|_{V_{ds}=0}$$

$$0 \le V_{GS} \le 5$$

where:
I_d = drain current
I_{dss} = drain-source saturation current
V_{gs} = gate source voltage
$V_p = V_{gs}$ (off), pinch-off voltage
$g_m = g_{fs}$, device transconductance
g_{mo} = the maximum ac gain parameter of the JFET

- **MOSFET**

$$I_{DS} = k(V_{GS} - V_{TH})^2$$

$$k = 0.3mA/V^2$$

- **FET biasing**

DC bias of a FET requires setting the gate-source voltage, which results in a desired drain current. V_{gg} is used to reverse bias the gate so that $I_g = 0$.

POWER SUPPLY

- **Transformer**

$$a = \frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \sqrt{\frac{Z_s}{Z_p}}$$

where: a = turns ratio
V_s = secondary induce voltage
V_p = primary voltage
N_s = no. of turns on the secondary windings
N_p = no. of turns on the primary windings
I_p = current in the primary windings
I_s = current in the secondary windings
Z_s = impedance of the load connected to the secondary winding
Z_p = impedance looking into the primary from source

- **Rectifier**

Half–wave signal

$$V_{dc} = 0.318V_m \qquad V_{rms} = \frac{V_m}{2}$$

$$V_{dc} = 0.636V_{rms} \qquad PIV = 2V_{rms}$$

Ripple frequency = AC input frequency

Full–wave rectified signal (bridge type)

$$V_{dc} = 0.636V_m \qquad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{dc} = 0.9V_{rms} \qquad PIV = \sqrt{2}V_{rms}$$

Ripple frequency = 2×AC input frequency

Full–wave with center-tapped transformer

V_{dc} = 0.9V_{rms} of the half the secondary
= 0.45V_{rms} of the full secondary
= 0.637V_{pk} of half of the secondary
= 0.637V_{pk} of the full secondary
PIV = 1.414V_{rms} of full secondary

$$r = \frac{AC}{DC} = \frac{V_r(rms)}{V_{dc}}$$

$$V_r(rms) = \sqrt{V_{rms}^{\ 2} - V_{dc}^{\ 2}}$$

where: r = ripple factor
$V_r(rms)$ = rms value of the ripple voltage
V_{dc} = average value of the filter's output voltage

$$V_r(rms) = 0.385V_m \qquad \textit{half–wave rectified signal}$$

$$V_r(rms) = 0.308V_m \qquad \textit{full–wave rectified signal}$$

- **Filter**

$$V_r(rms) = \frac{V_r(p)}{\sqrt{3}} = \frac{V_r(p-p)}{2\sqrt{3}}$$

$$V_r(rms) = \frac{I_{dc}}{4\sqrt{3}fC} = \frac{2.4I_{dc}}{C} = \frac{2.4V_{dc}}{R_L C}$$

$$V_{dc} = V_m - \frac{V_r(p-p)}{2} = V_m - \frac{I_{dc}}{4fC} = V_m - \frac{4.17I_{dc}}{C}$$

$$r = \frac{V_r(rms)}{V_{dc}} \times 100\% = \frac{2.4I_{dc}}{CV_{dc}} \times 100\% = \frac{2.4}{R_L C} \times 100\%$$

where: I_{dc} = the load current in mA
C = filter capacitor in µF
R_L = load resistance at the filter stage in kΩ
V_m = the peak rectified voltage
I_{dc} = the load current in mA
C = filter capacitor in µF
f = frequency at 60 Hz

- **Regulator**

Voltage Regulation

$$V.R. = \frac{V_{noload} - V_{fload}}{V_{fload}} \times 100\%$$

Stability factor (S)

$$S = \frac{\Delta V_{out}}{\Delta V_{in}} \quad \textit{(constant output current)}$$

Improved series regulation

$$V_o = \frac{R_1 + R_2}{R_2}(V_Z + V_{BE2})$$

INSTRUMENTATION

- **DC Ammeter**

Relationship between current without the ammeter and current with the ammeter

$$\frac{I_{wm}}{I_{wom}} = \frac{R_o}{R_o + R_m}$$

where: I_{wm} = current with meter
I_{wom} = current without meter
R_o = equivalent resistance

R_m = internal resistance of ammeter

Accuracy Equation of an ammeter

$$accuracy = \frac{I_{wm}}{I_{wom}}$$

Percent of loading error

$$\%error = (1 - accuracy) \times 100\%$$

Ammeter Shunt

$$R_{sh} = \frac{I_{fs} R_m}{I_t - I_{fs}} \qquad Rin_{sh} = \frac{R_m R_{sh}}{R_m + R_{sh}}$$

$$Rin_{sh} = \frac{V_{in}}{I_{in}} = \frac{I_{fs} R_m}{I_t}$$

where: R_{sh} = shunt resistance
I_{fs} = full scale current
R_m = meter resistance
I_t = total current
Rin_{sh} = input resistance of the shunted meter
V_{in} = voltage input
I_{in} = current input

- **Voltmeter**

For full scale current

$$V_{fs} = (R_s + R_m) I_{fs}$$

$$R_s = \frac{V_{fs}}{I_{fs}} - R_m$$

$$R_{in} = R_s + R_m$$

where: V_{fs} = full scale voltage
R_s = series resistor
R_{in} = input resistance

Sensitivity of Voltmeter

$$S = \frac{1}{I_{fs}} \qquad R_{in} = \frac{V_{fs}}{I_{fs}}$$

Voltmeter Loading Error

$$accuracy = \frac{V_{wm}}{V_{wom}} = \frac{R_{in}}{R_{in} + R_o}$$

$$V_{wm} = \frac{R_{in} V_{wom}}{R_{in} + R_o}$$

- **Ohmmeter**

$$I_{fs} = \frac{V_{oc}}{R_o} \qquad I = \frac{V_{oc}}{R_o + R_u}$$

$$D = \frac{I}{I_{fs}} = \frac{R_o}{R_o + R_u}$$

where: I_{fs} = full scale current
V_{oc} = open circuit voltage
R_o = internal resistance of ohmmeter
D = meter deflection
R_u = unknown resistance

- **AC Detection**

$$S_{ac} = \frac{0.45}{I_{fs}} \text{ Sensitivity for a half-wave rectifier}$$

$$S_{ac} = \frac{0.9}{I_{fs}} \text{ Sensitivity for a full-wave rectifier}$$

- **DC Bridges**

Wheatstone bridge ohmmeter

Bridge is balance if

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

- **Attenuators**

$$R_o = \sqrt{R_{ins} R_{ino}}$$

where: R_o = characteristic resistance
R_{ins} = input resistance with output terminals shorted
R_{ino} = input resistance with output terminals open

L type or the voltage divider

$$gain = \frac{R_2}{R_1 + R_2} \qquad attenuation = \frac{V_{in}}{V_{out}} = \frac{1}{gain}$$

$$\frac{R_1}{R_2} = \frac{X_{C1}}{X_{C2}} \qquad C_1 = \frac{R_2 C_2}{R_1}$$

Symmetrical Attenuator

$$m = \frac{R_2}{R_1}; R_2 = mR_1$$

Symmetrical T Analysis

$$R_0 = R_1\sqrt{1+2m} \qquad a = \frac{V_{in}}{V_{out}} = \frac{1+m+\sqrt{1+2m}}{m}$$

Symmetrical Pi Analysis

$$R_0 = \frac{R_2}{\sqrt{1+2m}} \qquad a = \frac{V_{in}}{V_{out}} = \frac{1+m+\sqrt{1+2m}}{m}$$

Design Formulas for T Attenuator

$$R_1 = \frac{a^2 - 1}{2a} R_o \qquad R_2 = \frac{a+1}{a-1} R_o$$

Design Formulas for T Attenuator

$$R_1 = \frac{a-1}{a+1} R_o \qquad R_2 = \frac{2a}{a^2 - 1} R_o$$

Variable Attenuator

Analysis

$$R_1 = R_0 \qquad a = \frac{R_1}{R_2} + 1$$

Design

$$R_1 = R_0 \qquad R_2 = \frac{R_0}{a-1} \qquad R_3 = \frac{a-1}{R_0}$$

COMPUTER FUNDAMENTALS

r's complement

$$(r^n)_{10} - N$$

(r – 1)'s complement

$$(r^n - r^{-m})_{10} - N$$

Types of Binary Coding

Binary Coded Decimal Code (BCD)

DECIMAL DIGIT	BCD Equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Excess-3-code

DECIMAL DIGIT	Excess-3
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

Gray Code (Reflected Code)

DECIMAL DIGIT	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000

DECIMAL	84-2-1	2421	Biquinary 5043210
0	0000	0000	0100001
1	0111	0001	0100010
2	0110	0010	0100100
3	0101	0011	0101000
4	0100	0100	0110000
5	1011	1011	1000001
6	1010	1100	1000010
7	1001	1101	1000100
8	1000	1110	1001000
9	1111	1111	1010000

OPERATIONAL AMPLIFIERS

$$V_D = V_+ - V_-$$

where: V_D = differential voltage
V_+ = voltage at the non-inverting terminal
V_- = voltage at the inverting terminal

$$CMRR = \frac{A_d}{A_c}$$

where: A_d = differential gain of the amplifier
A_c = common-gain of the amplifier

Slew rate

$$SR = \frac{\Delta V_o}{\Delta t} = 2pf_{\max} V_{pk}$$

where: f_{max} = highest undistorted frequency
V_{pk} = peak value of output sine wave

Differentiator

$$V_o = -RC\frac{dV_{in}}{dt}$$

Integrator

$$V_o = -\frac{1}{RC}\int V_{in}\,dt$$

Basic non-inverting amplifier

$$gain = 1 + \frac{R_2}{R_1}$$

Basic inverting amplifier

$$gain = -\frac{R_2}{R_1}$$

LOGIC GATES

- **Boolean Algebra**

Postulated and Theorems of Boolean algebra

$X + 0 = X$	$X \bullet 1 = X$
$X + X' = 1$	$X \bullet X' = 0$
$X + X = X$	$X \bullet X = X$
$X + 1 = 1$	$X \bullet 0 = 0$

(Commutative Law)

$X + Y = Y + X$ $\qquad$ $X \bullet Y = Y \bullet X$

(Associative Law)

$X + (Y + Z) = (X + Y) + Z$ $\qquad$ $X \bullet (YZ) = (XY) \bullet Z$

(Distributive Law)

$$X(Y + Z) = XY + YZ$$

$$X + (YZ) = (X + Y)(Y + Z)$$

(Law of Absorption)

$(X + Z)X + XY = X$ $\qquad$ $X + (X + Y) = X$

(De Morgan's Theorem)

$(X + Y)' = X'Y'$ $\qquad$ $(XY)' = X' + Y'$

- **Logic Family Criterion**

Propagation delay is the average transition delay time for a signal to propagate from input to output.

$$t_p = \frac{t_{PHL} + t_{PLH}}{2}$$

where: t_p = propagation delay
t_{PHL} = propagation delay high to low transition
t_{PLH} = propagation delay low to high transition

Power dissipation is the amount of power that an IC drains from its power supply.

$$I_{CC}(AVG) = \frac{I_{CCH} + I_{CCL}}{2}$$

$$P_D(AVG) = I_{CC}(AVG) \times V_{CC}$$

where: I_{CCH} = current drawn from the power supply at high level
I_{CCL} = current drawn from the power supply at low level

Noise Margin is the maximum noise voltage added to the input signal of a digital circuit that does not cause an undesirable change in the circuit output.

Low State Noise Margin

$$NM_L = V_{IL} - V_{OL}$$

where: NM = Noise Margin
V_{IL} = low state input voltage
V_{OL} = low state output voltage

High State Margin

$$NM_H = V_{OH} - V_{IH}$$

where: NM = Noise Margin
V_{IH} = high state input voltage
V_{OH} = high state output voltage

Logic Swing

$$V_{ls} = V_{OH} - V_{OL}$$

where: V_{ls} = voltage logic swing
V_{OH} = high state output voltage
V_{OL} = low state output voltage

Transition Width

$$V_{tw} = V_{IH} - V_{IL}$$

where: V_{tw} = voltage transition width
V_{IH} = high state input voltage
V_{IL} = low state input voltage

TYPICAL CHARACTERISTICS OF IC LOGIC FAMILIES

IC Logic Family	Fan out	Power Dissipation (mW)	Propagation Delay (ns)	Noise Margin (V)
Standard TTL	10	10	10	0.4
Schottky	10	22	3	0.4
Low power Schottky TTL	20	2	10	0.4
ECL	25	25	2	0.2
CMOS	50	0.1	25	3

LEVEL OF INTEGRATION

Level of Integration	No. of gates per chip
Small Scale Integration (SSI)	Less than 12
Medium Scale Integration (MSI)	12 – 99
Large Scale Integration (LSI)	100 – 9999
Very Large Scale Integration (VLSI)	10000 – 99999
Ultra Large Scale Integration (LSI)	100000 or more

CAPACITOR/INDUCTOR TRANSIENT CIRCUITS

- **Capacitors**

The Gauss Theorem

"The total electric flux extending from a closed surface is equal to the algebraic sum of the charges inside the closed surface."

$$y \equiv Q$$

Electric Flux Density

$$D = \frac{y}{A}$$

where: D = flux density, Tesla (T) or Wb/m^2
ψ = electric flux, Weber (Wb)
A = plate area, m^2

Electric field strength or intensity (ξ)

$$x = \frac{F}{Q} = \frac{V}{d}$$

where: ξ = field strength (N/C, V/m)
F = force (Newton)
Q = charge (Coulomb)
V = voltage across the plates (volt)
d = distance between plates (m)

Coulomb's Laws of Electrostatics

First Law:

"Unlike charges attract each other while like charges repel."

Second Law:

"The force of attraction or repulsion between charges is directly proportional to the product of the two charges but inversely proportional to the square of distance between them."

$$F = \frac{kQ_1Q_2}{r^2}$$

$$k = \frac{1}{4pe} \qquad e = e_r e_0$$

Permittivity

A measure of how easily the dielectric will "permit" the establishment of flux line within the dielectric.

$$e = \frac{D}{x}$$

For vacuum, $e_0 = \frac{10^{-9}}{36p} = 8.854 \times 10^{-12} \frac{F}{m}$

Capacitance

$$C = \frac{Q}{V} \qquad C = (n-1)e\frac{A}{d}$$

where: Q = charge
V = voltage
n = number of plates
A = plate area
d = distance between plates

Relative Permittivity (Dielectric Constant) of various dielectrics

Dielectric Material	ε_r(Average value)
Vacuum	1.0
Air	1.0006
Teflon	2.0
Paper, paraffined	2.5
Rubber	3.0
Transformer oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite	7.0

Glass	7.5
Distilled water	80.0
Barium-strontium titanite (ceramic)	7500.0

Dielectric strength of some dielectric materials

Dielectric Material	Dielectric Strength (Average Value) in V/mil
Air	75
Barium-strontium titanite (ceramic)	75
Porcelain	200
Transformer oil	400
Bakelite	400
Rubber	700
Paper, paraffined	1300
Teflon	1500
Glass	3000
Mica	5000

Energy stored

$$E = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Capacitors in Series

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}}$$

$$Q_T = Q_1 = Q_2 = Q_3 = \ldots = Q_n$$

Capacitors in Parallel

$$C_T = C_1 + C_2 + C_3 + \ldots + C_n$$

$$Q_T = Q_1 + Q_2 + Q_3 + \ldots + Q_n$$

Other capacitor configurations

Composite medium parallel-plate capacitor

$$C = \frac{e_0 A}{\left(\frac{d_1}{e_{r1}} + \frac{d_2}{e_{r2}} + \frac{d_3}{e_{r3}}\right)}$$

where: d_1, d_2 and d_3 = thickness of dielectrics with relative permittivities of ε_{r1}, ε_{r2} and ε_{r3} respectively

Medium partly air parallel-plate capacitor

$$C = \frac{e_0 A}{\left(d - \left[t - \frac{t}{e_r}\right]\right)}$$

Cylindrical capacitor

$$C = \frac{e_r \mathbf{l}}{41.4\left(\log\frac{b}{a}\right)} \times 10^{-9}$$

where: a = diameter of single core cable conductor and surrounded by an insulation of inner diameter b

ε_r = relative permittivity of the insulation of the cable

l = length of the cylindrical capacitor

Capacitance of an isolated sphere

$$C = 4\pi\varepsilon r$$

where: r = radius of the isolated sphere in a medium of relative permittivity ε_r

Capacitance of concentric spheres

a.) When outer sphere earthed

$$C = 4pe\frac{ab}{(b-a)}$$

Where: a and b are radii of two concentric spheres

ε = permittivity of the dielectric between two spheres

b.) When inner sphere is earthed

$$C = 4pe\frac{b^2}{(b-a)}$$

- **Inductors**

Inductance (L) is a measure of the ability of a coil to oppose any change in current through the coil and to store energy in the form of a magnetic field in the region surrounding the coil.

In terms of physical dimensions,

$$L = m\frac{N^2 A}{\mathbf{l}} \quad \text{Henry}$$

where: μ = permeability of the core (H/m)

N = number of turns

A = area of the core (m^2)

l = mean length of the core (m)

In terms of electrical definition,

$$L = N\frac{df}{di}$$

Faraday's Law

"The voltage induced across a coil of wire equals the number of turns in the coil times the rate of change of the magnetic flux."

$$e_{in} = N\frac{df}{dt}$$

where: N = number of turns of the coil

$\frac{df}{dt}$ = change in the magnetic flux

Lenz's Law

"An induced effect is always such as to oppose the cause that produced it."

$$e_{in} = -N\frac{df}{dt}$$

Induced voltage by Faraday's Law

$$e_L = L\frac{di}{dt}$$

Energy stored

$$W_L = \frac{1}{2}LI^2$$

Inductance without mutual inductance in series

$$L_T = L_1 + L_2 + L_3 + \ldots + L_n$$

With mutual inductance (M)

a.) when fields are aiding

$$L_{Ta} = L_1 + L_2 + 2M$$

b.) when fields are opposing

$$L_{To} = L_1 + L_2 - 2M$$

Total inductance without mutual inductance (M)

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}}$$

With mutual inductance (M)

a.) when fields are aiding

$$L_{T(a)} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

b.) when fields are opposing

$$L_{T(o)} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

Mutual inductance

It is a measure of the amount of inductive coupling that exists between the two coils.

$$M = k\sqrt{L_1L_2}$$

$$M = \frac{L_{Ta} - L_{To}}{4}$$

where: k = coupling coefficient

L_1 and L_2 = self-inductances of coils 1 and 2

L_{Ta} and L_{To} = total inductances with mutual inductance

Coupling coefficient (k)

$$k = \frac{M}{\sqrt{L_1L_2}}$$

$$k = \frac{flux\_linkage\_between\_L_1\_and\_L_2}{flux\_produced\_by\_L_1}$$

Formulas for other coil geometries

(a) LONG COIL

$$L = m\frac{N^2A}{\mathbf{l}}$$

(b) SHORT COIL

$$L = m\frac{N^2A}{\mathbf{l} + 0.45d}$$

where: L = inductance (H)

μ = permeability (4π×10^{-7} for air)

N = number of turns

A = cross-sectional area of the coil (m^2)

l = length of the core (m)

d = diameter of core (m)

(c) TOROIDAL COIL with rectangular cross-section

$$L = m\frac{N^2h}{2p}\ln\frac{d_2}{d_1}$$

where: h = thickness

d_1 and d_2 = inner and outer diameters

(d) CIRCULAR AIR-CORE COIL

$$L = \frac{0.07(RN)^2}{6R + 9\mathbf{l} + 10b}$$

$$R = \frac{d}{2} + \frac{b}{2}$$

where: L = inductance (μH)

N = number of turns

d = core diameter, in

b = coil build-up, in
$\mathbf{l}$ = length, in

(e) RECTANGULAR AIR-CORE COIL

$$L = \frac{0.07(CN)^2}{1.908C + 9\mathbf{l} + 10b}$$

where: L = inductance (μH)
C = d + y + 2b
d = core height, in
y = core width, in
b = coil build-up, in
$\mathbf{l}$ = length, in

(f) MAGNETIC CORE COIL (no air gap)

$$L = \frac{0.012N^2 mA}{\mathbf{l}_c}$$

(g) MAGNETIC CORE COIL (with air gap)

$$L = \frac{0.012N^2 A}{\mathbf{l}_g + \frac{\mathbf{l}_c}{m}}$$

where: L = inductance (μH)
N = number of turns
A = effective cross-sectional area, cm^2
$\mathbf{l}_c$ = magnetic path length, cm
$\mathbf{l}_g$ = gap length, cm
M = magnetic permeability

- **DC Transient Circuits**

Circuit Element	Voltage across	Current flowing
R	$v = iR$	$i = \frac{v}{R}$
L	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int vdt$
C	$v = \frac{q}{C} = \frac{1}{C}\int idt$	$i = C\frac{dv}{dt}$

Response of L and C to a voltage source

Circuit Element	@ t = 0	@ t = ∞
L	open	short
C	short	open

RL Transient Circuit
Storage Cycle:

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{E}{R}\left(1 - e^{-\frac{t}{t}}\right)$$

$$t = \frac{L}{R} \qquad v_R = E\left(1 - e^{-\frac{R}{L}t}\right) \qquad v_L = Ee^{-\frac{R}{L}t}$$

Decay Phase:

$$i = \frac{E}{R}e^{-\frac{R}{L}t} = \frac{E}{R}e^{-\frac{t}{t}}$$

$$t = \frac{L}{R_T} \qquad R_T = R_1 + R$$

RC Transient Circuit
Charging Cycle:

$$q = EC + (q_0 - EC)e^{-\frac{t}{RC}}$$

$$q = EC\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{with } q_0 = 0$$

$$i = \frac{E}{R}e^{-\frac{t}{RC}} \qquad v_R = Ee^{-\frac{t}{RC}}$$

$$v_C = E\left(1 - e^{-\frac{t}{RC}}\right) \qquad t = RC$$

Discharging Phase:

$$v_C = Ee^{-\frac{t}{RC}} \qquad t = RC$$

RLC Transient Circuits
Conditions for series RLC transient circuit:
(1) @ t = 0, i = 0
(2) @ t = 0, Ldi/dt = E

Current equations
Case 1 – Overdamped case

$$\text{when } \left(\frac{R}{2L}\right)^2 > \frac{1}{LC} \text{ then}$$

$$i = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

$$C_1 = -C_2 \qquad C_2 = -\frac{E}{2bL}$$

$$r_1 = a + b \qquad r_2 = a - b$$

$$a = -\frac{R}{2L} \qquad b = \sqrt{\left(\frac{R}{2L}\right) - \frac{1}{LC}}$$

Case 2 – Critically damped case

$$\text{when } \left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \text{ then}$$

$$i = e^{at}(C_1 + C_2 t)$$

$$C_1 = 0 \qquad C_2 = \frac{E}{L}$$

$$a = -\frac{R}{2L}$$

Case 2 – Underdamped case

$$\text{when } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC} \text{ then}$$

$$i = e^{at}(C_1 \cos + C_2 \sin bt)$$

$$a = -\frac{R}{2L} \qquad b = \sqrt{\left|\left(\frac{R}{2L}\right) - \frac{1}{LC}\right|}$$

$$C_1 = 0 \qquad C_2 = \frac{E}{bL}$$

AC CIRCUITS 1

- **Introduction to AC: Formulas**

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}} = \frac{0.707V_m}{0.637V_m} = 1.11$$

$$\text{Peak factor} = \frac{V_m}{V_{rms}} = \frac{V_m}{0.707V_m} = 1.4142$$

$$X_L = 2pfL \qquad X_C = \frac{1}{2pfC}$$

- **Series AC circuits**

Series RL Circuit

Total voltage, V_T

$$V_T = V_R + jV_L = |V_T|\angle q$$

$$|V_T| = \sqrt{V_R^2 + V_L^2} \qquad q = \tan^{-1}\frac{V_L}{V_R}$$

Total impedance, Z

$$Z = R + jX_L = |Z|\angle q$$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad q = \tan^{-1}\frac{X_L}{R}$$

Series RC Circuit

Total voltage, V_T

$$V_T = V_R - jV_C = |V_T|\angle q$$

$$|V_T| = \sqrt{V_R^2 + V_C^2} \qquad q = \tan^{-1}\frac{V_C}{V_R}$$

Total impedance, Z

$$Z = R - jX_C = |Z|\angle q$$

$$|Z| = \sqrt{R^2 + X_C^2} \qquad q = -\tan^{-1}\frac{X_C}{R}$$

Series RLC Circuit

Total voltage, V_T

$$V_T = V_R + jV_L - jV_C = |V_T|\angle q$$

$$|V_T| = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad q = \pm\tan^{-1}\frac{(V_L - V_C)}{V_R}$$

Total impedance, Z

$$Z = R + jX_L - jX_C = |Z|\angle q$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$q = \pm\tan^{-1}\frac{(X_L - X_C)}{R}$$

Total Current, I_T

$$I_T = \frac{V_T}{Z}$$

- **Parallel AC circuits**

Parallel RL Circuit

Total Current, I_T

$$I_T = I_R - jI_L = |I_T|\angle q$$

$$|I_T| = \sqrt{I_R^2 + I_L^2} \qquad q = -\tan^{-1}\frac{I_L}{I_R}$$

Total admittance, Y

$$Y = G - jB_L = |Y|\angle q$$

$$|Y| = \sqrt{G^2 + B_L^2} \qquad q = -\tan^{-1}\frac{B_L}{G}$$

Parallel RC Circuit

Total Current, I_T

$$I_T = I_R + jI_C = |I_T|\angle q$$

$$|I_T| = \sqrt{I_R^2 + I_C^2} \qquad q = \tan^{-1}\frac{I_C}{I_R}$$

Total Admittance, Y

$$Y = G + jB_C = |Y|\angle q$$

$$|Y| = \sqrt{G^2 + B_C^2} \qquad q = \tan^{-1}\frac{B_C}{G}$$

Parallel RLC Circuit
Total Current, I_T

$$I_T = I_R + jI_C - jI_L = |I_T| \angle q$$

$$|I_T| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$q = \pm \tan^{-1} \frac{(I_C - I_L)}{I_R}$$

Total admittance, Y

$$Y = G + jB_C - jB_L = |Y| \angle q$$

$$|Y| = \sqrt{G^2 + (B_C - B_L)^2}$$

$$q = \pm \tan^{-1} \frac{(B_C - B_L)}{G}$$

Total impedance, Z

$$Z = \frac{1}{Y}$$

Total voltage, V_T

$$V_T = I_T Z$$

Power of AC Circuits
True/Real/Average/Active Power

$$P = I_R{}^2 R = \frac{V_R{}^2}{R} = I_R V_R = V_T I_T \cos q$$

Reactive Power

$$Q = I_X{}^2 X_{eq} = \frac{V_x{}^2}{X_{eq}} = I_X V_X = V_T I_T \sin q$$

Apparent Power

$$Q = I_T{}^2 Z = \frac{V_T{}^2}{Z} = V_T I_T$$

$$\cos q = \frac{P}{S} = \textit{Power Factor (PF)}$$

$$\sin q = \frac{Q}{S} = \textit{Reactive Factor (RF)}$$

$$S = P \pm jQ = |S| \angle q$$

$$|S| = \sqrt{P^2 + Q^2}$$

$$q = \pm \tan^{-1} \frac{Q}{P}$$

- **Values of other alternating waveforms**

Symmetrical Trapezoid

$$V_{rms} = \frac{a + 0.577(b - a)}{b} V_p \qquad |V_{avg}| = \frac{a + b}{2b} V_p$$

DC Pulse

$$|V_{rms}| = V_p \sqrt{\frac{a}{b}} \qquad |V_{avg}| = V_p \frac{a}{b}$$

Triangular or Sawtooth

$$V_{rms} = 0.577 V_p \qquad |V_{avg}| = 0.5 V_p$$

Sine wave on dc level

$$V_{rms} = \sqrt{V_{DC}{}^2 + \frac{V_p{}^2}{2}}$$

Square wave

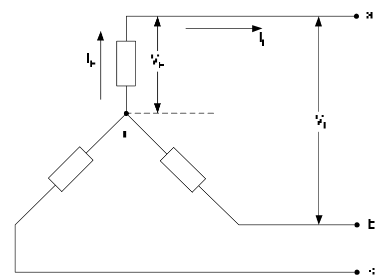
$$V_{rms} = V_p \qquad |V_{avg}| = V_p$$

White Noise

$$V_{rms} \approx \frac{1}{4} V_p$$

ENERGY CONVERSION

Types of three-phase alternators
A. Wye or Star-connected



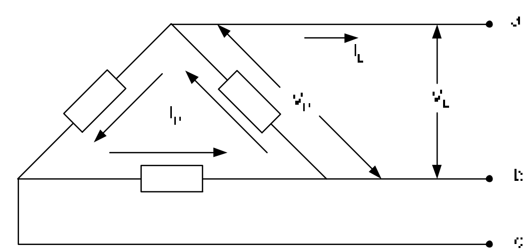
$$V_{Line} = \sqrt{3} V_{phase}$$

$$I_{Line} = I_{phase}$$

$$P_{3f} = \sqrt{3} V_L I_L \cos q$$

$$P_{3f} = 3 V_P I_P \cos q$$

B. Delta or Mesh-connected



$$V_{Line} = V_{phase}$$

$$I_{Line} = \sqrt{3} I_{phase}$$

$$P_{3f} = \sqrt{3} V_L I_L \cos q$$
$$P_{3f} = 3 V_P I_P \cos q$$

Frequency of the AC Voltage Generated in an Alternator

$$f = \frac{PN}{120}$$

where: f = frequency (Hz)
P = number of poles (even number)
N = speed of prime mover (rpm)

Speed Characteristics of DC Motors

$$H = k_s \frac{E_c}{f}$$

where: E_c = counter emf
k_s = speed constant
f = flux

Torque Characteristics of DC Motors

$$T = k_t f I_a$$

where: I_a = armature current
k_t = torque constant
f = flux

Speed of an AC Motor

$$N = \frac{120 f}{P}$$

where: N = synchronous speed (rpm)
f = frequency (Hz)
P = number of poles

OSCILLATORS

- **Introduction**

Oscillator Requirements

a. Amplifier
b. Tank circuit
c. Feedback

Overall gain with feedback

$$A_f = \frac{A}{1 + bA}$$

Barkhausen Criterion for Oscillation

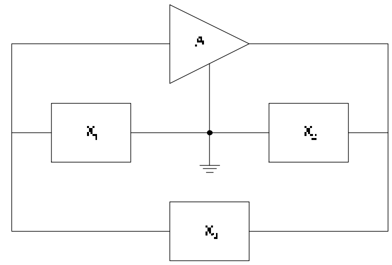
a. The net gain around the feedback loop must be no less than one; and
b. The net phase-shift around the loop must be a positive integer multiple of 2π radians or 360°.

Mathematically,

$$|bA| \geq 1$$

$$f = n \times 360° \quad n = 1,2,3...$$

Basic Configuration of a Resonant Circuit Oscillator



- **LC Oscillators**

Resonant-Frequency Feedback Oscillators

Oscillator Type	X_1	X_2	X_3
Hartley	L	L	C
Colpitts	C	C	L
Clapp	C	C	Series LC (net L)
Pierce Crystal	C	C	Crystal (net L)

A. Hartley Oscillator

Amplifier gain without feedback,

$$A_V = -\frac{R}{r_e}$$

for a common-emitter configuration

The feedback factor,

$$b = -\frac{L_2}{L_1}$$

To maintain the oscillation,

$$A_V = \frac{R}{r_e} = \frac{L_1}{L_2}$$

The frequency of oscillation is

$$f_0 = \frac{1}{2p\sqrt{L_{eq} C}}$$

where

$$L_{eq} = L_1 + L_2 + 2M$$
$$M = \sqrt{L_1 L_2}$$

B. Colpitts Oscillator
Amplifier gain without feedback,

$$A_V = -\frac{R}{r_e}$$

The feedback factor,

$$b = -\frac{C_1}{C_2}$$

The frequency of oscillation is

$$f_0 = \frac{1}{2p\sqrt{LC_{eq}}}$$

where

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

To maintain the oscillation,

$$A_V = \frac{R}{r_e} = \frac{C_2}{C_1}$$

C. Clapp Oscillator
The frequency of oscillation is

$$f_0 = \frac{1}{2p\sqrt{LC_{eq}}}$$

where

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

- **Crystal Oscillators**

Frequency drift
LC: 0.8%
Crystal: 0.0001% (1 ppm)

Natural frequency of vibration

$$thickness\, a \,\frac{1}{f}$$

The thicker the crystal, the lower its frequency of vibration

Series and Parallel Resonant Frequencies
Series

$$f_{rs} = \frac{1}{2p\sqrt{LC_s}}$$

Parallel

$$f_{rp} = \frac{1}{2p\sqrt{L\frac{C_s C_m}{C_s + C_m}}}$$

Note: Series resonant frequency, f_{rs} is slightly lower than parallel resonant frequency, f_{rp}.

- **RC Oscillators**

RC Phase-Shift Oscillator
The gain of the basic inverting amplifier is,

$$A_V = -\frac{R_f}{R_s}$$

The feedback factor is,

$$b = -\frac{1}{29}$$

To maintain the oscillation,

$$A_V = -\frac{R_f}{R_s} = -29$$

The frequency of oscillation is,

$$f_0 = \frac{1}{2pRC\sqrt{6}}$$

Wien Bridge Oscillator
The open-loop gain is

$$A_V = 1 + \frac{R_f}{R_s} = 3$$

The feedback factor is

$$b = \frac{1}{3}$$

To maintain the oscillation,

$$\frac{R_f}{R_s} = 2$$

The frequency of oscillation is,

$$f_0 = \frac{1}{2p\sqrt{R_1 C_1 R_2 C_2}}$$

Neglecting loading effects of the op-amp input and output impedances, the analysis of the bridge results in

$$\frac{R_f}{R_s} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \quad \text{(bridge-balance condition)}$$

Therefore, for the bridge to be balanced,

$R_1 = R_2 = R$ and $C_1 = C_2 = C$

The frequency of oscillation

$$f_0 = \frac{1}{2pRC}$$

FEEDBACK AMPLIFIERS

- **Types of Feedback Connections**

Equations of open-loop gain, feedback factor and closed-loop gain for different types of feedback

Feedback Connection	Source Signal	Output Signal	A	β	A_f
Voltage Series	Voltage	Voltage	$\frac{v_o}{v_i}$	$\frac{v_f}{v_o}$	$\frac{v_o}{v_s}$
Current Series	Voltage	Current	$\frac{i_o}{v_i}$	$\frac{v_f}{i_o}$	$\frac{i_o}{v_s}$
Voltage Shunt	Current	Voltage	$\frac{v_o}{i_i}$	$\frac{i_f}{v_o}$	$\frac{v_o}{i_s}$
Current Shunt	Current	Current	$\frac{i_o}{i_i}$	$\frac{i_f}{i_o}$	$\frac{i_o}{i_s}$

Note: Some references try to designate the following terms to describe the four main types of feedback equations.

2. Series-shunt = Voltage series
3. Series-series = Current series
4. Shunt-shunt = Voltage shunt
5. Shunt-series = Current-shunt

- **Negative Feedback Equations**

$$A_f = \frac{A}{1+bA}$$

where: A = gain without feedback (open-loop gain)

A_f = gain with feedback (closed-loop gain)

1 + βA = desensitivity or sacrifice factor

βA = loop gain

Equations of closed-loop gain for different types of feedback connections

Feedback Type	Gain with Feedback	Type of Amplifier
Voltage Series	$A_{vf} = \frac{A_v}{1+bA_v}$	Voltage Amplifier
Current Series	$G_{mf} = \frac{G_m}{1+bG_m}$	Transconductance Amplifier
Voltage Shunt	$R_{mf} = \frac{R_m}{1+bR_m}$	Transresistance Amplifier
Current Shunt	$A_{if} = \frac{A_i}{1+bA_i}$	Currrent Amplifier

- **Performance Characteristics of Negative Feedback Networks**

Equations of amplifier impedance levels when using negative feedback connection

Feedback Type	Input Resistance	Output Resistance
Voltage Series	$R_i(1+bA)$ increased	$\frac{R_o}{1+bA}$ decreased
Current Series	$R_i(1+bA)$ increased	$R_o(1+bA)$ increased
Voltage Shunt	$\frac{R_i}{1+bA}$ decreased	$\frac{R_o}{1+bA}$ decreased
Current Shunt	$\frac{R_i}{1+bA}$ decreased	$R_o(1+bA)$ increased

$$\left|\frac{dA_f}{A_f}\right| = \frac{1}{|1+bA|}\left|\frac{dA}{A}\right|$$

where: $\left|\frac{dA_f}{A_f}\right|$ = change in gain with feedback

$\left|\frac{dA}{A}\right|$ = change in gain without feedback

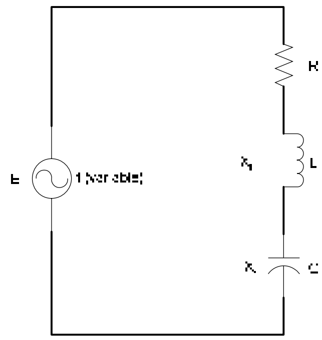
magnitude, $|\beta A| = 1$

phase-shift, $\theta = 180°$

The limiting condition is for the negative feedback amplifiers.

AC CIRCUITS 2

- **Series Resonance**



$$f_r = \frac{1}{2p\sqrt{LC}}$$

where: f_r = resonant frequency
L = Inductance
C = Capacitance

Characteristics of series resonance

1. At resonance, $X_L = X_C$, $V_L = V_C$.
2. At resonance, Z is minimum. Z = R.
3. At resonance, I is maximum. I = E/R.
4. At resonance, Z is resistive. $\theta = 0°$ (I in phase with E).
5. At f < f_r, Z is capacitive. θ = + (I Leads E).
6. At f > f_r, Z is inductive. θ = – (I Lags E).

Quality Factor (Q) of a resonant circuit:

$$Q = \frac{Reactive\_power\_of\_either\_L\_or\_C}{Active\_power\_of\_R}$$

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Resonant Rise in Voltage

$$V_L = V_C = QE$$

Bandwidth (BW) is the range of frequencies over which the operation is satisfactory and is taken between two half-power (3dB down) points.

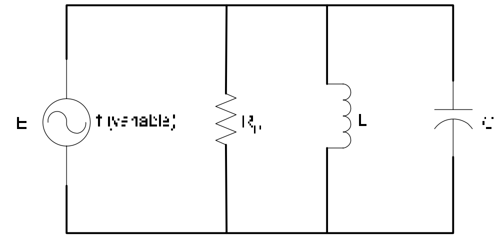
$$BW = f_2 - f_1 = \frac{f_r}{Q}$$

If Q ≥ 10; then f_r bisects BW

$$f_1 = f_r - \frac{BW}{2} \qquad f_2 = f_r + \frac{BW}{2}$$

- **Parallel Resonance**

A. Theoretical Parallel Resonant Circuit



Characteristics of parallel resonance

1. At resonance, $B_L = B_C$, $X_L = X_C$, $I_L = I_C$.
2. At resonance, Z is maximum. $Z = R_P$.
3. At resonance, I_T is minimum. $I_T = I_{RP}$.
4. At resonance, Z is resistive. $\theta = 0°$ (I in phase with E).
5. At f < f_r, Z is inductive. θ = – (I Lags E).
6. At f > f_r, Z is capacitive. θ = + (I Leads E).

Q of a Theoretical circuit:

$$Q = \frac{R_P}{X_L} = \frac{R_P}{X_C} = R_P\sqrt{\frac{C}{L}}$$

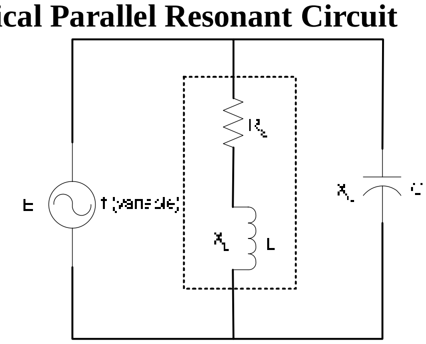
Resonant Rise in tank current

$$I_{tank} = QI_T = I_L = I_C$$

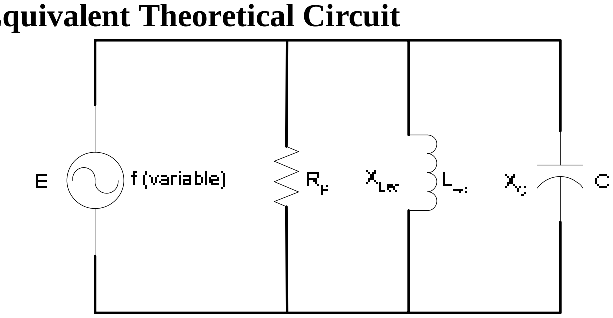
Bandwidth (BW)

$$BW = f_2 - f_1 = \frac{f_r}{Q}$$

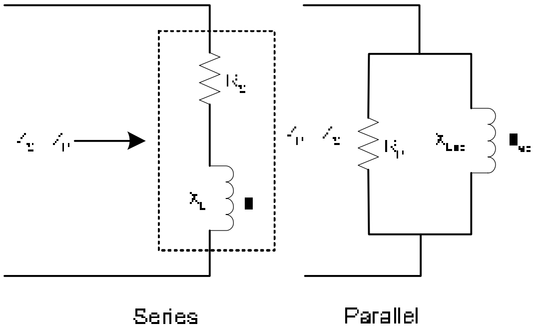
B. Practical Parallel Resonant Circuit



Equivalent Theoretical Circuit



Impedance transformation:



Q of Equivalent Theoretical Circuit

$$Q = \frac{R_P}{X_{Leq}}$$

Q of Practical Circuit

$$Q = \frac{X_L}{R_S}$$

Resonant frequency (practical circuit)

$$f_r = \frac{1}{2p\sqrt{LC}}\sqrt{1 - \frac{R_S^2 C}{L}}\ ;\ \text{if } R_S = 0;\ f_r = \frac{1}{2p\sqrt{LC}}$$

$$f_r = \frac{1}{2p\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}}\ ;\ \text{if } Q \geq 10;\ f_r = \frac{1}{2p\sqrt{LC}}$$

Total Impedance Z

$$Z = R_S(1+Q^2) \approx Q^2 R_S \qquad \text{if } Q \geq 10$$

MAGNETISM AND MAGNETIC CIRCUITS

- **Magnetism**

Curie temperature (Pierre Curie) – the critical temperature such that when ferromagnets are heated above that temperature their ability to possess permanent magnetism disappears.

Curie temperatures of ferromagnets

Ferromagnet	Temperature (°C)
Iron (Fe)	770
Nickel (Ni)	358
Cobalt (Co)	1130
Gadolinium	16

Alloys commonly magnetized

Alloy	Percentage Content
Permalloy	22% Fe, 78% Ni
Hipernik	40% Fe, 60% Ni
Perminvar	30% Fe, 45% Ni, 25% Co
Alnico	24% Co, 51% Fe

Coulomb's Laws

First Law

"The force of attraction or repulsion between two magnetic poles is directly proportional to their strengths."

Second First Law

"The force of attraction or repulsion between two poles is inversely proportional to the square of the distance between them."

$$F = k\frac{m_1 m_2}{r^2} \qquad \text{(Newtons, N)}$$

where: $k = \frac{1}{4pm}$ $\qquad m = m_r m_0$

Magnitude of the Force

$$F = BIl\sin q \qquad \text{(Newtons, N)}$$

where: B = flux density (Wb/m^2)

I = current (A)

l = length of conductor (m)

θ = angle between the conductor and field

Magnitude of the flux surrounding a straight conductor

$$\Phi = 14Il\log\frac{R}{r} \qquad \text{(Maxwells, Mx)}$$

where: I = current (A)

l = length of conductor (ft)

R = radius to the desired limiting cylinder

r = radius of the conductor

The force between two parallel conductors

$$F = \frac{2I_1 I_2 l}{d}\times 10^{-7} \qquad \text{(Newtons, N)}$$

where: l = length of each conductor (m)

d = distance between conductors (m)

I_1 = current carried by conductor A

I_2 = current carried by conductor B

Magnitude of the flux between two parallel conductors

$$\Phi = 28Il\log\frac{(d-r)}{r} \quad \text{(Maxwells, Mx)}$$

where: I = current (A)
l = length of conductor (ft)
r = radius of each conductor (m)
d = distance of the conductors from center to center (m)

- **Magnetic Circuits**

$$B = \frac{\Phi}{A}$$

where: B = Flux density in Tesla (T)
Φ = Flux lines in Webers (Wb)
A = Area in square meters (m^2)

Note: 1 Tesla = 1 Wb/m^2

Permeability

$$m_0 = 4p \times 10^{-7}\,\frac{Weber}{Ampere-meter}\; or\; \frac{H}{m}$$

Note: $\mu = \mu_0$; $\mu_r = 1$ → non–magnetic
$\mu < \mu_0$; $\mu_r < 1$ → diamagnetic
$\mu > \mu_0$; $\mu_r > 1$ → paramagnetic
$\mu >> \mu_0$; $\mu_r >> 1$ → ferromagnetic ($\mu_r \geq 100$)

$$\Re = \frac{L}{mA}$$

where: $\Re$ = reluctance
L = the length of the magnetic path
A = the cross-sectional area

Note: The t in the unit A-t/Wb is the number of turns of the applied winding.

Different units of Reluctance ($\Re$)

a.) $\frac{Ampere-turn}{Weber}$ b.) $\frac{Ampere-turn}{Maxwell}$

c.) $\frac{Gilbert}{Maxwell}$ d.) $\frac{Gilbert}{Weber}$

Note: 1 Weber = 1×10^8 maxwells
1 Gilbert = 0.7958 ampere-turns
1 Gauss = 1 maxwell/cm^2

Ohm's Law for Magnetic Circuits

$$Effect = \frac{Cause}{Opposition}$$

Then,

$$\Phi = \frac{\Im}{\Re}$$

where: $\Re$ = reluctance
$\Im$ = magnetomotive force, mmf (Gb or At)
Φ = flux (Weber or Maxwells)

Comparison bet. Magnetic and Electric Circuits

Electric Circuits	Magnetic Circuits
Resistance, R (Ω)	Reluctance, $\Re$ (Gb/Mx)
Current, I (A)	Flux, Φ (Wb or Mx)
emf, V (V)	mmf, $\Im$ (Gb or At)

Total reluctance in series

$$\Re_T = \Re_1 + \Re_2 + ... + \Re_n$$

Total reluctance in parallel

$$\frac{1}{\Re_T} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + ... + \frac{1}{\Re_n}$$

Total flux in series

$$\Phi_T = \Phi_1 = \Phi_2 = ... = \Phi_n$$

Total flux in parallel

$$\Phi_T = \Phi_1 + \Phi_2 + ... + \Phi_n$$

Energy stored

$$W_m = \frac{1}{2}\Re\Phi^2 \qquad \text{Joules}$$

Magnetomotive force (mmf, $\Im$)

$\Im = NI$ Ampere – turns, At
$\Im = 0.4pNI$ Gilberts, Gb

mmf of an air gap

$$mmf = \frac{dB}{m_0} \qquad \text{Ampere-turns}$$

Tractive force or lifting force of a magnet

$$F = \frac{1}{2}\left(\frac{AB^2}{m_0}\right) \qquad \text{Newtons}$$

Magnetizing Force (H)

$$H = \frac{\Im}{\mathbf{l}} \qquad H = \frac{NI}{\mathbf{l}}$$

Note: The unit of H is At/m

Permeability – the ratio of flux density to the magnetizing force.

$$m = \frac{B}{H}$$

B and H of an infinitely long straight wire

$$B = \frac{mI}{2pr} \qquad H = \frac{I}{2pr}$$

Steinmetz's Formula of Hysteresis Loss

$$W_h = hfB_m^{1.6} \qquad \frac{J}{m^3}$$

where: η = hysteresis coefficient

f = frequency

B_m = maximum flux density

Ampere's Circuital Law

"The algebraic sum of the rises and drops of the mmf a closed loop of a magnetic circuit is equal to zero; that is, the sum of the mmf rises equals the sum of the mmf drops around a closed loop."

$$\sum \cap \Im = 0 \qquad \text{(for magnetic circuits)}$$

Source of mmf is expressed by the equation

$$\Im = NI \qquad \text{(At)}$$

For mmf drop,

$$\Im = \Phi \Re \qquad \text{(At)}$$

A more practical equation of mmf drop

$$\Im = H\mathbf{l} \qquad \text{(At)}$$